Norwegian University of Life Sciences
School of Economics and Business

Philosophiae Doctor (PhD)
Thesis 2023:24

Four essays on oil price uncertainty, optimal investment strategies and cost transmission of an oil price shock

Fire artikler om oljeprisusikkerhet, optimale investeringsstrategier og kostnadsovervelting av et oljeprissjokk

Micah Lucy Abigaba

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Ås (2023)

Thesis number 2023:24
ISSN 1894-6402
ISBN 978-82-575-2054-0

Dedication

To my father, Dr. Fabius M. Byaruhanga, who has always believed in me more than I did in myself. To my mother, Mrs. Alice Muhairwe Byaruhanga, that instilled in me the will to push through any hurdles and never give up. Thank you for your love, making sure we never lacked and for giving us a safe healthy childhood that has made me who I am today. I am grateful for your love and care for my Flower and Martin.

To my brothers and sisters that have been my support system and parents to my children for the longest time. Ivan Bulemu, Esther Barungi, Martha Bayenda and Tom Basiima- *Rukundo egumeho!* To my cousin Joseline Agaba, who has been my children's school guardian. To my Uncle Zephania Mugisha and cousin Prof. Rose Namara for your continued guidance and being my source of inspiration. I am forever indebted to you!

To my children, Martin Amani Mpuuga, Flower Alice Nsiime, Mbonimpa Ethan Ochieng, Mugisha Ewald Ochieng. You are the reason I continue to pursue my dreams. To Martin and Flower- thank you for your love, understanding and patience for the long times that I have had to be away. I know that I cannot make up for my absence. Always know that my home is where you are.

To my partner, father to our children and designated 'Director of Studies', Robert Mbonimpa Ochieng, thank you for being my support system, enduring my emotional roller coaster, for the hours we have spent discussing my thesis and all things 'oil'. I am grateful to you for funding my extended stay in Norway and ensuring that I never had to worry about finances after my scholarship ended. I also thank you for being a good father to all our children!

In loving memory of my grandfather, Ernest Budala and my brother Tom Basiima.

Acknowledgements

I am enormously grateful to my main supervisor, Prof. Knut Einar Rosendahl for your dedicated supervision, critical reviews, mentorship, encouragement and believing in me. I am also grateful to Jens Bengtsson, who agreed to be my co-supervisor, and take time to shape my understanding of Real Options, give comments and feedback on my work. You have both always put my interests ahead of yours, for which I am forever indebted. I am honoured to have co-authored with you and been under your mentorship.

I am also grateful to my other two co-authors, Peter Kort and Martijn Ketelaars. I am thankful to you for sharing your knowledge, expertise, and insightful advice. To Peter Kort, thank you for your additional supervision without expecting anything in return. You made this thesis journey rewarding and less stressful.

My sincere gratitude goes out to Berit Pettersen, Prof. Olvar Bergland, Prof. Ragnar A. Øygard, Prof. Arild Angelsen, Lise Thoen, Kirsti Pettersen, Reidun Aasheim, and the rest of the academic and administrative staff at the NMBU School of Economics and Business (HH) for making my PhD experience worthwhile.

I would also like to thank my mentor, Dr. Isaac Nkote, for profoundly shaping my career in academia, right from my Undergraduate studies. I also thank my Head of Department at Makerere University Business School (MUBS), Dr. Atukunda Ronnette for her moral support and encouragement. I also appreciate Prof. Mohammed Ngoma, Dr. Livingstone and Bosco Amerit for their invaluable administrative support throughout my PhD. I would like to also thank Wilson Asiimwe for his mentorship, guidance and encouragement.

I would also like to recognise the Norwegian government for funding my PhD studies under the NORAD- NORHEAD I project- Capacity Building in Education and Research for Economic Governance in Uganda. I extend my gratitude to the management of Makerere University Business School and HH for believing in me and helping me advance my career in academia. My gratitude goes out to Robert Mbonimpa Ochieng for funding my extended stay in Norway.

I am also thankful to my officemates, other fellow PhD colleagues, the Ubuntu family, and friends in Ås for the good laughs, welcoming smiles, shared lunches and exchanged books and articles. My heartfelt thanks go to Ritah Kansiime, Doreen Auma, Phionah, Prof. Samuel Muyiwa Adaramola and Irene Namugenyi Anecho for the *Ubuntu*. To my friends in Oslo, Chavez, Lungu and Robert's friends, thank you for always being there to help, for the hearty laughs and good times.

To my dear friends, Miria Nakamya, Grace Namubiru Birungi, Patsy Katutsi, Hope Masaba, Yennie Katarina Bredin, Marjama Hassan, Diana Bamuhigire, Eve Nanyonga and Kauther Nantambi. Thank you for your words of encouragement, support and prayers.

To my family away from home- Doreen Ochieng, Ken Ochieng, Mama Monica Unzamukunda and Mzee Zachary Ochieng- thank you for your love, care and prayers that have lessened my burden of being far from home.

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Paper 3: Real options valuation of a Production Sharing Agreement – Conflicting strategies between government and IOCs? Co-authors: Jens Bengtsson and Knut Einar Rosendahl. Working paper, Ås, Norway

Paper 4: The potential inflationary impacts of an oil price shock on the economy: a social accounting matrix price multiplier analysis for Uganda. Working paper, Ås, Norway.

Summary

This thesis investigates the influence of oil price uncertainty on optimal investment decisions and the economy-wide effects of oil price shocks. This general objective is achieved in four research papers. The thesis is organised in five chapters; chapter one is the introduction, and the papers are in chapters two to five.

Amidst high crude oil volatility in the past two decades, there has been renewed interest by international oil companies (IOCs) in investments towards undeveloped oil reserves in Africa. With prospects for economic growth emanating from oil revenues, several host governments have initiated oil contracts with IOCs, as a key component in the overall regulatory framework for upstream oil operations. Oil investments are typically sequential, highly costly, lumpy and are irreversible, for the most part, once expended. In addition to oil price uncertainty, oil projects are faced with uncertainties about; time to complete stages that precede oil production, operating expenditures during the long lifetime of the project, economically viable oil reserves, the amount of oil produced from proven reserves, and future world demand for oil. These uncertainties are more pronounced in resource-endowed developing countries that are venturing into oil production for the first time due to their lack of technical know-how, technology, and limited capital. Inherently, the IOCs and host governments are faced with strategic decisions of whether to invest immediately or to defer investment, until the prevailing risk factors are favourable. Real options valuation methods that account for these complexities and uncertainties are most appropriate to estimate oil projects and to inform optimal investment decisions.

This thesis, therefore, contributes to the literature by developing three real options frameworks which are applied to a case study of Uganda's undeveloped reserves to generate numeral results. Paper 1 formulates a model to estimate the project values and optimal oil price thresholds for investing at the development and production stages, while accounting for completion of the development stage. Papers 2 and 3 apply a binomial lattices model. Paper 2 estimates the value of the oil project while accounting for the flexibility of deferring production and oil price uncertainty. Paper 3 builds on Paper 2 by assessing whether the IOCs and government have aligned or conflicting optimal investment strategies, arising from the design of the production sharing agreement (PSA) and variations in external risk factors. Overall, our findings establish that uncertainties faced by oil projects have profound impacts on the project values and optimal strategies. IOCs and host governments have conflicting interests and may bias their decisions if they neglect these project uncertainties. This thesis recommends that the

PSA should be designed in a way that allows for flexibilities in the event that these risks arise. This would in turn incentivise oil investments by the IOCs while stabilizing government revenues from the oil sector.

The volatility in crude oil prices not only impacts oil investments but is also an important determinant of the prices of refined petroleum. This is because the prices of refined petroleum co-move with the price of crude oil. Surges in crude oil prices are a concern for net-importing developing countries because they result in higher import prices of refined petroleum, which are transmitted to the final consumer in form of higher fuel pump prices. Rising fuel pump prices represent an increase in production costs of firms and higher cost of living for households, which ultimately have implications on inflation. It is therefore imperative to trace the transmission of an oil price shock through its impact on domestic production prices and consumer prices and ultimately measure the potential inflationary effects of an oil price shock. To achieve this objective, paper 4 constructs a SAM Price multiplier model which is applied to Uganda's economy. The results from Paper 4 inform government on the extent to which the absence of fuel price controls account for Uganda's vulnerability to oil price shocks. The findings show that the influence of oil prices is very asymmetric and depends to a greater extent on the specific sector and/or agent analysed. As expected, the activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. The results also establish that the distributional impacts of rising petroleum prices tend to be progressive since the poorest households are the least affected compared to the higher income households. The thesis recommends that policy interventions should be tailored in a way that enhances competitiveness in sectors most affected by oil price shocks and without worsening the welfare of households that are most sensitive to the oil price shocks.

Sammendrag

Denne avhandlingen undersøker hvordan usikkerhet omkring oljeprisen påvirker optimale investeringsbeslutninger, og makroøkonomiske effekter av oljeprissjokk. Dette studeres i fire forskningsartikler. Avhandlingen er inndelt i fem kapitler; kapittel 1 er innledning, mens kapitlene 2-5 består av artiklene.

Samtidig som det har vært betydelig volatilitet i oljeprisen de to siste tiårene, har det blitt fornyet interesse fra internasjonale oljeselskaper (IOCs) i å investere i utvikling av oljereserver i Afrika. Med utsikter til økonomisk vekst som følge av oljeinntekter, har flere afrikanske regjeringer initiert oljekontrakter med IOCs, som en sentral komponent i den overordnede politikken knyttet til oljeutvinning. Oljeinvesteringer er typisk sekvensielle, svært kostbare, og ofte irreversible når de først er utført. I tillegg til oljeprisusikkerhet, står oljeprosjekter overfor usikkerhet knyttet til utbyggingsfasen, driftskostnader gjennom den lange levetida for prosjektet, mengden utvinnbare reserver, hvor mye olje som kan produseres, og global etterspørsel etter olje. Disse usikkerhetene er mer utstrakte i ressursrike utviklingsland som går inn i oljeutvinning for første gang, som følge av mangel på teknologisk kompetanse, teknologi, og begrenset tilgang på kapital. IOCs og myndighetene står overfor strategiske beslutninger om hvorvidt man bør investere umiddelbart eller utsette investeringene inntil rådende risikofaktorer er gunstige. Metoder for realopsjonsverdsetting, som tar hensyn til disse kompleksitetene og usikkerhetene, er nyttige til å vurdere oljeprosjekter og hva som er optimale investeringsbeslutninger.

Denne avhandlingen bidrar til forskningslitteraturen ved å utvikle tre realopsjonsrammeverk som anvendes på en casestudie knyttet til Ugandas uutviklete oljereserver. Artikkel 1 anvender dynamisk programmering til å estimere prosjektverdier og optimale oljepristerskler for investeringer i utbyggings- og produksjonsfasene, mens det tas hensyn til fullføring av utbyggingsfasen. Artikkel 2 og 3 anvender en binomisk gitter modell. Artikkel 2 beregner verdien av oljeprosjektet mens det tas hensyn til fleksibiliteten i å utsette produksjon og usikkerheten i oljeprisen. Artikkel 3 bygger på Artikkel 2 og undersøker hvorvidt IOCs og myndighetene har samsvarende eller motstridende optimale investeringsstrategier som følge av hvordan produksjonsdelingsavtale (PSA) er utformet, samt variasjonen i eksterne risikofaktorer. Alt i alt finner vi at usikkerheten oljeprosjekter står overfor har betydelig innvirkning på prosjektverdier og optimale strategier. IOCs og myndighetene har motstridende interesser og dette kan virke inn på hvordan prosjektusikkerhet påvirker deres beslutninger. Avhandlingen anbefaler at PSA bør utformes slik at de åpner for fornuftig fleksibilitet i lys av

risikobildet. Dette kan videre gi IOCs insentiver til oljeinvesteringer, samtidig som det kan gi stabile oljeinntekter for myndighetene.

Volatiliteten i råoljeprisen påvirker ikke bare oljeinvesteringer men også prisen på oljeprodukter. Prisen på oljeprodukter går i takt med prisen på råolje. Økte råoljepriser er en bekymring for utviklingsland som er nettoimportører av olje fordi det fører til økte importpriser på oljeprodukter, som videre overføres til økte pumpepriser for sluttbrukere. Økte pumpepriser innebærer økte produksjonskostnader for bedrifter og økte levekostnader for husholdninger, og videre økt inflasjon. Det er viktig å spore hvordan et oljeprissjokk overføres gjennom økonomien via effekter på produktpriser og konsumentpriser, og til slutt potensielle inflasjonseffekter av et oljeprissjokk. For å undersøke dette, konstruerer Artikkel 4 en SAM (Social Accounting Matrix) prismultiplikatormodell som anvendes på Ugandas økonomi. Resultatene fra Artikkel 4 informerer myndighetene om i hvilken grad mangel på priskontroll for oljeprodukter kan gjøre Uganda utsatt for oljeprissjokk. Resultatene viser at påvirkningen av oljeprisen av svært asymmetrisk og avhenger i stor grad av den spesifikke sektoren og/eller husholdning som studeres. Som ventet blir effektene størst for sektorer med stort forbruk av oljeprodukter. Resultatene tyder også på at fordelingseffekten av økte petroleumspriser er progressiv, ettersom de fattigste husholdningene blir mindre berørt enn de med høyere inntekt. Avhandlingen anbefaler at politikkinnblanding bør innrettes slik at den forbedrer konkurransekraften i sektorer som er mest utsatt for oljeprissjokk, og unngår å forverre velferden for husholdninger som er mest utsatt.

Introductory chapter to PhD thesis

Four essays on oil price uncertainty, optimal investment strategies and cost transmission of an oil price shock

1. Introduction

1.1 Background to the thesis

Oil investments are typically characterised by high costs, numerous uncertainties, irreversibility, sequentiality and lumpiness. Investing in an oil project is a step-wise process, from exploration, appraisal of oil reserves, development of oil fields and support infrastructure, actual production of oil to decommissioning. Investments at each sequential stage of oil project lifecycle are made in lumps and are sunk, for the most part, once expended. More so, each successive stage prior to the production stage typically does not lead to immediate cash flows but opens up further investment opportunities. The capital intensity of oil investments, particularly at the development stage, makes them irreversible because the oil wells and operation facilities can only be used to produce oil. These complexities are exacerbated by the various uncertainties faced by these projects. Among these is the uncertainty about the oil price which significantly influences the value of an oil project. Another is the uncertain time to completion of the preceding stages to production stage, particularly the development stage that requires large capital-intensive investments in oil drilling facilities and support infrastructure that take a long time to build. The decision makers are faced with strategic decisions of whether to invest immediately or to postpone investment, until the prevailing risk factors are favourable. Therefore, project valuation methods that account for these complexities and uncertainties are most appropriate to estimate oil projects and inform optimal investment decisions.

The past two decades have been characterised by high crude oil price volatility (see Appendix A for crude oil price trend) that has shaped global oil investments. Particularly, the oil price up swings between 2003 and 2014 attracted new discoveries and an ongoing search for oil in various African countries. During this period, several host governments initiated oil contracts[1] with international oil companies (IOCs) as a key component in the overall regulatory framework for upstream oil operations (see Graham and Ovadia, 2019). There are two key motivations for pursuing these oil contracts: First, high costs, risks and uncertainties are more pronounced in resource-endowed developing countries that are venturing into oil production for the first time due to their lack of technical know-how, technology, and limited capital. Second, oil contracts serve the purpose of explicitly defining resource ownership, risk bearing, payments owed to each party and how to resolve any issues, should they arise. The

[1] Oil contracts are broadly categorised as production sharing agreements, service contracts, concession contracts and joint venture contracts. The contract types differ based on ownership of oil resources, the extent of government control over operations, the size of the national oil company's participation and risk bearing by each party to the contract.

effectiveness of an oil contract is thus assessed based on its ability to maximise returns for both the host government and the IOCs, without distortions in exploration, production and development activities.

Inherent in the oil contract, the parties also obtain the right to exercise different managerial flexibilities. For instance, parties to an oil contract have the option to defer investment, expand or contract the oil project, abandon for salvage, farm-out of a joint venture or switch to another plan. This enables the respective parties to strategically capitalize on revenue windfalls arising from best-case scenarios of the uncertainties while mitigating the risks associated with low revenues during the worst-case scenarios. Investing in an oil project is thus a real options investment problem.

Real options valuation is concerned with estimating the value of the flexibilities embedded in investment decisions with emergence of new information based on probabilistic market variables. This is contrary to the discounted cash flow valuation, which assumes a predetermined scenario that oil fields are based on deterministic market variables such as cost and revenue, which represent the future of the project, and that a project operates in each year of its duration. Real options valuation derived from the financial option theory, and was originally coined by Myers (1977) in response to the various limitations of the discounted cash flow valuation. Thereafter, the real options approach was first applied to the valuation of oil and gas production projects by Brennan and Schwartz (1985). Since then, there has been growing application of real options to analyse managerial flexibilities and uncertainties in oil investments.

Despite the renewed interests of IOCs in Africa’s vast oil reserves in the past two decades, most of the real options literature are focused on developed reserves in high-income countries (see Section 3.2). As prior cited, high costs, risks and uncertainties are more pronounced in developing countries that are venturing into oil investments for the first time. It is also expected that the stages prior to actual oil production would take longer. Due to lack of technical know-how, technology, and limited capital, the host governments have to rely on oil contracts with IOCs to realise oil investments. Challenges of information asymmetry during negotiations of oil contracts, such as distorted bargaining power, are expected since IOCs are often more informed than the host governments about oil market dynamics. These complexities are critical motivations for undertaking real options valuation of oil investments in developing countries.

Papers 1-3 of this thesis therefore contribute to the literature by applying real options in the context of a developing country with undeveloped reserves. The studies in the thesis are particularly motivated by the volatile trend of world crude oil prices. Uganda's oil project is chosen as the case study. The choice of the case study is fostered by: First, the discovery of Uganda's oil reserves in 2006. Second, the issuance of the first production license in 2012 under a production sharing agreement (PSA) between the government of Uganda and three IOCs. Third, the expensive development phase of the oil project with an estimated cost of USD12.5-USD15 billion. Fourth, the continuous delay in completion of the development stage which was expected to begin in 2018 and last for 3 years. Lastly, the persistent high volatility of global crude oil prices.

Upswings in crude oil prices not only impacts oil investments but also translates into high inflation for any economy. Particularly for oil-importing countries, a surge in crude oil prices results in higher import prices of refined petroleum, which are transmitted to the final consumer in form of higher fuel pump prices. The rise in fuel pump prices is inflationary in three ways (Ogwang *et al.*, 2019). First, households and firms pay more for petroleum products they consume directly. Secondly, higher oil prices increase the prices of all other goods that have oil as an intermediate input. Thirdly, higher fuel pump prices exacerbate the cost of doing business on account of higher transport costs. In addition, the rising prices of fuel pump prices have distributional implications. Empirical studies show that rising fuel prices tend to affect poor households and richer households differently (Saari *et al.*, 2016). It is therefore imperative to analyse the potential inflationary impacts of oil price shocks.

Uganda is chosen for our case study in Paper 4 for three main reasons. First, Uganda is entirely dependent on imports of refined petroleum, which exacerbates its vulnerability to external oil price shocks. Second, the recent crude oil price shocks in 2022 have led to substantial domestic fuel pump price hikes in Uganda. Lastly, Uganda's fuel market is fully liberalised such that fuel pump prices are determined purely by forces of demand and supply. In the absence of fuel price controls and fuel subsidies, high petroleum import prices are thus fully transmitted in form of high domestic fuel pump prices. This makes Uganda a good case in point for analysing potential inflationary impacts of crude oil price shocks on a developing oil-importing country.

1.2 Thesis objectives

The main objectives guiding this thesis, as addressed by each paper, are as follows:

Paper 1: To value an oil investment project with uncertain time to completion of the development stage.

Paper 2: To estimate the value of deferring production of Uganda's oil while accounting for oil price uncertainty.

Paper 3: To assesses whether the government and IOCs have aligned or conflicting optimal strategies, arising from the terms of the PSA, while accounting for flexibilities and uncertainties.

Paper 4: To estimate the potential inflationary impacts of oil price shocks on an economy, by constructing and employing a Social Accounting Matrix (SAM) Price multiplier model.

1.3 Snapshot of the thesis

Table 1: The models applied in this thesis and the main findings

Paper	Empirical model	Estimation method	Uncertain variable(s)	Model setting
1	Real options model	Dynamic programming	Crude oil price Time to completion of development stage	- Sequential irreversible investment - Two stage investment decisions: development and production - Four decision states - Single decision maker
Main findings: - The threshold price for the development stage is significantly higher than that of the production stage. - The project value for the development stage increases when the expected time to completion is shorter. - The threshold price may be non-monotonic in the expected time to completion - Higher oil price volatility increases the project value and higher threshold prices.				
2	Real options model	Binomial lattices	Crude oil price	- Decision to invest in development stage is already undertaken - One investment decision: whether to start or defer oil production - Single decision maker
Main findings: - The option to defer production by another year adds value to Uganda's oil.				

(continued) - The value of the option to defer production particularly increases at lower crude oil prices amidst higher crude oil price volatility. - A combination of a high net convenience yield rate and high oil price result in a lower option value. - At low oil prices, increases in cost inflation result in rejection of the project.				
3	Real options model	Binomial lattices	Crude oil price	- Investment decisions hinged on PSA - One investment decision: - whether to start or defer oil production - Two decision makers
Main findings: - The government's critical oil prices are lower than those of the IOCs. - As the oil price rises, the expanded NPV of the government rises faster than that of the IOCs. - A higher cost oil limit increases the expanded NPV of the IOC whereas the expanded NPV of the government declines. - Overall, the parties have conflicting optimal strategies. - A combination of a low cost oil rate and a low oil price, increases the risk that the IOC may not fully recover its incurred project costs.				
4	SAM Price Model	Multiplier analysis and Multiplier decomposition	Import price of refined petroleum	- Constant technical coefficient matrix - 8 household groups
Main findings: -The influence of petroleum import price shock is very asymmetric and depends on the specific sector and/or agent analysed. -The activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. - The production prices of manufacturing and processing sectors are, on average, the most affected. - The inflationary impact of the exogenous petroleum import price shock is mainly transmitted directly through increases in prices of activity sectors that use petroleum as an intermediate input in the production process. - For both urban and rural regions, the households in higher income quartiles are more affected by the petroleum import shock, as compared to the low-income quartiles.				

1.4 Thesis Outline

The thesis is organised in five chapters. In the subsequent sections of this chapter, I describe Uganda's oil market in section two, followed by a discussion of the data and methods used in section three. In section four, I present the main findings and contributions and draw policy implications and conclusions in section five. The limitations of the studies and recommendations for further research are discussed in section six. The four research papers are compiled in the remaining four chapters.

2. The Albertine oil project and the fuel market in Uganda

2.1 Albertine oil project

In 2006, Uganda joined the list of prospective oil producing countries with 6 billion proven oil reserves in the Albertine Graben[2] of which 1.4 billion barrels are economically viable for extraction. The peak production is projected to be between 200,000 and 250,000 barrels of oil per day with extraction lasting 25 years (Petroleum Authority of Uganda, 2018). The cost of extracting oil over this period will amount to approximately USD19 billion in capital expenditures and operating expenses. Prior to this production stage, the development of infrastructure, operation facilities and production wells will cost about USD12.5-15 billion. It is anticipated that oil revenues from oil production could generate approximately 10–15% of Uganda's GDP at peak production (World Bank, 2010). Wiebelt *et al.* (2018) show that, if well-managed, the oil revenues have the potential to significantly stimulate Uganda's economic growth and real household incomes.

The African Union Agenda 2063 advocates for expanded local ownership and increased control of oil and gas reserves[3] (African Union, 2014). However, like many resource-endowed Sub-Saharan countries, Uganda has limited capacity to solely finance and operate immense complex oil projects (Graham and Ovadia, 2019). Consequently, in 2016, the government of Uganda finalised a PSA with three IOCs. The 2012 Oil and Gas Revenue Management policy (Ministry of Finance, Planning and Economic Development, 2012) establishes the PSA among the fiscal instruments for managing Uganda's oil revenues. The PSA stipulates how risks and revenues are shared between the government and the IOCs, throughout the project lifespan. According to the PSA, the IOCs incur all expenditures on exploration, development and production of oil. Upon production of oil, the IOCs pay royalties and additional royalties to the government. The IOC recovers its costs, as per the cost recovery limit. The remainder after royalties and cost recovery is the profit oil that is shared between the IOCs and the government. The share of profit oil for either party depends on the volume of oil production. A corporate income tax is levied on the IOC's share of the profit oil. The net cashflow of the IOC is what is left after

[2] The Albertine Graben is approximately 500 km long, averaging 45 km in width and measures about 23,000 square kilometres in Western Uganda (see map in Appendix B).

[3] The African Union (AU) Agenda 2063 is a plan for Africa's structural transformation and was agreed upon by the Heads of AU member states in May 2013. The AU envisages 'Transformed Economies and Jobs' as its Goal 5. To achieve goal 5, one of the priority areas is 'Expanded ownership, control and value addition (local content) in extractive industries' (see African Union Commission, 2014).

deducting the total costs from the sum of cost oil and profit oil after taxes. The government yields revenues[4] from royalties, additional royalties, profit oil and income tax revenues.

In 2016, the three IOCs; Tullow Oil, Total Energies and China National Offshore Oil Corporation (CNOOC) initiated a joint venture agreement to operate the three oil fields: Buliisa, Kingfisher and Kaiso-Tonya. In 2022, Tullow oil finalised the sale of its entire stake in the oil project to Total Energies, bringing the project ownership to 56.67% for Total Energies; 28.33% for CNOOC. The state company, Uganda National Oil Company (UNOC) is mandated to manage the country's commercial interests in the oil sector and holds the remaining 15%, as per the PSA terms. The production licenses, as part of the comprehensive PSA, are valid for 25 years upon the extraction of the first oil (Petroleum Authority of Uganda, 2018). The issuance of these production licenses formed the basis for the Final Investment Decision (FID)[5] for the development phase. However, the FID that was initially expected in 2015, was continuously delayed. The reasons for this delay include; tax disputes over capital gains of IOCs, the sale of Tullow's stake in the oil project, delayed or deadlocked negotiations with other partners to the contract and the delay in the compensation and relocation of communities affected by the oil project. Another critical reason is oil price volatility. For instance, renegotiations of oil contract terms ensued from the surge in global crude oil prices between 2009 and 2014. Slumps in oil prices between 2014 to 2016 forced IOCs to drastically trim their local workforce and cut their investment budgets by 20 to 30 percent. The drop in oil prices due to the Covid-19 pandemic and the ensuing lockdowns in Uganda created deeper ambiguity about the timeline for Uganda's development phase and extraction of first oil.

In February 2022, Total Energies and CNOOC signed the FID committing to invest in the development of the oil fields and a crude oil pipeline. For purposes of modelling throughout the thesis, we have categorised the oil fields into three, based on their geographical location in the Albertine Graben[6].

[4] The government will yield additional streams of revenue flows from bonus payments, capital gains tax, surface rental charges and other fees.

[5] Final Investment Decision defines the financial commitment of an oil firm to towards investing in the development stage. It marks the beginning of engineering, procurement and construction. The FID was preceded by the Inter-Government Agreement (IGA) between the governments of Uganda and Tanzania for the EACOP, signed in May 2017, and the Front-End Engineering Design (FEED) for the EACOP approved by the Petroleum Authority of Uganda in October 2020.

[6] In recent official publications, upon the exit of Tullow Oil PLC exiting Uganda's oil project, the oil fields are categorised into two; Tilenga and Kingfisher oil fields (see Petroleum Authority of Uganda, 2022).

Buliisa oil fields

The Buliisa oil fields cover the EA2 North and EA1 blocks, North-East of Lake Albert, with eleven fields under development (see the map in Appendix B). Total Energies is the lead operator of the Buliisa oil fields. The Buliisa fields hold the highest reserves, estimated at 819 million barrels of recoverable oil, with its production peak in its fourth year of extraction. The total CAPEX is estimated to be USD6.5bn, of which USD5.1bn will be expended in the first 7 years prior to production. The total OPEX is assumed to be USD8.7bn over the entire 25 years of the production phase.

Kingfisher oil field

The Kingfisher oil field, operated by CNOOC, encompasses the EA3A Block, South of Lake Albert (see Appendix B) and is estimated to have 196 million barrels of recoverable oil, with expected peak production in its eighth year of extraction. The project's CAPEX is estimated to be USD1.5bn, 87% of which is spent in the 5 years prior to production and the rest in the first 3 years of oil production. The total OPEX expended over the 25 years of oil production is projected to be USD2bn.

Kaiso-Tonya fields

The Kaiso-Tonya fields cover EA2 Block, South East of Lake Albert, with three oil fields (see Appendix B). The oil fields are relatively small, with 39 million barrels of recoverable oil, and would not be economically viable on their own. The fields are thus the least complex and least costly as their production is tied-in to that of the Kingfisher oil field. The production from Kaiso-Tonya begins in the ninth year of Kingfisher's production to compensate for the decline in the latter field. The project's CAPEX is estimated to be USD483 million, which is all expended in the 4 years prior to production. The total OPEX is USD357 million over the 19 years of oil production from these oil fields.

An additional development cost to the oil project is the East Africa Crude Oil Export Pipeline (EACOP) that will transport the crude oil from Uganda's oil fields to the port of Tanga in Tanzania for export (see map in Appendix C). This pipeline will be constructed at a cost of USD3.5-5 billion and operated through a pipeline company with shareholding from the Uganda National Oil Company (15%), the Tanzania Petroleum Development Corporation (15%) and the two oil companies; Total Energies (62%) and CNOOC (8%) (EACOP,2022).

The CAPEX, OPEX and development costs of the EACOP are recoverable as per the terms of the cost oil in the PSA. As is the case for the oil field projects, the IOCs are required to incur all cost commitments of UNOC in the EACOP project during the development stage, which are deemed recoverable, based on the terms of the PSA. Other infrastructural requirements towards the oil project, including roads and the Hoima international airport, are expended by the government and are not recoverable as per the PSA.

2.2 Domestic fuel pump market and fuel price trends

Uganda is entirely dependent on imports of refined petroleum, which exacerbates its vulnerability to external oil price shocks. About 90% of Uganda's petroleum imports are transported by fuel trucks through Kenya and the remaining 10% come from Tanzania. In 2021, Uganda's petroleum accounted for about 14% of the total imports bill and recorded an average daily consumption of 6.5million litres of petroleum (Bank of Uganda, 2022).

Uganda's fuel market is fully liberalised such that fuel pump prices are determined purely by forces of demand and supply. In the absence of fuel price controls and fuel subsidies, high petroleum import prices are thus fully transmitted in form of high domestic fuel pump prices. Odokonyero and Bulime (2022) decomposed the fuel pump price for the year 2021, and showed that, on average, an increase in the global crude oil price by USD 1 results in an average pass-through of USD 2 for the fuel pump price in Uganda. The recent crude oil price shock from December 2021 that has continued into 2022, led to substantial domestic fuel pump price hikes in Uganda. Particularly, fuel pump prices of diesel and petrol recorded annual percentage changes of 72% and 56%, respectively, in June 2022. The domestic fuel pump prices follow the same pattern as the global crude oil prices, which reflects a transmission of an external oil price shock to the domestic fuel prices (Odokonye and Bulime, 2022).

3. Data and Methods

3.1 Data

The thesis employs three data sets. The first data set is used for analysis of the first three papers that value Uganda's oil investments. The data set consists of the oil production profile, cost of development, CAPEX and OPEX of the Kingfisher and Buliisa oil fields[7], as projected over the period of 25 years. All these cost data were obtained from estimates by Ward and Malov

[7] In the Papers 1-3, the Kaiso-Tonya fields are solely economically infeasible and are thus excluded from the analyses. This has no effects on the results and implications drawn from our studies.

(2016) and through interviews with officials at the Petroleum Authority of Uganda and Ministry of Energy and Mineral Development. The cost estimates exclude sunk costs towards; land acquisition, contingency, Front-End Engineering design (FEED), Environmental and Social Impact Assessment (ESIA), feasibility studies and other studies that are completed before the development phase commences. The oil project data was supplemented with data on the monthly historical spot prices of Nigeria's Bonny Light crude from January 2006 to December 2018. The spot price of Bonny Light crude is chosen as a proxy for Uganda's crude oil over Brent and WTI crude because of its similar characteristics in terms of API gravity and sulphur content, as well as the geographical location. The price data is obtained from the website of the Central Bank of Nigeria (Central Bank of Nigeria, 2018).

The second data set comprises of the structure/terms of the PSA such as the royalty rates, additional royalty rates, cost recovery limit, corporate tax rate and profit sharing rates. This data was obtained from the official PSA as published on the official website of the Uganda National Oil Company (2021). This data set complements the first data set to achieve the research objectives of Paper three.

The third data set is the 2016/17 official Social Accounting Matrix for Uganda. A SAM covers the entire economy and quantifies linkages between several production sectors and households[8]. This data is the basis for the fourth paper that estimates the potential inflationary impact of an oil price shock on Uganda's economy. The SAM was obtained from the Ministry of Finance, Planning and Economic Development. A detailed description of the SAM can be found in Tran *et al*. (2019). The original SAM consists of 186 activities and commodities, 2 accounts for trade and transport margins, 5 tax accounts, 17 factor accounts, 32 household groups, 2 enterprise accounts and lastly one account each for Non-Profit Institutions Serving Households (NPISH), government, investment-savings, changes in inventory, and rest of the world. For the purpose of analysis of Paper four, the SAM is aggregated into:

i. 45 activity accounts with 45 corresponding commodity accounts,

ii. 5 factor accounts consisting of 4 labour types and 1 capital account. The labour groups are classified according to gender and areas of residence,

[8] A SAM is a general equilibrium database that depicts the flows of income and expenditure among sectors, between sectors and institutions (such as households, enterprise and the government) and between these domestic entities and the rest of the world.

iii. 12 institutions including 8 household groups, NPISH account, enterprise account, government account and the Rest of the World. The household groups are categorised according to the income quartiles and areas of residence (Urban and Rural areas),

iv. 2 margin accounts (trade and transport margins), 5 tax accounts, 1 investment-savings account and 1 change in inventory account.

Appendix D presents Uganda's macro SAM that is constructed by aggregating the official 2016/17 SAM.

3.2 Methods

3.2.1 Real options methods

In the real world of uncertainties, the value generated by managerial flexibilities increases the value of an investment project, such that the true expected project value is the expanded NPV. The project value from real options valuation thus consists of two components: the traditional static NPV of expected cash flows, and the value of the flexibility component (Trigeorgis, 1996). That is; $Expanded\ NPV\ =\ static\ NPV\ +\ option\ value$

The first three papers of the thesis apply real options methods to value Uganda's oil project. Paper 1 develops an analytical[9] real options framework for valuation of a sequential irreversible oil project with uncertainties. We specifically apply dynamic programming to derive optimal investment rules for a two-stage problem. Earlier literature have applied real options methods to analyse option-like flexibility and value/uncertainty relationship, in the context of sequential investment projects. These studies include; McDonald and Siegel (1986), Majd and Pindyck (1987), Cortazar and Schwartz (1993), Dixit and Pindyck (1994), Cortazar and Schwartz (1997), and Huisman and Kort (2015). Paper 1 contributes to the literature by adding the dimension of uncertainty about time to completion to study sequential irreversible investments. Past literature that is closely related to our study are Miltersen and Schwartz (2007), Helland and Torgersen (2014) and Ketelaars and Kort (2022). There are however some distinct differences that define the novelty of our study. The analytical expressions of our model diverge from those of Ketelaars and Kort (2022), as we hinge ours on an oil project. Miltersen

[9] Analytical approaches solve partial differential equations subject to certain boundary conditions and are based on the premise that agents make economic decisions that aim at maximising the sum of their present net benefits and their discounted expected future net benefits. Analytical approaches to real options valuation include dynamic programming and contingent claims. These two techniques are closely related and often yield the same results but differ in the different assumptions they assert about financial markets and discount rates (Dixit and Pindyck (1994).

and Schwartz (2007) and Helland and Torgersen (2014) consider one decision state, *i.e.* the optimal investment decision upon completion of the first stage, whereas we extensively analyse the optimal decisions by modelling four different states and two investment decisions. Paper 1 then proceeds to undertake numerical analysis by applying the model to our case study.

Following Cox *et al*. (1979), Paper 2 constructs binomial lattices to value an oil project while accounting for oil price uncertainty. The bulk of real options literature apply numerical approaches[10] such as binomial lattices, binomial trees and Monte Carlo simulations to value oil projects and find the option value of a vast range of flexibilities (see for example, Smith and Mccardle, 1998; Lund, 1999; Fleten *et al*., 2011; Aleksandrov and Espinoza, 2011; Kobari, 2014; Elmerskog, 2016; Abadie and Chamorro, 2017). A binomial lattice model is the most suited technique to numerical approaches, as they offer simplicity and intuition (Bailey *et al.,* 2004; Smith, 2005; Bradao *et al*., 2005). The binomial lattices method is also proposed because it allows modelling of sequentiality in projects that require irreversible investments (Hauschild and Reimsbach, 2014). The real options considered in our analysis of Uganda's oil project is the possibility to defer production of the first barrel of oil. This is based on the premise that the option to defer production is the most relevant for analysis of undeveloped reserves (Dixit and Pindyck, 1994).

Numerous studies apply traditional discounted cash flow methods to assess how various factors influence the NPVs of parties to an oil contract. See, for example studies by Bindemann (1999), Liu *et al*. (2012), Cheng *et al*. (2018) and Farimani *et al*. (2020). However, such traditional valuation methods fail to recognise managerial flexibilities and uncertainties embedded in oil projects and thus underestimate the project value. Paper 3 is the first known study to apply real options methods to assess an oil contract. Specifically, Paper 3 replicates the binomial lattices framework developed in Paper 2 to achieve its research objectives. The difference in the two models is how the cashflows for the different agents are computed. The binomial lattice model is applied to explore how the value of option to defer production and the optimal strategy (to defer production versus to start production immediately) of the government and IOCs change with variations to the oil price, net convenience yield, cost oil limit and oil price volatility.

[10] Numerical methods use discrete time frameworks to approximate the solution to the partial differential equation. In cases where a closed-form solution is absent, numerical methods are required to solve for the option values.

3.2.2 SAM Price model

To analyse the potential inflationary impacts of petroleum price shocks on Uganda's economy in Paper 4, I develop a SAM price model framework following Roland-Holst and Sancho (1995). A SAM Price model is an economy-wide model of price impacts, hinged on the theoretical and empirical constructs of a SAM. The SAM price model is an extension to the Leontief Input-Output price model since it is constructed from the accounting identities of a SAM. Our study specifically assesses the distributional impact of a petroleum import price shocks on household groups. The Leontief Input-Output price model is limited to the analysis of cost linkages among production sectors, and is thus not the appropriate modelling approach for Paper 4. In comparison to the Computable General Equilibrium (CGE) models, the SAM price model is sufficient to achieve the objectives of Paper 4 without the tedious technical and empirical requirements of CGE modelling.

First formulated by Roland-Holst and Sancho (1995), the SAM Price model is the dual version of the quantity-based SAM model. The latter is widely employed to estimate income generating processes through the relationships of circular flow of income and expenditures in the economy. The difference between the two models lies in the assumptions imposed. In the quantity-based SAM model, the output levels are assumed to vary while the prices are held constant. To the contrary, in the SAM Price model, prices vary with cost changes while output levels are fixed. As is the case in the quantity-based SAM model, the assumptions of generalized homogeneity in activities and excess capacity, are imposed. These assumptions allow for endogenously defining the prices of production activities, prices of factors of production or costs faced by consumers and other agents/institutions, independent of output levels (Roland-Holst and Sancho,1995).

In addition to the model assumptions, the SAM accounts are categorised into endogenous and exogenous accounts[11], which facilitates the computation of the price multipliers. In my analysis, the exogenous components are; the government, investment-savings, change in inventory, tax accounts and the rest of the world. The remaining accounts are categorised as

[11] Endogenous accounts include those accounts where income-expenditure is governed by mechanisms that operate entirely within the SAM model. Exogenous accounts are those accounts where income and/or expenditure are influenced by forces external to the SAM framework. The distinction between endogenous and exogenous accounts comes from the limit to the endogenous responses that are captured in the SAM multiplier model. The exogenous accounts are only affected by the initial shock and by changes in the leakages from the endogenous to the exogenous accounts to balance the exogenous accounts as a group (Round, 2003).

endogenous. In the context of Paper 4's objective, the price multipliers measure the direct and indirect effects on prices of endogenous accounts resulting from an exogenous increase in the price of petroleum imports by 1%. This exogenous shock is introduced into the model as an increase in the import price of the refined petroleum sector. As a result, this cost shock results in higher domestic prices of refined petroleum (*i.e* higher fuel pump prices) which will then be reflected in higher prices for the endogenous accounts. The overall price effect of the shock on each endogenous account is estimated by a product of the price multiplier and the size of the shock. This is expressed in equation 1, as follows;

$$\Delta p = \Delta v M' \tag{1}$$

p is the vector of price indices for the endogenous accounts, M' is the price multiplier matrix and v is the vector of average exogenous costs to endogenous accounts.

Since the seminal work by Roland-Holst and Sancho (1995), there are a few known studies that apply the SAM Price model (see Llop, 2018 for a detailed review of literature). Among these are; Akkemik (2011), Saari *et al*. (2016), Llop (2018) and Xue *et al.* (2019). The earlier literature on impacts of petroleum price shocks that are closely related to this study are Saari *et al*. (2016) and Llop (2018). Similar to Saari *et al*. (2016), Paper 4 accounts for the distributional impacts of petroleum price shocks. Saari *et al*. (2016) tailored their extended SAM price model to Malaysia which has subsidies on petroleum products. To the contrary, I apply the standard SAM Price Model which departs from the extended SAM price model. In my study, the input coefficients of producers and consumption patterns of households are fixed, such that, producers and consumers do not respond to changes in relative prices by substituting certain primary inputs or commodities for others. The standard SAM Price model is sufficient to analyse the short-term impact of a petroleum price shock of an economy with a fully liberalised fuel market.

Paper 4 also departs from Saari *et al*. (2016), by decomposing the price multiplier matrix to extensively analyse the price transmission mechanism of the exogenous petroleum import price shock. The price multiplier decomposition is formulated as an additive construction[12], into transfer effects, open-loop effects and closed-loop effects, following Roland-Holst and Sancho (1995) and Miller and Blair (2022). Paper 4 extends the work of Llop (2018) by assessing the distributional impacts of the energy price shocks on household groups. Specifically, paper 4 categorises the household groups into 8, based on their income quartiles and geographical area

[12] The additive multiplier decomposition was first proposed by Stone (1985) and formulated for the SAM quantity model.

of residence (urban vs rural) and compares the changes in their respective costs of living resulting from the petroleum import price shock.

4. Main findings and Scientific contribution

Paper 1: Uncertain time to completion in a sequential investment problem: a theoretical and empirical analysis

In the first paper, we apply dynamic programming to derive analytical expressions of the values of the project and the optimal investment thresholds of the development stage and production stage. The novelty our study lies in how we conceptualize our real options problem. We consider that the oil project can be divided into four different states. The first state is when the firm is faced with the decisions to invest in development or to wait. The firm invests only when an oil price threshold is reached. The second state is when the first investment has been undertaken and the development phase is in progress, but not completed. Uncertainties about the time to completion of the development stage and oil price variations make it unclear when it is physically possible to begin oil production. The third state is when development is completed, and the firm can decide whether to make the second investment and start production or wait. As in the case with the first investment, the oil price must be above a certain threshold for the second investment to be undertaken. The final state is when oil production begins.

To generate numerical results, we employ the model to our case study of Uganda's oil project. From the base case analysis, we find that the threshold price for development stage is USD63 while the threshold price for the production stage is USD18. The divergence in the two thresholds is mainly because development costs are significantly higher than the production stage costs. We proceed to analyse how changes in important input parameters, such as the share of total investment cost, the expected time to completion, volatility affect the project value and threshold prices. First, we analyse the impact of the share of total investment cost between the development and production stages on project values and threshold prices. We find that increasing the cost share allocated to the production stage, increases the project value. We also establish that the firm would require a larger mark-up when the capital expenditures of the production stage increase as a share of total investment costs, and thus requiring higher threshold prices for oil production to start. Secondly, we derive the project values and investment thresholds for different expected times to completion of the development stage. As expected, we find that the project value for the development stage increases when the expected time to completion is shorter. This is because the firm would begin oil production at an earlier point in time and generate revenue, thus raising the project value. We observe that, for low values of drift rates, the threshold price for the development stage reduces as the expected time to completion is shortened. However, for high drift rates, we find that the threshold price instead increases with reducing expected time to completion. Thus, the threshold price may be non-monotonic in the expected time to completion. Lastly, we study the effect of oil price volatility on project values and threshold prices. In line with standard finance and real options theory and literature, we find that higher oil price volatility increases the project value and higher threshold prices.

Paper 2: How valuable is the option to defer Uganda's crude oil production?

In spite of the renewed interests of IOCs in Africa's vast oil reserves, amidst high oil price volatility in the past two decades, most of the previous studies are focused on developed reserves in high-income countries. We are aware of only three previous studies on Africa's undeveloped reserves (see Abid and Kaffel, 2009; Qui *et al*., 2015; Fonseca *et al*., 2017). It is against this premise that Paper 2 contributes to the limited literature on real options valuation of Africa's oil investment by constructing a replicable binomial lattices model that can be applied to value undeveloped reserves.

In this paper, we consider that the development stage is completed, and that the decision maker is faced with whether to begin oil production or to wait another year. We specifically quantify the value of deferring oil production and how the optimal strategy changes with different risk factors. Our base case results suggest that deferring production by another year adds value of USD0.9 billion to Uganda's oil project and it is thus optimal for the IOCs to defer production. The positive option value partly emanates from our assumption that IOCs can defer expending the remaining OPEX and CAPEX to the subsequent year. We further analyse the sensitivity of the option value to crude oil price level, crude oil price volatility, net convenience yield and cost inflation, and illustrate combinations when it is optimal to start production now or wait, respectively. Notwithstanding the volatility rate, at lower oil prices, the static NPV is comparably lower that the expanded NPV and further reductions in the oil price increase the value of the option to defer. This implies that the traditional discounted cashflow approach significantly undervalues the oil project. Our results also show that the option to defer is significant in the case with low oil prices and high volatility rates. This is an expected outcome since higher volatility increases the possibility for lower oil prices and thus increases the value of having the option to defer the project. However, the option values are insignificant in cases of low volatilities and high oil prices. The reason is that, in those cases, it is almost certain that the oil price will be at a level where the project is expected to be profitable. We also establish that, when the rate of net convenience yield is high and the oil price is high, the value of the option is lower and becomes worthless at a critical price of USD65 per barrel of crude oil. This implies that the option to defer adds meagre value at high oil prices and high rates of net convenience yield. This is natural, as oil production is almost certain to begin at a high price such that deferring has no value. We also find that low oil prices combined with increases in cost inflation result in rejection of the project, as both static and expanded NPV reduce to negatives. In principle, cost inflation raises the critical oil price required to make the oil project economically viable and renders the option to defer worthless. Our results also show that when cost inflation is equal to 4% and net convenience yield is equal to zero, the option to defer is worthless at oil prices of USD45 and above. We reach the same conclusion when the net convenience yield is 4% and cost inflation is equal to zero.

Paper 3: Real options valuation of a Production Sharing Agreement

The design of a PSA has a critical impact on the investment decisions of the IOCs and host governments. For the IOCs, a PSA may be a disincentive to investment if the contract terms are designed in a way that channels a significant share of the project cashflows to the host government. Whereas, for the host government, it is imperative to design a PSA in a way that the resultant cashflow shares maximise state revenue while incentivising IOCs to invest. It is therefore essential to assess how a PSA design influences the optimal investment strategy from the perspective of IOCs and the government.

In Paper 3, we replicate the binomial lattices model constructed in Paper 2 to assess the optimal investment decisions of the IOCs and the government, based on an actual PSA and the prevailing risk factors that influence oil investments. As prior mentioned in subsection 3.2.1, Paper 3 is the first known study to apply real options methods to assess an oil contract. Since the parties have different interests, their strategic investment decisions may be aligned or conflicting, depending on how the risk factors impact the value of the oil project. We also consider that the IOCs and government can exercise the option to start production immediately or to defer production. On account of these premises, real options methods are considered to be the appropriate valuation approach, compared to the traditional discounted cashflow methods that are widely used in related studies.

From the base case analysis, we find that the critical oil prices at which to start oil production, from both fields, differ for the parties. The government's critical oil prices are lower than those of the IOCs. Collectively, the critical oil price for beginning oil production is USD18 for the government and USD42 for the IOCs, which suggests that there may be conflicting interests between the IOCs and the government when it comes to realizing the project.

We proceeded to examine the sensitivity of each party's expanded NPV to changes in the crude oil price, volatility of oil prices, net convenience yield and cost oil limit. The results from the sensitivity analyses indicate that the PSA design is progressive in the sense that, as the oil price rises and the oil project becomes more profitable, the expanded NPV of the government rises faster than that of the IOCs. We also find that the government's expanded NPVs increase with rising oil price volatility. Contrary to real options theory, the expanded NPVs of the IOCs decline with a rise in oil price volatility. In line with real options theory, for both parties, the expanded NPVs is a decreasing function of the net convenience yield. As expected, a higher cost oil limit increases the expanded NPV of the IOC whereas the expanded NPV of the government declines.

Overall, we establish that the parties have conflicting optimal strategies. Particularly, the government has a strong preference to defer production, except in the cases when prices are low, and the project approaches the expiration of the defer option. To the IOCs, this is the reverse. A combination of a low cost oil rate and a low oil price increases the risk that the IOC may not fully recover its incurred project costs, and thus results in a negative expanded NPV for the IOCs. The reason for the conflicting strategies emanates from the design of the cost oil function and the expected oil price realizations. In cases of low oil prices, the cost oil will be limited by the cost oil limit, since net revenue will be low compared to total actual cost. Cost oil then turns into a function of oil prices, via net revenues.

Paper 4: The potential inflationary impacts of an oil price shock on the economy: a social accounting matrix price multiplier analysis for Uganda

Paper 4 particularly estimates the inflationary impact of an oil price shock by using the information provided by Uganda's SAM 2016/17. The study constructs a SAM Price multiplier model which enables us to trace the transmission of an oil price shock through its impact on production prices and consumer prices and ultimately measure the potential inflationary effect of oil-price shocks. An extended analysis about the extent and magnitude of petroleum cost linkages across sectors, production factors, and the institutions, is undertaken by decomposing the total price multipliers into additive components. The study also assesses the potential distributional impacts of the increase in petroleum prices on different household groups.

Paper 4 further makes two contributions to the international literature on the link between oil price shocks and inflation. First, it revives the analysis of distributional aspects of oil price shocks, for a particularly small net-importing developing economy. This study is one of the few on low-income countries, given that the economic structures are different from those of developed and emerging economies. Secondly, this study is timely, as the global economy currently faces very high oil prices causing significant global effects on households, business and the economy's demand for goods and services.

From my analysis, I find that the influence of an oil price shock on the rest of the economy very asymmetric and depends to a greater extent on the specific sector and institution analysed. As expected, the activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. The production prices are relatively more responsive to a petroleum import price shock than consumer prices. It is also noteworthy that the production prices of manufacturing and processing sectors are, on average, the most affected as compared to the other sector categories. This affirms that petroleum is an important component of production costs of the manufacturing and processing sectors, hence increases in petroleum prices could reduce the competitiveness of domestic products in global markets.

In regard to the price multiplier decomposition, transfer effects measure how the exogenous petroleum import price shock results in direct increases in prices of production activities emanating from the rising cost of petroleum as an intermediate input. The results confirm that transfer effects completely dominate price influences in the activity sectors which reflects the strong intersectoral linkages between the refined petroleum sector and other production sectors. This implies that the inflationary impact of the exogenous petroleum import price shock is mainly transmitted directly through increases in prices of activity sectors that use petroleum as an intermediate input in the production process.

The results also show that the distributional impacts of rising petroleum prices tend to be progressive. For both urban and rural regions, the households in higher income quartiles are more affected by the petroleum import shock, as compared to the low-income quartiles. Thus, the study recommends that equity considerations are accounted for, as a basis for exploring plausible policy interventions to mitigate the impacts of future oil price shocks.

5. Policy implications and conclusions

Based on our findings from Papers 1, 2, 3 and 4 the following policy implications and conclusions are drawn;

Uncertainties faced by oil projects have profound impacts on the project values. IOCs and host governments may bias their decisions if they neglect these project uncertainties. Taking our findings of Paper 2 as an example, the traditional discounted cashflow valuation indicates that the NPV of Uganda's oil project is \$36.5 billion. However, by applying real options valuation which accounts for project uncertainties and the embedded option to defer production, the expanded NPV is estimated to be \$37.4 billion, thus generating an option value of \$0.9 billion. Hence, the traditional discounted cashflow approach may undervalue oil projects and result in sub-optimal investment decisions.

In the Papers 1, 2 and 3, it is shown that the optimal investment decisions are highly dependent on external factors like oil price, net convenience yield, cost inflation and oil price volatility. In Paper 3, our findings provide an insight into how the PSA structure and the prevailing risk factors can potentially influence the optimal strategies taken by the IOCs and government. This information can be used by each party when negotiating contract terms and also aids decision making for both parties under uncertainty. We also recommend that the PSA should be designed in a way that allows for flexibilities in the event that these risks arise. This would in turn incentivise oil investments while stabilizing government revenues from the oil sector.

Our sensitivity analyses in Paper 3 indicate that the government expanded NPV is more sensitive to the external shocks, compared to the IOCs. These shocks may translate into erroneous fluctuations to government oil revenues and ultimately macroeconomic instability. Thus, the government must take account of these shocks when designing and negotiating oil contracts. Furthermore, the government should have fiscal policy measures that counteract the macroeconomic shocks that may arise from the instabilities in the oil sector.

The results from Paper 4 inform government on the extent to which the absence of fuel price controls account for Uganda's vulnerability to oil price shocks. Specifically, the results show that the distributional impacts of rising petroleum prices tend to be progressive since the poorest households are the least affected compared to the higher income households. Thus, the study recommends that equity considerations are accounted for, as a basis for exploring plausible policy interventions to mitigate the impacts of future oil price shocks. The simulation results from Paper 4 show that an oil price shock has asymmetric inflationary impacts on sectors and agents of Uganda's economy. Therefore, policy interventions should be tailored in a way that enhances competitiveness in sectors most affected by oil price shocks and without worsening the welfare of households that are most sensitive to the oil price shocks.

6. Limitations and future research

There are obviously some limitations that arise from the models employed, underlying assumptions made and bottlenecks encountered during data collection.

First, due to secrecy and confidentiality regarding Uganda's oil sector, the results from Papers 1, 2 and 3 are based on cost and production profile estimates by Ward and Malov (2016), to fill the data gap when official accurate data could not be obtained through primary data collection. Therefore, our results may not be a complete representative of the oil sector.

Second, the Albertine region is rich in biodiversity. The thesis is silent on the impact of the project on nature. A study by Byakagaba *et al*. (2019) on oil exploration in the Albertine Graben reported noise pollution due to blasting of rocks during exploration, soil erosion due to clearing of vegetation for road construction and wildlife disturbance due to increased human activity in the wildlife reserve as the major environmental impacts. Oil activities in the region thus raise concerns of environmental degradation, particularly their impact on the biodiversity of the natural habitats. For future research, it would be pertinent to account for environmental damage emanating from the oil project.

Third, the land acquisition for Uganda's oil project has resulted in undesirable socio-economic impacts on displaced and resettled communities of the Albertine region. Aboda *et al.* (2019) found that over 81% of households experiencing displacement from the Albertine region lost their land and experienced reduced resource access. The most affected were females and those with low or no education levels. For ease of modelling and limited access to accurate data, the thesis is silent on the monetary and external costs of the land acquisition for Uganda's oil project. For future research, it would be appropriate to incorporate these costs when valuing oil projects.

Fourth, the thesis focuses on uncertainties about oil prices and expected time to completion of the development stage as the risks faced by the government and oil firms during the project's lifetime. The studies exclude other risks such as geological risks, political risks, technological risk and future climate policies which would make the estimation of the real options multifaceted and a better reference for optimal decision making. It would also be interesting to apply the models to other stochastic processes such as mean reversion.

Fifth, as already highlighted, despite its usefulness in capturing price transmission mechanisms within an economy, the SAM Price model ignores the likely substitution effects that may emanate from an oil price shock. Therefore, the results in Paper 4 should be interpreted as an upper bound of the oil price shock impact in the short-term, before economic agents make responsive adjustments. For further research, it would be interesting to simulate long-term price effects of a petroleum import price shock on Uganda by allowing some degree of substitutability among production inputs and products for consumption as shown in Saari *et al.* (2016). It would also be of interest to adjust Uganda's SAM to include a refinery sub-sector supplied by the domestic crude oil sector. Comparing our Paper 4 results to import substitution effects of domestic oil production, would be insightful.

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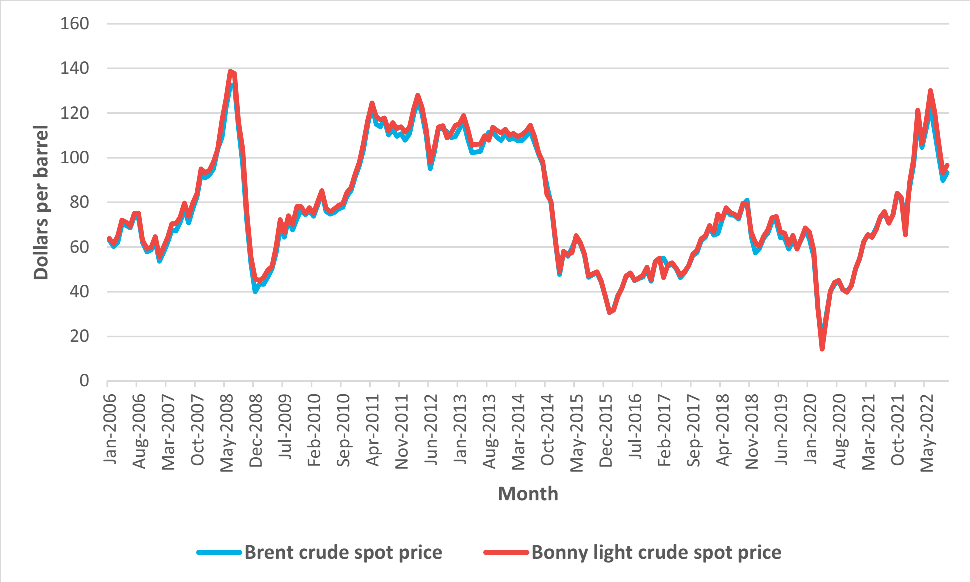
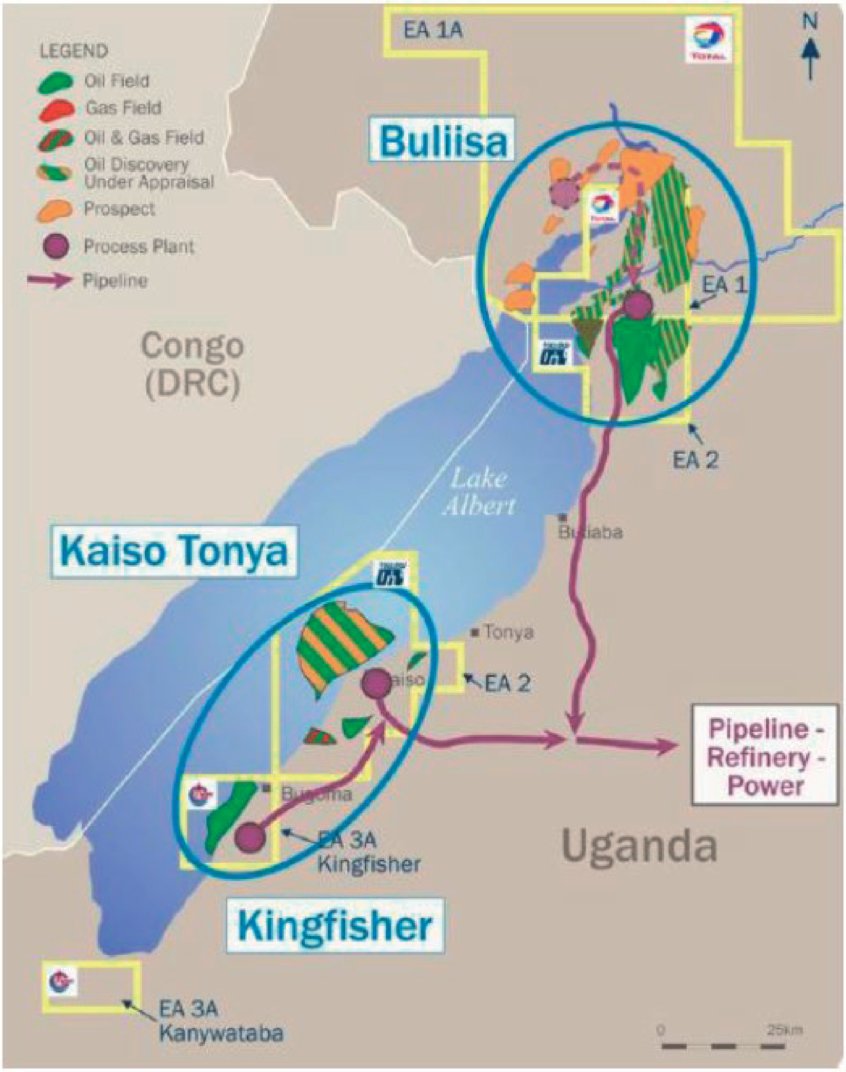
Appendices

A. Monthly trend in crude spot prices (for the years 2000-2022)



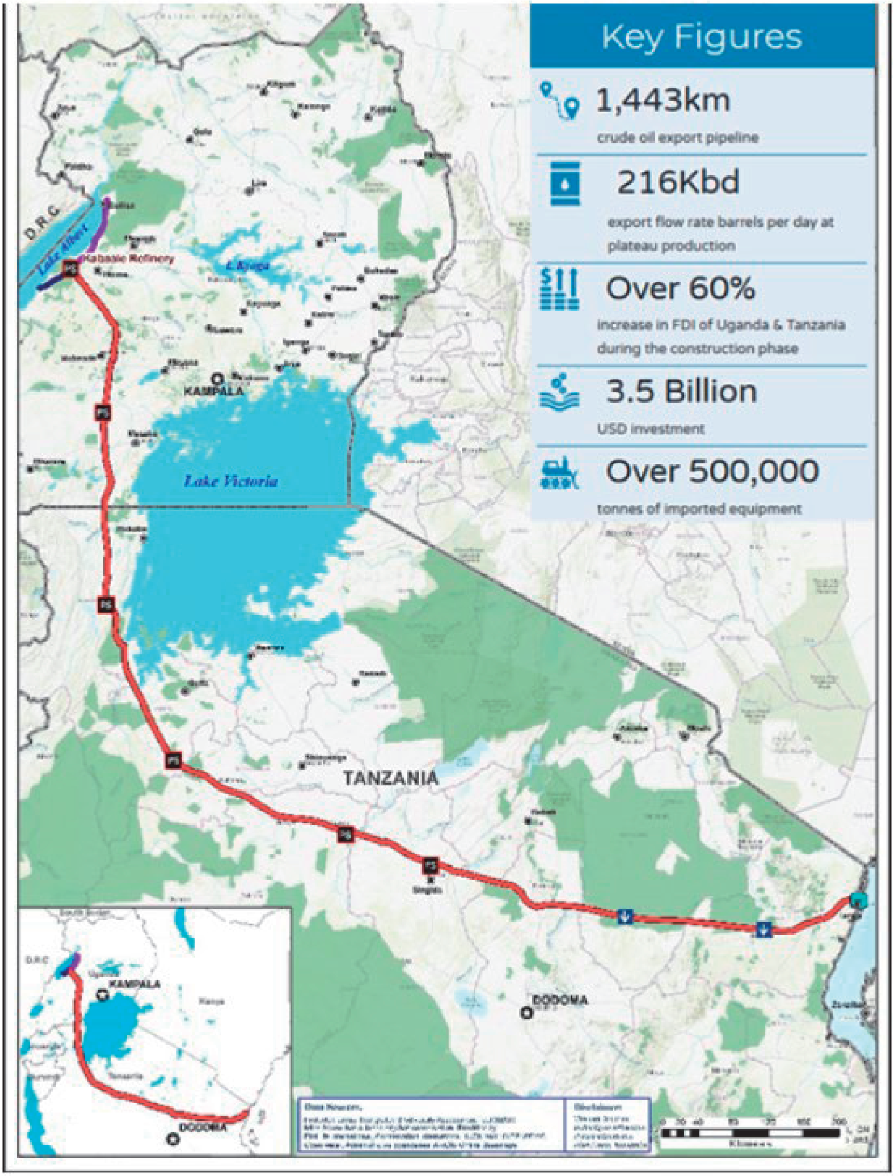
Note: The Brent crude spot price data is obtained from the website of US Energy Information Administration, EIA (2022). Despite being similar, the Bonny light crude spot price trend is included for reference because it is used as a proxy for Uganda's crude oil price in Papers 1, 2, & 3. The Bonny light crude spot price data is obtained from the website of the Central Bank of Nigeria (2022).

B. *Petroleum discoveries in the Albertine region*



Source: Tullow Oil PLC (2012)

C. East African Crude Oil Pipeline Route



Source: East African Crude Oil Pipeline (2022)

***D.** The 2016/17 Macro SAM for Uganda (UGX trillions)*

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
1	Activities	-	153	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	153
2	Commodities	53	-	-	-	-	-	-	-	-	-	-	75	2	-	-	9	25	1	20	184
3	Trade margin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	Transport margin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	Excise tax	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
6	Import duty	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
7	VAT	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
8	Labour	29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	29
9	Capital	72	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	72
10	Production taxes	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
11	Direct taxes	-	-	-	-	-	-	-	-	-	-	-	2	-	2	0	-	-	-	-	4
12	Households	-	-	-	-	-	-	-	28	43	-	-	3	-	17	2	2	-	-	4	98
13	NPISH	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	0	-	-	1	3
14	Non-financial Enterprises	-	-	-	-	-	-	-	-	26	-	-	-	-	-	1	0	-	-	-	27
15	Financial Enterprises	-	-	-	-	-	-	-	-	1	-	-	1	-	1	-	1	-	-	-	4
16	Government	-	-	-	-	3	1	4	-	-	0	4	0	-	0	0	-	-	-	1	14
17	Savings	-	-	-	-	-	-	-	-	-	-	-	16	0	7	1	1	-	-	1	25
18	Stock	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	1
19	Rest of the World	-	23	-	-	-	-	-	0	1	-	-	-	-	1	0	0	-	-	-	26
	Total	153	184	-	-	3	1	4	29	72	0	4	98	3	27	4	14	25	1	26	

Source: Author's compilation based on official 2016/17 Uganda SAM.

Chapter 1: Uncertain time to completion in a sequential investment problem: a theoretical and empirical analysis.

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Working Paper

Uncertain time to completion in a sequential investment problem: a theoretical and empirical analysis

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Martijn W. Ketelaars[3] Peter M. Kort[3]

Abstract

This paper develops an analytical real options framework to establish optimal investment rules for a sequential irreversible oil project. We consider that the firm has to decide when to invest in the production stage which is preceded by the firm's investment in the development stage. Uncertainties about the time to completion of the development stage and oil price variations make it unclear when it is possible to begin oil production. The two-stage problem is divided into four decision states and dynamic programming is applied to obtain analytical expressions for project values and threshold prices at which the firm should invest. To find numerical solutions, the model is applied to a case study of Uganda's oil project. From the base case analysis, we find that the threshold price for development stage is more than three times that of the production stage. From our sensitivity analysis, we find that the project value for the development stage increases when the expected time to completion is shorter. Whereas the threshold price may be non-monotonic in the expected time to completion. We also establish that the firm will wait longer to invest in the development stage at a higher oil price, when faced with higher oil price uncertainty.

Keywords: Sequential investment project; Uncertain time to completion; Real options; Oil project; Development stage

JEL Classification Number: C02; C61; D25; D81

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The authors are grateful to the two anonymous reviewers and participants of the 25th Annual International Real Options Conference in Porto, June 2022, for their helpful comments

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1 Introduction

Investment towards oil production is a complex step-wise process, from exploration, appraisal of oil reserves, development of oil fields and support infrastructure, actual production of oil to decommissioning. The exploration stage involves site surveys for potentially viable oil reserves, and is characterised by high uncertainties about the existing reserves. If no potentially viable reserves are discovered, the project is abandoned. Otherwise the discovery of possibly feasible oil sources results in further exploration, leading to the appraisal of the oil sites. At the appraisal stage, exploration wells are drilled to discover and map the oil reserves. Where oil reserves are confirmed to be commercially viable, the oil companies proceed to prepare to develop the site. The development phase involves significant investments towards well drilling, construction of production facilities, roads, oil pipelines, terminals, in preparation of actual oil production. During the production stage, the oil companies generate revenues from the sale of oil and recover the costs incurred in the preceding stages . The production phase commonly lasts from 20-30 years from the first oil to abandonment. Over this period, production may vary depending on available reserves, the number of active wells, oil price and extraction technology. The rate of extraction typically rises to a peak in the early years of production before declining significantly towards the end of the commercial lifetime of the oil field. Once it is no longer economically viable to extract the remaining oil reserves, the site is decommissioned. This final stage involves plugging oil wells, site remediation and removal of facilities and equipment with the aim to restore the site to its original state, as close as possible.

Investments at each sequential stage of oil project life-cycle are highly costly, lumpy and are sunk, for the most part, once expended. More so, each successive stage prior to the production stage typically does not lead to immediate cash flows but opens up further investment opportunities. The capital intensity of oil investments, particularly at the development stage, makes them irreversible because the oil wells and operation facilities can only be used to produce oil. These complexities are exacerbated by the various uncertainties faced by these projects. Among these is the uncertainty about the oil price which significantly influences the value of an oil project. Another is the uncertain time to completion of the preceding stages to production stage, particularly the development stage that requires large capital-intensive investments in oil drilling facilities and support infrastructure that take a long time to build.

In our study, we present a real options framework for evaluating the development stage of an oil investment project with uncertain time time to completion. Based on the costs of development and subsequent oil production, the oil price, and expected time to completion of the development stage, we estimate the value of the irreversible option to invest in the development stage. Accordingly, we find optimal oil price threshold levels for investment at the development and production stages. Real options methods have been applied to analyse option-like flexibility and value/uncertainty relationship, in the context of sequential investment projects. McDonald and Siegel (1986) examine the optimal timing of an irreversible investment project where the project value and the final investment cost are both dependent on stochastic variables. The variables follow a stochastic geometric Brownian motion (GBM) process, and the authors derive an optimal investment rule and a formula for valuation of the project. Their main findings are that the option to wait increases project value, and that it is optimal to postpone the final investment until the project value is twice the size of the investment cost. McDonald and Siegel (1986), however, ignore sequentiality of such complex investments. Majd and Pindyck (1987) analyse sequential construction projects by using contingent claims analysis to derive optimal decisions rules and values of investments with irreversible cash outlays and maximum construction rates. Their study presents the effects of time to build, opportunity cost and uncertainty on the investment decision. Their findings suggest that time-to-build has the greatest impact on the investment decision when the project is faced with high uncertainty. Their study also examines the economic value of construction time flexibility and find that the value of the investment project rises as the minimum construction time shortens. Cortazar and Schwartz (1993) provide analytical expressions for valuing a firm operating under two sequential stages, each with a different output capacity. Their study analyses the effects of price volatility and interest rates on firm value, optimal production and optimal inventory levels. Their findings show that an increase in the risk-free rate raises the critical spot price of the second stage. They also find that increases in the spot price volatility and in the risk-free interest rate decreases the first stage critical spot price. Dixit and Pindyck (1994) formulate a model for a two-stage investment that can be applied to a wide range of contexts under sequential investments. The model can be used to develop an investment rule to solve for threshold oil prices and project values.

Cortazar and Schwartz (1997) apply finite difference methods to solve for the value of an undeveloped Chilean oil field, that undergoes three states (*i.e.*, before committing to investment of development phase; development of the oil fields and actual oil production). They implement their model under the assumption of that the oil price follows a mean-reversion process. Their findings identify a critical spot price that triggers development of the field. The critical price lowers as the time to expiry of the delaying option shortens. They find that the value of the option to delay development investment decreases as the oil price increases. Huisman and Kort (2015) develop a stochastic dynamic duopoly framework to analyse investment timing and capacity decisions of firms, where undertaking the investment implies obtaining a production plant. They assume a linear demand function that follows a GBM process. In line with standard real options theory, they find that increased uncertainty about demand raises the optimal investment threshold and thus delays investment. They also find that increased uncertainty incentivises over-investment as an entrance deterrence strategy. Building on the model by Huisman and Kort (2015), Ketelaars and Kort (2022) recognises sequentiality and uncertainty of time to completion of R&D investment projects. The study assumes a Poisson process to model uncertainty of innovation break-through. The authors rely on numerical proofs to show that a firm invests more in the first stage (R&D stage) to innovate sooner if it observes a higher price. In consideration of the abandonment option, the authors find that investing more in R&D implies more costs and hence it is optimal for the firm to exit at a higher price. As regards, the option to defer the R&D investment, it is more likely that the firm immediately launches the new product upon innovation, when the price is above the product launch threshold. In line with standard real options theory, the study finds that increased market uncertainty causes the firm to delay R&D investment project and increases the likelihood of abandoning the project before innovation. Similar to Huisman and Kort (2015), the study also finds that the firm over-invests with increased uncertainty. Although similar, the analytical expressions of our model diverge from those of Ketelaars and Kort (2022), as we hinge ours on an oil project.

Our study contributes to the literature by adding the dimension of uncertainty about time to completion in valuing sequential irreversible investments. We consider that the oil project can be divided into four different states. The first state is when the firm is faced with the decisions to invest in development or to wait. The firm invests only when an oil price threshold is reached. The second state is when the first investment has been undertaken and the development phase is in progress, but not completed. Uncertainties about the time to completion of the development stage and oil price variations make it unclear when it is physically possible to begin oil production. The third state is when development is completed, and the firm is faced with the decision to make the second investment towards oil production or wait. As in the case with the first investment, the oil price must be above a certain threshold for the second investment to be undertaken. When the second investment is undertaken, the fourth state is entered and the project starts to generate revenues from oil sales. We assume that the oil price, and thus the value of the project follow a GBM stochastic process. We further use a Poisson process to model uncertainty of time to completion of the development stage. We proceed to apply our model to derive analytical expressions of the values of the project and the optimal investment thresholds at each stage. To generate numerical results, we employ our model to a case study of Uganda's oil project.

We undertake a sensitivity analysis of how changes in important input parameters, such as the share of total investment cost, the expected time to completion, volatility affect the project value and threshold prices. First, we analyse the impact of the share of total investment cost between the development and production stages on project values and threshold prices. We find that increasing the cost share allocated to the production stage, increases the project value. We also establish that the firm would require a larger mark-up when the capital expenditures of the production stage increase as a share of total investment costs, and thus requiring higher threshold prices for oil production to start. Secondly, we derive the project values and investment thresholds for different expected times to completion of the development stage. As expected, we find that the project value for the development stage increases when the expected time to completion is shorter. This is because the firm would begin oil production at an earlier point in time and generate revenue, thus raising the project value. We observe that for low values of drift rates, the threshold price for the development stage reduces as the expected time to completion is shortened. However, for high drift rates, we find that the threshold price instead increases with reducing expected time to completion. Thus, the threshold price may be non-monotonic in the expected time to completion. Lastly, we study the effect of oil price volatility on project values and threshold prices. In line with standard finance and real options theory and literature, we confirm that higher oil price volatility increases the project value and results in higher threshold prices.

Earlier literature that are closely related to our study are Miltersen and Schwartz (2007), and Helland and Torgersen (2014). Miltersen4 and Schwartz (2007) develop a real options valuation framework to derive closed form solutions for the value of a two-stage project and optimal abandonment and switching thresholds, with uncertain time to completion of the first stage. Time to completion of the exploration stage is modelled as a Poisson process. Their study assumes that the owner of the project pays on-going costs per unit of time until the first stage is completed and these costs are uncertain. They also assume that the value of the investment project follows a GBM process. The authors found that, for realistic values of on-going costs, the value of the investment project decreases with longer expected time to completion. However, for very low values of on-going costs, their results show that the value of the investment project increases in the expected time to completion. This is contrary to our findings, as we establish that the value of the oil project is monotonic in expected time to completion of the first stage. They also demonstrate that the abandonment thresholds are a non-monotonic function of the expected time to completion and that the switching thresholds increase with lower on-going investment costs and longer expected times to completion. Similar to Huisman and Kort (2015) , Miltersen and Schwartz (2007) also extend their analysis to a duopoly case, which is beyond the scope of our study. Our study is specifically applied to oil investment projects, which are often structured in a way that an oil company has monopoly rights to the contract area over which it operates. Helland and Torgersen (2014) replicate the Miltersen and Schwartz (2007) model to evaluate the exploration stage of a petroleum investment project while accounting for uncertainty of the oil price, exploration costs and the time to completion of the exploration. From the closed form solutions, they are able to compute the value of the investment option when the project owner has the flexibilities of abandoning and delaying the investment upon completion of the exploration stage. They find that decreasing the expected time to completion of the exploration stage increases the value of the project at all oil prices, and consequently reduces the abandonment threshold level. Another key difference with Miltersen and Schwartz (2007) and Helland and Torgersen (2014) is that both studies consider one decision state, *i.e.,* the optimal investment decision upon completion of the first stage, whereas we extensively analyse the optimal decisions by modelling four different states and two investment decisions.

The rest of the article is organised as follows; The model is presented in Section 2. Section 3 presents the numerical analysis of our case study. We conclude in Section 4. The proofs of all propositions can be found in the appendix.

2 Model

We consider a firm or a joint venture in which there are exclusive rights to develop and subsequently extract oil from an oil field over a certain period of time. The rights are stipulated in an oil contract with the resource owner, which is often a government. The oil contract is based on the premise that the firm takes all risks involved with the investments in development and oil production. We further assume that the firm produces oil until the end of its contract tenure, *i.e.*, the firm has no option to temporarily shut down operations or to abandon the project before its tenure expires. Within the contract tenure the firm has the flexibility to decide on the timing of the irreversible investments in development and subsequently in oil production. First, the firm holds the option to invest in the development of the oil field against a sunk cost. The firm faces technological uncertainty during the development stage in the sense that the time to completion is stochastic. Upon completion of the development stage the firm obtains the option to start the oil production stage against a sunk cost. The additional uncertainty the firm faces is the uncertainty of the crude oil price. Thus, the firm's investment decisions depend on the crude oil price at each of the successive stages. In the remaining sections we determine the optimal price level whereupon the firm starts development, and the optimal price level for the firm to start production.

2.1 Production stage

To determine the optimal crude oil price at which the firm should begin the development of the oil field, we work backwards by first considering the case in which the development phase has already been completed. We assume that the price of one barrel of crude oil at time, t, is given by $P(t)$ and follows a geometric brownian motion with a drift, which is given by:

$$dP(t) = \mu P(t)dt + \sigma P(t)dZ(t), \tag{2.1}$$

in which μ is the drift parameter, σ is the volatility parameter, and $dZ(t)$ is the increment of a Wiener process. The value of the price at time zero, $P(0)$, is denoted by P and is strictly positive. The instantaneous profit at time t is given by:

$$\pi(P(t)) = \max\{(P(t) - C)Q, 0\}, \tag{2.2}$$

in which Q is the expected annual production of barrels of oil, which is constant over the project lifetime, and C is the average operating cost of producing one barrel of oil. Discounting happens with a positive risk-free interest rate r. We impose that $r > \mu$, otherwise it is optimal for the firm to wait indefinitely and never undertake the project.

The value of the firm in the production stage is determined on the basis of two scenarios. First, if $P < C$, the current profit flow is zero, see (2.2), meaning that the firm suspends production, though it has an option to produce. Second, if $P > C$, the current profit is positive, see (2.2), meaning that the firm produces and receives a positive profit flow. At the point where these two regions meet, *i.e.*, at $P = C$, the firm is indifferent between producing and not producing. The following proposition states the value of the firm in the production stage.

Proposition 2.1. *The value of the firm in the production stage is given by:*

$$V_2(P) = \begin{cases} A_1 P^{\beta_1} & \text{if } P < C, \\ A_2 P^{\beta_2} + \dfrac{PQ}{r-\mu} - \dfrac{CQ}{r} & \text{if } P \geq C, \end{cases} \tag{2.3}$$

in which

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1,$$

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0,$$

$$A_1 = \frac{QC^{1-\beta_1}}{\beta_1 - \beta_2} \frac{r - \mu\beta_2}{r(r-\mu)} > 0,$$

and

$$A_2 = \frac{QC^{1-\beta_2}}{\beta_1 - \beta_2} \frac{r - \mu\beta_1}{r(r-\mu)} > 0. \tag{2.4}$$

Proof. See pages 186-189 of Dixit and Pindyck (1994). □

In the value function $V_2(P)$ given in (2.3), the value $A_1P^{\beta_1}$ is the option value to resume production in case the price P rises above C, whereas the value $A_2P^{\beta_2}$ is the option value to suspend production in case the price P falls below C. The second term in the value function $V_2(P)$ if $P \geq C$ is equal to the expected present value of the profit flow when the initial value is P, *i.e.*,

$$\mathbb{E}\left[\int_0^\infty (P(t) - C)Qe^{-rt}dt \middle| P = P(0)\right] = \frac{PQ}{r-\mu} - \frac{CQ}{r}.$$

The firm is required to incur a sunk cost, I_2, in order start the production stage. We determine a threshold price, P_2^*, for which it holds that it is optimal to wait if $P < P_2^*$, whereas it is optimal to start the production stage if $P \geq P_2^*$. Consequently, we can determine the value of the firm upon completion of the development phase, which is given in the following proposition.

Proposition 2.2. *The value of the firm upon completion of the development phase is given by:*

$$F_2(P) = \begin{cases} B_1 P^{\beta_1} & \text{if } P < P_2^*, \\ B_2 P^{\beta_2} + \dfrac{PQ}{r-\mu} - \left(\dfrac{CQ}{r} + I_2\right) & \text{if } P \geq P_2^*, \end{cases} \tag{2.5}$$

where B_2 is the constant given by (2.4) in Proposition 2.1, B_1 is equal to:

$$B_1 = \frac{\beta_2}{\beta_1} B_2 (P_2^*)^{\beta_2 - \beta_1} + \frac{Q}{\beta_1(r-\mu)} (P_2^*)^{1-\beta_1},$$

and where the investment threshold P_2^* is implicitly determined by:

$$(\beta_1 - 1)\frac{P_2^* Q}{r-\mu} + (\beta_1 - \beta_2) B_2 (P_2^*)^{\beta_2} - \beta_1 \left(\frac{CQ}{r} + I_2\right) = 0. \tag{2.6}$$

Proof. See pages 190-191 of Dixit and Pindyck (1994). □

In the previous proposition, the value of the firm is equal to $B_1P^{\beta_1}$ if $P < P_2^*$, which captures the value of the option to start the production stage, whereas the value of the firm is equal to $V_2(P) - I_2$ if $P \geq P_2^*$ (see also (2.3)). There exists a unique positive solution to (2.6) that is larger than C (see Lemma A.1 in Appendix A) — in fact, $P_2^* = C$ is the only solution if $I_2 = 0$. Moreover, P_2^* goes up as I_2 increases.

2.2 Development stage

The firm has to go through a development stage to create the option to start producing. The stochastic completion time of the development stage is denoted by T and by assumption follows an exponential distribution with mean $1/\lambda$ and probability density function $f(t) = \lambda e^{-\lambda t}$. Equivalently, the uncertainty in time to compelete the development stage is modelled by a Poisson process with parameter $\lambda > 0$; thus, the probability that the development stage is completed in the infinitesimal time interval dt is equal to λdt. The arrival rate of the Poisson process is assumed to be constant.

If the firm starts its development stage now, then upon completion of the development stage at time T, the firm acquires the value $F_2(P(T))$ as given in Proposition 2.2. As a consequence we have to distinguish between two cases that can occur. First, the firm waits before starting the production stage, which happens if $P(T) < P_2^*$; this situation is visualized in Figure 2.1.

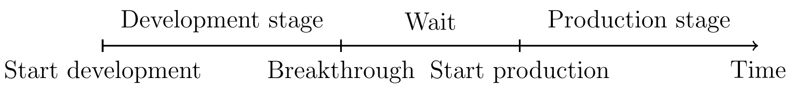

Figure 2.1: The firm waits before starting its production stage upon completion of the development stage.

Second, the firm immediately starts production upon completion of the development phase, which happens if $P(T) \geq P_2^*$; this situation is visualized in Figure 2.2.

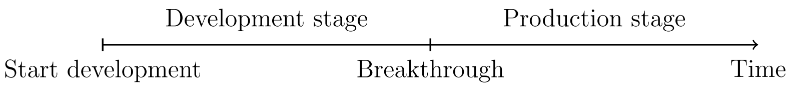

Figure 2.2: The firm immediately starts production upon completion of the development stage.

The value of the firm at the start of the development stage thus depends on the situation that arises upon completion of the development stage, which is represented by the threshold price P_2^*. The following proposition states the respective value of the firm

Proposition 2.3. *The value of the firm at the start of the development stage is given by:*

$$V_1(P) = \begin{cases} M_1 P^{\gamma_1} + B_1 P^{\beta_1} & \text{if } P < P_2^*, \\ M_2 P^{\gamma_2} + B_2 P^{\beta_2} + \dfrac{\lambda}{\lambda + r - \mu} \dfrac{PQ}{r - \mu} - \dfrac{\lambda}{\lambda + r}\left(\dfrac{CQ}{r} + I_2\right) & \text{if } P \geq P_2^*, \end{cases} \tag{2.7}$$

in which

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}},$$

$$\gamma_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}},$$

$$M_1 = \frac{1}{(P_2^*)^{\gamma_1}(\gamma_1 - \gamma_2)}\left[(\gamma_2 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_2 \frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2\right)\right], \tag{2.8}$$

and

$$M_2 = \frac{1}{(P_2^*)^{\gamma_2}(\gamma_1 - \gamma_2)}\left[(\gamma_1 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_1 \frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2\right)\right]. \tag{2.9}$$

Proof. See Appendix A. □

It follows that $M_1 < 0$ (see Proposition A.1 in Appendix A), which implies that $M_1 P^{\gamma_1}$ is negative so that it corrects for the fact that the firm will not start production upon completion of the development stage. The correction is larger if P is larger because the closer P is to P_2^*, the stronger the desire for the firm to start production. The value of M_2 can be both negative and positive. In fact, $\lim_{\lambda \downarrow 0} M_2 P^{\gamma_2} = -B_2 P^{\beta_2} < 0$.

We denote the investment cost of the development stage by I_1, which implies that the value of the firm when it starts development is equal to $V_1(P) - I_1$, in which $V_1(P)$ is given by (2.7). The threshold price whereupon the firm starts the development stage can either be smaller or larger than P_2^*. We denote these these threshold prices by P_{11}^* and P_{22}^*, respectively. The threshold price P_{11}^* is determined on the basis of the value function $V_1(P)$ for $P < P_2^*$, whereas the threshold price P_{12}^* is determined on the basis of the value function $V_1(P)$ for $P \geq P_2^*$. The relevant threshold price depends on the size of the investment I_1. A small investment cost I_1 implies that the firm prefers to start development relatively early, meaning that we are in the $P < P_2^*$ region, whereas a large investment cost I_2 implies the opposite.

Proposition 2.4. *The threshold price whereupon the firm starts the development stage is given by:*

$$P_1^* = \begin{cases} P_{11}^* & \text{if } I_1 < -M_1(P_2^*)^{\gamma_1}\dfrac{\gamma_1 - \beta_1}{\beta_1} \\ P_{12}^* & \text{if } I_1 \geq -M_1(P_2^*)^{\gamma_1}\dfrac{\gamma_1 - \beta_1}{\beta_1}, \end{cases} \tag{2.10}$$

in which M_1 is given by (2.8),

$$P_{11}^* = \left(\frac{\beta_1}{\gamma_1 - \beta_1}\frac{I_1}{-M_1}\right)^{\frac{1}{\gamma_1}}, \tag{2.11}$$

and in which P_{12}^* is implicitly determined by

$$(\beta_1 - 1)\frac{P_{12}^* Q}{r - \mu}\frac{\lambda}{\lambda + r - \mu} + (\beta_1 - \beta_2)B_2(P_{12}^*)^{\beta_2} + (\beta_1 - \gamma_2)M_2(P_{12}^*)^{\gamma_2} - \beta_1\left(I_1 + \frac{\lambda}{\lambda + r}\left(\frac{CQ}{r} + I_2\right)\right) = 0, \tag{2.12}$$

in which M_2 is given by (2.9).

Proof. See Appendix A. □

3 Numerical analysis

3.1 Project data and Base case analysis

Table 3.1 presents data on Uganda's Albertine Graben oil project, that is used throughout the numerical analysis. We have collected representative values such as expected time to completion and project costs from Ward and Malov (2016) and from interviews with people who have official knowledge about the project. As can be seen in Table 3.1, the first investment is substantially higher than the second. The first investment covers costs on infrastructure development and expenditures on development of oil production plants such as; on equipment, raw materials, prefabrications, construction, engineering designs, project management, insurance and certification whereas the second mainly covers additional well drilling that is undertaken upon commencement of oil production. The number representing OPEX per barrel of oil is obtained from Ward and Malov (2016) and is stated in real terms.

The development phase was initially meant to begin in 2018 and to last 3 years, but has since been delayed with an expected date of completion in 2025. We therefore set the expected time to completion of the development phase to $T = 3$, and increase it sequentially by 1 additional year up to $T = 9$, for our sensitivity analyses. Our theoretical model assumes an infinite time horizon when the project has started. However, in practice, the oil project has a limited life span, which in this case is expected to be around 25 years. To set a yearly production rate, we use the estimated total oil reservoir quantity (1 112 000 000 barrels) and divide it by 25. Thus, we assume that net present values beyond 25 years will not have a significant impact on threshold values, *i.e.,* optimal decisions and net present value of the project.

The monthly historical spot prices of Nigeria's Bonny Light crude from January 2006 to December 2018 were used to compute the estimates of the drift rate and volatility. The spot price of Bonny Light crude is chosen as a proxy for Uganda's crude oil over Brent and WTI crude because of its similar characteristics in terms of API gravity and sulphur content, as well as the geographical location. The price data is obtained from the website of the Central Bank of Nigeria (Central Bank of Nigeria, 2018). The risk-free interest rate of 5% and the drift rate of 0% are arbitrary values.

From our base case analysis, as shown in Figure 3.1, the threshold price for development is USD63 while the threshold price for investing in production is USD18. The significant gap between the two thresholds is mainly because the development costs are significantly higher than the production stage costs. It is of interest to assess how these thresholds change with variations to different project parameters. We thus proceed to undertake sensitivity analyses of the project value and threshold prices to changes in important input parameters, such as the share of total investment cost, the expected time to completion and oil price volatility.

Table 3.1: Baseline input data in numerical analysis

Parameter	Data	Description	Source
I_1	USD 12.5 billion	First stage investment. Capital expenditures before and during development.	Ward and Malov (2016) and personal communication.
I_2	USD 1.47 billion	Second stage investment. Capital expenditures related to production phase.	Ward and Malov (2016) and personal communication.
C	USD 9	Operating expenses per barrel of oil	Ward and Malov (2016) (and rounded)
Q	44,480,000 barrels	Yearly production rate	Total oil reservoir quantity from Ward and Malov (2016) and 25 years production
r	5%	Discount rate	
λ	$\frac{1}{3}$	Expected time to completed development stage is $1/\lambda$, i.e. 3 years.	Ward and Malov (2016) p.6 (adapted)
μ	0%	Oil price drift rate in GBM	Assumption that oil price in real terms has zero drift - Base case
σ	30%	Oil price volatility rate	Base case assumption

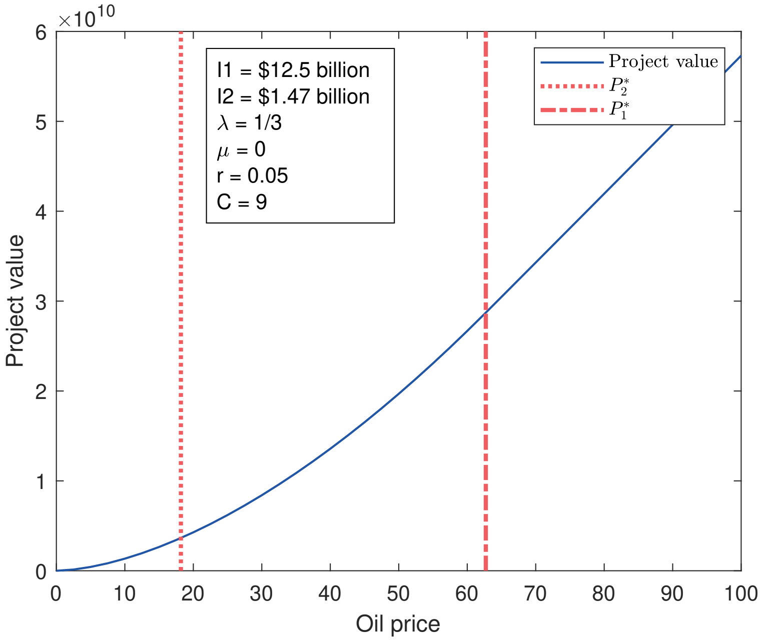

Figure 3.1: Project value F_1, and threshold prices P_1^* and P_2^* for the original project.

3.2 Distribution of total investment amount

Oil investments are capital-intensive. In our case study, the estimated total investment I is equal to USD 13.97 billion. This is the sum of I_1 and I_2 in Table 3.1 above. As already stated, even though I_1 is much larger than I_2 in the original case, it is of interest to analyze what impact the distribution of total invested capital between I_1 and I_2 makes on threshold prices P_1^* and P_2^*.

In addition, in section 2.2, theoretical results were also derived under the condition that $P_1^* > P_2^*$ and the condition that $P_2^* > P_1^*$, and it is therefore of interest to analyse when $P_1^* = P_2^*$. To visualize this, we change values for I_1 and I_2 by distributing the total investment amount I= USD 13.97 billion, between I_1 and I_2. Let ϵ be the share of total investment I distributed to I_2. 1-ϵ is then the share distributed to I_1, thus, (1-ϵ)*I $= I_1$ and ϵ*I $= I_2$. If ϵ=0, then I_2=0 and $I_1 =$ USD 13.97 billion. If $\epsilon = 0.5$ then $I_1 = I_2 =$ USD 6.985 billion and when $\epsilon = 1$, the I_1=0 and $I_2 =$ USD 13.97 billion. Increasing ϵ increases I_2's share of total investment giving that a larger investment amount moves forward in time.

Figure 3.2 presents threshold prices P_1^* and P_2^* for different values of ϵ. The figure shows that the threshold price lines cross each other at approximately $\epsilon = 0.85$. That means that when I_2 makes up approximately 85% of total I then $P_2^* > P_1^*$. It is also interesting to note that when $\epsilon = 0$, *i.e.*, I_2=0, then $P_2^* = 9$ which is the same value as C. This is expected since the project generates profit when the oil price is above C, otherwise, a loss is made.

Since the capital expenditures of the production stage are zero, the firm starts producing as soon as the oil price equals the marginal cost of producing oil. The firm requires a larger markup when the capital expenditures of the production stage increase, and therefore the threshold price of the production stage is increasing with I_2. The same applies for the development stage, e.g., if $I_1 = 0$, the threshold price is zero as well.

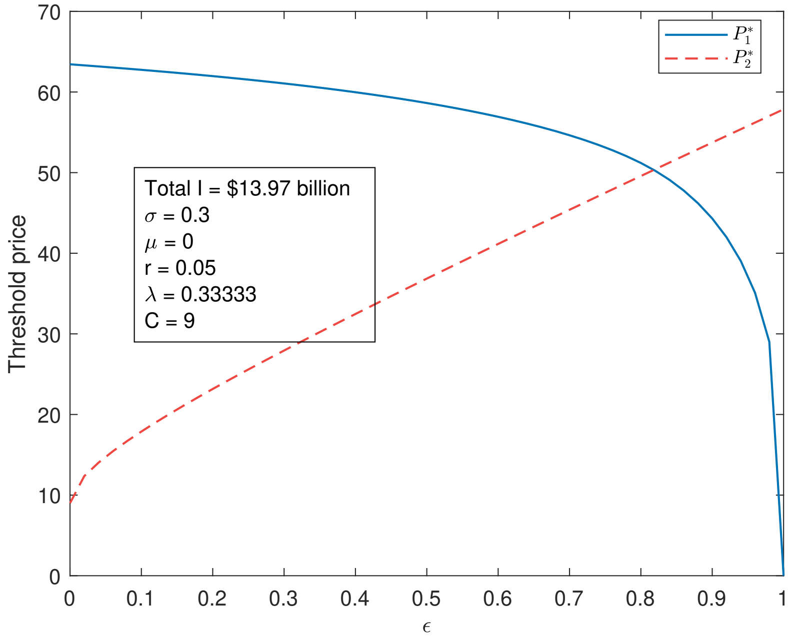

Figure 3.2: Threshold prices P_1^* and P_2^* for different values of ϵ when μ=0 and λ=1/3.

3.3 Oil price drift rate

Figure 3.3 illustrates how the project value is affected by the value of the oil price drift rate. Like Ketelaars and Kort (2022), we find that the project value is highly sensitive to changes in the drift rate, at all oil prices. This is especially true when μ is bigger than zero and as can be seen there is a significant difference between project values when $\mu = 0.04$ compared to when μ is zero.

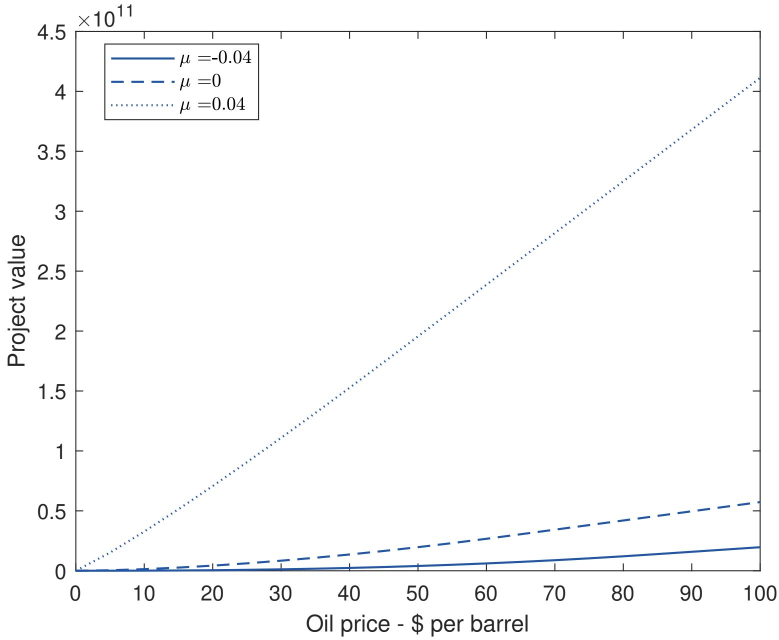

Figure 3.3: Project values F_1 for different oil prices and drift rate μ.

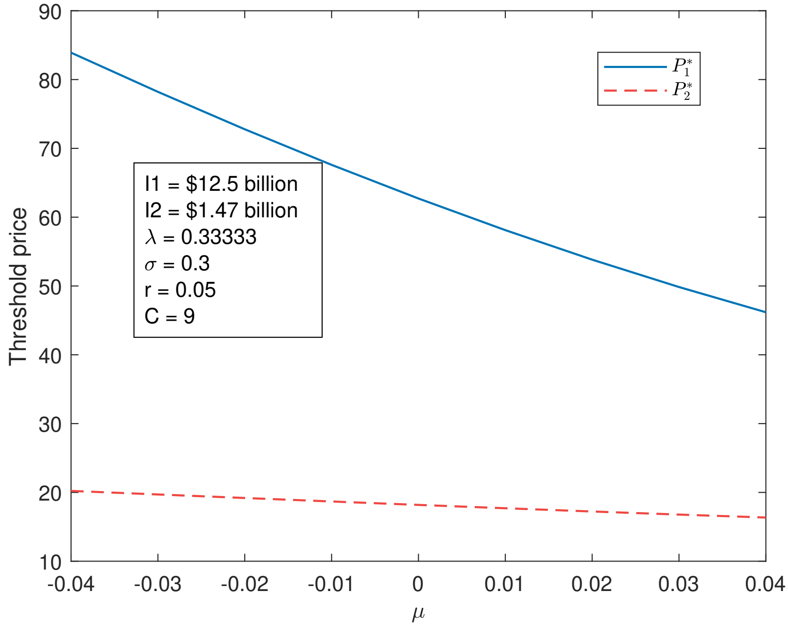

Figure 3.4: Threshold prices P_1^* and P_2^* for different oil price drift rates.

Figure 3.4 illustrates the threshold prices for P_1^* and P_2^* for different values of the drift rate mu. A negative μ means that the expected future oil prices reduce over time, and under such circumstances we require a higher threshold price P_1^* when μ is e.g., -4% instead of 0 %. When μ is -4% the expected oil price will decrease during the development stage. This is not the case when μ and the expected future oil price are at the same level over time. The same line of reasoning can be applied for increasing positive values of μ. If we expect higher price increase during the development stage, we are willing to invest at a lower threshold price P_1^*. In Figure 3.4, we also see that there is a significant difference between P_1^* and P_2^*. As discussed in the previous section, the difference between I_1 and I_2 is large and thus results in considerable differences between P_1^* and P_2^*. From the figure, we also observe that the oil price must drop significantly between the first and the second decision for it to be optimal not to invest I_2. Both thresholds P_2^* and P_1^* decrease with higher values of μ. The lower the μ the higher the threshold price since a lower μ will imply a lower oil price growth rate in the future, which ultimately affects cash flows and present values. To reduce the probability of negative cash flows during the production stage, the firm invests at a higher threshold price P_2^* when μ is reduced.

3.4 Volatility rate

Figure 3.5 shows the effect of oil price volatility on the project values. Overall, a higher volatility rate increases the project value and, as can be seen in the same figure, increasing volatility has the largest impact around an oil price equal to USD30. This result is a standard result within financial and real option literature (see Dixit, 1992; Dixit and Pindyck, 1994; Trigeorgis, 1996).

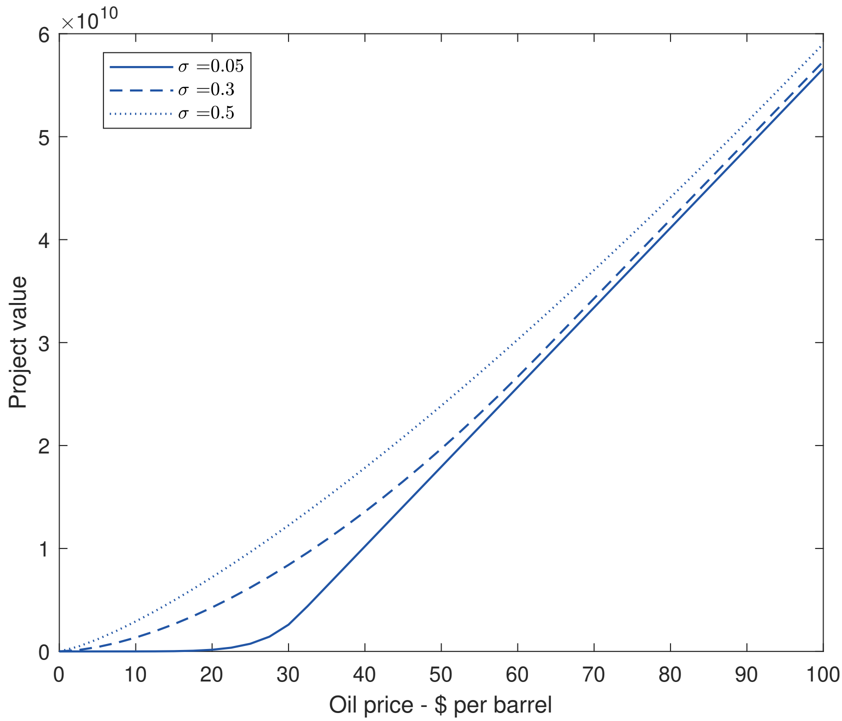

Figure 3.5: Project value F_1 for different volatility rates.

Figure 3.6 shows that P_1^* rises faster than P_2^* and remains above P_2^* when volatility increases. Similar to Huisman and Kort (2015) and Ketelaars and Kort (2022), and in line with standard real options theory, our results suggest that increased uncertainty results in higher investment thresholds. This implies that the firm will wait longer to invest in the development stage at a higher oil price, when faced with higher oil price uncertainty. The commencement of the development stage provides information which mitigates the impact of uncertainty (Majd and Pindyck, 1986), such that the firm can invest in the production stage at P_2^* far below P_1^*.

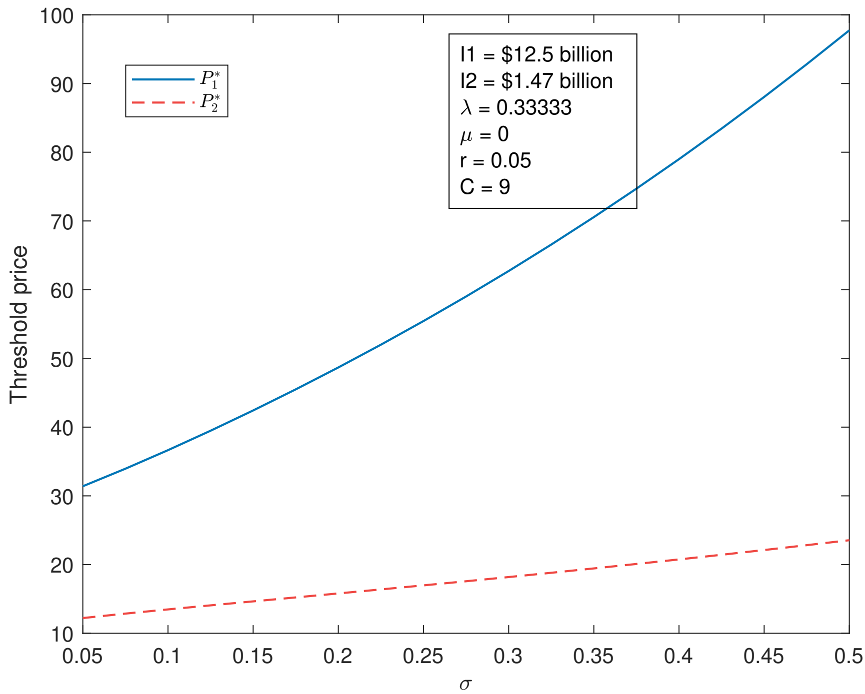

Figure 3.6: Threshold prices P_1^* and P_2^* for different oil price volatility rates.

3.5 Lambda values and expected time to completion

Figures 3.7 and 3.8 present the threshold prices P_1^* and P_2^* for different λ-values. The threshold prices in Figure 3.7 are based on μ=0% while the threshold prices in Figure 3.8 are based on μ=4%. An interesting observation when comparing the two figures is that the P_1^* curves are different. While P_1^* has a downward slope in Figure 3.7, it has an upward slope in Figure 3.8. In Figure 3.7, we observe that for low values of λ (λ=1/9, 2/9..), the slope is steep and flattens at higher values of λ beyond λ=0.4. Note that when λ=1/9, the expected development time is 9 years. An increase in the expected development time combined with a drift rate of zero apparently increases the value of the option to wait, thus creating an incentive to delay development and initiate the project at a higher threshold price, compared to when, for instance, λ=1/3 (expected development time = 3 years) or λ=1 (expected development time=1 year).

In Figure 3.8, one can observe that the optimal threshold price P_1^* increases slightly as λ increases. This is the opposite pattern compared to Figure 3.7. In Figure 3.8, the oil price drift rate is 4% such that the expected oil price will increase with a factor exp(0.04) each year. An increase in expected development time gives a longer time of exponential growth in the expected oil prices.

Thus, in case of longer development times, it is rational for the firm to initiate the project at a lower P_1^* since there would be a longer time to reach the price level where the project becomes profitable. These results demonstrate that the threshold prices may be a non-monotonic function of the expected time to completion. It is also noteworthy that, in the figures 3.7 and 3.8, the optimal P_2^* is unaffected by the changes in λ-values. This is anticipated since the expected time to complete the development stage is excluded from the analysis upon completion of the development stage.

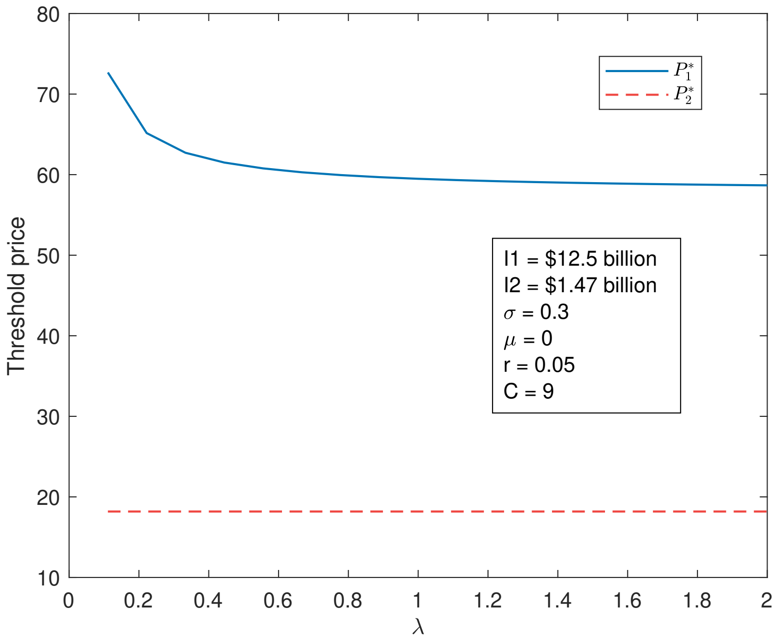

Figure 3.7: Threshold prices P_1^* and P_2^* when mu=0 and for different lambda values

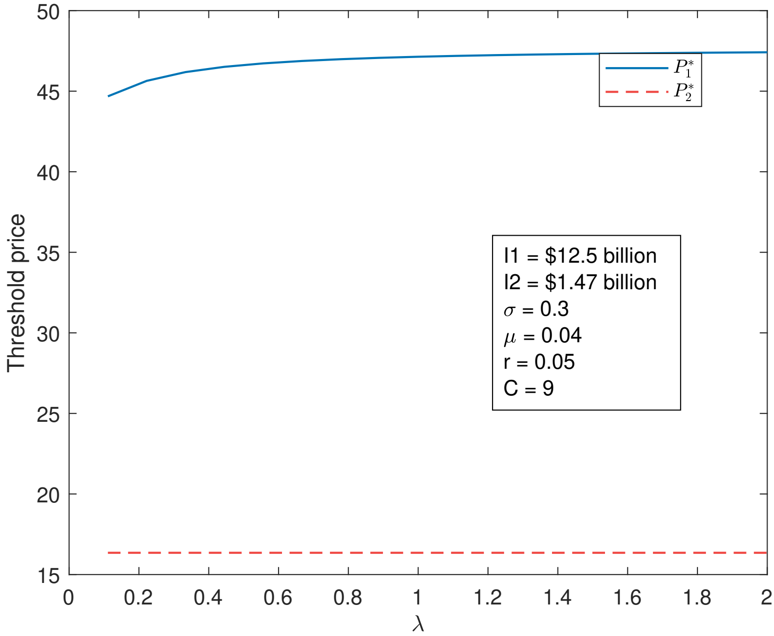

Figure 3.8: Threshold prices P_1^* and P_2^* when mu=0.04 and for different lambda values

3.6 Cost per barrel of oil

When we analyse how the project value is affected by changes in cost, we find that project costs have relatively small effect on the project values. The reason for this can partly be seen in Figure 3.9. To start the development stage, the oil price must be equal or above P_1^*. In this case, it means that the oil price must be above USD 63. The project will not start for C values within the range of USD 7.5 to USD 10.5. As such, there are no significant differences in project values at oil prices below USD 63. For the oil prices above P_1^*, the threshold prices are much higher than C and increase as C increases. An increase in C will trigger an almost corresponding increase in P_1^* and P_2^*.

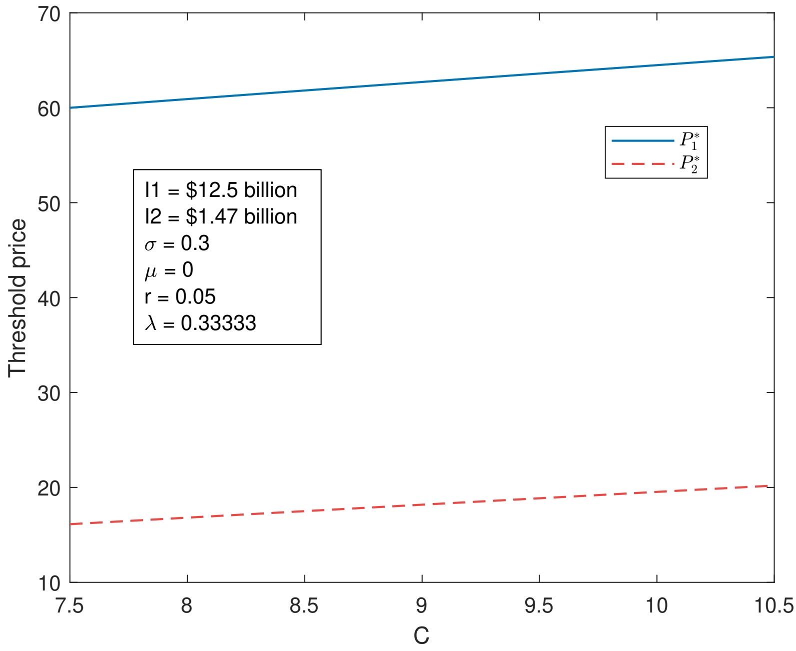

Figure 3.9: Threshold prices P_1^* and P_2^* for different values of cost per barrel

3.7 Production quantities

One of the key assumptions that underlies the valuation model is that we have an infinite time horizon over which project will generate a yearly production rate. Since future cash flows are discounted at a discount rate, larger than the oil price drift rate, there will be a point in time when additional future cash flow will only have a marginal impact in the project value. Thus, it is of interest to analyse the impact of yearly production rate on the project value and the corresponding threshold prices. Figure 3.10 illustrates the project value for three different yearly production rates. As expected, the project values increase as the yearly production rate increases. A proportionate increase in quantity extracted generates a proportionate increase in the project value. In Figure 3.11, we also observe that the threshold prices are a decreasing function of yearly production rates.

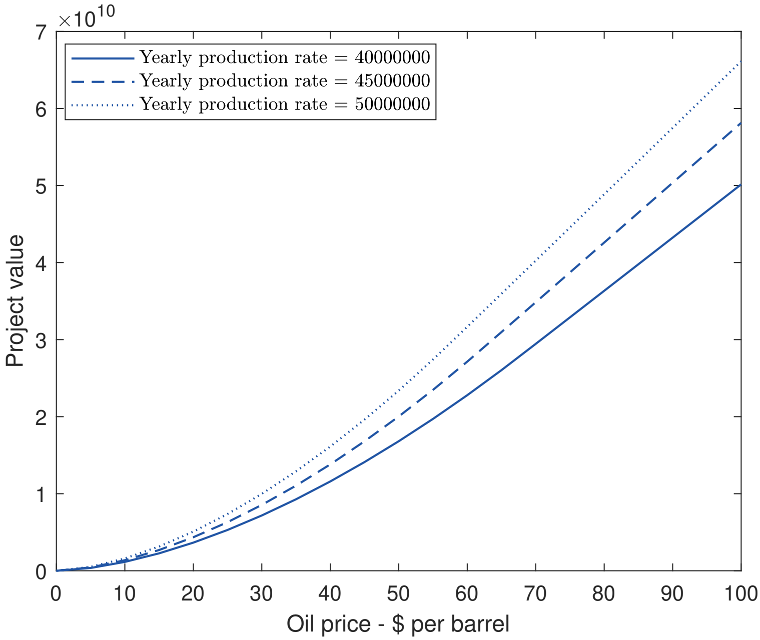

Figure 3.10: Project value for different estimates of yearly production rates

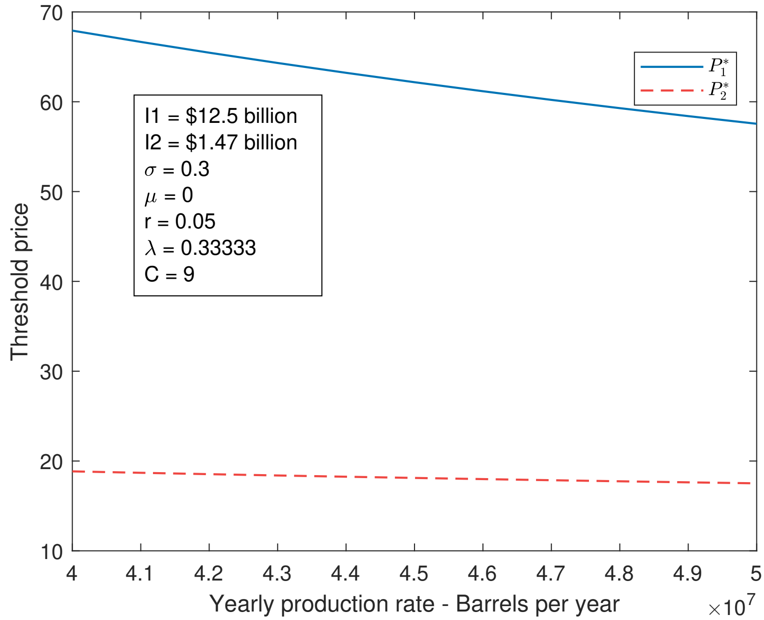

Figure 3.11: Threshold prices P_1^* and P_2^* for different estimates of yearly production rates.

4 Concluding remarks

The paper develops a real options framework for evaluating the development stage of an oil investment project with uncertain time to completion. Based on the costs of development and subsequent oil production, the oil price, and expected time to completion of the development stage, we find the optimal oil price threshold levels for investment at the development and production stages. We consider that the oil project can be divided into four different states and that two strategic decisions are made. We apply our model to derive analytical expressions of the values of the project and the optimal investment thresholds at each stage. To generate numerical results, we employ our model to a case study of Uganda's oil project. From the base case analysis, we find that the threshold price for development is more than three times the threshold price for production, because development costs are significantly higher than the costs of the production stage. From our sensitivity analyses, we find that increasing the cost share allocated to the production stage, increases the project value. We also establish that the firm would require a larger mark-up when the capital expenditures of the production stage increase as a share of total investment costs, and thus requiring higher threshold prices for oil production to start. As expected, we find that the project value for the development stage increases when the expected time to completion is shorter. We, however, observe that the threshold price may be non-monotonic in the expected time to completion. A limitation to our model is that we focus on uncertainties about oil prices and expected time to completion of the development stage as the risks faced by decision makers during the project's lifetime. Our model excludes other risks such as geological risks, technological risks and policy risks which would make the estimation of the real options multifaceted and a better reference for optimal decision making. It would also be interesting to extend the model to analysis of optimal investment strategies from the perspectives of the oil companies and host government to an oil contract.

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A Proofs

Proof of Proposition 2.3. The value of the firm at the start of the development stage, $V_1(P)$, must satisfy the following Bellman equation

$$rV_1(P) = \lim_{dt \downarrow 0} \frac{1}{dt} \mathbb{E}[dV_1(P)]. \tag{A.1}$$

Once there is a breakthrough at stochastic innovation time T, the value of the firm jumps to $F_2(P)$ as given by (2.5) in Proposition 2.2. The right-hand side of (A.1) can be expanded with the use of Itô's Lemma, which gives

$$\frac{1}{dt}\mathbb{E}[dV_1(P)] = \frac{1}{2}\sigma^2 V_1''(P)P^2 + \mu V_1'(P)P + \lambda(F_2(P) - V_1(P)) + \frac{o(dt)}{dt}. \tag{A.2}$$

Substituting (A.2) into (A.1) and rewriting gives the following non-homogeneous second-order differential equation

$$\frac{1}{2}\sigma^2 V_1''(P)P^2 + \mu V_1'(P)P - (\lambda + r)V_1(P) + \lambda F_2(P) = 0. \tag{A.3}$$

A general solution to the homogeneous part of (A.3) is given by

$$V_1(P) = M_1 P^{\gamma_1} + M_2 P^{\gamma_2}, \tag{A.4}$$

in which M_1 and M_2 are constants to be determined, and in which γ_1 is the positive and γ_2 is the negative root of the following quadratic equation

$$Q(\gamma) = \frac{1}{2}\sigma^2\gamma^2 + (\mu - \frac{1}{2}\sigma^2)\gamma - (\lambda + r) = 0. \tag{A.5}$$

It follows that $\gamma_1 > 1$ and $\gamma_2 < 0$ because $Q(\gamma)$ is strictly convex with $Q(0) = -(\lambda + r) < 0$ and $Q(1) = \mu - (r + \lambda) < 0$. To determine the solution (A.3), we distinguish between $F_2(P)$ for $P < P_2^*$ and $P \geq P_2^*$.

First, if $P < P_2^*$, then $F_2(P) = B_1 P^{\beta_1}$, which is also a particular solution to (A.3). Furthermore, the boundary condition $V_1(0) = 0$ implies that $M_2 = 0$ because $\gamma_2 < 0$. Therefore, the value of the firm at the start of the development stage if $P < P_2^*$ is equal to

$$V_1(P) = M_1 P^{\gamma_1} + B_1 P^{\beta_1}, \tag{A.6}$$

in which

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}.$$

Second, if $P \geq P_2^*$, then $F_2(P) = B_2 P^{\beta_2} + \frac{PQ}{r-\mu} - \left(\frac{CQ}{r} + I_2\right)$. To find a particular solution, we try $V_1(P) = B_2 P^{\beta_2} + aP + b$, which gives the particular solution

$$V_1(P) = B_2 P^{\beta_2} + \frac{\lambda}{\lambda + r - \mu}\frac{PQ}{r - \mu} - \frac{\lambda}{\lambda + r}\left(\frac{CQ}{r} + I_2\right).$$

The condition that rules out speculative bubbles (see page 181 of Dixit and Pindyck (1994)),

$$\lim_{P\to\infty}\frac{V_1(P)}{P}=\frac{\lambda}{\lambda+r-\mu}\frac{Q}{r-\mu},$$

implies that $M_1=0$, meaning that the fundamental component of the value if $P\geq P_2^*$ is captured by $\frac{\lambda}{\lambda+r-\mu}\frac{PQ}{r-\mu}$ alone. Hence, the value of the firm at the start of the development stage if $P\geq P_2^*$ is equal to

$$V_1(P)=M_2P^{\gamma_2}+B_2P^{\beta_2}+\frac{\lambda}{\lambda+r-\mu}\frac{PQ}{r-\mu}-\frac{\lambda}{\lambda+r}\left(\frac{CQ}{r}+I_2\right), \tag{A.7}$$

in which

$$\gamma_2=\frac{1}{2}-\frac{\mu}{\sigma^2}-\sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2(r+\lambda)}{\sigma^2}}.$$

The constants M_1 and M_2 follow from the value matching and smooth pasting conditions at $P=P_2^*$ with respect to (A.6) and (A.7), i.e.,

$$M_1(P_2^*)^{\gamma_1}+B_1(P_2^*)^{\beta_1}=M_2(P_2^*)^{\gamma_2}+B_2(P_2^*)^{\beta_2}+\frac{\lambda}{\lambda+r-\mu}\frac{P_2^*Q}{r-\mu}-\frac{\lambda}{\lambda+r}\left(\frac{CQ}{r}+I_2\right)$$

$$M_1\gamma_1(P_2^*)^{\gamma_1-1}+B_1\beta_1(P_2^*)^{\beta_1-1}=M_2\gamma_2(P_2^*)^{\gamma_2-1}+B_2\beta_2(P_2^*)^{\beta_2-1}+\frac{\lambda}{\lambda+r-\mu}\frac{Q}{r-\mu}.$$

Solving gives

$$M_1=\frac{B_2(P_2^*)^{\beta_2}(\beta_2-\gamma_2)+\frac{\lambda}{\lambda+r-\mu}\frac{P_2^*Q}{r-\mu}(1-\gamma_2)+B_1(P_2^*)^{\beta_1}(\gamma_2-\beta_1)+\gamma_2\frac{\lambda}{\lambda+r}\left(\frac{CQ}{r}+I_2\right)}{(P_2^*)^{\gamma_1}(\gamma_1-\gamma_2)},$$

and

$$M_2=\frac{B_2(P_2^*)^{\beta_2}(\beta_2-\gamma_1)+\frac{\lambda}{\lambda+r-\mu}\frac{P_2^*Q}{r-\mu}(1-\gamma_1)+B_1(P_2^*)^{\beta_1}(\gamma_1-\beta_1)+\gamma_1\frac{\lambda}{\lambda+r}\left(\frac{CQ}{r}+I_2\right)}{(P_2^*)^{\gamma_2}(\gamma_1-\gamma_2)}.$$

From Proposition 2.2 it follows that

$$B_1(P_2^*)^{\beta_1}=\frac{\beta_2}{\beta_1}B_2(P_2^*)^{\beta_2}+\frac{Q}{\beta_1(r-\mu)}P_2^*, \tag{A.8}$$

and that

$$\left(\frac{CQ}{r}+I_2\right)=\left(1-\frac{1}{\beta_1}\right)\frac{P_2^*Q}{r-\mu}+\left(1-\frac{\beta_2}{\beta_1}\right)B_2(P_2^*)^{\beta_2}. \tag{A.9}$$

Using (A.8) and (A.9) one can rewrite M_1 and M_2 to be equal to

$$M_1=\frac{1}{(P_2^*)^{\gamma_1}(\gamma_1-\gamma_2)}\left[(\gamma_2-1)\frac{P_2^*Q}{\lambda+r-\mu}-\gamma_2\frac{r}{\lambda+r}\left(\frac{CQ}{r}+I_2\right)\right],$$

and

$$M_2 = \frac{1}{(P_2^*)^{\gamma_2}(\gamma_1 - \gamma_2)} \left[(\gamma_1 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_1 \frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2 \right) \right],$$

respectively.

$\square$

Lemma A.1. *It holds that $P_2^* Q \geq CQ + rI_2$.*

Proof. Define

$$f(P) = (\beta_1 - 1)\frac{PQ}{r-\mu} + (\beta_1 - \beta_2)B_2 P^{\beta_2} - \beta_1 \left(\frac{CQ}{r} + I_2 \right).$$

Then, $f''(P) = (\beta_1 - \beta_2)B_2\beta_2(\beta_2 - 1)P^{\beta_2 - 2} > 0$ for $P > 0$, which implies that $f''(P)$ is strictly convex for $P > 0$. Let $I_2 > 0$. Plugging in $P = C + \delta$ with $\delta = r\frac{I_2}{Q}$ gives

$$\begin{aligned}
f(C+\delta) &= (\beta_1 - 1)\frac{(C+\delta)Q}{r-\mu} + (\beta_1 - \beta_2)B_2(C+\delta)^{\beta_2} - \beta_1 \left(\frac{CQ}{r} + I_2 \right) \\
&< (\beta_1 - 1)\frac{(C+\delta)Q}{r-\mu} + (\beta_1 - \beta_2)B_2 C^{\beta_2} - \beta_1 \left(\frac{CQ}{r} + I_2 \right) \\
&= CQ\left[\frac{\beta_1 - 1}{r-\mu} + \frac{r - \mu\beta_1}{r(r-\mu)} - \frac{\beta_1}{r} \right] + (\beta_1 - 1)\frac{\delta Q}{r-\mu} - I_2\beta_1 \\
&= 0 + (\beta_1 - 1)\frac{\delta Q}{r-\mu} - I_2\beta_1 \\
&= -I_2 \frac{r - \beta_1\mu}{r-\mu} \\
&< 0.
\end{aligned}$$

The first inequality follows from the fact that $\beta_2 < 0$ and $\delta > 0$. Hence, because $f(C+\delta) < 0$, $\lim_{P\to\infty} f(P) = \infty$, and the fact that f is strictly convex for $P > 0$, there must exist a positive root which is strictly larger than $C + \delta$; thus, we must have that $P_2^* Q > CQ + rI_2$. If $I_2 = 0$, then $P_2^* = C$. $\square$

Proposition A.1. *It holds that $M_1 < 0$.*

Proof. The denominator of M_1 is positive which implies that it suffices to show that

$$(\gamma_2 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_2 \frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2 \right) < 0.$$

It holds that

$$\begin{aligned}
(\gamma_2 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_2 \frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2 \right) &< (CQ + rI_2)\left[\frac{\gamma_2 - 1}{\lambda + r - \mu} - \frac{\gamma_2}{\lambda + r} \right] \\
&= (CQ + rI_2)\frac{\gamma_2\mu - (\lambda + r)}{(\lambda + r - \mu)(\lambda + r)} \\
&< 0.
\end{aligned}$$

The first inequality follows from Lemma A.1 and the fact that $\gamma_2 - 1 < 0$; the second inequality follows from the fact that $(\lambda + r) - \gamma_2\mu > 0$. $\square$

Proof of Proposition 2.4. Let the option values to start the development stage be given by $K_{11}P^{\beta_1}$ in case $P < P_2^*$ and by $K_{12}P^{\beta_1}$ in case $P \geq P_2^*$, in which K_{11} and K_{12} are constants to be determined.

First, suppose that $P < P_2^*$ such that (see (2.7))

$$V_1(P) = M_1 P^{\gamma_1} + B_1 P^{\beta_1}.$$

Then, the value matching and smooth pasting conditions at $P = P_{11}^*$, given by

$$\begin{aligned} K_{11}(P_{11}^*)^{\beta_1} &= M_1(P_{11}^*)^{\gamma_1} + B_1(P_{11}^*)^{\beta_1} - I_1, \\ K_{11}\beta_1(P_{11}^*)^{\beta_1-1} &= M_1\gamma_1(P_{11}^*)^{\gamma_1-1} + B_1\beta_1(P_{11}^*)^{\beta_1-1}, \end{aligned}$$

respectively, imply that

$$P_{11}^* = \left(\frac{\beta_1}{\gamma_1-\beta_1}\frac{I_1}{-M_1}\right)^{\frac{1}{\gamma_1}}.$$

Clearly, $P_{11}^* = 0$ if $I_1 = 0$ and $P_{11}^* = P_2^*$ if $I_1 = -M_1(P_2^*)^{\gamma_1}\frac{\gamma_1-\beta_1}{\beta_1}$. Moreover, it follows that

$$\frac{\partial P_{11}^*}{\partial I_1} = \frac{\gamma_1}{I_1}P_{11}^* > 0.$$

Therefore, $P_{11}^* \leq P_2^*$ which means that P_{11}^* is well-defined

Next, suppose that $P \geq P_2^*$ such that (see (2.7))

$$V_1(P) = M_2P^{\gamma_2} + B_2P^{\beta_2} + \frac{\lambda}{\lambda+r-\mu}\frac{PQ}{r-\mu} - \frac{\lambda}{\lambda+r}\left(\frac{CQ}{r} + I_2\right).$$

Then, the value matching and smooth pasting conditions at $P = P_{12}^*$, given by

$$\begin{aligned} K_{12}(P_{12}^*)^{\beta_1} &= M_2(P_{12}^*)^{\gamma_2} + B_2(P_{12}^*)^{\beta_2} + \frac{\lambda}{\lambda+r-\mu}\frac{P_{12}^*Q}{r-\mu} - \frac{\lambda}{\lambda+r}\left(\frac{CQ}{r} + I_2\right) - I_1, \\ K_{12}\beta_1(P_{12}^*)^{\beta_1-1} &= M_2\gamma_2(P_{12}^*)^{\gamma_2-1} + B_2\beta_2(P_{12}^*)^{\beta_2-1} + \frac{\lambda}{\lambda+r-\mu}\frac{Q}{r-\mu}, \end{aligned}$$

respectively, imply that P_{12}^* is implicitly determined by

$$\begin{aligned} (\beta_1-1)\frac{P_{12}^*Q}{r-\mu}\frac{\lambda}{\lambda+r-\mu} &+ (\beta_1-\beta_2)B_2(P_{12}^*)^{\beta_2} + (\beta_1-\gamma_2)M_2(P_{12}^*)^{\gamma_2} \\ &- \beta_1\left(I_1 + \frac{\lambda}{\lambda+r}\left(\frac{CQ}{r} + I_2\right)\right) = 0. \end{aligned}$$

Define

$$\begin{aligned} f(P) = (\beta_1-1)\frac{PQ}{r-\mu}\frac{\lambda}{\lambda+r-\mu} &+ (\beta_1-\beta_2)B_2P^{\beta_2} + (\beta_1-\gamma_2)M_2P^{\gamma_2} \\ &- \beta_1\left(I_1 + \frac{\lambda}{\lambda+r}\left(\frac{CQ}{r} + I_2\right)\right) = 0. \end{aligned}$$

From the expressions of P_2^* and M_2, given in Proposition 2.2 and Proposition 2.3, respectively, it follows that

$$(\beta_1 - \beta_2)B_2(P_2^*)^{\beta_2} = -(\beta_1 - 1)\frac{P_2^* Q}{r-\mu} + \beta_1\left(\frac{CQ}{r} + I_2\right),$$

and

$$(\beta_1 - \gamma_2)M_2(P_2^*)^{\gamma_2} = \frac{\beta_1 - \gamma_2}{\gamma_1 - \gamma_2}\left[(\gamma_1 - 1)\frac{P_2^* Q}{\lambda + r - \mu} - \gamma_1\frac{r}{\lambda + r}\left(\frac{CQ}{r} + I_2\right)\right].$$

Using these expressions one can show that

$$f(P_2^*) = -M_1(P_2^*)^{\gamma_1}(\gamma_1 - \beta_1) - \beta_1 I_1,$$

which implies that $P_{12}^* = P_2^*$ if $I_1 = -M_1(P_2^*)^{\gamma_1}\frac{\gamma_1 - \beta_1}{\beta_1}$. Moreover, it holds that

$$f''(P) = P^{-2}\left[(\beta_1 - \beta_2)\beta_2(\beta_2 - 1)B_2 P^{\beta_2} + (\beta_1 - \gamma_2)\gamma_2(\gamma_2 - 1)M_2 P^{\gamma_2}\right]. \tag{A.10}$$

If $M_2 > 0$, then $f''(P) > 0$. Otherwise $M_2 < 0$ and, due to the complicated expression for $f''(P)$, it is not possible to analytically prove that $f''(P) > 0$, though extensive numerical experimentation suggests that $f''(P) > 0$. Moreover, notice that an increase in I_1 shifts the function $f(P)$ downwards. Therefore, because the function $f(P)$ is strictly convex in P and the fact that $f(P_2^*) < 0$ for $I_1 > -M_1(P_2^*)^{\gamma_1}\frac{\gamma_1 - \beta_1}{\beta_1}$, it must hold that P_{12}^* increases as I_1 increases. Thus, $P_{12}^* \geq P_2^*$ which means that P_{12}^* is well-defined.

□

Chapter 2: How valuable is the option to defer Uganda's crude oil production?

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Published in Scientific African, 13, e00868

Scientific African 13 (2021) e00868

Contents lists available at ScienceDirect

Scientific African

journal homepage: www.elsevier.com/locate/sciaf

How valuable is the option to defer Uganda's crude oil production?

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ARTICLE INFO

Article history:
Received 23 November 2020
Revised 20 June 2021
Accepted 22 July 2021

Editor: DR B Gyampoh

JEL classification:
D81
E22
G11
G32
L71
Q40

Keywords:
Crude oil price
Real options
Binomial lattices
Option to defer
Geometric Brownian motion
Ugandan oil project

ABSTRACT

Our study contributes to the limited literature on real options valuation of Africa's oil investments. We establish binomial lattices to assess the value of the option to defer crude oil production in Uganda. We assume that oil prices follow a Geometric Brownian Motion (GBM) stochastic process. In our base case, we find that deferring production by another year adds value of $0.9 billion to the oil project. The value of the option to defer production particularly increases at lower crude oil prices amidst higher crude oil price volatility. When the rate of net convenience yield is high and the oil price is high, the value of the option is lower. At low oil prices, increases in cost inflation result in rejection of the project. We conclude that the Uganda oil project is generally profitable, and that deferring oil production is justified except in the cases where the net convenience yield or cost inflation is high.

Introduction

The upswings in crude oil prices between 2004 and 2013 renewed interests of international oil companies (IOCs) in Africa's vast oil reserves. For instance, between 2011 and 2014, African countries accounted for around 20% of global oil discoveries [22]. In 2006, Uganda joined the list of prospective oil producing countries with 6 billion proven oil reserves in the Albertine Graben of which 1.4 billion barrels are economically viable for extraction [29]. Uganda's peak production is projected to be between 200,000 and 250,000 barrels of oil per day with extraction lasting 25 years (see Appendix B). The cost of extracting oil over this period will amount to approximately $19 billion in capital expenditures and operating expenses. Prior to this production stage, the development of infrastructure, operation facilities and production wells will cost about $12.5 billion.

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https://doi.org/10.1016/j.sciaf.2021.e00868

In addition to being highly costly, numerous uncertainties, irreversibility and lumpiness of investments in oil projects add to their complexity. Investment towards oil production is a step-wise process, from exploration, appraisal of oil reserves, development of oil fields and support infrastructure, actual production of oil to decommissioning. Investments at each sequential stage of oil project lifecycle are made in lumps and are sunk, for the most part, once expended. More so, each successive stage prior to the production stage typically do not lead to immediate cash flows but open up further investment opportunities. The capital intensity of oil investments makes them irreversible because the oil wells and operation facilities can only be used to produce oil. The complexities of oil projects are further exacerbated by market and technical uncertainties about; oil price volatility, operating expenditures during the long lifetime of the project, economically viable oil reserves, the amount of oil produced from proven reserves, and future world demand for oil [11,14].

The African Union Agenda 2063 advocates for expanded local ownership and increased control of oil and gas reserves[1] [4]. However, like many resource-endowed Sub-Saharan countries, Uganda has limited capacity to solely finance and operate immense complex oil projects [19]. Consequently, in the years 2012 and 2016, the government of Uganda issued oil production licenses to three IOCs under a contractual arrangement in the form of a Production Sharing Agreement (PSA). The issuance of the production licenses set track for investment in the development phase in preparation of oil production. However, slumps in oil prices beginning in 2014 forced IOCs to downsize their prior optimistic investment plans. IOCs significantly trimmed their local workforce and cut their investment budgets by 20 to 30% in response to the erratic downswings of global crude oil prices from USD 95 in mid-2014 to USD 30 in the first quarter of 2016 [28].

On the global front, the recent unprecedented collapse of world oil markets due to the prevalent global Covid-19 pandemic and Russia-Saudi Arabia oil price war in March, 2020, have widely disrupted oil investment activities. The global oil price fell from an average of USD 64 in 2019 to USD 25 in the second quarter of 2020, amidst a record deep in oil demand. Thus, resulting in an estimated drop in 2020 global upstream investment of about one quarter compared with 2019 [23]. New potential oil-producers are likely to be worst hit as they present IOCs with ease of abandoning or downsizing the investments. Although oil prices partially recovered to USD 50 by the close of 2020, the looming ambiguity about the duration of the pandemic has exacerbated the uncertainty of future global oil investments and oil price movements. Another uncertainty facing the oil market is future climate policies, illustrated by the large difference when it comes to global oil demand between the Stated Policy Scenario and the Sustainable Development Scenario in the IEA [24].

These developments in the oil markets have awakened concerns about whether investment in extracting Uganda's oil can deliver sufficient returns to all parties amidst periods of high oil market uncertainty. To address these concerns, this study applies real options methods which recognise the lumpiness, irreversibility, sequentiality [13] of investments in oil projects and the prevailing uncertainty of the economic environment in which those projects are undertaken [15].

Embedded in the specificities of oil contracts, oil investment decisions are treated as real options. As stipulated by the PSA, the government of Uganda retains the ownership rights to the oil resource while the IOCs assume the risks of investing in exploration, development and production. Inherent in their production licenses, the IOCs have the right to exercise different managerial flexibilities in order to strategically capitalize on revenue windfalls arising from periods of high oil prices while mitigating the risks associated with low revenues during times of low oil prices. Among the potential flexibilities, we can identify; i) the option to defer production; ii) the option to expand production to smaller fields; iii) the option to indefinitely abandon production.

How valuable are these flexibilities and how can their value be quantified [35]? Similar studies have applied real options theory to address these research questions (e.g [3,16,17,25,26,34]). Despite the renewed interests of international oil companies (IOCs) in Africa's vast oil reserves in the past two decades, most of these studies are focused on developed reserves in high-income countries. We are aware of only three previous studies on Africa's undeveloped reserves (see [2,18,32]). It is against this premise that this study applies real option methods to value Uganda's undeveloped reserves, while accounting for uncertainty of crude oil price volatility.

We specifically quantify the value of deferring production and how this value changes at different levels of crude oil price volatility. Our base case results suggest that deferring production by another year adds value of $0.9 billion to Uganda's oil project. The value of the option to delay production particularly increases at lower crude oil prices amidst higher crude oil price volatility. The value of the option to defer production reduces as the net convenience yield rises and reaches zero at a critical price of $65 per barrel of crude oil. At low oil prices, increases in cost inflation result in negative values of both static Net Present Value (NPV) and expanded NPV. We conclude that the Uganda oil project is generally profitable, while deferring oil production is justified except in scenarios where the net convenience yield or cost inflation is high.

In section two, a literature review of the application of real options to the analysis of oil investments and the contribution of our paper are presented. The third section describes Uganda's oil extraction project. The fourth section describes the data and discusses the binomial lattice model, as applied to the project. The fifth section presents the analysis and results of the option value of deferring production. The last section draws some conclusions.

[1] The African Union (AU) Agenda 2063 is a plan for Africa's structural transformation and was agreed upon by the Heads of AU member states in May 2013. The AU envisages 'Transformed Economies and Jobs' as its Goal 5. To achieve goal 5, one of the priority areas is 'Expanded ownership, control and value addition (local content) in extractive industries' (see African Union Commission, 2014).

Literature review

Real options theory was invented in 1977 in response to the various limitations of the traditional discounted cash flow (DCF) theory [27] and first applied to the valuation of oil and gas production projects by Brennan and Schwartz [8]. Since then, there has been growing applications of real options valuation techniques to analyse managerial flexibility in oil exploration and production investments. The literature reviewed presents diverse techniques to real option valuation and the modelling of the stochastic process of crude oil prices. For instance, Lund [26] applied a binomial option pricing model and Geometric Brownian Motion (GBM) as the stochastic process of the oil price to measure the value of initiation, termination and capacity flexibility in Norway, finding that the role of flexibility adds significant value to the oil projects. Fleten et al. [17] applied the Least Squares Monte Carlo (LSMC) method to value expansion of an offshore oil field by tying in a satellite field, and the option of early shut down in Norway, modelling the oil price as a GBM. Their study found that the tie in option has significant value if the oil price increases, while early shutdown has insignificant value.

Aleksandrov and Espinoza [3], estimated a multiple real option optimization problem for Brazil and United Arab Emirates by employing the Least squares Monte Carlo method, assuming that the oil price follows a mean-reverting stochastic process. Their results showed that the net present value of both countries' oil reserves increases significantly when production decisions are made conditional on oil prices. A related study by Elmerskog [16] applied the binomial option pricing model with a GBM price model and the Least Squares Monte Carlo method to estimate the value of co-producing adjacent oil fields in Norway. The study found that including the option of timing production adds significant value while early shut down adds meagre value, in line with Fleten et al. [17].

Kobari et al. [25] estimates the value of an oil sand plant in Canada, while accounting for oil price uncertainty by employing a trinomial tree technique under the assumption of a mean reverting stochastic process for oil prices. The study also found the critical spot oil price should be significantly low for the plant to shut down. Also Abadie and Chamorro [1] applied Monte Carlo simulations to estimate managerial flexibilities in production from oil wells in Canada, drawing on an Integrated Geometric Brownian Motion (IGBM). The authors showed that the value of deferring production was significant while the abandonment option was less valuable.

For the specific regional case of Africa; Abid and Kaffel [2] present a methodology to evaluate an option to defer an oilfield development and apply it to a Tunisian oil project. After identifying the appropriate stochastic processes for three risk factors (crude oil price, convenience yield and risk-free interest rate), they applied LSMC to estimate the value of the option to defer by means of one-factor, two-factor and three-factor pricing models. Their results showed that the value of the option to defer reduced with the number of stochastic risk factors included in the model. Qui et al. [32] also developed a multi-factor real options model and applied it to an offshore oil project located in West Africa. Similar to Abid and Kaffel [2], their results under a multi-factor real options model were more conservative than those given by the single-factor model. These deductions by Qui et al. [32] and Abid and Kaffel [2] are based on a theoretical comparison of single-factor and multi-factor real options models. Our study deviates from their approach by making a base case analysis with oil price as our only stochastic variable followed by sensitivity analyses to estimate how the option to delay production changes with variations in cost inflation, net convenience yield and volatility.

Fonseca et al. [18] applied a binomial tree model to value the option to delay development of an oil field in Africa[2], under the assumption that crude oil prices follow a GBM stochastic process. They also assume that the expiration time for the option is five years. Their results showed that the value of the option to delay development increased with volatility. They also show that the trigger price reduces as the volatility increases and as they near the expiration of the option. Contrary to Fonseca et al. [18], the real options considered in our framework is the possibility to defer production of the first barrel of oil. This is based on the premise that the option to defer production is the most relevant for analysis of undeveloped reserves [15].

Our study establishes binomial lattices to quantify the real options values. A binomial lattice model is considered to be the most suited technique to numerical approaches, as they offer simplicity and intuition [5,7,33]. Binomial lattices method is also proposed because it allows modelling of sequentiality in projects that require irreversible investments [21].

Description of Uganda's oil project

Oil discoveries in the Albertine Graben

After a century of on-and-off oil exploration due to political instability, insecurity, oil price volatility, social concerns, contractual and regulatory disputes, a series of oil discoveries emerged as successful in 2006. The first commercial discovery was made by Hardman Resources on its Mputa-1 well, followed by Heritage Oil with its Kingfisher discovery and other multiple drilling successes in the Albertine Graben thereafter. In the same year, Tullow Oil Uganda acquired the assets of both Harman Resources and Heritage Oil. From 2006 to mid-2014, overall, there was substantial exploration success of 88%, with 102 out of 116 wells yielding proven hydrocarbons. The estimated resources have increased from 300 million barrels in 2006 to 2 billion in 2010 and 3.5 billion barrels two years later. As of 2018, official reports indicated that there are 6.5 billion

[2] The authors did not specify the African country or region.

proven oil resources in the Albertine Graben of which 1.4 billion barrels are recoverable reserves [29]. The Albertine Graben is approximately 500 km long, averaging 45 km in width and measures about 23,000 square kilometres in Western Uganda (see map in Appendix D). The discoveries in these areas are reported to be the largest onshore oil discoveries in Sub-Saharan Africa in over 20 years [28] and place Uganda as the eighth country with the highest proven oil reserves in Africa. Only 40% of the total prospective area in the Albertine Graben has been explored, which indicates potential for additional oil resources upon further exploration.

It is however noteworthy that this region has the highest biodiversity in Uganda and is host to 70% of Uganda's protected area encompassing natural forests, national parks, fresh water bodies, game reserves and biosphere reserves (Plumptre et al., 2018). A study by Byakagaba et al. [9] on oil exploration in the Albertine Graben reported noise pollution due to blasting of rocks during exploration, soil erosion due to clearing of vegetation for road construction and wildlife disturbance due to increased human activity in the wildlife reserve as the major environmental impacts. Oil activities in the region thus raise concerns of environmental degradation, particularly their impact on the biodiversity of the natural habitats. Notwithstanding these pertinent issues, environmental concerns are beyond the scope of this study.

Issuance of production licences

By 2016, the Government of Uganda, had granted production licenses to three IOCs; Tullow Oil Uganda, Total E&P Uganda and China National Offshore Oil Corporation (CNOOC) Uganda Limited. The two latter firms acquired a third of Tullow's equity each in a farm-down and formed a joint venture partnership. The production licenses, as part of the comprehensive PSA, are valid for 25 years upon the extraction of the first oil [29]. After the required investments are made and oil is extracted, costs of expenditures on exploration, development and extraction are recovered by the IOCs and the remainder of the rent is shared between the IOCs and the government. The government also receives revenue in form of royalties, bonuses and taxes owed by the IOCs, as per the PSA.

The issuance of these production licenses forms the basis for the Final Investment Decision (FID)[3] in preparation for the development phase, structured under three major oil fields; the Kingfisher, Buliisa and Kaiso-Tonya.

Projects, forecasted production and cost profiles

For purposes of modelling, we present a generic timeline for the overall oil project (see Fig. 1). We consider that development starts from Year -1 to year 0 and thereafter production commences from Year 1 through to Year 25 (see Fig. 1) for Buliisa and Kingfisher oil fields and at a later year for Kaiso-Tonya.

The Buliisa oil fields cover the EA2 North and EA1 blocks, North-East of Lake Albert, with eleven fields under development (see Appendix A). Total is the operator and lead investor, with CNOOC and Tullow as equal partners. The Buliisa fields hold the highest reserves among all three projects, estimated at 819 million barrels of recoverable oil, with its production peak in its fourth year of extraction (see Fig. 2).

The Kingfisher oil field encompasses the EA3A Block, South of Lake Albert (see Appendix A) and is estimated to have 196 million barrels of recoverable oil, with expected peak production in its eighth year of extraction (see Fig. 2). Although CNOOC is the operator and lead investor, equal shares are held by Tullow and Total.

The Kaiso-Tonya fields cover EA2 Block, South East of Lake Albert, with three oil fields (see Appendix A). The oil fields are relatively small, with 39 million barrels of recoverable oil, and would not be economically viable on their own. The fields are thus the least complex and least costly as their production is tied-in to that of the Kingfisher oil field. For instance, Kaiso-Tonya has no central processing facility, as the extracted oil is transported to the facility of Kingfisher. The production from Kaiso-Tonya begins in the ninth year of Kingfisher's production to compensate for the decline in the latter field (see Fig. 2). Tullow is the operator and lead investor in the Kaiso-Tonya fields, with equal stakes held by the two other joint venture partners.

A key precondition for the production stage is the construction of a pipeline that will transport the crude oil for export through Tanzania (see Appendix D for details). Other infrastructural requirements include roads and the Hoima international airport. The initial investment cost at the development stage will amount to $12.5 billion. Appendix B shows the rest of the costs (i.e. the capital expenditures (CAPEX) and operating expenses (OPEX)) expended on the three oil fields. The CAPEX begins two years prior to production and includes all costs on development of oil production plants such as; expenditures on equipment, raw materials (e.g. steel and concrete), prefabrications, construction, engineering designs, project management, insurance and certification. CAPEX will be retrospective costs at the year of production start-up, and hence will be sunk if a decision to delay production is made that year. The OPEX entails the cost of running the oil production plants over their lifetime after construction is completed towards personnel for maintenance and operations, chemicals and fuels, spare parts, well servicing and other expenses to maintain production. Due to the larger number of reservoirs in Buliisa oil fields, contributing to its higher complexity, the absolute CAPEX and OPEX are higher than those of the other two oil fields.

[3] The FID on the oil project has been considerably delayed since 2006 due to tax-related disputes, alterations to contractual terms and political issues. We do not explicitly address the FID delays as they are beyond the scope of our study.

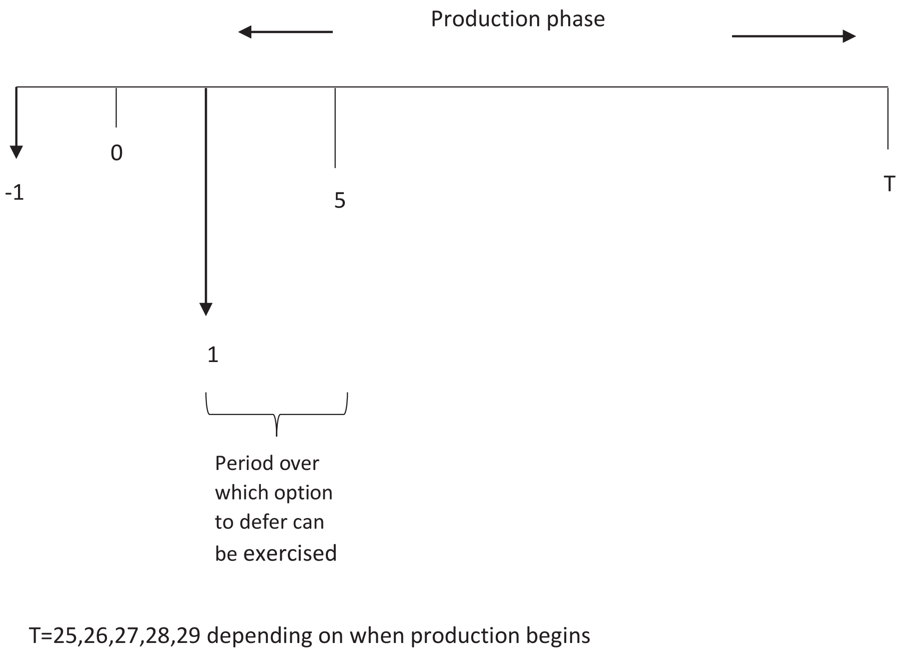

Fig. 1. Generic timeline for the overall oil project showing production phase and when there is an option to defer.

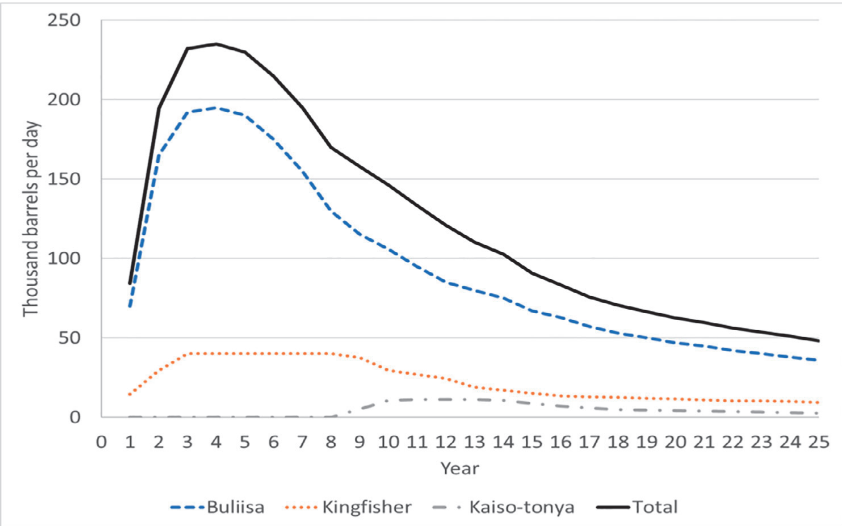

Fig. 2. Production profile for each oil field and in total after start-up of production ('000 Barrels of oil per day)

Valuation framework

The option to defer production and assumptions

Our study focuses on the option to defer production of the Kingfisher and Buliisa oil fields for up to five years (while we do not consider the Kaiso-Tonya fields in our analysis) beyond the planned commencement year 1. We assume that the option to defer is exercisable within the first five years (beyond which it expires) since the IOCs can renew their produc-

Year -1		Year 0		Year 1
				149.94
			118.31	
		93.36		93.36
	73.67		73.67	
58.13		58.13		58.13
	45.87		45.87	
		36.19		36.19
			28.56	
				22.54

Fig. 3. Binomial lattice modelling future oil prices in USD during the first two years of the project.

tion licenses for an additional five years only. A related assumption is that; upon commencement of production, extraction continues without any disruption, since restarting of production would be very costly and thus deemed economically unviable. For every subsequent year, management would save the planned outlays if the conditions are not favourable. Since the option to defer implies sacrificing accruing revenues early, the option is only justifiable if the value of deferring actually exceeds the value of beginning production earlier by a substantial premium [25]. We further assume that the decision to defer production has no impact on the production and costs profiles. For instance, if production is deferred from year 1 to year 2, then all the costs shown in Appendix B, except the two first years before production start-up, are deferred by one year. The valuation framework is finally coded and implemented in Python in order to compute values.

Data and variable description

The data used consists of the oil production profile, cost of development, CAPEX and OPEX of the Kingfisher and Buliisa oil fields, as projected over the period of 25 years (see Figs. 2 and 3). All these cost data were obtained from estimates by Ward and Malov (2016) and through interviews with officials at the Petroleum Authority of Uganda and Ministry of Energy and Mineral Development. The cost estimates exclude sunk costs towards; land acquisition, contingency, Front-End Engineering design (FEED), Environmental and Social Impact Assessment (ESIA), feasibility studies and other studies that are completed before the development phase commences.

The monthly historical spot prices of Nigeria's Bonny Light crude from January 2006 to December 2018 were used to compute the annualised volatility. The spot price of Bonny Light crude is chosen as a proxy for Uganda's crude oil over Brent and WTI crude because of its similar characteristics in terms of API gravity and sulphur content,[4] as well as the geographical location. The price data is obtained from the website of the Central Bank of Nigeria [10].

Geometric Brownian Motion and Risk-neutrality

In this study binomial lattice model based on Cox et al. [12] is established to estimate the value of the project under flexibilities of deferring production. To construct the binomial lattices, we begin by presenting the embedded assumptions and follow by deriving the formulae for the parameters of interest.

We assume that crude oil prices follow a Geometric Brownian Motion (GBM), such that;

$$dP = \mu P dt + \sigma P dW \tag{1}$$

Where $P$is the crude oil spot price at time t, dW is the increment of the Wiener process with $dW = \epsilon * \sqrt{dt}, \varepsilon \sim N(0, 1)$. μ is ε is a normally distributed variable with mean equal to zero and variance equal to 1. the drift rate and σ is volatility rate. μ and σ are taken as constants. In contrast to mean-reverting process, our choice of GBM as the appropriate stochastic process for modelling crude oil price movements is premised on three arguments. First, it is easier to model real options with the underlying assumption of GBM [15,32]. Second, extremely long price series (i.e 100 years and more) are required to correctly confirm that a series is mean reverting (Dixit and [30,31]). Our price series of 13 years is, in this regard, relatively short which limits us from ascertaining whether the series is mean reverting. Third, while empirical studies have shown that oil prices exhibit mean reverting behaviour (see [2,6]), GBM is less likely to result in significant evaluation errors when compared to mean reversion [20,31]. A mean reverting model faces challenges with half-life rates and equilibrium price levels, which when wrongly applied could give evaluation errors [31].

Further, at any time interval $(t,\ t + \Delta t)$ along the lattices, the price may go up by the multiplier u with a probability of q or fall by the multiplier d with a probability of $1 - q$, at the end of the time interval $t + \Delta t$. Following the model by Cox

[4] Bonny Light crude has an API gravity of 32.9°-34.5° and a sulphur content of 0.16%. Uganda's crude has an API gravity of 33° and a sulphur content of 0.16%. WTI's quality is characterised as API gravity of 39.6° and 0.24% sulphur compared to Brent crude with API gravity of 38° and 0.37% sulphur.

et al. [12], the up and down multipliers are computed from the volatility and the time step (Δt) such that;

$$u = \exp(\sigma \Delta t) \quad (2)$$

and

$$d = 1/u \quad (3)$$

Thus, an increase from P_t to $P^+_{t+\Delta t}$ and a decrease from P_t to $P^-_{t+\Delta t}$ are calculated as

$$P^+_{t+\Delta t} = P_t u \quad (4)$$

$$P^-_{t+\Delta t} = P_t d \quad (5)$$

The risk-neutral probability q is calculated as;

$$q = (\exp(r_f.\Delta t) - d)/(u - d) \quad (6)$$

where r_f is the risk-free rate.

The risk-neutral probabilities q and $1 - q$ are constant at all steps of the lattices since we take volatility, σ, to be constant in our model.

The parameters u , d and q are derived such that in case of infinitesimal Δt, the spot price follows a log-normal distribution where its mean and variance are; $E[ln(\frac{\widetilde{P}_t}{P_0})] = (r_f - \frac{1}{2}\sigma^2)t$ and $Var[ln((\frac{\widetilde{P}_t}{P_0}))] = \sigma^2 t$.

Parameter values

In order to compute the up and down multipliers and the risk-neutral probabilities, we proceed to determine our parameters of interest (i.e volatility, risk-free rate and length of time steps). The historical spot price data of the Bonny light is used to compute the annualised volatility rate and is estimated to be 33.5% per annum (see Appendix E for details on computations). The time step Δt is set to be 0.5 year, and the continuous risk-free rate r_f is 2.39% per year. The risk-free rate corresponds to the US 3-month treasury bill rate (US Department of the Treasury, 2018) since all the project costs and revenues are expressed in US Dollars. Using these parameter values in formula (2), (3) and (6) gives that $u = 1.2673$, $d = 0.7891$ and q=0.4662. Thus, $1 - q = 0.5338$.

Fig. 3 shows the initial oil price at Year -1 (i.e. two years before first decision at Year 1) and the modelled price outcomes until Year 1 when using half-year size time steps in the binomial lattice. The initial crude oil price used in the binomial lattice is set to $ 58.13 per barrel which corresponds to the average daily oil price in December 2018. For the base case, we apply the December 2018 price but apply varying prices in our sensitivity analysis (see Appendix C). Based on Eqs. (4) and (5), the price goes up by the multiplier u or falls by the multiplier d.

Analysis and results

Present value of starting production at Year 1

At Year 1, the oil prices are observed, and decisions are made to start production or not. For instance, if the oil price at Year 1 is 149.94 and production starts, then future expected revenues in Year 2 will be dependent on the expected oil price in Year 2, and so on for each of the subsequent years. Since an option pricing framework is used, it is the expected oil prices under the risk neutral measure that are of interest. The expected risk-neutral oil price at time m+n, given oil price realization j in Year m i.e. E[$P_{j,m,n,}$], can be determined from the formula $E[P_{j,m,n}] = P_{j,m} \times \exp(r_f * n)$. Where $P_{j,m}$ represents the different prices in the binomial lattice at different start year m, m=[1..5]. This formula is applied to all the remaining years until the 25th year after production starts, i.e. n=[0..24]. Appendix C.1 shows the expected risk neutral prices E[$P_{j,m,n}$] for n=0..4 when m=1, i.e., given $P_{1,1}$ = 149.94, $P_{2,1}$ = 93.36, ..., $P_{5,1}$ = 22.54. That is, given the outcome from the binomial lattice at m=1, the expected risk neutral prices for Year 2-5 are presented. The expected prices for Year 6-25 are omitted from the table for ease of exposition.

Appendix C.3 shows the net cashflows $C_{j,m,n}$ from the first five years (given that production starts at Year 1), obtained by subtracting the annual CAPEX and OPEX expended on the two oil fields (see Appendix B for CAPEX and OPEX values) from the revenue shown in Appendix C.2.

Appendix C.4 shows the expected present value of the net cash flows ($S_{j,m,n}$) in the case m=1 given as;

$$S_{j,m,n} = C_{j,m,n} + S_{j,m,n+1,}/exp(r_f * 2 * \Delta t) \quad (7)$$

In general, $S_{j,m,n}$ is computed from the sum of the present value of all future expected cash flows after year m+n and the cash flow generated during year m+n. At Year 1, given m=1 and n=0 in this case, the project may have a total expected present value equal to either 150, 90, 52, 29 or 14 billion US dollar (see Appendix C.4). Which of these five expected values will be realized depends on the realization j of the oil price at Year 1 (see Appendix C.1).

j	Year 1	Year 2	Year 3	Year 4	Year 5
1	150.5	247.8	404.0	655.0	1058.0
2	89.9	150.5	247.8	404.0	655.0
3	52.2	89.9	150.5	247.8	404.0
4	28.7	52.2	89.9	150.5	247.8
5	14.1	28.7	52.2	89.9	150.5
6		14.1	28.7	52.2	89.9
7		4.9	14.1	28.7	52.2
8			4.9	14.1	28.7
9			-0.7	4.9	14.1
10				-0.7	4.9
11				-4.3	-0.7
12					-4.3
13					-6.4

Fig. 4.. Future present value of the project's cashflows if production is started in a given year (Year 1 to Year 5) and a given oil price realization j (in billions of US Dollars).

The present value of starting production at Year 2 to Year 5 and the option to defer

In order to determine the value of the project when there is an option to postpone the production for up to five years, the respective values for starting production at Year 2 to Year 5 must be computed. Then one can analyse if it is the best strategy to start at a given year or defer until the next year and take a new start/defer decision. Since the oil price is modelled as a binomial process, the number of possible price realizations will increase with the number of half-years, i.e. for Year 2 there are seven possible prices, for Year 3 nine prices and so on. In Fig. 4, the present values of the project, for given start years and price realizations, are presented.

In Fig. 5, the present values of the project, when there is an option to defer, are presented. The value of the project, option value to defer included, at each node ($V_{j,m}$) is computed by taking;

$$V_{j,m} = max\left\{S_{j,m,0};\, 0\right\} m = 5 \tag{8}$$

$$V_{j,m} = max\left\{S_{j,m,0};\, \left(q^2.V^{uu}_{j,m+1} + 2q(1-q)V^{ud}_{j+1,m+1,} + (1-q)^2 V^{dd}_{j+2,m+1}\right)/exp\left(r_f * 2\Delta t\right)\right\} m = [1..4]$$

Equations 8 mean that the IOCs are faced with the choices between starting at year m to receive cash flow generated during m and the expected present value of future cash flow, or deferring another year before taking a new decision. If the expected present value of deferring to start the project is higher than starting immediately, beginning production is postponed.

Finally, using backward induction, the expanded NPV at year -1 is computed as;

$$\begin{aligned} ENPV_{-1} &= C_{-1,} + C_0/exp\left(r_f * 2\Delta t\right) \\ &+ \left(q^4 V_{1,1} + 4 * q^3(1-q)V_{2,1} + 6q^2(1-q)^2 V_{3,1} + 4q(1-q)^3 V_{4,1} + (1-q)^4 V_{5,1}\right)/exp\left(r_f * 4\Delta t\right) \end{aligned} \tag{9}$$

The variables C_{-1} and C_0 typically represent investment outlays that are made during Year -1 and Year 0, e.g., necessary infrastructure and further explorations. The option value to defer at each node is computed by subtracting the NPV with no consideration of options (in Fig. 4) from the expanded NPV (in Fig. 5). In comparison with Year values in Fig. 4, the values in Fig. 5 are slightly higher (151.4; 90.8; 53.1; 29.7 and 15.3 compared to 150.5; 89.9; 52.2; 28.7 and 14.1), which indicates that the value of deferring is slightly higher than starting production immediately. The results also indicate that the relative value of the option to defer is rather small, under the assumptions made in this analysis.

The value of option to defer

The results in the previous sub-section are taken as our base case analysis. From the results of traditional Discounted Cash Flow (DCF) method, the NPV at year -1 is found to be $ 36.5 billion indicating that the oil project is viable. When the option to defer production by another year is considered, the expanded NPV is estimated to be $37.4 billion, thus generating an option value of $0.9 billion. The positive option value emanates from the flexibility to defer expending the remaining OPEX and CAPEX to the subsequent year and management's ability to benefit from random oil price rises while minimising the risks from unfavourable oil price falls. Therefore, production should be postponed. These findings are similar to those of Abadie and Chamorro [1].

j	Year 1	Year 2	Year 3	Year 4	Year 5
1	151.4	248.5	404.5	655.2	1058.0
2	90.8	151.2	248.2	404.2	655.0
3	53.1	90.6	150.9	248.0	404.0
4	29.7	52.9	90.4	150.7	247.8
5	15.3	29.4	52.6	90.1	150.5
6		14.9	29.1	52.4	89.9
7		6.4	14.6	28.9	52.2
8			5.9	14.3	28.7
9			1.7	5.4	14.1
10				1.1	4.9
11				0.0	0.0
12					0.0
13					0.0

Fig. 5. Future present values with when there is an option to defer (in billions of US Dollars).

Sensitivity analysis

Behind the figures in the result section there are a number of assumptions that affect the outcomes. In this section we analyse to what extent changes in volatility rates, net convenience yield and cost inflation affect the net present value of the project and the option to defer. In the sensitivity analysis we have also, unless nothing else is stated, applied a binomial tree with 10 timesteps per year, comparted to the two time steps per year illustrated in chapter 4.

Sensitivity to changes in volatility rates

Volatility is a main driver to the option value. In order to illustrate its impact on the net present values for the project, with and without options, we compute the static NPV and option values for different oil price and volatility rates as presented in Appendix C.5. According to our results, notwithstanding the volatity rate, the static NPV is comparably low at lower oil prices and further reductions in the oil price increase the value of the option to defer. This implies that the traditional DCF significantly undervalues the oil project. Our finding is consistent with postulations by Smith [33]; Dixit and Pindyck [15]; and Trigeorgis [35] on justification for real options theory over traditional DCF. Similar to findings by Fleten et al. [17], Abadie and Chamorro [1] and Fonseca et al. [18],[5] our results also show that the option to defer is significant in the case with low oil prices and high volatility rates. This is also an expected outcome since higher volatility increases the possibility for lower oil prices and thus increases the value of having the option to defer the project. Intuitively, when the oil price is low and oil price volatility is high, it is optimal to delay oil production.

Another is that there are equal and approximately equal option values in cases of low volatilities, i.e. 25% and below, and at moderate to high oil prices. The reason is that, in those cases, it is almost certain that the oil price will be at a level where the project is expected to be profitable (when investment decision is made), while the option value comes from the fact that we can push expenses one year ahead. In the base case, it is assumed that there is no cost inflation, thus pushing them forward will increase the NPV. The impact of cost inflation is analysed in Section 5.4.3.

Sensitivity to changes in net convenience yield

In the base case analysis, it is assumed that the net convenience yield is equal to zero during the time horizon. However, since oil is a consumption asset, it is likely that the net convenience yield is different from zero. For instance Qui et al.

[5] Fonseca et al. [18] only analyse the influence of volatility on the option to defer an oil project.

Table 1
Comparing net present values of starting immediately and defer start for different values of oil price, net convenience yield and cost inflation. The optimal decision for each combination is highlighted in grey.

Oil price at Year 1	Net conv. Yield =0% Cost inflation=0%		Net conv. Yield = 0% Cost inflation= 4%		Net conv. Yield=4% Cost inflation = 0%		Net conv. yield =4% Cost inflation =4%	
Action	Start now	Defer	Start now	Defer	Start now	Defer	Start now	Defer
121.10	119 588	120 507	115 333	115 112	84 654	81 178	80 400	76 464
73.82	68 965	69 884	64 711	64 489	47 670	45 644	43 416	40 930
45.00	38 106	39 047	33 852	33 744	25 125	23 983	20 870	19 354
27.43	19 295	20 401	15 040	15 657	11 382	11 122	7 127	7 357
16.72	7 828	9 507	3 573	6 036	3 004	4 272	-	2 125

[32] calculated a value of 1.5% and modelled their net convenience yield as a function of change in oil production, after-tax profit of oil sales and oil value of developed reserves. Abid and Kaffel [2] also showed that the net convenience yield can be negative. For our sensitivity analysis, we assume that net convenience yield varies from -6 to +6%. In Appendix C.6, we therefore present static and expanded NPV for different values of oil price and net convenience yield. Our results show that the NPV, both static and expanded, drops rather significantly as the net convenience yield increases. This is natural since increased net convenience yield reduces the expected oil price increase in the option pricing models and thus will reduce expected future revenues. At an oil price of $65 per barrel and net convenience values of 4-6%, the expanded NPV is almost equal to the static NPV. This implies that the option to defer adds meagre value at high oil prices and high rates of net convenience yield. This is natural, as oil production is almost certain to begin at a high price such that deferring has no value. Our results are similar to those of Abid and Kaffel [2] who found that adding net convenience yield to the real options model reduced the value of the option to defer.

Sensitivity to changes in cost inflation

In the original analysis it is assumed that OPEX and (remaining) CAPEX are not changing if the project is deferred. That is, if a start-up decision is postponed by one year, OPEX and CAPEX are just moved one year ahead in time. So, in case of no inflation and positive discount rate there is an incentive to push OPEX and CAPEX forward in time. In Appendix C.7, the static and expanded NPV are shown for different values of the cost inflation for OPEX and CAPEX. Two different values of the cost inflation, in addition to the base case of no inflation, are incorporated. We assume a cost inflation rate of 2.42% equal to the risk-free rate of return (discrete) and thus identical to the expected price increase of the oil price in the option model. The NPV is, not surprisingly, negatively affected by increasing cost inflation. An oil price of $25 per barrel and an annual cost inflation rate of 4% result in negative NPVs, both static and expanded, and thus rejection of the project (see Appendix C.7). In case of oil prices equal to $45 and $65, the NPVs decline by approximately 10-20% when cost inflation increases from 0% to 4%. In addition, at all oil prices, the value of the option to defer drops with rising cost inflation. In principle, cost inflation rises the oil price required to make the oil project economically viable, and renders the option to defer worthless.

Optimal exercise policy

For most cases in the base case analysis the optimal exercise policy is to defer until year 5 (the last opportunity) before exercising the option to start extraction. There are two reasons for that. First, in the original analysis the expected oil price increases more than the increase in cost over time, which has been dealt with in 5.4.3. Second, in case of low initial oil price, starting the project immediately might have a lower NPV than deferring a year and ending up either at a higher NPV or zero NPV in the case of no start-up. The expected NPV from deferring is higher due to limited downside.

It is of interest to see how net convenience yield and cost inflation affect the optimal exercise policy. Table 1 illustrates the optimal exercise policy at year 1, given oil price $45 at year -1 and for different values of the net convenience yield and cost inflation. In this case a binomial tree with two time steps per year is used in order to ease exposition. The table presents the NPV of investing immediately at year 1 and the NPV of deferring for different combinations of oil price realization, net convenience yield and cost inflation.

Each cell in Table 1 presents the NPV of investing immediately (i.e. Start now) to the left of the NPV of deferring. For each combination the highest NPV is marked with a grey shade and it can be seen that there will be different exercise policies. If both net convenience yield and cost inflation are equal to zero, then it will always be optimal to defer, which is what we concluded before in the sensitivity analysis. In case of cost inflation equal to 4% and net convenience yield equal to zero, it will instead be optimal to invest immediately at year 1 if the oil price is 45, 73.82 or 121.10. On the other hand, if a lower price is realized in Year 1, then it is optimal to defer as the best outcome may be to not start the project at all. If the net convenience yield is 4% and cost inflation is equal to zero, then it will be optimal to invest immediately unless the oil price is very low (16.72), in which case it is optimal to defer and consider not extracting at all. There is also a similar pattern for net convenience yield and cost inflation equal to 4%.

As can be seen from Table 1, there will be several factors that will affect whether it is optimal to start or defer production. In the real options literature, see e.g. Dixit and Pindyck [15], it is common to analytically derive a trigger price where it is optimal to invest but these models typically rely on restricting assumptions regarding the project. Our research is based on an existing oil project where it is required to use numerical methods to determine a value and thus no trigger price is determined in an analytical way. The pattern seen in Table 1 is the same as in analytical models, in that when oil prices are higher, this will trigger an investment. The actual trigger price will however be dependent on a combination of oil price, net convenience yield, cost inflation and the characteristics of the project. In the case of existing projects, one has to perform a detailed numerical analysis to find this out. We also conclude that incorporating flexibilities in strategic decision making would give government a higher bargaining power in petroleum licensing, and also ensure high returns from upswings while mitigating losses emanating from downswings in the oil markets.

Conclusion

The upswings in crude oil prices between 2004 and 2013 renewed interests of international oil companies (IOCs) in Africa's vast oil reserves. During this period, Uganda with an estimated 1.4 billion barrels of economically viable reserves, joined the list of prospective oil producers. In 2012 and 2016, the government of Uganda effected its first PSAs with three IOCs. The erratic downswings of crude oil prices between mid-2014 and 2016, however, awakened concerns about whether investment in extracting Uganda's oil can deliver sufficient returns to all parties amidst periods of high crude oil price volatility.

Inherent in the PSA, these IOCs have a number of potential real options. Our study addresses the valuation of the option to defer production, as we consider this most suited as for the analysis of undeveloped reserves. Following Cox et al. [12], we establish binomial lattices to quantify the real options values, and model the stochastic oil price process as GBM. Our study contributes to the scanty literature on real options valuation of Africa's oil projects, as most existing studies are focused on developed reserves in high-income countries.

Our results from the base case analysis show that deferring production by another year adds value of $0.9 billion to the oil project and it is thus optimal for the IOCs to defer production. The positive option value partly emanates from our assumption that management is able to defer expending the remaining OPEX and CAPEX to the subsequent year.

We further analyse the sensitivity of the option value to crude oil price, crude oil price volatility, net convenience yield and cost inflation, and illustrate combinations when it is optimal to start production now or wait, respectively. The value of the option to defer production particularly increases at lower crude oil prices amidst higher crude oil price volatility. In this case, the static NPV is comparably low and further reductions in the oil price may result in a project with negative NPV implying that the value of the project is undervalued by the traditional DCF approach. When the rate of net convenience yield and oil price are high, the value of the option is lower and becomes worthless at a critical price of $65 per barrel of crude oil. At low oil prices, increases in cost inflation result in rejection of the project, as both static and expanded NPV reduce to negatives. When cost inflation is equal to 4% and net convenience yield is equal to zero, the option to defer is worthless at oil prices of $45 and above. We reach the same conclusion when the net convenience yield is 4% and cost inflation is equal to zero.

In this research it is shown using data from an existing oil project case that the option values and optimal exercise policies, i.e. invest or defer, are highly dependent on external factors like oil price, net convenience yield, inflation and volatility. Further numerical analysis can be performed to identify the trigger price for each combination of factors. However, in general, trigger prices will also be affected by project characteristics like investment outlays and oil production profile and thus differ between projects. In case of another project, the analysis must be carried out based on that project's data in order to determine the correct trigger price.

There are some ways in which the research study can be extended. One way to extend the study would be to estimate the value of flexibility under a Production Sharing Agreement (PSA). In our model, the terms of the PSA are excluded and the value of the option to defer does not depict what accrues to the host government and the oil companies, respectively. Also, the study can be extended to analyse how the value of real options depends on the tax policy by examining the implications of the magnitude of taxes on the firm's incentives to invest. Another extension of the study would be to model oil prices as jump diffusion or mean-reverting stochastic processes.

Funding

This paper was supported within Norwegian Programme for Capacity Development in Higher Education and Research for Development (NORHED I) under the project- Capacity Building in Education and Research for Economic Governance, a partnership between Makerere University Business School and Norwegian University of Life Sciences. The funders had no role in study design; in the collection, analysis and interpretation of data; in the writing of the report; or in the decision to submit the article for publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.sciaf.2021.e00868.

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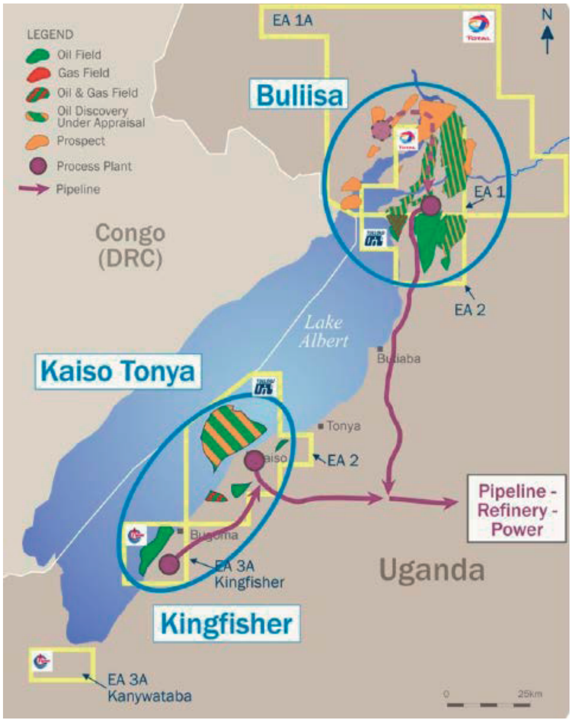
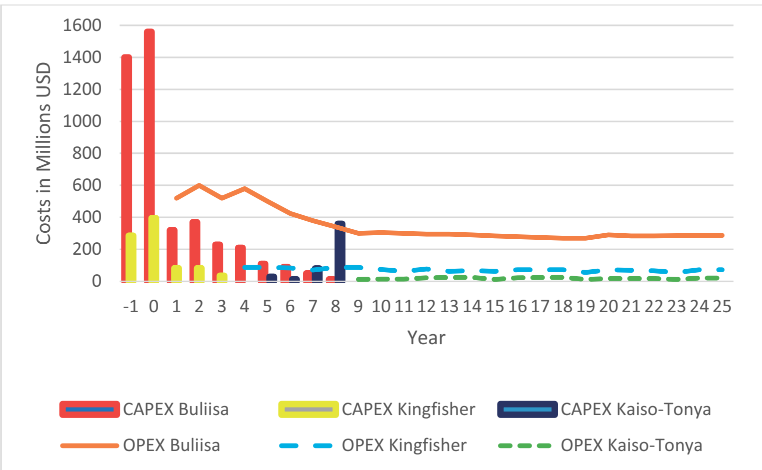
Appendices

A. Petroleum discoveries in the Albertine region



Source: Tullow Oil PLC (2012)

B. Costs profile (CAPEX and OPEX) for the three fields (Million US Dollars per year)



C. Supplementary tables

Table C.1: Expected future oil prices $E[P_{j,m,n,}]$ in Year 2-5, given $P_{j,1}$, i.e oil price realization *j* in Year 1.

	Oil price realization in binomial tree When m = 1	Expected oil price in USD given oil price realization in Year 1			
j	Year 1	Year 2	Year 3	Year 4	Year 5
1	149.94	153.56	157.28	161.08	164.98
2	93.36	95.62	97.93	100.30	102.72
3	58.13	59.54	60.98	62.45	63.96
4	36.19	37.07	37.97	38.89	39.83
5	22.54	23.08	23.64	24.21	24.80

Table C.2: Expected annual revenue in the first five years respectively, if production begins in Year *m*=1 at oil price realization *j* (in billions of US Dollars)

Oil price realization *j*	Year 1 (n=0)	Year 2 (n=1)	Year 3 (n=2	Year 4 (n=3)	Year 5 (n=4)
1	4.6	10.9	13.3	13.8	13.8
2	2.9	6.8	8.3	8.6	8.6
3	1.8	4.2	5.2	5.4	5.4
4	1.1	2.6	3.2	3.3	3.3
5	0.7	1.6	2.0	2.1	2.1

The expected revenue from the first 5 years when production starts at Year 1 is shown in Appendix C.2. The revenue is obtained by multiplying the oil production (from Kingfisher and Buliisa oil fields) presented in Figure 2 by the corresponding expected oil prices shown in Appendix C.1.

Table C.3: Expected net annual cashflows in the first five years if production begins in Year 1, m=1 at oil price realization j (in billions of US Dollars)

Oil price realization j	Year 1 (n=0)	Year 2 (n=1)	Year 3 (n=2	Year 4 (n=3)	Year 5 (n=4)
1	3.6	9.7	12.4	12.9	13.1
2	1.8	5.6	7.4	7.7	7.9
3	0.7	3.0	4.2	4.5	4.7
4	0	1.4	2.3	2.5	2.6
5	-0.3	0.4	1.1	1.2	1.4

Table C.4: Expected present value of remaining future net cashflows at different points in time during the first five years, if production begins in Year 1 at oil price realization j (in billions of US Dollars)

Oil price realization j	Year 1 (n=0)	Year 2 (n=1)	Year 3 (n=2	Year 4 (n=3)	Year 5 (n=4)
1	150	150	144	135	125
2	90	90	87	81	75
3	52	53	51	48	44
4	29	29	29	27	25
5	14	15	15	14	13

Table C.5: Option values in the project in Millions of USD at varying crude oil prices and volatility rates

		Option value (MUSD) Volatility rate p.a.							
Oil price	Static NPV (MUSD)	*15 %*	*20 %*	*25 %*	*30 %*	*35 %*	*40 %*	*45 %*	*50 %*
25	1 066	877	903	994	1 169	1 432	1 743	2 120	2 496
30	6 420	876	885	934	1 050	1 251	1 526	1 835	2 223
35	11 774	876	879	906	984	1 135	1 363	1 661	1 989
40	17 127	876	877	892	948	1 068	1 257	1 503	1 833
45	22 481	876	876	885	923	1 012	1 173	1 408	1 677
50	27 835	876	876	881	908	982	1 118	1 314	1 583
55	33 188	876	876	879	898	952	1 064	1 248	1 499
60	38 542	876	876	878	891	938	1 036	1 199	1 415
65	43 896	876	876	877	887	924	1 010	1 151	1 350
70	49 249	876	876	877	884	912	983	1 108	1 308
75	54 603	876	876	876	881	906	968	1 085	1 266

Table C.6: The project's expanded NPV and static NPV at varying net convenience yield rates

Net present value in MUSD at 35% volatility Net convinience yield								
Oil price	NPV (000')	*-6 %*	*-4 %*	*-2 %*	*0 %*	*2 %*	*4 %*	*6 %*
25	Expanded NPV	44 841	25 000	11 614	2 498	-3 074	-6 762	-9 439
25	Static NPV	29 063	16 648	7 671	1 066	-3 880	-7 649	-10 570
45	Expanded NPV	100 496	64 652	40 308	23 493	13 717	6 931	1 715
45	Static NPV	72 875	50 528	34 370	22 481	13 578	6 793	1 535
65	Expanded NPV	156 186	104 395	69 190	44 820	31 062	21 261	13 676
65	Static NPV	116 686	84 408	61 069	43 896	31 035	21 236	13 641

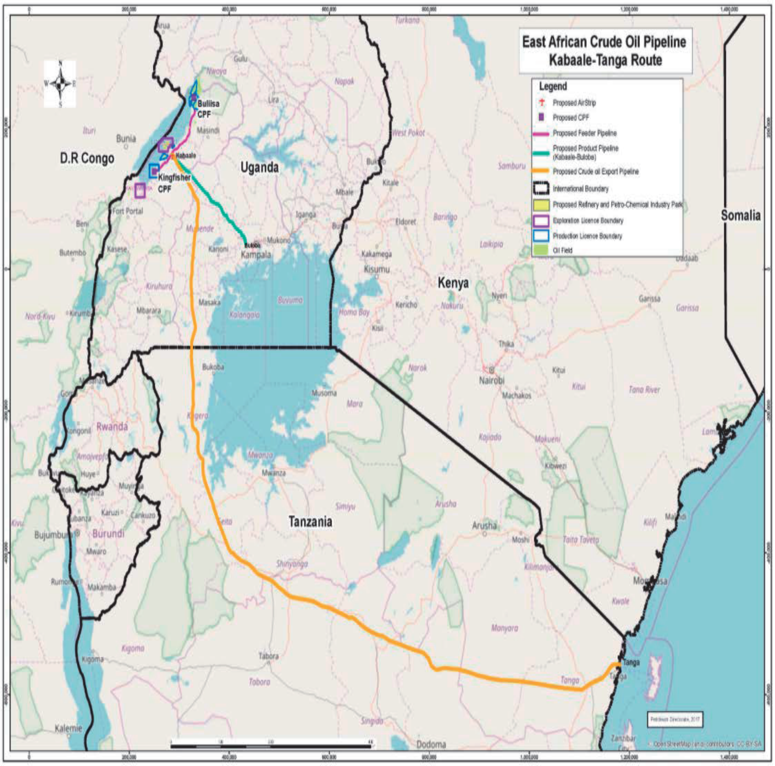
Table C.7: The project's expanded NPV and static NPV at varying cost inflation rates

Net present value in MUSD at 35% volatility

		Cost inflation p.a		
Oil price	NPV	*0 %*	*2.42 %*	*4 %*
25	Expanded NPV	2 498	175	-1 439
25	Static NPV	1 066	-1 112	-2 990
45	Expanded NPV	23 493	20 704	18 787
45	Static NPV	22 481	20 303	18 425
65	Expanded NPV	44 820	41 875	39 938
65	Static NPV	43 896	41 717	39 839

D. Development of the East African Crude Oil Pipeline

Figure D.1: East African Crude Oil Pipeline Route



Source: Petroleum Authority of Uganda (2019)

In May 2017, the governments of Uganda and Tanzania signed a \$3.5 billion agreement for the construction of the East African Crude Oil Pipeline (EACOP) that will transport Uganda's crude oil to Tanzania's Tanga sea port for export to the rest of the world. The EACOP has an export flow rate capacity of 216,000 barrels per day; and a length of 1,445km crude oil pipeline beginning from Kaabale, Hoima district, to Chongoleani in Tanga, Tanzania. Due to the waxy nature of Uganda's crude oil, with an average pour point of 40^0C, the 24-inch diameter pipeline will require heating along the entire route. The EACOP project will be financed through debt and equity from the joint venture partners and national oil companies of the two countries, with Total E&P Uganda as the lead investor.

E. Estimation of Volatility

We estimate the volatility from the annualised standard deviation of monthly log-returns on monthly Bonny Light spot prices. This approach is appropriate since we assume that the oil price series follow a GBM stochastic process with constant volatility. Taking price data from n+1 periods (156 months, in our case), then the estimate for historical volatility is given by;

$$\sigma = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_i - \bar{R})^2}}{\sqrt{\tau}} \qquad \text{(eq. E.1)}$$

Where monthly log-returns in period i, $R_i = InP_{t+1} - lnP_t$, $\bar{R}$ is the sample mean for all R_i and τ is the total length of each period in years (which is 12 in this case). The numerator is the sample standard deviation of the R_i and the denominator is used to annualise this estimate.

Table E.1: Descriptive statistics of Monthly Bonny Light crude oil prices

Summary Statistics		**Significance Test**	**Test**	**P-value**
		P-value	White-noise	0.037965
Mean	-0.00022	0.48853	Normal	7.83E-42

			distribution	
Annualised mean	-0.00264		Arch effects	0.286594
Standard deviation	0.096739			
Annualised Standard deviation	0.335114			

Table E.1 reports the descriptive statistics used to analyse whether the Bonny Lights returns. The annualised average returns stands at 0.264 percent, the lowest and highest returns recorded are -172.97 percent and 64.19 percent. The historical volatility is 33.5 percent and is computed by multiplying the standard deviation by the square root of 12.

Chapter 3: Real options valuation of a Production Sharing Agreement – Conflicting strategies between government and IOCs?

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Working paper

Real options valuation of a Production Sharing Agreement – Conflicting strategies between government and IOCs?

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Abstract

Although real options valuation accounts for the embedded flexibilities and uncertainties of oil projects, most existing literature relies on the traditional discounted cashflow approach to evaluate oil contracts. This study applies real options model and provides a numerical example on how to assess the effectiveness of a production sharing agreement (PSA) design. We specifically construct binomial lattices and apply the model to analyse Uganda's PSA with data on two oil field projects. We consider that the IOCs and government can exercise the option to start production immediately or to defer production. We proceed to explore how the value of option to defer production and optimal investment strategy change with different variations to project parameters, under different oil price scenarios. The study also examines whether the parties have aligned or conflicting optimal strategies, arising from variations in the contract elements. From our base case analysis, we find that the government is willing to start the project at a lower critical oil price than the IOCs, since the government experiences a positive expanded NPV at lower oil prices than the IOCs do. Sensitivity analyses of each party's expanded NPV to changes in the crude oil price, volatility of the oil price, net convenience yield and cost oil limit, are undertaken. Overall, the results suggest that there may be conflicting optimal strategies between the IOCs and the government when it comes to realizing the project. The conflicting strategies emanate in particular from the design of the cost oil function and expected oil price realizations.

Keywords: Production Sharing Agreement, Oil price uncertainty; Real options; Binomial lattices; Option to defer; International Oil Companies; Ugandan oil project

JEL Classification: C61; D81; E22; G11; H32; K32; L71; Q02

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1. Introduction

The past two decades have been characterised by high crude oil price volatility that has influenced global oil investments. Particularly, the oil price up swings between 2003 and 2014 attracted new discoveries and an ongoing search for oil in various African countries. During this period, several host governments initiated oil contracts with international oil companies (IOCs) as a key component in the overall regulatory framework for upstream oil operations (see Graham and Ovadia, 2019). There are two key motivations for pursuing these oil contracts: First, high costs, risks and uncertainties are more pronounced in resource-endowed developing countries that are venturing into oil production for the first time due to their lack of technical know-how, technology, and capital. Second, oil contracts serve the purpose of explicitly defining resource ownership, risk bearing, payments allocated to each party and how to resolve any issues, should they arise. The effectiveness of an oil contract is thus assessed based on its ability to maximise returns for both the host government and the companies, and how the government can increase its share of economic rent without distortions in exploration, production and development activities.

Production Sharing Agreements (PSA) have become the popular choice of contract type across Sub-Saharan Africa (see Graham and Ovadia, 2019). Anchored in a PSA, the oil resource remains state-owned and controlled by the host government, while the IOC bears all the risks and costs of exploration, development and production. After oil is extracted, the IOC recovers its costs, as per the terms of cost recovery, and receives a share of the remaining revenues, so-called profit oil. The host government, in turn, receives royalties and taxes levied on the IOCs and the remaining share of the profit oil. The design of a PSA thus has a critical impact on the investment decisions of the IOCs and host governments. For the IOCs, a PSA may be a disincentive to investment if the contract terms are designed in a way that channels a significant share of the project cashflows to the host government. Whereas, for the host government, it is imperative to design a PSA in a way that the resultant cashflow shares maximise state revenue while incentivising IOCs to invest. It is therefore essential to assess how a PSA design influences the optimal investment strategy from perspectives of both IOCs and the government.

In this study, we apply the real options approach to examine the implications of the PSA contract design to the government and IOCs, given the prevailing risk factors that influence the project. Oil projects command high costs that are expended over long contract periods. This exacerbates the risks of oil investments due to the high uncertainties faced over a project lifetime. The uncertainties that pervade oil projects are in form of ambiguities about; future oil price volatility, expenditures during the long lifetime of the project, economically viable oil reserves, the amount of oil produced from proven reserves and future world demand for oil. Upon the initiation of a PSA, an IOC bears all the risks pertaining to the project while the government retains ownership and management over the oil fields. Embedded in the oil contract, the parties also obtain the right to exercise different managerial flexibilities[1]. This enables the respective parties to strategically capitalize on revenue windfalls arising from best-case scenarios of the uncertainties while mitigating the risks associated with low revenues during the worst-case scenarios. Investing in an oil project is thus a real options investment problem.

Since the parties have somewhat different interests, their strategic investment decisions may be aligned or conflicting, depending on how the risk factors impact the value of the oil project. Thus, our study contributes to the literature by applying real options valuation to account for flexibilities and uncertainties while assessing the effectiveness of a PSA design. We specifically consider that the IOCs and government can exercise the option to start production immediately or to defer production. We proceed to explore how the value of option to defer production and optimal strategy (to defer production versus to start production immediately) change with different variations to project parameters, under different oil price scenarios. Lastly, the study examines whether the parties have aligned or conflicting optimal strategies, arising from variations in risk factors and contract elements.

Our results from the base case analysis show that the critical oil prices required to start oil production differ for the parties which suggest that there may be conflicting optimal strategies between the IOCs and the government when it comes to realizing the project. The sensitivity analyses of each party's expanded NPV to changes in the crude oil price, volatility of the oil price, net convenience yield and cost oil limit, confirm these findings. Specifically, the government has a strong preference to defer production, except in the cases when prices are low, and the project approaches the expiration of the defer option. To the IOCs, this is the reverse. The conflicting strategies particularly arise from the design of the cost oil function and expected oil price realizations.

[1] For example, parties to an oil contract have the option to defer investment, expand or contract the oil project, abandon for salvage, farm-out of a joint venture or switch to another plan.

The rest of the paper is organised as follows: Sections 2 presents the review of literature. Section 3 describes the case study and the structure of the PSA. Section 4 presents the empirical model specification. The results are discussed in Section 5. Lastly, Section 6 concludes.

2. Literature review

Numerous studies apply traditional discounted cash flow methods to assess how various contract elements influence the NPV of oil projects. However, such traditional valuation methods fail to recognise managerial flexibilities and uncertainties embedded in oil projects and thus underestimate the project value. In the real world of uncertainties, the value generated by managerial flexibilities increases the value of an investment project, such that the true expected project value is the expanded NPV. The project value from real options valuation thus consists of two components: the traditional static NPV of expected cash flows, and the value of the flexibility component (Trigeorgis, 1996). That is, the expanded NPV is simply the sum of the static NPV and the option value.

We, however, present an extensive literature review of previous studies that apply traditional discounted cashflow methods, to attain an insight into the implications of oil contract designs to the respective parties. Bindemann (1999) was the first to undertake an economic analysis of PSA contract terms and how the contract elements affect the returns from the project. The study found that variations in the profit oil sharing cause significant changes in the Internal Rate of Return (IRR) and NPV. They also found that the higher the royalty, the earlier production will be stopped. However, the study shows that if the oil price increases, the IRR increases significantly despite the royalty payment.

Liu *et al*. (2012) investigated the effect of different oil contract elements on the net present value (NPV) by applying Monte Carlo simulations. To achieve this, they model the PSA in 11 different scenarios by changing the value of each contract element. They compared results of Geometric Brownian Motion (GBM) and Mean reversion stochastic process (MRP) for the oil price. The authors found that the probability density of NPVs does not change greatly with an increase of royalty. They, however, found that the contract would result in a better return if oil price followed a GBM process. They also showed that the effect of cost oil on NPV is significant, especially under MRP and that profit oil had a significant effect on NPV under both GBM and MRP.

Cheng *et al*. (2018) apply Monte Carlo simulations and Value at Risk (VaR) to assess the influences of different oil contract elements on the NPV of the project. The paper considers an international oil project to study NPV frequency histogram of the project under a royalty contract, PSA and a service contract. Their study finds that, under a royalty contract, the oil price has the greatest impact on NPV, the royalty rate ranks second; estimate of proven reserves also has a certain impact, while OPEX per barrel has a small impact. Under a PSA, the cost recovery rate and oil price volatility were found to have the greatest influence on NPV. Lastly, under a service contract, the randomness of oil price has the greatest impact, estimate of proven reserves ranks second; while OPEX per barrel and compensation rate have small impacts.

Farimani *et al*. (2020) study the Iranian Petroleum Contract and examine the sensitivity of each party's profitability and takes on changes in oil price, CAPEX, OPEX and the remuneration fee. For each sensitivity case, they vary the parameters of interest, *i.e*., oil price, remuneration fee, CAPEX and OPEX, from their base case by 50–200%, while holding other parameters constant from the base case. They found that, as the oil price varies from 50% to 200% of the base case ($65/bbl), both the government and the contractor's NPVs increase. Their results show that, when the base remuneration fee increases, the NPV of the contractor increases and that of the government decreases. Additionally, they found that increases in the project expenses (CAPEX or OPEX) have a larger negative impact on the contractor's NPV than on that of the government.

Our study contributes to the literature by applying real options valuation to assess the effectiveness of a PSA. To achieve this, we employ a binomial lattices model to determine the optimal investment decisions with respect to the IOCs and the government, while accounting for oil price uncertainties. To generate numerical results, we apply our model to a case study - Uganda's oil project in the Albertine region. As far as we know, this is the first study to apply real options valuation in assessing whether IOCs and the government have different or aligned interests in the context of a PSA.

3. Case study and Structure of PSA

3.1 Description of Uganda's oil project and PSA Structure

Our study is applied to Uganda's operational PSA. In 2006, Uganda joined the list of prospective oil producing countries with 6.5 billion proven oil reserves in the Albertine Graben of which 1.4 billion barrels are economically viable for extraction (Petroleum Authority of

Uganda, 2022). Based on these discoveries, Uganda's production capacity is projected to be between 200,000 and 250,000 barrels of oil per day over a period of 25 years (Ward and Malov, 2016). In 2016, the government of Uganda finalised a PSA with three IOCs. The three IOCs; Tullow Oil, Total Energies and China National Offshore Oil Corporation (CNOOC) would operate under a joint venture to operate three oil fields - Buliisa, Kingfisher and Kaiso-Tonya. In 2022, Tullow oil finalised the sale of its entire stake in the oil project to Total Energies, bringing the project ownership to 56.67% for Total Energies and 28.33% for CNOOC. The state company Uganda National Oil Company (UNOC) is mandated to manage the country's commercial interests in the oil sector and holds the remaining 15%, as per the PSA terms. A crude oil pipeline will transport oil to Tanzania for export. For more details about the fields and the pipeline, see Section 4.3.

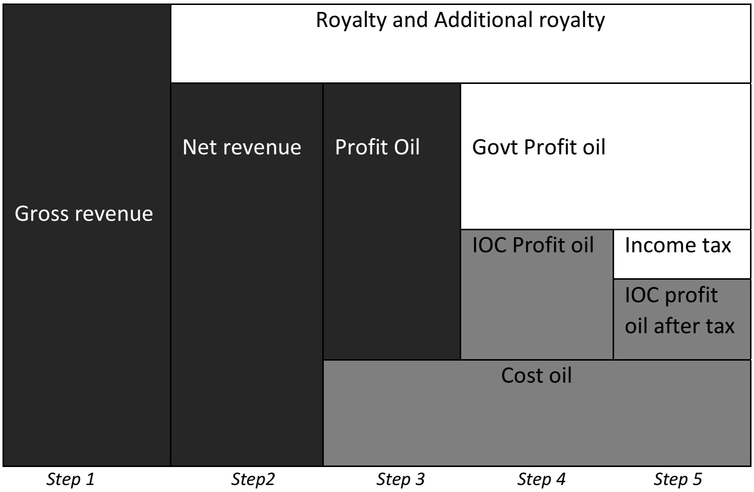
Figure 1 presents a visual illustration of how revenues are distributed between Uganda's government and the IOCs. According to the PSA, the IOCs incur all expenditures towards exploration of oil, development of oil fields and the crude oil pipeline, and production of oil. The IOCs are required to meet the 15% UNOC cost commitments to each of the projects. The IOCs are entitled to recover these costs from the cost oil, as per the PSA terms. Other project costs such as infrastructural development of roads and an airport are incurred by the government. A detailed description of the costs and cost responsibilities is presented in 4.3.1.

Upon production and sale of oil, the IOCs pay royalties and additional royalties to the government. The royalty rates are based on daily production by the IOCs in a particular month while Additional royalty rates are dependent on the cumulative recovered reserves. The royalty and additional royalty payments are computed using a sliding scale approach (see Tables A1 and A2).

The IOCs recover their costs, from oil field revenues net of the royalties and additional royalties, with a cost recovery limit of 60%. Any unrecovered costs beyond the limit are carried forward to the subsequent period until all costs are fully recovered. Ringfencing restrictions also apply to cost recovery such that costs incurred by the IOCs on one oil field project cannot be recovered from net revenues accrued from another oil field project.

Profit oil is the remainder of the net field revenues after the deduction of royalties, additional royalties and cost recovery. The profit oil is shared between the IOCs and the government according to the volume of production and is also structured based on a sliding scale (see Table A3). A corporate income tax of 31 percent is levied on the IOC's share of the profit oil. Step 5 of Figure 1 shows the distribution of net field revenues among the parties. The IOCs recovers its costs in form of cost oil and receives a share of the profit oil. The government, in turn, receives royalties, additional royalties, the remaining share of the profit oil, and taxes levied on the IOCs.

Figure 1: Revenue allocation between government and IOCs



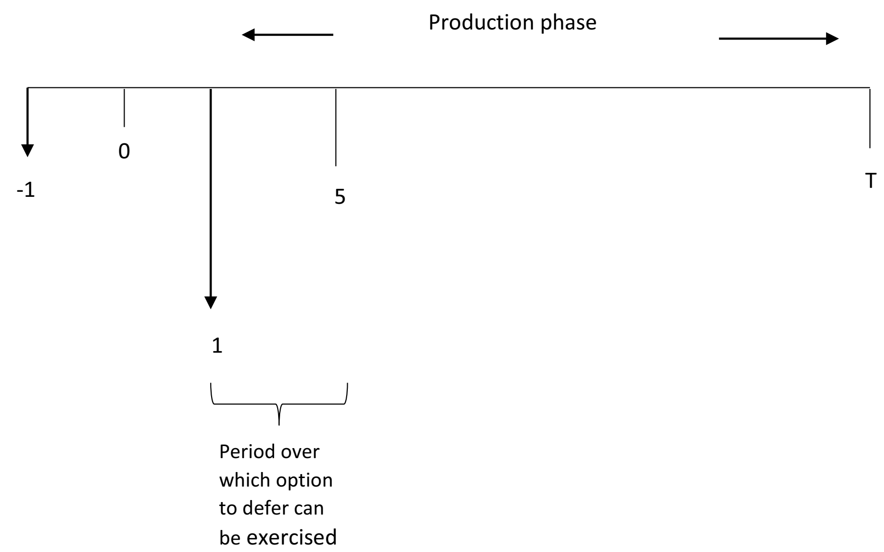
Source: *Authors' compilation based on Kasriel and Wood (2013)*

Black=unallocated; White= to Government (Govt); Grey= to IOCs

4. Empirical Model specification: Binomial lattices Model

Our study applies real options valuation to analyse the implications of the PSA contract design to the government and IOCs, while accounting for the flexibility of deferring production that arises from oil price uncertainty. We specifically apply a binomial lattice model based on Cox *et al*. (1979) to estimate the value of the project and resulting net cashflows for each party. We modify the binomial lattices model as presented in Abigaba *et al*. (2021) and extend the application of the model to the economic analysis of a PSA. We construct binomial lattices to estimate a party's option to defer the production by one more year. The IOCs are licensed to produce oil for 25 years and are eligible for contract extension of 5 years. We thus consider that the IOCs and the government can evaluate whether to begin production immediately or to defer production by another year. The option to defer production by another year can be exercised until the 5th year, upon which the option expires (see Figure 2). If the IOCs begin oil production in a given year, oil is produced with no disruptions throughout the 25 years of the production contract tenure.

Figure 2: Generic timeline of oil project



T=25,26,27,28,29 depending on when production begins

Source: *Adopted from Abigaba et al. (2021)*

4.1 GBM and risk neutrality assumptions

The binomial lattices model is established based on assumptions of Geometric Brownian Motion (GBM) of oil price process and risk neutrality, such that;

$$dP = \mu P dt + \sigma P dW \qquad (1)$$

Where $P$is the crude oil spot price at time, t, dW is the increment of the Wiener process with $dW = \epsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$. μ is the drift rate and σ is volatility. μ and σ are taken as constants.

We further assume risk neutrality such that at any time interval ($t, t + \Delta t$) along the lattices, the oil price (P) and thus the project value (V) behave as a binary random walk process. The values are affected by an up movement multiplier u with a probability of q or fall movement by the multiplier d with a probability of $1 - q$, at the end of the time interval $t + \Delta t$. The risk-neutral probability q is calculated as;

$$q = (exp(r_f \Delta t) - d)/(u - d) \quad (2)$$

where r_f is the risk-free rate, and

$$u = exp(\sigma \Delta t) \quad (3)$$

$$d = 1/u. \quad (4)$$

The parameter domains are such that; $u > 1, d < 1$, and $d < 1 + r_f < u$. In addition, the risk-neutral probabilities q and $1 - q$ are constant at all steps of the lattices since we take volatility, σ, to be constant in our model.

4.2 Establishment of binomial lattices for oil prices and cashflows

In this study, the value of the flexibility is determined by comparing the values of the project with and without the flexibility in deferring oil production. We assume that Uganda is a small potential oil producer and will have no influence on the crude oil price. We further assert that the expected oil prices do not influence the oil production rate, once production has started. Thus, the oil production profile for each field over the 25 years is fixed (see figure C1 for the forecasted oil production). We begin by generating binomial lattices for oil prices by assuming a GBM stochastic price process. For each binomial lattice of oil prices, with the given oil production profile, a cash flow lattice is generated. As prior described in Section 3, the royalty, additional royalty and profit oil rates are determined by the daily oil production in a given year, on a sliding scale basis. The net cash flows for the oil fields are thus computed as per the official PSA for the oil project. The lattices of the project values are calculated node by node from the expected present value of the net cash flows. Lastly the expanded NPVs are computed. The expanded NPV reflects the project value from both the expected future cash flows and the strategic option value of deferring oil production which accounts for the uncertainties and flexibilities embedded in the oil project.

4.2.1 Binomial lattices for oil prices

First, the binomial lattices are generated for the oil prices for the years -1 to 1. The oil price lattices indicate how possible future oil prices could evolve. Following the binomial option-pricing model by Cox *et al*. (1979), as described in Section 4.1, in each time interval $(t, t + \Delta t)$, the oil prices for the years 0 to 1 could increase from P_t to $P_t u$ with probability q or decrease to $P_t d$ with probability $1 - q$. The up (u) and down (d) multipliers are determined from equations (3) and (4), while the risk neutral probability q is computed from equation (2). Thus, beginning with an initial oil price at year -1, P_{-1}, the expected oil price in each node of the binomial lattices for the years 0 and 1 are determined (see Abigaba *et al.*, 2021 for a detailed description). At year 1, the oil prices are observed, and the parties make strategic decisions on whether to start oil production immediately or defer oil production for another year. In line with the theory of option-pricing, the expected return on a barrel of oil is assumed to be risk-free such that the expected future oil prices for the years 2-25 are assumed to grow at a rate equivalent to the risk-free rate;

$$E\left[P_{j,m,n}\right] = P_{j,m} exp\ (r_f n) \tag{5}$$

Where $P_{j,m}$ represents all the possible oil price realizations, j, in year $m (m = 1,2, \dots \dots ,25)$. The formula in equation (5) is applied to all the remaining years of production, n $(n = 0,1, \dots \dots ,24)$ (see Abigaba *et al.*, 2021 for details).

4.2.2 Binomial lattices for net cash flows

We modify the net cash flow binomial lattices as presented in Abigaba *et al.* (2021) and extend the application of the model to the economic analysis of a PSA. For the IOCs and the government, the expected net annual cashflows are computed based on the year that oil production starts, how future oil prices evolve over the first five years (2-6), and the embedded PSA elements. The details of the formulae for computing net cash flows are presented in Appendix B.[2] Once production begins, the terms of the PSA are initiated. With reference to Figure 1, the oil project yields gross field revenue from the extraction and sale of oil. The IOCs then pay royalties and additional royalties to the government. Before the profit sharing is effected, the IOCs recover their costs as per the cost oil limit and the unrecovered costs are carried forward and recovered in the subsequent year. After subtraction of royalty, additional royalty and cost oil, the remainder of the net field revenue is the profit oil, which is shared between the government and the IOCs. The IOCs are required to pay an income tax on their share of profit oil.

For each expected price realization *j* in year *m* with *n* remaining years of production, the expected net cash flows $C_{j,m,n}$ of the IOCs and the government are expressed in the equations (6) and (7). The detailed derivations of these two equations are shown in Appendix B.

[2] In the implementation of the numerical model, the principal of ring-fencing as per Uganda's PSA is applied such that we construct lattices and evaluate optimal decisions for each field separately. As mentioned above, the royalties, additional royalties and profit oil shares are computed using a sliding scale approach (see Appendix A fore details), so the expressions in the numerical model are more complicated than shown in the equations (6) and (7).

$$C_{j,m,n}^{IOC} = 365[(1-\alpha)[\theta(Q_m)P_m(1-\tau_m-a_{rm})Q_m] + (1 - \theta(Q_m) + \alpha\theta(Q_m))CO_m] - OPEX_m - CAPEX_m \quad (6)$$

$$C_{j,m,n}^{GVT} = 365[P_mQ_m - (1-\alpha)\theta(Q_m)[P_m(1-\tau_m-a_{rm})Q_m] + (\theta(Q_m) - \alpha\theta(Q_m) - 1)CO_m] \quad (7)$$

where Q_m is the daily oil production in a given year m,

τ_m is the royalty rate, paid on the gross total daily production in terms of barrels of oil per day for each field,

a_{rm} is the additional royalty rate, taken as a percentage of the value of the recovered reserves on the basis of gross total daily production in barrels of oil per day for each field,

$\theta(Q_m)$ is the IOCs share of the profit oil, and is expressed as a function of daily oil production, as stipulated by the PSA,

α is the corporate income tax paid on the IOCs' operating profit,

CO_m is the cost oil which is determined from:

$$CO_m = min(C_m + max(C_{rec,m-1} - CO_{capm-1}, 0), CO_{capm}),$$

in which C_m is the sum of OPEX and CAPEX incurred by the IOCs in year m; $C_{rec,m-1}$ is the recoverable costs from the previous year m. $CO_{capm}$is the maximum amount available for cost recovery and is calculated as a proportion of net field revenue and a fixed cost recovery rate of 60%.

In equation (6), $C_{j,m,n}^{IOC}$ conveys the expected net cashflow of the IOCs in a given year m, which is determined by deducting the annual OPEX and CAPEX from the IOCs' PSA revenue share (accruing from profit oil after tax and cost oil). Whereas, Equation (7) expresses the government's expected net cash flow, $C_{j,m,n}^{GVT}$ in a given year m. The government's net cashflow is equivalent to the sum of its PSA revenue earned from royalties and additional royalties, profit oil and corporate tax revenue.

Based on the computed net cashflows, we proceed to calculate the expected present value of the net cash flows ($S_{j,m,n}$) in year m for each decision maker i ($i = IOCs, government$). In a general case, $S_{j,m,n}^{i}$ is computed from the sum of the present value of all future expected cash flows after year $m+n$ and the cash flow generated during year $m+n$. Taking $m = 1$ and $n = 0$ for instance, the present value of all future expected cash flows is given as,

$$S_{j,m,n}^{i} = C_{j,m,n}^{i} + S_{j,m,n+1}^{i}/exp\ (r_f 2\Delta t) \tag{8}$$

Which of these five expected values will be realized depends on the realization j of the oil price at year 1.

The presence of the option to defer production at each node changes the project values. Using backward induction, the project values with the option to defer at year m and price realization j are determined starting with year 6. At each terminal node of year 6, at which the option to defer production expires, the values of the project at each node are computed as,

$$V_{j,m}^{i} = max\{S_{j,m,0}^{i}; 0\} \qquad m=6 \tag{9}$$

Equation (9) means that at each terminal node, a party wants to start production when the present value of net cashflows, $S_{j,m,0}^{i}$ is above 0, otherwise the party wants to abandon the project.

For each node before the terminal nodes, the maximum value between the expected present values of deferring oil production and starting production immediately must be evaluated. This provides information about exercising the real option or keeping the option open. The value of the project, at each node for the years 2-5, is computed by taking:

$$\begin{aligned} V_{j,m}^{i} = max\{S_{j,m,0}^{i}; (q^2 V_{j,m+1}^{uu} + 2q(1-q)V_{j+1,m+1,}^{ud} \\ + (1-q)^2 V_{j+2,m+1}^{dd})/exp\ (r_f 2\Delta t)\} \qquad m = [2..5] \end{aligned} \tag{10}$$

Equations (10) shows that at each node, each party is faced with the choices between starting at year m to receive $S_{j,m,0}$, or deferring production by another year. If the party holds the option to defer production by another year, they do not earn an immediate reward but will have the option in the following period worth $(q^2 V_{j,m+1}^{uu} + 2q(1-q)V_{j+1,m+1,}^{ud} + (1-q)^2 V_{j+2,m+1}^{dd})/ exp\ (r_f 2\Delta t)$.

Finally, using backward induction, the expanded NPV for each party at year -1 is computed as,

$$\begin{aligned} ENPV_{-1} = C_{-1,} + C_0/exp\ (r_f 2\Delta t) \\ + \big(q^4 V_{1,1} + 4q^3(1-q)V_{2,1} + 6q^2(1-q)^2 V_{3,1} \\ + 4q(1-q)^3 V_{4,1} + (1-q)^4 V_{5,1}\big)/exp\ (r_f 4\Delta t) \end{aligned} \tag{11}$$

The variables C_{-1} and C_0 typically represent development costs that are incurred during year -1 and year 0. At each node, the optimal strategy for each party is determined by comparing

NPV with no consideration of options (*i.e.* the static NPV) to the expanded NPV. This process is undertaken for each field.

4.3 Data and variable description

4.3.1 The production profile and cost parameters

The data used consists of the oil production profile, cost of development, CAPEX and OPEX of the Kingfisher and Buliisa oil fields, as projected over the period of 25 years (see Figs. C1 and C2). The Kaiso-Tonya oil fields are excluded from our analysis since they are comparably small, economically infeasible on their own and are considered a tie-in to the Kingfisher oil field. All these cost data were obtained from estimates by Ward and Malov (2016) and through interviews with officials at the Petroleum Authority of Uganda and Ministry of Energy and Mineral Development. All the production, cost and price data are adopted as used in Abigaba *et al*. (2021).

The CAPEX begins 5-7 years prior to production and 3-8 years after production has started, depending on the oil fields. The CAPEX includes all costs on development of oil production plants such as; expenditures on equipment, raw materials (e.g. steel and concrete), prefabrications, construction, engineering designs, project management, insurance and certification. The OPEX entails the cost of operating the oil production plants over their lifetime after development is completed and include costs towards labour for maintenance and operations, chemicals and fuels, spare parts, well servicing and other expenses to production. In our analysis, the cost estimates exclude sunk costs towards; land acquisition, contingency, Front-End Engineering design (FEED), Environmental and Social Impact Assessment (ESIA), feasibility studies and other studies that are completed before the development phase commences.

The Buliisa oil fields cover the EA2 North and EA1 blocks, North-East of Lake Albert, with eleven fields under development (see the map in Appendix A). Total is the operator with a share of 56.67%, while its partners CNOOC and UNOC hold 28.33% and 15%, respectively. The Buliisa fields hold the highest reserves, estimated at 819 million barrels of recoverable oil, with its production peak in its fourth year of extraction (see Fig.C1). The total CAPEX is estimated to be USD6.5bn, of which USD5.1bn will be expended in the first 7 years prior to production. The total OPEX is assumed to be USD8.7bn over the entire 25 years of the production phase.

The Kingfisher oil fields, operated by CNOOC, encompasses the EA3A Block, South of Lake Albert (see Appendix A) and is estimated to have 196 million barrels of recoverable oil, with expected peak production in its eighth year of extraction (see Fig.C1). The Kingfisher project ownership is such that CNOOC holds 28.33% while Total holds 56.67% and the remaining 15% is held by UNOC. The project's CAPEX is estimated to be USD1.5bn, 87% of which is spent in the 5 years prior to production and the rest in the first 3 years of oil production. The total OPEX expended over the 25 years of oil production is projected to be USD2bn.

An additional development cost to the oil project is the East Africa Crude Oil Export Pipeline (EACOP) that will transport the crude oil from Uganda's oil fields to the port of Tanga in Tanzania for export[3]. This pipeline will be constructed at a cost of USD3.5bn and operated through the EACOP Company[4] with shareholding from the Uganda National Oil Company (15%), the Tanzania Petroleum Development Corporation (15%) and the two oil companies; Total Energies (62%) and CNOOC (8%) (EACOP,2022).

The CAPEX and OPEX costs of the oil fields, and development costs of the EACOP are recoverable as per the terms of the PSA. As is the case for the oil field projects, the IOCs are required to incur all cost commitments of UNOC in the EACOP project during the development stage, which are deemed recoverable for the entire production phase, as per the PSA terms. Other project costs towards infrastructural requirements, such as roads and the Hoima international airport, are incurred by the government but are not recoverable, as per the PSA. These costs amount to USD9bn and are deducted from the government's total PSA revenue.

The data on the structure/terms of the PSA is obtained from the official PSA document as published on the official website of UNOC (2021).

[3] EACOP is a 1,443km crude oil export pipeline that will transport Uganda's crude oil from Kabaale, Hoima in Uganda to the Chongoleani peninsula near Tanga port in Tanzania. It will have a peak capacity of 246,000 barrels of oil per day. The first 296 km of EACOP are in Uganda and the remaining 1147 km are in Tanzania (EACOP, 2022).

[4] The shareholders and the EACOP Company entered into a Transport and Tariff Agreement which stipulates the terms and conditions for the EACOP to transport oil. According to the agreement, the EACOP Company will charge the owners of the oil a tariff of USD12.77 for each barrel of oil transported through the pipeline. The EACOP Company assumes all risks of transporting the oil from the terminal in Hoima, Uganda up until marine oil terminal in Tanga bay. The ownership of the oil remains with the Government of Uganda, Total Energies E&P Uganda, CNOOC Uganda and UNOC. In our analysis, the tariff costs are included in the OPEX of the oil fields.

4.3.2 Real options model parameters

Similar to Abigaba *et al*. (2021), the monthly historical spot prices of Nigeria's Bonny Light crude from January 2006 to December 2018 were used to compute the annualised volatility. The spot price of Bonny Light crude is chosen as a proxy for Uganda's crude oil over Brent and WTI crude because of its similar characteristics in terms of API gravity and sulphur content, as well as the geographical location. The price data is obtained from the website of the Central Bank of Nigeria.

The parameter values as applied to the base case, are presented in Table 1.

Table 1: Real options Parameter values

Volatility, σ	33.5%
Time step, Δt	0.25 year
Risk-free rate, r_f	2.39%
Up multiplier, u	1.2673
Down multiplier, d	0.7891
Risk-neutral probability of up-movement, q	0.4662
Risk-neutral probability of down-movement, $1 - q$	0.5338
Net convenience yield	0.0%
Initial oil price	USD60

The time step Δt is set to 0.25 year and the risk-free rate r_f is 2.39% per year. The risk-free rate corresponds to the US 3-month treasury bill rate (US Department of the Treasury, 2018) since all the project costs and revenues are expressed in US Dollars. Using the formulae in equation (2) and expressions of u and d in equations (3) and (4), the estimated volatility, time step and risk-free rate, we proceed to determine the rest of the parameters of interest. We also assume a net convenience yield of 0%.

4.4 Implementation of the model in Python

The Python program that computes the value of the PSA contract to the government and the IOCs is basically built around the steps shown in Figure 1.

First, the royalties and additional royalties are calculated from daily and cumulative production quantities. As the royalty rates are independent of the oil price, we begin by computing royalties for a unitary oil price, taking into account the royalty and additional royalty structures in Tables A1 and A2.

Second, the government's and the IOCs' shares of profit oil are calculated. The profit oil share is determined from daily production quantities on the basis of a sliding scale shown in Table A3, *i.e.*, also independent of the oil price.

Third, a binomial tree representing the development in oil prices is built and factored in. This binomial tree forms the basis for future oil prices after production is initiated, and the real option valuation procedure.

Fourth, for the whole production period after production is initiated and for different oil prices, the profit oil and how the profit oil is split between the IOCs and the government are computed. In order to do that, net revenues and cost oil are first estimated to calculate profit oil. Next, the profit oil is split between the IOCs and the government, using previously computed profit shares. The IOCs then pay tax on their share. Subsequently, the cash flows that go to the IOCs and government, respectively, are summed up for each year and the net present values of the project to all parties can be calculated. The most complex calculation in this step is determining the cost oil. The cost oil is constrained by the maximum cost oil limit and also depends on the oil price, unrecovered cost, production profile, and must be recalculated for different oil prices at each node in the binomial tree. If the cost oil is not sufficient for cost recovery by the IOCs, the balance is carried forward and recovered in the subsequent period.

Finally, a stochastic dynamic programming procedure is initiated to determine the optimal start/wait/abandon-decision and to determine the expanded NPV, which takes the real option value into account.

5. Results

5.1 Base case analysis

We use base case parameters to generate the real options valuations presented in Table 2. The crucial difference between real option valuation and a traditional discounted cash flow method is that the project owner has the right, and not the obligation to exercise the option at a particular time. Therefore, as the project progresses over time, the parties can revise their estimates of NPV and change their future investment decisions, based on new information about the project risks. From our results, for both parties, the static NPVs of the two fields at year -1 are found to be positive indicating that the oil project is viable. For both parties, the expanded NPVs are greater than the static NPVs, which implies that the consideration of deferring oil production is valuable to each party.

Table 2: Real Options Valuations in billions of US Dollars

		Kingfisher	**Buliisa**
IOCs	Expanded NPV	0.416	3.34
	Static NPV	0.383	3.24
	Option value	0.03	0.10
Government	Expanded NPV	2.31	29.20
	Static NPV	1.70	27.20
	Option value	0.61	2.0

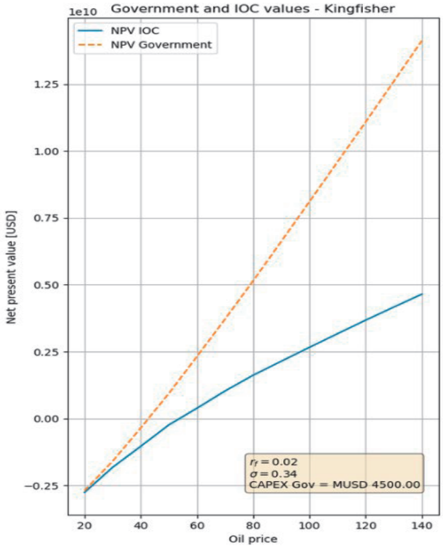
In our study, the additional value from the option to defer oil production emanates from the design of the cost oil function and the expected oil price realizations. On the one hand, from the perspective of the government, deferring oil production by another year minimizes the downside risks of negative cashflows due to high cost oil when the future oil price is high. On the other hand, it is valuable for IOCs to defer production to minimize the probability of revenue losses from cost oil when the future oil price is low. We discuss this in more detail in Section 6.

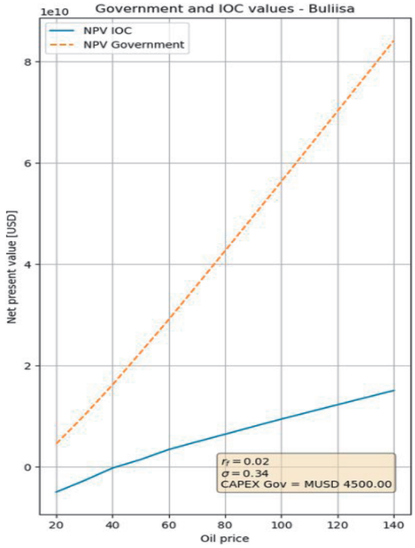
From our real options valuation, we also derive critical oil prices at which it is optimal for the parties to initiate oil production. The critical oil prices at which to start oil production, from both fields, differ for the parties. The critical prices for the Kingfisher field are USD52 and USD42 per barrel for the IOCs and the government, respectively. For Buliisa oil fields, the critical price for the government is significantly lower at USD16 compared to that of the IOCs (USD40). Collectively, the critical oil price for the entire project is USD18 for the government and USD42 for the IOCs. This clearly suggests that there may be conflicting interests between the IOCs and the government when it comes to realizing the project.

5.2 Sensitivity to changes in the oil price

The initial oil price has a direct influence on expected future oil prices and thus the expected project revenue flows, for each party, making it a key determinant of expanded NPV. The sensitivity of each party's expanded NPVs to changes in the initial oil price is depicted in Figures 3. For the two oil fields, as one would expect, a price increase results in major increases of expanded NPVs for both parties. At all oil prices, the expanded NPV of the government rises faster than that of the IOCs. Higher oil prices translate into higher expected royalty revenues and higher expected profit oil shares, which increase the government's expanded NPV. Because of the structure of the profit sharing, the government's profit share is significantly higher when the oil production is higher, which is the case for the Buliisa fields. This makes the Buliisa fields significantly more profitable for the government at all oil prices, as compared to the Kingfisher oil fields with comparatively lower production. The upward expanded NPV curve for the IOCs exhibits some kinks, which occur as the oil price passes through different tranches of royalty, additional royalty and profit oil share structures (see Kasriel and Wood, 2013). An analysis of the combined oil field projects reveals the same findings. The government's expanded NPV curve is linearly increasing with rising oil prices whereas that of the IOCs exhibits a slight kink when the oil price reaches USD60, and rises more slowly beyond this price.

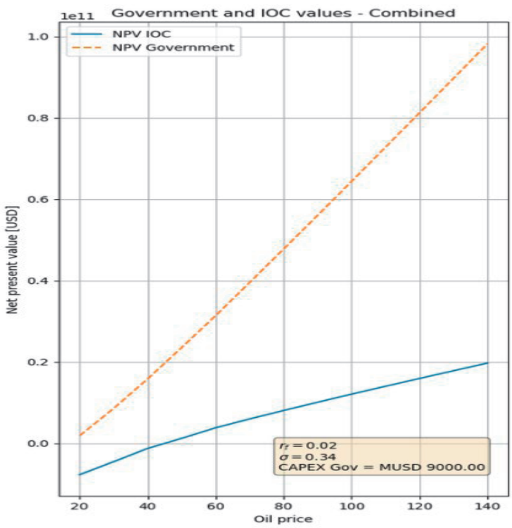
Figures 3: Sensitivity of each party's NPV to oil price changes





(a) Expanded NPVs of IOCs and government for the Kingfisher field

(b) Expanded NPVs of IOCs and government for the Buliisa fields



(c) Expanded NPVs for the IOCs and government when the fields are combined.

Note: *The figures show the sensitivity of each party's expanded NPV to changes in the initial oil price. The base case initial oil price is USD60 per barrel of oil. All other parameters are presented in Table 1. In Figures 3a and 3b, the parties' expanded NPVs are expressed in 10 billions of US Dollars. In Figure 3c, the expanded NPVs are in 100 billions of US Dollars. The oil price is in USD/barrel.*

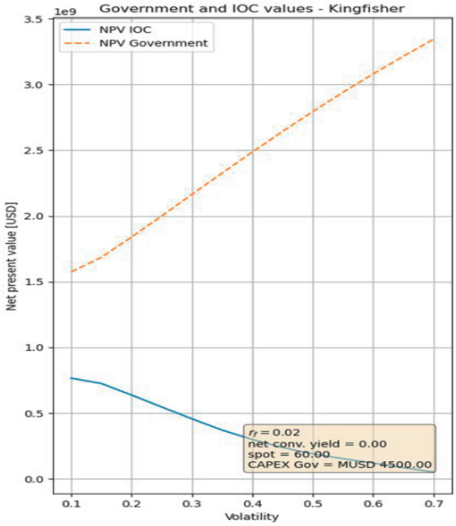
5.3 Sensitivity to changes in volatility rates

In order to illustrate the impact of changes in oil price volatility on the expanded NPVs of each party, a range of volatility rates from 10% to 70% are considered, as illustrated in Figures 4. The benchmark initial oil price is USD60. We observe that, for both fields, the government's expanded NPVs increase with rising oil price volatility. This is in line with conventional real options theory. High volatility increases the project value because it raises the possibility of windfall returns on high price outcomes, without increasing the probability of large losses due to low price outcomes (Dixit & Pindyck, 1994; Trigeorgis, 1996; Fleten *et al.,* 2011).

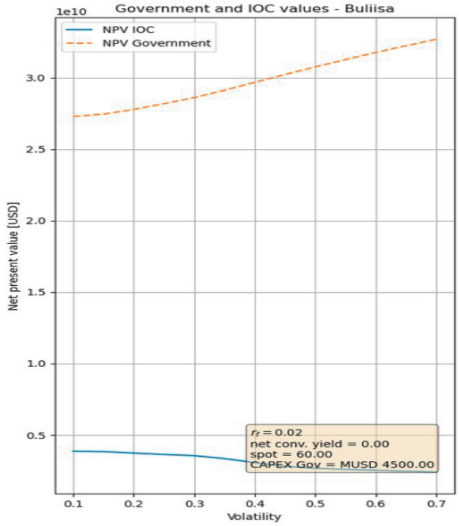
On the other hand, Figures 4 show that a rise in the oil price volatility *reduces* the expanded NPVs of the IOCs. Although contrary to the standard result in options valuation problems, this is not an unrealistic outcome. In our study, expanded NPV consists of two components: the value of starting production immediately and the value of the option to defer production by another year. According to real options theory, the value of deferring production must become more valuable with greater volatility, since an increase in volatility increases the likelihood that the option to defer will be exercised. However, the value of starting production immediately decreases with an increase in the volatility because a higher volatility raises the probability of the IOCs not fully recovering their project costs. The overall impact in this case is a decline in the expanded NPVs with a rise in oil price volatility. Henriques and Sadorsky (2011) use a large panel data set of US companies to assess the linear relationship between oil price volatility and strategic investment. They find that the relationship between oil price volatility and investment is U-shaped and complex due to the effects of different interacting options.

We find the same impact of volatility on the expanded NPVs of IOCs at higher oil prices (see figures D1 and D2)

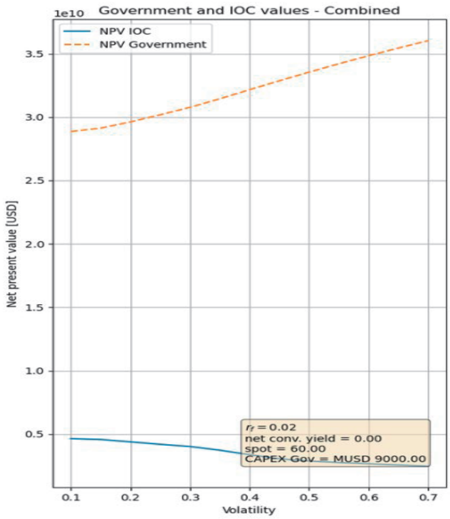
Figures 4: Sensitivity of each party's expanded NPV to volatility changes



(a) Expanded NPVs of IOCs and government for the Kingfisher field



(b) Expanded NPVs of IOCs and government for the Buliisa fields



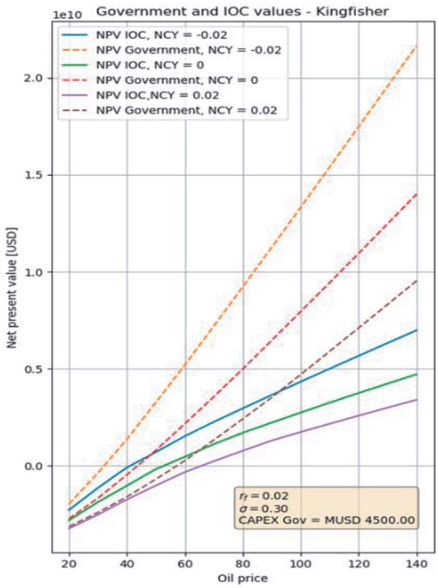
(c) Expanded NPVs for the IOCs and government when the fields are combined.

Note: *The figures show the sensitivity of each party's expanded NPV to changes in the oil price volatility. The base case initial oil price is USD60 per barrel of oil. All other parameters are presented in Table 1. In Figure 4a, the parties' expanded NPVs are expressed in billions of US Dollars. In Figures 4b and 4c, the expanded NPVs are in 10 billions of US Dollars. The volatility is in percentages per annum.*

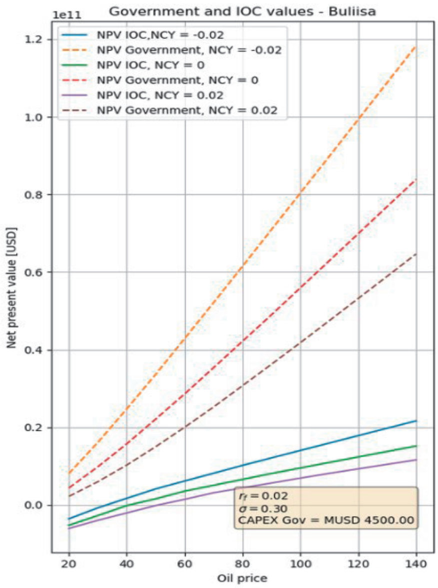
5.4 Sensitivity to changes in net convenience yield

Briefly defined, the net convenience yield represents the intrinsic marginal value associated with holding an additional unit of crude oil as inventories and accrues to the owners of the physical crude oil (Brennan and Schwartz, 1985). A high net convenience yield implies that crude oil prices are expected to fall (Alquist *et al*., 2014). For our sensitivity analysis, as depicted in Figures 5, we consider that the net convenience yield varies from -2.0 to +2.0%. As expected, for both oil fields and both parties, the expanded NPVs is a decreasing function of the net convenience yield. We also observe that the critical oil price increases with rising net convenience yield.

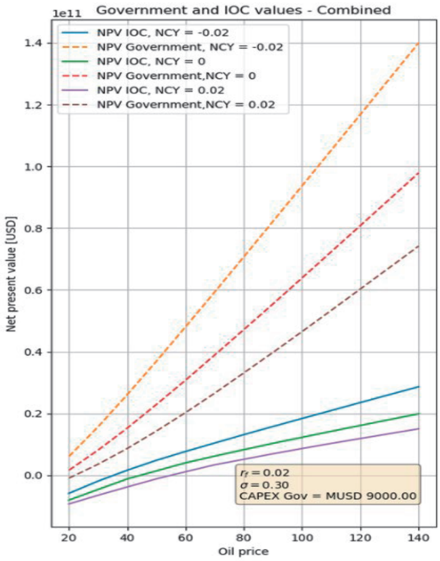
Figures 5: Sensitivity of each party's expanded NPV to changes in the net convenience yield



(a) Expanded NPVs of IOCs and government for the Kingfisher field



(b) Expanded NPVs of IOCs and government for the Buliisa fields



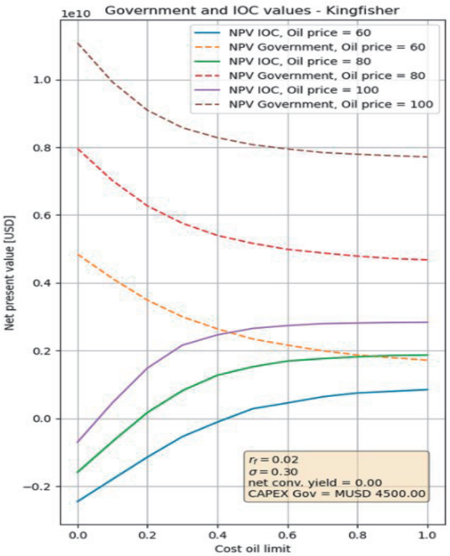
(c) Expanded NPVs for the IOCs and government when the fields are combined.

Note: *The figures show each party's expanded NPV as a function of the initial oil price, for three different net convenience yield rates, -2%, 0% and 2% per annum. The base case initial oil price is USD60 per barrel of oil. All other parameters are given in Table 1. In Figure 5a, the parties' expanded NPVs are expressed in 10 billions of US Dollars. In Figures 5b and 5c, the expanded NPVs are in 100 billions of US Dollar. The oil price is in USD/barrel.*

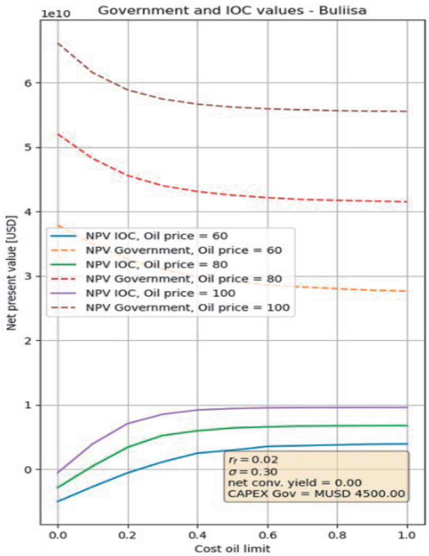
5.5 Sensitivity to changes in the cost oil limit

An important parameter in the PSA is the cost oil limit, which is set to 60%. Here we consider the implication of varying the cost oil limit at three different initial oil prices (USD60, USD80 and USD100). The results presented in Figures 6 indicate that the cost oil limit has a significant effect on the expanded NPVs of the IOCs and government. A higher cost oil limit increases the expanded NPV of the IOCs whereas the expanded NPV of the government declines. A combination of a low cost oil rate and a low oil price, increases the risk that the IOCs may not fully recover its incurred project costs, and thus results in a negative expanded NPV for the IOCs. For example, this is the case when the cost oil limit is below 40% at a low oil price of USD60 for the Kingfisher field. This would render the option value worthless and the Kingfisher field project infeasible.

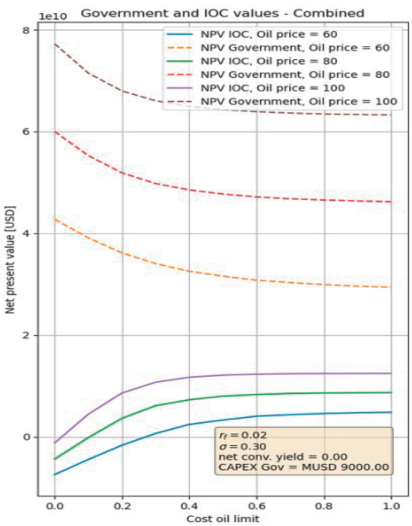
Figures 6: Sensitivity of each party's expanded NPV to changes in the cost oil recovery limit



(a) Expanded NPVs of IOCs and government for the Kingfisher field



(b) Expanded NPVs of IOCs and government for the Buliisa fields



(c) Expanded NPVs for the IOCs and government when the fields are combined.

Note: *The figures show each party's expanded NPV as a function of the cost oil limit, for three different initial oil prices, USD 60, USD 80 and USD 100. The base case initial oil price is USD60 per barrel of oil. All other parameters are given in Table 1. The parties' expanded NPVs are expressed in 10 billions of US Dollars. The oil price is in USD/barrel.*

Considering the combined fields, at any given oil price, further increases in the cost oil rate beyond 40%, do not yield significant changes in the NPVs for the IOCs and government. This implies that the official cost oil limit of 60% is a sufficient rate at which most of costs incurred by the IOCs can be recovered, such that further increases in the cost oil limit would have meagre influence on the NPVs for all parties.

It is also noteworthy that, for all cost limit rates, higher oil prices increase the expanded NPV of the IOCs and minimise the risk of yielding negative NPVs and unrecovered project costs. For instance, in Figure 6c, at a low cost oil rate of 20%, an increase in the oil price from USD60 to USD80 would result in a positive expanded NPV for the IOCs.

6. Optimal decisions during the second stage

In previous sections several figures illustrate discrepancies between investment decisions of the government and the IOCs. Typically, the government is willing to start the project at a lower oil price than the IOCs since the government experience a positive NPV before the IOCs does. However, after the first stage investment decision is made there is a second stage decision after two years. The second investment decision can be postponed up until five years (see figure 2). In this section we take a closer look at the second stage investment decision and highlight the optimal decisions of the government and IOCs. For ease of exposition, we utilize binomial price lattices with four time steps per year. The results are the same also if we increase the number of time steps per year. In addition, we only show the oil prices in the binomial tree when the government and the IOCs make decision, *i.e.*, at $m = 2, 3, 4, 5, 6$ year.

6.1 Optimal investment decisions when net convenience yield is 0%

Figure 7 shows optimal government and IOCs investment decisions for the Buliisa project for all modelled price and time combinations during the 5 years, where either invest-now, wait or abandonment decision can be made. The corresponding figure for the Kingfisher project can be seen in Figure E1 and shows the same major trends as the Buliisa project. In the last period, the decision is either to invest or abandon project. Figure 7 shows that it is only the IOCs that are interested in an abandonment decision, and this happens if the oil price goes below a certain level. The government will not be interested in an abandonment decision at the second stage since all government's investment outlays occur during the first stage. As an effect, the government will only face positive net cashflow, and positive NPV, when the project starts during the second stage.

Figure 7: Optimal actions for government and IOCs for different oil prices when net convenience yield=0%, for the Buliisa project.

Oil price - Binomial tree					Government decision - Buliisa					IOC decision - Buliisa				
				2195.89					1					1
				1626.76					1					1
			1205.13	1205.13				0	1				1	1
			892.78	892.78				0	1				1	1
		661.39	661.39	661.39			0	0	1			1	1	1
		489.97	489.97	489.97			0	0	1			1	1	1
	362.98	362.98	362.98	362.98		0	0	0	1		1	1	1	1
	268.90	268.90	268.90	268.90		0	0	0	1		1	1	1	1
199.21	199.21	199.21	199.21	199.21	0	0	0	0	1	1	1	1	1	1
147.58	147.58	147.58	147.58	147.58	0	0	0	0	1	1	1	1	1	1
109.33	109.33	109.33	109.33	109.33	0	0	0	0	1	1	1	1	1	1
80.99	80.99	80.99	80.99	80.99	0	0	0	0	1	1	1	1	1	1
60.00	60.00	60.00	60.00	60.00	0	0	0	0	1	1	1	1	1	1
44.45	44.45	44.45	44.45	44.45	0	0	0	0	1	1	1	1	1	1
32.93	32.93	32.93	32.93	32.93	0	0	0	0	1	1	1	1	1	1
24.39	24.39	24.39	24.39	24.39	0	0	0	0	1	0	0	0	0	1
18.07	18.07	18.07	18.07	18.07	0	0	0	1	1	0	0	0	0	-1
	13.39	13.39	13.39	13.39		0	0	1	1		0	0	0	-1
	9.92	9.92	9.92	9.92		0	1	1	1		0	0	0	-1
		7.35	7.35	7.35			1	1	1			0	0	-1
		5.44	5.44	5.44			1	1	1			0	0	-1
			4.03	4.03				1	1				0	-1
			2.99	2.99				1	1				0	-1
				2.21					1					-1
				1.64					1					-1
						1	: invest		0	: wait		-1	:abandon	

Note: *Green indicates invest immediately, yellow indicates wait, and red indicates abandon project.*

Looking at Figure 7, one can see that often there are situations where optimal decisions between the government and the IOCs differ. The government has a strong preference to wait except in the cases when prices are low, and the project approaches the last period. To the IOCs the situations are reversed. To understand why this is this case one must look at what constitutes the NPV to the government and IOCs. The government NPV comes from royalties, income tax and government share of profit oil. Government share of profit oil, and its NPV, will decrease with increasing cost oil whereas the IOCs' NPV will increase with increasing cost oil.

As depicted in Table 3, looking more into the cost oil calculation one sees that cost oil is limited by the lowest number between actual total cost and the cost oil limit. Cost oil limit, in turn, is calculated from a percentage of net revenue. An increase in the oil price increases net revenues and hence the cost oil limit. When the oil price is high, there is a high probability that the actual cost is lower than the cost oil limit, and thus cost oil is equal to actual total cost. Actual cost is unaffected by oil prices and as such cost oil is unaffected by oil price changes when oil prices are high (see last column of Table 3).

Since we in this case assume no increase in actual total cost from one year to another, *i.e.*, no inflation, it will have a positive impact on NPV to push the cost one year forward, and this is also the case for the government. Waiting another year reduces the government's NPV of the negative cash flow associated with cost oil. The situation is reversed for the IOCs since they are the receiver of cost oil (see also Figure 1 describing how revenues are allocated). Waiting another year reduces their NPV.

In case of low oil prices, the cost oil will be limited by the cost oil limit, since net revenue will be low compared to total actual cost (see second column of Table 3). Cost oil then turns into a function of oil prices, via net revenues, and given that the drift rate of the oil price is equal to the discount rate, the advantage of waiting will decrease. It will then be optimal for the government to start the project instead of waiting. To the IOCs, it is the other way around.

Table 3: Relationship among Cost oil limit, profit oil share, recovered costs by IOCs and oil price. Unrecovered costs are left out in this overview for ease of exposition.

	Actual cost > Cost oil limit **Cost recovery limit is 60%** **(cost oil affected by oil price)**	**Cost oil limit > actual cost (cost oil not affected by oil price)**
Profit oil	$Profit\ oil = net\ revenue - cost\ oil\ limit$ $[net\ revenue = (1 - royalties)PQ]$ $[cost\ oil\ limit = net\ revenue.cost\ recovery\ limit]$ $Profit\ oil = (1 - cost\ recovery\ limit)net\ revenue = 0.4net\ revenue$	$Profit\ oil = (1 - royalties)PQ - actual\ cost$ $Profit\ oil = net\ revenue - actual\ cost$
IOCs recovered cost	$0.6net\ revenue$	$actual\ cost$
IOCs share of profit oil	$0.4\theta net\ revenue$	$\theta(net\ revenue - actual\ cost)$
Government share of profit oil.	$0.4(1 - \theta)net\ revenue$	$(1 - \theta)(net\ revenue - actual\ cost)$

Note: *The benchmark net convenience yield is 0%. From the perspective of the government, the last column of the table shows when it is optimal for the government to wait and is similar to the upperpart of fig 7. The second column depicts the case when it is optimal for government to choose to start producing oil immediately, and is similar to the lower part of fig 7. It is the reverse for the IOCs.*

Seen from the perspective of the IOCs it is also important to highlight that the IOCs also have another incentive to wait. As can be seen in Figure 7, there is a probability that the project might end up with a negative NPV and in such cases it has a value to wait and see how the oil price develops.

In the analysis above, the net convenience yield was assumed to be zero. From a valuation perspective this implies that the oil price drift rate in the risk neutral valuation approach is equal to the discount rate, *i.e.*, the risk-free rate of return. However, the net convenience yield sometimes has a negative or a positive value (Abid and Kaffel, 2009), and it is therefore of interest to analyze how optimal decision making is affected by the net convenience yield.

6.2 Optimal investment decisions when net convenience yield is -2%

Figure 8 shows optimal decisions in case of a net convenience yield equal to -2%. A negative net convenience yield implies that the oil price drift rate is larger than the risk-free rate of return and that expected future oil prices increase at a higher rate than the risk-free rate. This makes a wait-strategy more interesting since waiting will increase net present value compared to starting immediately. Figure 8 shows this pattern for both the government and the IOCs.

Figure 8: Optimal actions for the government and IOCs for different oil prices, when net convenience yield is -2%, for the Buliisa project.

Oil price - Binomial tree					Government decision - Buliisa					IOC decision - Buliisa				
				2195.89					1					1
				1626.76					1					1
			1205.13	1205.13				0	1				0	1
			892.78	892.78				0	1				0	1
		661.39	661.39	661.39			0	0	1			0	0	1
		489.97	489.97	489.97			0	0	1			0	0	1
	362.98	362.98	362.98	362.98		0	0	0	1		0	0	0	1
	268.90	268.90	268.90	268.90		0	0	0	1		0	0	0	1
199.21	199.21	199.21	199.21	199.21	0	0	0	0	1	0	0	0	0	1
147.58	147.58	147.58	147.58	147.58	0	0	0	0	1	0	0	0	0	1
109.33	109.33	109.33	109.33	109.33	0	0	0	0	1	0	0	0	0	1
80.99	80.99	80.99	80.99	80.99	0	0	0	0	1	0	0	0	0	1
60.00	60.00	60.00	60.00	60.00	0	0	0	0	1	0	0	0	0	1
44.45	44.45	44.45	44.45	44.45	0	0	0	0	1	1	1	1	1	1
32.93	32.93	32.93	32.93	32.93	0	0	0	0	1	1	1	1	1	1
24.39	24.39	24.39	24.39	24.39	0	0	0	0	1	0	0	0	0	1
18.07	18.07	18.07	18.07	18.07	0	0	0	0	1	0	0	0	0	1
	13.39	13.39	13.39	13.39		0	0	0	1		0	0	0	-1
	9.92	9.92	9.92	9.92		0	0	0	1		0	0	0	-1
		7.35	7.35	7.35			0	0	1			0	0	-1
		5.44	5.44	5.44			0	0	1			0	0	-1
			4.03	4.03				0	1				0	-1
			2.99	2.99				0	1				0	-1
				2.21					1					-1
				1.64					1					-1
						1	: invest		0	: wait		-1	:abandon	

Note: *Green indicates invest immediately, yellow indicates wait, and red indicates abandon project.*

There is a range of oil prices where the IOCs want to start instead of waiting. The reason for this is once again the design of the cost oil function. Below a certain oil price, the cost oil will be lower than the actual total cost, *i.e.*, 60% of net revenue will be lower than the actual total cost. In such a case, the recovered amount is lower than the actual total cost, which affects the NPV negatively. Thus, waiting exposes IOCs for a situation where they might end up with actual total cost recovered. However, it is also likely that they end up in a situation where oil prices are lower than today, affecting revenues and recovered cost negatively. In fact, at some prices unrecovered costs are adding up throughout the project, which of course affect profitability negatively. This situation is also illustrated in Table 4 below showing how unrecovered costs are increasing significantly as oil prices go below a certain level. In total, this means that at some prices it will be more profitable to start immediately and reduce the risk of negative impacts from reduced revenue and increased unrecovered costs. This means that the present value of waiting a year is lower than starting immediately at these prices. Considering a higher cost oil limit equal to e.g. 70%, the price range where it is optimal to start instead of wait is lowered, according to our computations.

Table 4: Unrecovered costs during the first 15 of 25 years, at different years from production start, given different initial oil prices and futures prices based on initial oil price (in Millions of US dollars).

	Years from start decision in second stage														
Oil spot price	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
80,99	1049	759	37	0	0	0	0	0	0	0	0	0	0	0	0
60	1322	1695	1738	1824	1870	395	0	0	0	0	0	0	0	0	0
44,45	1526	2389	2998	3666	4289	3359	2482	1698	928	186	0	0	0	0	0
32,93	1677	2903	3932	5031	6081	5555	5039	4571	4092	3629	3197	2800	2415	2041	1708
24,39	1789	3283	4624	6042	7409	7182	6934	6700	6436	6180	5941	5724	5515	5313	5141

In Figure 8, one can also see that the IOCs want to wait when oil prices are even lower, as also can be seen in Figure 7. This is again an effect due to the risk of ending up with negative NPV and high value of the option to wait.

6.3 Optimal investment decisions when net convenience yield is 2%

Figure 9 shows optimal actions when the net convenience yield is equal to 2%. In this case the oil price drift rate is identical to the risk-free rate and expected future oil price will increase at a lower rate than the risk-free rate of return, *i.e.*, there is zero drift in this case where the risk-free rate is equal to the net convenience yield. This will initially call for a strategy where the decision maker prefers starting the project earlier rather than later. However, as with the case illustrated in Figure 8, there is a price range where the decision maker wants to deviate from the typical start immediately strategy. In this case it is the government who wants to wait. Again, as in the case with net convenience yield equal to -2%, the explanation can be found in cost oil and unrecovered cost. Lower cost oil and higher unrecovered cost will to some extent be advantageous to the government, since it is reducing its cash outflows. In the price range where it is optimal to wait, the unrecovered cost increases significantly if future oil prices go down. Thus, it is more valuable to the government to wait, having the possibility of drawing advantage from such an outcome, compared to starting immediately.

Figure 9: Optimal actions for the government and IOCs for different oil prices, when net convenience yield is 2%, for the Buliisa project.

Oil price - Binomial tree					Government decision - Buliisa					IOC decision - Buliisa				
				2195.89					1					1
				1626.76					1					1
			1205.13	1205.13				1	1				1	1
			892.78	892.78				1	1				1	1
		661.39	661.39	661.39			1	1	1			1	1	1
		489.97	489.97	489.97			1	1	1			1	1	1
	362.98	362.98	362.98	362.98		1	1	1	1		1	1	1	1
	268.90	268.90	268.90	268.90		1	1	1	1		1	1	1	1
199.21	199.21	199.21	199.21	199.21	1	1	1	1	1	1	1	1	1	1
147.58	147.58	147.58	147.58	147.58	1	1	1	1	1	1	1	1	1	1
109.33	109.33	109.33	109.33	109.33	1	1	1	1	1	1	1	1	1	1
80.99	80.99	80.99	80.99	80.99	1	1	1	1	1	1	1	1	1	1
60.00	60.00	60.00	60.00	60.00	0	0	0	1	1	1	1	1	1	1
44.45	44.45	44.45	44.45	44.45	0	0	0	0	1	1	1	1	1	1
32.93	32.93	32.93	32.93	32.93	0	0	0	1	1	0	0	0	1	1
24.39	24.39	24.39	24.39	24.39	1	1	1	1	1	0	0	0	0	1
18.07	18.07	18.07	18.07	18.07	1	1	1	1	1	0	0	0	0	-1
	13.39	13.39	13.39	13.39		1	1	1	1		0	0	0	-1
	9.92	9.92	9.92	9.92		1	1	1	1		0	0	0	-1
		7.35	7.35	7.35			1	1	1			0	0	-1
		5.44	5.44	5.44			1	1	1			0	0	-1
			4.03	4.03				1	1				0	-1
			2.99	2.99				1	1				0	-1
				2.21					1					-1
				1.64					1					-1

1 : invest　0 : wait　-1 : abandon

Note: *Green indicates invest immediately, yellow indicates wait, and red indicates abandon project.*

7. Concluding remarks

As far as we know, this is the first study to apply real options valuation in assessing whether IOCs and the government have different or aligned interests in the context of a PSA. We extend the binomial lattices framework by Abigaba *et al*. (2021) to determine the optimal strategies with respect to the IOCs and the government, while accounting for flexibilities and uncertainties embedded in oil contracts. We consider that the IOCs and government can exercise the option to start production immediately or to defer production for up to 5 years, beyond which the option to defer expires. To generate numerical results, we apply our model to analyse Uganda's PSA with technical data on the Kingfisher and Buliisa oil fields.

From our base case analysis, the critical oil prices at which to start oil production, from both fields, differ for the parties. Collectively, the critical oil price for the entire project is USD18 for the government and USD42 for the IOCs, which suggests that there may be conflicting interests between the IOCs and the government when it comes to realizing the project. The results from the sensitivity analyses indicate that the PSA design is progressive in the sense that, as the oil price rises, the expanded NPV of the government rises faster than that of the IOCs. We also find that the government's expanded NPVs increase with rising oil price volatility. Contrary to real options theory, the expanded NPVs of the IOCs decline with a rise in oil price volatility. In line with real options theory, for both parties, the expanded NPV is a decreasing function of the net convenience yield. As expected, a higher cost oil limit increases the expanded NPV of the IOC whereas the expanded NPV of the government declines.

Overall, we establish that the parties have conflicting optimal strategies. Particularly, the government has a strong preference to defer production, except in the cases when prices are low, and the project approaches the expiration of the defer option. To the IOCs, this is the reverse. A combination of a low cost oil rate and a low oil price, increases the risk that the IOC may not fully recover its incurred project costs, and thus results in a negative expanded NPV for the IOCs. The reason for the conflicting strategies emanates from the design of the cost oil function and the expected oil price realizations. In cases of low oil prices, the cost oil will be limited by the cost oil limit, since net revenue will be low compared to total actual cost. Cost oil then turns into a function of oil prices, via net revenues.

Our results have significant policy implications. Since the cost oil is a fundamental determinant of IOCs' expected returns and optimal strategy, it is imperative that the cost oil limit is flexible such that IOCs can negotiate for a favourable cost oil limit, especially in cases when the future oil prices are expected to be low.

Second, our sensitivity analysis indicates that the government expanded NPV is more sensitive to external shocks, such as the oil price, net convenience yield and oil price volatility, compared to the IOCs. Thus, the government must focus on these external factors and how they affect the returns and incentives, also for the IOCs, when negotiating the contract elements of the PSA.

There are some ways in which the research study can be extended. One way to extend the study could be to consider a multi-factor model that includes both the oil price, and geological and policy uncertainties. In this study, the tax rate, project costs and field production profiles have been held constant, based on the data of the case study. The study can also be extended to analyse how the optimal strategies of the parties depend on variations in these contract elements. This study assumes that the oil price and project value follow a GBM stochastic process. Another extension of the study could be to model oil prices as jump diffusion or mean-reverting stochastic processes.

Acknowledgements

We are thankful to the officials at Petroleum Authority of Uganda (PAU) and the Ministry of Energy and Mineral Development (MoEMD), for providing us with data.

Funding

This paper was supported within Norwegian Programme for Capacity Development in Higher Education and Research for Development (NORHED I) under the project- Capacity Building in Education and Research for Economic Governance, a partnership between Makerere University Business School and Norwegian University of Life Sciences. The funders had no role in study design; in the collection, analysis and interpretation of data; in the writing of the report; or in the decision to submit the article for publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could influence the work reported in this paper.

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Appendices

Appendix A. Royalty, additional royalty and profit oil

The royalty and additional royalty payments are further structured based on tranches. If, for instance, daily production in a given month lies between 2,500 and 5,000; 5% is levied on the first 2,500 bopd and 7,5% on what's above 2,500 bopd.

Table A1: Royalty structure of the oil project

Gross Total Daily Production (BOPD)	Royalty rate
(i) Where the production does not exceed 2,500	5%
(ii) Where the production is higher than 2,500 but does not exceed 5,000	7.5%
(iii) Where the production is higher than 5,000 but does not exceed7,500	10%
(iv) Where the production exceeds7,500	12.5%

Source: *Uganda National Oil Company (2021)*

Table A2: Additional royalty structure of the oil project

Recovered Cumulative Petroleum (Million Barrels)	Additional royalty rate
i) Where the recovered cumulative Petroleum does not exceed 50	2.5%
(ii) Where the recovered cumulative Petroleum is higher than 50 but does not exceed 100	5%
(iii) Where the recovered cumulative Petroleum is higher than 100 but does not exceed 150	7.5%
(iv) Where the recovered cumulative Petroleum is higher than 150 but does not exceed 250	10%

(continued)

Table A2 continued	
(v) Where the recovered cumulative Petroleum is higher than 250 but does not exceed 350	12.5%
(vi) Where the recovered cumulative Petroleum is higher than 350	15%

Source: *Uganda National Oil Company (2021)*

After the payment of royalties and additional royalties; and cost recovery, the following Government/IOC production sharing will apply on the profit oil.

Table A3: Production sharing structure of the oil project

Production BOPD	**Government Production Share**	**IOC Production Share**
(i) Where production does not exceed 5,000	46%	54%
(ii) Where production is higher than 5000 but does not exceed 10,000	48.5%	51.5%
(iii) Where production is higher than 10,000 but does not exceed 20,000	53.5%	46.5%
(iv) Where production is higher than 20,000 but does not exceed 30,000	58.5%	41.5%
(v) Where production is higher than 30,000 but does not exceed 40,000	63.5%	36.5%
(vi) Where production is higher than 40,000	68.5%	31.5%

Source: *Uganda National Oil Company (2021)*

Appendix B. Mathematical expression of Uganda's PSA and Project cashflows

The structure of Uganda's PSA can be expressed mathematically as follows;

Consider that for each oil field, the daily gross field revenue is defined as the average volume of production multiplied by the average price per unit of production:

$$Gross\ field\ revenue, G_m = P_m Q_m \tag{B1}$$

where P_t is average crude oil price on year *m,* and Q_m is the average daily production of crude oil (in barrels) in year *m.*

The royalty (additional royalty) is a product of the royalty rate (additional royalty rate) and the daily gross field revenue:

$$Royalty, R_m = G_m \tau_m \tag{B2}$$

where τ_m is the royalty rate which is dependent on the daily oil production.

$$Additional\ royalty, A_{Rm} = G_m a_{rm} \tag{B3}$$

where a_{rm} is the additional royalty rate determined based on the cumulative recovered reserves in year *m.*

The net field revenue is the remnant of the gross field revenue after royalties and additional royalties have been deducted, such that:

$$Net\ revenue = G_m - R_m - A_{Rm} \tag{B4}$$

The IOCs are entitled to recover their incurred costs from the net field revenue in form of cost oil. The maximum amount available for cost recovery is calculated as a proportion of the net field revenue, that is,

$$Cost\ recovery\ limit, CO_{capm} = (\ G_m - R_m - A_{Rm})t_c \tag{B5}$$

where t_c is the cost oil limit.

Any unrecovered costs beyond the cost recovery limit are carried forward to the subsequent period until all costs are fully recovered. Thus the cost oil in year *m* is given by:

$$CO_m = min(C_m + max(C_{rec,m-1} - CO_{capm-1}, 0), CO_{capm}) \tag{B6}$$

where C_m is the sum of CAPEX and OPEX costs incurred by the IOC in year *m*; $C_{rec,m-1}$ is the recoverable costs from the previous year *m*.

The proportion of the daily net field revenue which is left after the IOCs receive their cost oil is known as profit oil, and is distributed between the IOCs and government, according to the volume of production based on a sliding scale (see Table A3). Profit oil is equal to

$$Profit\ oil, PO_m = G_m - R_m - A_{Rm} - CO_m = PO_{IOCm} + PO_{GVTm} \tag{B7}$$

where PO_{IOC} is the IOCs' share of profit oil on day t and PO_{GVT} is the government share of profit oil on day t

Consider $\theta(Q_m)$ is the IOCs' share of the profit oil, and is expressed as a function of daily oil production, as stipulated by the PSA. Then the IOCs' share of the profit oil is

$$PO_{IOCm} = \theta(Q_m)(G_m - R_m - A_m - CO_m)$$

also expressed as;

$$PO_{IOCm} = \theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m - \theta(Q_m)CO_m \tag{B8}$$

A corporate income tax is levied on the IOCs' share of the profit oil at a rate α.

$$IOC\ tax\ payment;\ T_m = \alpha\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m - \alpha\theta(Q_m)CO_m \tag{B9}$$

then the IOCs' share of profit oil after tax is given by:

$$POAT_{IOCm} = (1 - \alpha)\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m - (1 - \alpha)\theta(Q_m) \tag{B10}$$

Taking the sum of cost oil and the share of the profit oil after tax, the IOCs' average daily share of net field revenue is given by:

$$IOC\ total\ revenue, Rev_{IOCm}$$
$$= (1 - \alpha)\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m - (1 - \alpha)\theta(Q_m)CO_m + CO_m$$

which can be expressed as

$$IOC\ total\ revenue, Rev_{IOCm}$$
$$= (1 - \alpha)\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m + (1 - \theta(Q_m) + \alpha\theta(Q_m))CO_m \tag{B11}$$

Then the average annual IOC net field revenue is given by:

$$= 365[(1 - \alpha)\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m + \left(1 - \theta(Q_m) + \alpha\theta(Q_m)\right)CO_m] \tag{B12}$$

The net cashflow of the IOCs in a given year m (for price realization j and remaining n years of production), is determined by deducting the annual OPEX and CAPEX from the IOCs' PSA revenue share such that;

$$C_{j,m,n}^{IOC} = 365\left[(1 - \alpha)\theta(Q_m)P_m(1 - \tau_m - a_{rm})Q_m + \left(1 - \theta(Q_m) + \alpha\theta(Q_m)\right)CO_m\right] - OPEX_m - CAPEX \tag{B13}$$

The government's net cashflow is equivalent to the sum of its PSA revenue earned from royalties and additional royalties, profit oil and corporate tax revenue.

The government's share of profit oil is equal to:

$$PO_{GVTm} = (1 - \theta(Q_m))P_m(1 - \tau_m - a_{rm})Q_m - (1 - \theta(Q_m))CO_m \tag{B14}$$

Government's revenue from royalty and additional royalty payments is given by

$$P_m(\tau_m + a_{rm})Q_m \tag{B15}$$

Government's revenue from taxes is given by equation (B9).

Taking the sum of expressions in equations (B14), (B15) and (B9) gives government's average daily net cash flow as follows:

$$\begin{aligned} C_m^{GVT} = \big(1 - \theta(Q_m)\big)P_m(1 - \tau_m - a_{rm})Q_m - \big(1 - \theta(Q_m)\big)CO_m \\ + P_m(\tau_m + a_{rm})Q_m + \alpha\theta(Q_m)P_m(1 - \tau_m \\ - a_{rm})Q_m - \alpha\theta(Q_m)CO_m \end{aligned}$$

which can also be expressed as:

$$\begin{aligned} C_m^{GVT} = P_mQ_m - (1 - \alpha)\theta(Q_m)[P_m(1 - \tau_m - a_{rm})Q_m] + (\theta(Q_m) \\ - \alpha\theta(Q_m) - 1)CO_m \end{aligned} \tag{B16}$$

Such that, the average annual net cashflow of government in a given year m (for price realization j and remaining n years of production), is determined by:

$$\begin{aligned} C_{j,m,n}^{GVT} = 365[P_mQ_m - (1 - \alpha)\theta(Q_m)[P_m(1 - \tau_m - a_{rm})Q_m] \\ + (\theta(Q_m) - \alpha\theta(Q_m) - 1)CO_m] \end{aligned} \tag{B17}$$

Appendix C. Forecast Production profiles and cost profiles for the three fields

Figure C1: Production profiles for the oil fields after start-up of production ('000 Barrels of oil per day)

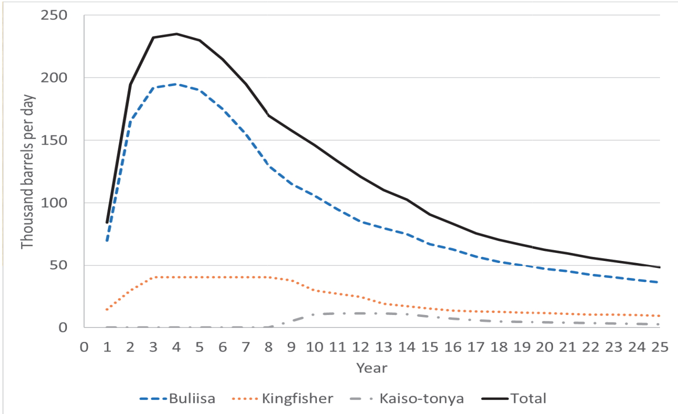
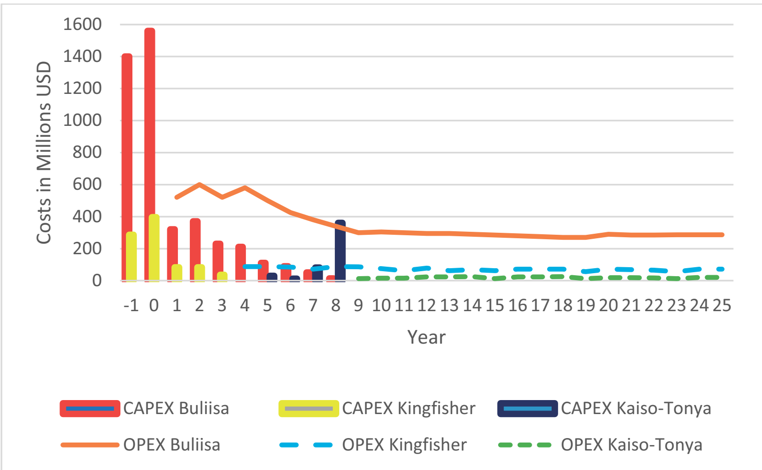
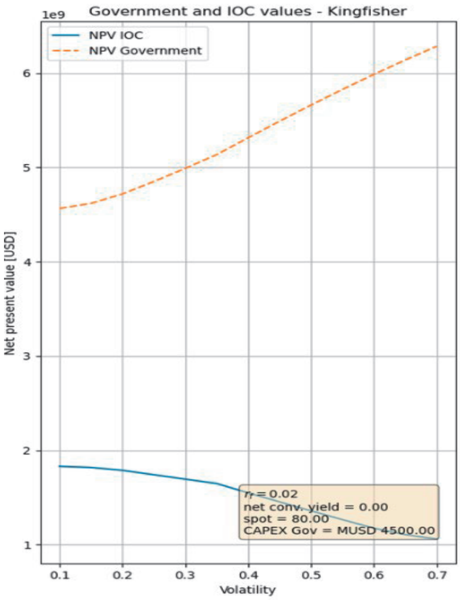


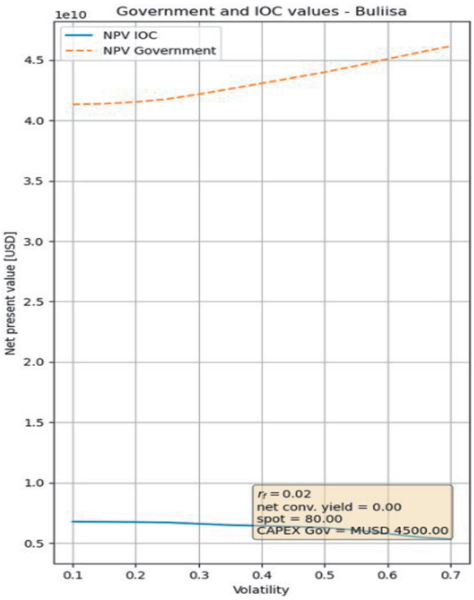
Figure C2: Costs profile (CAPEX and OPEX) for the three fields (Million US Dollars per year)



Appendix D. Additional result figures

Figure D1: The impact of oil price volatility on expanded NPVs of IOCs and government at an oil price of USD 80





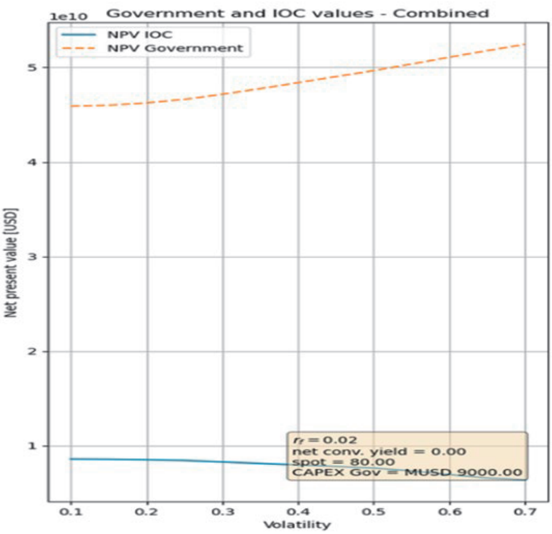
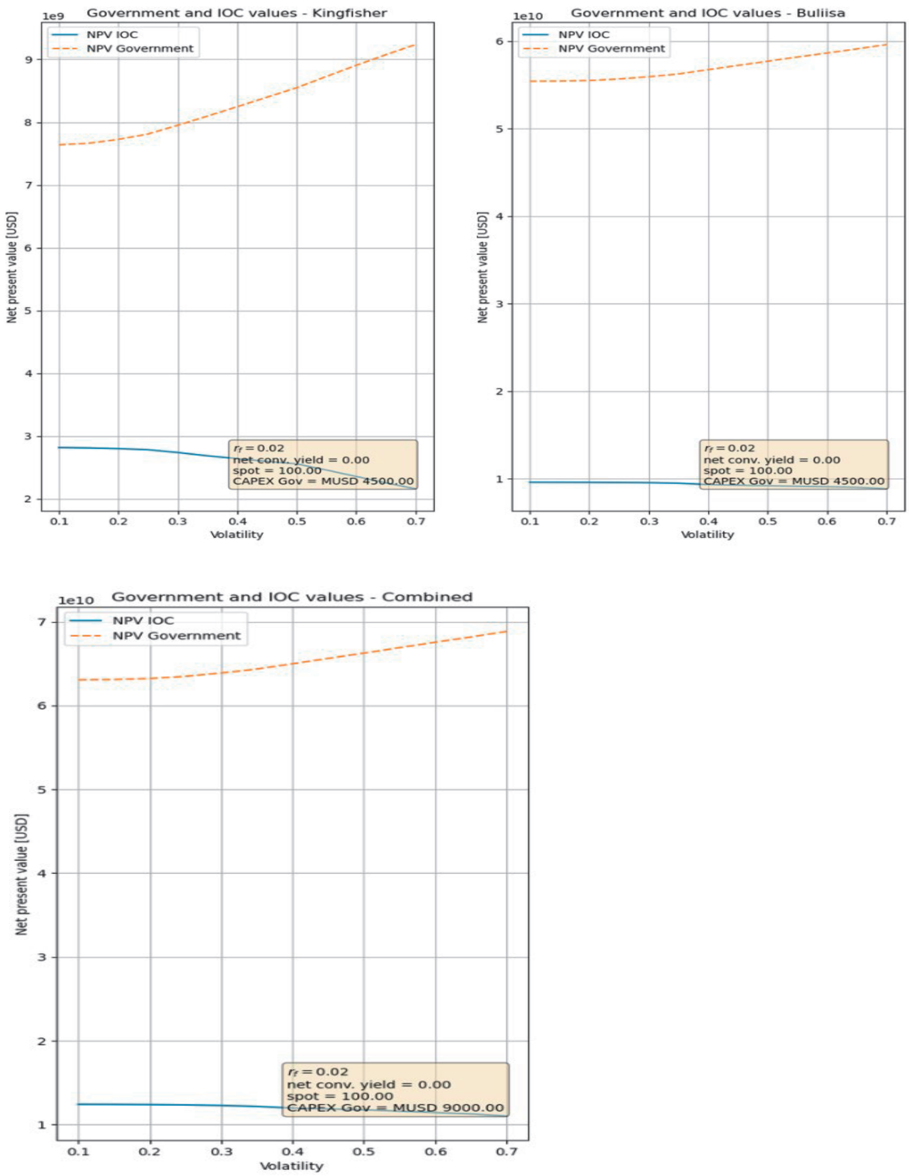


Figure D2: The impact of oil price volatility on expanded NPVs of IOCs and government at an oil price of USD 100



Appendix E. Optimal investment decision – Kingfisher

Figure E1: Optimal actions for government and IOCs for different oil prices when net convenience yield=0%, for the Kingfisher project.

Oil price - Binomial tree					Government decision - Kingfisher					IOCs decision - Kingfisher				
				2195.89					1					1
				1626.76					1					1
			1205.13	1205.13				0	1				1	1
			892.78	892.78				0	1				1	1
		661.39	661.39	661.39			0	0	1			1	1	1
		489.97	489.97	489.97			0	0	1			1	1	1
	362.98	362.98	362.98	362.98		0	0	0	1		1	1	1	1
	268.90	268.90	268.90	268.90		0	0	0	1		1	1	1	1
199.21	199.21	199.21	199.21	199.21	0	0	0	0	1	1	1	1	1	1
147.58	147.58	147.58	147.58	147.58	0	0	0	0	1	1	1	1	1	1
109.33	109.33	109.33	109.33	109.33	0	0	0	0	1	1	1	1	1	1
80.99	80.99	80.99	80.99	80.99	0	0	0	0	1	1	1	1	1	1
60.00	60.00	60.00	60.00	60.00	0	0	0	0	1	1	1	1	1	1
44.45	44.45	44.45	44.45	44.45	0	0	0	0	1	1	1	1	1	1
32.93	32.93	32.93	32.93	32.93	0	0	0	0	1	0	0	0	0	1
24.39	24.39	24.39	24.39	24.39	0	0	0	1	1	0	0	0	0	1
18.07	18.07	18.07	18.07	18.07	0	0	0	1	1	0	0	0	0	1
	13.39	13.39	13.39	13.39		0	1	1	1		0	0	0	-1
	9.92	9.92	9.92	9.92		0	1	1	1		0	0	0	-1
		7.35	7.35	7.35			1	1	1			0	0	-1
		5.44	5.44	5.44			1	1	1			0	0	-1
			4.03	4.03				1	1				0	-1
			2.99	2.99				1	1				0	-1
				2.21					1					-1
				1.64					1					-1
						1	: invest		0	: wait		-1	:abandon	

Note: *Green indicates invest immediately, yellow indicates wait, and red indicates abandon project.*

Chapter 4: The potential inflationary impacts of an oil price shock on the economy: a social accounting matrix price multiplier analysis for Uganda.

Micah Lucy Abigaba

Working paper

The potential inflationary impacts of an oil price shock on the economy: a social accounting matrix price multiplier analysis for Uganda

Micah Lucy Abigaba*

Abstract

Surges in crude oil prices are a concern for net-importing developing countries because they result in higher import prices of refined petroleum, which are transmitted to the final consumer in form of higher fuel pump prices. This paper constructs a SAM Price multiplier model to trace the transmission of an oil price shock through its impact on domestic production prices and consumer prices and ultimately measures the potential inflationary effects of an oil price shock. The study further assesses the likely distributional impacts of the increase in petroleum prices on different household groups. The model is applied to Uganda's economy. Based on the estimated price multipliers, the activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. The production prices are relatively more responsive to a petroleum import price shock than consumer prices. The analysis of the decomposed multipliers shows that the inflationary impact of an exogenous petroleum import price shock is mainly transmitted through increases in prices of activity sectors that use petroleum as an intermediate input in the production process. The results also show that the distributional impacts of rising petroleum prices tend to be progressive. For both urban and rural regions, the households in higher income quartiles are more affected by the petroleum import shock, as compared to the low-income quartiles.

Key words: Oil price shock; Petroleum import price; Inflationary impacts; Social Accounting Matrix (SAM); Price Multiplier decomposition; Distributional impacts; Uganda

JEL Classification: C63; D58; E16; R29

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1. Introduction

In the past two years, the global crude oil price has been significantly volatile. At the beginning of 2020, in the wake of the global COVID-19 pandemic, oil prices plunged to USD 18 per barrel of oil [1] in the first quarter of 2020. By the end of 2020, oil prices had assumed an upward trend and soared to a record high of USD 122 per barrel of oil in June 2022. The oil price crisis poses significant challenges for global recovery from the COVID-19 economic down-turn.

Particularly for oil-importing countries, a surge in crude oil prices results in higher import prices of refined petroleum, which are transmitted to the final consumer in form of higher fuel pump prices. The rise in fuel pump prices is inflationary in three ways (Ogwang *et al*., 2019). First, households and firms pay more for petroleum products they consume directly. Secondly, higher oil prices increase the prices of all other goods that have oil as an intermediate input. Thirdly, higher fuel pump prices exacerbate the cost of doing business on account of higher transport costs. In addition, rising fuel pump prices have distributional implications. Empirical studies show that rising fuel prices tend to affect low-income and high-income households differently (for example, see Saari *et al*., 2016). It is therefore imperative to analyse the inflationary impacts of oil price shocks.

Uganda is a good case in point for an analysis of potential inflationary impacts of oil price shocks on a net-importing developing economy. First, the fuel market is fully liberalised and there is no government intervention to cushion customers from effects of high fuel prices. Second, the country is landlocked and imports all its refined petroleum [2]. Our study particularly estimates the potential inflationary impact of an oil price shock by using the information provided by Uganda's Social Accounting Matrix (SAM) 2016/17. A SAM covers the entire economy and quantifies linkages between several production sectors and households. The study constructs a SAM Price multiplier model which enables us to trace the transmission of an oil price shock through its impact on production prices and consumer prices and ultimately

[1] This study uses the Brent crude oil spot price as a proxy for the global crude oil price. The data on Brent crude oil spot prices is obtained from the U.S Energy Information Administration, EIA (2022).
[2] Uganda has no refinery and all the country's imported refined petroleum is delivered by fuel trucks through Kenya (80%) and Tanzania (20%).

measure the potential inflationary effect of oil-price shocks. Our study also assesses the potential distributional impacts of the increase in fuel prices on different household groups.

Roland-Holst and Sancho (1995) developed the SAM Price Multiplier model and applied it to capture the interdependence among activities, households, and factors and provided an analysis of the price mechanisms in Spain. Since their seminal work, there are a few known studies that apply the SAM Price model (see Llop, 2018). For instance; Akkemik (2011) studied the impact of changes in electricity prices on production and consumer prices in Turkey. Saari *et al*. (2016) provided an extension of the SAM Price model to incorporate substitution possibilities among production inputs and consumption goods. The authors applied their model to analyse the distributional impacts of rising petroleum prices among ethnic groups in Malaysia. Llop (2018) evaluated the contribution of five energy activities in the price formation mechanism by quantifying the extent to which energy costs affect production and consumer prices. Xue *et al.* (2019) estimated the impacts of carbon pricing on sectoral prices and household groups in Beijing, China.

The earlier literature on impacts of petroleum price shocks that are closely related to this study are Saari *et al*. (2016) and Llop (2018). Similar to Saari *et al*. (2016), this study accounts for the distributional impacts of petroleum price shocks. Saari *et al*. (2016) tailored their extended SAM price model to Malaysia, that has subsidies on petroleum products. To the contrary, I apply the standard SAM Price Model as it is sufficient to analyse Uganda's fully liberalised fuel market in the short-term. This study also departs from Saari *et al*. (2016), by employing the standard SAM Price multiplier decomposition approach to trace the price transmission mechanism of an exogenous petroleum import price shock. The decomposition approach is in relation to Llop (2018). I extend the work of Llop (2018) by assessing the distributional impacts of the energy price shocks on household groups. Specifically, this study categorises the household groups into 8, based on their income quartiles and geographical area of residence (urban vs rural) and compares the changes in their respective costs of living resulting from the petroleum import price shock.

This study further makes three contributions to the international literature on the link between oil price shocks and inflation. First, it revives the analysis of distributional aspects of oil price shocks, for a particularly small net-importing developing economy. This is one of the few studies on low-income countries, given that the economic structures are different from those of developed and emerging economies. Secondly, this study is timely, as the global economy faces an oil price crisis and unprecedented global effects on households, business and the economy's demand for goods and services. Lastly, this study has policy implications as it informs government on the extent to which the absence of fuel price controls account for Uganda's vulnerability to oil price shocks. The findings of this study are a basis for exploring plausible policy interventions to mitigate the impacts of future oil price shocks.

The rest of the paper is organised as follows: Sections 2 highlights the patterns of price movements, refined petroleum imports and household expenditure shares. Section 3 presents the empirical model specification. The results are discussed in Section 4. Lastly, Section 5 concludes.

2. Price movements, Refined petroleum Imports and household expenditure shares.

Across the globe, the unprecedented crude oil price surges have transmitted into higher domestic fuel pump prices. Particularly, in the first half of 2022, fuel pump prices rose to record highs in most countries around the world (International Energy Agency, IEA, 2022). Chinele *et al*. (2022) documented fuel shortages and oil price crisis across Africa. On average the fuel pump price increased by at least 37% for landlocked countries compared to 29% for countries with access to the open seas, from April 2021 to April 2022. The exceptions were Algeria, Angola, Chad and Gabon, that are large net-exporters of crude oil, and managed to keep the fuel pump prices unchanged. Over the same period, Uganda's fuel pump prices of diesel and petrol prices rose by 38% and 33% (Uganda Bureau of Statistics, 2022).

Uganda's domestic fuel pump prices follow the same pattern as the global crude oil prices (see figure A1), which reflects a transmission of an external oil price shock to the domestic fuel (Odokonye and Bulime, 2022). It is, however, noteworthy, that the fuel pump prices in Uganda increased higher than those of other countries within the East African region (see Figure A2 and Figure A3 in Appendix A). All other countries (*i.e.* Kenya, Tanzania and Rwanda) have some form of price control to cushion consumers from f uel price effects.

Based on the 2016/17 Uganda SAM, Uganda is heavily reliant on refined petroleum imports which account for 14% of the total imports bill and amount to 6% of GDP. 78% of the refined petroleum demand is allocated to intermediate input requirements, 14% is directly consumed by households, while the remaining proportion is re-exported.

Table 1 presents the consumption expenditure shares on refined petroleum and five other consumer goods. The five consumer goods are commodities that account for a relatively large share of household consumption expenditure. The fuel intensities and the consumption shares are computed from the 2016/2017 Uganda SAM. On average, expenditure share of petroleum by households is only 1.03%. For both rural and urban areas, the richer households spent more on petroleum compared to the poor. Based on the preliminary analysis of Table 1, households spend a large share of their incomes on less petroleum-intensive products. Thus, the increase in petroleum prices may have a limited impact on household cost of living. This study investigates this hypothesis further in subsection 4.4.

Table 1: Fuel intensity and consumption shares for consumer goods by different household groups

	Fuel intensity	Urban Q1	UrbanQ 2	UrbanQ 3	UrbanQ 4	RuralQ1	RuralQ2	RuralQ3	RuralQ4
Refined petroleum		0.08%	0.10%	0.39%	2.13%	0.04%	0.15%	0.50%	1.83%
Food crops	0%	23.5%	20.36%	14.74%	7.52%	24.92%	22.41%	17.82%	10.54%
Manufactured foods and beverages	1.5%	18.7%	20.97%	23.48%	16.19%	22.62%	22.40%	24.42%	19.10%
Pharmaceuticals	0.8%	5.0%	6.54%	5.11%	4.0%	5.17%	6.02%	6.97%	7.36%
Firewood	2.2%	4.7%	2.40%	0.93%	0.22%	6.30%	4.38%	2.88%	1.25%
Real estate services	0.4%	7.8%	5.73%	7.98%	10.91%	6.19%	5.82%	6.03%	9.66%

Source: Author's computations from Uganda 2016/17 SAM. The income quartiles Q1-Q4 measure the average household income of urban and rural residents, with Q1 being poorest and Q4 being richest. Fuel intensity is computed as the cost share of petroleum (refined petroleum products) in total output of a particular industry. For each household group, the consumer shares are obtained by dividing the expenditure on a given commodity by the total household consumption expenditure, respectively.

3. Method

This section begins by describing a SAM in subsection 3.1. To analyse the potential inflationary impacts of petroleum price shocks on Uganda's economy, I develop a SAM price model framework following Roland-Holst and Sancho (1995), as shown in subsection 3.2. For an extensive analysis of cost interdependences among activities, factors and households, the price multipliers are decomposed into three price effects. The decomposition of the price multipliers is derived in 3.3. The SAM price model is applied to the 2016/17 Uganda SAM, which is described in subsection 3.4.

3.1 Social Accounting Matrix: Structure and interlinkages among economic agents

A SAM[3] is a general equilibrium database that depicts the flows of income and expenditure among sectors, between sectors and institutions (such as households, enterprise and the government) and between these domestic entities and the rest of the world. It is presented as a square matrix in which each row and column is called an account. Table 2 depicts a simplified macro SAM showing an economy's income and expenditure flows across 5 accounts. The accounts of the SAM are categorised as; endogenous accounts and exogenous accounts[4]. Conventionally, the production activities, factors, households are endogenous accounts, along with other institutions such as enterprises, whereas the rest of the accounts such as government, savings-investment and the rest of the world are consolidated into the exogenous accounts. All the flows are included in the square matrix, in which the rows show receipts and the columns depict payments. These income and expenditure flows across accounts portray both direct and indirect linkages among sectors and institutions, which makes it possible to analyse the full impacts of specific changes to the economy[5].

Table 2: Structure of Macro SAM

	1.Activities	2.Factors	3.Households	4.Other endogenous accounts	5.Exogenous accounts	Total
1.Activities	X_{11}	0	X_{13}	X_{14}	X_{15}	Y_1
2.Factors	X_{21}	0	0	0	X_{25}	Y_2
3.Households	0	X_{32}	X_{33}	0	X_{35}	Y_3
4.Other endogenous accounts	0	X_{42}	X_{43}	X_{44}	X_{45}	Y_4
5.Exogenous accounts	X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	Y_5
Total	Y_1	Y_2	Y_3	Y_4	Y_5	

X_{11} is a square matrix containing the intermediate input requirements; X_{13} shows consumption of domestic products by households, X_{14} is domestic consumption expenditure by other

[3] A SAM is a generalisation of the System of National Accounts (SNA) and is constructed as a representation of the economy.

[4] Endogenous accounts include those accounts where income-expenditure is governed by mechanisms that operate entirely within the SAM model. Exogenous accounts are those accounts where income and/or expenditure are influenced by forces external to the SAM framework. The distinction between endogenous and exogenous accounts comes from the limit to the endogenous responses that are captured in the SAM multiplier model. The exogenous accounts are only affected by the initial shock and by changes in the leakages from the endogenous to the exogenous accounts to balance the exogenous accounts as a group (Round, 2003).

[5] See Pyatt and Round (1979), for a detailed description of the SAM

endogenous institutions, X_{15} contains all other forms of final demand (such as exports, public expenditure, investment).

X_{21} is value added while X_{25} is factor income received abroad. X_{32} is factor income distribution for households, X_{33} represents income transfers among households and X_{35} is a matrix of all other domestic and foreign transfers to households.

Matrices X_{42} , X_{43} , X_{44} and X_{45} show the transactions corresponding to the rest of the endogenous accounts (such as enterprises).

X_{51} corresponds to a matrix of payments for imports and taxes by production activities, X_{52} is factor income paid to the rest of the world, X_{53} is the consumption of imports, private savings and taxes paid by households while X_{54} contains all consumption of imports, savings and taxes expended by other endogenous institutions. Lastly, X_{55} combines all other imports, taxes and balance of payments.

For each group, the column totals (total payments) must equal the row totals (total receipts).

3.2 SAM Price model: Model framework and derivation of Multipliers

The theoretical framework of the SAM Price model is based on the structure of the SAM described in subsection 3.1 and theoretical constructs about the relationship between economic actors in markets. The main assumption is that prices vary with cost changes while output levels are fixed. The mechanisms of the flows of incomes and payments are assumed to be constant. In the conclusions, I highlight the implications of these two assumptions.

To construct the SAM price model, first, average expenditure propensities, A_{ij} are computed by dividing the column entries in the SAM (X_{ij}) by the corresponding column total (Y_{ij}). The average expenditure propensities consist of two types;

i. those corresponding to the endogenous accounts ($A_{ij} = X_{ij}/Y_j \; for \; i,j = 1,2,3,4$), which capture all the relationships among the endogenous accounts. The coefficient Matrix A of these average expenditure propensities is expressed as:

$$A = \begin{pmatrix} A_{11} & 0 & A_{13} & A_{14} \\ A_{21} & 0 & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 \\ 0 & A_{42} & A_{43} & A_{44} \end{pmatrix} \quad (1)$$

Where; submatrix A_{11} denotes the share of intermediate inputs used to produce a monetary unit worth of output. Submatrix A_{21} is the share of factors of production required to produce a monetary unit of output. Submatrix A_{13} is the average household consumption expenditure as a share of total household expenditure. Submatrix A_{32} is the average factor income received by households for every monetary unit of factors of production provided. Submatrix A_{33} denotes average transfers among households as a share of total household expenditure. Submatrix A_{14} is the average consumption spending by all other endogenous groups as a share of total spending by all other endogenous groups. Submatrix A_{42} is the factor income received by other endogenous accounts (e.g enterprises) for every monetary unit worth of factors produced (i.e capital). Submatrix A_{43} expresses the average transfers from households to other endogenous accounts as a share of total household expenditure. Lastly, Submatrix A_{44} depicts the average transfers among other endogenous accounts as a share of total spending by other endogenous groups.

ii. those corresponding to the exogenous accounts ($\bar{A}_5 = X_{5j}/Y_j \; for \; j = 1,2,3,4$). The average expenditure propensities in submatrix A_j express exogenous per unit costs to the endogenous accounts. $\overline{A_5} = (\overline{A_{51}} \quad \overline{A_{52}} \quad \overline{A_{53}} \quad \overline{A_{54}})$ is thus a vector of leakages of expenditures from endogenous to exogenous accounts in the form of per unit costs towards imports, factor payments to the rest of the world, savings, and taxes. To distinguish, the exogenous accounts from the endogenous accounts, a bar is placed on top of the vector of exogenous costs and exogenous price shock parameter.

By construction, the sum of each column of computed average expenditure propensities should be equal to 1. For instance, for the activities account, $A_{11} + A_{21} + \overline{A_{51}} = 1$

Second, the assumption of fixed quantities and varying prices is imposed. This allows for prices to be computed independently of the output level (Roland-Holst and Sancho,1995). Let p_i denote a price index for account i's activity. Utilizing the average expenditure propensities and reading down the SAM columns of the endogenous accounts yields a set of linear equations in terms of prices:

$$
\begin{aligned}
p_1 &= p_1 A_{11} + p_2 A_{21} + \bar{p}_5 \bar{A}_{51} \qquad (2) \\
p_2 &= p_3 A_{32} + p_4 A_{42} + \bar{p}_5 \bar{A}_{52} \\
p_3 &= p_1 A_{13} + p_3 A_{33} + p_4 A_{43} + \bar{p}_5 \bar{A}_{53}
\end{aligned}
$$

$$p_4 = p_1 A_{14} + p_4 A_{44} + \bar{p}_5 \bar{A}_{52}$$

p_1 denotes the price indices of production activities, p_2 stands for price indices of factors of production, p_3 represents price indices for the expenditures of households, and p_4 is the price indices for the expenditure of other endogenous accounts (such as enterprises). $\bar{p}_5$ expresses the price indices for primary inputs and is the exogenous price shock parameter. In Table 2, the primary inputs are imports, factor payments to the rest of the world, savings, and taxes.

The set of linear equations in (2) can be rearranged and expressed as;

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{21} & 0 & 0 \\ 0 & 0 & A_{32} & A_{42} \\ A_{13} & 0 & A_{33} & A_{43} \\ A_{14} & 0 & 0 & A_{44} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} + \bar{p}_5 \begin{pmatrix} \bar{A}_{51} \\ \bar{A}_{52} \\ \bar{A}_{53} \\ \bar{A}_{54} \end{pmatrix} \quad (3)$$

taking $p = (p_1, p_2, p_3, p_4)$ as the vector of price indices for the endogenous accounts. The vector of exogenous costs is expressed as, $v = \bar{p}_5 \bar{A}_5$.

Third, matrix notation in (3) is expressed in a reduced form which yields:

$$p = pA' + v - - \rightarrow v(I - A')^{-1} \quad (4)$$

Where A' is the transpose of the coefficient matrix, A.

Equation (4) depicts that the prices of the endogenous accounts are determined by exogenous costs (v) and the cost linkages among the endogenous accounts (A). The coefficient matrix A remains fixed, based on the prior assumption of constant structure of incomes and payments. The base-year solution corresponds to the SAM from which the coefficients were obtained, therefore all base-year prices are equal to unity (Saari *et al*., 2016).

Fourth, Equation (4) is rewritten as;

$$p = v(I - A')^{-1} = vM' \quad (5)$$

where $M' = (I - A')^{-1}$ is the price multilpier matrix and its entries are interpreted by reading M' across the rows[6].

[6] This is contrary to the SAM Quantity model where the entries of M are read down the columns, as they measure income impacts.

Equation (5) is the SAM price model and is useful in analysing the cost linkages that results from the relationships among households, factors of production, and producers. An increase in the exogenous price of petroleum imports, for example, raises the cost of supplying petroleum domestically and thus results in an increase in the fuel pump prices. This prompts an increase in output prices of activity sectors that use petroleum as an intermediate input in the production process (A_{11}) and higher consumer prices paid by households (A_{13}) and other endogenous groups (A_{14}). Consequently, wages and capital costs rise due to cost-of- living adjustments by households (owners of labour) and enterprises (owners of capital), (A_{32} and A_{42}), respectively, resulting in further price increases. We can thus deduce that, the price variation in the exogenous account of petroleum imports, in this case, affects values of v which translates into higher prices for the endogenous accounts by M'. Assuming linearity of the SAM price model and equating all price indices in the baseline SAM to 1, we can infer that equation (5) also holds for changes in these accounts, such that;

$$\Delta p = \Delta v M' \qquad (6)$$

3.3 Decomposition of SAM Price Multipliers

Roland-Holst and Sancho (1995) were the first to show how the price multiplier matrix M' can be decomposed into three multiplicative (and additive) matrices. This decomposition distinguishes the degree to which price effects arise from sectoral interlinkages, household consumption expenditures, and factor prices. The existing literature on the SAM Price model shows variations in approaches to decomposing the matrix M', emanating from the subdivision of the coefficient matrix, A (Llop, 2018). This study follows Miller and Blair (2022) in their approach to subdividing Matrix A into two submatrices, $A_1$and A_2:

$$A = A_1 + A_2 = \begin{pmatrix} A_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & A_{13} & A_{14} \\ A_{21} & 0 & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \qquad (4)$$

Where A_1 is the submatrix for the activity sector coefficients only, and A_2 is the submatrix for coefficients involving all other endogenous accounts.

Equation (1) is then further modified to incorporate the submatrices in (4) as follows;

$$p = v(I - A_1' - A_2')^{-1} = vM_1'M_2'M_3' = vM' \quad (5)$$

Following Miller and Blair (2022), $M_1' = (I - A_1')^{-1}$, $M_2 = (I + T + T^2)$, $M_3 = (I - T^3)^{-1}$, and $T = A_2'(I - A_1')^{-1}$.

For ease of analysis and interpretation of results, the multiplicative multiplier decomposition is further formulated as an additive construction[7], into transfer effects, open-loop effects and closed-loop effects, following Roland-Holst and Sancho (1995) and Miller and Blair (2022):

$$M' = I + N_1 + N_2 + N_3 \quad (6)$$

Where $N_1 = M_1' - I$, $N_2 = M_1'M_3'M_2' - M_1'M_3'$, $N_3 = M_1'M_3' - M_1'$

Transfer effects refer to the price multiplier effects originating from an exogenous shock to the same group of accounts. From equation (6), $I + N_1$ defines the transfer effects, which measures how an exogenous cost shock affecting the production activities multiplies itself through the inter-sectoral cost linkages. This component excludes the multiplier effects associated with other endogenous accounts such as the factor and household accounts. In this study, the transfer effect measures the price impacts due to direct cost linkages between the petroleum sector and production activities. An increase in the price of petroleum imports results in an increase in the fuel pump prices. This prompts an increase in output prices of activity sectors that use petroleum as an intermediate input in the production. Thus, $I + N_1$ measures how the exogenous petroleum import price shock results in direct increases in prices of production activities emanating from the rising cost of petroleum as an intermediate input.

The open-loop effect refers to the indirect effect of the same exogenous cost shock, as it estimates the impact of a price shock on the factor prices after affecting endogenous institutions (such as households and enterprises). In this analysis, the open-loop effect, N_2 reflects the indirect effects of the petroleum import price shock as it estimates the impact of the petroleum import price shock on the factor costs that is preceded by an increase in households' costs of living.

Lastly, the closed-loop effect is an estimate of the feedback effects of the same exogenous price shock, as the economy adjusts to equilibrium. Thus, N_3 quantifies the impact on activity prices of the exogenous price shock after first affecting the endogenous institutions, then moving onto factor prices then finally back to activity prices. The increase in factor costs, as a consequence

[7] The additive multiplier decomposition was first proposed by Stone (1985) and formulated for the SAM quantity model.

of the initial exogenous petroleum import price shock causes further increases in activity prices as producers adjust their output prices in response to higher production costs.

3.4 Data: The SAM for Uganda

The SAM price model analysis is based on the official Uganda SAM for 2016/2017, obtained from the Ministry of Finance, Planning and Economic Development. A detailed description of the SAM can be found in (Tran *et al*., 2019).The original SAM consists of 186 activities and commodities, 2 accounts for trade and transport margins, 5 tax accounts, 17 factor accounts, 32 household groups, 2 enterprise accounts and lastly one account each for Non-Profit Institutions Serving Households (NPISH), government, investment-savings, changes in inventory, and rest of the world.

For the purpose of this study, the SAM is aggregated into:

i. 45 activity accounts with 45 corresponding commodity accounts,
ii. 5 factor accounts consisting of 4 labour types and 1 capital account. The labour groups are classified according to gender and areas of residence,
iii. 12 institutions including 8 household groups, NPISH account, enterprise account, government account and the Rest of the World. The household groups are categorised according to the income quartiles and areas of residence (Urban and Rural areas),
iv. the remaining accounts of the SAM include; 2 margin accounts (trade and transport margins), 5 tax accounts, investment-savings account and change in inventory account.

These accounts are further categorised into endogenous and exogenous accounts. The exogenous components are; the government, investment-savings, change in inventory, tax accounts and the rest of the world. The rest of the accounts are categorised as endogenous. Table 3 is Uganda's macro SAM that is constructed by aggregating the official 2016/17 SAM.

Table 3: The 2016/17 Macro SAM for Uganda (UGX trillions)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
1	Activities	-	153	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	153
2	Commodities	53	-	-	-	-	-	-	-	-	-	-	75	2	-	-	9	25	1	20	184
3	Trade margin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	Transport margin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	Excise tax	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
6	Import duty	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
7	VAT	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
8	Labour	29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	29
9	Capital	72	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	72
10	Production taxes	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
11	Direct taxes	-	-	-	-	-	-	-	-	-	-	-	2	-	2	0	-	-	-	-	4
12	Households	-	-	-	-	-	-	-	28	43	-	-	3	-	17	2	2	-	-	4	98
13	NPISH	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	0	-	-	1	3
14	Non-financial Enterprises	-	-	-	-	-	-	-	-	26	-	-	-	-	-	1	0	-	-	-	27
15	Financial Enterprises	-	-	-	-	-	-	-	-	1	-	-	1	-	1	-	1	-	-	-	4
16	Government	-	-	-	-	3	1	4	-	-	0	4	0	-	0	0	-	-	-	1	14
17	Savings	-	-	-	-	-	-	-	-	-	-	-	16	0	7	1	1	-	-	1	25
18	Stock	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	1
19	Rest of the World	-	23	-	-	-	-	-	0	1	-	-	-	-	1	0	0	-	-	-	26
	Total	153	184	-	-	3	1	4	29	72	0	4	98	3	27	4	14	25	1	26	

Source: Author's compilation based on official 2016/17 Uganda SAM.

4. Empirical Results and discussion

This section discusses the extent to which an oil price shock impacts sectors and purchasers of petroleum products. First the price multipliers and the results of the multiplier decomposition are presented. For this purpose, the consequences of an exogenous increase of 1% in the price of petroleum imports are examined. Second, the results of a price shock simulation of a 35% increase in petroleum import price on the producer and consumer price indices are presented in subsections 5.3 to 5.4. The average annual fuel pump price change for Uganda, as at April 2022 is used as the reference for the price simulation shock.

For an extensive analysis of the price transmission mechanism, in this study, the prices for the endogenous accounts are distinguished into production and consumer prices. The production price is the sum of the basic price and taxes on production activities while the consumer price is a sum of the production price, trade and transport margins, taxes on product, and import price.

4.1 Impact on production and consumer prices

In this subsection, the impacts of a 1% increase in the petroleum import price on production and consumer prices, are examined.

The results reported in Table 4 are the price multipliers of production and consumer prices by each economic activity. For instance, a 1% increase in the petroleum import price generates a 0.096% increase in the production price of refined petroleum sector. The consumer price of the refined petroleum sector increases by 1.0003%. The difference in the effects on the production price and consumer price emanates from the composition of each of these prices. The consumer price includes the import price of petroleum and thus a cost increase in the imports of petroleum influences the consumer price of petroleum the most, as expected. Since all Uganda's refined petroleum is imported, a shock to the petroleum import price is fully absorbed in form of a higher consumer price of petroleum. The production price of the refined petroleum sector entails the production costs of distributing refined petroleum and the production taxes. Thus, the price shock is transmitted to the rest of the economy mainly through higher consumer prices for petroleum.

The price multipliers of the other energy sectors are of moderate magnitude which range from 0.0519% to 0.0753%, with the electricity supply sector and firewood being the next most sensitive to the petroleum import price shock, after refined petroleum. Charcoal is the least responsive to the price shock. Not surprising, as charcoal has the lowest fuel intensity (0.97%), compared to the rest of the energy sectors (see Table B1 for the estimated fuel intensities for all activity sectors).

Table 4: Price Multipliers on refined petroleum and other energy sectors- the impact of a 1% increase in the import price of petroleum

Energy sectors		**Price Multiplier (M')**	
		Production price	**Consumer price**
	Refined Petroleum	0.0963	1.0003
Other energy sectors	1. Firewood	0.0736	0.0591
	2. Charcoal	0.0614	0.0487
	3. Electricity Supply	0.0753	0.0747

Table 5 presents the influence of exogenous petroleum import price shock on primary sectors. The four agriculture sub-sectors, that is, cash crops, other food crops, rice and animal, poultry, and bees, experience the least price effects. As expected, the total influence on prices is expected to be higher in the sectors that use petroleum more intensively. These sectors are forestry sector, and fishing and aquaculture with fuel intensities of 2.76% and 2.37%, respectively.

Table 5: Price Multipliers on primary sectors- the impact of a 1% increase in the import price of petroleum

Primary sectors	Price Multiplier (M')	
	Production price	Consumer price
1. Cash crops	0.0514	0.0367
2. Other Food crops	0.0490	0.0416
3. Rice	0.0487	0.0282
4. Animal, poultry and bees	0.0494	0.0474
5. forestry (timber, poles and other forestry)	0.0853	0.0746
6. Fishing and aquaculture	0.0736	0.0597

The results in Table 6 depict a varied range of price multipliers for both production and consumer prices of the manufacturing and processing sectors. Regarding production prices, the total effect is up to 0.207%. The lowest corresponds to the impact on manufacturing of tobacco products while the highest corresponds to the impact on mining and quarrying sector. For consumer prices, the lowest net price effect is on Manufacture of machinery and motor products (0.0025%) whereas the highest effect falls on the mining and quarrying sector. Naturally, the impact on the mining and quarrying sector is as expected since it is the most fuel intensive sensitive sector (with a fuel intensity of 14.34%). There are some sectors with high price multipliers despite recording low fuel intensities because they use intermediate inputs which are produced in sectors with high fuel intensities. For instance, 34% of the intermediate inputs used by the Manufacture of cement and ceramic products sector are from the mining and quarrying sector which explains its high multiplier (0.1103% for the production price), even though it has a low fuel intensity of 0.96%.

Table 6: Net Price Multipliers on Manufacturing and Processing sectors- the impact of a 1% increase in the import price of petroleum

Manufacturing and Processing sectors	Price Multiplier (M')	
	Production price	Consumer price
1. Mining and quarrying	0.2068	0.1726
2. Manufacturing of Food and Beverages	0.0636	0.0476
3. Manufacturing of Tobacco Products	0.0523	0.0304
4. Manufacturing of Textile, wearing apparel and leather products	0.1116	0.0460

(continued)

Table 6 continued

Manufacturing and Processing sectors	**Price Multiplier (M')**	
	Production price	**Consumer price**
5. Manufacture of Wood Straw and paper Products	0.1510	0.0528
6. Light Manufacturing of Other Products	0.0861	0.0644
7. Manufacturing of paint and other chemical products	0.1008	0.0276
8. Manufacture of Pharmaceuticals, medecinal Chemical &Botanical Products	0.0642	0.0285
9. Manufacturing of Rubber and Plastic Prods	0.1109	0.0411
10. Manufacturing of cement and ceramic products	0.1103	0.0653
11. Manufacture of metalic products	0.1061	0.0538
12. Manufacture of electrical and electronic products	0.1126	0.0067
13. Manufacture of machinery and motor products	0.1497	0.0025
14. Manufacturing of furniture	0.1315	0.1026
15. Heavy Manufacturing of Other Products	0.0975	0.0423

Table 7 presents the influence of a petroleum import price shock on the prices of services. Similar to the manufacturing and processing sectors, the services sectors report a wide range of price multipliers. Not surprisingly, the greatest influence is on road transport services at 0.2714 for its production price and 0.3233 for its consumer price, explained by its high fuel-intensity of 13.9%.

Table 7: Net Price Multipliers on Manufacturing and Processing sectors- the impact of a 1% increase in the import price of petroleum

Services	**Price Multiplier (M')**	
	Production price	**Consumer price**
1. Water Supply; Sewerage and Waste Management Activities	0.0656	0.0656
2. Agriculture support Services	0.0000	0.0494
3. Construction	0.1419	0.1406

(continued)

Table 7 continued	Price Multiplier (M')	
Services	**Production price**	**Consumer price**
4. Trade and repairs like retail, wholesale, repair of household items, etc	0.0825	0.8612
5. Transport by Rail, water, air transport and storage	0.1364	0.0700
6. Road transport services	0.2714	0.3233
7. Information and Communication	0.1159	0.1030
8. Accommodation and Food Service Activities	0.0583	0.0554
9. Recreational services	0.0899	0.0809
10. Business support services(consultancies, legal and accounting services, advertising research etc)	0.0823	0.0483
11. Financial and Insurance Activities	0.0966	0.0947
12. Real State Activities	0.0587	0.0580
13. Other Service Activities	0.0768	0.0669
14. Public Administration	0.0799	0.0785
15. Education	0.0636	0.0636
16. Human Health and Social Work Activities	0.0677	0.0676

Overall, the results in Tables 4-7 show that the production prices are more responsive to a petroleum import price shock, than consumer prices. This implies that the petroleum import price shock effect is mainly transmitted to the rest of the economy through an increase in the cost of production of activities. Naturally, the total influence on prices is expected to be highest in the sectors that are the most fuel-intensive and have strong linkages to the refined petroleum sectors, as shown by the magnitudes of their multipliers. This is aligned with our finding that the production prices of manufacturing and processing sectors are, on average, the most affected as compared to the other categories.

4.2 Decomposition of Price Multipliers

As described in section 3, the additive decompositions of the price multipliers in Table 8 show the 3 different categories of interdependence through which the impact of a petroleum import shock is transmitted. This information allows an extended analysis about the extent and magnitude of petroleum cost linkages across sectors, production factors, and the institutions. The table presents results for 10 sectors with the highest production price multipliers.

The results should be interpreted as follows: When the petroleum import price increases by 1%, it will generate an increase in the production price of road transport services amounting to 0.271% emanating from a 0.19 percentage point increase arising from transfer effects $(1 + N_1)$ between refined petroleum and road transport services (70% of the total effect), a 0.032 percentage point increase due to open-loop effects, N_2 (12% of the total effect) and finally a percentage point increase of 0.0498 originating from the closed-loop effects, N_3 (18% of the total effect).

Transfer effects $(1 + N_1)$ completely dominate the open-loop effects (N_2) and closed-loop effects (N_3) in these sectors which reflects the strong intersectoral cost linkages between refined petroleum and other activity sectors. This implies that the inflationary impact of the exogenous petroleum import price shock is mainly transmitted directly through increases in output prices of activity sectors that use petroleum as an intermediate input in the production process.

Table 8: Decomposition of Price Multipliers- the impact of a 1% increase in the import price of petroleum

Activity sector	$\boldsymbol{M'}$	$\boldsymbol{I + N_1}$	$\boldsymbol{N_2}$	$\boldsymbol{N_3}$
Road transport services	0.2714	0.1899 70%	0.0316 12%	0.0498 18%
Mining and quarrying	0.2068	0.1643 79%	0.0180 9%	0.0245 12%
Manufacture of Wood Straw and paper Products	0.1510	0.1064 70%	0.01624 11%	0.0283 19%
Manufacture of machinery and motor products	0.1497	0.0824 55%	0.0270 18%	0.0403 27%
Construction	0.1419	0.1041 73%	0.0138 10%	0.0240 17%

(Continued)

Table 8 (continued)

Activity sector	M'	$I + N_1$	N_2	N_3
Transport by rail, water, air transport and storage	0.1364	0.0887 65%	0.0187 14%	0.0290 21%
Manufacturing of furniture	0.1315	0.0876 67%	0.0171 13%	0.0268 20%
Information and communication	0.1159	0.0719 62%	0.0173 15%	0.0268 23%
Manufacture of electrical and electronic products	0.1126	0.0651 58%	0.0191 17%	0.0284 25%
Manufacturing of Textile, wearing apparel and leather products	0.1116	0.0708 63%	0.0162 15%	0.0246 22%

4.3 Impact on Production Price Index of a 35% petroleum import price shock

The Production price index (PPI) measures the change in prices paid by producers for inputs of goods and services used in the production of output. In this analysis, the change in PPI caused by a 35% petroleum import shock, is estimated. First, the changes in production and consumer prices of each sector are computed. Secondly, for each sector, the weighted share of output that is used for intermediate demand as a proportion of total value of intermediate demand, is determined. This weighted average is multiplied by the corresponding change in prices for each sector. The sum of these products gives us the change in PPI. The results show that PPI increases by a total of 3.24% due to a 35% petroleum import price shock. As mentioned in Section 2, based on the SAM, 78% of the refined petroleum demand is allocated to intermediate input requirements while 14% is directly consumed by households. Therefore, as expected, the PPI is more responsive than the CPI to the petroleum import price shock.

4.4 Impacts on Consumer Price Indices and distributional effects of a 35% petroleum import price shock

The consumer price index (CPI) measures the cost of acquiring the baseline basket of goods, before and after the price shock. The change in CPI thus captures the increase in the cost of living for households and thus provides useful information on the welfare effects on households as measured by changes in their consumption expenditures. Data on the specific expenditure patterns of the household groups classified by income quartiles and geographical area is obtained from the SAM, as depicted in Table B2. The change in CPI is computed by multiplying the household's consumption expenditure shares on commodities by the increase in the respective commodity prices (as caused by the petroleum import price shock). The sum of the products equates to the change in CPI. This implies that the change in the CPI depends on the relative shares of petroleum products and petroleum-intensive products in the household consumption bundle.

Table 9: Changes in CPI due to a 35% petroleum import price shock across household groups

Household type	Change in the CPI
Rural Q1	1.03%
Rural Q2	1.12%
Rural Q3	1.26%
Rural Q4	1.62%
Urban Q1	1.05%
Urban Q2	1.10%
Urban Q3	1.23%
Urban Q4	1.70%
All households	**1.38%**

In Table 9, on average, the increase in consumer price index is 1.38%. Based on household groups, the variations in the changes in CPI range between 1.03% for the rural households in the first quartile to 1.7% for the urban households in the fourth quartile. As expected for both urban and rural regions, the households in higher income quartiles are more affected by the petroleum import price shock, as compared to the low-income quartiles because of their higher consumption expenditure shares of petroleum. The increase in the CPI is largest for the urban households in the fourth quartile. The results show that the direct effects contribute the bulk of the total effects, even though refined petroleum only accounts for 1.03% of national household consumption demand (see Table B2). For the urban households in the fourth quartile, their direct consumption expenditure of refined petroleum is 2.13% which is much larger than for other groups (see Table B2). As regards the indirect effects, the differences in consumption patterns across the household groups are mostly on commodities, such as food crops, food and beverages and real estate (see Table B2), whose prices are minimally sensitive to changes in petroleum prices.

Our analysis implies that the direct consumption of petroleum products is responsible for the higher increase in CPI and the variations in the CPI across the household groups.

5. Concluding remarks

This paper particularly estimates the potential inflationary impact of an oil price shock on a net-importing developing economy. The study constructs a SAM Price multiplier model which enables us to trace the transmission of an oil price shock through its impact on production prices and consumer prices and ultimately measure the potential inflationary effects of an oil-price shock. The study further assesses the potential distributional impacts of the increase in petroleum prices on different household groups. An extended analysis about the extent and magnitude of petroleum cost linkages across sectors, production factors, and the institutions, is undertaken by decomposing the total price multipliers into additive components. The model is applied to Uganda's 2016/17 SAM.

From the analysis, the findings show that the influence of oil prices is very asymmetric and depends on the specific sector and/or agent analysed. As expected, the activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. The production prices are relatively more responsive to a petroleum import price shock than consumer prices. It is also noteworthy that the production prices of manufacturing and processing sectors are, on average, the most affected as compared to the other sector categories. This affirms that petroleum is an important component of production costs of the manufacturing and processing sectors, hence increases in petroleum prices could reduce the competitiveness of domestic products in global markets.

In this study, transfer effects measure how the exogenous petroleum import price shock results in direct increases in prices of production activities emanating from the rising cost of petroleum as an intermediate input. The results confirm that transfer effects completely dominate price influences in the activity sectors which reflects the strong intersectoral linkages between the refined petroleum sector and other production sectors. This implies that the inflationary impact of the exogenous petroleum import price shock is mainly transmitted directly through increases in prices of activity sectors that use petroleum as an intermediate input in the production process.

The results also show that the distributional impacts of rising petroleum prices tend to be progressive. For both urban and rural regions, the households in higher income quartiles are more affected by the petroleum import shock, as compared to the low-income quartiles. Thus, the study recommends that equity considerations are accounted for, as a basis for exploring plausible policy interventions to mitigate the impacts of future oil price shocks.

However, the results of this study should be interpreted with caution as there are limitations to the standard SAM Price Model applied, despite its usefulness in capturing price transmission mechanisms within an economy. The model ignores the likely substitution effects that may emanate from an oil price shock. For, instance an increase in domestic petroleum prices can induce firms to substitute petroleum for alternative inputs and for households to switch to cheaper energy products. Therefore, the results obtained from the model should be interpreted as an upper bound of the oil price shock impact in the short-term, before economic agents make responsive adjustments (Llop, 2018). The degrees of substitution may possibly differ somewhat across sectors, and among household groups. For further research, it would be interesting to analyse long-term price effects by simulating some degree of substitutability among production inputs and products for consumption as shown in Saari *et al*. (2016).

Uganda has 6.5 billion barrels of proven crude oil reserves, 1.4 billion of which are recoverable. Crude oil production is projected to begin in 2025, upon completion of the development stage. It would also be of interest to adjust Uganda's SAM to include a refinery sub-sector supplied by the domestic crude oil sector. Comparing our Paper 4 results to import substitution effects of domestic oil production, would be insightful.

Acknowledgements

I am thankful to Enock Bulime of Economic Policy Research Centre (EPRC), Uganda, for providing me with data. I am also grateful to Knut Einar Rosendahl, Wilson Asiimwe and Jens Bengtsson for their helpful comments.

Funding

This paper was supported within Norwegian Programme for Capacity Development in Higher Education and Research for Development (NORHED I) under the project- Capacity Building in Education and Research for Economic Governance, a partnership between Makerere University Business School and Norwegian University of Life Sciences. The funders had no

role in study design; in the collection, analysis and interpretation of data; in the writing of the report; or in the decision to submit the article for publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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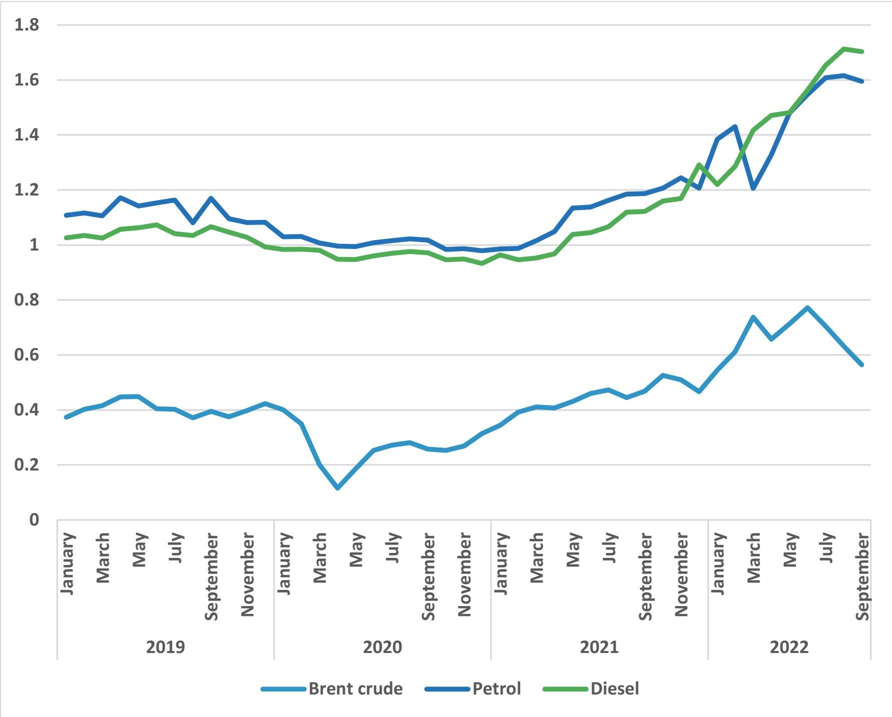
Appendices

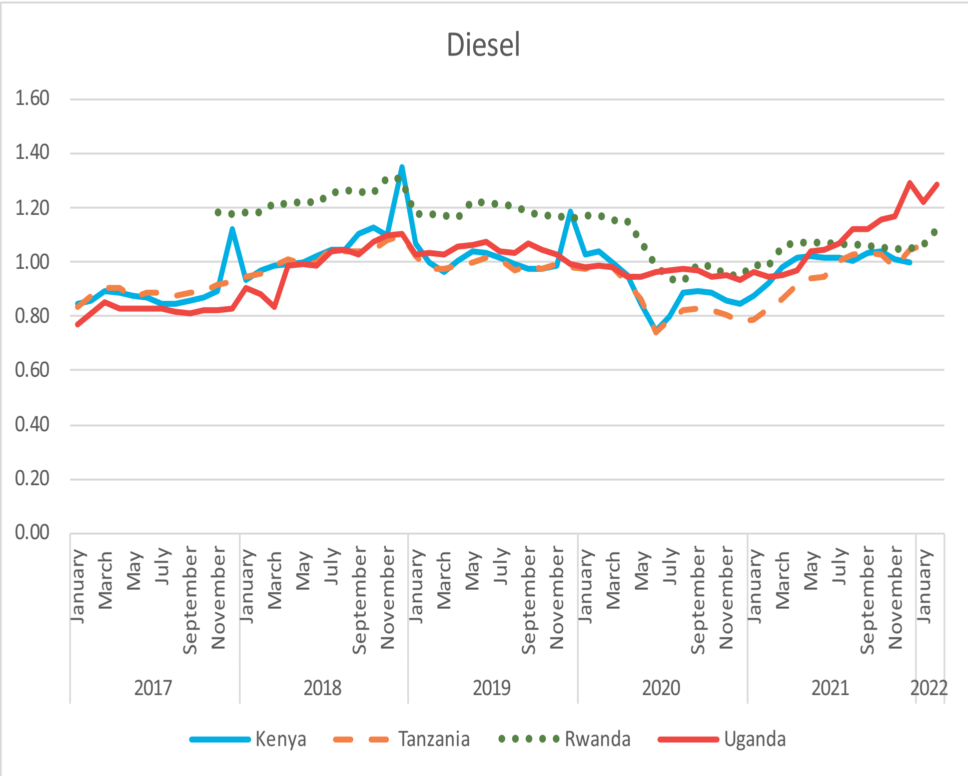
A: Trends of global crude oil prices and domestic fuel pump prices across East Africa

FigureA1: Relationship between trends of international oil price and domestic fuel pump prices in Uganda (USD per litre)



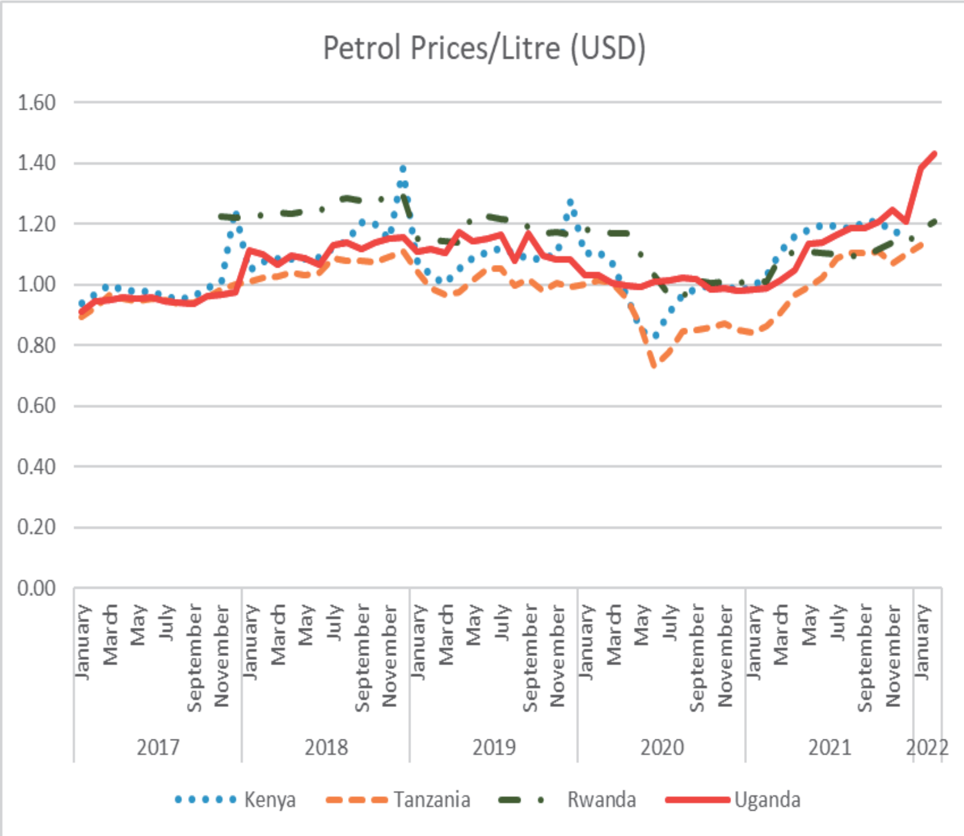
Source: Authors compilation. The Brent crude oil price data (per barrel of oil) was obtained from the U.S Energy Information Administration, EIA (2022). For comparison purposes with the domestic fuel pump prices, the Brent crude oil price was expressed as USD per litre (1 barrel = 159 litres). The data on Uganda's Petrol and diesel prices (as expressed in Uganda shillings) was obtained Uganda Bureau of Statistics (2022). The price data was then converted into USD using a period-specific average exchange rate. The official data on average exchange rate data was obtained from the website of the Central Bank of Uganda (Bank of Uganda, 2022).

Figure A2: Trends in average diesel prices (USD per litre) for 4 East African countries (for the period of January 2017-January 2018)



Source: With permission from (Odokonye and Bulime, 2022).

Figure A3: Trends in average gasoline prices (USD per litre) for 4 East African countries (for the period of January 2017-January 2022)



Source*:* With permission from (Odokonye and Bulime, 2022).

B. Fuel intensities and Consumption expenditure shares

Table B1: Fuel intensity for all activity sectors.

Refined petroleum	4.10%	Manufacture of machinery and motor products	5.56%
Cashcrops	0.02%	Manufacture of furniture	7.50%
Other Foodcrops	0.00%	Heavy manufacturing of other products	3.66%
Rice	0.00%	Electricity supply- Hydro	2.02%
Animal, Poultry and bees	0.00%	Electricity supply- Solar	2.02%
Agriculture support Services	0.00%	Electricity supply- Thermal	2.02%
Forestry (timber, poles and other forestry)	2.76%	Electricity supply- Other	2.02%
Firewood	2.24%	Water supply, Sewage and waste management activities	1.25%
Charcoal	0.97%	Construction	7.99%
Fishing and aquaculture	2.37%	Trade and Repairs (like retail, wholesale, repair of household items, motors, computers etc)	2.81%
Mining and quarrying	14.34%	Road transport services	13.88%
Manufacture of food and beverages	1.49%	Information and Communication	5.05%
Manufacture of tobacco products	0.65%	Accommodation and Food Services	0.15%
Manufacture of Leather, wearing apparel and leather products	5.48%	Recreational services	2.05%
Manufacture of wood straw and paper products	8.91%	Business support services (consultancies, legal and accounting services, advertising, research etc)	1.94%
Light manufacturing of other products	2.95%	Financial and insurance activities	2.66%
Manufacture of paint and other chemical products	5.14%	Real estate activities	0.36%
Manufacture of Pharmaceuticals, medicinal, chemical and botanical products	0.83%	Other service activities	1.42%
Manufacture of rubber and plastic products	5.60%	Public Administration	1.11%
Manufacture of cement and ceramic products	0.96%	Education	0.71%
Manufacture of metallic products	1.14%	Human Health and Social work activities	1.18%
Manufacture of electrical and electronic products	5.18%	Transport by Rail, water, air transport and storage	7.33%

Source: Author's computations based on Uganda's SAM 2016/17

Table B2: Consumption expenditure shares for each household group (%)

Commodity	Rural Q1	Rural Q2	Rural Q3	Rural Q4	Urban Q1	Urban Q2	Urban Q3	Urban Q4	Overall
Refined petroleum	0.04	0.15	0.50	1.83	0.08	0.10	0.39	2.13	1.03
Cashcrops	4.00	1.92	0.90	0.26	2.89	1.01	0.60	0.14	1.11
Other Foodcrops	24.92	22.41	17.82	10.54	23.53	20.36	14.74	7.52	15.24
Rice	0.10	0.14	0.15	0.11	0.21	0.36	0.32	0.15	0.16
Animal, Poultry and bees	1.43	2.20	2.61	2.12	1.01	1.53	2.10	1.82	2.01
Agriculture support Services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Forestry (timber, poles and other forestry)	0.05	0.04	0.03	0.01	0.04	0.02	0.01	0.00	0.02
Firewood	6.30	4.38	2.88	1.25	4.72	2.40	0.93	0.22	2.36
Charcoal	0.28	0.59	0.94	0.83	2.53	3.58	3.32	1.69	1.30
Fishing and aquaculture	5.35	4.44	3.44	1.80	4.47	3.90	3.40	1.78	3.11
Mining and quarrying	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Manufacture of food and beverages	22.62	22.40	24.42	19.10	18.72	20.97	23.48	16.19	20.50
Manufacture of tobacco products	0.77	0.67	0.56	0.31	0.59	0.48	0.32	0.15	0.43
Manufacture of Leather, wearing apparel and leather products	2.54	2.89	3.01	2.97	3.03	3.23	4.11	3.72	3.21
Manufacture of wood straw and paper products	0.02	0.07	0.25	0.54	0.10	0.43	0.82	1.10	0.52
Light manufacturing of other products	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Manufacture of Pharmaceuticals, medicinal, chemical and botanical products	5.17	6.02	6.97	7.36	5.01	6.54	5.11	3.99	5.72
(continued)									

Table B1 continued **Commodity**	**Rural Q1**	**Rural Q2**	**Rural Q3**	**Rural Q4**	**Urban Q1**	**Urban Q2**	**Urban Q3**	**Urban Q4**	**Overall**
Manufacture of rubber and plastic products	1.30	1.09	0.97	1.20	0.84	0.89	0.87	0.76	1.00
Manufacture of cement and ceramic products	0.10	0.09	0.10	0.08	0.13	0.15	0.14	0.12	0.10
Manufacture of metallic products	0.04	0.04	0.06	0.05	0.03	0.03	0.04	0.03	0.04
Manufacture of electrical and electronic products	1.54	1.62	1.60	1.31	1.18	1.20	1.45	1.30	1.43
Manufacture of machinery and motor products	0.01	0.05	0.13	0.85	0.02	0.05	0.06	0.65	0.36
Manufacture of furniture	0.11	0.17	0.23	0.41	0.09	0.07	0.21	0.41	0.28
Heavy manufacturing of other products	0.27	0.25	0.27	0.22	0.37	0.42	0.38	0.32	0.29
Electricity supply-Hydro	0.01	0.10	0.21	0.33	0.25	0.71	0.96	1.05	0.49
Electricity supply-Solar	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00
Electricity supply-Thermal	0.00	0.01	0.02	0.03	0.02	0.06	0.09	0.09	0.04
Electricity supply-Other	0.00	0.01	0.01	0.02	0.01	0.04	0.05	0.06	0.03
Water supply, Sewage and waste management activities	4.45	4.06	3.67	2.51	5.09	4.69	3.71	2.45	3.35
Construction	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Trade and Repairs	0.12	0.18	0.26	0.55	0.16	0.13	0.20	0.42	0.32
Transport by Rail, water, air transport and storage	0.58	0.99	1.39	1.56	0.42	0.75	1.21	1.49	1.24

(continued)

Table B2 continued **Commodity**	**Rural Q1**	**Rural Q2**	**Rural Q3**	**Rural Q4**	**Urban Q1**	**Urban Q2**	**Urban Q3**	**Urban Q4**	**Overall**
Road transport services	1.10	1.88	2.65	2.98	0.80	1.44	2.30	2.83	2.36
Information and Communication	0.41	0.79	1.46	2.70	0.59	0.94	1.40	4.40	2.20
Accommodation and Food Services	0.99	1.57	2.35	6.91	1.12	1.97	4.20	14.51	6.20
Recreational services	0.07	0.11	0.24	0.36	0.02	0.28	0.41	0.92	0.41
Business support services	0.04	0.07	0.08	0.14	0.03	0.06	0.12	0.21	0.12
Financial and insurance activities	0.34	0.44	0.62	0.91	0.49	0.75	0.97	1.12	0.78
Real estate activities	6.19	5.82	6.03	9.66	7.80	5.73	7.98	10.91	8.13
Other service activities	0.65	1.27	2.40	5.37	0.94	1.60	2.34	3.50	2.80
Public Administration	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Education	3.73	4.64	5.28	6.49	5.33	7.32	6.29	7.15	5.87
Human Health and Social work activities	2.29	4.74	4.14	5.24	4.93	3.92	3.35	3.53	3.99
Total	100	100	100	100	100	100	100	100	100

Source: Author's computations based on Uganda's 2016/17 SAM. The last column presents the overall household consumption expenditure by all households for each commodity.

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Thesis Number: 2023:24
ISSN: 1894-6402
ISBN: 978-82-575-2054-0

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Her thesis investigates the influence of oil price uncertainty on optimal investment decisions and the economy-wide effects of oil price shocks in four independent related research papers.

The first three papers employ real options methods to assess when and whether decision makers should invest in the development stage and subsequently when to begin oil production. To generate numerical results, the models are applied to the case of Uganda's undeveloped oil reserves. Overall, the thesis establishes that uncertainties faced by Uganda's oil project have profound impacts on the project values and optimal strategies. The IOCs and the government have conflicting interests and may bias their decisions if they neglect these project uncertainties. This thesis recommends that the PSA should be designed in a way that allows for flexibilities in the event that risks arise.

The last paper applies the Social Accounting Matrix (SAM) Price Model to estimate the potential inflationary effects of an oil price shock in Uganda's economy. The thesis finds that the activity sectors with high fuel-intensities recorded the highest responses to the petroleum import price shock. The results also establish that the distributional impacts of rising petroleum prices tend to be progressive since the poorest households are the least affected compared to the higher income households. The thesis recommends that policy interventions should be tailored in a way that enhances competitiveness in sectors most affected by oil price shocks and without worsening the welfare of households that are most sensitive to the oil price shocks.

Supervisors: Prof. Knut Einar Rosendahl and Assoc. Prof. Jens Bengtsson

ISBN:978-82-575-2054-0
ISSN: 1894-6402

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