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Squaring the Circle Is Possible When Taking into Consideration the Heisenberg Uncertainty Principle

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Abstract

Squaring the circle is one of the oldest challenges in mathematical geometry. In 1882, it was proven that π was transcendental, and the task of squaring the circle was considered impossible. Demonstrating that squaring the circle was not possible took place before discovering quantum mechanics. The purpose of this paper is to show that it is actually possible to square the circle when taking into account the Heisenberg uncertainty principle. The conclusion is clear: it is possible to square the circle when taking into account the Heisenberg uncertainty principle.

Keywords

Squaring the Circle, Quantum Mechanics, Heisenberg Uncertainty Principle

1. Background

Squaring the circle is the challenge of making a square and a circle with the same area using only a compass and straightedge in a finite number of steps; see [1] [2]. There was a long series of attempts to square the circle until in 1882 Lindemann [3] proved that π was a transcendental number and that, for this reason, it was impossible to square the circle. Hobson [4] in 1913 published an excellent book with a well-documented history of squaring the circle. He said:

"*The history of our problem falls into three periods marked out fundamentally* [*with*] *distinct differences in respect of method, immediate aims, and of equipment in possession of intellectual tools.*"

He also concluded with the following:

"*It has thus been proved that π is a transcendental number…the impossibil-*

ity of 'squaring the circle' has been effectively established."

However, since then, we have come to understand the physical world at a much deeper level than it was understood in 1882. Relativity theory and quantum mechanics were invented in the early 20th century, and we now have the tools to consider that we are in a fourth, fundamentally distinct time with respect to methods and tools for squaring the circle. Haug [5] has recently shown that while it is impossible to square the circle in space, it is possible to square it in relativistic space-time. However, as he discussed in that paper, recent claims have simply moved the problems in space over into time. One then needs clocks with infinite precision, something that is likely impossible to achieve. That paper used special relativity theory to discuss how new insights in geometry, namely Minkowski's space-time, have provided new tools and new opportunities to look at the squaring the circle problem. There have also been recent claims of squaring the circle; see Barton [6] and Samuel and America [7], that naturally should be scrutinized by more researchers over time before any final conclusion is made, but the research in squaring the circle is clearly not over.

This paper looks at how quantum mechanics and the Heisenberg uncertainty principle will affect the squaring the circle challenges. As we will see, if the Heisenberg uncertainty principle always holds, then it seems one can square the circle.

In the next section, we discuss the Heisenberg uncertainty principle's implications on the squaring the circle challenge. In section three, we discuss possible limitations of standard quantum mechanics and what implications they can have for our analysis, before we summarize important findings from the paper in the conclusion.

2. Heisenberg's Uncertainty Principle and Squaring the Circle

In 1927, Heisenberg [8] introduced what today is known as the Heisenberg uncertainty principle, which basically states:

$$\Delta p \Delta x \geq \hbar \tag{1}$$

where Δp is uncertainty in momentum, and Δx is uncertainty in position, and $\hbar$ is the reduced Planck constant. In the same year, Kennard [9] claimed it should be $\Delta p \Delta x \geq \frac{\hbar}{2}$; however, if one should use $\hbar$ or $\hbar/2$ in the uncertainty principle will not be important for our main findings. Max Planck [10] [11] in 1899 introduced the Planck length $l_p = \sqrt{\frac{G\hbar}{c^3}}$. The Planck length is assumed by many physicists to be the smallest possible length; see, for example, [12]-[17]. Similar views are held in superstring theory [18]. The Planck length plays an important role in quantum gravity theories, and to unify gravity with quantum mechanics, it is also assumed the Planck length must be incorporated in the Heisenberg uncertainty principle—which leads to what we can call various gravitational uncertainty principles. This is basically the Heisenberg principle modified to consider the Planck scale. For example, Adler and Santiago [19], based on such an extended uncertainty principle, argued that:

"*From the modified or gravitational uncertainty principle, it follows that there is an absolute minimum uncertainty in the position of any particle, of the order of the Planck length.*"

Garay [20] similarly writes that:

"*We are facing a resolution limit, a minimum length for relativistic quantum mechanics: it is not possible to localize a particle with better accuracy than its Compton wavelength.*"

And since the Planck length is, by many, considered the shortest possible Compton wavelength, then the minimum uncertainty in the position can likely be seen as the Planck length; see [21] [22] [23] [24]. This is not the maximum uncertainty, but it is the minimum uncertainty according to standard physics.

Let us now assume that the minimum uncertainty for a particle is the Planck length. To draw a circle on a sheet of paper or the ground, we basically put out or localize particles. The line is particles, and the minimum uncertainty in the position of any particle is the Planck length. This means the minimum uncertainty in the diameter of the circle is $2l_p$. Assume we try to draw a circle with a radius of one meter, a unit circle. There will now be an uncertainty in the radius equal to the Planck length. The area of a circle is:

$$A = \pi r^2 \tag{2}$$

But under the uncertainty principle, we must have:

$$A = \pi \left(r \pm l_p \right)^2 \tag{3}$$

That again means the area must be between $\pi \left(r + l_p \right)^2$ and $\pi \left(r - l_p \right)^2$. We can move over to uncertainty in π instead, because we can set up the following equation:

$$\pi \left(r \pm l_p \right)^2 = \pi_u r^2 \tag{4}$$

where π_u is the uncertain π. This means we can move the uncertainty from the radius to uncertainty in π and that this must be:

$$\pi_u = \frac{\pi \left(r \pm l_p \right)^2}{r^2} \tag{5}$$

And the difference between the real π and this uncertain π must be:

$$\Delta \pi_u = \pi - \pi_u = \pi - \frac{\pi \left(r \pm l_p \right)^2}{r^2} \tag{6}$$

Suppose we use the CODATA (NIST) 2019 Planck length value 1.616255 × 10^{-35} m, then we find that for a unit circle ($r = 1$), we need to only know π to 34 decimal places to stay inside the uncertainty. In π, caused by the uncertainty in the line of the circle, that again creates an uncertainty in the area of the circle. In other words, Lindemann's 1882 proof that π was transcendental becomes irrelevant for squaring the circle "in practice" (under ideal conditions) when one considers what we today know of quantum mechanics and its uncertainty principle.

3. Possible Limitations

Not all physicists agree that the Planck length is a minimum length or the minimum uncertainty. For example, Unzicker [25] has argued that since the Planck length can only be found from dimensional analysis based on knowledge of G, $\hbar$ and c, and not measured more directly, then it is more like a mathematical artifact. This was also a view held by, for example, Bridgman [26]. However, Haug [27] has recently also shown how the Planck length can be extracted from gravity observations with no knowledge of G or $\hbar$ or even c, something that strengthens the view that the Planck length is something very fundamental. This experimental finding strongly supports the majority view that the Planck length is fundamental and linked to something "real". The question is still what it truly represents.

If the uncertainty principle leads to a minimum uncertainty, then we have shown we can square the circle. This view is, however, rooted in the Heisenberg uncertainty principle always holding. Haug [28] [29] has recently argued that the uncertainty principle breaks down at the Planck scale and that it, at the very Planck scale, is replaced with a certainty principle, and that this again is directly linked to gravity. This is a new and controversial view, and it will take time before itis fully investigated. However, if it should hold true, then one can likely still not square the circle. It is, therefore, in our view, still an open question whether one can square the circle in practice, even under ideal conditions. This is because we know from standard physics theory that there is no agreement yet on how to unify gravity with quantum mechanics, and such unification could indeed lead to modifications of quantum mechanics. Therefore, to truly know if we can square the circle will likely not be fully settled until we have settled on a unified quantum gravity theory.

4. Conclusion

Squaring the circle has been one of the great challenges in geometry, with attempts to do so for hundreds, if not thousands, of years. In 1882, it was proven that π was transcendental, and the task of squaring the circle has been considered impossible since then. However, new epochs in science lead to new tools. Quantum mechanics and the uncertainty principle were unknown in 1882. If one should try to square the circle even under ideal conditions, one must consider quantum mechanics to, at least, explore whether it should be physically possible and not merely a geometrical challenge in an imaginary geometrical world. When considering the uncertainty principle, it becomes irrelevant that π is transcendental, so for any given circle size, one ends up with that one only needs to know π for a given and limited number of decimal places so as to square the circle. An outstanding issue is if the Heisenberg uncertainty principle always holds, and also does so at the Planck scale. One of the biggest challenges in physics is to make a quantum gravity theory that unifies with quantum mechanics, so this can also lead to changes in quantum mechanics. So, the final answer to whether we can square the circle can likely be settled when we have a unified quantum gravity theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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