Hedging salmon price risk

Daumantas Bloznelis[1]

Research Centre for Operations Research and Business Statistics

Katholieke Universiteit Leuven

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School of Economics and Business

Norwegian University of Life Sciences

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[1] Mailing address: Research Centre for Operations Research and Business Statistics (ORSTAT), Leuven, Naamsestraat 69 - box 3555, 3000 Leuven, Belgium. E-mail: dbloznelis@gmail.com.

Abstract

Salmon price is highly volatile and hard to predict. This obscures planning decisions and raises financing costs for market participants. This study considers hedging the spot price uncertainty with salmon futures contracts. It employs a new framework of hedging under square loss, consisting of a new objective function, an optimal hedge ratio and a measure of hedging effectiveness. The new framework aims at minimizing the expected squared forecast error. It generalizes the classical minimum variance hedging as it relaxes the assumption of known expected prices. The salmon futures contracts deliver satisfactory hedging performance, albeit constrained by low liquidity. Therefore, I suggest holding the contract through maturity rather than closing the futures and the spot positions simultaneously. This strategy alleviates the liquidity issue and saves transaction costs. All things considered, hedging with salmon futures is a moderately effective way of handling the salmon price uncertainty. Importantly, the empirical results differ starkly under the two different hedging frameworks. Hence, it is crucial to choose the new framework when expected prices are unknown.

(**JEL:** Q14, G13, C53)

Keywords: hedging, square loss, uncertainty, risk, futures, salmon price.

1. Introduction

Salmon price is highly volatile and hard to predict (Oglend, 2013; Bloznelis, 2016b, 2017b). Large unexpected price fluctuations complicate business management and financial planning for market participants. Salmon farmers and processors tailor their operation schedule to match the profit-maximizing production path, which depends on the future spot price of salmon. Exporters and retailers buy fish in the spot market and need to plan their future expenditures based on price expectations. The high uncertainty over future realizations of the spot price requires them to be flexible and financially solid enough to withstand large unexpected increases in costs or decreases in revenues. Unsurprisingly, the high price uncertainty is perceived negatively by market participants (Jensen, 2013), and there is broad demand for risk management solutions such as hedging.

The objective of this study is to design a feasible hedging strategy for the spot price of salmon and evaluate its effectiveness. The study employs a new framework of hedging under square loss aimed at minimizing the expected squared forecast error, which was first outlined in Bloznelis (2016a). It considers hedging the spot price uncertainty with salmon futures contracts four, eight and 13 weeks ahead. Three different hedging strategies are compared, two of them classical and one unorthodox (in which the futures contract is not liquidated simultaneously with the spot position but rather kept until maturity); the latter is especially useful when the hedging instrument has low liquidity. Out-of-sample performance evaluation suggests that hedging with salmon futures is a moderately effective way of managing the price uncertainty.

The novelty of this work is twofold. First, it presents a new framework of hedging under square loss, defined by a new objective function, a corresponding optimal hedge ratio and a measure of hedging effectiveness. The new framework generalizes the classical minimum variance hedging (Johnson, 1960; Stein, 1961; Ederington, 1979) which is inappropriate when assets and/or hedging instruments have unknown expected prices. The new framework naturally accommodates both known and unknown expected prices and thus applies broadly in commodity and financial markets.

Its relevance is only limited by the choice of the loss function (but the underlying idea extends to alternative loss functions as well).

Second, the study suggests a simple yet effective hedging strategy that is feasible under low liquidity, unlike its traditional counterparts. It is also applicable to a wide spectrum of futures markets with low or high liquidity where the futures contracts are cash settled. On the whole, this work is of immediate relevance to hedgers in commodity and financial markets and may bring particularly large benefits when dealing with unknown expected prices, cash settled futures contracts, and illiquid hedging instruments. Also, policymakers will benefit from the new hedging framework as it elicits the fundamentals of hedging under square loss and exposes a major fallacy of the standard approach. Avoiding such fallacies and having a solid grasp of the principles of hedging is instrumental in designing functional incentive schemes and regulating the markets effectively.

Academic literature on hedging the salmon price uncertainty is nascent. Bergfjord (2007) offers a hypothetical discussion on hedging the spot price risk with salmon futures contracts as he examines the perspectives of several newly established futures exchanges; however, no empirical investigation is supplied. Bergfjord (2009) reports a survey on risk perception and risk management of Norwegian salmon farmers. They appear to be only moderately risk averse, but they consider the future spot price of salmon to be the most important source of uncertainty. Misund and Asche (2016) is perhaps the only applied study focusing directly on price hedging in the salmon market. The authors employ the classical minimum variance hedging framework and consider using salmon futures contracts as a hedging instrument for salmon price risk. Several different models are applied for estimating optimal hedge ratios, some of them only in-sample (making the resulting hedge ratios impossible to use in real time) while other also out of sample (and thus applicable in real time hedging). For the period 2006-2014, the out-of-sample optimal hedge ratios yield almost 30% reduction in variance when hedging 4-5 weeks ahead, while a naïve ratio of unity yields almost 40%. While Misund and Asche (2016) operate under the classical framework of minimum variance hedging, this study shows that this framework is inappropriate when expected prices may be unknown, such as in the case of salmon. Therefore, market participant demand for risk reduction might not be fully met. Meanwhile, the current study applies the new, appropriate framework, and thus offers a timely response to a pressing issue.

The remainder of the article is structured as follows. Section 2 focuses on measures of uncertainty. Section 3 presents hedging and discusses objective functions, optimal hedge ratios and measures of hedging effectiveness. Section 4 provides an introduction to the salmon market and overviews the peculiarities of hedging the future spot price of salmon with salmon futures contracts. It also presents three hedging strategies and explains the rationale behind them. Econometric methodology and a review of the data are laid out in Section 5. Empirical results are to be found in Section 6, followed by a conclusion in Section 7.

2. Measuring uncertainty

2.1 Uncertainty and forecast error

Market participants such as salmon farmers face uncertainty over what the salmon spot price will be in the future. Currently, at time t, the price is s_t. In the future, at time $t+h$, where $h>0$, the price will be s_{t+h}. At time t, a farmer does not know s_{t+h}, but he/she has some idea of what it could be. He/she may have a density forecast or at least a point forecast for s_{t+h}. Let us consider the point forecast and let us denote it $\hat{s}_{t+h|t}$, indicating that it is a point forecast of s_{t+h} made at time t.

Since forecasts are hardly ever perfect, the farmer makes a forecast error $err_{t+h|t} \coloneqq s_{t+h} - \hat{s}_{t+h|t}$. The error is realized (becomes known) at time $t+h$; before that, at time t, $err_{t+h|t}$ is a random variable. It is the probabilistic properties of $err_{t+h|t}$ that characterize the price uncertainty the farmer is facing at time t. These properties allow us to investigate the uncertainty from a quantitative perspective, to measure it, and to connect the practical interpretation of uncertainty with its mathematical characterization.

Any and all probabilistic properties of $err_{t+h|t}$, including those characterizing uncertainty, can be extracted from the probability distribution function of $err_{t+h|t}$ and the corresponding probability density function. For example, if large (negative or positive) errors are relatively likely to be realized, i.e. their probability density is high, then the uncertainty is high. If they are unlikely, i.e. their probability density is low, the uncertainty is low. Conversely, by high uncertainty we mean that large (negative or positive) errors are relatively likely to be realized, i.e. their probability density is high. Meanwhile, by low uncertainty we mean that such errors are unlikely, i.e. their probability density is low. This is how we interpret the probability distribution of $err_{t+h|t}$ in terms of uncertainty, and also how we translate the practical understanding of uncertainty into probabilistic characteristics of $err_{t+h|t}$.

2.2 Distributional moments that reflect uncertainty

It is not always convenient to work with the probability distribution function of a random variable. Instead, some summary characteristics may be simpler to handle yet still serve the purpose of characterizing uncertainty. See Table 1 for a schematic overview. For example, one such characteristic is the first absolute moment $\mathrm{E}_t(|err_{t+h|t}|)$, where $\mathrm{E}_t(\cdot)$ denotes the mathematical expectation conditional on the information available at time t. When $\mathrm{E}_t(|err_{t+h|t}|)$ is large, the probability density of large errors must be high and hence the uncertainty is high; when $\mathrm{E}_t(|err_{t+h|t}|)$ is small, the probability density of large errors must be low and hence the uncertainty is low. Conversely, high uncertainty translates into high probability density of large errors and thus large values of $\mathrm{E}_t(|err_{t+h|t}|)$; and low uncertainty translates into low probability density of large errors and hence small values of $\mathrm{E}_t(|err_{t+h|t}|)$. Thus $\mathrm{E}_t(|err_{t+h|t}|)$ is informative of the magnitude of uncertainty and in general is a sensible measure of uncertainty. See Figure 1 for some illustrations of diverse forecast error distributions, their summary characteristics, and their implied uncertainty.

[Table 1 about here]

[Figure 1 about here]

Similarly, consider the second moment $\mathrm{E}_t(err^2_{t+h|t})$. When $\mathrm{E}_t(err^2_{t+h|t})$ is large, the probability density of large errors must be high and hence the uncertainty is high; when $\mathrm{E}_t(err^2_{t+h|t})$ is small, the probability density of large errors must be low and hence the uncertainty is low. Conversely, high uncertainty and the corresponding high probability density of large errors produce large values of $\mathrm{E}_t(err^2_{t+h|t})$; and low uncertainty produces small values. Just like the first absolute moment, also the second moment is informative of the magnitude of uncertainty and thus makes sense as a measure of uncertainty. Whether to use the first absolute moment or the second moment depends on the loss function that is relevant in a particular application. The first absolute moment is applicable under absolute loss, while the second moment applies under square loss.

2.3 Distributional moments that fail to reflect uncertainty

On the other hand, not all summary characteristics of the forecast error distribution adequately reflect uncertainty. That is, some or all values of these characteristics are not informative of the magnitude of uncertainty. Consider the first moment, or the mathematical expectation $\mathrm{E}_t(err_{t+h|t})$ as an example. If $\mathrm{E}_t(err_{t+h|t})$ is large (negative or positive), the uncertainty is high, because a large $\mathrm{E}_t(err_{t+h|t})$ implies that large errors are relatively likely to be realized and that large positive errors do not outweigh large negative errors nor the other way around. But if $\mathrm{E}_t(err_{t+h|t})$ is small (close to zero), the uncertainty may be either low or high. E.g. if $\mathrm{E}_t(err_{t+h|t}) = 0$, i.e. $\mathrm{E}_t(err_{t+h|t})$ is the smallest possible, $err_{t+h|t}$ may be zero with probability one, that is, there may be no uncertainty at all as the price may be forecast with perfect accuracy (zero error); see panel c) of Figure 1. Alternatively, if $\mathrm{E}_t(err_{t+h|t}) = 0$, then $err_{t+h|t}$ may have a symmetric heavy-tailed distribution where large errors are likely and thus the uncertainty is high; see panel b) of Figure 1. Since a small $\mathrm{E}_t(err_{t+h|t})$ is perfectly compatible with both low and high uncertainty, it is not informative of the magnitude of uncertainty. Hence, $\mathrm{E}_t(err_{t+h|t})$ does not make sense as a measure of uncertainty.

Similarly, consider the second central moment, or variance $\mathrm{Var}_t(err_{t+h|t}) \coloneqq \mathrm{E}_t\left(\left(err_{t+h|t} - \mathrm{E}_t(err_{t+h|t})\right)^2\right)$. When it is large, large errors (either negative or positive, or both, depending on $\mathrm{E}_t(err_{t+h|t})$) are likely and the uncertainty is high. But if variance is small, the uncertainty may be either low or high. E.g. if variance is zero, $err_{t+h|t}$ may be zero with probability one, indicating complete certainty; see panel c) of Figure 1. On the other hand, if variance is zero, the forecast error distribution may have all of its mass concentrated at a large (negative or positive) value, meaning high uncertainty because large errors are guaranteed; see panel d) of Figure 1. Indeed, small variance is compatible with both low and high uncertainty. Therefore, small variance is not informative of the magnitude of uncertainty, and hence variance is not a valid measure of uncertainty.

However, there is one condition under which variance becomes a sensible measure of uncertainty; the condition is that the mathematical expectation be zero. If the expectation is zero, variance equals the second moment: $\mathrm{E}_t(err_{t+h|t}) = 0 \Rightarrow \mathrm{Var}_t(err_{t+h|t}) = \mathrm{E}_t(err_{t+h|t}^2)$. Since the second moment adequately reflects uncertainty, the expectation being zero ensures that variance does, too. The expectation being zero is an important special case in which variance is transformed from an otherwise invalid measure of uncertainty to a valid one; it happens as variance turns into the second moment. Consequently, for all practical purposes of measuring uncertainty, it is always safer and simpler to use the second moment in place of variance. First, if the two coincide, there is no loss by using the second moment. Second, if they do not coincide, it is only the second moment that is a valid measure of uncertainty while variance is not. Section 3 will scrutinize the second moment and variance in the context of hedging.

2.4 Asset price vs. forecast error

Uncertainty and its measurement have been explained above with the help of the forecast error. One may ask whether it is necessary to employ the forecast error in the explanation. For example, could uncertainty be characterized by the probabilistic properties of the asset price alone, without referring to the forecast error? Would the first, the first absolute, the second or the second central moment of the asset price itself reflect uncertainty?

The answer is, no. Fundamentally, uncertainty arises when a market participant does not know how the asset price is generated.[2] Uncertainty refers to the gap between the market participant's beliefs about the price-generating process (and thus his/her price forecasts) and the actual process (the realizations). Hypothetically, if the asset price were generated in a deterministic way and there were no randomness involved, there would still be uncertainty as long as the market participant did not know the deterministic mechanism that generated the price. In summary, considering the asset price alone is not sufficient to characterize uncertainty.

3. Hedging under square loss

3.1 Objective of hedging

3.1.1 Classical objective

Hedging is "making an investment to reduce the risk of adverse price movements in an asset" (Investopedia, n.d.).[3] The goal of hedging is to lower price risk, or price uncertainty (Hull, 2012, p. 11). More precisely, the purpose of hedging is defined by an objective function. When hedging the price uncertainty of an asset h periods ahead, a popular objective is to minimize the variance (conditional on the information available at time t) of the portfolio price,

$$\mathrm{Var}_t(p_{t+h}) = \mathrm{E}_t\left(\left(p_{t+h} - \mathrm{E}_t(p_{t+h})\right)^2\right) \to \min_{\beta}, \quad (1)$$

where $p = p(\beta) = s - \beta f$ is the price of the hedge portfolio; s is the price of the original asset; f is the price of the hedging instrument; and $-\beta$ is the portfolio weight of the hedging instrument; β is also known as the hedge ratio. The (negative of the) hedge ratio reflects the hedger's exposure to the

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[2] In this work, the notion of uncertainty is different from the notion of risk. *Risk* arises when the price generating mechanism is known but is stochastic, such as a roulette or a dice throw. Meanwhile, *uncertainty* refers to situations where the mechanism is unknown, regardless of whether it is deterministic or stochastic. (However, the term *risk* will appear in fixed expressions such as *price risk, risk reduction* and *risk management*.)

[3] See Bloznelis (2017a) for an introduction into hedging under square loss and a detailed presentation of a new hedging framework based on Bloznelis (2016a).

hedging instrument's price as a fraction (or a multiple) of the exposure to the price of the original asset.

The objective in equation (1) does not directly relate to uncertainty, as discussed in Section 2.4. So how is it linked to minimization of uncertainty, if at all? An additional assumption is necessary to make the connection.

Assumption 1: The hedger knows the mathematical expectation (conditional on the information available at time t) of the portfolio price at a future time $t+h$, and uses it at time t as a point forecast for the portfolio price at $t+h$: $\hat{p}_{t+h|t} = \mathrm{E}_t(p_{t+h})$.

Under Assumption 1, we can rewrite $\mathrm{Var}_t(p_{t+h})$ as

$$\mathrm{Var}_t(p_{t+h}) = \mathrm{E}_t\left(\left(p_{t+h} - \mathrm{E}_t(p_{t+h})\right)^2\right) = \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right), \tag{2}$$

which is the second moment of the forecast error $p_{t+h} - \hat{p}_{t+h|t}$. As discussed in Section 2.2, this is a valid measure of uncertainty under square loss. Let us call it the expected squared forecast error of the portfolio price p_{t+h} and denote it

$$\mathrm{ESFE}_t(p_{t+h}) \coloneqq \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right). \tag{3}$$

Assumption 1 turns the hedging objective in equation (1) into minimization of the second moment of the forecast error, or the expected squared forecast error,

$$\mathrm{ESFE}_t(p_{t+h}) \to \min_{\beta}, \tag{4}$$

which corresponds to minimization of uncertainty under square loss.

How restrictive is Assumption 1? It requires knowledge of the expectation of the future portfolio price, which is not innocuous in the context of financial markets. There are markets and forecast horizons (and thus hedging horizons) where such an assumption might be plausible; e.g. stock or foreign exchange markets in short time horizons can be assumed to have zero expected returns, implying that $\mathrm{E}_t(p_{t+h}) = p_t$, which allows using p_t as a point forecast at time t. But there are other markets where this assumption is too strong and likely to be violated, e.g. markets of seasonal commodities such as wheat, live cattle, gas or electricity in forecast horizons equal to a fraction of the seasonal period; there the expected return is clearly nonzero but generally unknown, so a point forecast satisfying $\hat{p}_{t+h|t} = \mathrm{E}_t(p_{t+h})$ is unavailable. The salmon market also belongs among the latter. When the expected value is unknown, the objective in equation (1) does not correspond to uncertainty minimization in equation (4) and is not a valid goal from a hedger's perspective. Thus a replacement objective is needed.

3.1.2 New objective

When Assumption 1 holds, minimization of portfolio variance coincides with uncertainty minimization under square loss. Let us generalize the hedging objective to correspond to uncertainty minimization under square loss when the assumption is relaxed. Such a general objective is given in equation (4) and is to minimize the second moment of the forecast error, or the expected squared forecast error. Thus the objective in equation (4) corresponds to uncertainty minimization regardless of whether Assumption 1 holds. As such it is a valid replacement of variance minimization in hedging exercises.

While the classical and the new objective coincide under Assumption 1, let us examine how they differ when the assumption is violated. It is straightforward to show that the expected squared forecast error is greater than or equal to the variance of the portfolio price,

$$
\begin{aligned}
\mathrm{ESFE}_t(p_{t+h}) &= \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) \\
&= \mathrm{E}_t\left(\left(p_{t+h} - \mathrm{E}_t(p_{t+h}) + \mathrm{E}_t(p_{t+h}) - \hat{p}_{t+h|t}\right)^2\right) \\
&= \mathrm{E}_t\left(\left(p_{t+h} - \mathrm{E}_t(p_{t+h})\right)^2\right) + \mathrm{E}_t\left(\left(\hat{p}_{t+h|t} - \mathrm{E}_t(p_{t+h})\right)^2\right) \\
&= \mathrm{E}_t\left(\left(p_{t+h} - \mathrm{E}_t(p_{t+h})\right)^2\right) + \left(\mathrm{E}_t\left(\hat{p}_{t+h|t} - p_{t+h}\right)\right)^2 \\
&= \mathrm{Var}_t(p_{t+h}) + \mathrm{Bias}_t^2\left(\hat{p}_{t+h|t}\right) \\
&\geq \mathrm{Var}_t(p_{t+h}),
\end{aligned}
\tag{5}
$$

where $\mathrm{Bias}_t^2(\hat{p}_{t+h|t}) \coloneqq \left(\mathrm{E}_t(\hat{p}_{t+h|t} - p_{t+h})\right)^2 \geq 0$ denotes the squared bias of the forecast.[4] This decomposition also illustrates that low variance is perfectly compatible with high uncertainty whenever there is a high squared forecast bias. Moreover, a decrease in variance due to hedging is compatible with an arbitrarily large increase in the expected squared forecast error and thus uncertainty! This happens when the increase in the squared forecast bias is greater than the reduction in variance:

$$\mathrm{Var}_t(p_{t+h}) < \mathrm{Var}_t(s_{t+h}) \quad \text{holds simultaneously with} \quad \mathrm{ESFE}_t(p_{t+h}) > \mathrm{ESFE}_t(s_{t+h}) \tag{6}$$

if

$$\mathrm{Bias}_t^2(\hat{p}_{t+h|t}) - \mathrm{Bias}_t^2(\hat{s}_{t+h|t}) > \mathrm{Var}_t(s_{t+h}) - \mathrm{Var}_t(p_{t+h}). \tag{7}$$

Therefore, pursuing variance minimization due to equation (1) when aiming at reduced uncertainty may be a dangerous and detrimental strategy, bringing an increase in uncertainty despite a reduction in variance.

3.2 Optimal hedge ratio

3.2.1 Derivation

The hedge ratio β that optimizes the objective function is called the optimal hedge ratio (OHR) or the risk-minimizing hedge ratio. Chen et al. (2003) provide a review of solving for and estimating optimal hedge ratios under different assumptions for a variety of objective functions. The optimal hedge ratio due to the objective in equation (4) for hedging h weeks ahead, $\beta^*_{h,\mathrm{ESFE}}$, is not included there. It can be found by taking the derivative of the objective function with respect to the hedge ratio and setting it to zero, which yields

[4] This is an unorthodox definition of forecast bias. Here, the forecast itself is fixed (nonrandom) and the expectation is taken over the distribution of the target variable. It is opposite to the classical notion of bias of a forecast-generating process, where the target is fixed and the expectation is taken over the distribution of the forecast which is a random variable.

$$\beta^*_{h,\mathrm{ESFE}} \coloneqq \frac{\mathrm{E}_t(s_{t+h}f_{t+h}) - \hat{s}_{t+h|t}\mathrm{E}_t(f_{t+h}) - \hat{f}_{t+h|t}\mathrm{E}_t(s_{t+h}) + \hat{s}_{t+h|t}\hat{f}_{t+h|t}}{\mathrm{E}_t(f^2_{t+h}) - 2\hat{f}_{t+h|t}\mathrm{E}_t(f_{t+h}) + \hat{f}^2_{t+h|t}}. \tag{8}$$

A feasible sample counterpart, $\hat{\beta}^*_{h,\mathrm{ESFE}}$, is obtained by substituting the unknown theoretical quantities in equation (8) by their sample analogues: $\mathrm{E}_t(s_{t+h})$ with $\hat{s}_{t+h|t}$, $\mathrm{E}_t(f_{t+h})$ with $\hat{f}_{t+h|t}$, $\mathrm{E}_t(s_{t+h}f_{t+h})$ with $\widehat{\mathrm{E}}_t(s_{t+h}f_{t+h})$, and $\mathrm{E}_t\left(f^2_{t+h}\right)$ with $\widehat{\mathrm{E}}_t\left(f^2_{t+h}\right)$, to yield

$$\begin{aligned}
\hat{\beta}^*_{h,\mathrm{ESFE}} &\coloneqq \frac{\widehat{\mathrm{E}}_t(s_{t+h}f_{t+h}) - \hat{s}_{t+h|t}\hat{f}_{t+h|t} - \hat{f}_{t+h|t}\hat{s}_{t+h|t} + \hat{s}_{t+h|t}\hat{f}_{t+h|t}}{\widehat{\mathrm{E}}_t(f^2_{t+h}) - 2\hat{f}_{t+h|t}\hat{f}_{t+h|t} + \hat{f}^2_{t+h|t}} \\
&= \frac{\widehat{\mathrm{E}}_t(s_{t+h}f_{t+h}) - \hat{s}_{t+h|t}\hat{f}_{t+h|t}}{\widehat{\mathrm{E}}_t(f^2_{t+h}) - \hat{f}^2_{t+h|t}} \\
&= \frac{\widehat{\mathrm{Cov}}_t(s_{t+h}, f_{t+h})}{\widehat{\mathrm{Var}}_t(f_{t+h})}.
\end{aligned} \tag{9}$$

Here, $\widehat{\mathrm{Cov}}_t(s_{t+h}, f_{t+h}) \coloneqq \widehat{\mathrm{E}}_t(s_{t+h}f_{t+h}) - \hat{s}_{t+h|t}\hat{f}_{t+h|t}$ and $\widehat{\mathrm{Var}}_t(f_{t+h}) \coloneqq \widehat{\mathrm{E}}_t\left(f^2_{t+h}\right) - \hat{f}^2_{t+h|t}$ are estimates of conditional covariance $\mathrm{Cov}_t(s_{t+h}, f_{t+h})$ and conditional variance $\mathrm{Var}_t(f_{t+h})$, respectively. Given a conditional mean model for s_t and f_t with respective additive innovations u_t and v_t, we may further express $\hat{\beta}^*_{h,\mathrm{ESFE}}$ as

$$\hat{\beta}^*_{h,\mathrm{ESFE}} = \frac{\widehat{\mathrm{Cov}}_t\left(\sum_{i=1}^h u_{t+i}, \sum_{i=1}^h v_{t+i}\right)}{\widehat{\mathrm{Var}}_t\left(\sum_{i=1}^h v_{t+i}\right)}. \tag{10}$$

Assuming that the innovations from different time periods are conditionally uncorrelated, $\mathrm{Cov}_t(u_t, u_s) = \mathrm{Cov}_t(u_t, v_s) = \mathrm{Cov}_t(v_t, v_s) = 0$ for $t \neq s$, we obtain a simplification of $\hat{\beta}^*_{h,\mathrm{ESFE}}$,

$$\hat{\beta}^*_{h,\mathrm{ESFE}} = \frac{\sum_{i=1}^h \widehat{\mathrm{Cov}}_t(u_{t+i}, v_{t+i})}{\sum_{i=1}^h \widehat{\mathrm{Var}}_t(v_{t+i})} \eqqcolon \hat{\beta}^{**}_{h,\mathrm{ESFE}} \tag{11}$$

which we denote $\hat{\beta}^{**}_{h,\mathrm{ESFE}}$. This hedge ratio can be readily used in applications, its inputs being forecasts of the conditional variance matrices of the additive errors from the conditional mean model for s_t and f_t.

3.2.2 Comparison with classical optimal hedge ratio

The optimal hedge ratio stemming from the objective in equation (4) differs from the optimal hedge ratio due to the objective in equation (1), the latter being

$$\beta^*_{h,\mathrm{Var}} = \frac{\mathrm{Cov}_t(s_{t+h}, f_{t+h})}{\mathrm{Var}_t(f_{t+h})} \tag{12}$$

(Johnson, 1960, Ederington, 1979). At the same time, the empirical optimal hedge ratio in equation (9) is the same as the commonly used empirical counterpart of equation (12), $\hat{\beta}^*_{h,\mathrm{Var}}$:

$$\hat{\beta}^*_{h,\mathrm{Var}} = \frac{\widehat{\mathrm{Cov}}_t(s_{t+h}, f_{t+h})}{\widehat{\mathrm{Var}}_t(f_{t+h})} = \hat{\beta}^*_{h,\mathrm{ESFE}}. \tag{13}$$

This might appear paradoxical since the theoretical optimal hedge ratios due to equations (8) and (12) differ. But there is nothing wrong with that; both empirical ratios, $\hat{\beta}^*_{h,\mathrm{ESFE}}$ and $\hat{\beta}^*_{h,\mathrm{Var}}$, are derived by substituting the population quantities by their sample counterparts. The differences between the theoretical ratios $\beta^*_{h,\mathrm{ESFE}}$ and $\beta^*_{h,\mathrm{Var}}$ happen to cancel out because of the substitution, which then leads to the empirical ratios becoming the same, $\hat{\beta}^*_{h,\mathrm{ESFE}} = \hat{\beta}^*_{h,\mathrm{Var}}$. However, $\hat{\beta}^*_{h,\mathrm{ESFE}}$ and $\hat{\beta}^*_{h,\mathrm{Var}}$ need not always be the optimal estimators of $\beta^*_{h,\mathrm{ESFE}}$ and $\beta^*_{h,\mathrm{Var}}$, and hence other empirical counterparts of the theoretical optimal hedge ratios may be considered. There is no reason to maintain that all empirical counterparts of $\beta^*_{h,\mathrm{ESFE}}$ and $\beta^*_{h,\mathrm{Var}}$ should coincide, because the latter are not equivalent.

3.3 Hedging effectiveness

3.3.1 Measures of hedging effectiveness

Given an estimated optimal hedge ratio and point forecasts of the prices of the original asset and the hedging instrument, a hedge portfolio can be formed and its price predicted. Hedging effectiveness is assessed by comparing the predicted price with its actual realization in the future. In order to match the hedging objective function due to equation (4) with a measure of hedging effectiveness, *absolute* hedging effectiveness is defined as the expected squared forecast error of the hedge portfolio's price. In other words, absolute hedging effectiveness is the realized value of the objective function. As the measure depends on the scale of the variables, its interpretation might not be universally intuitive. Therefore, this study proposes a *relative* measure of hedging effectiveness that has a more convenient interpretation. It is the relative reduction in the price uncertainty due to replacing the unhedged spot position with the hedge portfolio. More precisely, the relative measure of hedging effectiveness is the relative reduction in expected squared forecast error (RRESFE):

$$\mathrm{RRESFE} := \frac{\mathrm{ESFE}_t(s_{t+h}) - \mathrm{ESFE}_t(p_{t+h})}{\mathrm{ESFE}_t(s_{t+h})} = \frac{\mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right) - \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right)}{\mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right)}. \tag{14}$$

RRESFE compares the expected squared forecast error of the portfolio price with the expected squared forecast error of the spot price. The measure is theoretical as it involves theoretical moments of random variables. If the theoretical quantities are replaced by their empirical counterparts based on a sample of forecasts and corresponding realizations, we obtain an *empirical relative* measure of hedging effectiveness, the relative reduction in mean squared forecast error (RRMSFE):

$$\mathrm{RRMSFE} := \frac{\frac{1}{n}\sum_{\tau=1}^{n}\left(s_{\tau+h} - \hat{s}_{\tau+h|\tau}\right)^2 - \frac{1}{n}\sum_{\tau=1}^{n}\left(p_{\tau+h} - \hat{p}_{\tau+h|\tau}\right)^2}{\frac{1}{n}\sum_{\tau=1}^{n}\left(s_{\tau+h} - \hat{s}_{\tau+h|\tau}\right)^2}. \tag{15}$$

Here, the subscript τ indexes the time periods in which forecasts are made and n is the sample size.

3.3.2 Properties and use of effectiveness measures

RRESFE lies in the interval $(-\infty, 1]$, and larger values indicate greater hedging effectiveness. If the uncertainty over the portfolio price is zero, RRESFE is at its upper bound of unity, or 100%, $\mathrm{RRESFE} = 1 \Leftrightarrow \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) = 0$. This is the case of perfect hedging effectiveness when all of the uncertainty is eliminated by hedging, which may be rare in practice. However, it is enough to achieve positive effectiveness to conclude that hedging works in the desired direction, i.e. reduces uncertainty, $\mathrm{RRESFE} > 0 \Leftrightarrow \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) < \mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right)$. Meanwhile, a value of zero suggests that hedging has no effect on uncertainty, $\mathrm{RRESFE} = 0 \Leftrightarrow \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) = \mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right)$. In other words, the uncertainty over the price of the portfolio is as large as the uncertainty over the price of the original asset. Finally, negative values of RRMSFE signal that hedging is detrimental, i.e. the uncertainty associated with the portfolio price is greater than the uncertainty over the original asset's price, $\mathrm{RRESFE} < 0 \Leftrightarrow \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) > \mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right)$. If the uncertainty over the price of the original asset is zero, $\mathrm{E}_t\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right)^2\right) = 0$, RRESFE is ill-defined; but then no hedging is needed since there is no uncertainty to be reduced. The range of the empirical measure RRMSFE and the interpretation of its values are analogous to those of RRESFE.

When measuring hedging effectiveness empirically, estimation errors are unavoidable, and hence the measured values are imperfect reflections of the true underlying values. Given a measured value one may be interested in whether the corresponding underlying value is different from zero. This is equivalent to asking whether hedging has any genuine effect on price uncertainty. The null hypothesis of equally great price uncertainty under hedging versus no hedging can be tested by the Diebold-Mariano test of equal predictive accuracy (Diebold & Mariano, 1995; Harvey et al., 1997). A rejection of the null hypothesis would attest the effect of hedging is genuine, while a failure to reject would indicate the evidence is insufficient to conclude so. Testing the difference between the underlying effectiveness and an arbitrary value other than zero is a trivial extension.

The hedging effectiveness measured by the RRESFE or RRMSFE depends on the forecast accuracy of the original asset's price. *Ceteris paribus*, the lower the accuracy, the higher the measured effectiveness. The best available forecast of the original asset's price should be used to obtain a fair estimate of hedging effectiveness, as otherwise the measured hedging effectiveness could be artificially inflated by using an unnecessarily poor forecast for the original asset.

Caution is needed when comparing RRESFEs or RRMSFEs across models. If the point forecasts of the future spot price, $\hat{s}_{t+h|t}$, differ across models, the models with less accurate spot price forecasts will by construction tend to yield higher hedging effectiveness. Therefore, direct comparability of RRESFEs or RRMSFEs requires $\hat{s}_{t+h|t}$ to be the same across models.

3.3.3 New measures vs. classical measure

Even though the use of the RRESFE and RRMSFE is justified by the objective function of hedging under square loss, we may want to consider it in light of the classical measure of hedging effectiveness. The golden standard in the hedging literature is Ederington's measure defined as the relative reduction in variance (RRV; Johnson, 1960, Ederington, 1979),

$$\mathrm{RRV} \coloneqq \frac{\mathrm{Var}_t(s_{t+h}) - \mathrm{Var}_t(p_{t+h})}{\mathrm{Var}_t(s_{t+h})}. \tag{16}$$

Clearly, RRV is a special case of RRESFE; using equation (5), the latter can be expressed as

$$\mathrm{RRESFE} = \frac{\left(\mathrm{Var}_t(s_{t+h}) + \mathrm{Bias}_t^2(\hat{s}_{t+h|t})\right) - \left(\mathrm{Var}_t(p_{t+h}) + \mathrm{Bias}_t^2(\hat{p}_{t+h|t})\right)}{\mathrm{Var}_t(s_{t+h}) + \mathrm{Bias}_t^2(\hat{s}_{t+h|t})}. \tag{17}$$

Under Assumption 1, i.e. when the expected prices are known and are used as point forecasts, $\mathrm{Bias}_t(\hat{s}_{t+h|t}) = \mathrm{Bias}_t(\hat{p}_{t+h|t}) = 0$, and RRESFE collapses to RRV. Therefore, RRV may be a natural measure for applications in markets where both the asset price and the portfolio price have known expected returns (e.g. zero expected returns in liquid stock markets over short time horizons) which are used as point forecasts. However, RRV does not accommodate cases where prices of individual assets (such as salmon) and/or hedging portfolios have unknown expected returns and thus where $\mathrm{Bias}_t(\hat{s}_{t+h|t}) \neq 0$ and/or $\mathrm{Bias}_t(\hat{p}_{t+h|t}) \neq 0$. It is because RRV neglects the squared bias terms as can be seen by comparing equations (16) and (17). In other words, RRV is not a valid measure of the reduction in uncertainty due to hedging whenever Assumption 1 fails. Hence, RRV does not suit the case of the salmon spot price. See also Lien (2005) for a discussion on appropriateness (or lack thereof) of Ederington's measure under different conditions.

3.3.4 Determinants of hedging effectiveness

When measured by RRESFE, hedging effectiveness is to a large extent determined by the relevance of the hedging instrument. In turn, the relevance depends on the magnitude of correlation between the unexpected shocks to the price of the original asset and the hedging instrument. If the unexpected shocks are strongly correlated between the two, hedging effectiveness will be high;

there will exist a hedge ratio β such that $s_{t+h} - \hat{s}_{t+h|t}$ and $-\beta\left(f_{t+h} - \hat{f}_{t+h|t}\right)$ will be close in expectation, i.e. $\mathrm{E}_t\left(\left(\left(s_{t+h} - \hat{s}_{t+h|t}\right) - \beta\left(f_{t+h} - \hat{f}_{t+h|t}\right)\right)^2\right) = \mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right)$ will be low, yielding a value of the RRESFE close to unity.

For a given asset and a fixed hedging instrument, hedging effectiveness will depend on two properties, (1) the accuracy of the point forecasts $\hat{s}_{t+h|t}$ and $\hat{f}_{t+h|t}$, and (2) the accuracy of the conditional variance forecast for the pair of random variables s_{t+h} and f_{t+h}. First, accurate point forecasts of the spot price and the hedging instrument's price will yield an accurate forecast of the price of the hedge portfolio, for any given hedge ratio. The direction of this factor's effect on hedging effectiveness is unclear; while an accurate spot price forecast alone reduces the RRESFE, an accurate forecast of the portfolio price increases the RRESFE. Second, the conditional variance forecast determines the estimate of the optimal hedge ratio. The more accurate the forecast, the more accurate the estimate and therefore the higher the hedging effectiveness. In conclusion, achieving high hedging effectiveness requires finding a relevant hedging instrument and accurately predicting s_{t+h}, f_{t+h}, and their conditional variance matrix.

3.4 Workflow of hedging

The workflow of hedging can be summarized as follows. Consider a system of prices of the original asset and the hedging instrument. First, estimate a statistical model for the conditional mean vector and the conditional variance matrix. Second, obtain point forecasts and forecasts of the conditional variance matrix h steps ahead, where h is the hedging horizon. Third, calculate the optimal hedge ratio from the forecasted conditional variance matrix. Then form a hypothetical hedge portfolio of the original asset and the hedging instrument based on the optimal hedge ratio, and calculate its point forecast. Fourth, assess the forecast accuracy out of sample for the original asset's price and the price of the hedge portfolio. Fifth, evaluate hedging effectiveness in absolute and/or relative terms based on the forecast accuracy.

4. Hedging in the salmon market

4.1 Salmon farming industry and price of salmon

Commercial farming of salmon started in 1970s in Norway.[5] The industry expanded rapidly so that by late 1990s the salmon aquaculture production had soared past the capture fisheries. In 2014, the global farmed salmon production stood at around two million metric tons, valued roughly at 14 billion dollars. About 55% of the volume is of Norwegian origin; the other major salmon producers are Chile, Canada and Scotland. Farmed salmon is consumed all over the world; the largest markets are the EU (51% of global consumption) and the U.S. (24%).

The price of the Norwegian farmed Atlantic salmon experienced a continuous decline from the 1980s until the break of the millennia. The decrease is attributed to technological improvement and fast supply growth (Asche & Bjorndal, 2011, p. 43-48). The price trend reversed in the early 2000s likely because of slower technological progress, reduced supply growth due to limited availability of suitable production sites, increasing raw material costs (Asche and Oglend, 2016) and increased demand from the emerging markets.

The price of salmon is volatile and difficult to predict (Bloznelis, 2016b, Bloznelis, 2017b). In other words, there is high uncertainty over the future spot price of salmon. The uncertainty may arise due to supply and demand factors. On the supply side, there are several production risks, such as infectious disease (e.g. infectious salmon anaemia), biological conditions (e.g. presence or absence of toxic blooming algae), parasites (e.g. salmon lice) and the uncertainty in future water temperature (suboptimal temperature leads to slower growth of fish). On the demand side, price uncertainty may be affected by unexpected changes in consumer tastes resulting from positive or negative publicity (e.g. reports on health gains from a salmon-rich diet or concerns over pollution from the salmon farms); or changes in prices of substitutes or complements, or purchasing power. In the short term,

[5] This section is largely based on the information in Marine Harvest (2015).

the supply factors likely dominate the demand factors in causing large unexpected price fluctuations; see Oglend and Sikveland (2008) and Oglend (2013) for further discussion.

4.2 Salmon futures market

A large share of salmon is sold on the spot market, but forward contracting is also substantial (Larsen & Asche, 2011). Since mid-2006 there exists a futures exchange for salmon, Fish Pool, based in Bergen, Norway. Monthly contracts for one to 60 months ahead are available for trading. The contracts are cash settled; hence, no physical fish is bought or sold at Fish Pool. The underlying of a futures contract is the average price of (one tonne of) salmon sold in the spot market over a given month. The contract reference price is the Fish Pool Index (FPI), a weighted average of three (previously two to five) price indices that may be considered representative of the spot price of 3-6 kg salmon. Empirically the FPI is almost indistinguishable from the regular spot price. The futures market suffers from low liquidity as its turnover matches only about a tenth of the physical market volume (Fish Pool, 2015) and the trades are rather infrequent. Fish Pool also provides clearing services for salmon forward contracts.

4.3 Salmon futures as a hedging instrument

Since the FPI is very similar to the spot price, the underlying of a futures contract is approximately equal to the average spot price over a given month. Therefore, shocks to the futures price should to a large extend coincide with shocks to the spot price over that month. This makes the salmon futures contract a natural hedging instrument for the spot price over monthly periods. That is, the exposure to the uncertainty over the future spot price may be hedged by taking an offsetting position in the futures market. The hedging effectiveness should be the highest for a farmer selling an equal amount of fish every week of the month, because then the spot position matches the underlying of the futures contract the closest. The hedging effectiveness may be lowered by deviations in volume sold from week to week and/or variations in fish quality resulting in discrepancies between the FPI and the price obtained by the farmer.

4.4 Hedging strategies

When hedging a given spot position with salmon futures contracts, a market participant has to decide which contract to trade and at what times. This study examines three alternative hedging strategies. Two of them are classical (Hull, 2012, p. 55), while the third one is unorthodox and is suggested so as to avoid some undesirable features of the first two. Suppose the original asset to be hedged is salmon that will be sold on the spot market on the fourth week of January, 2017. Suppose for concreteness that the current date is the last week of December, 2016. The first (classical) hedging strategy is to sell a January contract in the last week of December and buy it back in the fourth week of January; this strategy is denoted F0. The second (classical) hedging strategy is to sell a February contract in the last week of December and buy it back in the fourth week of January (denoted F1). The third (unorthodox) hedging strategy is to sell a January contract in the last week of December and keep it until expiration (maturity), which is in the second week of February[6]; this strategy will be denoted M (due to "maturity").[7] The only difference between M and F0 is that the January contract is held until maturity rather than bought back in the fourth week of January. For an illustration of the contracts used in F0, F1 and M and their respective holding periods, see Figure 2. The three strategies are generally applicable when hedging the price uncertainty of any commodity or financial asset for which a futures market exists and the relevant contracts are available for trading. The M strategy also requires that the futures contract be cash settled.

[Figure 2 about here]

The first strategy, F0, may provide an effective hedge as the unpredictable shocks to the price of a futures contract (here the January contract) close to maturity should be highly correlated to the unpredictable shocks hitting the spot price. However, F0 may be difficult to implement due to the

[6] The salmon futures contracts are traded not only until the end of the underlying month (in this case, January) but also for up to two additional weeks (in this case, until the second week of February).

[7] I am not aware of previous studies mentioning the M strategy, thus the design of the strategy might be a new contribution to the hedging literature.

lack of liquidity in the salmon futures market, especially as the trading volume might decrease when the contract approaches the expiration date. Also, low liquidity may make the contract price volatile and the shocks to it less correlated with the shocks to the spot price, thus reducing the hedging effectiveness. The two other strategies circumvent these issues.

The second strategy, F1, partly bypasses the problems of liquidity and price volatility as the February contract is relatively far from expiration in the fourth week of January. (The liquidity problem is reduced but not entirely solved, because even the most liquid salmon futures contracts are only thinly traded regardless of the time to maturity.) However, this comes at a price of potentially lower shock correlation; the underlying of the February contract is the average FPI over February, and the unexpected shocks affecting February's FPI may not be highly correlated with the shocks affecting the spot price of the fourth week of January, against which we wish to hedge. Nevertheless, using F1 is common practice in some futures markets (Hull, 2012, p. 55).

The third strategy, M, also mitigates the liquidity problem as the contract is never bought back but rather held until expiration. M is also cost effective since commissions associated with buying the contract back are avoided. However, the unpredictable component of the January contract price at its expiration date could be less correlated with the unpredictable shocks to the spot price of the fourth week of January. It is difficult to foresee which of the three strategies should be the most effective in the case of salmon. Their relative performance will be revealed by empirical analysis in Section 6.

M is different from F0 and F1 in that the outcome of hedging becomes known with a delay, as the futures contract is not liquidated simultaneously with the spot position but rather kept until maturity. In the case of salmon futures, the contract expires in the second week after the underlying month. Thus in practice the delay is between two and six weeks, depending on which week of the month is being hedged. Consequently, the uncertainty over the price of the hedge portfolio due to M is prolonged, and the hedger needs to attend to margin calls on his/her futures position for a longer period. Furthermore, the effectiveness of M as measured by RRESFE is not directly comparable to that of F0 or F1 because the nominal difference in effectiveness between M and F0 or F1 should be considered in light of the prolonged exposure to uncertainty due to M.

A peculiar feature of M is that for a period of time the futures contract is held without a matching position in the spot market. On the first look, this could be perceived as speculative behaviour and a source of added uncertainty. However, this is precisely what facilitates hedging in illiquid futures markets where buying the contract back shortly before maturity is difficult or impossible. Furthermore, it leads to significantly reduced uncertainty, as will be seen from the empirical results. Whenever the hedging effectiveness of M exceeds that of F0 it is precisely because of the "speculative" element, as it is the only difference between F0 and M. In summary, a hedger will accept M and benefit from it regardless of the "speculative" element as long as M will reduce uncertainty, which is the goal of hedging.

4.5 Hedging one week vs. one month

Hedging the price uncertainty for a particular week using a monthly futures contract is less effective than hedging the price uncertainty for a whole month, as long as the trading volume is approximately equally distributed across the different weeks of the month. This is because the underlying of a futures contract closely matches the physical position for the given month rather than any particular week. If the price faced by a market participant closely matches the market average price, the salmon futures contract kept until maturity would offer a nearly perfect hedge for a whole month's physical position. More formally, we would have $s_{t+h} \approx f_{t+h}$ and $\hat{s}_{t+h|t} \approx \hat{f}_{t+h|t}$, and hence $p_{t+h} \approx \hat{p}_{t+h|t} \approx 0$ for $\beta = 1$. Thus there are good grounds to expect $\mathrm{E}_t\left(\left(p_{t+h} - \hat{p}_{t+h|t}\right)^2\right) \approx 0$, which means the objective function given in equations (4) reaching its global optimum. However, if the production volume is distributed unequally across the weeks in a month, the physical position will not match the underlying of the futures contract closely and hedging with salmon futures will be less effective.

5. Methods and data

Hedging requires predicting the prices of the original asset and the hedge portfolio, which involves estimating the optimal hedge ratio. The latter is based on the predicted conditional variance matrix of the original asset's price and the price of the hedging instrument. Therefore, we need to model the conditional variance matrix and hence also the conditional mean vector of the spot price and the hedging instrument's price, as outlined in Section 3.4. The conditional mean and variance models are estimated in rolling windows spanning 209 weekly observations (four years), and out-of-sample forecasts are obtained. Forecast accuracy is estimated on out-of-sample data and used to assess the hedging effectiveness.

5.1 Conditional mean model

Salmon price is known to be seasonal (Asche & Guttormsen, 2001). Seasonal adjustment is performed using regression with ARMA errors following Hyndman (2014). The regressors are Fourier terms and Christmas and Easter dummies. The set of the Fourier terms is made of pairs of $\sin(\cdot)$ and $\cos(\cdot)$ series with periodicity of 1 year, ½ year, ¼ year, etc. The lag order of the ARMA model for the regression errors is allowed to be any subset of an ARMA(4,4) specification. The number of the Fourier terms and the ARMA lags are selected simultaneously using the Akaike's information criterion (AIC) (Akaike, 1974). The salmon futures prices are seasonally adjusted using the same method.

Asche et al. (2016) find cointegration between the spot and the futures price of salmon. By the Granger representation theorem (Engle & Granger, 1987), the (seasonally-adjusted) spot and futures prices follow a bivariate vector error correction model (VEC model, or VECM):

$$\Delta\begin{pmatrix} s_t \\ f_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} (1 \quad \theta) \begin{pmatrix} s_{t-1} \\ f_{t-1} \end{pmatrix} + \Gamma_1 \Delta \begin{pmatrix} s_{t-1} \\ f_{t-1} \end{pmatrix} + \cdots + \Gamma_p \Delta \begin{pmatrix} s_{t-p} \\ f_{t-p} \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}; \tag{18}$$

here Δ denotes difference operator such that $\Delta x_t \coloneqq x_t - x_{t-1}$; α_1, α_2 are loading coefficients; $(1 \quad \theta)'$ is the cointegrating vector, such that $(1 \quad \theta)\begin{pmatrix} s_t \\ f_t \end{pmatrix}$ is a stationary process while s_t and f_t

individually are integrated processes; Γ_1 to Γ_p are 2×2 coefficient matrices; and u_t and v_t are zero-mean, non-autocorrelated error terms. A cointegrating vector $(1, \theta)' = (1, -1)'$ is arbitrarily imposed as the seasonally-adjusted spot and futures prices are close to being equal. Fixing θ at 1 will prevent imprecise estimation of the cointegrating vector due to the small sample size of the rolling windows. (Unrestricted VEC models where θ is determined by the data have also been estimated, and the modelling results are very similar to the case of $\theta = 1$.) The lag order of the VECM is selected using AIC. AIC-based choice tends to yield the model that produces the smallest squared one-step-ahead forecast error among the set of candidate models (Konishi & Kitagawa, 2008, p. 249-250), which is exactly what is needed when selecting a forecasting model under square loss.

5.2 Conditional variance model

The conditional mean model will produce point forecasts and also in-sample residuals. The latter will be used for modelling the in-sample conditional variance matrix. Its diagonal elements are specified using univariate generalized autoregressive conditional heteroskedasticity (GARCH) models (Bollerslev, 1986), and the off-diagonal elements by a dynamic conditional correlation (DCC) model (Engle, 2002). This is a standard approach in financial applications due to the model's relatively simple specification, fast estimation and good forecasting performance (*ibid*.). The DCC model has been used for modelling commodity prices in Bekkermann (2011), Creti et al. (2013) and Mensi et al. (2014), among other.

Let z_t denote the scalar error term (the innovation) of the conditional mean model and let σ_t^2 denote the conditional variance of a time series process z_t given the information up until the time period $t - 1$. A GARCH(1,1) model for z_t is specified as follows:

$$\begin{aligned} z_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \varphi z_{t-1}^2 + \psi \sigma_{t-1}^2, \\ \varepsilon_t &\sim i.i.d.(0,1), \end{aligned} \tag{19}$$

where ω, φ and ψ are nonnegative constants and d is a probability distribution. The index (1,1) in the model name indicates the use of lagged innovation and lagged conditional variance from one period before. In principle, higher-order lags indexed by (q, r) could be invoked; however, the (1,1) specification is often found to be sufficient in financial applications (Hansen & Lunde, 2005) and is widely used in practice; see e.g. Dawson et al. (2000), Szakmary et al. (2003), Yang et al. (2005), and references therein.

A bivariate DCC(1,1) model treats the conditional correlation between a pair of innovations in a similar way as to how a GARCH(1,1) model treats the conditional variance of an innovation. Given the standardized innovations $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ from the GARCH models for the error terms u_t and v_t of the spot price and the hedging instrument's price, respectively, a DCC(1,1) model specifies the law of motion for the conditional correlation matrix R_t with a typical element $\rho_{i,j,t}$ as follows:

$$q_{i,j,t} = (1 - \gamma - \delta)\rho_{i,j} + \gamma\varepsilon_{\mathrm{i},t-1}\varepsilon_{\mathrm{j},t-1} + \delta q_{i,j,t-1} \tag{20}$$

with γ and δ being nonnegative constants, $\rho_{i,j}$ the unconditional correlation, and $q_{i,j,t}$ the so-called conditional quasi-correlation, where $i, j \in \{1,2\}$. The proper conditional correlation is obtained as scaled conditional quasi-correlation,

$$\rho_{1,2,t} = \frac{q_{1,2,t}}{\sqrt{q_{1,1,t}}\sqrt{q_{2,2,t}}}. \tag{21}$$

Scaling due to equation (21) ensures the conditional correlation lies strictly between negative one and one. If the univariate GARCH models producing $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ from u_t and v_t fit well, scaling should have a negligible effect. Therefore, for illustrative purposes one could think of the conditional correlation being approximately a GARCH-type process,

$$\rho_{1,2,t} \approx (1 - \gamma - \delta)\rho_{1,2} + \gamma\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \delta\rho_{1,2,t-1}. \tag{22}$$

Similarly to GARCH, the index (1,1) in the DCC model indicates use of one period's lag of the standardized innovations and the conditional (quasi-) correlation. While higher lag orders could be considered, the relatively short time series at hand prompt the use of the most parsimonious (1,1) specification.

5.3 Data on salmon spot and futures prices

The spot price of the Norwegian farmed Atlantic salmon is obtained from NASDAQ. It is due to a weekly survey from 2007-2015, reflecting prices in Norwegian kroner per kilogram (NOK/kg) paid by exporters to salmon farmers (until 2013 week 13) and received by exporters from foreign buyers (from 2013 week 14). The difference between the former and the latter period is adjusted for by subtracting NOK 0.75/kg from the latter period's prices, following the practice of Fish Pool (Fish Pool, 2014). The price, its seasonally adjusted version and the seasonal component are depicted in the top row of Figure 3.

Data on the salmon futures prices is provided by Fish Pool. The prices are converted from daily to weekly by taking the last price of each week. Taking the last instead of the average price should be beneficial for forecasting as the last price reflects the most recent information. (In practice, taking weekly averages instead does not materially change the results.) The front month salmon futures price is shown in the bottom row of Figure 3.

[Figure 3 about here]

6. Results

Out-of-sample hedging results are discussed in this section.[8] Hedging effectiveness and optimal hedge ratios are compared across the strategies (F0, F1 and M) and the horizons (four, eight and 13 weeks ahead). A limited comparison is also made with Misund and Asche (2016) who employ a different hedging framework. Empirical results reveal that the optimal hedge ratios are around 0.5-0.6 on average and that hedging is a rather effective way of mitigating the spot price uncertainty, especially in the longer horizons.

[8] See Bloznelis (2016a) for results from alternative specifications of the conditional mean and variance models, and performance comparisons across different models and weight classes of salmon.

6.1 Hedging effectiveness

Hedging effectiveness as measured by RRMSFE is given in the first column of Table 2. The values range from 0.16 to 0.58 indicating a 16% to 58% reduction in the mean squared forecast error when using a hedge portfolio in place of an unhedged spot position. To assess whether the reduction in uncertainty due to hedging is genuine or just incidental, the Diebold-Mariano test (Diebold & Mariano, 1995; Harvey et al., 1997) is employed. The null hypothesis of no reduction in uncertainty, or equal predictive accuracy of the unhedged spot position and the portfolio, is rejected in all cases except one (for the 4-week hedge using the F0 strategy); see Table 2. Therefore, hedging is indeed fairly effective, especially in the longer horizons.

[Table 2 about here]

Hedging effectiveness measured by RRV is reported in the second column of Table 2. The numbers differ starkly from those due to RRMSFE. This is unsurprising as Assumption 1 is violated in the salmon market and the forecast bias for the spot as well as the portfolio price is not zero; see columns 3 and 4 of Table 2. For example, the effectiveness of M in the 4-week horizon is as high as 0.74 according to RRV but only 0.26 when measured by RRMSFE. Therefore, mistakenly using an inappropriate measure (RRV) would suggest that three quarters of uncertainty is removed by hedging, while the actual number is barely one quarter. The salient mismatch between the empirical values of RRV and RRMSFE underlines the importance of choosing the appropriate measure (RRMSFE) in applications.

Misund and Asche's (2016) rolling-window based strategies yield RRV values of 0.27 to 0.29, which are more than twice lower than here. This is likely due to their use of a constant conditional mean model as opposed to VECM and also shorter rolling windows (either 20 or 52 weeks against 209 weeks) which may yield lower estimation accuracy for the optimal hedge ratios.

Regarding the strategies, F0 beats F1 in the 8-week and 13-week horizons, while F1 is more effective in the 4-week horizon. Thus the choice between F0 and F1 should depend on how far in advance a position is hedged. The overall most effective hedging strategy is M. However, its effectiveness is not directly comparable to that of F0 and F1, because M yields longer exposure to uncertainty due to the delayed liquidation of the futures contract (see Section 4.4). However, for a hedger who is willing to wait a few extra weeks until the futures position is closed, the M strategy is an effective alternative to F0 and F1.

Hedging effectiveness increases with the hedging horizon, as should be expected when the spot and the futures prices are cointegrated. When two time series share a common stochastic trend, their paths are roughly the same in the long run, in spite of any short-term deviations from the equilibrium. For an integrated process such as the spot price or the futures price by itself, there is high uncertainty (thus low forecast accuracy) over where it might end up in the distant future. However, there is considerably less uncertainty (higher forecast accuracy) over the future outcome of a stationary combination of two cointegrated processes. This is the intuition behind the increasing hedging effectiveness for cointegrated processes in ever longer horizons.

6.2 Optimal hedge ratios

The optimal hedge ratios are depicted in Figure 4. In any given panel of the figure, each point in the curve corresponds to a hedge ratio obtained from a different rolling window. E.g. the first point in the top left graph is dated 2011 week 4 and shows the 4-week hedge ratio calculated from the rolling window spanning from 2007 week 1 to 2010 week 52; the second point is dated 2011 week 5 and shows the 4-week hedge ratio due to the rolling window covering 2007 week 2 to 2011 week 1; etc. Note that the hedge ratio is constant over the lifetime of any given hedge. Meanwhile, the time variation in the hedge ratios across the rolling windows is due to the changing period that is being hedged and the changing information content based on which the optimal hedge ratio is being calculated.

[Figure 4 about here]

The variation within each curve in Figure 4 reflects the variation in the optimal hedge ratios across the rolling windows. It is quite high for the 4-week hedge but decreases with the hedging horizon. This is to be expected; the h-week hedge ratio is the ratio of an h-element sum of predicted covariances over an h-element sum of predicted variances as per equation (11). For the hedging horizon $h = 4$, one element out of four changes in each of the sums when moving from a given rolling window to the next one. Meanwhile, for the 13-week hedge only one element out of thirteen changes in each of the sums. Thus naturally there is more variation in the 4-week than in the 13-week hedge ratio.

The time variation of the optimal hedge ratios in Misund and Asche (2016) is considerably higher, most likely because of their shorter rolling windows. Also, the average level of the optimal hedge ratios in Misund and Asche (2016) is higher because of the high unconditional correlation between the prices of the original asset and the hedging instrument that is a key determinant of their optimal hedge ratio.

From a longer perspective, we see that the hedge ratios for F0 and F1 (and to a small extent also for M) were higher in the beginning of the sample but decreased over the first two years, representing the rolling windows covering 2007-2010 to 2009-2012. There has been no clear trend afterwards. Examining the predicted covariances and variances as in equation (11) does not allow drawing broad conclusions with regards to what the underlying driving force has been, since there is considerable variation in the covariances and variance across the hedging strategies.

The estimated optimal hedge ratios differ visibly across the hedging strategies. The average hedge ratios are higher for F0 and F1 (around 0.6) than for M (around 0.5). There are substantial high-frequency oscillations in the case of M, which can be explained by the nature of keeping the futures contract until maturity. Consider using M to hedge h weeks ahead. Take a spot position that is in week 1 (the first week) of a month. The futures contract will be acquired $h - 1$ weeks ahead of the target week. It will not be liquidated simultaneously with the spot position in the target week but rather kept for another five or six weeks until the contract expires. The value of the spot and the futures positions will move together only until and including the target week; after that, only the value of the futures position will move as the spot position will have already been liquidated. As per equation (11), the sum of the predicted covariances in the numerator will contain only h non-zero elements. Meanwhile, the sum of the predicted variances in the denominator will have $h + 5$ or $h + 6$ elements that will all be positive. Thus the hedge ratio, which is the value of the fraction in equation (11), will be relatively low in absolute value. Now consider hedging a spot position that is in week 4 or week 5 (the last week) of a month. The sum of the predicted covariances in the numerator will now contain $h + 3$ or $h + 4$ non-zero elements, while the denominator will stay the same as before. Since the predicted covariances are likely to be positive for each hedging horizon, their $h + 3$- or $h + 4$-element sum will be greater than the sum of just the first h elements. Thus, when hedging the last week of a month, the fraction representing the hedge ratio will be relatively large. For the weeks in between (week 2 and week 3 in all months, and week 4 in five-week months) the effect will be in between these two cases. This explains the observed high-frequency oscillations with periods of four or five weeks in the estimated optimal hedge ratios of the M strategy.

6.3 Economic significance of hedging effectiveness

How attractive is hedging with salmon futures contracts in practical terms? The 4-week hedging effectiveness of the F1 strategy yields a RRMSFE of 28%. For ease of interpretation, let us view this in terms of root mean squared forecast error ($\mathrm{rootMSFE}$); the reduction in uncertainty due to hedging is from $\mathrm{rootMSFE}$ of NOK 3.9/kg to NOK 3.3/kg, which is not substantial; see the last three columns of Table 2. Adding the poor liquidity of the futures market and the transaction costs, the 4-week hedge is unlikely to attract considerable interest. Meanwhile, the RRMSFE of 54% (F0) or 58% (M) for the 13-week hedge yields a reduction in the $\mathrm{rootMSFE}$ from NOK 6.4/kg to NOK 4.4/kg (F0) or NOK 4.1/kg (M), respectively. A reduction of NOK 2.0/kg or higher is tangible from a practical perspective; e.g. it is on the scale of a salmon farmer’s net profit (or loss) per kilogram in times of medium or low spot prices. Also, recall that hedging effectiveness would likely increase if monthly rather than weekly spot positions were hedged, and nearly perfect hedges could be expected (see Section 4.5). This makes hedging with salmon futures even more attractive.

Why then are there so few trades at Fish Pool? The possible reasons could be, (1) a lack of speculative interest and thus a lack of counterparties to the potential hedgers, (2) an increasing vertical integration in the salmon industry that provides implicit hedging within a vertically integrated company, and (3) a lack of independent studies assessing the hedging effectiveness and supplying reliable and timely information for the futures market participants. The latter reason is addressed by the current study.

7. Conclusion

There is high price uncertainty in the Norwegian farmed salmon market. The spot price is volatile and hard to predict, which complicates operations management and financial planning and is regarded as a major problem by the market participants. A standard way of risk management in commodity and financial markets is hedging. This study considers hedging the uncertainty in the future spot price of salmon with salmon futures contracts. A new framework of hedging under square loss is presented. It generalizes the classical framework of minimum variance hedging and includes a new objective function, a new optimal hedge ratio and a new measure of hedging effectiveness. The new objective is to minimize the expected squared forecast error of the hedge portfolio. It is naturally applicable not only under known but also unknown expected prices, where the classical objective of variance minimization loses relevance. The new optimal hedge ratio is similar to the classical one; the same estimator for the optimal hedge ratio can be used in both the classical and the new framework. Hence, the standard approach to choosing the optimal hedge ratio may still be employed. The new effectiveness measure is the relative reduction in the expected squared forecast error. It assesses the predictability of the portfolio price relative to that of the original asset. While the classical effectiveness measure – the relative reduction in variance – fails in absence of unknown expected prices, the new measure works well regardless. The differences in hedging effectiveness measured by the classical versus the new measure are stark, and empirical results due to the two measures may lead to considerably different economic implications, as evidenced by the case of salmon. All things considered, choosing the new hedging framework is critical in applied work, especially when the expected prices are unknown.

The salmon futures contracts are found to be a moderately effective hedging instrument for the salmon spot price; however, their appeal may be limited by low liquidity. Fortunately, holding the contract through maturity is not only easier in presence of liquidity problems but also cheaper than closing the contract simultaneously with the spot position. If low liquidity of the salmon futures contracts were still a problem even when holding the contract through maturity, hedgers would have to rely on the ongoing vertical integration in the salmon industry, which provides implicit hedging within a company, or on increased use of salmon forward contracts.

Overall, the failure of the classical hedging framework in absence of known expected prices suggests that hedging under square loss should be reconsidered in a number of commodity markets such as wheat, live cattle, gas and electricity. Hedging effectiveness should be re-evaluated using the proper effectiveness measure, which may lead to finding different optimal hedging instruments and/or hedging strategies than before. Accordingly, policymakers might need to review the current regulations and incentive schemes so as to encourage effective measurement and management of risk. The crucial questions to be answered are, how the optimal hedging instruments and strategies due to the new framework differ from the classical ones, and what implications this bears for hedgers, speculators and the practice of risk management in commodity and financial markets.

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Appendix A: Tables

Table 1 Some measures of uncertainty and their characteristics

Measure (moment of forecast error distribution)	Value	Magnitude of uncertainty	Is informative of the magnitude of uncertainty	Minimizing the measure is equivalent to minimizing uncertainty
First moment, i.e. mathematical expectation	Close to zero	Unknown	**No**	**No**
	Far from zero	High	Yes	
First absolute moment	Close to zero	Low	Yes	Yes
	Far from zero	High	Yes	
Second moment	Close to zero	Low	Yes	Yes
	Far from zero	High	Yes	
Second central moment, i.e. variance	Close to zero	Unknown	**No***	**No***
	Far from zero	High	Yes	

Note: * except when the first moment is zero.

Table 2 Hedging results

Hedging strategy and horizon	RRMSFE	RRV	Bias (spot)	Bias (portf.)	rootMSFE (spot)	rootMSFE (instr.)	rootMSFE (portf.)
4 weeks ahead							
F0	0.16	0.64	0.35	0.44	3.88	3.07	3.54
F1	0.28**	0.76	0.35	0.41	3.88	2.63	3.28**
M†	0.26**	0.74	0.35	0.40	3.88	3.49	3.34**
8 weeks ahead							
F0	0.43**	0.73	0.50	0.48	5.10	3.81	3.85**
F1	0.40**	0.78	0.50	0.62	5.10	3.40	3.96**
M†	0.45***	0.75	0.50	0.45	5.10	4.69	3.78***
13 weeks ahead							
F0	0.54**	0.67	0.57	0.56	6.39	5.26	4.35**
F1	0.44**	0.77	0.57	0.78	6.39	4.37	4.77**
M†	0.58**	0.69	0.57	0.46	6.39	6.07	4.13**

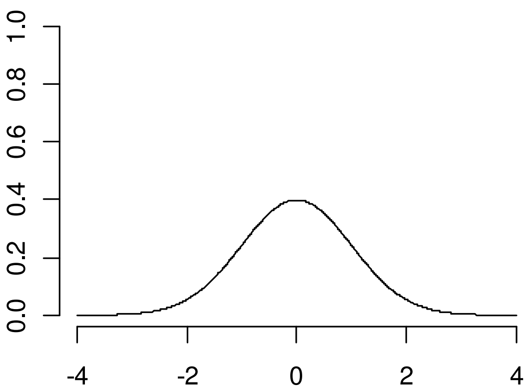
Note: Relative reduction in mean squared forecast error (RRMSFE), relative reduction in variance (RRV), forecast bias of the spot and the portfolio prices, and root mean squared forecast error (rootMSFE) of spot price, hedging instruments' price and hedge portfolios, 2011 week 1 - 2015 week 17. *, ** and *** mark significance at 10%, 5% and 1%, respectively, with regards to the Diebold-Mariano test. The null hypothesis in the first column is that hedging effectiveness is zero. The null hypothesis in the last column is that the forecast accuracy (measured by mean squared forecast error) of the portfolio is equal to that of the spot price. Note that the two hypotheses are algebraically equivalent. † The effectiveness of the M strategy is not directly comparable to the other two strategies because the futures contract is liquidated with a delay; see Section 4.4 for details.

Appendix B: Figures

Figure 1 Measures of uncertainty for different forecast error distributions

Low uncertainty

a) Light-tailed distribution, symmetric around zero

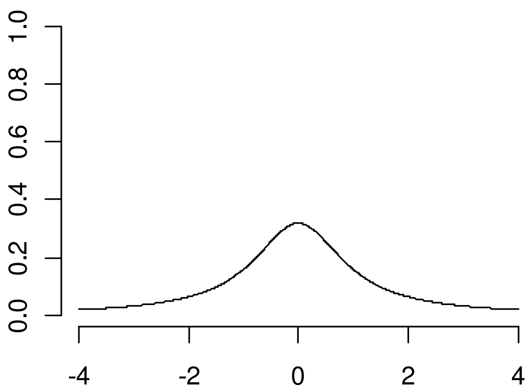


$\mathrm{E}_t\left(err_{t+h|t}\right) = 0$, $\mathrm{E}_t\left(err_{t+h|t}^2\right)$ small,

$\mathrm{E}_t\left(\left|err_{t+h|t}\right|\right)$ small, $\mathrm{Var}_t\left(err_{t+h|t}\right)$ small.

High uncertainty

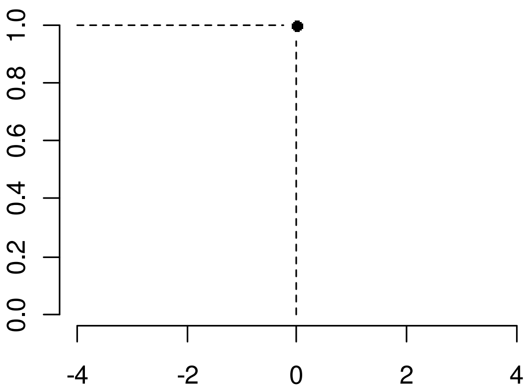
b) Heavy-tailed distribution, symmetric around zero



$\mathrm{E}_t\left(err_{t+h|t}\right) = 0$ (!), $\mathrm{E}_t\left(err_{t+h|t}^2\right)$ large,

$\mathrm{E}_t\left(\left|err_{t+h|t}\right|\right)$ large, $\mathrm{Var}_t\left(err_{t+h|t}\right)$ large.

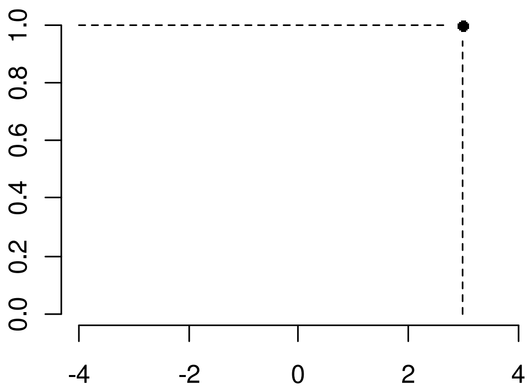
c) Distribution with a unit mass at zero



$\mathrm{E}_t\left(err_{t+h|t}\right) = 0$, $\mathrm{E}_t\left(err_{t+h|t}^2\right) = 0$,

$\mathrm{E}_t\left(\left|err_{t+h|t}\right|\right) = 0$, $\mathrm{Var}_t\left(err_{t+h|t}\right) = 0$.

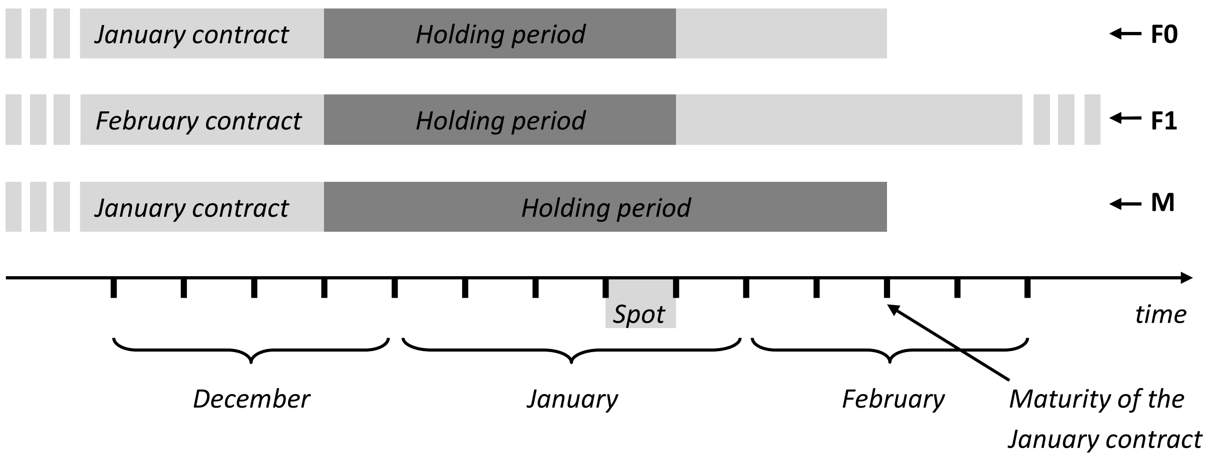
d) Distribution with a unit mass at a large value



$\mathrm{E}_t\left(err_{t+h|t}\right)$ large, $\mathrm{E}_t\left(err_{t+h|t}^2\right)$ large,

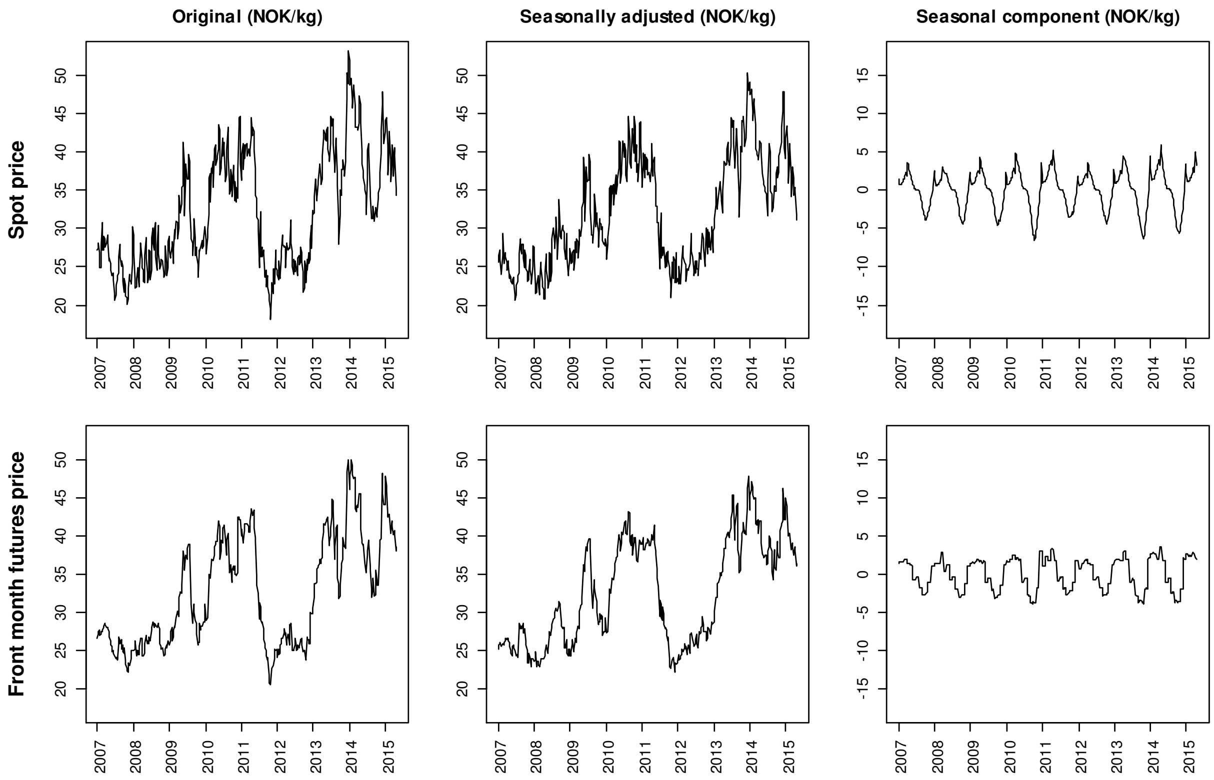
$\mathrm{E}_t\left(\left|err_{t+h|t}\right|\right)$ large, $\mathrm{Var}_t\left(err_{t+h|t}\right) = 0$ (!).

Figure 2 Contracts and holding periods for different hedging strategies



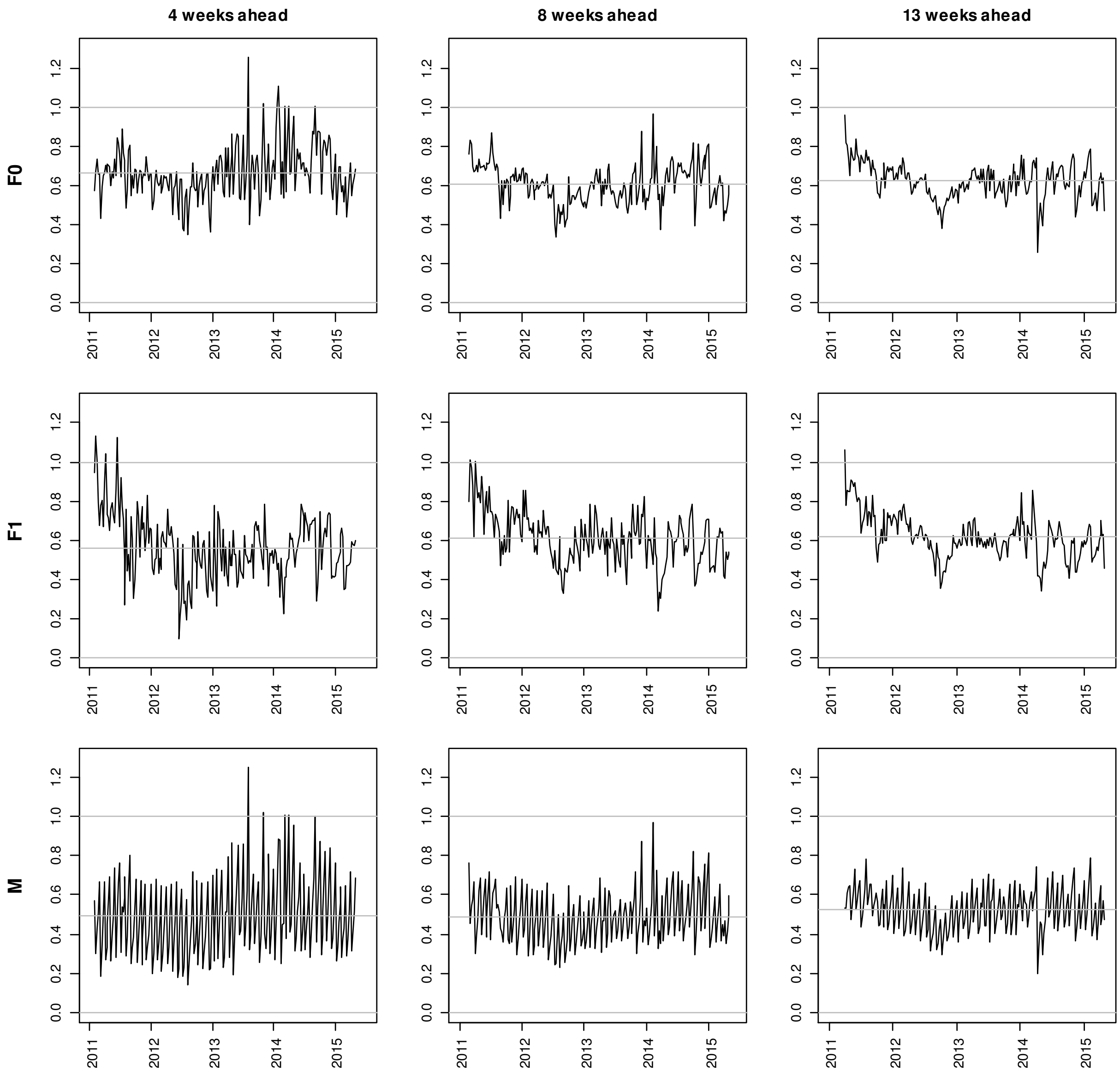
Note: Contracts and holding periods for strategies F0 (top), F1 (middle) and M (bottom) for a hedge starting in the last week of December, targeted at hedging a spot position in the fourth week of January.

Figure 3 Salmon spot price and front month futures price



Note: Salmon spot price (top row) and salmon front month futures price (bottom row); original (left), seasonally adjusted (middle), and seasonal component (right); 2007 week 1 - 2015 week 17.

Figure 4 Optimal hedge ratios



Note: Optimal hedge ratios for the different hedging strategies (across rows) and hedging horizons (across columns), 2011 week 1 - 2015 week 17. Horizontal lines mark 0, 1 and the mean of the optimal hedge ratio over the period.