A Note on the Dimensionless Gravitational Coupling Constant Down to the Quantum Level

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Abstract

In this paper, we rewrite the dimensionless gravitational coupling constant: α_G, in a different form than has been shown before (without changing its value). We demonstrate that the dimensionless gravitational coupling constant is simply related to the Planck length squared, divided by the reduced Compton wavelength squared, of the mass in question. This could be useful for understanding how the ratio of the gravitational force versus the Coulomb force is linked to the quantum scale, which is linked to the Compton wavelength and the Planck length.

Index Terms

Compton wavelength, dimensionless gravitational coupling constant, electron mass, Planck mass, proton mass.

I. THE DIMENSIONLESS GRAVITATIONAL COUPLING CONSTANT

The gravitational coupling constant, often described as the dimensionless gravitational constant or the gravitational fine-structure constant, has been discussed in a series of papers in theoretical physics; see, for example, [2]-[7]. The gravitational coupling constant α_G is defined as the gravitational attraction between a pair of electrons divided by the Coulomb [8] force of two Planck charges, and is normally given by:

$$\alpha_G = \frac{G\frac{m_e^2}{R^2}}{k_e\frac{q_p q_p}{R^2}} = \frac{Gm_e^2}{\hbar c} = \left(\frac{m_e}{m_p}\right)^2 \approx 1.7518 \times 10^{-45} \tag{1}$$

where $\hbar$ is Planck's reduced constant, m_e is the electron mass, m_p is the Planck mass, k_e is the Coulomb constant, and q_p is the Planck charge. The Compton [9], [10] wavelength is given by:

$$\lambda = \frac{h}{mc} \tag{2}$$

and the reduced Compton wavelength is linked to the reduced Planck constant $\hbar = \frac{h}{2\pi}$, which is also known as the Dirac constant by the following relation:

$$\bar{\lambda} = \frac{\hbar}{mc} \tag{3}$$

Compton also experimentally found the Compton wavelength for electrons by using what today is known as Compton scattering. We can solve this formula for m, and this gives:

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{4}$$

In addition, we take adventage of the fact that Newton's gravitational constant[1] can be written on composite form $G = \frac{l_p^2 c^3}{\hbar}$ (see [12]). This formula we simply get by solving the Planck [13], [14] length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ for G. The composite view of G has been discussed for several decades and a review study of it can be found in [15].

An important point about the decades-long discussion in relation to expressing G from the Planck units was that it was not considered fruitful until a very recent breakthrough. The reason was that no one knew how to measure the Planck units without first finding G, and then to express G in the form of Planck units, such as the Planck length, then just led to a circular problem, as pointed out by Cohen [16] in 1987; see also the interesting paper by McCulloch [17].

However, since 2017, we have been able to demonstrate that the Planck length can surprisingly be found totally independently of G see [18]. Later, it was also demonstrated that the Planck length can be measured independently of knowledge of G, c, and $\hbar$; see [19]-[21].

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[1]Actually, Newton never invented or used a gravity constant. It was invented in 1873 by Cornu and Baille [11].

We can now replace the electron mass with $m_e = \frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}$, and the gravitational constant with $G = \frac{l_p^2 c^3}{\hbar}$ in the gravitational coupling constant formula, and we get:

$$\begin{aligned}
\alpha_G &= \frac{G m_e m_e}{\hbar c} \\
\alpha_G &= \frac{\frac{l_p^2 c^3}{\hbar} m_e^2}{\hbar c} \\
\alpha_G &= \frac{m_e^2}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_G &= \frac{m_e^2}{m_p^2} \\
\alpha_G &= \frac{\frac{\hbar^2}{\bar{\lambda}_e^2}\frac{1}{c^2}}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_G &= \frac{\frac{\hbar^2}{\bar{\lambda}_e^2}}{\frac{\hbar^2}{l_p^2}} \\
\alpha_G &= \frac{\frac{1}{\bar{\lambda}_e^2}}{\frac{1}{l_p^2}} \\
\alpha_G &= \frac{l_p^2}{\bar{\lambda}_e^2}
\end{aligned} \tag{5}$$

where $\bar{\lambda}_e$ is the reduced Compton wavelength of the electron. The Compton wavelength of the electron is the rest-mass wavelength of the electron and the reduced Compton wavelength is this length divided by 2π; that is, $\frac{\lambda_e}{2\pi}$. Further, l_p is the Planck length, which is actually the reduced Compton wavelength of the Planck mass. The gravitational coupling constant is simply the reduced Compton wavelength of the Planck mass squared, divided by the square of the reduced Compton wavelength of the electron.

$$\alpha_G = \frac{(\text{Reduced Compton length Planck mass})^2}{(\text{Reduced Compton length mass of interest})^2} \tag{6}$$

The meaning of the term "dimensionless" in the gravitational coupling constant simply infers that its value will not change even if we change our unit systems for length, time, and kg, and thereby change the units for mass, the speed of light, and the Planck constant.

As Max Planck [13] showed us that the Planck mass is given by $m_p = \sqrt{\frac{\hbar c}{G}}$, we naturally have $m_p^2 = \frac{\hbar c}{G}$, so we naturally also have:

$$\alpha_G = \frac{G m_e^2}{\hbar c} = \frac{m_e^2}{\frac{\hbar c}{G}} = \frac{m_e^2}{m_p^2} \tag{7}$$

This result has been pointed out for example [22]. What is new here is that at the deepest level, this simply means we have $\frac{l_p^2}{\bar{\lambda}_e}$, since we must have $\frac{l_p^2}{\bar{\lambda}_e} = \frac{m_e^2}{m_p^2}$, since all kilogram masses can be expressed as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$; see [25]. For electrons, this is perhaps not so surprising, but we will next look at protons and see that this principle holds there.

It is important to be aware that the gravitational coupling constant is not the same for different masses. Silk also mentioned the gravitational coupling constant for two protons, and he described this as:

$$\alpha_G = \frac{G m_{pr}^2}{\hbar c} \approx 5.9 \times 10^{-39} \tag{8}$$

where m_{pr} is the proton mass. Silk, in this paper, stated that this can be seen as "*the ratio of gravitational to electromagnetic forces between a pair of protons multiplied by the atomic fine structure constant* $\alpha(= e^2/\hbar c)$*, and equal to* 5.9×10^{-39}".

The literature that also discusses the Compton wavelength of the proton reaches quite far back in time; see Levitt [23] as well as Trinhammer and Bohr [24]. Actually, all kilogram masses can be written as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ as demonstrated by Haug [25]. This mean we can re-write the gravitational coupling constant between two protons as:

$$\alpha_G = \frac{G m_{pr}^2}{\hbar c} = \frac{l_p^2}{\bar{\lambda}_{proton}^2} = \frac{1.61623 \times 10^{-35}}{2.10309 \times 10^{-16}} \approx 5.90595 \times 10^{-39} \tag{9}$$

It has also been pointed out by Burrows and Ostrike [6] that this is equal to $\alpha_G = \frac{Gm_{pr}^2}{\hbar c} = \frac{m_{pr}^2}{m_p^2}$, this has likely been mention long time before them, it would be interesting to find the first source mentioning this.

That is, the dimensionless gravitational coupling constant at the deepest level is the Planck length squared, divided by the reduced Compton wavelength of the proton squared. The Planck length has been suggested as being related to the reduced Compton wavelength of the Planck mass particle that again seems to be the building block of all matter in a recent quantum gravity theory; see [26].

Further, the gravitational coupling constant between two Planck masses can be seen as the gravitational force between two Planck masses divided by the Coulomb force of two Planck charges:

$$\alpha_G = \frac{\frac{Gm_pm_p}{R^2}}{k_e\frac{q_pq_p}{R^2}} = \frac{Gm_pm_p}{\hbar c} \tag{10}$$

This we can re-write as:

$$\begin{aligned}
\alpha_G &= \frac{Gm_pm_p}{\hbar c} \\
\alpha_G &= \frac{\frac{l_p^2c^3}{\hbar}m_p^2}{\hbar c} \\
\alpha_G &= \frac{m_p^2}{\frac{\hbar^2}{l_p^2}\frac{1}{c^2}} \\
\alpha_G &= \frac{m_p^2}{m_p^2} \\
\alpha_G &= \frac{l_p^2}{l_p^2} = 1
\end{aligned} \tag{11}$$

That the gravitational coupling constant is normally assigned such a low value 1.7518×10^{-45} between two electrons, or as 5.90595×10^{-39} between two protons, simply has to do with the low masses of electrons and protons relative to the Planck mass. It would make more sense to say that the fundamental gravitational coupling constant is linked to two Planck masses and that its value is one. It basically shows that the gravitational force is very strong between two dense bodies that are very close to each other.

II. Conclusion

We have demonstrated that, at the deepest level, the dimensionless gravitational coupling constant can be represented simply as the Planck length squared, divided by the reduced Compton wavelength of the relevant mass squared. This seems to bring us closer to understanding the gravitational force relative to the Coulomb force at the Planck scale.

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