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# Planck Scale Fluid Mechanics: Measuring the Planck Length from Fluid Mechanics Independent of *G*

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## Abstract

We demonstrate how to extract the Planck length from hydrostatic pressure without relying on any knowledge of Newton's gravitational constant, $G$. By measuring the pressure from a water column, we can determine the Planck length without requiring knowledge of either $G$ or the Planck constant. This experiment is simple to perform and cost-effective, making it not only of interest to researchers studying gravity but also suitable for low-budget educational settings. Despite its simplicity, this has never been demonstrated to be possible before, and it is achievable due to new theoretical insights into gravity and its connection to quantum gravity and the Planck scale. This provides new insights into fluid mechanics and the Planck scale. We are also exploring initial concepts related to what we are calling "Planck fluid", which could potentially play a central role in quantum gravity and quantum fluid mechanics.

## Keywords

Planck Length, Hydrostatic Pressure, Pascal's Law, Gravity, Planck Fluid

## 1. Theory

Blaise Pascal's law is directly related to the well-known hydrostatic pressure formula (Granger [1]):

$$p = \rho g H \tag{1}$$

where $\rho$ is the liquid density (an "incompressible" fluid), $H$ is the height of the liquid column, $p$ is the pressure, and $g$ is the gravitational acceleration. As $g = \frac{GM}{r^2}$, we can rewrite this as:

$$p = \rho \frac{GM}{r^2} H \tag{2}$$

The Planck length was first described by Max Planck [2] [3] and is traditionally given by $l_p = \sqrt{\frac{G\hbar}{c^3}}$. Max Planck found this formula by assuming $G$, $c$, and $\hbar$ were the most important universal constants, and then applied dimensional analysis to derive this formula as well as formulas for the Planck time, Planck mass, and Planck temperature. We can solve the Planck length formula for $G$ and get $G = \frac{l_p^2 c^3}{\hbar}$ and then claim the Planck length is the important fundamental constant, and that $G$ is a composite constant. Such suggestions were made already in 1984 when Cahill [4] [5] suggested expressing the gravitational constant from the Planck mass as $G = \frac{\hbar c}{m_p^2}$. In 1987, Cohen [6] pointed out correctly that this would only lead to a circular problem as long as no one at that time knew how to find the Planck mass or Planck length independently of first knowing $G$. However, in recent years, we have shown how to find the Planck length independent of $G$ and also $\hbar$ using a Newton force spring or a Cavendish apparatus, see [7] [8]. In this paper, we will show a similar approach utilizing fundamental principles of fluid mechanics. Not only will we outline the theory to do so, but also we will perform simple experiment using a manometer in a water column to demonstrate that this is more than theory.

Another important point we will utilize is that we can solve the Compton [9] wavelength formula, $\lambda = \frac{h}{Mc}$, with respect to mass. This gives:

$$M = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{3}$$

where $\hbar$ is the reduced Planck constant, also known as the Dirac constant ($\hbar = h/2\pi$), and $\bar{\lambda}$ is the reduced Compton wavelength. We will claim that this formula holds for any mass, even astronomical-sized objects like the Earth, the Sun, stars, and galaxies. It might seem questionable as the Compton wavelength formula was originally developed in relation to Compton scattering of electrons. However, the mass of any object (or mass-equivalent) can be described by this formula for mass. Composite masses do not have a single Compton wavelength, but the reduced Compton wavelength in the formula then represents the aggregate of all the wavelengths in the particles making up the mass, including energy, which we can treat as having a Compton wavelength. We have the relation:

$$\bar{\lambda} = \frac{1}{\sum_i^n \frac{1}{\bar{\lambda}_i}} \tag{4}$$

This is fully consistent with:

$$M = m_1 + m_2 + m_3 + m_n + \frac{E_1}{c^2} + \frac{E_2}{c^2} + \frac{E_3}{c^2} + \frac{E_n}{c^2}$$

$$\frac{h}{\lambda}\frac{1}{c}=\frac{h}{\lambda_1}\frac{1}{c}+\frac{h}{\lambda_2}\frac{1}{c}+\frac{h}{\lambda_3}\frac{1}{c}+\frac{h}{\lambda_n}\frac{1}{c}+\frac{h\frac{c}{\lambda_1}}{c^2}+\frac{h\frac{c}{\lambda_2}}{c^2}+\frac{h\frac{c}{\lambda_3}}{c^2}+\frac{h\frac{c}{\lambda_n}}{c^2} \tag{5}$$

$$\lambda=\frac{1}{\sum_i^n\frac{1}{\bar{\lambda}_i}}$$

So, even binding energy can be taken into account, be aware the energy can be both added as done above or subtracted. Ignoring nuclear binding energy will, however, introduce less than a 1% error in the predicted Compton wavelength and, therefore, also in the mass we are working with. This is discussed in detail in [7] [10].

Based on the analysis above, we can replace $G$ with $G=\frac{l_p^2c^3}{\hbar}$ and $M$ with $M=\frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ in Equation (2). This gives:

$$p=\rho\frac{c^2l_p^2}{\bar{\lambda}r^2}H \tag{6}$$

Next, we simply solve this with respect to the Planck length and we get:

$$l_p=\frac{r}{c}\sqrt{\frac{p\bar{\lambda}}{\rho H}} \tag{7}$$

To find the Planck length, we need to know the speed of light $c$, the radius of the Earth $r$, the reduced Compton wavelength of the Earth $\bar{\lambda}$, the density $\rho$ of the fluid, and the height of the fluid column $H$. In the next section, we will demonstrate that this is easily possible, even in practice.

This seems to be fully in line with a new quantization of general relativity theory. Haug [11] [12] recently re-written Einstein's [13] [14] field equation as:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\frac{8\pi l_p^2}{\hbar c}T_{\mu\nu}. \tag{8}$$

And further demonstrated that the Schwarzschild solution can be re-written as:

$$\mathrm{d}s^2=-\left(1-\frac{2GM}{rc^2}\right)c^2\mathrm{d}t^2+\left(1-\frac{2GM}{rc^2}\right)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega^2$$

$$\mathrm{d}s^2=-\left(1-\frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)c^2\mathrm{d}t^2+\left(1-\frac{2l_p}{r}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega^2 \tag{9}$$

where $\bar{\lambda}_M$ is the reduced Compton wavelength of the mass $M$, and $\mathrm{d}\Omega^2=\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2$. Further the term $\frac{l_p}{\bar{\lambda}_M}$ represents the reduced Compton frequency per Planck time. This provides the same predictions as the standard Schwarzschild metric, but it offers a deeper insight in our view. The re-written metric, expressed solely in terms of constants, requires only the Planck length and the speed of light for utilization, excluding the gravitational constant $G$. This paper demonstrates that we can determine the Planck length through gravitational phenomena, such as hydrostatic pressure, without relying on knowledge of *G*. This appears to establish a closer connection between fluid mechanics and gravity, and even extends to quantum gravity.

## 2. Experiment

We will measure the pressure in a water column and, based on this, deduce the Planck length without knowledge of the gravitational constant. To do this, we will use two low-budget manometers: the Klein Tools ET 180 digital manometer and the RISEPRO Digital manometer (see **Figure 1**). Any decent manometer should suffice; we are using two manometers just to reduce the chance of errors due to something unique to a specific manometer. Both of these manometers cost less than $50 each, and we mention this to highlight that this experiment could easily be conducted in almost any classroom. Our goal is not to achieve the highest possible precision, nor to measure the Planck length more accurately than has been done indirectly by other methods, such as a Cavendish apparatus. Instead, we aim to demonstrate how simple laws of fluid mechanics can be utilized in practice to find the Planck length without knowing the value of *G*. This has never been done before.


**Figure 1.** The figure illustrates the setup of our simple yet powerful experiment. We measure the pressure in a fluid cylinder filled with 15 cm of water using three different manometers. Based on new and deeper insights into gravity, we can extract the Planck length without any knowledge of the gravitational constant.

Figure 1 illustrates our setup. We use a fluid cylinder filled with 15 cm of water, and then we measure the hydrostatic pressure with three different manometers. The water was measured to temperature 21 celcius, which was the same as the room temperature. The measured pressure at the bottom of the vessel was 1.37 kilopascals with the RISEPRO Digital manometer and 1.38 with the Klein Tools ET 180 digital manometer. This is after adjusting for air pressure, which is done automatically by the RISEPRO Digital manometer but had to be done manually with two readings from the Klein Tools ET 180 digital manometer.

Next, we need to determine the speed of light, which we can measure without any knowledge of gravity or simply look up. As of today, the speed of light is defined exactly as 299,792,458 m/s, and we will use this value.

We also need to calculate the reduced Compton wavelength of the Earth. To do this, we take the mass of the Earth in kilograms and use the formula $\bar{\lambda}=\frac{\hbar}{M_E c}\approx 5.89\times 10^{-68}\ \text{m}$. Although determining the Earth's mass in kilograms typically requires knowledge of $G$, we can independently off knowledge of $G$ or even $\hbar$ find the reduced Compton wavelength of any mass using the procedure described in [7] and [10], which is also repeated in detail in **Appendix A**.

Furthermore, we need to determine the density of water, which is approximately 997 kg/m$^3$. With all the necessary inputs, we can use formula 7 to compute the Planck length:

$$l_p=\frac{r}{c}\sqrt{\frac{p\bar{\lambda}}{\rho H}}=\frac{6371000}{299792458}\sqrt{\frac{1.57\times 5.89\times 10^{-68}}{997\times 0.15}}\approx 1.56\times 10^{-35}\ \text{m}$$

This result is, as expected, slightly lower than the official CODATA NIST (2019) value of $1.616255\times 10^{-35}$ m, with a one standard deviation uncertainty of $0.000018\times 10^{-35}$ m. The reason for the lower value in our experiment is likely due to considerably higher uncertainty in our low-budget measurement tools. However, it's important to note that the primary aim of this experiment is not to establish a more accurate value of the Planck length but simply to demonstrate the remarkable fact that we can extract the Planck length from pressure alone, without prior knowledge of Newton's gravitational constant. This is in line with other recent research on the Planck length [7]. To accurately identify uncertainty in the measurement, a more thorough study must be conducted with careful control of temperature and air pressure during various measurements. This could serve as a basis for future research. However, the purpose of our study here is simply to demonstrate that one can indeed find the Planck length independent of the knowledge of $G$ from hydrostatic pressure.

## 3. Properties of a Hypothetical Planck Fluid

What we have discussed and investigated, even experimentally, above, is the relation between fluid pressure and the Planck length through standard gravitational observations, such as measuring pressure in a macroscopic setting. Since we can derive the Planck length from macroscopic gravitational phenomena, one would expect that the formula:

$$l_p = \frac{r}{c}\sqrt{\frac{p\bar{\lambda}}{\rho H}} \tag{10}$$

also potentially holds all the way down to the Planck scale. We are naturally far from observing something directly at the Planck length or Planck time, as these scales are significantly shorter than any current experimental devices can directly observe. However, the validity of formula 10 can be theoretically tested to some extent at the Planck scale by assuming the existence of a superfluid, which we will refer to as the Planck fluid. We will link the properties of the Planck fluid to Planck unit properties, see Unnikrishnan and Gillies [15] for an overview of Planck unit properties. For instance, we will assume that the pressure in the Planck fluid equals the Planck pressure, and that gravitational acceleration is measured at a height of one Planck length. Furthermore, the reduced Compton wavelength of the Planck mass is equal to the Planck length. Let's input these assumptions into the formula and see what results we obtain:

$$l_p = \frac{r}{c}\sqrt{\frac{p\bar{\lambda}}{\rho H}} = \frac{l_p}{c}\sqrt{\frac{p_p l_p}{\rho_p l_p}} = t_p\sqrt{\frac{p_p l_p}{\rho_p l_p}} \tag{11}$$

Here, $p_p$ represents the Planck pressure, defined as the Planck force divided by the Planck surface area, *i.e.*, $p_p = \frac{F_p}{l_p^2}$, where $F_p = G\frac{m_p m_p}{l_p^2}$ (see [15]). Further $\rho_p$ denotes the Planck (mass) density (see [15] [16]) of the Planck fluid, $\rho_p = \frac{m_p}{l_p^3}$. When we substitute these values into the formula above, we obtain:

$$l_p = \frac{l_p}{c}\sqrt{\frac{\frac{F_p}{l_p^2}l_p}{\frac{m_p}{l_p^3}l_p}} = \frac{l_p}{c}\sqrt{\frac{\frac{\frac{Gm_p^2}{l_p^2}}{l_p^2}l_p}{\frac{m_p}{l_p^3}l_p}} = \frac{l_p}{c}\sqrt{\frac{\frac{m_p c^2}{l_p^3}l_p}{\frac{m_p}{l_p^3}l_p}} = l_p \tag{12}$$

In gravitational theory, it is often assumed that the gravitational field behaves like a perfect fluid or superfluid. We propose that this superfluid can be referred to as the Planck fluid.

The Reynolds number plays a central role in various aspects of fluid mechanics, including the interpretation of the Navier-Stokes equation. The Reynolds number is defined as:

$$Re = \frac{\rho u L}{\mu} \tag{13}$$

Here, $\rho$ represents the kilogram density of the fluid, $u$ is the flow speed, $L$ is a characteristic length, and $\mu$ is the dynamic viscosity of the fluid.

An intriguing question arises: what is the Reynolds number for the Planck fluid? The Planck mass density is given as $\rho_p = \frac{m_p}{l_p^3}$. Furthermore, the flow speed of Planck mass particles is equal to the speed of light, denoted as *c*. Moreover, we assume that the characteristic length is equal to the Planck length, *i.e.*, $L = l_p$. The dynamic viscosity of the fluid can be seen as the product of pressure and time. The Planck pressure is typically expressed as

$p = \frac{\text{Planck force}}{\text{Planck area}} = \frac{F_p}{l_p^2} = \frac{G\frac{m_p m_p}{l_p^2}}{l_p^2} = \frac{m_p c^2}{l_p^3}$, and we assume it only exists for the Planck time. Therefore, the dynamic viscosity of the Planck fluid is:

$$\mu_p = \frac{m_p c^2}{l_p^3} t_p = \frac{m_p c}{l_p^2} \tag{14}$$

This means the Reynolds number for the Planck fluid is given by:

$$Re = \frac{\rho u L}{\mu} = \frac{\rho_p c l_p}{\frac{m_p c}{l_p^2}} = 1 \tag{15}$$

Chaotic behavior of fluids is typically associated with a high Reynolds number: $Re \gg 1$. Gravity, as we observe it at macroscopic scales, appears to be highly ordered and deterministic, unlike many chaotic fluids. A Reynolds number of 1 for this Planck fluid, which we associate with gravity, is therefore somewhat consistent with our observations.

The Reynolds number compares inertial forces to viscous forces. When the inertial force equals the viscous force, resulting in a Reynolds number of one, the fluid does not move. Initially, this may seem to pose a dilemma. We assumed a flow speed of *c*, so the Planck fluid should move at the speed of light, seemingly contradictory to the idea that it doesn't move. However, this is entirely consistent with a new quantum gravity model [17], where the Planck mass particle is essentially a collision between building blocks involving two photons. This collision lasts only for the Planck time, and the collision itself is what we consider as mass in this model. So, the superfluid can both remain completely still and move at the speed of light. We propose the hypothesis that the Planck fluid exists within all matter, including any fluids that has rest-mass, which are all known real fluids.

We expect, or perhaps more accurately, hope, that the Reynolds number for the Planck fluid could potentially provide new insights into the Navier-Stokes equations. Fluid mechanics at the Planck scale may behave in a binary manner, either moving at *c* or not moving at all-switching between these states, essentially causing the Planck fluid to vibrate. Admittedly, we are entering speculative territory here, but we have dedicated many years to studying quantum gravity theory, so there is more thought behind this than we can explore fully in a single article. As far back as 1916, Einstein [14] suggested in one of his general relativity papers that the next significant step in gravity would involve the development of quantum gravity. Eddington [18], in 1918, was likely the first to propose that such a quantum gravity theory had to be linked to the Planck length (Planck scale). Now, over 100 years later, there is still no consensus on a unified quantum gravity theory [19]. Perhaps new ideas concerning Planck scale superfluids can take us a step further. At the very least, this quantum Planck fluid introduced here appears to be consistent with a new way to quantize general relativity theory; see [12] [20], and connect it to the Planck scale. However, we don't ask anyone to take any of this for granted, but rather to investigate for themselves and join the discussions about the Planck scale, which should also include the potential existence of a Planck fluid.

## 4. Conclusion

We have presented a theory on how the Planck length can be determined from fluid mechanics without requiring knowledge of the gravitational constant $G$. Specifically, we achieve this by utilizing Pascal's law and new insights in quantum gravity. Additionally, we conducted a simple experiment using a manometer to measure the pressure in a water column and deduced the Planck length from the results. This demonstration shows that it can be easily done in practice. Furthermore, we briefly outline some properties of a hypothetical fluid that we have coined the "Planck fluid." We find that its Reynolds number is likely to be one, and that we could hypothetically extract the Planck length from it if we could measure its pressure to be the Planck pressure.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix

We will demonstrate how one can determine the Compton wavelength for any mass, even very large ones, without the need to know the mass in kilograms or the Planck constant. We have detailed this approach in multiple papers, but because it is so central, we are reiterating much of this methodology here in this appendix. The Compton wavelength is typically given by

$$\lambda = \frac{h}{mc} \tag{16}$$

In this case, we usually require knowledge of both the Planck constant and the mass, denoted as $m$, along with the speed of light to calculate the Compton wavelength. However, Compton, based on Compton scattering of electrons, provided the following formula:

$$\lambda_{\gamma,1} - \lambda_{\gamma,2} = \frac{h}{mc}(1-\cos\theta) \tag{17}$$

We can replace $\frac{h}{mc}$ with the Compton wavelength $\lambda$ and solve for it, yielding:

$$\lambda = \frac{\lambda_{\gamma,1} - \lambda_{\gamma,2}}{1-\cos\theta} \tag{18}$$

To find the Compton wavelength of the electron, we need to measure the wavelengths of the incoming and outgoing photons that collided with the electron, as well as the angle between the incoming and outgoing photons. This process is known as basic Compton scattering.

Next, we will utilize the fact that the absolute value of the charge is the same for an electron and a proton. The Cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \tag{19}$$

The ratio of the Cyclotron frequency of the proton to that of the electron is expressed as:

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_P}}{\frac{qB}{2\pi m_e}} = \frac{m_P}{m_e} = \frac{\bar{\lambda}_e}{\bar{\lambda}_P} \approx 1836.15 \tag{20}$$

Therefore, all we need to do to find the Compton wavelength of the proton relative to the electron is to measure the Cyclotron frequency of a proton and an electron (see also [21] and [22]). The Compton wavelength is then $\bar{\lambda}_P = \frac{1}{1836.15}$. However, since the proton is a composite particle, it is unlikely to have a single Compton wavelength, even though the proton wavelength has been discussed in the literature for many years (see [23] [24], and others). Nevertheless, the method above provides the correct relative mass of the proton to the electron.

To find the Compton wavelength of a larger macroscopic mass, all we need to do is count the number of protons and neutrons in it. Since neutrons have almost the same mass as protons, the Compton wavelength of such a mass is simply the Compton wavelength of the proton divided by the total number of protons and neutrons in the mass of interest. Additionally, we need to consider a small correction factor for the number of electrons in the mass. Counting atoms in a macroscopic object is not easy, but it is entirely feasible and was actually one of the competing methods for establishing a new kilogram standard (see [25] [26] [27] [28]). Other methods are also available for counting atoms; for example, refer to [29].