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Analysis of Climate Policy and Monetary Policy Nexus in the Norwegian Context: A DSGE Approach

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Abstract

This thesis aims to analyze the nexus between climate policy and monetary policy using DSGE approach in the context of Norway. It considers two central themes: first, whether subsidization on electric vehicle could substantiate to emission reduction, and second, whether it could be a better idea to involve government in direct economic activities, such as production. New Keynesian macroeconomics provided essential theoretical and methodological foundations. Simulations over the selected variables followed by estimation using Dynare version 5.5 within the Matlab environment offered several results. The observed data series included the gross domestic product, consumer price index, returns on bonds, and emissions from Norwegian economic activities. All the series are related to Norway during 1990Q1:2022Q4. Data analysis occurred especially through impulse response function and historical decomposition methods.

The results show that climate policy through incentivization of electric vehicles does not contribute to emission reductions. On the other hand, public investment resulting from direct involvement of government in the production activity contributes to influencing the evolution of several key macro variables such as employment and output along with greenhouse gases emissions. To conclude Monetary policy has substantial effects on causing variation of emission level over time.

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Any errors are my own.

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1. Introduction

Incorporation of climate concerns to monetary policy domain is relatively a novel frontier. Climate policies, which aim at emissions reduction either through increased taxes (price regulation) or cap-and-trade (quantity regulation), are standard economic tools to regulate climate issues. However, recent findings suggest that these tools usually administered by fiscal regulators, may not be adequate in isolation to address challenges induced by climate change that impact output, employment, and prices in the economy. Hence, the central bank appears to have its role through conduct of monetary policy.

Recent studies provide some insights into potential relationships between climate policy and monetary policy. Boneva, Ferrucci and Mongelli (2022) state that greening monetary policy is a most viable option to achieve climate goals. They discuss several techniques through which central banks can engage with climate challenges, including protective actions aimed at enhancing green finance and transition to a low carbon economy. Schoenmaker (2021) argues that carbon intensive firms can significantly reduce costs of capital as far as the European Central Bank's (ECB) assets and collateral profile tilts towards low carbon companies. Giving proper emphasis to counteract climate induced uncertainties and vulnerabilities in the economy involves green investment through the support of monetary policy instruments (Batten, Sowerbutts, & Tanaka, 2020; Boneva, Ferrucci, & Mongelli, 2021; Dafermos, Nikolaidi, & Galanis, 2018).

The existing literature primarily emphasizes the European Union (EU), yet the same occurrence can also be observed in other countries (Chan, 2020; Dietrich, Müller, & Schoenle, 2021; Jourdan & Kalinowski, 2019). Dietrich, Müller and Schoenle (2021) even claimed that "First, climate-change related disaster expectations lower the natural rate of interest substantially. Second, time-variation in disaster expectations contributes to cyclical fluctuations." Economides and Xepapadeas (2018) found some robust and non-trivial implication in design of monetary policy taking climate change into account. These findings could have implications for conduct of both climate policy and monetary policy as the disconnection in the relationship challenges the relevance of the climate targets and sustainability. This further calls for a reassessment of traditional banking roles central banks play in an economy.

This thesis explores how the relationship among some crucial macro variables vis-à-vis climate and environmental variables have evolved over time in Norway especially after 1990s. Thus, it also examines the anticipated trajectories for macro variables, especially in the long run. In doing so, I aim to answer the following questions:

1. To what extent does incentivization of Electric Vehicle (EV) contribute to the reduction of emissions in Norway?
2. Does injection of public investment help achieve high and stable output and employment coupled with low and stable inflation i.e., divine coincidence, in the Norwegian economy while also reducing GHG emissions?

To answer these questions, I apply Dynamic Stochastic General Equilibrium (DSGE) approach as it can capture a synchronized dynamics inherent in macroeconomic variables also incorporating the climate and environmental realms.

Given the wider scope of the topic, with multiple premises and theoretical interpretations regarding the plausible connection among variables at hand, I will set discrete boundaries to remain inconspicuous. These boundaries encompass a concentrated emphasis on empirical evidence, deliberate omission of alternative hypotheses, and narrowed-down focus on the topic itself. The research questions are chosen due to lack of assessment of this topic in Norway. The aspiring climate targets enshrined in climate policy goals for 2030 by Norwegian government further motivated this research.

This thesis aims to provide a robust empirical foundation for the findings it presents. The study analysis will provide insights for policymakers in Norway and beyond helping them navigate through a complex landscape of climate-macroeconomic policy nexus. Furthermore, my thesis brings forth a contextual understanding given Norway's unique economic context, featured by substantial oil reserves, a strong orientation to welfare state, and ambitious climate goals.

The rest of the thesis is organized as follows: Chapter 2 presents how this thesis connects to the contextual and empirical literature on the DSGE framework and recent climate-monetary policy literature. Chapter 3 provides empirical and conceptual framework that guides DSGE analysis in this study. Additionally, it discusses the empirical data processing techniques and, discusses on the simulation and estimation strategies. This is where the proposed DSGE models appears in. Chapter 4 includes overview of the time series and provides guidance for further processing of data. This will be followed by presentation of results in chapter 5. Finally, chapter 6 includes discussion and conclusion along with some empirical learnings, and rooms for further studies.

2. Literature Review

2.1 Historical Background

Norway has been at the forefront of advancing climate actions. Since the establishment of the Brundtland Commission in 1983, which was chaired by the then Prime Minister of Norway, there has been a call for nations worldwide to prioritize for sustainable future. Norway was the first country to pledge, as early as 2007, to become carbon neutral by 2050 (*OECD Environmental Performance Reviews: Norway*, 2022). From a domestic perspective, the Norwegian parliament in 2016 approved the most ambitious goal of emission reduction and carbon offsetting to materialize by 2030 (Ibid).

Norway's aggressive electric mobility promotion through substantial subsidies on electric vehicles (EVs) is a cornerstone to climate policies in Norway. A generous tax incentives and subsidies for EVs and EV-related infrastructures marks Norway a most notable market for Zero-emission vehicles (ZEVs)[1]. Notably, the government is said to have spent billions of Kroner – counterfactually – over time to combat climate change in general, and to abate emissions from greenhouse gases (GHGs) in particular. Such a fiscal incentives, however, can have various far-reaching consequences in the economy such as prolonged inflation and/or lower output in the long-run (Aissa & Rebei, 2012; Harris, 1943). It's because in the long-run, producers, and consumers both realize that government subsidies which they receive as 'transfer income' is financed through increase in taxes. Consequently, rational producers will increase prices over and above what is required to cover their tax burden (immediate effects) as well as tax incidence (ultimate effects).

Not surprisingly, Norwegian government's direct efforts towards green capital formation through massive investment, for instance, in hydroelectricity sector, railways; and in petroleum exploration, production and follow-up activities as per the *Petroleum Act of Norway* (1996) opens avenues for further analysis in terms of its consequences in overall macroeconomy, although the legal framework is getting more flexible regarding entry of private and/or foreign sectors in these sectors[2].

[1] See https://www.bloomberg.com/news/articles/2023-07-26/norway-pulls-the-plug-on-ev-tax-incentives-and-subsidies.
[2] See https://www.state.gov/reports/2023-investment-climate-statements/norway/.

In consistence with the fiscal apparatus, the monetary authority of Norway (Norges Bank) also designs monetary policy aiming primarily at economic stabilization. Overall, the historical background creates space for studying possible nexus between climate policy and monetary policy in Norway.

2.2 Empirical Review

Modeling growth and business cycle fluctuations with DSGE models is a widespread practice. One classic quantitative DSGE model is the Real Business Cycle (RBC) model introduced by Kydland and Prescott (1982) and Long Jr and Plosser (1983). Following this, Backus, Kehoe and Kydland (1993) and Bils and Cho (1994) extended the seminal RBC model of Kydland and Prescott to analyze international aspects and investment-specific technological progress respectively. RBC models depict an economy characterized by perfect competition in goods markets, factor markets, and asset markets populated by a representative agent. Technology shocks are the primary sources of uncertainty in RBC models. The assumption of representative consumers can be viewed literally or reflected in Gorman's aggregation of heterogeneous consumers experiencing idiosyncratic income shocks (Gorman, 1953). According to these models, fluctuations in aggregate economic activity are the economy's efficient response to exogenous shocks. Consequently, no government intervention is required. The government's policy of stabilizing the business cycle reduces a country's welfare eventually. This finding – in the present context of global climate crisis – may, however, differ when we have a public good (bad) like GHGs emissions we need to control for, and thus, monetary (as well as fiscal) policy interventions may still be welfare enhancing.

Gali (1999) developed a model that incorporated nominal rigidities into the standard RBC which helped popularize the New Keynesian (NK) DSGE framework in analysis of business fluctuations. Gali's model includes price and wage stickiness, to analyze monetary policy and its effects on economic fluctuations. The author claims that in the presence nominal rigidities, the central bank can influence the real economic activities – such as employment and output – using interest rate and inflation targeting.

Ma and Zimmermann (2023) reveal that government intervention in economy through monetary policy can have substantial consequences upon innovations. Using impulse response function analysis, they argue that monetary policy influences innovation activities through changing aggregate demand and profitability of business and financial market conditions. Their finding further affirms that monetary policy affects productive capacity of the economy in the long run while also revealing near-term effects on economic outcomes such as GDP, investment, venture capital accumulation and so on. They clearly set a sharp departure from neoclassical notion of 'monetary neutrality' – meaning changes in money supply can have no real-term effects in the long run.

Based on a DSGE model with financial frictions, Benmir and Roman (2020) examine different fiscal, monetary, and macroprudential policies aimed at reducing CO_2 emissions. They indicate that CO_2 emissions and CO_2 mitigation policies create two inefficiencies: risk premiums and welfare distortions. Their first finding is that a substantial carbon tax is needed in the Euro Area to comply with the Paris Agreement, but it significantly impacts welfare. They explore monetary and macroprudential tools to dampen this effect and stop emissions shocks from distorting monetary policy. They assert that sectoral time-varying macroprudential weights on loans can help boosting green capital and output with a minimal welfare loss. Implementing a carbon tax enhances the advantages associated with both green and dirty asset acquisition under Quantitative Easing (QE). A QE rule would reduce the impact of emissions on risk premia by drastically reducing the environmental externality. This work shows the importance of including central banks in the fight against global warming by providing them with tools to mitigate it.

Del Negro and Federal Reserve Bank of Atlanta. (2004) review the forecasting records of DSGE models, showing how they can be used for forecasting, storytelling, and policy experiments. Furthermore, they conducted a real-time analysis of Smets and Wouters' (2007) model, compared it to Blue Chip and Greenbook forecasts, and demonstrated how the model changes as it incorporates external data such as interest rate forecasts, and long-term inflation and growth forecasts. With the help of counterfactual interest rate paths, they examine methods for generating forecasts with a zero-low-bound constraint on nominal interest rates. They demonstrate that DSGE forecasts of the Great Recession using spreads as an observable, augmented by financial frictions, are comparable to Blue Chip forecasts.

Christiano, Eichenbaum and Trabandt (2018) examine mainstream DSGE models prior to the financial crisis and the Great Recession. The authors then describe the process of estimating and evaluating DSGE models. In this article, the authors explain why DSGE modelers could not predict the Great Recession and the financial crisis the way other economists and policymakers did. They also examine what DSGE modelers did after the financial crisis. Their discussion focuses on how policymakers use current DSGE models in practice. The authors respond briefly to some criticisms of DSGE models, with particular attention to criticisms raised by Joseph Stiglitz, and conclude with a few remarks.

Blanchard (2016) points out that scenario analysis is one of the crucial merit of DSGE models. Since the models explicitly fit the data, they are less vulnerable to the Lucas critique (Rudebusch, 2005). To illustrate their point, the authors use several examples, including de-anchoring inflation expectations, identifying the shocks that resulted in the development of growth and inflation expectations, and assessing the drivers of financial market movements.

In one of their most cited articles, Smets and Wouters (2004) demonstrate how sticky-price dynamic stochastic general equilibrium (DSGE) models can be used by central banks as an additional tool for forecasting. First, they show that these models perform well with respect to forecasting performance compared with theoretical vector autoregressive models. The authors demonstrate how the posterior distribution of the model can be used to calculate different inflation risk measures. Furthermore, the model's structural features allow forecasts to be based on a policy path. This method permits for a detailed assessment of the structural causes of forecast errors and their implications for monetary policy. After the launch of the Economic and Monetary Union (EMU), they have used these tools to analyze macroeconomic developments in the eurozone. Based on their argument, the use of conditional and unconditional forecasts of inflation and output plays a key role in monetary policy strategies aimed at maintaining price stability.

Millard (2011) estimates a dynamic stochastic general equilibrium (DSGE) model for the United Kingdom considering three consumption goods: non-energy output, petrol, and utilities. The author discusses how the model, when estimated, may serve as an additional input within the policymaker's suite of models by analyzing its implications for the responses of macroeconomic variables to varying economic shocks and by decomposing recent changes in energy and non-energy outputs and inflation into proportions of these shocks.

Kravik and Mimir (2019) estimated NEMO, the dynamic stochastic general equilibrium model employed at the central bank of Norway, for monetary policy analysis and forecasting. With a

Bayesian approach to estimation, they assessed the model's dynamic properties. They conducted impulse response function analysis, historical shock decompositions, forecast error-variance decomposition, and a suite of empirical models to evaluate its forecasting performance. Forecasts for key variables in the Norwegian economy are based on NEMO in conjunction with a broad set of data, empirical models, and judgment. The authors admit that it is essential to continue to revise and develop NEMO to ensure that the model remains valid for analyzing monetary policy.

Golosov et al. (2014) analyze a DSGE model with an externality resulting from the use of fossil fuel. They present a simple formula for calculating the marginal externality damage caused by emissions and calculating the mal carbon tax equivalently. Under plausible assumptions, their formula reveals that the damage is proportional to GDP, depending merely on three factors, including – discounting, the expected damage elasticity (the percentage of loss in output flow in response to an additional carbon released), and the structure of carbon depreciation in the atmosphere. The formula eliminates stochastic values for future output, consumption, atmospheric CO2 concentration, and paths associated with technology and population. Based on their study, the optimal tax rate should be higher than the median or most well-known estimate. Additionally, they present a parsimonious and comprehensive model that can be easily solved for determining optimal paths for various energy sources. Due to its abundance, they consider coal – rather than oil – to be the primary threat to economic welfare. They also found that the costs of inaction are highly dependent on assumptions regarding the substitutability of different energy sources and technological advancements. However, authors fail to explore how the economy responds to CO2 tax given a tighter or more flexible monetary policy.

Xiao et al. (2022) developed an E-DSGE model that incorporated international trade, asymmetric economies, and heterogeneous industries within an open economy. They examine the macroeconomic effects of linking standalone national carbon markets based on the example of the linkage between the EU and China carbon markets. According to their findings, the linked carbon markets improved China's and the EU's overall social welfare but led to an unequal distribution of social welfare across the regions. Under a linked carbon market, China and the EU performed better economically than under two different carbon markets; the linked carbon market also amplified expansionary effects and mitigated adverse spillover effects across borders. The linked and separate carbon markets functioned as automatic economic stabilizers; the linked carbon markets reduced the economy's volatility in response to supply-side shocks, and the different carbon markets reduced the volatility of the economy in response to demand-side shocks. The authors agreed that linking carbon markets in different economies makes international climate cooperation possible. However, the study fails to consider monetary policy's contribution or deterrence in amplifying results of international climate negotiations.

3. Methodology

3.1 Theoretical Framework

3.1.1 New Keynesian DSGE Model

DSGE models are a class of state-space models in macroeconomics. State-space modeling assumes that evolution of system under investigation depends on unobserved series of vectors $\{\alpha_1, \ldots, \alpha_n\}$, that are associated with observed series $\{y_1, \ldots, y_n\}$. State-space models specifiy the relationship between α_t's and y_t's.

The New Keynesian DSGE (NK-DSGE) modeling basically relies on three things: dynamic IS curve, the Dynamic Philips Curve, and the Monetary Rule. The dynamic IS curve and Dynamic Philips Curve take into account the 'inflation expectations' of agents in the economy. Further, NK-DSGE is more pragmatic modeling technique in that it can incorporate issues such as habits persistence of consumers, underutilization of the installed capacity of the firms, investment adjustment costs, and wage and price rigidities. These issues collectively give rise to frictions on the variables of interest, and NK-DSGE tries to precisely manage and navigate through these issues, demonstrating its efficiency in addressing complexities inherent in the economy. The following diagram offers a glimpse of NK-DSGE.

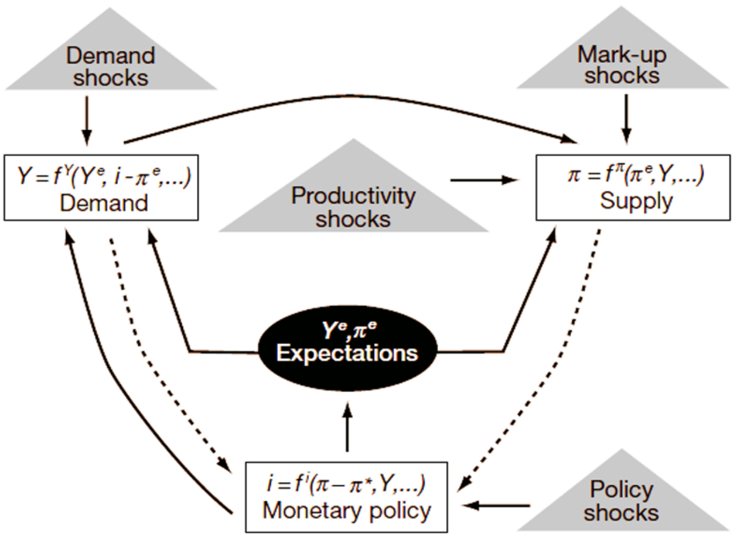


Fig. 3. 1: Basic NK-DSGE model

Source: Sbordone et al. (2010, p. 25)

The three principal agents are households, firms, and the central bank representing demand, supply, and monetary policy rule respectively. Firstly, the central bank administers changes in interest rate as one of its policy instruments. Following this change, households and firms make expectations regarding their intertemporal consumption and production decisions respectively over time. As agents are assumed to be rational, they make choices optimally given their prevailing constraints. The sources of deviations of variables of interest from the steady state values are economic shocks that might enter as demand shocks, productivity shocks, mark-up shock arising from prices, or any other shocks arising from policy change. The model can be extended to include other agents. I extend this model to include government to represent fiscal authority in addition to monetary authority, as well as emission trading system to as part of climate policy framework.

3.1.2 The model

3.1.2.1 Ricardian households

A representative consumer seeks to maximize lifetime utility derived from consumption of basket of goods and leisure given by the following expression:

$$\max_{C_{R,t}, K_{t+1}^{P}, U_t, I_t^{P}, B_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \tag{1.1}$$

where, $C_{R,t}$ is consumption at the period t, ϕ_c is coefficient of habit persistence, where $\phi_c > 0$, which represents the degree to which preferences cannot separate over time. $L_{R,t}$ represents leisure a consumer enjoys beyond workhours. $\sigma > 0$ is the coefficient of relative risk aversion, and $\varphi > 0$ is the marginal disutility associated with supply of labor. $\beta < 1$ is the discount factor that measures intertemporal preference rate – the greater value of β, the more patient the households are with regards to consumption. E_t is the expectation operator. The consumers are infinitely lived so that model depicts time range from today (*t=0*) to ∞.

However, the representative consumer is subject to some constraints. Equation (1.2) describes the consumer's budget constraint, which can be expressed as follows:

$$P_t(1+\tau_t^c)(C_{R,t}+I_t^P)+\frac{B_{t+1}}{R_t^B}=W_tL_{R,t}(1-\tau_t^l)+R_tU_tK_t^P(1-\tau_t^k)$$
$$-P_tK_t^P\left[\Psi_1(U_t-1)+\frac{\Psi_2}{2}(U_t-1)^2\right]+B_t+\omega_RP_t\,TRANS_t \tag{1.2}$$

where, P_t, W_t, and R_t, and B_t are prices of products, wages, rental cost of capital, and nominal value of Green Bonds respectively. So, an explicit assumption is that the government as well as private institutions sell 'green bonds'[3] as part of climate policy to promote a more sustainable transition of the economy. $I_t^P$and K_t^P are private investment and private capital stock at period t. U_t represents level of utilization of installed capacity and usually, $0 < U_t < 1$, which means that not all the productive capital that is available for use gets utilized due to market imperfections and presence of shocks hitting the economy in different periods. τ_t^c, τ_t^l, and τ_t^k denote the distortionary taxes levied on consumption, labor income, and capital income respectively. Similarly, $\Psi_1 > 1$ and $\Psi_2 > 1$ are functions of U_t. ω_R denotes a fraction of consumers that are Ricardian by nature i.e., they have access to financial markets and therefore, can allocate their consumptions intertemporally whilst the remaining fraction of consumers, $1-\omega_R$, consume just in the current period and not in the future because they are financially constrained. Finally, $TRANS_t$ denotes transfer of income by government to the public at the period *t*.

The law of motion of the private capital tomorrow (K_{t+1}^P) can is given by:

$$K_{t+1}^P=(1-\delta)K_t^P+I_t^P\left[1-\frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P}-1\right)^2\right] \tag{1.3}$$

Where, I_t^P is the gross private investment today, δ is the rate of depreciation of capital which depends on level of utilization of installed capacity, and $\left[1-\frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P}-1\right)^2\right]$ is the adjustment cost function associated with given gross investment. $X > 0$ is a sensitivity parameter for investment adjustment.

Equations (1.1), (1.2), and (1.3) give the following Lagrangian:

[3] https://www.kbn.com/om-oss/

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t & \left\{ \left[\frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{R,t}^{1+\varphi}}{1+\varphi} \right] \right. \\
& -\lambda_{R,t} \left[P_t (1+\tau_t^c)\left(C_{R,t} + I_t^P\right) + \frac{B_{t+1}}{R_t^B} - W_t L_{R,t}\left(1 - \tau_t^l\right) \right. \\
& -R_t U_t K_t^P \left(1 - \tau_t^k\right) + P_t K_t^P \left[\Psi_1 (U_t - 1) + \frac{\Psi_2}{2} (U_t - 1)^2 \right] \\
& \left. -B_t - \omega_R P_t TRANS_t \right] \\
& \left. -Q_t \left[K_{t+1}^P - (1-\delta) K_t^P - I_t^P \left[1 - \frac{\chi}{2} \left(\frac{I_t^P}{I_{t-1}^P} - 1 \right)^2 \right] \right] \right\}
\end{aligned}
\tag{1.4}
$$

where Q_t is a Lagrange multiplier known as Tobin's Q.[4] The symbol $\lambda_{R,t}$ is Ricardian household's Lagrange multiplier.

The first order conditions will be:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_{R,t}} = & \left(C_{R,t} - \phi_c C_{R,t-1}\right)^{-\sigma} - \lambda_{R,t} P_t (1+\tau_t^c) \\
& -\phi_c \beta \left(E_t C_{R,t+1} - \phi_c C_{R,t}\right)^{-\sigma} = 0
\end{aligned}
\tag{1.5}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{t+1}^P} = & \beta E_t \{ \lambda_{R,t+1} R_{t+1} U_{t+1} \left(1 - \tau_{t+1}^k\right) - \beta \lambda_{R,t+1} P_{t+1} \\
& \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \\
& -Q_t + \beta Q_{t+1} (1-\delta) \} = 0
\end{aligned}
\tag{1.6}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial U_t} = & \lambda_{R,t} R_t K_t^P \left(1 - \tau_t^k\right) - \lambda_{R,t} P_t K_t^P \Psi_1 \\
& -\lambda_{R,t} P_t K_t^P \Psi_2 (U_t - 1) = 0
\end{aligned}
\tag{1.7}
$$

[4] Tobin's Q is calculated as the market value of the firm's assets divided by the replacement cost.

$$\frac{\partial \mathcal{L}}{\partial I_t^P} = -\lambda_{R,t} P_t (1+\tau_t^c) + Q_t \left[1 - \frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)^2 - \chi \frac{I_t^P}{I_{t-1}^P}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)\right] + \chi\beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}^P}{I_t^P}\right)^2 \left(\frac{I_{t+1}^P}{I_t^P} - 1\right)\right] = 0 \tag{1.8}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = -\frac{\lambda_{R,t}}{R_t^B} + \beta E_t \lambda_{R,t+1} = 0 \tag{1.9}$$

From equation (1.5),

$$\lambda_{R,t} = \frac{\left(C_{R,t} - \phi_c C_{R,t-1}\right)^{-\sigma}}{P_t(1+\tau_t^c)} - \phi_c \beta \frac{\left(E_t C_{R,t+1} - \phi_c C_{R,t}\right)^{-\sigma}}{P_t(1+\tau_t^c)} \tag{2.0}$$

From equation (1.6), one can calculate Tobin's Q

$$Q_t = \beta E_t \Big\{ (1-\delta) Q_{t+1} + \lambda_{R,t+1} R_{t+1} U_{t+1} \left(1 - \tau_{t+1}^k\right) - \lambda_{R,t+1} P_{t+1} \left[\Psi_1 (U_{t+1} - 1) + \frac{\Psi_2}{2} (U_{t+1} - 1)^2 \right] \Big\} \tag{2.1}$$

From equation (1.7), one can get real return to capital.

$$\frac{R_t}{P_t} = \left(\frac{1}{1-\tau_t^k}\right) [\Psi_1 + \Psi_2 (U_t - 1)] \tag{2.2}$$

From equation (1.8),

$$\lambda_{R,t} P_t (1+\tau_t^c) - Q_t \left[1 - \frac{\chi}{2}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)^2 - \chi \frac{I_t^P}{I_{t-1}^P}\left(\frac{I_t^P}{I_{t-1}^P} - 1\right)\right] = \chi\beta E_t \left[Q_{t+1} \left(\frac{I_{t+1}^P}{I_t^P}\right)^2 \left(\frac{I_{t+1}^P}{I_t^P} - 1\right)\right] \tag{2.3}$$

From equation (1.9),

$$\frac{\lambda_{R,t}}{R_t^B} = \beta E_t \lambda_{R,t+1} \tag{2.4}$$

3.1.2.2 Non-Ricardian households

Non-Ricardian households do not have access to financial borrowing. They are supposed to have income which they consume on the current period and there is no borrowing. These households do not meet the so-called Ricardian equivalence – which states that households are forward looking and base their consumptions not just on the current period's income but also on the expected income. To state differently, the non-Ricardian agent's current consumption must equal their current income. Accordingly, they seek to maximize utility given by the following expression:

$$\max_{C_{NR,t}} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{NR,t}^{1+\varphi}}{1+\varphi} \right] \tag{2.5}$$

subject to,

$$P_t(1+\tau_t^c)C_{NR,t} = W_t L_{NR,t}\left(1-\tau_t^l\right) + (1-\omega_R)P_t \text{ TRANS }_t \tag{2.6}$$

where, symbols have usual meanings as already mentioned. The subscript NR stands for the non-Ricardian agents and $1-\omega_R$ is a scaling factor showing the proportion of agents who are non-Ricardian by nature.

Using Lagrangian:

$$\begin{aligned} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \left[\frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{NR,t}^{1+\varphi}}{1+\varphi} \right] \\ & -\lambda_{NR,t}\left[P_t(1+\tau_t^c)C_{NR,t} - W_t L_{NR,t}\left(1-\tau_t^l\right) \right. \\ & \left. -(1-\omega_R)P_t TRANS_t\right]\Bigg\} \end{aligned} \tag{2.7}$$

The first order condition:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_{NR,t}} = & \left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{-\sigma} - \lambda_{NR,t} P_t (1+\tau_t^c) \\ & -\phi_c \beta \left(E_t C_{NR,t+1} - \phi_c C_{NR,t}\right)^{-\sigma} = 0 \end{aligned} \tag{2.8}$$

Solving equation (2.8) for λ,

$$\lambda_{NR,t} = \frac{\left(C_{NR,t} - \phi_c C_{NR,t-1}\right)^{-\sigma}}{P_t(1+\tau_t^c)} - \phi_c \beta \frac{\left(E_t C_{NR,t+1} - \phi_c C_{NR,t}\right)^{-\sigma}}{P_t(1+\tau_t^c)} \tag{2.9}$$

3.1.2.3 Wage Definition

Both Ricardian and non-Ricardian agents share common property in terms of labor supply and definition of the wage, so that x = {R , NR}. They aim at maximizing wages represented by:

$$\max_{W_{j,t}^{*}} E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left\{ -\frac{1}{1+\varphi}\left[L_{x,t+i}\left(\frac{W_{t+i}}{W_{j,t}^{*}}\right)^{\psi_W}\right]^{1+\varphi} \right. \\ \left. +\lambda_{x,t+i}\left[W_{j,t}^{*}L_{x,t+i}\left(\frac{W_{t+i}}{W_{j,t}^{*}}\right)^{\psi_W}\left(1-\tau_{t+i}^{l}\right)\right]\right\} \tag{3.0}$$

where θ_W denotes the fraction of households (labor suppliers) who can readjust wages as expectations change, $0 < \beta < 1$ is still the discount factor, and L is how much labor the household chooses to supply. Presence of θ_W in the expression that reveals Calvo-wage setting mechanism since the model assumes wage rigidities. Intuitively, the sum of infinite geometric series is given by:

$$\sum_{i=0}^{\infty} (\beta\theta_W)^i = \frac{1}{1-\beta\theta_W}$$

Partial differentiation of equation (3.0) with respect to L gives FOC as:

$$E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \{\psi_W \left[L_{x,t+i}\left(\frac{W_{t+i}}{W_{j,t}^{*}}\right)^{\psi_W}\right]^{\varphi} L_{x,t+i}\left(\frac{W_{t+i}}{W_{j,t}^{*}}\right)^{\psi_W}\frac{1}{W_{j,t}^{*}} \\ \left. +(1-\psi_W)\lambda_{x,t+i}L_{x,t+i}\left(\frac{W_{t+i}}{W_{j,t}^{*}}\right)^{\psi_W}\left(1-\tau_{t+i}^{l}\right)\right\} = 0$$

On simplifying,

$$E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left\{\psi_W L_{x,j,t+i}^{\varphi}\frac{1}{W_{j,t}^{*}} + (1-\psi_W)\lambda_{x,t+i}\left(1-\tau_{t+i}^{l}\right)\right\} = 0 \tag{3.1}$$

With a little manipulation of equation (3.1) gives the optimal wage by the households $W_{j,t}^{*}$:

$$W_{j,t}^{*} = \left(\frac{\psi_W}{\psi_W - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\frac{L_{x,j,t+i}^{\varphi}}{\lambda_{x,t+i}\left(1-\tau_{t+i}^{l}\right)}\right] \tag{3.2}$$

Thus, optimal wages for Ricardian HHs will be:

$$W_{j,t}^{*} = \left(\frac{\psi_W}{\psi_W - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\frac{L_{R,j,t+i}^{\varphi}}{\lambda_{R,t+i}\left(1-\tau_{t+i}^{l}\right)}\right] \tag{3.3}$$

and for the non-Ricardian agents will be:

$$W_{j,t}^{*} = \left(\frac{\psi_W}{\psi_W - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta_W)^i \left[\frac{L_{NR,j,t+i}^{\varphi}}{\lambda_{NR,t+i}\left(1-\tau_{t+i}^{l}\right)}\right] \tag{3.4}$$

Finally, the Calvo-rule wage setting gives the following wage rate:

$$W_t = \left[\theta_W W_{t-1}^{1-\psi_W} + (1-\theta_W) W_t^{*1-\psi_W}\right]^{\frac{1}{1-\psi_W}} \tag{3.5}$$

The aggregated values for C_t and L_t are given by average of Ricardian and non-Ricardian agents:

$$C_t = \omega_R C_{R,t} + (1-\omega_R) C_{NR,t} \tag{3.6}$$

$$L_t = \omega_R L_{R,t} + (1-\omega_R) L_{NR,t} \tag{3.7}$$

3.1.2.4 Firms

I assume, as several others have done before me, two types of firms within the standard New-Keynesian DSGE literature – final goods and intermediate goods producing firms. The final-goods firm acts as a representative firm (unique) and sells aggregated products under a competitive environment. On the other hand, the intermediate goods firms operate under monopolistic competition and accordingly they can affect prices set by the market. However, the market for factors of production remains competitive. This assumption is by default a standard norm in NK-DSGE literature. Instinctively, perfect competition ensures greater efficiency in product market.

The aggregation technology of the unique final producer (retailer) is given by Dixit-Stiglitz aggregator (Dixit & Stiglitz, 1977) which can be expressed as:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj\right)^{\frac{\psi}{\psi-1}} \tag{3.8}$$

where, Y_t is the total output associated with a retailer that assembles each wholesale good $Y_{j,t}$ at period *t* where $j \in [0, 1]$ and $\psi > 1$ is the elasticity of substitution between wholesale goods. The objective here is to maximize profits given total production summed over each good j. Assuming that the demand function is given, the expression for maximization of profit is represented by:

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{3.9}$$

where, P_t is the nominal price level of the retail goods whereas $P_{j,t}$ is the price of a wholesale good. Substituting the value of Y_t from equation (3.8) in equation (3.9) will give:

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}} - P_{j,t} \int_0^1 Y_{j,t} dj \tag{4.0}$$

Differentiating equation (4.0) with respect to $Y_{j,t}$ and then equating to 0 to get the FOC:

$$\frac{\psi}{\psi-1} P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{\psi}{\psi-1}-1} \frac{\psi-1}{\psi} Y_{j,t}^{\frac{\psi-1}{\psi}-1} - P_{j,t} = 0$$

Some mathematical manipulation now gives:

$$P_t \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{1}{\psi-1}} Y_{j,t}^{\frac{-1}{\psi}} - P_{j,t} = 0$$

Note that equation (3.8) can also take the following form:

$$Y_t^{\frac{1}{\psi}} = \left(\int_0^1 Y_{j,t}^{\frac{\psi-1}{\psi}} dj \right)^{\frac{1}{\psi-1}}$$

Inserting this in the expression above, it turns to:

$$P_t Y_t^{\frac{1}{\psi}} Y_{j,t}^{\frac{-1}{\psi}} - P_{j,t} = 0$$

which leads to:

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{j,t}}\right)^{\psi} \tag{4.1}$$

Equation (4.1) represents the demand for wholesale good which depends inversely on relative price $\left(\frac{1}{\frac{P_{j,t}}{P_t}}\right)$ and directly on aggregate demand Y_t.

Now, substituting equation (4.1) into equation (3.8) and solving for the price of the final goods P_t, will result in:

$$Y_t = \left\{\int_0^1 \left[Y_t \left(\frac{P_t}{P_{j,t}}\right)^{\psi}\right]^{\frac{\psi-1}{\psi}} dj\right\}^{\frac{\psi}{\psi-1}}$$

$$Y_t = Y_t P_t^{\psi} \left\{\int_0^1 \left[\left(\frac{1}{P_{j,t}}\right)^{\psi}\right]^{\frac{\psi-1}{\psi}} dj\right\}^{\frac{\psi}{\psi-1}}$$

$$P_t^{\psi} = \left[\int_0^1 \left(P_{j,t}^{\psi}\right)^{\frac{\psi-1}{\psi}} dj\right]^{\frac{\psi}{\psi-1}}$$

$$P_t = \left[\int_0^1 P_{j,t}^{1-\psi} dj\right]^{\frac{1}{1-\psi}} \tag{4.2}$$

Equation (4.2) represents the mark-up rule for the retail products. In other words, it is a price aggregator.

As mentioned above, due to differentiated nature of the products, intermediate or wholesale firms exercise a certain degree of market power and are price setters. They sell their products to retail firms. If one rules out the existence of fixed cost, average variable cost equals average total cost which also coincides with the marginal cost. Further, assumption is that average production cost remains constant regardless of scale of production resulting in constant returns to scale.

The retail firms optimize pricing strategy in two stages. The first stage involves cost minimization under the given technology and in the second stage, there will be price readjustment.

They minimize production costs i.e.,

$$\min_{L_{j,t},K_{j,t}} W_t L_{j,t} + R_t K_{j,t} \tag{4.3}$$

subject to given Cobb-Douglas production technology:

$$Y_{j,t} = A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \tag{4.4}$$

where α and 1- α are the elasticity coefficients of capital and labor respectively and law of motion of productivity (A_t) is defined as:

$$\log\, A_t = (1-\rho_A)\log\, A_{ss} + \rho_A \log\, A_{t-1} + \varepsilon_t \tag{4.5}$$

where A_{ss} is productivity at steady state, $|\rho_A| < 1$ autoregressive parameter of productivity that ensures steady state, and $\epsilon_t \sim N(0, \sigma_A)$ is the productivity shock.

The production Lagrangian now becomes:

$$\mathcal{L} = W_t L_{j,t} + R_t K_{j,t} + \mu_{j,t}\left(Y_{j,t} - A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}\right) \tag{4.6}$$

where $\mu_{j,t} = MC_{j,t}$ (Marginal Cost of j^{th} firm at period *t*) is the Lagrange multiplier. It will follow that $MC_{j,t} = MC_t = P_t$ as marginal cost solely depends on prices of factors of production under perfect competition.

Obtaining the FOC by taking partial derivatives of $\mathcal{L}$ w.r.t labor and capital gives rise to equations (4.7), and (4.8) respectively.

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = W_t - (1-\alpha)\mu_{j,t} A_t K_{j,t}^{\alpha} L_{j,t}^{\alpha} = 0 \tag{4.7}$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t}} = R_t - \alpha \mu_{j,t} A_t K_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha} = 0 \tag{4.8}$$

Further simplifications of equation (4.7) and (4.8) result in demand for labor and capital by wholesale firm j at period t.

$$L_{j,t} = (1-\alpha)MC_{j,t}\frac{Y_{j,t}}{W_t} \tag{4.9}$$

$$K_{j,t} = \alpha MC_{j,t}\frac{Y_{j,t}}{R_t} \tag{5.0}$$

Intuitively,

$$MC_{j,t} = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{5.1}$$

Equation (5.1) also meets the assumption above regarding relationship between marginal and average total cost i.e.,

$$MC_{j,t} = \frac{CT_{j,t}}{Y_{j,t}}$$

However, the basic tenet of New-Keynesian models lies in the nominal price rigidity. To model price-sticky behavior of firms, the Calvo-pricing framework is a convenient choice. Accordingly, firms are assumed to have a random probability of adjusting their prices as expectations of the agents change. It means that some firms are able to change prices whereas others remain "sticky" (Calvo, 1983). According to the Calvo rule, a fraction $0 < \theta < 1$ of firms are randomly selected which can update their prices in each period t. The rest of the firms retain their prices as before so that $P_{j,t} = P_{j,t-1}$.

So, the problem of price-adjusting firms $1-\theta$ will be to maximize discounted profits defined as sum of discounted revenues over discounted total costs (TC) in the given period which can occur through maximization of the wholesale price $P^*_{j,t}$.

$$\underset{P^*_{j,t}}{\mathrm{Max}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left(P^*_{j,t}Y_{j,t+i} - CT_{j,t+i}\right) \tag{5.2}$$

From equation (4.1), it is possible to express $Y_{j,t+i}$ in terms of Y_{t+i} and relative prices as below:

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[P^*_{j,t}Y_{t+i}\left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\psi} - Y_{t+i}\left(\frac{P_{t+i}}{P^*_{j,t}}\right)^{\psi} MC_{j,t+i}\right] \tag{5.3}$$

Partially differentiating equation (5.3) w.r.t. $P^*_{j,t}$ gives:

$$E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[(1-\psi)Y_{j,t+i} + \psi \frac{Y_{j,t+i}}{P^*_{j,t}} MC_{j,t+i}\right] = 0$$

Solving for $P^*_{j,t}$:

$$P^*_{j,t} = \left(\frac{\psi}{\psi - 1}\right) E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{j,t+i} \tag{5.4}$$

$P^*_{j,t}$ is an optimal price, and it is same for all the price setters in all periods *t*. Now applying the markup rule from equation (4.2):

$$P_t^{1-\psi} = \left[jP_{t-1}^{1-\psi}\right]_0^{\theta} + \left[jP_t^{*1-\psi}\right]_{\theta}^{1}$$

$$P_t^{1-\psi} = \left[jP_{t-1}^{1-\psi}\right]_0^{\theta} + \left[jP_t^{*1-\psi}\right]_{\theta}^{1}$$

$$P_t^{1-\psi} = \theta P_{t-1}^{1-\psi} + (1-\theta)P_t^{*1-\psi}$$

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta)P_t^{*1-\psi}\right]^{\frac{1}{1-\psi}} \tag{5.5}$$

Equation (5.5) represents the general price level. There is a continuum of firms, and whichever group has the ability to alter its prices (and which group does not) is chosen randomly, regardless of when the firms last altered their prices. Accordingly, the distribution of prices across firms is unchanged between periods.

Equations (4.3) – (5.5) represent systems of equations of the firms whose technology basically depends on labor and capital. However, to model a climate setup, I include government not only as the regulator but also as the supplier of public capital (Barro, Mankiw, & Sala-i-Martin, 1992; Campiglio, 2014; Futagami, Morita, & Shibata, 1993; Shioji, 2001) in the production process. I explicitly assume that government participates in production of goods and services that are less carbon intensive in nature (green projects). I consider this assumption as a necessary condition to realize climate targets envisioned both in national climate plans as well as in international climate negotiations. In this context, the technology of the intermediate goods producers will be:

$$Y_{j,t} = A_t K_{j,t}^{P\ \alpha_1} L_{j,t}^{\alpha_2} K_{j,t}^{G\ \alpha_3}$$

where $K^G_{j,t}$ is the public capital used by the *j^th^* intermediate firm at period *t*; $\alpha_1 + \alpha_2 + \alpha_3 = 1$, implying a CRS technology – just to ensure linear relationship between inputs and output – with productivity following first order autoregressive process around the steady state as in the equation (4.5). Hence, everything remains the same other than simply considering the three-inputs production function instead of two-inputs. However, inclusion of public capital has consequences on demand for labor *($L_{j,t}$),* private capital *($K^P_{j,t}$),* and marginal costs *($MC_{j,t}$)* as represented by equations (5.6), (5.7), and (5.8) below:

$$L_{j,t} = \alpha_2 MC_{j,t} \frac{Y_{j,t}}{W_t} \tag{5.6}$$

$$U_t K^P_{j,t} = \alpha_1 MC_{j,t} \frac{Y_{j,t}}{R_t} \tag{5.7}$$

$$MC_{j,t} = \frac{1}{A_t K^{G\alpha_3}_{j,t}} \left(\frac{W_t}{\alpha_2}\right)^{\alpha_2} \left(\frac{R_t}{\alpha_1}\right)^{\alpha_1} \tag{5.8}$$

3.1.2.5 The Government

The government represents two authorities – fiscal and monetary. The fiscal authority is responsible for administering taxes and offering government bonds. By taxes, I assume both lumpsum tax and distortionary taxes levied on carbon intensive firms' capital (τ^k_t), investment goods, and their products (τ^c_t). It is so because the government is supposed to have profound orientation towards net-zero targets. It implies that the government administers no taxes or very nominal taxes to green initiatives at all levels from production to final consumption. The tax on individual income (τ^l_t) will be, however, irrespective of the nature of firms – whether carbon-intensive or less carbon-intensive since households are the sole suppliers of labor services, and thus, it does not really matter to which firms they provide labor as long as the consumers pay income taxes. In addition, government subsidies climate friendly initiatives directly through transfers payments $TRANS_t$.

The government budget constraint, therefore, will be:

$$\frac{B_{t+1}}{R^B_t} - B_t + T_t = P_t G_t + P_t I^G_t + P_t TRANS_t \tag{5.9}$$

where B_t is nominal value of bonds, $\frac{B_{t+1}}{R_t^B}$ represents discounted value of nominal bonds in the following period, and T_t is the total tax revenue. P_t represents general price level and G_t, I_t^G, and $TRANS_t$ denote government consumption, public investment, and government transfer payments to climate friendly activities. The left-hand side of the equation (5.9) shows government revenues whereas the right-hand side shows the government expenditure. The total tax revenue (T_t) is composed of:

$$T_t = \tau_t^c P_t(C_t + I_t^P) + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta) K_t^P \tag{6.0}$$

Following (Junior, 2016), the fiscal policy rule can be written as:

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}}\right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1}P_{t-1}} \frac{Y_{ss}P_{ss}}{B_{ss}}\right)^{(1-\gamma_Z)\phi_Z} s_t^Z \tag{6.1}$$

Where $Z = \{G_t, I_t^G, TRANS_t, \tau_t^c, \tau_t^l, \tau_t^k\}$ represents fiscal policy instruments that government uses both to generate revenue and to disburse it. s_t^Z is the fiscal policy shock which follows first order autoregressive process as:

$$\log S_t^Z = (1 - \rho_Z) \log S_{ss}^Z + \rho_Z \log S_{t-1}^Z + \varepsilon_{Z,t} \tag{6.2}$$

with $\varepsilon_{Z,t}$ having normal distribution.

Additionally, the law of motion of public capital is given by:

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G \tag{6.3}$$

Where δ_G is the depreciation rate of public capital.

On the other hand, the monetary authority or the central bank administers interest rate (r_t) basically to stabilize the economy and to achieve economic growth. The central bank does so by following Taylor rule (Taylor, 1993) formula (in the original form):

$$r_t = \theta + \beta \pi_t + \varphi x_t \tag{6.4}$$

where $\theta = r^* + (1 - \beta)\pi^*$, r_t is the interest rate targeting, π_t denotes the average annual inflation rate, π^* is targeted inflation, x_t is output gap which is the difference between actual and potential output, and r^* is the interest rate at equilibrium. When $\beta > 1$ and $\varphi > 0$, the real interest rate adjusts itself to stabilize inflation and the output. However, if $\beta < 1$, some inflation is acceptable. Exactly similar logic applies to φ as well, that must always be strictly positive for the Taylor rule to generate a stabilization in the economy. For the purpose of modeling, the following Taylor rule principle is utilized:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{ss}}\right)^{\gamma_Y}\right]^{(1-\gamma_R)} S_t^m \tag{6.5}$$

where R_t^B is the real bonds rate considered as real rate of interest, γ_π and γ_Y are sensitivity parameters of inflation and output respectively, γ_R is the interest rate smoothing parameter, superscripts ss denotes the steady state values of the respective variables, and S_t^m is the monetary policy shock which follows first order autoregressive process defined by:

$$\log S_t^m = (1-\rho_m)\log S_{ss}^m + \rho_m \log S_{t-1}^m + \varepsilon_{m,t} \tag{6.6}$$

with $\varepsilon_{m,t}$ being normally distributed.

3.1.2.6 Market Clearance

The economy attains the equilibrium condition when aggregate income (Y_t) equals aggregate expenditure in the form of consumption (C_t), private investment (I_t^P), public investment (I_t^G), and the government consumption (G_t) i.e.,

$$Y_t = C_t + I_t^P + I_t^G + G_t \tag{6.7}$$

3.1.2.7 Emissions

I consider the simplistic emission model proposed and estimated by Nordhaus and Yang (1996) and Barrage and Nordhaus (2023) that directly links CO_2 emissions with level of output in economy. Moreover, I assume the output as contributed specifically from brown (carbon-intensive) technology though there exists a certain scale of green GDP in Norway produced by low-carbon-intensive and/or zero-carbon-intensive firms. It is because it's not easy to get segregated data on GDP labeled as green or non-green in the National Account database.

Following Nordhaus and Yang (1996), CO_2 emission (E) produced in country *j* at period *t* can be expressed as:

$$E_{j,t} = [1 - \mu_{j,t}]\, \sigma_{E,t} Y_{j,t} \tag{6.8}$$

where $\mu_{j,t}$ is the utility of per capita consumption, and $\sigma_{E,t}$ denotes emission-output ratio, or emission intensity of country j at period t. It is to be noted that authors considered both parameters $0 \leq \mu_{j,t} \leq 1$ and $\sigma_E > 0$ as time varying, and they in fact change as time elapses. However, for simplicity, I consider emission intensity (σ_E) fixed over a specified range of period and calculate as an average from within that range (1990-2022; as it reflects my sample period). The authors themselves treat μ as fixed in their updated version[5] of the Dynamic Integrated Climate-Economy (DICE) model.

3.2 Conceptual Framework

I design a simple four-panel diagram to grasp how macroeconomic variables interact with emissions and vice-versa. I borrow the basic concept from IS-LM model and extend to emission-axis assuming linear marginal abatement cost (MAC) and marginal cost of damage (MD) curves. Next, I delve into analysis of macroeconomic dynamics vis-à-vis climate emission reduction, the model presented below is based on static IS-LM framework as it is not unusual to extrapolate (Bénassy, 2007; Gerlach, 2017; Hicks, 2017) static IS-LM concept into the realm of New Keynesian paradigm. In the similar manner, following Romstad (2016), it is also relevant to extend the static MAC-MD framework to dynamic efficiency considerations.

The figure below illustrates responses of the key macroeconomic variables due to change in emission through climate policy. Starting from the top-right panel, the economy equilibrates initially at point e_0 where IS curve intersects LM curve corresponding to output level y_0 and interest rate r_0 ensuring both goods market and money market equilibrium. From a microeconomic perspective, all the economic agents including households and firms, as well as fiscal and monetary authorities optimize their problems at e_0. However, emissions at e_0 will be at their unregulated (highest) level (M_0) along the emission axis as taxes are zero. These unregulated emissions are not desirable and therefore, regulators impose taxes on emissions corresponding to T_1 which is consistent with the static equilibrium in emission trading system (bottom-left) where MAC (M_t*) = MD (M_t*). Introduction of a lumpsum tax T_1 can help reduce emissions from unregulated level M_0 to M_1, but this will result in disequilibrium in goods market and/or money market. Bouncing back to equilibrium in goods market and money market can occur under three different scenarios:

[5] See Barrage, L., & Nordhaus, W. D. (2023). *Policies, Projections, and the Social Cost of Carbon: Results from the DICE-2023 Model*.

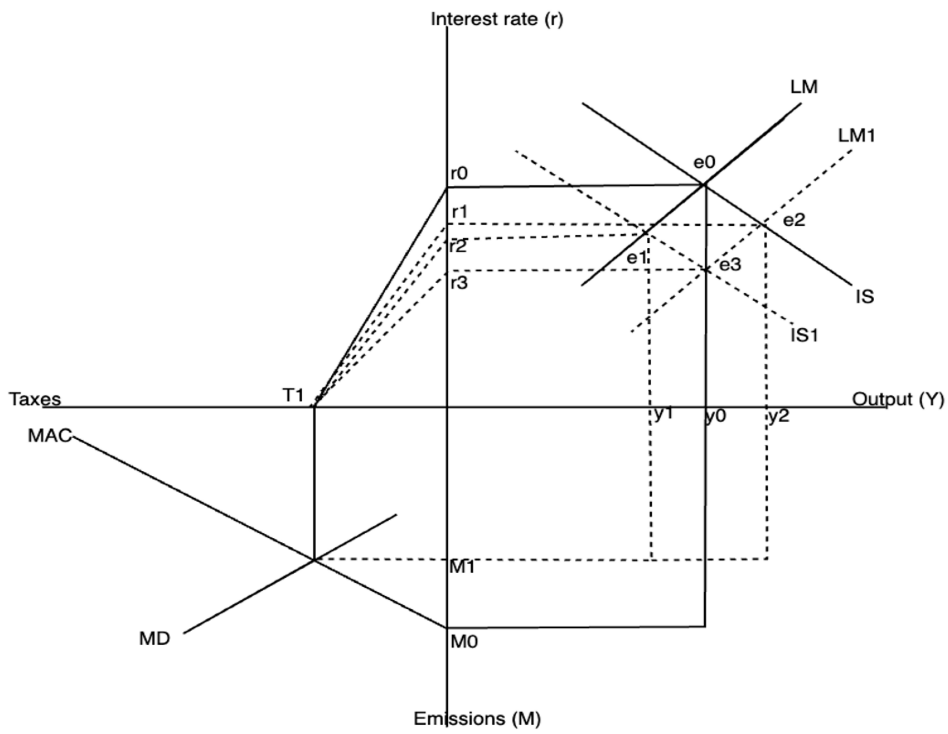


Fig. 3. 2: IS-LM-Emission optimality.

Possibility 1

If the LM curve remains unchanged with increase in the taxes, it is possible that the economy equilibrates at e_1 where output (y_1) and interest rate (r_2) both will be lower than those before intervention. This follows because the increased tax will reduce investment demand (I) [in other words, investment is less than savings] that subsequently shifts IS curve to IS_1.

Possibility 2

Assuming that IS curve remains unchanged with an increase in the taxes, it is likely that economy equilibrates at e_2 where output (y_2) will be higher than before, but interest rate falls to r_1 from r_0. This can happen as an increase in taxes reduces disposable income which further reduces money demand by agents. When money demand (*Md*) is less than the money supply (*MS*) at the prevailing interest rate (r_0), the LM curve shifts to the right as LM_1 where $r_1 < r_0$.

Possibility 3

The next possible equilibrium point may be at e_3 where output remains unchanged. If the increased taxes simultaneously shift IS curve to the left (as IS_1) and LM curve to the right (as LM_1) [scenarios 2 and 3 occurring simultaneously], then $IS_1 = LM_1$ at point e_3 where interest rate will be much lower (r_3), but GDP remains intact.

Of all the possibilities, the equilibrium at e_2 looks very promising as increased tax not only lowers interest rate but also increases the output thereby still having equilibrium in emission trading market. This improves economic well-being when output grows. Decreased interest rate further propels private investment. However, there is inflationary pressure in the economy as money supply exceeds money demand in the economy. This is not desirable for economic stability. Additionally, there are other forces at work (such as productivity shocks) which may hinder the general equilibrium in all the markets over time which necessitates the dynamic equilibrium analysis.

3.3 Empirical Framework

3.3.1 Simulation Strategy

I considered thirty-eight endogenous variables of the model equations for simulation, as (log) deviations from their steady states, using Dynare version 5.5 on MATLAB 2023b. The crucial endogenous variable of interest include level of output (*Y*), private investment (*IP*), government or public investment (*IG*), consumption I, government expenditure (*G*), private capital (*KP*), public capital (*KG*), labor supplied (*L*), rate of return on capital (*R*), wage rate (*W*), marginal costs (*MC*), price level (*P*), interest rate expressed as rate of return on bonds (*RB*), total tax revenue (*T*), transfer income/payment (*TRANS*), and greenhouse gas emission (*E*), among others [here, symbols are not to be confused with the symbols used in previous sections because these symbols are compatible with the Dynare notation and also appear in the results section]. Likewise, I defined seven exogenous variables that contained stochastic shocks. As Dynare performs first-order Taylor approximation of the linear models, it is essential that all the model equations (and variables) are linearized around their steady states.

An endogenous variable x_t will be in the steady state x_{ss} if $E_t x_{t+1} = x_t = x_{t-1} = x_{ss}$ which implies that variable is constant over time in the absence of shocks, i.e., $E(\epsilon_t) = 0$.

I applied Uhlig's method (Uhlig, 1995) to log-linearize the equations. According to this method, it is possible to replace a variable of interest X_t by $X_{ss}e^{\tilde{X}_t}$ where $\tilde{X}_t = \log\ X - \log X_{ss}$. $\tilde{X}_t$ represents the log of deviation of variable of interest from its steady state. The other forms of Uhlig's log linearization techniques are:

$$e^{(\tilde{X}_t + a\tilde{Y}_t)} \approx 1 + \tilde{X}_t + a\tilde{Y}_t$$
$$\tilde{X}_t\tilde{Y}_t \approx 0$$
$$E_t\left[ae^{\tilde{X}_{t+1}}\right] \approx a + aE_t[\tilde{X}_{t+1}]$$

where E_t is the expectation operator, $\tilde{Y}_t = \log\ Y - \log Y_{ss}$, and a is a constant.

3.3.1.1 Parameter Calibration

Before simulating the model, its essential to calibrate parameters. I fixed discount factor to 0.9861 in consistence with social discount rate of 1.4 percent (Stern, 2007)[6]. But later I allowed it to vary when I tried to estimate for Norway. It is about assigning the best possible values to the structural parameters used in the models. I utilized the available literature regarding DSGE modeling to calibrate the parameters. The list of calibrated parameters is presented below:

Table 3. 1: Calibrated parameters

Parameter	Value	Meaning
σ	2	Relative Risk Aversion Coefficient
ϕ	3.5	Marginal Disutility Regarding Supply of Labor
α_1	0.2	Elasticity of Production in Relation to Private Capital
α_2	0.55	Elasticity of Production in Relation to Labor
α_3	$1 - \alpha_1 - \alpha_2$	Elasticity of Production in Relation to Public Capital
β	0.9861	Discount Factor
δ	0.02	Depreciation Rate
θ	0.85	Price Stickiness Parameter
ψ	3.5	Elasticity of Substitution Among Intermediate Goods
θ_W	0.85	Wage Stickiness Parameter

[6] See https://www.regjeringen.no/en/dokumenter/nou-2012-16/id700821/?ch=6.

ψ_W	14	Elasticity of Substitution Between Differentiated Labor
τ_{ssc}	0.12	Tax on Consumption in Steady State
τ_{ssl}	0.12	Tax on Income from Labor in Steady State
τ_{ssk}	0.08	Tax on Income from Capital in Steady State
ω_R	0.4	Participation of Ricardians in Consumption and Labor
ϕ_c	0.8	Habit Persistence
χ	1	Sensitivity of Investments in Relation to Adjustment Cost
Ψ_1	$\frac{1}{\beta-(1-\delta)}$	Sensitivity of Cost of Under-utilization of Installed Capacity 1
Ψ_2	1	Sensitivity of Cost of Under-utilization of Installed Capacity 2
δ_G	0.02	Rate of Depreciation of Public Capital
γ_R	0.79	Interest Rate Persistence
γ_Y	0.16	Sensitivity of Interest Rate in Relation to GDP
γ_π	2.43	Sensitivity of Interest Rate in Relation to Inflation
ϕ_{TRANSss}	0.01	Proportion of Transfers in Relation to GDP
$\phi_{B_{ss}}$	1	Proportion of Public Debt in Relation to GDP
$\phi_{IG_{ss}}$	0.02	Proportion of Public Investment in Relation to GDP
γ_G	0	Public Spending Persistence
γ_{IG}	0.1	Persistence of Public Investment
γ_{TRANS}	0.1	Persistence of Income Transfer
γ_{τ_c}	0	Persistence of Tax on Consumption
γ_{τ_l}	0	Persistence of Tax on Labor Income

γ_{τ_k}	0	Persistence of Tax on Capital Income
ϕ_G	0	Proportion of Public Spending Over GDP
ϕ_{IG}	0.1	Proportion of Public Investment Over GDP
ϕ_{TRANS}	0.1	Proportion of Income Transfer Over GDP
ϕ_{τ_c}	0	Tax on Consumption Over GDP
ϕ_{τ_l}	0	Tax on Labor Income Over GDP
ϕ_{τ_k}	0	Tax on Capital Income Over GDP
μ	0.95	Emission Control Rate, Nordhaus Parameter[7]
σ_E	14.90	Average Emission Intensity for Norway

I used impulse response function (IRF) analysis to explore the responses of key endogenous variables to stochastic shocks, over time. Historical shocks decomposition is another technique which I employed to observe the share of variation on endogenous variables of interest attributed to particular shocks.

Further, I conducted Bayesian estimation of some of the structural parameters and standard error of shocks using the Random Walk Metropolis-Hastings (MH) algorithm. Parameter estimation serves two things. First, it provides the quantification of relationship among variables of interest. Second, as the MH algorithm is a powerful Markov Chain Monte Carlo (MCMC) method, it provides a robust framework for sampling from posterior distribution of the model parameters. It is worth mentioning that the number of observed endogenous variables cannot exceed the total number of shocks. Otherwise, it can lead to identification issues and most often, the system turns out to be singular. The sub-section below offers a detailed description regarding estimation process.

[7] See Barrage, L., & Nordhaus, W. D. (2023). *Policies, Projections, and the Social Cost of Carbon: Results from the DICE-2023 Model.*

3.3.2 Bayesian Estimation of DSGE Models

The DSGE literature provides various approaches to determine the model parameters. They include pure calibration as in Kydland and Prescott (1982), Generalized Methods of Moments (GMM) by Ruge-Murcia (2013), or the full information maximum likelihood by Lindé (2005). Contemporary research on DSGE modeling increasingly relies on Bayesian method for parameter estimation. This surge in popularity of Bayesian method is attributed to its computational strength and practical approach it offers in estimation of parameters. Notably, various central banks including European Central Bank (ECB), Federal Reserve Bank of St. Louis, and others across the world use Bayesian method in DSGE analysis. The central bank of Norway uses Bayesian methods for parameter estimation in its NEMO[8] which is another variant of DSGE modeling.

As opposed to the frequentists' approach which treats model parameters as 'fixed', Bayesian approach considers parameters as 'random' variables. Bayesian technique incorporates the prior information from the distribution of model parameters that stem from previous studies. In case there are mis-specified or wrongly generated parameter distributions through DSGE estimation – which is often the case with DSGE due to its highly stylized modeling techniques – the priors help in re-weighting those parameters so that they do not contradict with the common observations. Thus, at very basic sense, Bayesian approach utilizes the calibrated parameters – as priors – to carry out the Maximum Likelihood Estimation (MLE). MLE enters the into DSGE as we feed real world observations or data into the models. Let the prior density function takes the form:

$$p(\boldsymbol{\theta}_q \mid q)$$

Where q is the model under consideration, $\boldsymbol{\theta}_q$ is the parameter vector of the model, and $p(.)$ denotes the probability density function (pdf). The MLE function, then, can be written as:

$$\mathcal{L}(\boldsymbol{\theta}_q \mid \boldsymbol{y}_T, q)$$

where $\boldsymbol{y}_T$ is vector of observed data available at period T. In a recursive manner, the likelihood function can take the form:

[8] See Kravik, E. M., & Paulsen, K. (2017). A complete documentation of Norges Bank's policy model NEMO. In *Technical report*. Norges Bank.

$$p(\boldsymbol{y}_T \mid \boldsymbol{\theta}_q, q) = p(y_0 \mid \boldsymbol{\theta}_q, q) \prod_{t=1}^{T} p(y_t \mid \boldsymbol{y}_{t-1}, \boldsymbol{\theta}_q, q)$$

where $p(\boldsymbol{\theta})$ is the prior density that is available and $p(\boldsymbol{y}_T \mid \boldsymbol{\theta})$ is likelihood function. As we try to estimate the posterior distribution, we can use Bayes theorem as:

$$p(\boldsymbol{\theta} \mid \boldsymbol{y}_T) = \frac{p(\boldsymbol{\theta}; \boldsymbol{y}_T)}{p(\boldsymbol{y}_T)}$$

which can be generalized as:

$$p(\boldsymbol{y}_T \mid \boldsymbol{\theta}) = \frac{p(\boldsymbol{\theta}; \boldsymbol{y}_T)}{p(\boldsymbol{\theta})} \Leftrightarrow p(\boldsymbol{\theta} \mid \boldsymbol{y}_T) = p(\boldsymbol{y}_T \mid \boldsymbol{\theta})p(\boldsymbol{\theta})$$

Now, we can obtain the posterior density given prior density and likelihood function as:

$$p(\boldsymbol{\theta}_q \mid \boldsymbol{y}_T, q) = \frac{p(\boldsymbol{y}_T \mid \boldsymbol{\theta}_q, q)p(\boldsymbol{\theta}_{q|q})}{p(\boldsymbol{y}_T \mid q)}$$

where $p(\boldsymbol{y}_T \mid q) = \int_{\Theta_q} p(\boldsymbol{\theta}_q; \boldsymbol{y}_T \mid q)d\boldsymbol{\theta}_q$ is the marginal density of observations given our model(s). The posterior kernel density or non-normalized posterior density is directly proportional to the posterior density, i.e.,

$$p(\boldsymbol{\theta}_q \mid \boldsymbol{y}_T, q) \propto p(\boldsymbol{y}_T \mid \boldsymbol{\theta}_q, q)p(\boldsymbol{\theta}_{q|q}) \equiv \mathcal{K}(\boldsymbol{\theta}_q \mid \boldsymbol{y}_T, q)$$

where $\mathcal{K}$ is the kernel and $\propto$ is the sign of proportionality. We can generate the posterior moments using this relation. To obtain likelihood, Kalman filter is used to simulate values of the posterior kernel using Markov Chain Monte Carlo (MCMC) method with the help of Metropolis-Hastings (MH) algorithm.

In the context of DSGE, the first order equilibrium conditions can be represented as:

$$\begin{aligned} E_t\{f(x_{t+1}, x_t, x_{t-1}, u_t)\} &= 0 \\ E(u_t) &= 0 \\ E(u_t u_t') &= \Sigma_u \end{aligned}$$

where is x represents vector of endogenous variables, and u_t is the vector of exogenous (stochastic) shocks. These vectors can take on any dimensions $m * n$ where $m > 0$ and $n > 0$. Further, we define the policy function:

$$x_t = h(x_{t-1}, u_t)$$

This policy function gives solution to the system today ($t = 0$) that is dependent on the past state of the system and the shocks today. The solution of DSGE models will be given by the following expressions:

$$\begin{aligned} x_t^* &= G\bar{x}(\boldsymbol{\theta}) + G\hat{x} + H(\boldsymbol{\theta})y_t + \epsilon_t \\ \hat{x} &= h_x(\boldsymbol{\theta})\hat{x}_{t-1} + h_u(\boldsymbol{\theta})u_t \\ E(\epsilon_t\epsilon_t') &= \mathrm{R}(\boldsymbol{\theta}) \\ E(u_tu_t') &= S(\boldsymbol{\theta}). \end{aligned}$$

where $\bar{x}$ is the steady state value of endogenous variables, is vector of structural parameters that are supposed to be estimated, and $\hat{x}_t$ denotes the percentage deviation of variables from respective steady states if x is in logarithmic form, and absolute deviation from steady states if x is in Levels. x_t^* denotes observed real world series with and error ϵ_t. $H(\boldsymbol{\theta})y_t$ is the trend component that is subject to structural parameters $\boldsymbol{\theta}$.

The model equations can be (and they are in this thesis) inherently non-linear in both parameters and endogenous and exogenous variables. However, we can use log-linearization using Uhlig's method (Uhlig, 1995) to make them linear. In the next step, we can estimate likelihood of the system. To do this, we use Kalman filter (Kalman, 1960) which is the 'linear prediction error algorithm' in computer. For $t = 0, 1, \dots, T$ with initial values x_1 and P_1 given, the Kalman recursive model is given by:

$$\begin{aligned} v_t &= x_t^* - \bar{x}^* - G\hat{x}_t - Hx_t \\ F_t &= GP_tG' + R \\ L_t &= h_xP_th_t'F_t^{-1} \\ \hat{x}_{t+1} &= h_x\hat{x}_t + L_tv_t \\ P_{t+1} &= h_xP_t(h_x - L_tG)' + h_uSh_u'. \end{aligned}$$

Given this, the log-likelihood function can be expressed as follow:

$$\ln \mathcal{L}(\boldsymbol{\theta} \mid \boldsymbol{x_T^*}) = -\frac{Tk}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} |F_t| - \frac{1}{2}v_t'F_t^{-1}v_t$$

where $\boldsymbol{\theta} = \theta, T(\theta)$ and $S(\theta)$ are to be estimated, and $\boldsymbol{x}_T^*$ denotes the vector of observed endogenous variables x_t in the observation equation. The log-likelihood expressed above, is used to obtain the posterior parameter distribution. We can denote the log posterior kernel given by:

$$\ln \mathcal{K}(\boldsymbol{\theta} \mid \boldsymbol{x}_T^*) = \ln \mathcal{L}(\boldsymbol{\theta} \mid \boldsymbol{x}_T^*) + \ln p(\boldsymbol{\theta})$$

where both the terms on the right-hand side are known – the first term is obtained after applying the Kalman filter and the second term is from our priors. A key aspect of Bayesian estimation is obtaining the mode of the posterior distribution. Nevertheless,'this isn't straightforward due to the non-Gaussian nature of the log-likelihood function with respect to θ because θ comes from state-space equations. The Metropolis-Hastings algorithm, which is a rejection sampling technique, is commonly applied for this purpose. The MH algorithm is handled by Dynare within the MATLAB environment.

The algorithm involves four steps:

1. Choose a starting point – that usually involves the posterior mode.
2. Draw a proposal from a jumping distribution or proposal distribution from the parameter space.
3. Compute the acceptance ratio based on posterior kernel.
4. Accept or reject the proposal and update as needed.

The acceptance rule allows exploration of the entire posterior distribution. The scaling in the jumping distribution is critical – if it is very small, the chain will mix slowly, and if it is very high, it may spend quite a bit of time in low-probability regions. Hence, it is important to tune up or down the scaling factor to obtain a suitable range of acceptance ratio which is generally between 25% and 33% [9].

3.3.3 Identification Strategy

DSGE modeling depends on Blanchard-Kahn condition to address issues related to the identification, existence, and convergence of solution to the system of equations. For instance, a linearized model in state-space representation takes the following form:

$$E\begin{bmatrix} \mathrm{z_t} \\ \mathrm{E_t x_{t+1}} \end{bmatrix} = \mathrm{A_0}\begin{bmatrix} \mathrm{z_{t-1}} \\ \mathrm{x_t} \end{bmatrix} + D\mathrm{r_{n,t}} + G\varepsilon_\mathrm{t} \qquad \text{(BK.1)}$$

Where x_t is vector of forward-looking variables, z_t is vector of predetermined variables, E, A_0, D, and G are matrices and ε_t is shock vector. One can define $r_{n,t}$ as:

$$r_{n,t} = K\begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix}$$

[9] See https://www.dynare.org/manual/the-model-file.html#displaying-and-saving-results.

where value of K differs depending on the model.

So, the equation (BK.1) becomes:

$$E\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = [A_0 + DK]\begin{bmatrix} Z_{t-1} \\ x_t \end{bmatrix} + G\varepsilon_t$$

Replacing $[\ A_0 + DK\ ]$ by A, gives:

$$E\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = A\begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + G\varepsilon_t$$

which results in,

$$\begin{bmatrix} z_t \\ E_t x_{t+1} \end{bmatrix} = \bar{A}\begin{bmatrix} z_{t-1} \\ x_t \end{bmatrix} + \bar{G}\varepsilon_t$$

Where, $\bar{A} = E^{-1}A$ and $\bar{G} = E^{-1}G$.

Hence, agents make rational expectations based on a set of information $\{z_s,\ x_{s+1},\ \varepsilon_s\}$, where $s \leq t$.

The existence of a unique and stable equilibrium in DSGE modeling is defined by eigenvalues corresponding to the matrix (A_0 + DK). According to Blanchard and Kahn (1980), when the number of eigenvalues exceeding absolute value of 1 equals the number of forward-looking variables, the system has a unique solution and is stable on saddle paths. This satisfies the rank condition. The fig. 3.5 reveals this scenario where a unique solution exists, and the system has a stable saddle path.

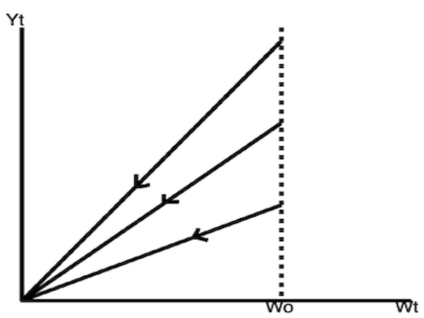


Fig. 3. 3: Multiple solutions with no unique path.

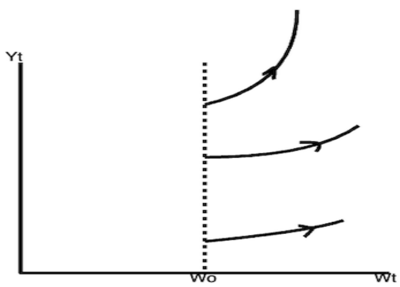


Fig. 3. 4: No solutions with divergent paths.

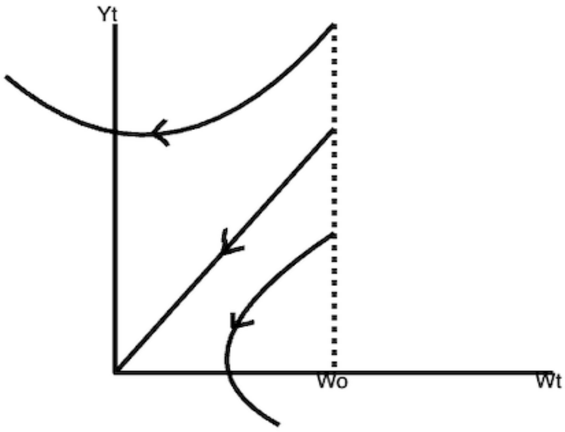


Fig. 3. 5: Unique solution with unique saddle path

On the contrary, indeterminacy occurs in the system if the number of eigenvalues with the absolute value greater than 1 is less than the number of forward-looking variables, in which case there will be several stable roots as in the figure 3.3. Further, fig. 3.4 shows the possibility of existence of many unstable roots illustrating that equilibrium paths are not only explosive but also has no solution.

4. Data Acquisition and Processing

I utilized four time series data in this thesis. The time series included real GDP (Y), returns on long-term (10-years) government bonds (RB) – which will replace real interest rate, emissions (E), and Consumer Price Index (CPI) – all pertaining to Norwegian economy and spanning from 1990 to 2022. The selection of this specific timeframe is due to unavailability of emission series prior to 1990 – so that I intended to create an equal-length vector of all observables, which is a prerequisite at times before executing operations in Dynare. I procured emission (E) time series from Statistics Norway (SSB)[10] and other three time series from Federal Reserve Bank of St. Louis' database[11] which is the largest public data-bank repository for statistical information in the USA.

The reason for selection of specific series is twofold. First, the variables that I have selected can best serve to represent my model variables under investigation. Second, they embody the general economic well-being of a country and possess a relatively higher degree of familiarity in common discourse. However, I used only two time series – real GDP, and CPI – for the estimation. I utilized the remaining two series – emissions and returns on bonds (interest rate) – to conduct the comparative analysis in relation to their smoothed and filtered series after estimation. The point to be noted is that I have all the model variables that have been depicted as log deviation from their steady states. Consequently, it is important that every variable in observation equation must capture the manner the model variables appear in the modeling equations.

I provide a short description of each of the series in the subsequent paragraphs:

Real GDP: This time series is available from 1978Q1 to 2023Q2 under the index CLVMNACSCAB1GQNO. The table delineates the GDP of mainland Norway at 2010 constant prices. The real GDP figures are in Millions of Euros. This is a quarterly time series and is seasonally adjusted. I converted this series as percentage change from the steady state which seems as in the figure 4.1 below and has legend as blue. This series is integrated of order 1 at 5% under all stationarity test models – such as autoregressive (AR), autoregressive with drift (ARD), and trend stationary (TS).

[10] See https://www.ssb.no/en/statbank/

[11] See https://fred.stlouisfed.org/series/IRLTLT01NOM156N

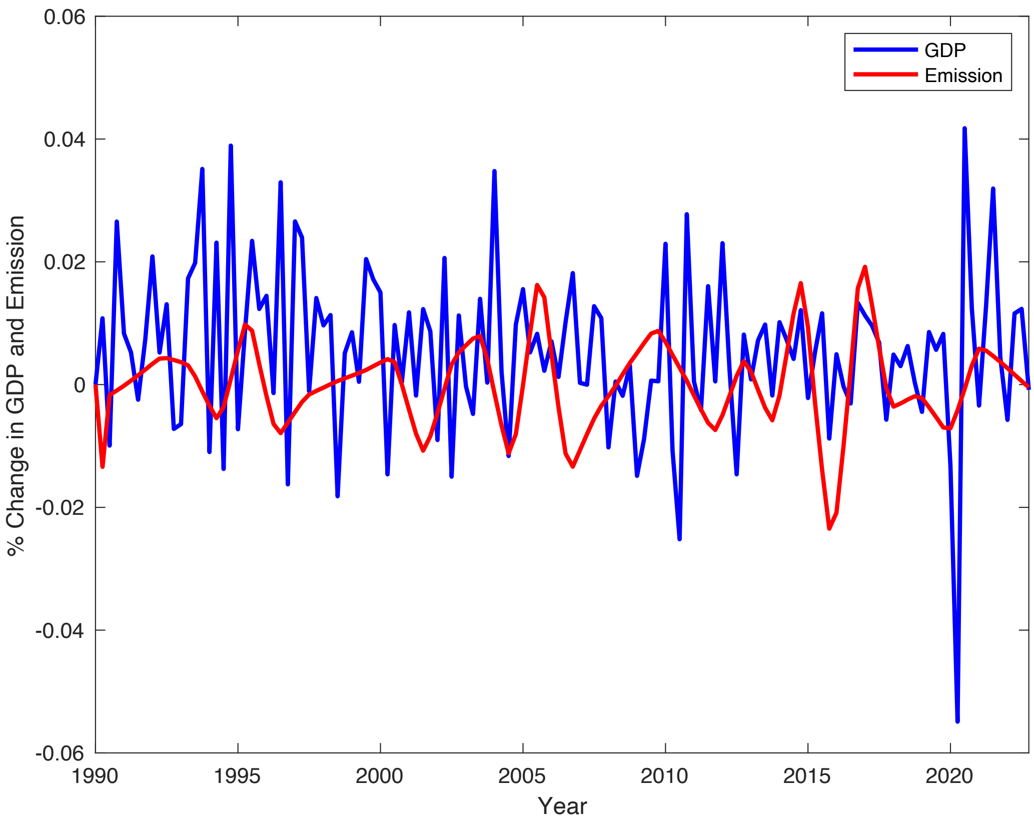


Fig. 4. 1: Percentage deviation of real GDP and Emission

Emissions: Table No. 13932 in SSB shows the greenhouse gases produced from Norwegian economic activities from 1990 to 2022. It is a yearly series. The time series shows total emissions during a particular year and is expressed as 1000 tones CO2-equivalents, AR5, where AR5 refers to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC). This series exhibits first-order integrated properties i.e., *I(1)* at 5% using AR, ARD, and TS. As the database was available on yearly basis, I used HP-filter to manipulate series from yearly to quarterly values which occurred through interpolation in MATLAB. The linearly approximated quarterly values were then expressed as percentage deviation from the steady state. The series in red legend in figure 4.1 depicts percentage change in emission from the steady state over the specified period.

4.1 Hodrick-Prescott Filter

HP filter is used to decompose a time series x_t into trend τ_t and cyclical c_t component so that $x_t = \tau_t + c_t$. Using a high pass filtering technique, this method penalizes the trend variations to an extent as defined by smoothing parameter called lambda (λ). The filter uses the following objective function:

$$g(\tau_t) = \sum_{T} \sum_{t=1} (x_t - \tau_t)^2 + \lambda_{\,T-1} \sum_{t=2} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$$

where T is the sample size. Here, the objective will be to minimize $g(\tau_t)$ over time t. The function seeks to penalize the sum of squared deviation of observed variable from its long-term trend with sum of squared second order difference for trend component which is also known as trend acceleration penalty. When $\lambda = 0$, the objective function tends to zero when $x_t = \tau_t$. As λ increases, it restricts the flexibility for the trend to increase and thereby gives more smoother trend. When λ is sufficiently larger, the trend acceleration tends to zero which results into a linear trend. Hodrick and Prescott (1997) as well as Ravn and Uhlig (2002) suggest $\lambda = 1600$ for the quarterly data.

CPI: This time series can be found with symbol NORCPIALLMINMEI under Fred's website. The time series is available from January 1960 until September 2023. It is an index where figures denote an average for the specific month. The base year chosen is 2015 wherein the index is designated as 100. I changed the periodicity from monthly to quarterly as Fred facilitates users with that option. The series was not seasonally adjusted. I did seasonal adjustment using Econometric toolbox in the MATLAB. As expected, the time series contained unit roots in level form, but the log difference of CPI was stationary in all respect – AR, ARD, and TS. It is to be noted that the natural log of ratio of current CPI to CPI one period ago will give price inflation. The figure 4.2 below depicts the percentage point change in inflation from the steady state.

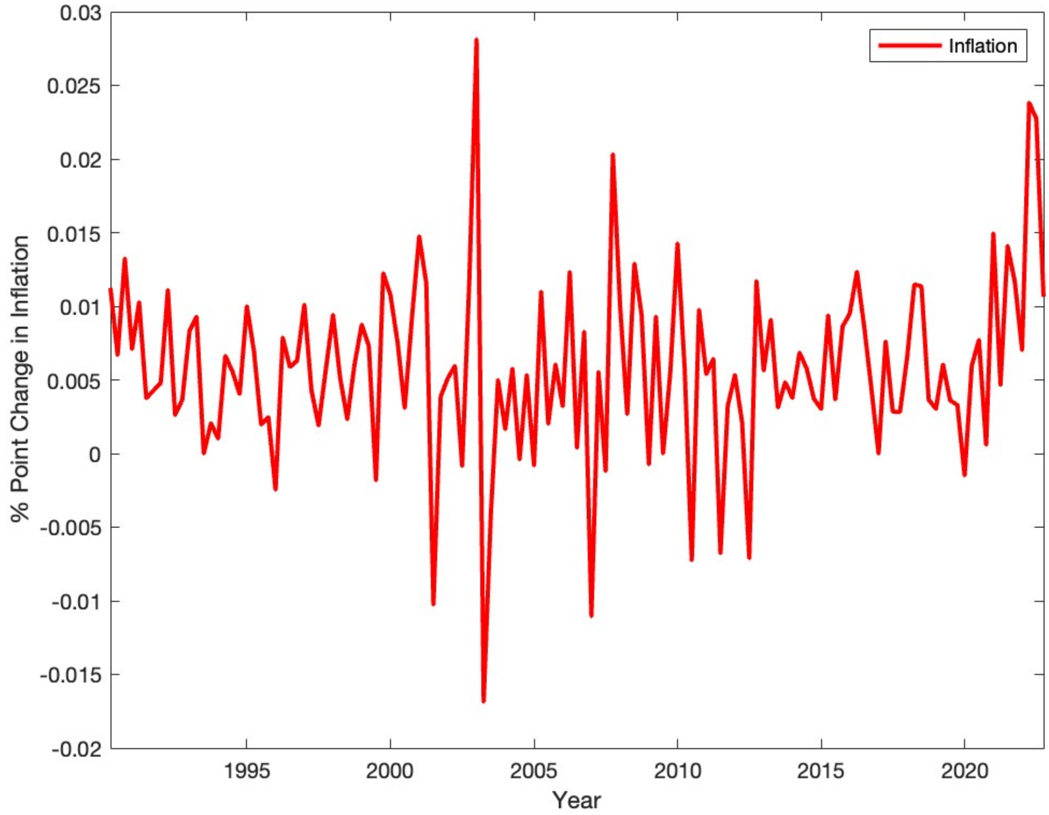


Fig. 4. 2: Percentage point change in inflation

Returns to Bonds (RB): I retrieved this time series from Fred's website. The series is indexed as IRLTLT01NOM156N. It is presented on monthly basis from January 1985 until October 2023 and is expressed in percentage as an average. I changed the series from monthly to quarterly to match series with my model dynamics. After HP filtering for seasonal adjustment, and first differencing, it turned to stationary at 5% under AR, ARD, and TS. I did not use the log-transformation to this data as it was already in percentage.

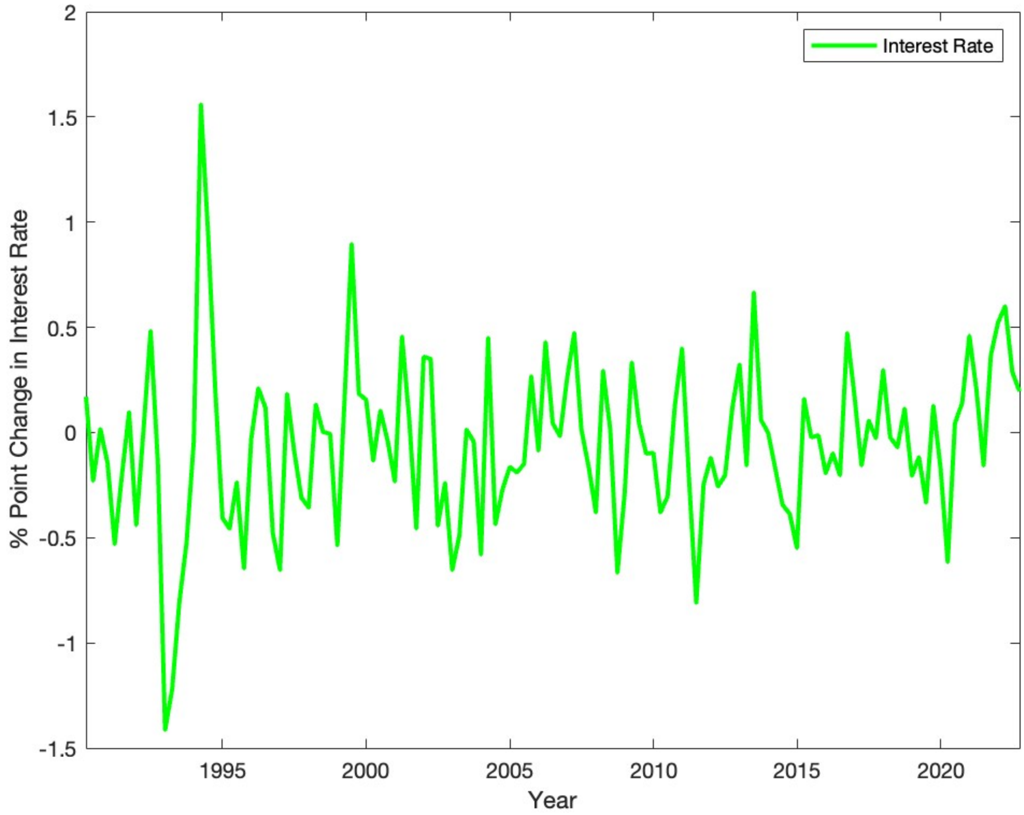


Fig. 4. 3: Percentage point change in bonds yield

Missing Observations: There are no missing observations. The only missing values are those that were lost due to differencing and seasonal adjustment. Consequently, any observation that were NaN were recognized by MATLAB as missing values and treated accordingly.

4.2 Breakpoints in Time Series

I used BEAST (Bayesian Estimator of Abrupt change, Seasonality, and Trend) (Zhao et al., 2019) technique in MATLAB to assess the breakpoints on real GDP and CPI. Like Bayesian method, this model assumes the order of the polynomials for individual segments as uknowns and the trend fitting occurs using piecewise polynomial modality. As the authors claim, "the orders of the polynomial needed to adequately fit the trend are estimated over time", as depicted in the tOrder sub-plot below.

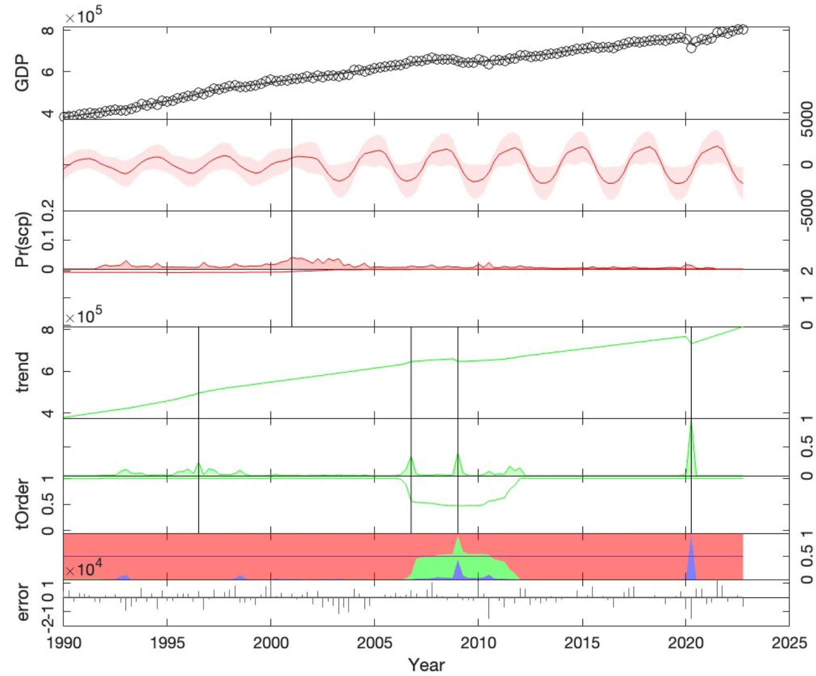


Fig. 4. 4: BEAST decomposition and changepoint detection of real GDP (Y)

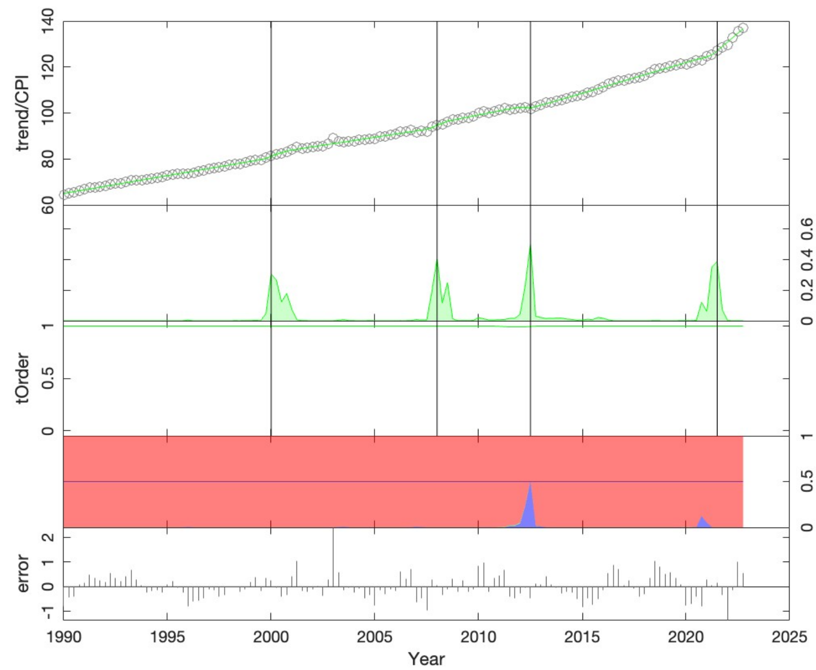


Fig. 4. 5: BEAST decomposition and changepoint detection of CPI

5. Results

5.1 The Simulated Model Summary

The focus of this thesis, as already mentioned, is on analysis of stochasticity and dynamics associated with key variables of interest. Therefore, I undertook stochastic simulations of the model to analyze the short run behavior of the variables. The theoretical rationale for this choice is guided by my assumption that the economic agents do not inherently possess all available information in the long run, but they might have some information in the short run. I simulated 18 variables – including available monetary policy variables. In my view these policy variables together with other reliable macroeconomic indices reflect most of the macroeconomic phenomena. The simulated endogenous variables appear along with IRF graphs in Appendix B. The IRF in stochastic simulation are based on parameter calibration.

The simulated dynamic model worked with no striking diagnostic problems in Dynare. Upon checking for residuals, none of the static model equations revealed non-zero residuals. Further, the theoretical means of simulated variables were found to be zero for all endogenous variables, which is expected. In other words, the steady state results for all the endogenous variables equaled to zero. There are 11 eigenvalues larger than 1 in modulus for 11 forward looking variables – implying that the rank condition was verified.

The simulation produced matrix of policy and transition function (VAR1) and matrix of correlation which I did not wish to replicate here due to their extremely larger size (see .log file in the appendix).

Figure 5.1 shows the coefficients of autocorrelation of the simulated endogenous variables up to 5th order. This diagram holds significance as it provides insights into the speed of effectiveness of shocks to bring about changes in the behavior of endogenous variables. Higher value of autocorrelation coefficients extending to the distant past time periods are indicative of lower responsiveness of the variables being examined.

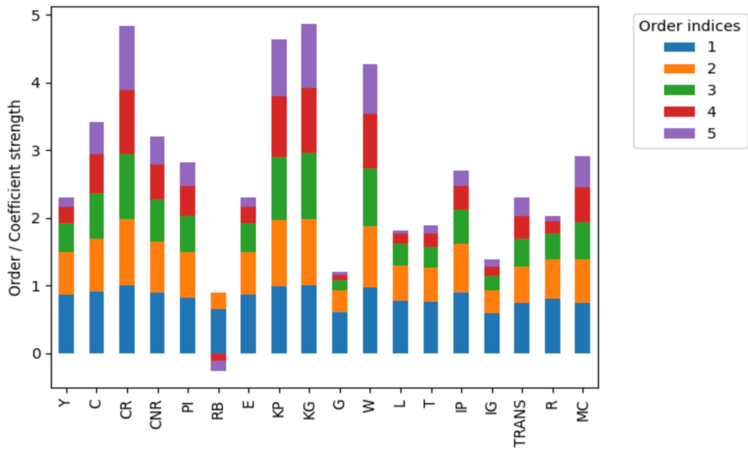


Fig. 5. 1: Stacked bars for coefficients of autocorrelation

The figure 5.1 reveals that consumption of the Ricardian agents (CR), private capital (KP), Public capital (KG), wages (W), and marginal costs (MC) possess significant autocorrelation up to and beyond third order. This may be evident due to presence of frictions such as wage stickiness, habit persistence in consumption, large investment adjustment costs, including others which can influence the concerned variables of interest. Notice that nominal interest rate (RB) has an inverted bar after period 3, meaning that autocorrelation coefficients after 3^{rd} order turn to be negative which signifies a flipping effect (alternative positive and negative effects) of lag orders – on the current interest rate.

5.2 Estimation Results

In this section I first discuss how I estimated the selected structural parameters and the processes that governed eight exogenous shocks. Then, I present the main estimation results under different sub-sections.

I applied 200,000 iterations in five-parallel chains while carrying out the Random Walk Metropolis Hastings (MH) algorithm. Although the model variables were log-linearized, they might still contain unit roots. Therefore, I used the *diffuse_filter* option in Dynare to control for unit roots – on endogenous variables which were prone to non-stationarity. Each estimate was confined to 95% MH confidence interval while applying the Bayesian estimation. As I introduced 11 forward-looking variables – those that appear with a lead in the model – the j-scale = 0.67 (decided after a series of tuning) was found to produce the 'acceptance ratio' which was within the designated optimum limits i.e., between 25% to 33%[12] (Adjemian et al., 2011). The acceptance ratio for five different chains in this model ranged between 29% to 30% which signaled that MH algorithm under the operation was optimally efficient.

5.2.1 Posterior moments information

Table 5.1 displays the posterior modes and standard deviation of the shock processes. The table also shows the priors which are our best guess before estimation.

Table 5. 1: Standard Deviation of Shocks with the inverse gamma distribution

Shocks	Prior Mean	Mode	Posterior Standard Deviation	Prior Standard Deviation
Transfer payment shock	0.1	0.0975	0.0094	0.01
Public Investment shock	0.1	0.0986	0.0097	0.01
Money supply shock	0.1	0.0601	0.0031	0.01
Technology shock	0.1	0.1019	0.0054	0.01

The table 5.2 shows the posterior means with 95% HPD (highest probability density) interval.

Table 5. 2: Prior and Posterior means of Parameters with the beta distribution

Parameters	Prior mean	Posterior Mean	95% HPD interval		Posterior Standard Deviation
betta	0.986	0.988	0.986	0.989	0.001
theta	0.550	0.550	0.548	0.552	0.001
thetaW	0.350	0.349	0.347	0.352	0.001
siggma	0.770	0.770	0.768	0.772	0.001

[12] See Dynare Reference Manual 5.5

It also reports the posterior standard deviation of parameters estimated. Prior means, as already mentioned, are our own best guesses. The information provided in both tables above create posterior distribution of the samples that appear in the subsequent sub-section.

5.2.2 Prior and Posterior Distributions

The following figure shows prior and posterior distributions of the selected estimated parameters and standard errors of shocks. The horizontal axis displays the segment of support of prior distribution, while the vertical axis represents corresponding density. The gray curves depict the prior densities, and the black curves show posterior densities. The vertical green dashed line corresponds to the posterior mode. The exact overlapping of the two curves is an indication that, either the prior accurately reflected the information in the data or, the parameter under consideration is only weakly identified and the data does not offer much information to update the prior.

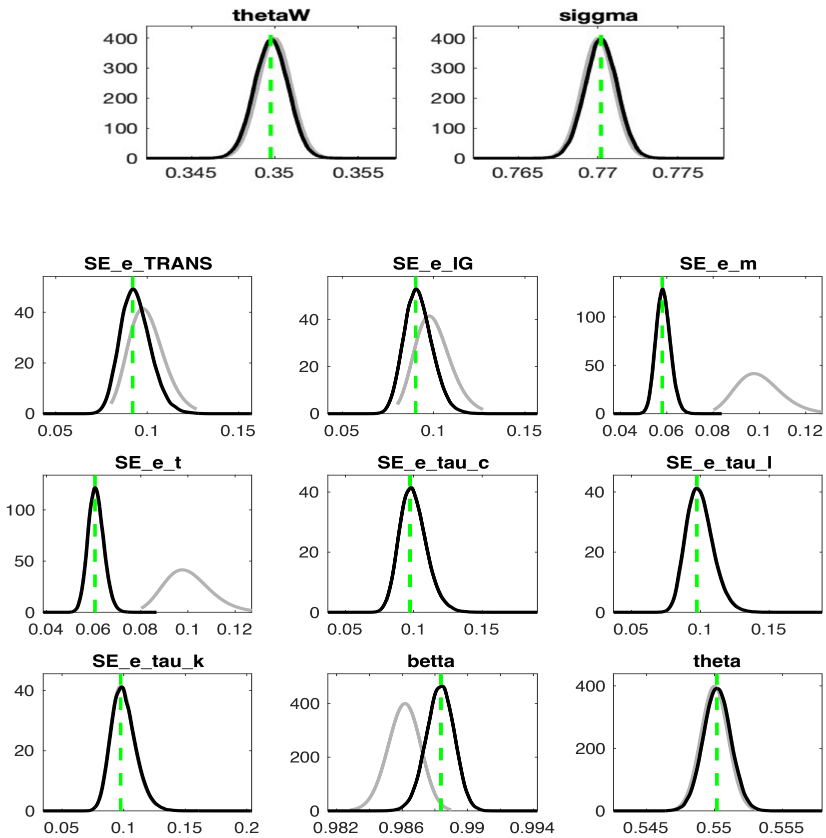


Fig. 5. 2: Priors and posterior distribution.

The figure reveals that the discount factor (betta) was well identified, whereas the other parameters such as intertemporal elasticity of substitution (siggma), Calvo parameter or price stickiness parameter (theta), and elasticity of substitution between differentiated labor (thetaW) showed lower degree of identification. A more detailed assessment of identification and convergence issues have been presented on sub-section 5.2.6. The choice behind the selection of just four parameters for estimation was rather random because this thesis does not promise for a full estimation of DSGE model for Norway but focusses on analyzing the nexus pertaining to Norwegian climate and monetary policy rules.

5.2.3 Posterior IRFs

Figures 5.3 to 5.6 denote posterior IRFs produced after the Bayesian estimation. In the ensuing figures, the vertical axis displays deviations from the steady state of the endogenous variables, either absolute deviations for linearized models or percentage deviations for log linearized models. The horizontal axis represents time periods (in quarters). The reason for reporting Bayesian IRFS is that these IRFs also convey the measure of uncertainty as against simulated IRFs that simply convey parametric information. It is, further, worth noting that Dynare employs Cholesky decomposition technique in the case it finds any correlated shocks. The order of Cholesky decomposition will be consistent with the declaration order of the structural shocks.

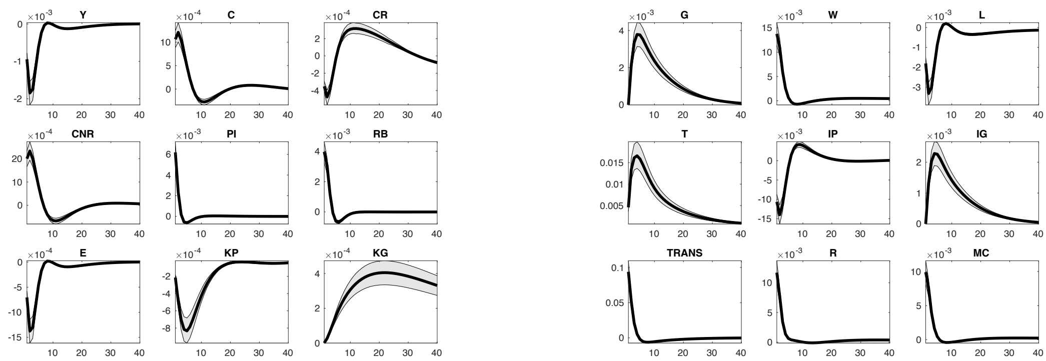


Fig. 5. 3: Orthogonalized shock to Transfer payments (Subsidies)

The figures above shows that about 1 percent increase in Transfer payment/income (TRANS) shock leads to 0.1 percent decrease in output (*Y*) at period 1 that induces around 0.07% fall in emissions (*E*) and it takes about 30 periods for output and emissions to return to the steady state. This pattern (decreasing but with varying quantities) is also evident for labor supplied by households (*L*), private investment (*IP*), and private capital stock (*KP*). However, the transfer payment shock increases inflation (*PI*), returns on capital (R), marginal cost (*MC*), aggregate consumption (*C*), wages (*W*), and total tax revenue (*T*). Given the model setting, the public investment (*IG*), and hence, the public capital (*KG*) will increase as well.

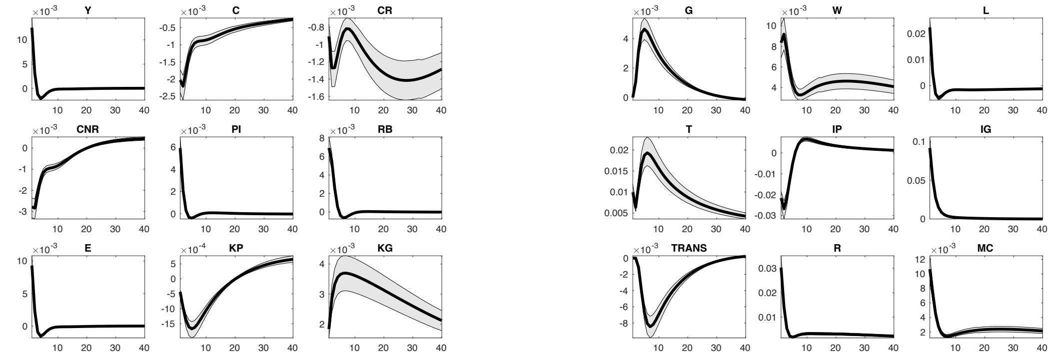


Fig. 5. 4: Orthogonalized shock to public investment

The public investment shock, on the contrary, will lead to increase almost all the variables in the diagrams except private investment, private capital stock, and total consumption. Tax revenue falls instantly for a short period, but it increases for almost 10 quarters before returning to the steady state. The fall in private investment in this scenario potentially explains the crowding out effect.

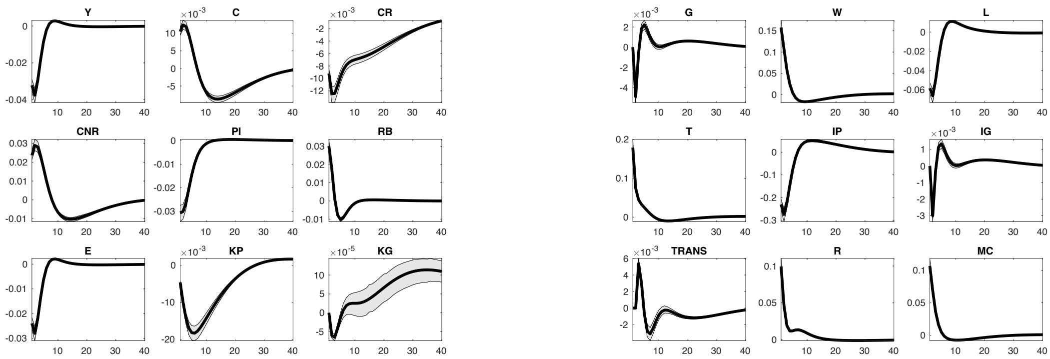


Fig. 5. 5: Orthogonalized shock to monetary policy

The money supply shock lowers output, emissions, labor supply, and private investment before they stabilize after 5 to 8 quarters. Private investment takes an instant increase within 2nd to 3rd quarter as interest rate (RB) now becomes lower after 2nd to 3rd quarter even though it rises initially. The strange behavior of interest rate rising with increase in money supply can be attributed to persistence of earlier shocks or potentially due to uneven information dissemination – that everyone does not, at once, get to know that banks have reduced the cost of borrowing following increased money supply by the central bank.

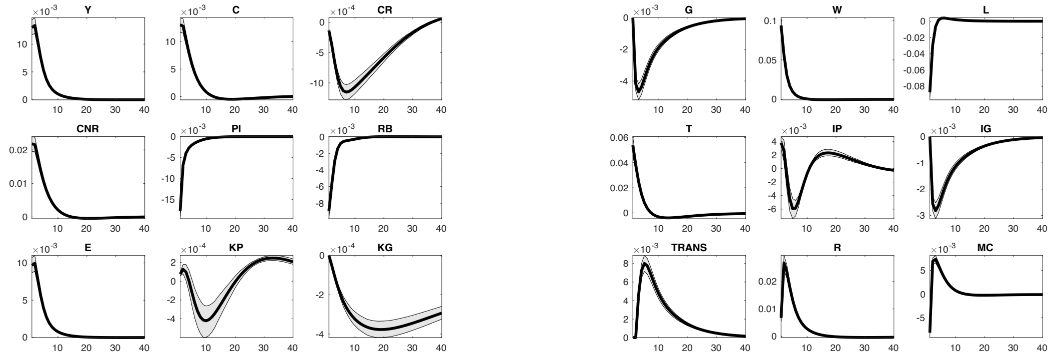


Fig. 5. 6: Orthogonalized shock to technoloy

The figure above displays the responses of selected variables given a unit of technology shock i.e., 1% technology shock. With positive technology shock, output goes up almost instantly followed by corresponding increase in emissions level, wages, rate of returns on capital, taxes (*T*), and aggregate consumption (*C*). Inflation and interest rate both decrease although they are very short-lived. However, the private capital completely overshoots before it returns to its steady state after more than 30 quarters. Moreover, technology shock increases marginal cost (*MC*) due to increase in rate of return on capital (*R*). On the contrary, the government will be more reluctant to undertake public investment – possibly due to two reasons. First, the government may be facing budget constraints or some fiscal pressures, and therefore, will prioritize on other areas such as transfer payments. Second, the government may want to divert funds to non-profit projects such as schools, health care, and any other socially essential sectors.

Note that that Dynare plots the variables being decided at period *t*. Due to this, the initial value of capital stock (K) – both KP and KG – are displayed as jumping while responding to shocks in all the figures above (the variables plotted are KP_{t+1} and KG_{t+1} not KP_t and KG_t).

5.2.4 Forecast

Figure 5.7 below shows point forecasts with various deciles for the endogenous variables for four quarters.

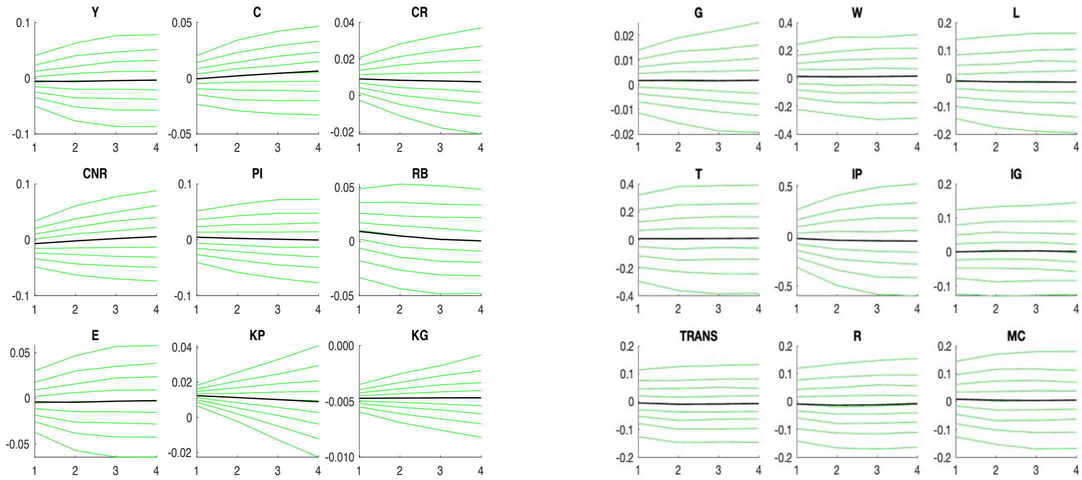


Fig. 5. 7: Point forecasts and forecast deciles for four quarters.

The black lines reveal the point-based forecast of variables of interest. On the horizontal line we have the number of forecast periods, here quarters. The point forecasts depict percentage change in GDP, aggregate consumption, stock of emission, evolution of private capital, supply of labor, and marginal cost whereas they depict percentage point change in inflation, returns to capital, and bond yields. The green lines are point forecast deciles. The point forecasts consider both parameter uncertainty and uncertainty about occurrence of future shocks.

The figure illustrates that GDP, wages, labor supplied, inflation, emissions, transfer payments, returns on capital, and marginal cost will average on to their steady state. However, the private capital, consumption for Ricardian households, and interest rate show some deviations initially but tend to return to the steady state after four quarters.

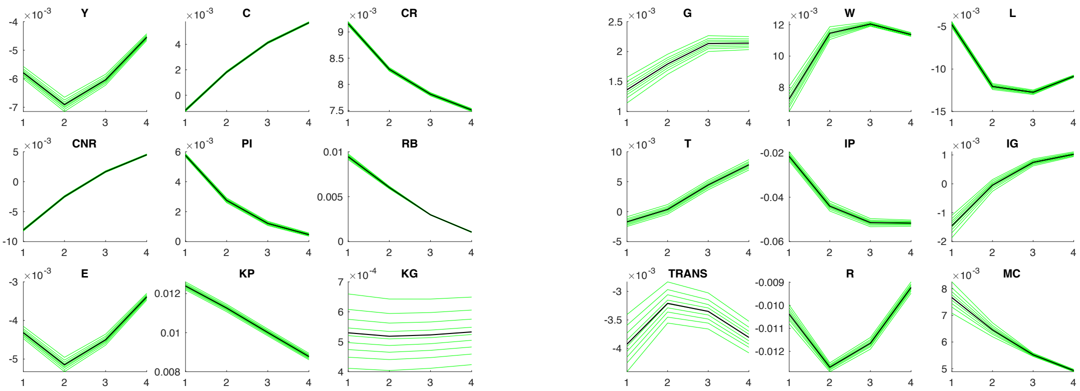


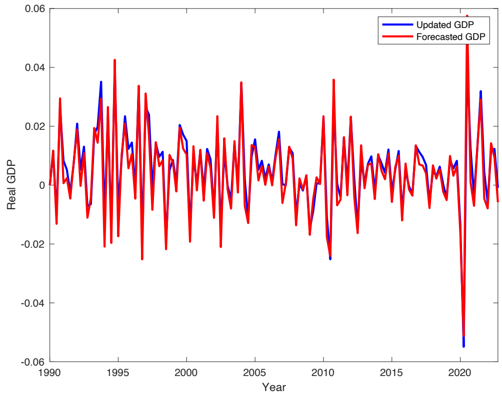
Fig. 5.8: Mean forecast for four quarters ahead.

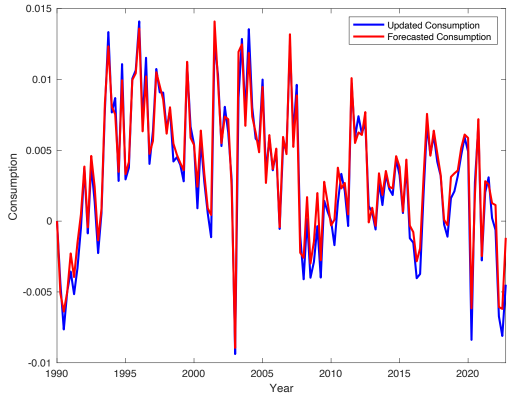
The black lines in the figure 5.8 show mean forecast values of the variables. The forecast begins at the last observation of the sample and goes onto as many steps in the future as mentioned in the forecast-option while estimating the model – 4 quarters in this case. The green lines are mean forecast deciles. The mean forecasts ignore the potential uncertainties emerging out of the future shocks and take only the parameter uncertainty into account. Future shocks are averaged out and are assumed to be zero.

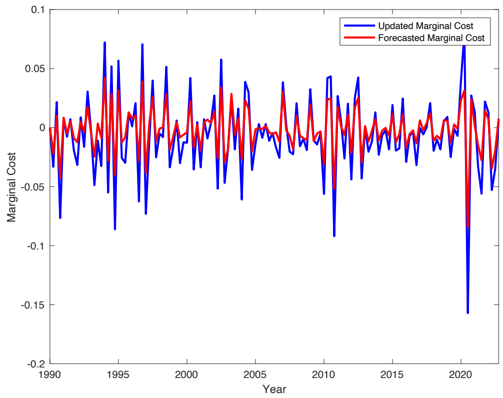
Thus, given that there are no shocks at all during the next year (4 quarters), that is after 2022:4 which is last quarterly observation of the sample, GDP, emissions, returns to capital, total consumption, tax revenue, wages, and public investment seem to increase whereas private capital, interest rate, inflation, marginal cost, private investment, and consumption by Ricardian agents seems to decrease. Transfer payments first increase, and then start to fall after the 2nd quarter. Public capital seems to remain above the steady state throughout the year. However, note that scale of measurement is quite small (10^{-3}) in most of the cases.

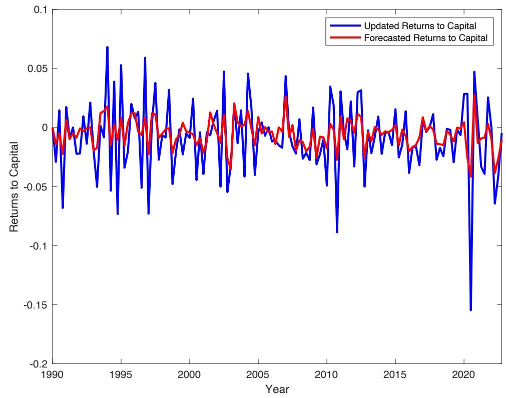
5.2.5 Updated or observed versus Filtered or forecasted variables

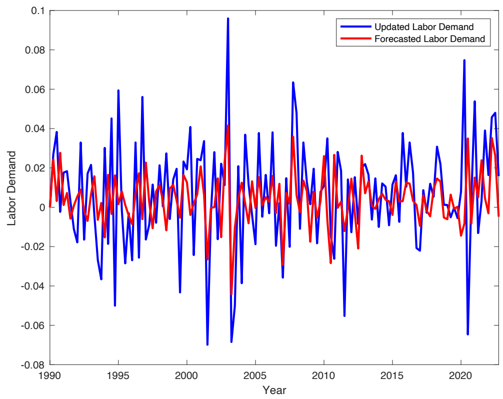
Figure 5.9 shows quarterly observed variables versus filtered/forecasted variables from 1990:1 to 2023:4. There were only two observed variables – that entered the estimation model. They included output or GDP (*Y*) and inflation (*PI*). The remaining eight endogenous/state variables which have been plotted below are simply the updated values after the estimation given observed variables. This is the most striking feature of the state-space or DSGE modeling.

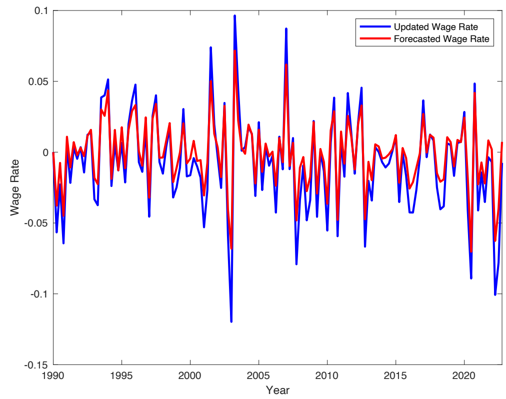

Updated GDP
Forecasted GDP
Real GDP
Year


Updated Consumption
Forecasted Consumption
Consumption
Year


Updated Marginal Cost
Forecasted Marginal Cost
Marginal Cost
Year


Updated Returns to Capital
Forecasted Returns to Capital
Returns to Capital
Year


Updated Labor Demand
Forecasted Labor Demand
Labor Demand
Year


Updated Wage Rate
Forecasted Wage Rate
Wage Rate
Year

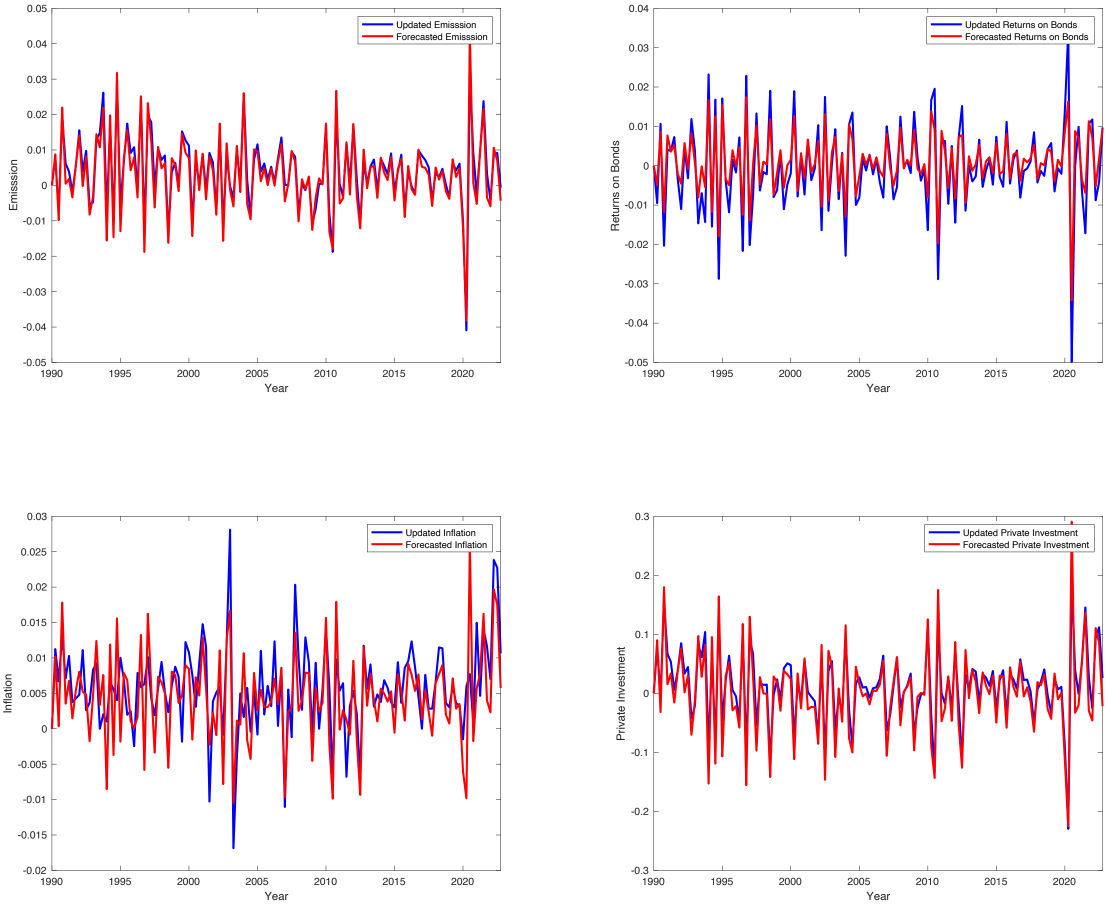


Fig. 5. 9: Updated and observed vs. forecasted variables.

The plots display that updated variables are centered around their steady states apart from aggregate consumption which typically seems to follow a random or conspicuous pattern – though it still seems to have mean zero over the sample period. The historical evolution of the plotted variables portrays a close alignment with the forecasted values. Endogenous variables including GDP, aggregate consumption, wages, emissions, interest rate (returns on bonds), and private investment showed a closer matching with corresponding forecasts. The rest of the plotted variables such as inflation, returns to capital, and labor demand showed decent matching with their corresponding forecasts but not perfectly.

5.2.5 Historical Shocks Decomposition

Figures 5.10 - 5.25 below illustrate the historical decomposition of several key variables for 120 periods. Figure caption shows which variable each figure relates to. Colored bars represent the contribution of each smoothed shock to the deviation of the smoothed endogenous variable from its steady state, i.e., the model's 'best guess' for unobserved variables resulting from each smoothed shock. An 'initial value' is simply the unknown value of the state variables when smoothed shocks are unable to explain a deviation from a steady state. Usually, the influence of the initial values disappears relatively quickly as shocks get more operative over time.

The figures illustrate that the monetary supply shock (light green legend) and the technology shock (dark blue legend) are the key contributors of several endogenous variables. In other words, these two shocks play pivotal roles in influencing the evolution of multiple endogenous variables over time. Not surprisingly, the private capital was remarkably influenced by money supply shock – and the depicted trajectory is due to overshooting, whereas both money supply shock and technology shock – including the transfer payment shock contributed to deviation of public capital below the steady state. The consumption tax shock and public investment shock tended to nullify the effects of money supply shock, technology shock, and transfer payment shock – which eventually returned the public capital being just below the steady state to the steady state almost after 120 periods.

Figure 5.22 shows that the public investment shock (black legend) itself was the key mover of deviation of public investment both below and above the steady state. The monetary policy shock and the technology shock, in this case, played very minimal roles.

Aggregate consumption evolved mainly with joint effect of money supply shock and technology shock. Additionally, the evolution was subtly shaped by the transfer payment shock and public investment shock. The real variables such as GDP and employment (labor supply and labor demand) tended to evolve primarily through contribution of technology shock and public investment shock while nominal variables such as inflation and wages were shaped mainly by money supply shock. The role of the transfer payment shock (pink legend) in evolution of the GHGs emissions seemed almost zero. The transfer payment shock just contributes primarily on itself, i.e., "transfer payment begets transfer payment."

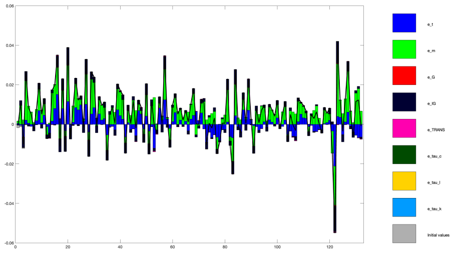


Fig. 5. 10: Output (GDP)

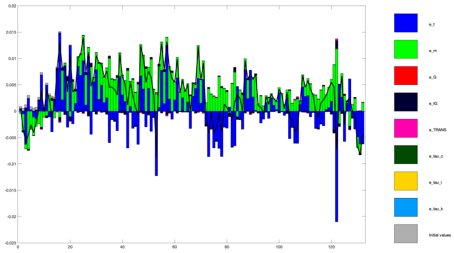


Fig. 5. 11: Consumption

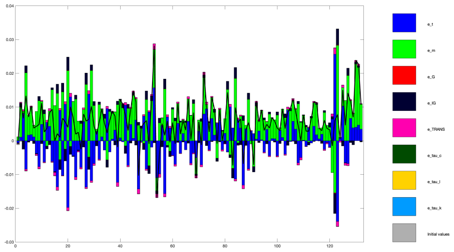


Fig. 5. 12: Inflation

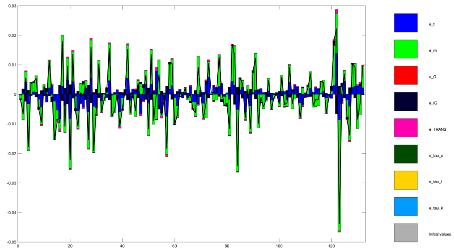


Fig. 5. 13: Interest rate (Returns on Bonds)

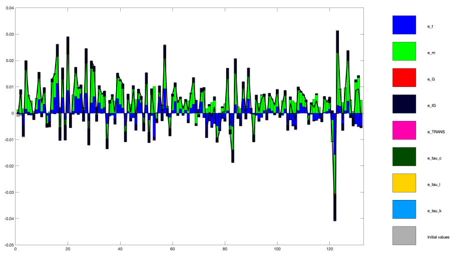


Fig. 5. 14: Emissions

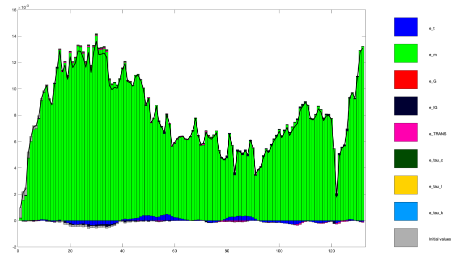


Fig. 5. 15: Private capital

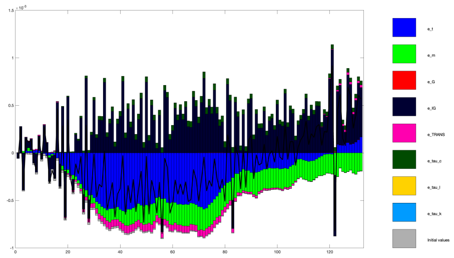


Fig. 5. 16: Public capital

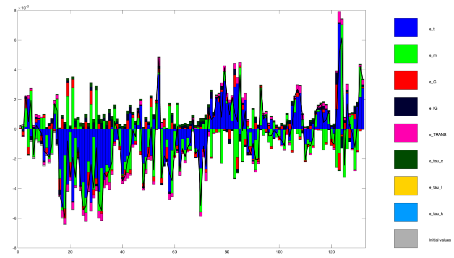


Fig. 5. 17: Government expenditure

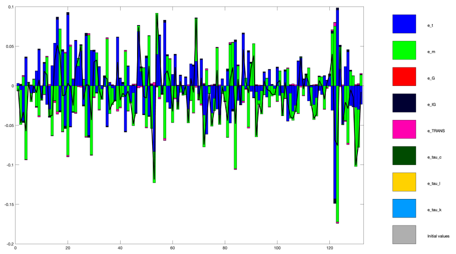


Fig. 5. 18: Wage rate

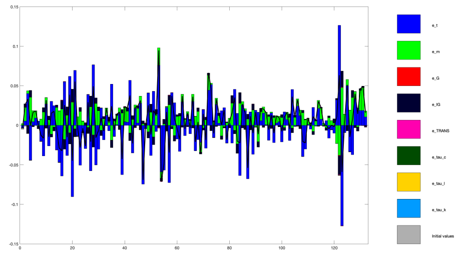


Fig. 5. 19: Labor supply

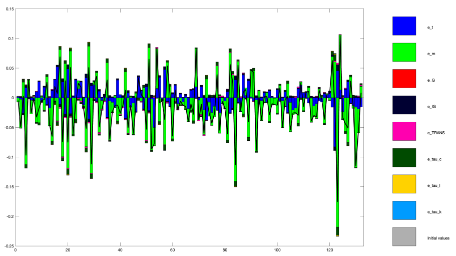


Fig. 5. 20: Tax Revenue

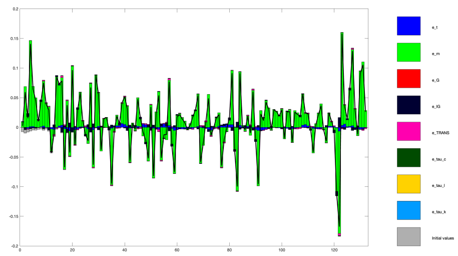


Fig. 5. 21: Private investment

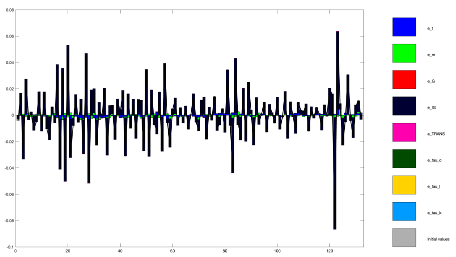


Fig. 5. 22: Public investment

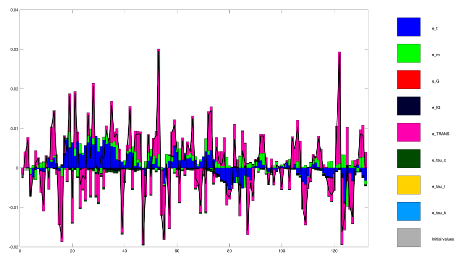


Fig. 5. 23: Transfer Payment

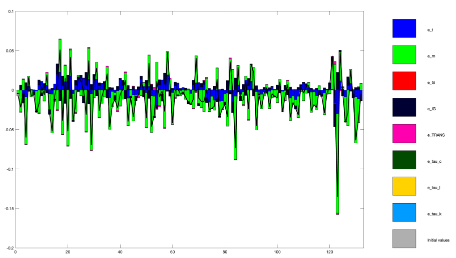


Fig. 5. 24: Returns on capital

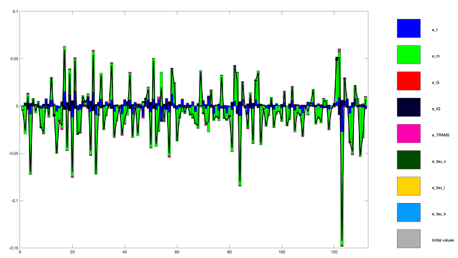


Fig. 5. 25: Marginal cost.

5.2.6 Diagnostics, Convergence and Identification

There are specific techniques to handle issues concerning diagnostics, convergence, and identification of model equation (in reduced form). Figure 5.26 below shows the mode check points for the parameters and standard errors that I estimated.

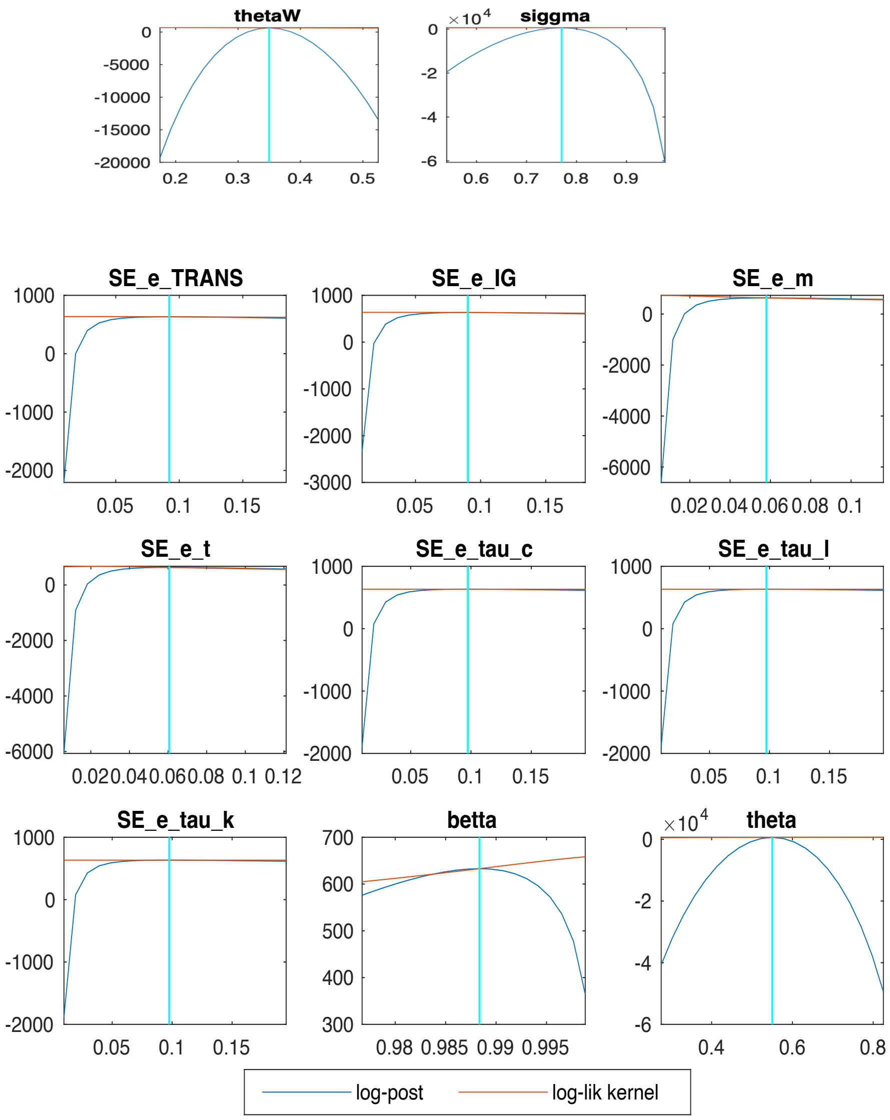
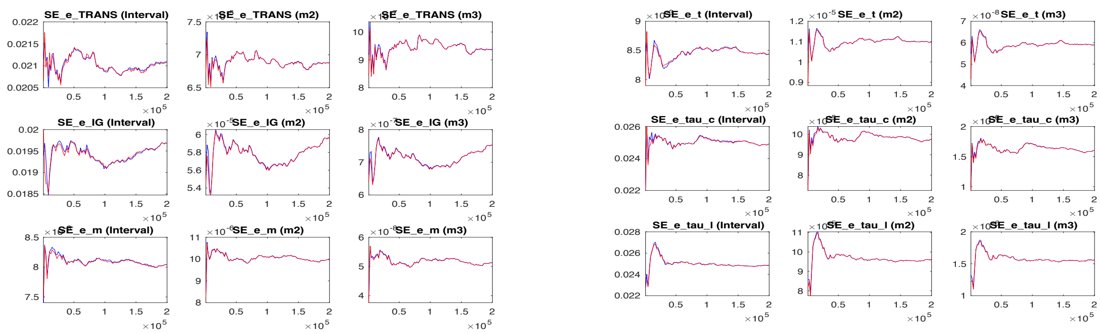
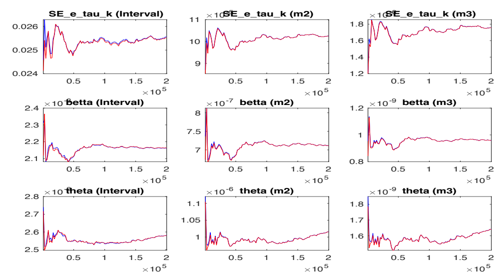


Fig. 5. 26: Mode check plots

This figure allows for checking whether the mode computation found the local mode. The horizontal axis of each panel displays an interval of parameter values centered around the estimated mode. The vertical axis, on the other hands, displays the corresponding value of the log-likelihood kernel shifted up or down by the prior values at the posterior mode (green line) and the posterior likelihood function (blue line). Differences in the shape between the likelihood kernel and the posterior likelihood indicate the prior's role in influencing the likelihood function's curvature. Ideally, the estimated mode must be present at the maximum of the posterior likelihood. Any presence of red dots indicates a parameter for which the model solution is impossible due to violations of the Blanchard-Kahn conditions (indeterminacy or no bounded solution). In figure 5.26 above, no such points are obtained. However, theta, thetaW, and sigma are running almost on the boundary region that might cause weak identification issues – but still identified!

Regarding convergence issues, Dynare reports both MCMC univariate and multivariate convergence diagnostics, as suggested by Brooks and Gelman (1998).





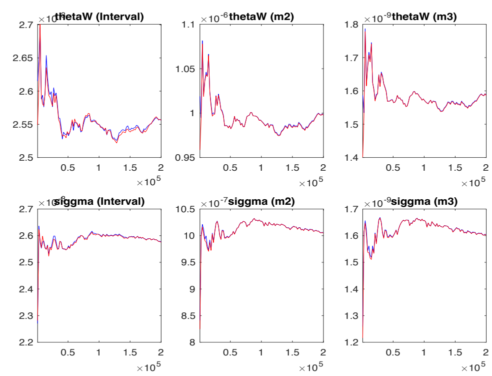


Fig. 5. 27: MCMC univariate convergence diagnostics, Brooks and Gelman (1998)

The first column with 'Interval' reveals the convergence diagnostics with an 80% interval. The blue line is the 80% quantile or range based on pooled draws from all sequences, whilst the red line shows the mean interval range based on the draws of individual series. The second and third columns with 'm2' and 'm3' denote an estimation of the same statistic for the second and third central moments - that is, the squared and cubed deviations from the pooled sequence in absolute terms and the within-sample mean, respectively. The two lines stabilizing horizontally, being close to each other, reflect that the chains converged during MCMC.

Figure 5.28 shows convergence diagnostics associated with the range of posterior likelihood function of the parameters under estimation. The posterior kernel aggregates the parameters in this case. As above, the stable convergence of the blue and red lines striking the rightmost vertical axis ensures that MCMC converged during estimation.

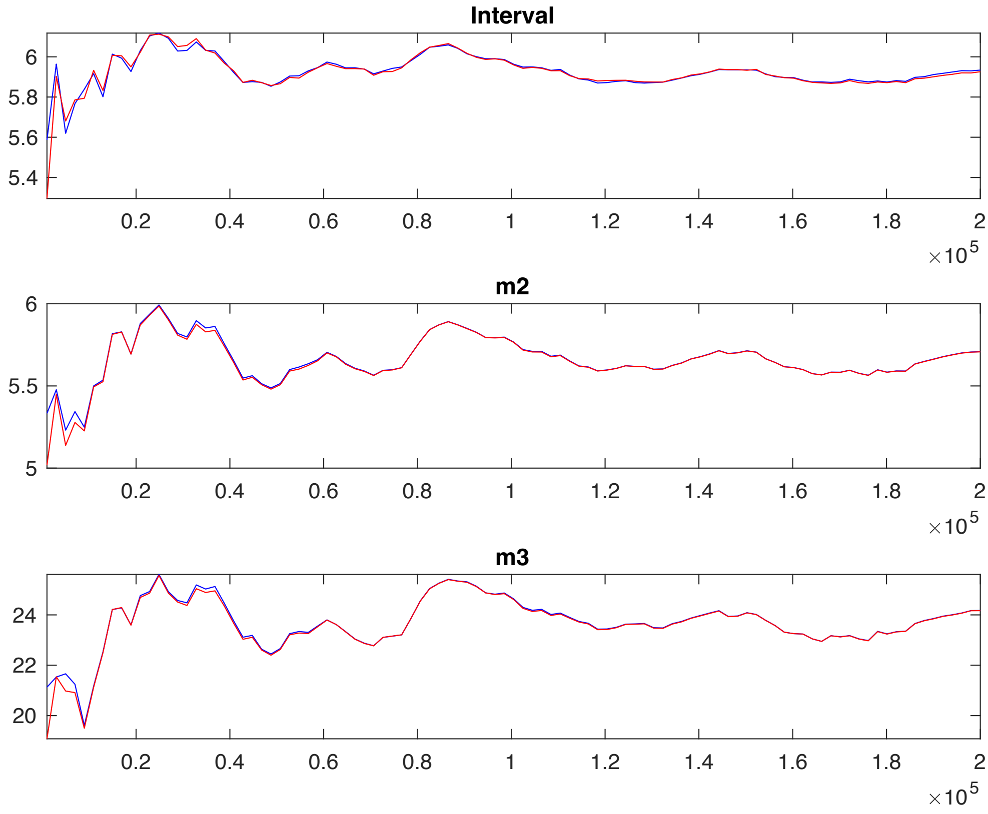


Fig. 5. 28: Multivariate convergence diagnostics

As both red and blue lines seem to converge successfully after sufficiently large iterations (i.e., 200,000), there was no convergence issue in this model estimation. However, in the case of the parameters including theta, thetaW, siggma, and standard errors of transfer payment shock (SE_e_TRANS), there is lack of efficacy as the two lines struggle to converge especially during earlier phase of replication (i.e., before 50,000). This does, not however, pose this model to indeterminacy bounds.

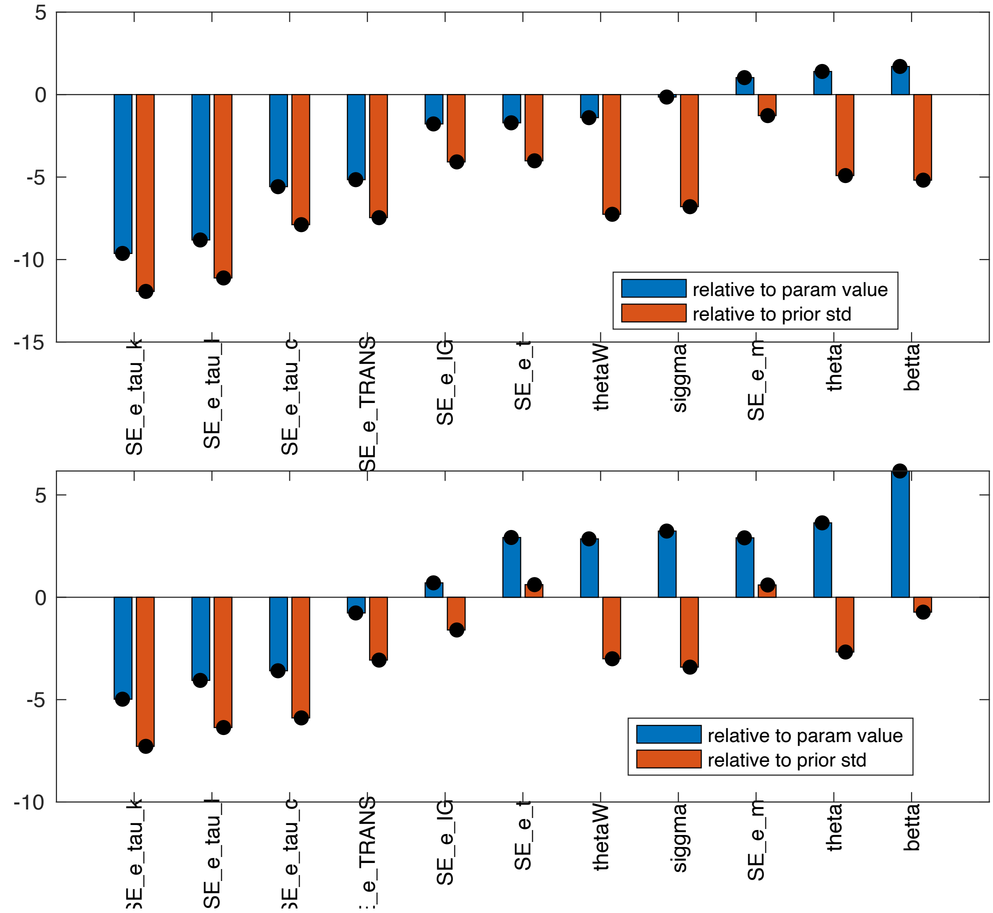


Fig. 5. 29: Prior-mean Identification strength (log-scale) - Upper Panel, and Sensitivity component (log-scale) - Lower Panel

Upper Panel: The bar graphs display the identification strength of the parameters on the basis of Fischer's information matrix[13] normalized by either the parameter at the prior mean as represented by blue bars or the standard deviation at the prior mean represented by the red bars. Intuitively, the bars represent the normalized curvature of the log-likelihood function at the prior mean in the direction of the parameter. Graphs are usually log-scaled but not for the unidentified parameters, which are displayed with a bar length of 0 (i.e., no bar at all). If the strength is 0 the parameter is unidentified. In contrast, the larger the value in log scale, the lesser is the poor identification issue. The parameters appear as ordered in the direction of increasing identification strength relative to parameter value. SE of monetary policy shock (SE_e_m), theta, and betta have higher identification strength, i.e., greater than 1, while the others have less than one in the figure above.

[13] See Dynare Reference Manual, Version 4.0-5.5

Lower Panel: This graph provides a further detailed analysis of the impact depicted in the upper panel. Weak identification can stem from two main sources. First, parameters might be linearly related, indicating that these parameters have a similar impact on the likelihood, and second, the likelihood remains unchanged for the parameter. This latter effect is termed as sensitivity by Ratto and Iskrev (2012). The panel shows that the sensitivity of none of the parameters is 0. Thus, all the parameters have been identified at the local mean, because it affects the likelihood and hence the model moments.

6. Discussion and Conclusion

6.1 Discussion

This section offers a discussion pertaining to the analysis of potential nexus between climate policy and monetary policy in the Norwegian context. My two research questions are:

1. To what extent does incentivization of Electric Vehicle (EV) contribute to the reduction of emissions in Norway?
2. Does injection of public investment help achieve high and stable output and employment coupled with low and stable inflation i.e., divine coincidence, in the Norwegian economy while also reducing GHG emissions?

My results yielded several key findings that shed light on the closer interconnection between climate policy and monetary policy. I start my discussion with the second research question as those results are most striking.

My main finding is that GHG emissions inherently followed the GDP trajectory. This nearly one-to-one linkage between emissions and GDP signifies that expansionary economic policies increase GHG emissions. During the period I have studied economic policy in Norway has been highly expansionary for two reasons. First, without large transfers from the Norwegian public sovereign fund «oljefondet» Norwegian public spending would have been far less. In other words, multiple Norwegian governments have pursued policies that entail negative public savings, i.e., state tax revenues are less than public spending.

A caveat with this perspective is that Norwegian economy consists of two parts: The mainland economy, and the petroleum related offshore activities. These two parts are connected through the available work force as an increase in the offshore activities lead to less labor available in the rest of the economy. Because domestic demand remains high in such settings and parts of domestic demand need to come from domestic production. Specifically, this relates to goods and services that need to be produced close to the consumers, so-called “naturally protected” activities. Hairdressers is one example of such a service. Hence, it is the so-called competitive sectors, i.e., export industries, that experiences a decline in employment (Aukrust, 1977). There may actually be an increase of employment in the “naturally protected” activities.

Second, the central bank of Norway (Norges Bank) has pursued relatively expansionary monetary policies, primarily through a low governing interest rate, that has resulted in overall lower interest rates, which again has led to a gradual decline in the value of the Norwegian krone (NOK). This has reduced the fall of employment in export industries. Here, I note that most Norwegian political parties have been concerned about declining employment in traditional export industries. The weaker NOK has also made imports more expensive, and hence helped to maintain employment in production geared towards the domestic markets.

In summary, expansionary fiscal and monetary policies have contributed to a growth in GDP. This brings me back to my main finding: results consistently indicate that emissions level depends solely on GDP. This finding aligns with arguments advocated by Barrage and Nordhaus (2023) and Hamilton and Turton (2002).

Regarding my first research question, the transfer payments which were supposedly treated to represent EV tax incentives revealed no contributions towards GHG emission reduction. This nuanced finding challenges the existing EV policy of the Norwegian government. However, it also opens doors for a new discourse within the Norwegian climate policy domain.

Returning to the general economic policies of Norway. There was strong evidence that government involvement is essential in production activities as it contributed significantly on evolution of crucial macro variables including GDP and employment. Further the public investment shock was also found to impact inflation in the long run.

Surprisingly, money supply shock was found to reduce both GDP and employment in the short run. This anomaly can be attributed to the role of expected inflation which may add uncertainties and induce producers to adjust their investment behavior, leading to a temporary downturn in economic activities. However, the money supply shock was observed as a prime contributor to evolution of key macroeconomic variables and GHG emission along their balanced growth paths or the steady states.

Furthermore, technology shock was found to be the second important driver of several macro variables under consideration. It was found to influence crucially to determination of employment decisions by firms in the economy. In other words, given a positive technology shock, firms increased employment opportunities over time.

6.2 Empirical learning and reflections

This thesis has been a significant learning experience for me, personally. I encountered notable methodological challenges such as inclusion of climate policy frontier within the broader DSGE framework. Next, modeling EV policy within that framework posed a big challenge initially as I encountered data gaps, i.e., no time series were readily available on fiscal incentives which EV users enjoyed in Norway. However, DSGE estimation generated essential series that helped assess the desired patterns. Further, I tried to incorporate as many frictions or rigidities as possible within the modeling as this lies in the heart of New Keynesian macroeconomics. This made modeling more-lively and pragmatic. Overall, it was a pleasant experience delving into this somewhat ambiguous yet essential topic area.

6.3 Limitations of the model and further studies

1. This model ignores the role of money supply in household's utility function.
2. The model features only the domestic economy whereas there can be pronounced effects of external sector in domestic economy i.e., imported inflation or exchange rate dynamics associated with NOK might significantly influence domestic activities.
3. Findings pertain to specific time periods only 1990:1 – 2022:4. Thus, it may not be generalizable for all other finite or infinite time horizons.

Thus, it is expected that someone could continue with this model in order to incorporate money supply as a component in utility function of the households. Additionally, a complete DSGE model emerges only if one could include external sector of the economy. Future researchers are hopefully expected to delve into those areas.

6.4 Conclusion

This thesis aimed at analyzing the potential connections between climate policy and monetary policy in the context of Norway using DSGE approach. Given that quite a few research existed within the intended area, it was interesting to delve into the field. The research interest within the climate-monetary nexus was further propelled given distinctive position of Norway in terms of climate policy implementation. Owing to her long legacy for advocacy and implementation of sustainable climate policies, Norway introduced EV policy that subsidized EV users resulting in remarkable progress towards GHG emissions reduction. It was, however, not empirically convincing that such subsidies or transfer payment could contribute to emission reductions or influence key macroeconomic scenarios. However, increased public investment and capital stock fiscal stimulus can contribute to emission reductions but achievement of divine coincidence appears to be unattainable as higher public investment might crowd out private investment thereby lowering actual output and employment, while further increasing inflation.

Appendix

A. log-linearized model equations

1. Lagrangian for Ricardian HHs

$$\tilde{\lambda}_{R,t} + \tilde{P}_t + \left(\frac{\tau_{ss}^c}{1+\tau_{ss}^c}\right)\tilde{\tau}_t^c$$
$$= \left[\frac{\sigma}{(1-\phi_c\beta)(1-\phi_c)}\right]\left[\phi_c\beta\left(E_t\tilde{C}_{R,t+1} - \phi_c\tilde{C}_{R,t}\right) - \left(\tilde{C}_{R,t} - \phi_c\tilde{C}_{R,t-1}\right)\right]$$

2. Phillips curve for Ricardian HHs

$$\tilde{\pi}_{W,t} = \beta E_t\tilde{\pi}_{W,t+1} + \left[\frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W}\right]\left[\varphi\tilde{L}_{R,t} - \tilde{\lambda}_{R,t} + \left(\frac{\tau_{ss}^l}{1-\tau_{ss}^l}\right)\tilde{\tau}_t^l\right]$$

3. Gross wage inflation

$$\tilde{\pi}_{W,t} = \widetilde{W}_t - \widetilde{W}_{t-1}$$

4. Budget constraint for Ricardian HHs

$$P_{ss}C_{R,ss}\left[\left(\tilde{P}_t + \tilde{C}_{R,t}\right)(1+\tau_{ss}^c) + \tau_{ss}^c\tilde{\tau}_t^c\right] + P_{ss}I_{ss}^P\left[\left(\tilde{P}_t + \tilde{I}_t^P\right)(1+\tau_{ss}^c) + \tau_{ss}^c\tilde{\tau}_t^c\right] + \frac{B_{ss}}{R_{ss}^B}\left(\tilde{B}_{t+1} - \tilde{R}_t^B\right) = W_{ss}L_{R,ss}\left[\left(\widetilde{W}_t + \tilde{L}_{R,t}\right)(1-\tau_{ss}^l) - \tau_{ss}^l\tilde{\tau}_t^l\right] + R_{ss}K_{ss}^P\left[\left(\tilde{R}_t + \widetilde{K}_t^P\right)(1-\tau_{ss}^k) - \tau_{ss}^k\tilde{\tau}_t^k\right] + B_{ss}\tilde{B}_t + \omega_R TRANS_{ss}\widetilde{TRANS}$$

5. Tobin's Q

$$\left(\frac{Q_{ss}}{\beta}\right)\tilde{Q}_t = E_t\Big\{(1-\delta)Q_{ss}\tilde{Q}_{t+1} + \lambda_{R,ss}R_{ss}U_{ss}(1-\tau_{ss}^k)$$
$$\left[\tilde{\lambda}_{R,t+1} + \tilde{R}_{t+1} + \widetilde{U}_{t+1} - \left(\frac{\tau_{ss}^k}{1-\tau_{ss}^k}\right)\tilde{\tau}_{t+1}^k\right] - \lambda_{R,ss}P_{ss}\Psi_1 U_{ss}\widetilde{U}_{t+1}\Big\}$$

6. Demand for installed capacity

$$(1-\tau_{ss}^k)\frac{R_{ss}}{P_{ss}}\left[\tilde{R}_t - \tilde{P}_t - \left(\frac{\tau_{ss}^k}{1-\tau_{ss}^k}\right)\tilde{\tau}_t^k\right] = \Psi_2 U_{ss}\widetilde{U}_t$$

7. Investment demand

$$(1+\tau^c_{ss})\lambda_{R,ss}P_{ss}\left[\tilde{\lambda}_{R,t}+\tilde{P}_t+\left(\frac{\tau^c_{ss}}{1+\tau^c_{ss}}\right)\tilde{\tau}^c_t\right]-\ Q_{ss}\tilde{Q}_t+\chi Q_{ss}(\tilde{I}^P_t-\tilde{I}^P_{t-1})$$
$$=\chi\beta Q_{ss}(E_t\tilde{I}^P_{t+1}-\tilde{I}^P_t)$$

8. Law of motion of private capital

$$\tilde{K}^P_{t+1}=(1-\delta)\tilde{K}^P_t+\delta\tilde{I}^P_t$$

9. Euler equation (Bonds)

$$\tilde{\lambda}_{R,t}-\tilde{R}^B_t=\tilde{\lambda}_{R,t+1}$$

10. Lagrangian for non-Ricardian HHs

$$\tilde{\lambda}_{NR,t}+\tilde{P}_t+\left(\frac{\tau^c_{ss}}{1+\tau^c_{ss}}\right)\tilde{\tau}^c_t$$
$$=\left[\frac{\sigma}{(1-\phi_c\beta)(1-\phi_c)}\right]\left[\phi_c\beta\left(E_t\tilde{C}_{NR,t+1}-\phi_c\tilde{C}_{NR,t}\right)-\left(\tilde{C}_{NR,t}-\phi_c\tilde{C}_{NR,t-1}\right)\right]$$

11. Phillips curve for non-Ricardian HHs

$$\tilde{\pi}_{Wt}=\beta E_t\tilde{\pi}_W t+1+\left[\frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W}\right]$$
$$\left[\varphi\tilde{L}_{NR,t}-\tilde{\lambda}_{NR,t}+\left(\frac{\tau^l_{ss}}{1-\tau^l_{ss}}\right)\tilde{\tau}^l_t\right]$$

12. Aggregate consumption

$$C_{ss}\tilde{C}_t=\omega_R C_{R,ss}\tilde{C}_{R,ss}+(1-\omega_R)C_{NR,ss}\tilde{C}_{NR,ss}$$

13. Aggregate labor supplied and demanded

$$L_{ss}\tilde{L}_t=\omega_R L_{R,ss}\tilde{L}_{R,ss}+(1-\omega_R)L_{NR,ss}\tilde{L}_{NR,ss}$$

14. Production function

$$\tilde{Y}_t=\tilde{A}_t+\alpha_1\left(\tilde{U}_t+\tilde{K}^P_t\right)+\alpha_2\tilde{L}_t+\alpha_3\tilde{K}^G_t$$

15. Firm's trade-off (marginal rate of substitution = relative prices)

$$\tilde{L}_t-\tilde{U}_t-\tilde{K}^P_t=\tilde{R}_t-\tilde{W}_t$$

16. Marginal cost (MC)

$$\widetilde{MC}_t = \alpha_2 \widetilde{W}_t + \alpha_1 \tilde{R}_t - \tilde{A}_t - \alpha_3 \widetilde{K}_t^G$$

17. Phillips curve

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right] \left(\widetilde{MC}_t - \tilde{P}_t\right)$$

18. Gross inflation

$$\tilde{\pi}_t = \tilde{P}_t - \tilde{P}_{t-1}$$

19. Govt. budget constraint

$$\frac{B_{ss}}{R_{ss}^B} \left(\tilde{B}_{t+1} - \tilde{R}_t^B\right) - B_{ss} \tilde{B}_t + T_{ss} \tilde{T}_t = P_{ss} G_{ss} \left(\tilde{G}_t + \tilde{P}_t\right) +$$
$$P_{ss} I_{ss}^G \left(\tilde{P}_t + \tilde{I}_t^G\right) + P_{ss} TRANS_{ss} \left(\tilde{P}_t + \widetilde{TRANS}_t\right)$$

20. Total tax revenue

$$T_{ss} \tilde{T}_t = \tau^c P_{ss} \left[C_{ss} \left(\tilde{C}_t + \tilde{P}_t\right) + I_{ss}^P \left(\tilde{I}_t^P + \tilde{P}_t\right)\right] +$$
$$\tau^l W_{ss} L_{ss} \left(\widetilde{W}_t + \tilde{L}_t\right) + \tau^k K_{ss}^P \left[R_{ss} \left(\tilde{R}_t + \widetilde{K}_t^P\right) - \delta \widetilde{K}_t^P\right]$$

21. Law of motion of public capital

$$\widetilde{K}_{t+1}^G = (1 - \delta_G) \widetilde{K}_t^G + \delta \tilde{I}_t^G$$

22. Fiscal policy rule

$$\tilde{Z}_t = \gamma_Z \tilde{Z}_{t-1} + (1 - \gamma_Z) \phi_Z \left(\tilde{B}_t - \tilde{Y}_{t-1} - \tilde{P}_{t-1}\right) + \tilde{S}_t^Z$$

23. Taylor's rule

$$\tilde{R}_t^B = \gamma_R \tilde{R}_{t-1}^B + (1 - \gamma_R) \left(\gamma_\pi \tilde{\pi}_t + \gamma_Y \tilde{Y}_t\right) + \tilde{S}_t^m$$

24. Market clearance

$$Y_{ss} \tilde{Y}_t = C_{ss} \tilde{C}_t + I_{ss}^P \tilde{I}_t^P + I_{ss}^G \tilde{I}_t^G + G_{ss} \tilde{G}_t$$

25. Productivity shock

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_t$$

26. Fiscal policy shock

$$\tilde{S}_t^Z = \rho_Z \tilde{S}_{t-1}^Z + \varepsilon_{Z,t}$$

27. Monetary policy shock

$$\tilde{S}_t^m = \rho_m \tilde{S}_{t-1}^m + \epsilon_{m,t}$$

28. Emission equation

$$\tilde{E}_t = [1-\mu]\sigma_E \tilde{Y}$$

B. IRFs based on calibrated parameter combination.

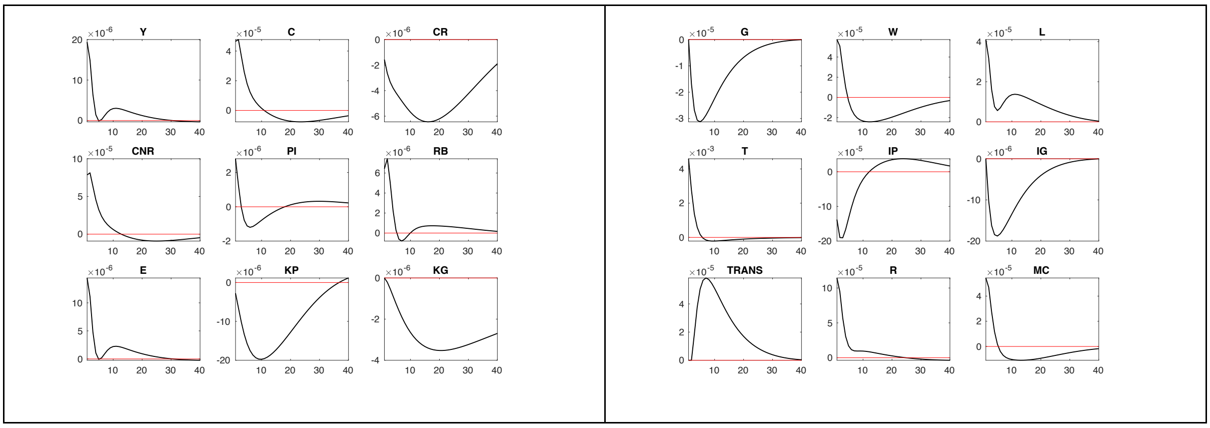


Fig. Orthogonalized shock to capital tax

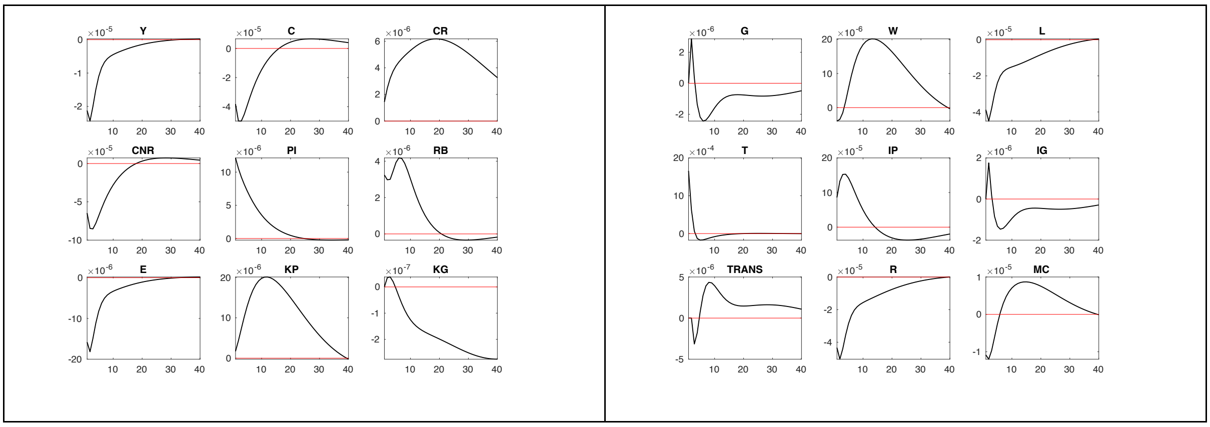
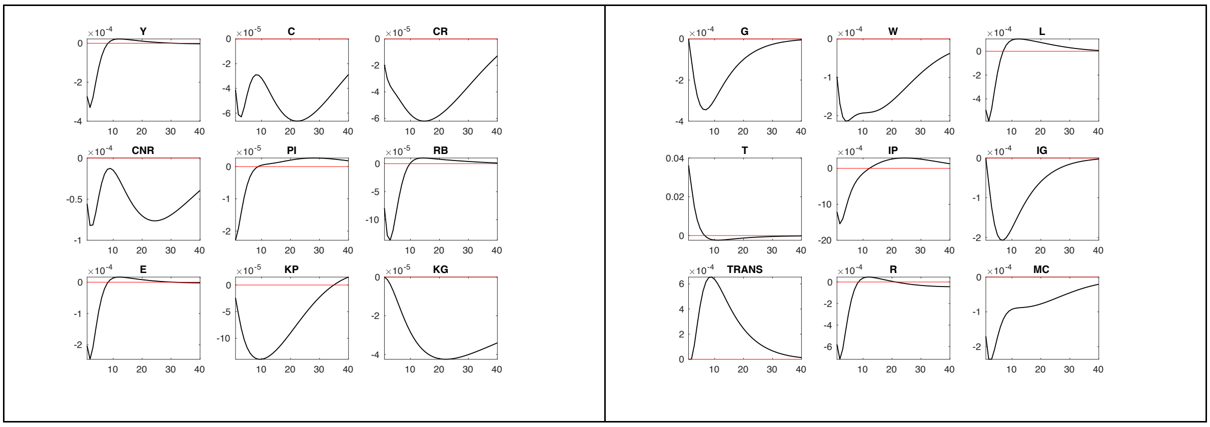
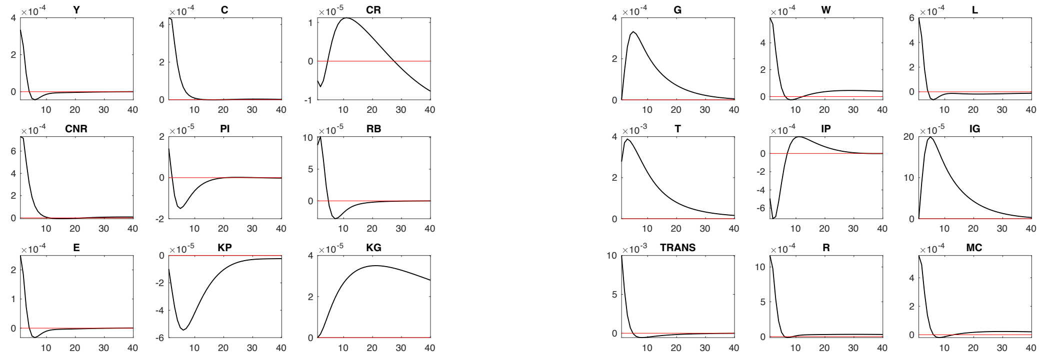


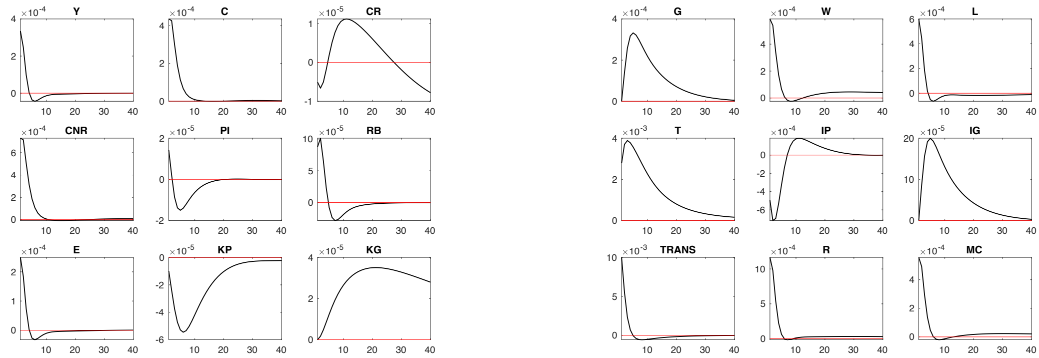
Fig. Orthogonalized shock to labor tax



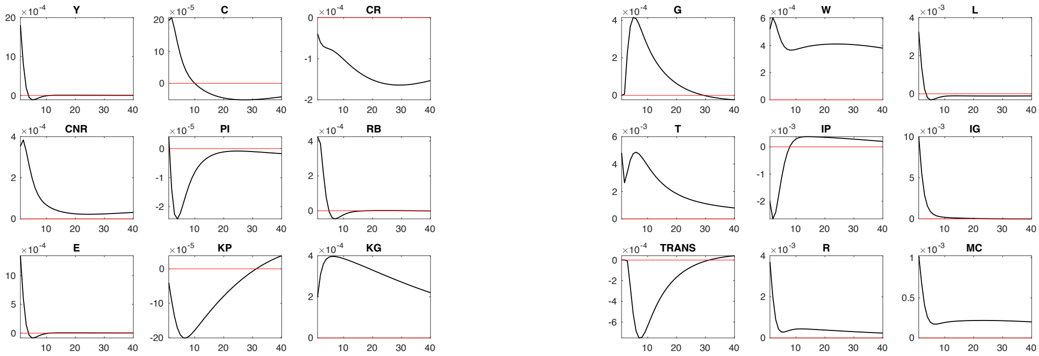
Orthogonalized shock to consumption



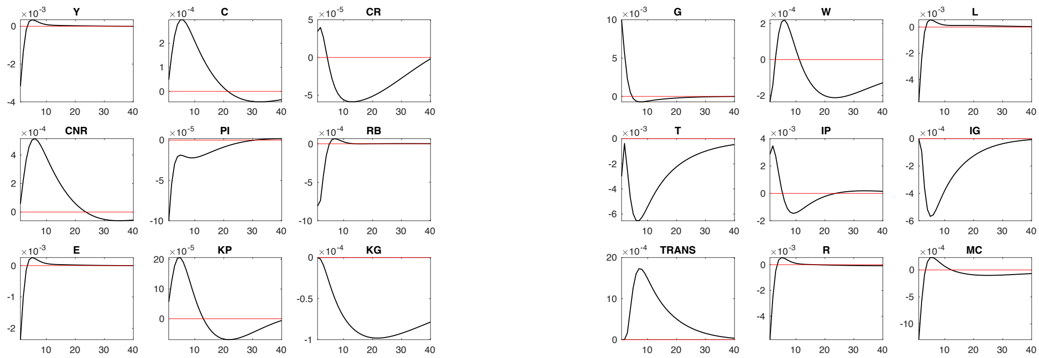
Orthogonalized shock to transfer payment



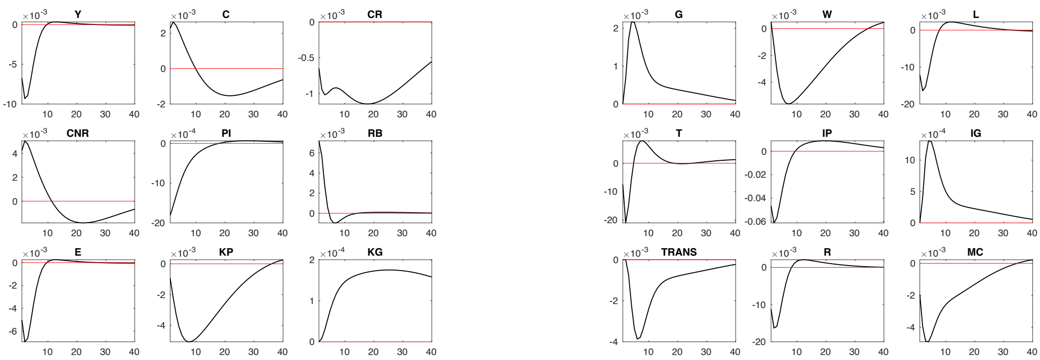
Orthogonalized shock to public investment



Orthogonalized shock to government expenditure

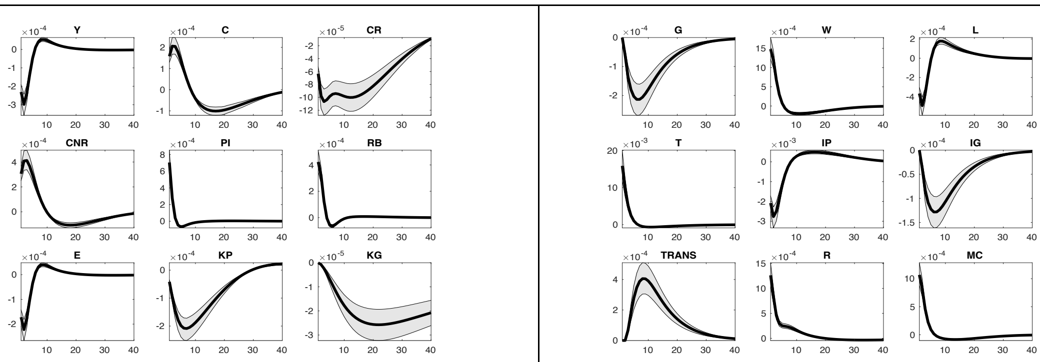


Orthogonalized shock to money supply

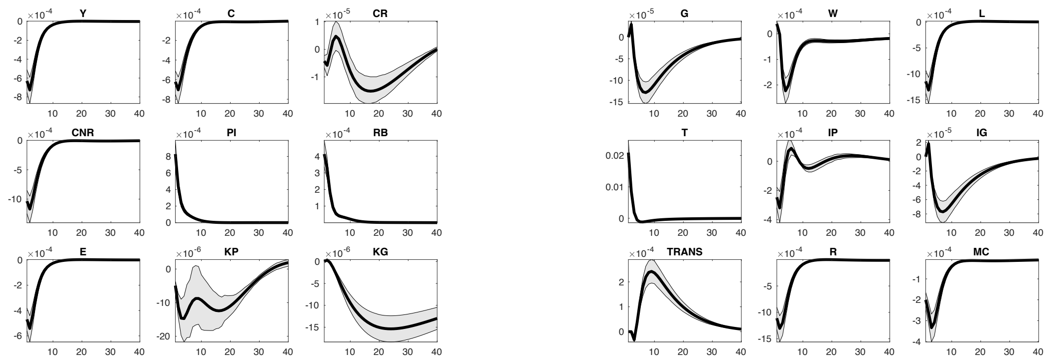


Orthogonalized shock to technology

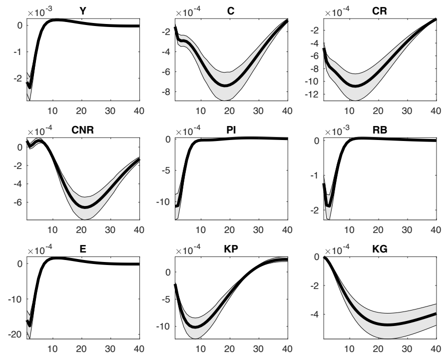
C. Some Bayesian IRFs



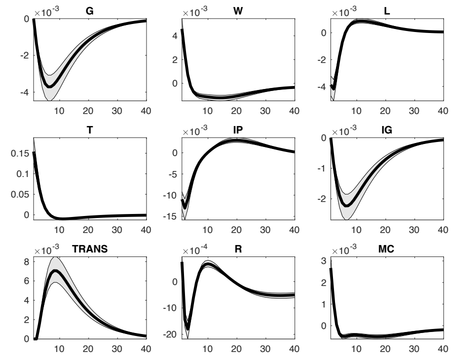
Orthogonalized shock to capital tax



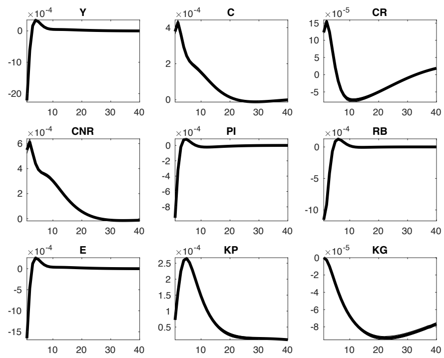
Orthogonalized shock to labor tax



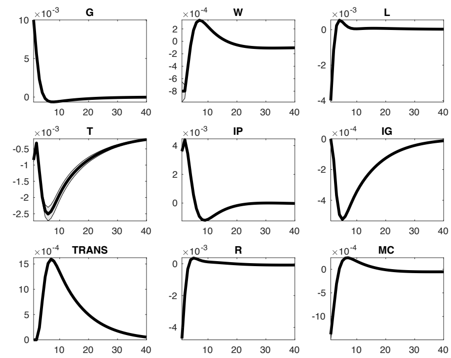
Orthogonalized shock to consumption tax



Orthogonalized shock to consumption tax

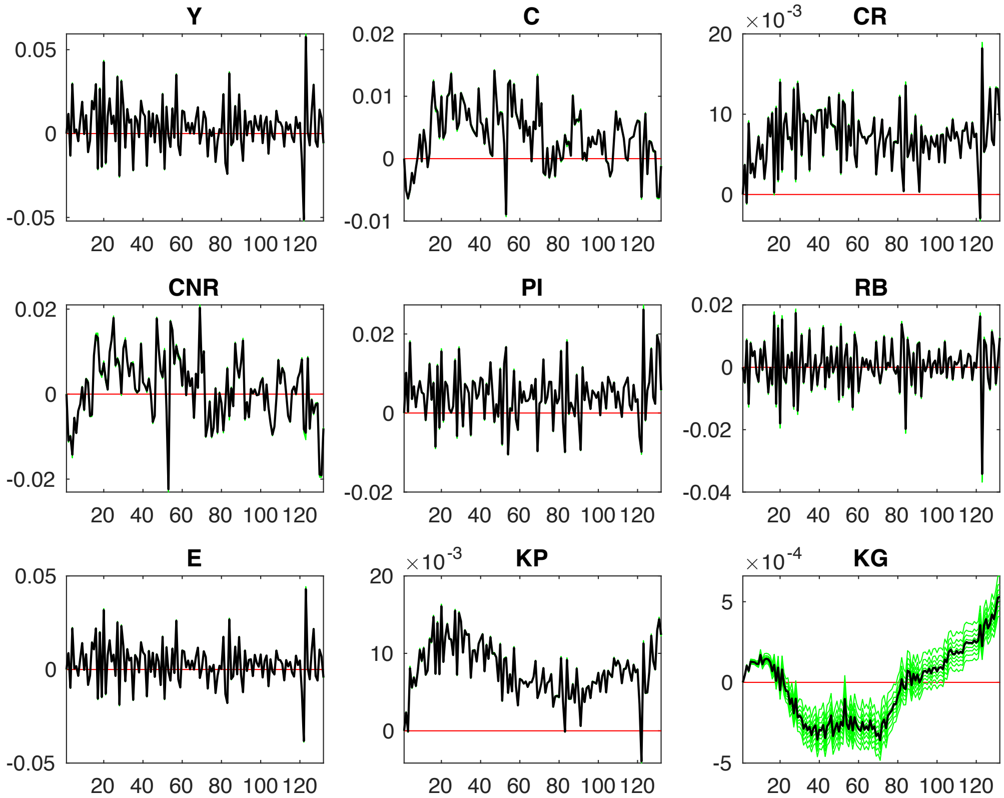


Orthogonalized shock to government expenditure



Orthogonalized shock government expenditure

D. One-quarter ahead forecasts of filtered variables



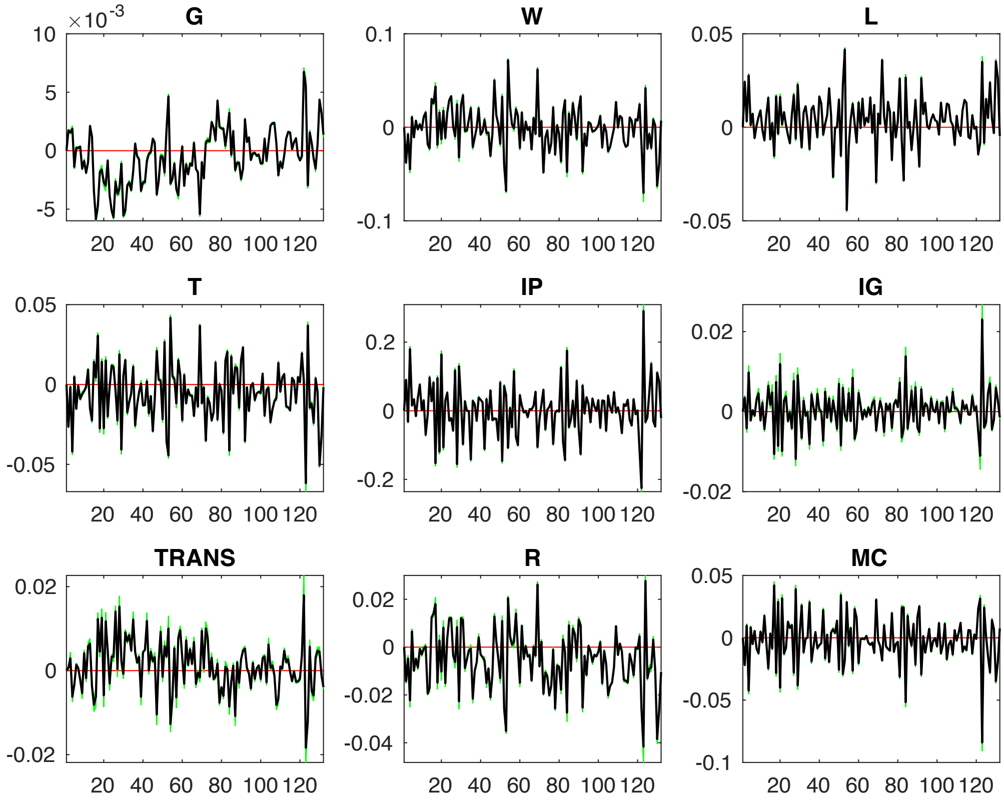


Fig. One-step ahead forecast of the filtered variables

E. Smoothed variables

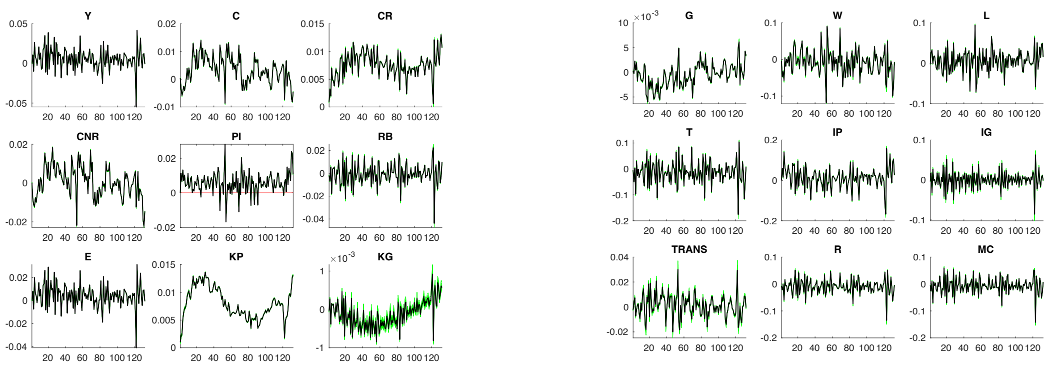


Fig. Smoothed variables for 120 quarters

F. Smoothed shock processes

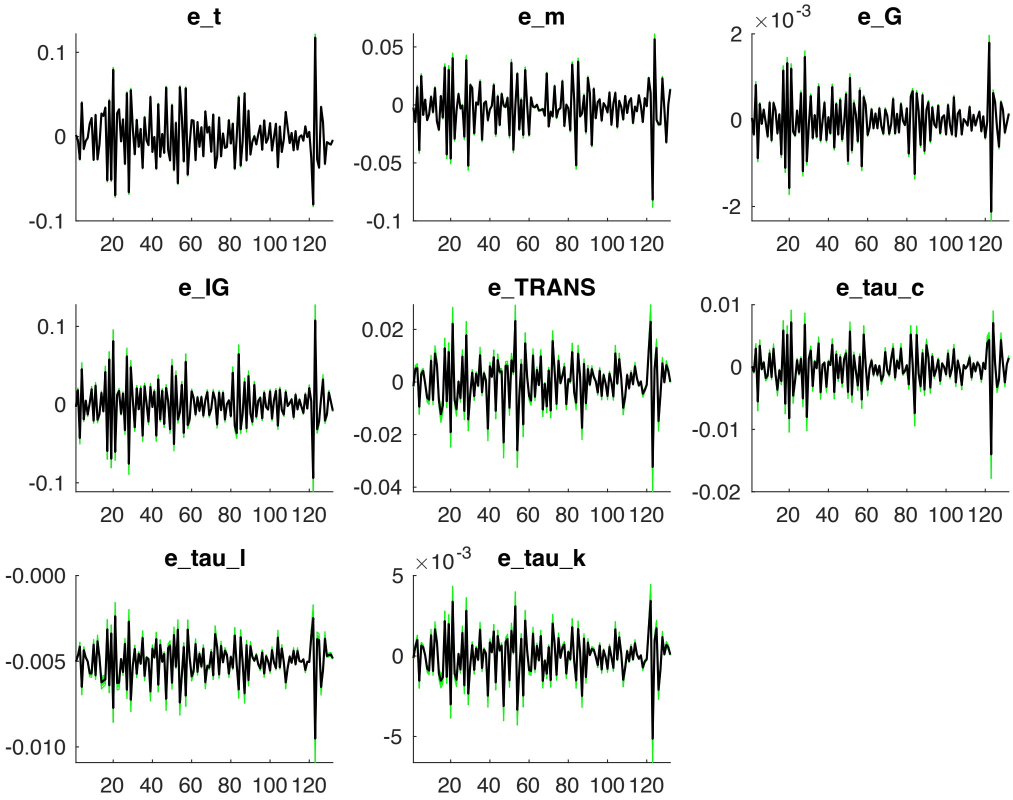


Fig. Smoothed shock processes

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