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Quantum Gravitational Energy Simplifies Gravitational Physics and Gives a New Einstein Inspired Quantum Field Equation without *G*

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Abstract

We show the simplest form with which one can express the gravity force, and that still gives all the same predictions of observable phenomena as does standard Newton gravity and general relativity theory. In addition, we show a new field equation that gives all the same predictions as general relativity theory, but that it is simpler as the only constant needed is the speed of light and that also gives quantum gravity. This new form to express gravity, through quantum gravitational energy, requires less constants to predict gravity phenomena than standard gravity theory. This alone should make the physics community interested in investigating this approach. It shows that gravitational energy, and other types of energy are a collision-length in their most complete and deepest form and that quantization of gravity is related to the reduced Compton frequency of the gravitational mass per Planck time. While general relativity theory needs two constants to predict gravity phenomena, that is G and c, our new theory, based on gravity energy, only needs one constant, c_g, that is easily found from gravitational observations with no prior knowledge of any constants. Further, we will show that, at the deepest quantum level, quantum gravity needs two constants, c_g and the Planck length, while the standard formulation here needs c, $\hbar$ and l_p. Thus our theory gives a reduction in constants and simpler formulas than does standard gravity theory. Most important we by this seems to have a fully consistent framework for quantum gravity.

Keywords

Quantum Gravity, Gravity Force, Newton Gravity, General Relativity Theory, Gravitational Energy, Gravity Constant

1. Short about the Modern Newtonian Formula

Today, Newton's gravitational force formula, as found in modern text books and research papers, is given by:

$$F = G\frac{Mm}{R^2} \tag{1}$$

where M and m are the mass of the large and the small mass in kilograms, G is the gravitational constant, and R is the center-to-center distance from M to m. It is of great importance to understand that the gravity force itself has never been directly observed, but only indirectly through observable gravitational phenomena, and the gravity force is not among these. In all observable gravitational phenomena predicted from this formula, the small mass m always cancels out in the derivation of something that actually can be observed, something we soon will look at in detail[1]. Also be aware that this version of the gravity formula with a gravity constant was introduced in 1873 by Cornu and Baille [1]. They were the first to introduce the gravitational constant and used notation f for it. Boys [2] in 1894 were likely the first to use the symbol G for the gravity constant. Isaac Newton's original gravity force formula was:

$$F = \frac{M_n m_n}{R^2} \tag{2}$$

as he only stated by word in *Principia* [3]. However, Newton's mass definition was very different than today's kilogram mass definition (so M_n and m_n, in this formula, mean something different); see [4] for an in-depth analysis of the original Newton gravity force formula. This formula was used all the way to 1873 as is clear also from the book of Maxwell [5] "*A Treatise on Electricity and Magnetism*" where Maxwell is clear on that astronomical masses has dimensions $L^3 \cdot T^{-2}$. Maxwell is clear on that this mass can be found from any observable gravitational formula (Newton) for example from gravitational acceleration that he describe as $g = \frac{M_n}{R^2}$. Since g easily can be measured without knowledge of G, for example by dropping a ball from the height H. The gravitational acceleration can then be found as $g = \frac{2H}{T_d^2}$ where T_d is the time it took.

The so-called Newton's gravitational constant was pointed out by Thüring [6] in 1961 to have been introduced somewhat ad-hoc; see also Gillies [7]. Thüring pointed out that the gravitational constant cannot be associated with a unique property of nature. The gravitational constant has dimensions $|G| = \frac{L^3}{MT^2}$ or, in SI units, m^3·kg^{-1}·s^{-2}. In nature this is, at a fundamental level, length cubed divided by mass times time squared. One must have a good imagination to come up with something physical that fits the bill. We think it is no coincidence that the gravitational constant has such output units. We have good reasons to think

[1]In real, two-body problems where both masses act significantly on each other, then the gravity parameter is changed to $G(M_1 + M_2) = GM_1 + GM_2$.

it is a composite constant, something that is discussed in detail in Haug [8].

Einstein's [9] general relativity theory took Newton's gravitational constant for granted. Already, in the same 1916 paper on general relativity, Einstein pointed out that the next step in gravity was to develop a quantum gravity theory or, in his own words:

"*Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation*. —A. Einstein

One should clearly still be allowed to question even the very foundation of gravity theories. We think that in particular one should be allowed to question the gravitational constant, what it represents, and if it can be replaced, as it was inserted ad-hoc in 1873. Einstein also mentioned *gravitational energy*. Here we will demonstrate that one can easily come up with a formula that replaces the modern Newton's formula using G and kilogram masses as well as the Einstein field equation, with two formulas based on gravitational energy and the speed of this gravitational energy, which we will see is the speed of gravity that again is identical to the speed of light. Further, our suggestions in this paper are directly linked to a recent quantum gravity theory known as collision-space-time [10] [11] [12], something we will get back to soon.

2. A New and Simpler Gravitational Formula Rooted in Gravitational Energy

We introduce a new formula for gravitational force that can replace Newton's formula. Later in the paper, we also present an improved field equation that can replace Einstein's general relativistic field equation. Our new gravity force formula is as follows:

$$F = c_g \frac{E_g \mathcal{E}_g}{R^2} \tag{3}$$

where E_g and $\mathcal{E}_g$ are the gravitational energy of the large and small mass (M_g and m_g). The gravitational energy is in form of collision-length as defined in collision space-time [10] [11] [12]. This length is unknown for any mass, but we will soon show how to measure it. Secondly, c_g is the speed of gravity, which we can also easily find from gravity observations. We will also demonstrate that the formula above will remarkably give exactly the same predictions for observable gravity phenomena as Newton's gravitational force formula $F = G\frac{Mm}{r^2}$, both in values and in output units. The only exception is for non-observable phenomena, such as the gravity force itself, something we will soon come back to.

This new gravity force formula has output units m·s^{-1} in SI units, or in dimensions $|F| = \left[L \cdot T^{-1}\right]$. In other words, the gravity force is a speed in this formulation. This is in contrast to the standard modern version of Newton's gravitational force which has output kg·m·s^{-2}. So, one could easily make the mistake of thinking that our gravity force formula must be wrong as it doesn't even match the output units for the standard gravity force. A basic first check in physics, that even I myself typically use when coming up with a formula, is whether one has at least got the output units right. If not, that is typically a sign one has done something wrong or based the derivation on wrong assumptions. Still, the gravity force has never been observed, so the output units have partly been arbitrary chosen, they where for example not the same in Newton's time, he did not even have a gravity constant in his formula. What is important is that the formula predicts accurately everything that can be observed with respect to both values and, naturally, the correct output units. Also, our new gravitational energy has dimension of length. This seems totally inconsistent with standard joule energy. However, as we [12] have already demonstrated in a paper on a new quantum gravity theory, this view is fully consistent with such things as the standard relativistic energy momentum relation.

It is, as we have demonstrated, true that both standard energy (joule) and Einstein's relativistic energy momentum relations are derivatives of simpler and deeper relations.

We will show that this gravitational model can be calibrated and used to predict a long series of observable gravitational phenomena. We have previously [10] [13] shown that a formula that predicts the same as Newton's gravitational force formula is given by:

$$F = c_g^3 \frac{M_g m_g}{R^2} \tag{4}$$

where M_g is the collision-time mass. That is, indeed mass as time which we come to by also incorporating gravity in the mass, something that is missing in the standard kilogram mass. Well, it is the duration of the aggregates of the collisions of the indivisible particles making up the mass; see [10] for detailed discussion. Further, this means we have $E_g = M_g c_g$, which at first glance seems to be totally inconsistent with $E = mc^2$, but it is not; it is fully consistent also with this. The reason for the difference is simply different energy and mass definitions.

For example, we could define a new energy $E_2 = E/c$. Then we would have $E_2 = mc$. There is nothing mathematically wrong with this as it is simply a change of units in the energy done in a consistent way. However, why should energy be re-defined as joule divided by the speed of light? A re-definition of energy and/or mass must be able to explain something new or make things more intuitive. Simply by taking the joule energy and dividing by c to define a new energy does not seem to simplify intuition or teach us something new. Our new energy definition of E_g, on the other hand, is just a length, and we can go from the joule energy to this energy (collision-length) by multiplying the joule energy, E, with $\frac{l_p^2}{\hbar c}$, and in $E = Mc^2$ we need to do the same on both sides, so we get

$M_g = M\frac{l_p^2}{\hbar}$. It is hard to see intuition here yet, except that we end up with a length for energy and time for mass. Length is something most of us find easier to understand than joule. To go from E and M to E_g and M_g in the way just described, one needs to know $\hbar$ and l_p and also c. So, is this not some fancy change of units? As we soon will see, E_g (and also M_g) can easily be extracted from gravity phenomena with no knowledge of G, h or c. We will end up needing knowledge of less constants than are used in Newton's and Einstein's gravity to make the same predictions. Just to briefly demonstrate that our new energy and mass definitions are consistent with the relativistic energy momentum relation, we must have:

$$\begin{aligned}
E_g &= m_g \gamma c_g \\
E_g^2 &= m_g^2 c_g^2 \gamma^2 \\
E_g^2 &= m_g^2 c^2 \gamma^2 - m_g^2 c^2 + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - m_g^2 c^2 + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - \frac{m_g^2 c^2 \left(1 - v^2/c^2\right)}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - \frac{m_g^2 c^2 - m_g^2 v^2}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 v^2}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= p_g^2 + m_g^2 c^2 \\
E_g &= \sqrt{p_g^2 + m_g^2 c^2} \qquad (5)
\end{aligned}$$

where $p_g = m_g v \gamma$ is the relativistic gravitational momentum and γ is, as usual, the Lorentz factor, $\gamma = 1/\sqrt{1 - v^2/c^2}$. That is, it's the same as the standard relativistic momentum except m is replaced with m_g. Now it is only necessary to multiply each side of Equation (5) with $\frac{\hbar}{l_p^2}c$ (or $\frac{c^4}{G}$) as well as setting $c_g = c$ (as we know it is from measurements and theory) and we end up with the standard $E = \sqrt{p^2c^2 + m^2c^4}$. We will claim the standard mass and standard energy are incomplete mass and energy definitions; they can almost be seen as derivatives of a deeper theory, where the deeper relation is the first line in the derivation above. The incomplete mass and energy are enough to describe energy and mass relations not related to gravity, but they fall short when we work with gravity.

The standard mass and energy have no information about the Planck scale as the Planck length embedded in E_g is taken out, something that will soon be clearer. We will claim the Planck scale is the essence of gravity. When the Planck scale is not incorporated in standard mass and energy, this is, in our view, one of the main reasons why one has not been able to unify gravity and quantum mechanics, at least until perhaps very recently; see [12] for a much more in-depth discussion about this point.

Actually the speed of gravity c_g constant which by calibration only from gravity phenomena can be shown to be equal to the speed of light can be seen as a pure scaling factor that adjust for human conventions in how we define length in relation to time. The speed of gravity can be found without knowledge of the speed of light and without any detection of gravitational waves, this have recently been demonstrated, see [13] [14]. If we take the standard gravity force formula and multiply it with R we get $G\frac{Mm}{R}$ which has output units joule, which is energy. In our formula if we multiply it with R we get $c_g\frac{E_g E_g}{R}$ which gives output units when using meters and seconds as meters squared divided by second. However if we set $c_g = 1$ then we get output dimensions $\frac{E_g E_g}{R}$ which is length as one of the collision-length energies dimensions cancel with the length dimensions of R, which is collision-length energy, when space and time are connected through the speed of light.

3. Finding the Speed of Gravity and the Gravitational Energy without any Knowledge off *G*, *h* or *c*

In our new gravitational model, we need to know E_g and c_g. They are both unknown, so even if we can assume $c_g = c$, we want to see if we can find it "experimentally", with no knowledge of c, and in a simple way by utilizing the implications of our theory. The radius R from the center of the gravitational object to the center of the small mass the gravitational field acts on, can typically be easily measured directly or indirectly. Remarkably, there is an easy way to find both the speed of gravity c_g and the gravitational energy without any prior knowledge of any constants or of the mass of the gravitational object. Also, in our formulation we must have:

$$m_g a = F \tag{6}$$

That, when we replace F with our new gravity force formula, leads to:

$$m_g a = c_g \frac{E_g E_g}{R^2} \tag{7}$$

and in our theory we have $m_g = \frac{E_g}{c}$, and as discussed in the section above this is fully consistent with $E = mc^2$, so we get

$$\frac{E_g}{c_g} a = c_g \frac{E_g E_g}{R^2}$$

$$a = c_g^2 \frac{E_g}{R^2} \tag{8}$$

That is, the gravitational acceleration field is given by $g = a = c_g^2 \frac{E_g}{R^2}$. This only dependent on one constant: the speed of gravity, c_g. There are two unknowns here, both c_g and the gravitational energy E_g of the mass in question. We could as standard theory assume $c_g = c$, something that also have been experimentally tested to at least be very close, but we will based on our own theory show it is a much easier way to extract c_g from gravity observations and therefore demonstrate we are totally independent on any assumptions about the value for c_g. Let us solve the gravitational acceleration field with respect to E_g; this gives:

$$E_g = \frac{gR^2}{c_g^2} \tag{9}$$

However, we still do not know c_g so we cannot, from this, find E_g yet. The gravitational red shift for a beam of light sent in a gravitational acceleration field from R_h to R_L ($R_h > R_L$) is given by:

$$z = \frac{f_h - f_L}{f_L} = \frac{\sqrt{1 - \frac{2E_g}{R_L}}}{\sqrt{1 - \frac{2E_g}{R_h}}} - 1 \tag{10}$$

Next, replace E_g with $E_g = g\frac{R^2}{c_g^2}$ in the formula above and we get:

$$z = \frac{f_h - f_L}{f_L} = \frac{\sqrt{1 - \frac{2g_L \frac{R_L^2}{c_g^2}}{R_L}}}{\sqrt{1 - \frac{2g_L \frac{R_L^2}{c_g^2}}{R_h}}} - 1 \tag{11}$$

$$z = \frac{\sqrt{1 - \frac{2g_L R_L}{c_g^2}}}{\sqrt{1 - \frac{2g_L R_L^2}{R_h c_g^2}}} - 1 \tag{12}$$

Solved with respect to c_g this gives:

$$c_g = \frac{\sqrt{2g_L R_L \left(2R_L + R_L z - \frac{R_h - R_L}{z}\right)}}{\sqrt{R_h (2 + z)}} \tag{13}$$

That is, to find the speed of gravity all we need to do is to measure the gravitational acceleration at the surface of the Earth, for example at sea level (R_L), and then also measure the gravitational red-shift from a laser beam going from R_h to R_L, where $R_h > R_L$. This result in itself is remarkable, because it means one can measure the speed of gravity easily by combining two types of gravitational observations, and thus there is no need for advanced LIGO measurements of gravitational waves to do this; see also [13]. Inputting measured values into this formula reveals that the speed of gravity, c_g, is indeed identical to the speed of light, c, as also assumed in general relativity theory and it seems confirmed by complex experiments; see [15] [16].

To measure the speed of gravity c_g, all we need is a measurement of the gravitational acceleration and the gravitational red shift. This can easily be done without any knowledge of any other constants. One can easily misunderstand here and think we are simply getting out c as we have inputted c, but this is not the case. In standard physics, we have $g = \frac{GM}{R^2}$ and $z \approx \frac{GM}{c^2 R}$, so one could think we are here getting c out since it is an input in the red shift formula. However, this is only if one predicts the gravitational red shift. There is no need to know c to measure the frequency shift in a light beam in a gravitational field; we only need to know c to predict the gravitational red shift in standard theory. However, here we are not predicting these, but measuring them and then finding c_g, which indeed has the same value as c. Based on our deeper understanding of our theory, we also know that $c_g = c$, naturally based on some assumptions; see [12].

Next, we can now find E_g from formula 9 as we now know c_g, g, and R. As soon as we know E_g and c_g, we can predict all kinds of other observable gravitational phenomena from the gravitational object's gravitational energy, E_g, which is the gravitational energy of the large mass. For example, we can predict all types of gravitational effects from the Earth. A long series of predictions we can do are illustrated in **Table 1**. Some of these can only be predicted by our new field equation that we soon will come to. Here we show both the modern standard Newton and GR formulations as well as the predictions from our new framework. For all observable phenomena, they give the same output in values and in terms of output units. Actually, the standard method is less accurate as it needs G that typically is calculated first from a smaller test mass, for example by using a Cavendish apparatus. This will add an additional measurement error when we work with larger masses such as, for example, the mass of the Earth, see [17] for a detail analysis. The formulas on the right hand side of the table below the line *Frequency Newton spring* we can still not get from our Newton type formula, for that we need a more advanced theory with a field equation that will be given in Section 7.

4. What about Quantum Gravity Energy?

At a deeper level, the gravitational energy (collision-length) is given by (see [12]):

$$E_g = l_p \frac{l_p}{\bar{\lambda}} \tag{14}$$

where l_p is the Planck length first described by Max Planck [18] [19] in 1899, and $\bar{\lambda}$ is the reduced Compton [20] wavelength. Max Planck introduced the

Table 1. The table shows that all observable gravity is linked to GM in the standard Newton and general relativity formulation. Pay attention to how standard gravity theory needs knowledge of two constants to predict gravity phenomena, namely G and the speed of light. The alternative theory only needs one constant, namely c_g. The gravitational energy E_g as well as c_g can be found directly from gravity observations without knowledge of any known physical constants. Standard theory needs g, c, and M. Both theories naturally, in addition, need to know the distance to the center of the gravitational object.

	Standard:	**Alternative:**
Mass	M (kg)	M_g (collision-time mass)
Energy	$E = Mc^2$ (joule)	$E_g = M_g c_g$ (gravitational energy)
Gravitational constant	G	c_g m/s
Non observable (contains GMm)		
Gravity force	$F = G\frac{Mm}{R^2}\left(\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2}\right)$	$F = c_g \frac{E_g E_g}{R^2}$ m/s
Observable predictions, identical for the two methods: (contains only GM)		
Gravity acceleration	$g = \frac{GM}{R^2}$	$g = c_g^2 \frac{E_g}{R^2}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}}$	$v_o = c_g\sqrt{\frac{E_g}{R}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$	$T = \frac{2\pi\sqrt{R^3}}{c_g\sqrt{E_g}}$
Velocity ball Newton cradle	$v_{out} = \sqrt{2\frac{GM}{R^2}H}$	$v_{out} = \frac{c_g}{R}\sqrt{2E_g H}$
Periodicity Pendulum (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi R\sqrt{\frac{L}{GM}}$	$T = \frac{2\pi R}{c_g}\sqrt{\frac{L}{E_g}}$
Frequency Newton spring	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}}$	$f = \frac{c_g}{2\pi R}\sqrt{\frac{E_g}{x}}$
Gravitational red shift	$z = \frac{\sqrt{1-\frac{2GM}{R_1 c^2}}}{\sqrt{1-\frac{2GM}{R_2 c^2}}} - 1$	$z = \frac{\sqrt{1-\frac{2E_g}{R_L}}}{\sqrt{1-\frac{2E_g}{R_h}}} - 1$
Time dilation	$T_R = T_f\sqrt{1-\frac{2GM}{Rc^2}}$	$T_R = T_f\sqrt{1-\frac{2E_g}{R}}$
Gravitational deflection (GR)	$\delta = \frac{4GM}{c^2 R}$	$\delta = \frac{4E_g}{R}$
Advance of perihelion	$\sigma = \frac{6\pi GM}{a\left(1-e^2\right)c^2}$	$\sigma = \frac{6\pi E_g}{a\left(1-e^2\right)}$
Micro lensing	$\theta = \sqrt{\frac{4GM}{c^2}\frac{d_s - d_L}{d_s d_L}}$	$\theta = 2\sqrt{E_g\frac{d_s - d_L}{d_s d_L}}$
Indirectly/"hypothetical" observable predictions: (contains only GM)		
Gravitational parameter	$\mu = GM$	$\mu = c_g^2 E_g$
Two body problem	$\mu = G\left(M_1 + M_2\right)$	$\mu = c_g^2\left(E_{g,1} + E_{g,2}\right)$
Constants needed	G, c	Only c_g $(c_g = c)$

Planck length in 1899 by the formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$. So, it looks like we need to know G and $\hbar$ to find the Planck length. In recent years it has been shown this is not the case. The Planck length can be found totally independently of G and $\hbar$; see [13] [21] [22]. We will also, in this paper, devote a short section on how to find the Planck length independently of knowledge of G and h.

So the first l_p in the gravitational energy formula represents the collision-length of a single collision (Planck event). This collision-length of one Planck event is always the Planck length; see [11] [23] for a discussion of why it is invariant. The next part, $\frac{l_p}{\bar{\lambda}}$, is the number of Planck mass events in the gravitational mass over an observational time window of the Planck time. It is not that we need to observe anything in the Planck time, but it is what this represents, and we can measure it indirectly. The factor $\frac{l_p}{\bar{\lambda}}$ is the reduced Compton frequency per Planck time. The reduced Compton frequency per second is $\frac{c}{\bar{\lambda}}$, and to get the reduced Compton frequency per Planck time we need to multiply with the Planck time that gives us $\frac{c}{\bar{\lambda}} t_p = \frac{c}{\bar{\lambda}} \frac{l_p}{c} = \frac{l_p}{\bar{\lambda}}$.

For a Planck mass then, the reduced Compton wavelength is the Planck length, and then this factor is 1. For a mass smaller than the Planck mass, then $\frac{l_p}{\bar{\lambda}}$ is less than one. It is then a probability for a Planck mass event in the Planck time. Elementary particles consist, in this view, of Planck mass events (collisions) happening at the reduced Compton frequency. If the gravitational mass is larger than the Planck mass (and it is typically much larger) then $\frac{l_p}{\bar{\lambda}}$ typically consist of a large integer number plus a small fraction. The integer part then represents the number of Planck mass events in the Planck time. In other words, this is where the quantization comes in. The quantization is linked to the Planck length, that in our view is the diameter of an indivisible particle; see [10] [11].

Our simple gravitational force formula $F = c_g \frac{E_g E_g}{R^2}$ has embedded quantum gravity. That is, gravitational energy and gravitational mass come in quanta. The quanta is linked to the Planck length and the reduced Compton wavelength through the factor $\frac{l_p}{\bar{\lambda}}$ that is embedded in the gravitational energy as it is, at the deepest level, described by Equation (14). We will even claim standard Newton gravity has hidden quantum gravity in it, not on purpose or by design, but by coincidence, as gravity when calibrating G to observations incorporates l_p; see a lengthy discussion and review of the composite view of the gravitational constant view by Haug [8].

Further the gravitational mass is equal to

$$M_g = \frac{E_g}{c} = \frac{l_p}{c}\frac{l_p}{\bar{\lambda}} = t_p \frac{l_p}{\bar{\lambda}} \tag{15}$$

In other words mass is time, it is collision-time, It is how many collisions we have in the gravitational mass per Planck time and each collision last the Planck time. Where again $\frac{l_p}{\bar{\lambda}}$ gives the numbers off such events, so even mass is quantized. Again one can easily think it must be inconsistent that mass is simply energy divided by c as it in standard theory is c^2, but this is fully consistent with that as demonstrated in the derivation in the end of section two.

5. Finding the Planck Length and the Compton Wavelength Independently of Any Knowledge of G, $\hbar$, and Even c

We can do all gravitational predictions simply from the gravitational energy, E_g, and the speed of gravity, c_g, and these we have already shown how to find. It is only when we want to understand the deeper aspects of E_g and M_g that we need the Planck length. The Planck length can easily be found without any knowledge of G, c and $\hbar$; something that is controversial in standard gravity theory, but that we have recently demonstrated, in a series of published papers, is possible. Still, we will also demonstrate here that it is also possible when we write our formulas from gravitational energy.

The gravitational acceleration as before (Section 3) is given by:

$$g = c_g^2 \frac{E_g}{R^2} \tag{16}$$

Further, E_g at the quantum level is given by $E_g = l_p \frac{l_p}{\bar{\lambda}}$, where $\bar{\lambda}$ is the reduced Compton wavelength of the gravitational energy in question. We can now simply solve the formula above for l_p, and this gives:

$$l_p = \sqrt{g \frac{R^2}{c_g^2} \bar{\lambda}} \tag{17}$$

That is, to find the Planck length independently of G and $\hbar$, we need to find g and the reduced Compton wavelength independently of these. The gravitational acceleration we can find simply by measuring how long it takes for a ball dropped at height H above the ground to hit the ground. It is given by $g = \frac{2H}{T^2}$.

Still, how do we find the reduced Compton wavelength of, for example, the Earth? The reduced Compton wavelength [20] for a fundamental particle like the electron is given by:

$$\bar{\lambda} = \frac{\hbar}{mc} \tag{18}$$

That is, if you know the kilogram mass of the particle m, the reduced Planck constant, and the speed of light. However, it is not necessary to know the kilogram mass of the particle or the Planck constant to find the Compton wavelength (or reduced Compton wavelength). In photon scattering of an electron, we have:

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos\theta} \tag{19}$$

where λ_e is the Compton wavelength of the electron, and $\lambda_{\gamma,1}$ and $\lambda_{\gamma,2}$ are respectively the wavelength of the incoming and outgoing photon; see [24]. So, there is no need to know the Planck constant to measure the Compton wavelength. The reduced Compton wavelength is simply this divided by 2π.

Next we can utilize the knowledge that electrons and protons have the same absolute value of the charge. The cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \tag{20}$$

This means we must have:

$$\frac{f_e}{f_{Pr}} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_{Pr}}} = \frac{m_{pr}}{m_e} = \frac{\bar{\lambda}_e}{\bar{\lambda}_P} \approx 1836.15 \tag{21}$$

That is, to find the reduced Compton wavelength of the proton we can simply take the reduced Compton wavelength of the electron and divided it by 1836.15. This is why cyclotron experiments have also been used to find the proton electron mass ratio; see [25] [26] [27]. Research on the proton Compton wavelength goes back to at least 1958 in the paper of Levitt [28]. Next, we can simply count the number of protons in the mass in question, divide the Compton wavelength of the single proton by this number, and we have the Compton wavelength of the mass in question. For simplicity and even practical purposes, we can treat neutrons as the same mass as protons, or alternatively by doing the small corrections for the slightly different mass. This way of finding the Compton wavelength of a large mass ignores nuclear binding energy (see for example D'Auria [29]), but this will, at a maximum, give an error in the Compton wavelength of about 1%. This is naturally a considerable additional error, but we can even more-or-less remove it by treating the binding energy as mass equivalent, $m = E/c^2$, and adjust for it.

To count the number of atoms in a mass is, in practice, no easy task, but for smaller macroscopic masses one can also, in practice, count the number of atoms. This was one of the competing methods to re-define the kilogram; see [30] [31] [32] [33]. Silicon 28 has a very uniform crystal structure, so if one can count the number of atoms in a very small volume of this material and next create a very accurate sphere of such material, then one can find with high precision the number of atoms in this sphere. Such a sphere can then, for example, be used in a Cavendish apparatus to measure gravitation effects, such as the gravitational acceleration from this uniform sphere.

Next, to find the Compton wavelength of much larger objects like, for example, the Earth, we can utilize the following relation:

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1} \tag{22}$$

After we have done this, we know the Planck length as well as the Compton wavelength of the gravitational object in question and we can next predict all gravity phenomena by using only two constants, namely the Planck length and the speed of gravity (light) plus variables such as R. We can now actually also directly see that the modern Newton formulation and our new gravity theory are, at the deepest level, identical. To see this, we need to replace G in the standard gravitational framework with $G = \frac{l_p^2 c^3}{\hbar}$, which is simply the Max Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ solved with respect to G; see [8] [34]. One should be aware that the idea of expressing G from Planck units goes back to at least 1984 when Cahill [35] [36] suggested $G = \frac{\hbar c}{m_p^2}$. However, as pointed out by Cohen [37] in 1987, this seemed to lead to a circular problem, as it seemed one had to know G to find m_p, so to express G from Planck units seemed to be useless. This has been a view held until very recently. It was first in 2017 that we showed one could find the Planck length independently of any knowledge off G. In addition to writing G as a composite constant, one needs to solve the reduced Compton wavelength formula with respect to m. This means we can express any kilogram mass as $m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c}$. Now, by inserting this composite gravity constant as well as this way to express the mass in the standard Newtonian framework, we see that it leads to exactly the same formulas for all observable phenomena as our new gravitational energy framework. In other words, they give the same output both in terms of values and units as can be seen from **Table 2**. Non-observable phenomena such as the gravitational force itself have different output units in the two approaches.

Table 2. The table shows that all observational gravity phenomena are predicted from GM and not GMm. The gravity constant is needed to remove the embedded Planck constant in the kilogram mass and to get in the Planck length. Our alternative formula only has a gravity constant that is simply the speed of gravity c_g. From a deeper understanding, we see the two models are the same and both can be represented by the Planck length and the speed of gravity as constants and, naturally, variables.

	Standard:	**Alternative:**
Mass	$M = \frac{\hbar}{\bar{\lambda}_M} \frac{1}{c}$ (kg)	$M_g = t_p \frac{l_p}{\bar{\lambda}_M}$ (collision-time)
Energy	$E = Mc^2$ (joule)	$E_g = M_g c_g = l_p \frac{l_p}{\bar{\lambda}_M}$ (collision-length)
Gravitational constant	$G = \frac{l_p^2 c^3}{\hbar}$	c_g
Gravity force	$F = G\frac{Mm}{R^2}$ $\left(\mathrm{kg \cdot m \cdot s^{-2}}\right)$	$F = c_g \frac{E_g E_g}{R^2}$ m/s

Continued

Observable predictions, identical for the two methods: (contains only *GM*)		
Gravity acceleration	$g=\frac{GM}{R^2}=\frac{c^2 l_p}{R^2}\frac{l_p}{\bar{\lambda}_M}$	$g=c_g^2\frac{E_g}{R^2}=\frac{c_g^2 l_p}{R^2}\frac{l_p}{\bar{\lambda}_M}$
Orbital velocity	$v_o=\sqrt{\frac{GM}{R}}=c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$	$v_o=c_g\sqrt{\frac{E_g}{R}}=c_g\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Orbital time	$T=\frac{2\pi R}{\sqrt{\frac{GM}{R}}}=\frac{2\pi\sqrt{\bar{\lambda}_M R^3}}{c l_p}$	$T=\frac{2\pi\sqrt{R^3}}{c_g\sqrt{E_g}}=\frac{2\pi\sqrt{\bar{\lambda}_M R^3}}{c_g l_p}$
Velocity ball Newton cradle	$v_{out}=\sqrt{2\frac{GM}{R^2}H}=\frac{c}{R}\sqrt{2Hl_p\frac{l_p}{\bar{\lambda}_M}}$	$v_{out}=\frac{c_g}{R}\sqrt{2E_g H}=\frac{c_g}{R}\sqrt{2Hl_p\frac{l_p}{\bar{\lambda}_M}}$
Periodicity pendulum (clock)	$T=2\pi\sqrt{\frac{L}{g}}=2\pi R\sqrt{\frac{L}{GM}}=\frac{2\pi R}{c_g l_p}\sqrt{L\bar{\lambda}_M}$	$T=\frac{2\pi R}{c_g}\sqrt{\frac{L}{E_g}}=\frac{2\pi R}{c_g l_p}\sqrt{L\bar{\lambda}_M}$
Frequency Newton spring	$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi R}\sqrt{\frac{GM}{x}}=\frac{c_g}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\bar{\lambda}_M}}$	$f=\frac{c_g}{2\pi R}\sqrt{\frac{E_g}{x}}=\frac{c_g}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\bar{\lambda}_M}}$
Gravitational red shift	$z=\frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}}-1=\frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\bar{\lambda}_M}}}-1$	$z=\frac{\sqrt{1-\frac{2E_g}{R_1}}}{\sqrt{1-\frac{2E_g}{R_2}}}-1=\frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\bar{\lambda}_M}}}-1$
Time dilation	$T_R=T_f\sqrt{1-\frac{\sqrt{\frac{2GM}{R}}^2}{c^2}}=T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$	$T_R=T_f\sqrt{1-\frac{2E_g}{R}}=T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Gravitational deflection (GR)	$\delta=\frac{4GM}{c^2R}=\frac{4l_p}{R}\frac{l_p}{\bar{\lambda}_M}$	$\delta=\frac{4E_g}{R}=\frac{4l_p}{R}\frac{l_p}{\bar{\lambda}_M}$
Advance of perihelion	$\sigma=\frac{6\pi GM}{a(1-e^2)c^2}=\frac{6\pi l_p}{a(1-e^2)}\frac{l_p}{\bar{\lambda}_M}$	$\sigma=\frac{6\pi E_g}{a(1-e^2)}=\frac{6\pi l_p}{a(1-e^2)}\frac{l_p}{\bar{\lambda}_M}$
Micro lensing	$\theta=\sqrt{\frac{4GM}{c^2}\frac{d_s-d_L}{d_sd_L}}=2\sqrt{l_p\frac{l_p}{\bar{\lambda}_M}\frac{d_s-d_L}{d_sd_L}}$	$\theta=2\sqrt{E_g\frac{d_s-d_L}{d_sd_L}}=2\sqrt{l_p\frac{l_p}{\bar{\lambda}_M}\frac{d_s-d_L}{d_sd_L}}$
Indirectly/"hypothetical" observable predictions: (contains only *GM*)		
Gravitational parameter	$\mu=GM=c^2l_p\frac{l_p}{\bar{\lambda}_M}$	$\mu=c_g^2E_g=c_g^2l_p\frac{l_p}{\bar{\lambda}_M}$
Two body problem	$\mu=G(M_1+M_2)=c^2l_p\frac{l_p}{\bar{\lambda}_1}+c^2l_p\frac{l_p}{\bar{\lambda}_2}$	$\mu=c_g^2(E_{g,1}+E_{g,2})=c_g^2l_p\frac{l_p}{\bar{\lambda}_1}+c_g^2l_p\frac{l_p}{\bar{\lambda}_2}$
Cosmology ($\bar{\lambda}_c$: reduced Compton wavelength Friedmann critical universe mass M_c) (contains only GM_c)		
Cosmological red shift	$z_H\approx\frac{dH_0}{c}=\frac{1}{\frac{2GM_c}{c^2d}}=\frac{d\bar{\lambda}_c}{2l_p^2}$	$z_H\approx\frac{R}{E_g}=\frac{d\bar{\lambda}_c}{2l_p^2}$
Hubble constant	$H_0=\frac{c^3}{2GM_c}=\frac{\bar{\lambda}_c}{2t_pl_p}$	$H_0=\frac{c}{E_g}=\frac{\bar{\lambda}_cc}{2l_p^2}=\frac{\bar{\lambda}_c}{2t_pl_p}$
Hubble radius	$R_H=\frac{c}{H_0}=\frac{2GM_c}{c^2}=\frac{2ct_pl_p}{\bar{\lambda}_c}$	$R_H=E_g=\frac{2l_p^2}{\bar{\lambda}_c}=\frac{2c_gt_pl_p}{\bar{\lambda}_c}$
Quantum analysis:		
Constants needed	G, $\hbar$, and c or l_p, $\hbar$, and c	l_p and c_g, for some phenomena only l_p ($c_g=c$)
Variable needed	one for mass size	one for mass size

6. Finding the Gravitational Energy and the Planck Length Using a Cavendish Apparatus

Here we will look at how to find the collision-length, that again is the gravitational energy, by using a Cavendish [38] apparatus. Moment of force, better known as torque, is given by:

$$\kappa\theta \tag{23}$$

where θ is the deflection angle of the balance and κ is the torsion coefficient of the suspending wire. Next, we have the following well-known relationship:

$$\kappa\theta = LF \tag{24}$$

where L is the length between the two small balls in the Cavendish apparatus. Further, F can be set equal to the gravitational force given by:

$$F = c_g \frac{E_g E_g}{R^2} \tag{25}$$

This means we have:

$$\kappa\theta = Lc_g \frac{E_g E_g}{R^2} \tag{26}$$

Further, the natural resonant oscillation period of a torsion balance is given by:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \tag{27}$$

The moment of inertia I of the balance is given by:

$$I = m_g\left(\frac{L}{2}\right)^2 + m_g\left(\frac{L}{2}\right)^2 = \frac{m_g L^2}{2} \tag{28}$$

from this we must have

$$T = 2\pi\sqrt{\frac{m_g L^2}{2\kappa}} \tag{29}$$

We now solve this with respect to κ, and this gives:

$$\frac{T^2}{2^2\pi^2} = \frac{m_g L^2}{2\kappa}$$

$$\kappa = \frac{m_g L^2 2\pi^2}{T^2} = \frac{E_g L^2 2\pi^2}{c_g T^2} \tag{30}$$

Now in the Equation (26) replace κ with this expression, and then we solve this equation with respect to the gravitational energy, and this gives:

$$\kappa\theta = LF$$

$$\frac{E_g L^2 2\pi^2}{c_g T^2}\theta = Lc_g \frac{E_g E_g}{R^2}$$

$$E_g = \frac{L 2\pi^2 R^2\theta}{T^2 c_g^2} \tag{31}$$

And the collision-time mass of the large ball in the Cavendish apparatus is given by

$$M_g = \frac{E_g}{c_g} = \frac{L 2\pi^2 R^2 \theta}{T^2 c_g^3} \tag{32}$$

Again, T is the oscillation time; further, L is the distance between the small balls in the Cavendish apparatus, and R is the distance from center to center between the small ball and the large ball in the Cavendish apparatus. Further, θ is the angle of the arm when the arm is deflected. In other words, there is no need to know G or h to measure this in a Cavendish apparatus. However, there the speed of gravity is, as we already know from previous sections, identical to the speed of light.

That is, we know the gravitational energy in the large ball in the Cavendish apparatus. It is indeed an incredibly short length, the collision-length. It is the aggregated collision-length of all the collisions in the mass making up the gravitational energy in the mass during only the Planck time. Pay attention to how we need less information to find this than to find G or M from a Cavendish apparatus. To find G in a Cavendish apparatus, one uses:

$$G = \frac{L 2\pi^2 R^2 \theta}{T^2 M} \tag{33}$$

That is, one needs to know M in addition to L, R, and T (that is needed to find E_g). One can only find the large mass in the Cavendish apparatus by simply weighting the mass, but doing so adds measurement errors compared to only finding E_g. If the G found from the Cavendish apparatus is only used in combination with the large mass in the Cavendish apparatus, then this error from measure M will cancel out with the error this gave to G, but if G measured from the Cavendish apparatus is next used in combination with the mass from the Earth to predict observable gravitational phenomena from the gravitational field of the Earth, then this will give bigger errors than simply using E_g from the Earth directly.

We can also find M in a Cavendish by:

$$M = \frac{L 2\pi^2 R^2 \theta}{T^2 G} \tag{34}$$

But then one needs G, and one needed M to find G so this makes little sense. To find the gravitational energy (or the collision-time mass) of the large mass in the Cavendish apparatus requires no G and no kilogram measurements.

To separate out the Planck length, we additionally need to know the reduced Compton wavelength of the gravitational mass, and we get:

$$E_g = l_p \frac{l_p}{\bar{\lambda}_M} = \frac{L 2\pi^2 R^2 \theta}{T^2 c_g^2}$$
$$l_p = \sqrt{\frac{\bar{\lambda}_M L 2\pi^2 R^2 \theta}{T^2 c_g^2}} \tag{35}$$

That is, the Planck scale (here the Planck length) can be measured (detected) without any knowledge off G or $\hbar$, but also here we see that we need the reduced Compton wavelength to do so, and this can be found independently of knowledge of G, $\hbar$, and c. This is rather amazing as it has been thought for more than 100 years that to detect the Planck scale is almost impossible, despite large efforts to do so. That we can indirectly measure the Planck length without knowledge of G, c or $\hbar$ means, in our view, simply that to detect gravity is to detect the Planck scale. This is highly controversial and perhaps a shocking view. What one has been searching for, the Planck scale, is already something one has detected all the time, namely almost any observable effects of the gravity force. However, we are measuring an aggregate of these Planck events, and it is when one first understands the relationship between Compton frequency and Planck mass events that one really gets to the depth of it.

7. New Field Equation

Einstein's [9] field equation is given by:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{36}$$

We [8] have demonstrated that G is a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$, and that the Planck length can be found independent of G. Replacing G with the composite constant gives:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi \frac{l_p^2 c^3}{\hbar}}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi l_p^2}{\hbar c} T_{\mu\nu} \tag{37}$$

This way to write Einstein's field equation were first noted in 2016 by [39], but with very limited discussion or implications. At that time, we had not yet determined how the Planck length could be derived independently from G. However, we were able to do so in 2017, as outlined in the preceding sections.

From equation (37) we also see that the Planck constant is a component of the field equation, but any useful derivation will negate it due to the Planck constant being embedded in both the Joule energy and kilogram mass measurements, that again is embedded in the stress-energy tensor. For practical purposes Einstein's field equation must be solved with respect to some boundary conditions to obtain the desired output. The most commonly utilized and practical solution is the Schwarzschild [40] [41] solution for a spherical non rotating gravitational object in polar coordinates, when the cosmological constant is set to zero, where Schwarzschild got

$$\mathrm{d}s^2 = \alpha\left(1 - \frac{k}{R}\right)\mathrm{d}t^2 - \left(1 - \frac{k}{R}\right)^{-1} \mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{38}$$

To find the α and k one end up with two equations (see for example [42])

$$A(R)=\alpha\left(1+\frac{k}{R}\right) \text{ and } B(R)=\left(1+\frac{k}{R}\right)^{-1} \tag{39}$$

The parameters k and α is then identified by considering the weak-field limit, that requires

$$\frac{A(R)}{c^2}\to 1+\frac{2\Phi}{c^2} \tag{40}$$

where Φ is the Newton gravitational potential $\Phi=-\frac{GM}{R}$. This means $\alpha=c^2$ and $k=-\frac{2GM}{c^2}$. This gives the well known Schwarzschild metric

$$c^2\mathrm{d}\tau^2=\left(1-\frac{2GM}{R}\right)c^2\mathrm{d}t^2-\left(1-\frac{2GM}{R}\right)^{-1}\mathrm{d}R^2-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right) \tag{41}$$

It is also essential to keep in mind that pure Joule energy can be expressed in the form of $E=h\frac{c}{\lambda_\gamma}$, where λ_γ is the wavelength of a photon. Furthermore, a kilogram of mass can be expressed as follows:

$$m=\frac{h}{\lambda}\frac{1}{c}=\frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{42}$$

As discussed in our recent papers [43] [44], simply setting $G=\frac{l_p^2c^3}{\hbar}$ and $M=\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$ in both Newton's law of gravitation and general relativity leads to full quantization of gravity. This simple yet overlooked approach enables the Schwarzschild solution to be rewritten as:

$$\mathrm{d}s^2=\left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)c^2\mathrm{d}t^2-\left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)^{-1}\mathrm{d}R^2-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)$$

$$\mathrm{d}s^2=\left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)c^2\mathrm{d}t^2-\left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}R^2-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right) \tag{43}$$

We believe there is no coincidence that the term $l_p\frac{l_p}{\bar{\lambda}_M}$ in the metric is identical to our quanitized collision length energy $E_g=l_p\frac{l_p}{\bar{\lambda}}$. The $\frac{G}{c^4}$ component of the Einstein constant in Einstein's field equation is from a more fundamental understanding necessary to convert Joule energy into collision-length energy. That is we have:

$$\frac{G}{c^4}E=\frac{G}{c^4}Mc^2=\frac{\frac{l_p^2c^3}{\hbar}}{c^4}\hbar\frac{c}{\bar{\lambda}_M}=l_p\frac{l_p}{\bar{\lambda}_M}=E_g \tag{44}$$

Note that the Planck constant embedded in G cancels out with the Planck constant embedded in M (or E), resulting in the Schwarzschild metric as understood from the deepest level is quantized, but still require no information about the Planck constant. The quantization is now in the factor $\frac{l_p}{\bar{\lambda}_M}$, which represent the reduced Compton frequency per Planck time.

We can actually propose a new field equation that yields the same predictions as Einstein's field equation, but requires less input. It can be expressed as:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi E_{\mu\nu} \tag{45}$$

The difference between this new general relativity inspired field equation is that the stress-energy tensor: $E_{\mu\nu}$, is now not linked to Joule energy or kilogram, but to collision-length energy and collision-time mass. One could mistakenly think we only have set $G = c = 1$. This is not the case. Our new gravitational framework is totally independent on G. We now get the same solution as Schwarzschild got from Einsteins field equation for spherical polar coordinates:

$$\mathrm{d}s^2 = \alpha\left(1 - \frac{k}{R}\right)\mathrm{d}t^2 - \left(1 - \frac{k}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \tag{46}$$

To determine the values of α and k, we must solve two equations:

$$A(R) = \alpha\left(1 + \frac{k}{R}\right) \text{ and } B(R) = \left(1 + \frac{k}{r}\right)^{-1} \tag{47}$$

These parameters can be identified by considering the weak-field limit, which requires that:

$$\frac{A(R)}{c_g^2} \to 1 + \frac{2\Phi}{c_g^2} \tag{48}$$

Here, Φ is the gravitational potential, given by $\Phi = -c_g^2\frac{E_g}{R}$. This leads to the identification of $\alpha = c_g^2 = c^2$ and $k = -2E_g$. Substituting these values gives us the metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2E_g}{R}\right)c_g^2\mathrm{d}t^2 - \left(1 - \frac{2E_g}{R}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \tag{49}$$

As we can see also from out metric only one constant is needed in our theory, namely the speed of gravity: c_g. This provides a fully functional metric that yields the same predictions as general relativity theory, the results are shown in Table 2. However, general relativity theory and the standard Schwarzschild metric requires two constants, namely G and c, and additionally requires the kilogram mass M that is linked to finding G. In our new field equation, we only need to determine E_g, which we have demonstrated in section 3 can be easily obtained without knowledge of G, as well as the speed of light to get to the full metric. While Einstein's field equation relies on the gravitational constant, which must first be determined, for example, using a Cavendish apparatus, our theory is simpler and require no gravity constant, but ultimately yields the same output predictions as can be seen in Table 2.

As we know also that $E_g = l_p \frac{l_p}{\bar{\lambda}}$ we must have

$$\mathrm{d}s^2 = \left(1 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)c_g^2\mathrm{d}t^2 - \left(1 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}R^2 - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right) \quad (50)$$

At the deepest level, we observe that even our metric is quantized as $\frac{l_p}{\bar{\lambda}}$, which is the reduced Compton frequency per Planck time. We also see that our gravity model is directly linked to the Planck scale. To detect any gravity phenomena is to detect the Planck scale. Our metric at the deepest quantum level requires two constants, l_p and c_g $(c_g = c)$, as well as a new mass size-dependent variable, $\bar{\lambda}$. The full term, $l_p \frac{l_p}{\bar{\lambda}} = E_g$, requires less information to be found than to separate out l_p and $\bar{\lambda}_M$. Therefore, the metric presented in 49 is the more practical and useful one at least for astronomical objects. The deeper metric in 50 requires more calibration work but provides full insight into the quantum gravity world.

We can also see that our new field equation is ultimately the same as that of general relativity theory as Equation (50) is identical to Equation (43), but only after someone has already shown how to quantize and understand general relativity theory from a deeper perspective, as we recently demonstrated in [43] [44]. This latest quantized Schwarzschild type metric is only of interest when working with gravity close to the subatomic level or to understand that gravity is quantized even in the metric. Finding the Planck length and $\bar{\lambda}$ independent of $\hbar$ requires significant work, as demonstrated in the preceding sections. Additionally, we need the Cavendish apparatus to find the Planck length.

While Einstein's field equation is consistent with $F = G\frac{Mm}{R^2}$ as the weak field limit, our new Einstein inspired field equation is consistent with $F = c^3\frac{M_g m_g}{R^2}$ and $F = c_g\frac{E_g E_g}{R^2}$ as weak field approximation limits. Our new field equation gives all the same predictions as Einstein's field equation.

Table 3 sumarizes our findings.

There exists one more interesting field equation that gives identical predictions to Einstein's field equation, which has not been mentioned in the literature before. It is given by:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi}{c^4}N_{\mu\nu} \quad (51)$$

Be aware this is not the same as setting $G = 1$. Also this field equation simply does not need G just as Newton's original gravity force formula not had any gravity constant. The stress-energy tensor: $N_{\mu\nu}$, in this equation must be related to

Table 3. Comparison of Einstein gravitational theory and the new way to understand gravity.

	Einstein:	**New:**
Field equation	$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}$	$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi E_{\mu\nu}$
Corresponding "Newton"	$F=G\frac{Mm}{R^2}$	$F=c_g\frac{E_gE_g}{R^2}$
Mass	Kilogram $M=\frac{\hbar}{\bar{\lambda}}\frac{1}{c}$	Collision time $M_g=t_p\frac{l_p}{\bar{\lambda}}$
Energy	Joule $E=h\frac{c}{\lambda}$	Collision length $E_g=l_p\frac{l_p}{\bar{\lambda}}$
Constants needed practice	G and c	only c_g ($c_g=c$)
Constants deeper level	$\hbar$, l_p and c	c_g and l_p

what we call Newtonian energy, which is the Newtonian mass M_n multiplied by c^2, that is $E_n=M_nc^2$. Using this, the Schwarzschild-type metric becomes (for a spherical non rotating gravitational body when the cosmological constant is set to zero):

$$\begin{aligned}\mathrm{d}s^2&=\left(1-\frac{2M_n}{c^2R}\right)c^2\mathrm{d}t^2-\left(1-\frac{2M_n}{c^2R}\right)^{-1}\mathrm{d}R^2-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\\ \mathrm{d}s^2&=\left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)c^2\mathrm{d}t^2-\left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}R^2-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\end{aligned} \quad (52)$$

This field equation is consistent with the original Newton formula: $F=\frac{M_nm_n}{R^2}$ as a weak field approximation. Notably, neither this equation nor the Equation (51) require a gravitational constant, but only depend on the speed of light (the speed of gravity). While it is possible to come up with other mass and energy definitions, doing so would lead to unnecessarily complex gravity constants similar to those encountered when using the kilogram mass and joule. We explore this idea further in our analysis of different mass definitions in relation to gravity, which can be found in [43].

8. Equivalence Principle

Our model must be indistinguishable from general relativity when it comes to the equivalence principle. However, we will briefly discuss a potential misunderstanding surrounding the weak equivalence principle in relation to standard Newtonian theory (pre-1873) and general relativity.

One aspect and test of the weak equivalence principle goes all the way back to Galileo Galilei, and is simply related to that all bodies should fall at the same rate in a gravitational field in a vacuum. That this holds is due to the fact that it has been extremely accurately tested in recent times; see, for example, [45] [46] [47]. This is also fully in line with our theory. Our theory is, like standard theory, also in line with recent tests of sub-millimeter scale deviations of the Newtonian

$1/R^2$, which have confirmed this law is also valid at such short scales; see [48].

The weak equivalence principle is, in addition, linked to so-called inertial mass and gravitational mass through the following well known relation:

$$m_i a = G\frac{Mm}{R^2} \tag{53}$$

That is, the weak equivalence principle is also about the inertial mass being equal to the gravitational mass, $m_i = m$ (see, for example, [49] [50] [51]). The mass m_i, on the left side of Equation (53), is thought of as inertial mass and is why we have marked the mass with subscript *i*. The mass *M* and *m* on the right side of the equation are considered gravitational masses. The gravitational force formula used here in modern physics is the 1873 modified Newton formula. This basically means a mass acted upon by a uniform gravitational field *g* behaves identically to a mass of the same size acted upon by a force (pseudo-force) different than the gravitational force (used for acceleration) are indistinguishable, something that has also been well tested. At least it appears to be so, and we are not questioning the experimental test results, but we will here actually question the interpretation of this in relation to the standard Newton gravity force formula (Equation (53)).

In our view there is, however, actually only one form of mass; it is the collision-time mass, which is given by $M_g = \frac{l_p}{c}\frac{l_p}{\bar{\lambda}} = \frac{G}{c^3}M$. This means we can just as well write:

$$\begin{aligned} m_g a &= c_g^3\frac{M_g m_g}{R^2} = c_g\frac{E_g E_g}{R^2} \\ \frac{G}{c^3}ma &= c_g^3\frac{\frac{G}{c^3}M\frac{G}{c^3}m}{R^2} \end{aligned} \tag{54}$$

After dividing by $\frac{G}{c^3}m$ on each side, we are left with:

$$a = \frac{GM}{R^2} \tag{55}$$

So, we see our theory is identical to standard theory when it comes to $a = g$. Still, it is an illusion that the inertial mass m_i, if expressed as just kilograms, as in the standard theory, is identical to the (in our view) true gravitational mass $m_g = \frac{G}{c^3}m$; that is, we naturally have $m \neq m_g$. The true gravitational mass is not *M* and *m*, but $M_g = \frac{l_p^2}{\hbar}M = \frac{G}{c^3}M$ and $m_g = \frac{l_p^2}{\hbar}m = \frac{G}{c^3}m$. The reason, in our view, that modern physics mistakenly thinks the kilogram mass, which is the unit of inertial mass, is the same as the gravitational mass, is that using the modern 1873 version of Newton's theory entails unknowingly using two different mass definitions in the same gravity force formula. That is, in the formula $F = G\frac{Mm}{R^2}$, the gravitational mass should in reality not be seen as *M* and *m*, but rather as *M* fixed for its lacking information about the mass components needed to model gravity by its multiplication with *G*; see also [43]. The other incomplete mass, *m*, is only used for derivations with another incomplete inertial mass m_i, that always cancels out in derivations to get formulas for something that can also be observed, such as *g* (see **Table 2**). The kilogram mass is incomplete and does not have enough information to also account for gravity, and is why any kilogram mass that has a significant gravitational field needs to be fixed (multiplied) with a gravitational constant. However, for standard energy mass relations such as $E = mc^2$, then the kilogram mass has enough information embedded in it, as this does not incorporate any information about gravitational mass or gravitational energy.

The inertial mass and the gravitational mass are the same when both are correctly expressed as collision-time mass, but they are not the kilogram mass. In standard theory "*even the unit of inertial mass is the kilogram is now per definition via h the unit of inertial mass. However, only the atom count determines the inertial mass of the kilogram realization without any reference to the equivalence principle*"; see Mana and Schlamminger [52]. This is in line with the research undertaken to decide on a new kilogram standard, culminating in the new 2019 NIST CODATA standard to link the kilogram to the Planck constant and the watt balance; see [53] [54]. The other alternative and competing method for defining the kilogram was counting atoms; see [30] [31] [32] [33]. Still, none of these methods make the kilogram contain any information needed to make it a gravitational mass, so it is not the same as a gravitational mass.

Inertial mass is, however, identical to gravitational mass when it is defined as collision-time mass. We can easily find the collision-time mass of any macroscopic-sized object in a Cavendish apparatus, as demonstrated in Section 6. This can be done without any knowledge of any constants except *c* and no need for counting atoms or including the watt balance and the Planck constant. This contrasts with the kilogram mass of the large ball in the Cavendish apparatus that we cannot find directly from the Cavendish apparatus without knowing *G* first. Further, we basically need to know the kilogram mass of the large ball in the apparatus to find *G*, so this leads to a circular problem. The way one avoids this circular problem in standard physics is to find the kilogram mass of the large ball used in the Cavendish apparatus from an independent method, such as the watt balance, or to count atoms. These methods to define the kilogram are directly or indirectly constructing an arbitrary human-made clump of matter as the standard kilogram mass, but we do not need any of this to find the collision-time mass or collision-length energy of the large balls in the Cavendish apparatus, as it can be found directly in the Cavendish apparatus by only knowing one constant, namely the speed of light *c*. This constant also has no uncertainty in it, unlike *G*.

For larger astronomical objects, we can also easily find the collision-time mass for any object such as planets, the sun, and other stars without finding *G* first.

Not only that, but also the uncertainty in these mass measures will be smaller than in the kilogram mass measure for the same objects. This is because to find the kilogram mass of astronomical objects, we generally need to first find G; see [17]. For example, the collision-time mass of the Earth we can simply find by measuring gravitational acceleration from a drop ball; the collision-time mass of the Earth is then given by:

$$M_g = \frac{2HR^2}{T_b^2 c^3} = g\frac{R^2}{c^3} \tag{56}$$

where H is the height of the drop from above the ground, R is the radius of the Earth, T_b is the time it took from when the ball was dropped until it hit the ground, and c is the speed of light (or gravity, as they are the same). We naturally do not claim this method is very accurate, The point is that all we need is to measure g accurately without any knowledge of G; for a detailed discussion of why the collision-time mass (and thereby the gravitational energy) can be found with higher precision than the kilogram mass, see [17].

Even if the kilogram mass (and therefore the kilogram inertial mass) is not the same as the gravitational mass, it is true, in our theory, that two masses with the same amount of kilogram also have the same gravitational mass, since any kilogram mass is simply a collision-time mass multiplied by a constant $m = m_g \frac{\hbar}{l_p^2} = m_g \frac{c^3}{G}$. This is, however, much more than just a change of units. This unit change when going from kilogram to collision-time is made to get information about the Planck scale into the mass definition, and the Planck scale is what is needed to do any gravity modeling, but this is done unknowingly in standard theory through the calibrated G. We need to know the Planck constant and the Planck length, or G and c, to find the kilogram mass from the collision-time mass.

The kilogram mass has not incorporated the Planck length (or Planck time), which is related to the essence of gravity, but by multiplying G with M one unknowingly gets the Planck length into the mass. This is also discussed in more detail in section 5 of the recently published paper [8].

The kilogram mass is incomplete and contains no information about the Planck scale (except for the Planck mass itself) and therefore is an incomplete mass definition, which is why the only mass that is significant for gravity in two-body problems, wherein $m \ll M$ problems the mass M is multiplied with G, so the real gravitational mass is linked to GM not just M in the Newton formula. In two-body problems where we do not have $m \ll M$, then the gravitational parameter is $\mu = G(M+m) = GM + Gm$; in other words, then both the kilogram masses are corrected by being multiplied by G, and one is thus unknowingly turning the kilogram mass into collision-time mass, which is the real gravitational mass in our view, but in standard theory this has not been discovered and therefore not been understood. Be aware that when G is multiplied by M, as is needed for prediction of any directly observable gravitational phenomena, then the kilogram unit falls out. This is because G has output units $m^3 \cdot kg^{-1} \cdot s^{-2}$ and the kilogram mass naturally outputs unit kg, so the kilogram cancels. This is because gravity does not depend on humanly-constructed mass units, but rather depends on the foundation of mass and energy that is linked to the Planck length and Planck time.

9. Conclusion

We have demonstrated how the well-known Newtonian formula, $F = G\frac{Mm}{R^2}$, can be replaced with a simpler and more intuitive formula: $F = c_g \frac{E_g E_g}{R^2}$. Moreover, we have shown how Einstein's field equation can be replaced with a slightly simpler field equation: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi E_{\mu\nu}$, which do not relay on the so-called Newton gravitational constant G, which was neither invented nor used by Newton. The stress-energy tensor ($E_{\mu\nu}$) must now be linked to collision-length energy and collision-time mass, not kilogram and Joule measurements. Our new field equation leads to a quantized Schwarzschild metric based on quantized gravitational energy. Furthermore, as recently demonstrated by us [43] [44], Einstein's general relativity can be easily quantized once one understands that the gravitational constant is a composite constant and rewrites the kilogram mass. Ultimately, at the quantum scale, it is the same gravitational theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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