Gravity Without Newton's Gravitational Constant and No Knowledge of the Mass Size

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Abstract

In this paper, we show that the Schwarzschild radius can be extracted easily from any gravitationally-linked phenomena without having knowledge of Newton's gravitational constant or the mass size of the gravitational object. Further, the Schwarzschild radius can be used to predict a long series of gravity phenomena accurately, again without knowledge of Newton's gravitational constant and also without knowledge of the size of the mass, although this may seem surprising at first.

Hidden within the Schwarzschild radius are the more fundamental mass of the gravitational object, the Planck length, which we will assert contain the secret essence related to gravity, in addition to the speed of light (the speed of gravity). This seems to support that gravity is quantized, even at the cosmological scale, and this quantization is directly linked to the Planck units. This also supports our view that Newton's gravitational constant is a universal composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$, rather than relying on the Planck units as a function of G. This does not mean that Newton's gravitational constant is not a universal constant, but rather that it is a composite universal constant, which depends on the Planck length, the speed of light, and the Planck constant.

This is, to our knowledge, the first paper[1] that shows how a long series of major gravity predictions and measurements can be completed without any knowledge of the mass size of the object, or Newton's gravitational constant. At minimum, we think it provides an interesting new angle for evaluating existing theories of gravitation.

Index Terms

Newton's gravitational constant, Planck length, Planck mass, Schwarzschild radius.

I. Introduction

Let's examine the following formula:

$$r_h = \frac{gR^2}{c^2} \tag{1}$$

Here, g is the gravitational acceleration, R is the radius from the center of the planet (gravitational mass) to the surface, and c is the speed of light. The gravitational acceleration is easy to measure without any knowledge of gravity; it is about $9.8\ m/s^2$. For example, we can simply drop a ball from height H above the ground and measure the time this took before it hit the ground; now we have the gravitational acceleration from the well known relation: $g = \frac{2H}{T^2}$.

The radius of Earth is about 6,371,000 meters. As for the speed of light, we can measure it with a low cost kit and or just take the standard accepted speed, which is defined as exactly 299,792,458 m/s. The main point is that one needs no knowledge of gravity or the gravitational constant to measure each of these input factors.

Next we plug these values into the formula above and get:

$$r_h = \frac{9.8 \times 6371000^2}{299792458^2} \approx 0.0044 \tag{2}$$

Some will recognize that this is very similar to half of the value of the Schwarzschild radius of Earth; this is not a coincidence, as that is indeed exactly what it is. Half the Schwarzschild radius is identical to the radius where the escape velocity is c when we take into account Lorentz [2] relativistic mass as has been derived by Haug [3]. That is, we also have $r_h = \frac{gR^2}{c^2} = \frac{\frac{GM}{R^2}R^2}{c^2} = \frac{GM}{c^2} = \frac{1}{2}r_s$.

Next, we can use this value of r_h and plug it into any of the formulas below to calculate almost any major gravity predictions. We can predict the orbital velocity of a satellite or the moon, for example, by the formula:

$$v_o = c\sqrt{\frac{r_h}{R_o}} \tag{3}$$

where R_o is the radius from the center of Earth to the object for which we want to predict orbital velocity. Further, the time dilation between two clocks at different altitudes around a planet is given by:

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$$\frac{T_h}{T_L} = \frac{\sqrt{1 - 2\frac{r_h}{R_L}}}{\sqrt{1 - 2\frac{r_h}{R_h}}} \tag{4}$$

where R_h is the radius further from the center of Earth than R_L. We can test this by placing one atomic clock at sea level and one at the top of a 2,000-meter mountain. We need to synchronize the clocks before performing this task. The clocks will be consistent with our gravity prediction. Again, all we need is r_h, which we can easily extract from the gravitational acceleration on the surface of Earth.

Next we can predict the redshift; it is given by using the following formula:

$$\lim_{r \to +\infty} z(r) = \frac{1}{\sqrt{1 - \frac{2r_h}{R}}} - 1 \tag{5}$$

If both the emitter and the receiver are inside the gravitational field and we focus on frequency rather than change in wavelength, we have the well-known formula:

$$f_h = f_L \frac{\sqrt{1 - \frac{2r_h}{R_L}}}{\sqrt{1 - \frac{2r_h}{R_h}}} \tag{6}$$

Based on this, we can accurately predict the results of experimental set-ups equal to the Pound Rebka experiment [4]. This is just one example of how we can perform a series of gravitational predictions that can be confirmed by experiment without any knowledge of Newton's gravitational constant or the mass size of any object. What we have relied on instead is r_h, which can simply be obtained from the gravitational acceleration at the surface of Earth, the speed of light, and the radius of Earth.

II. The Schwarzschild Radius in a New Perspective

The Schwarzschild radius comes from the Schwarzschild metric [5], [6] solution of Einstein's field equation [7] and is given by:

$$r_s = \frac{2GM}{c^2} \tag{7}$$

where G is Newton's gravitational constant, M is the mass of the object in kilogram, and c is the speed of light. In other words, we need to know the mass of the object of interest and Newton's gravitational constant in order to find its Schwarzschild radius using this formula. The escape velocity (see [8]) from a mass M at the radius R from the center of the mass is given by:

$$v_e = \sqrt{\frac{2GM}{R}} \tag{8}$$

When we replace the radius in the escape velocity with the Schwarzschild radius $r = r_s = \frac{2GM}{c^2}$, we get:

$$v_e = \sqrt{\frac{2GM}{\frac{2GM}{c^2}}} = c \tag{9}$$

Thus, if an object with mass M is packed inside the Schwarzschild radius, then we will have a mass from which even light cannot escape. This phenomenon is commonly known as a black hole, and the Schwarzschild radius is often linked to black holes.

It is important to be aware of the history of science on this topic. In 1783, the geologist John Michell noted that when the escape velocity calculated from Newton's gravitational theory was equal to the speed of light, the star would be dark [9]. That is, not even light could escape such a gravity-dense object. Since the escape velocities one gets from Newtonian theory and from general relativity theory are the same [8], this means that one can easily calculate a radius identical to the Schwarzschild radius from Newtonian theory. One could even argue that it should be called the "Michell radius", as he was the first to point out the possibility of an escape velocity equal to the speed of light and its corresponding radius (equivalent to the Schwarzschild radius), and noted how this would lead to something special: dark stars.

However, the two theories differ strongly in their interpretation of what this special "dark" object is. In general, both theories agree that the gravitational force would be so strong that either only light, or perhaps not even light, could escape, while GR has a large theory around black holes. In this paper, we will not address the interpretation of black holes, but the background information is important because when we discuss the Schwarzschild radius, we are speaking about it in broad terms—there is a special radius that can also be found from Newtonian-type gravity theory where the escape velocity is equal to the speed of light.

Any object we have observed directly in the sky or on Earth has mass for which the radius extends outside the Schwarzschild radius. In other words, no mass has directly been detected that has all of its mass inside the Schwarzschild radius, even though recent gravitational wave detections may have observed collisions of black hole-like objects.

The Schwarzschild radius can be found, as described above, from the gravitational acceleration of Earth, or from the measured orbital velocity of a satellite, such as the moon, by simply using the formula:

$$r_s = 2r_h = 2\frac{v_o^2 R}{c^2} \tag{10}$$

where R is now the radius from the center of Earth to the orbital object of interest. Further, v_o is the "easily" observed orbital velocity of the moon, for example. Alternatively, we could use two atomic clocks, measure the time dilation between them, and then plug the values into this formula to find the Schwarzschild radius:

$$r_h = R\left(1 - \frac{T_0^2}{T_f^2}\right) \tag{11}$$

Where T_f is a clock far distant from the gravity field and T_0 is a clock placed at radius R relative to the gravitational object: $T_0 = T_f\sqrt{1-\frac{2r_h}{R}} = T_f\sqrt{1-\frac{r_s}{R}}$. Naturally, we do not have access to a far-away clock T_f from Earth, but we can certainly have two clocks on Earth at altitude R_L and R_h, and from this we can calculate r_h by solving the following equation with respect to r_h:

$$\frac{T_h}{T_L} = \frac{\sqrt{1-2\frac{r_h}{R_L}}}{\sqrt{1-2\frac{r_h}{R_h}}} \tag{12}$$

this gives

$$r_h = \frac{R_h R_L (T_h^2 - T_L^2)}{2(R_L T_h^2 - R_h T_L^2)} \tag{13}$$

In 2016, Haug [10], [11] suggested that the gravitational constant is likely a universal composite constant of the form:

$$G = \frac{l_p^2 c^3}{\hbar} \tag{14}$$

This is simply the Planck length formula of Max Planck [12], [13]: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, solved with respect to G. This formula is similar to the composite constant suggested by Cahill [14], [15] in 1984 and, for example, also independently by McCulloch [16] in 2014: $G = \frac{\hbar c}{m_p^2}$, which is simply the Planck mass formula, $m_p = \sqrt{\frac{\hbar c}{G}}$, solved for G. However, it is generally assumed that we must know G in order to calculate any Planck units such as the Planck length and the Planck mass. McCulloch [17] has also assumed that one needs to know G, and therefore we get a circular problem.

> *In the above gravitational derivation, the correct value for the gravitational constant G can only be obtained when it is assumed that the gravitational interaction occurs between whole multiples of the Planck mass, but this last part of the derivation involves some circular reasoning, since the Planck mass is defined using the value for G.*
> — McCulloch 2016

Similarly, it was pointed out, at least by 1987, by Cohen [18] that one needs to know G to find the Planck units. So this just reflects what has been the view until recent years.

However, recently we have shown how the Planck length, the Planck time, and the Planck mass, both theoretically and experimentally, can be found easily with no knowledge of Newton's gravitational constant [19], [20], [21]; something that we, in all modesty, must say we think is quite a breakthrough. This means the way of representing the gravity constant as a composite constant is fully valid and very useful.

That G is a composite constant and that the Planck length can be found independently of G leads to an evaluation of the Schwarzschild radius at a deeper level by:

$$r_h = \frac{1}{2}r_s = \frac{GM}{c^2} = \frac{\frac{l_p^2 c^3}{\hbar}M}{c^2} = Nl_p \tag{15}$$

where N is the number of Planck masses in the mass M. The end result in the equation above is easier to see if we first solve the reduced Compton wavelength formula, $\bar{\lambda} = \frac{\hbar}{Mc}$ with respect to M, as this gives:

$$M = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{16}$$

This means we have:

$$r_h = \frac{1}{2} r_s = \frac{GM}{c^2} = \frac{\frac{l_p^2 c^3}{\hbar}}{c^2} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = \frac{\frac{\hbar}{\bar{\lambda}} \frac{1}{c}}{\frac{\hbar}{l_p} \frac{1}{c}} l_p = \frac{M}{m_p} l_p = N l_p \tag{17}$$

It is interesting to note that this also is equal to:

$$r_h = \frac{1}{2} r_s = \frac{GM}{c^2} = \frac{\frac{l_p^2 c^3}{\hbar}}{c^2} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = \frac{\frac{c}{\bar{\lambda}}}{\frac{c}{l_p}} l_p = \frac{t_c}{t_p} l_p = \frac{l_p}{\bar{\lambda}} = \frac{M}{m_p} l_p = N l_p \tag{18}$$

That is, the Compton time t_c is divided by the Planck time t_p multiplied by the Planck length. Haug [22], [23], [24] has recently shown that $\frac{l_p}{\bar{\lambda}}$ corresponds to the number of Planck mass events in the mass of interest (in this case the gravitational mass M), and when it is an integer it represents the number of such events in the Planck mass over a hypothetical observation window equal to the Planck time. When smaller than one, it represents the probability to observe such an event in the Planck time observational time-window. For masses smaller than the Planck mass, this is less than one. This means large masses are deterministic in form of gravity, and masses considerably smaller than the Planck mass are dominated by probability. In other words, this binds the quantum and probability aspects of physics together in one concept.

The idea that we can find the Schwarzschild radius with no knowledge of G or even the mass and use this to predict "all" known gravity phenomena is new. For gravity phenomena, the combined measurement of N and l_p is important; in this case, we do not need to know N (the gravity mass quantization number) or l_p separately, but the combination of the two, Nl_p, will be sufficient to predict observable gravity phenomena. It has, however, recently been shown that the gravitational quantization number N can be found independently of any knowledge of G, $\hbar$ or c, see [25].

III. Is Newton's Gravitational Constant a Composite Constant?

There are several reasons to question whether or not Newton's gravitational constant is a composite constant [26], including the following points.

1) If we "never" need Newton's gravitational constant for any gravitation observations, not even when calibrating a model, does this imply that it is not central for gravity either? See Table 1 for a series of calculations and observations that can be completed without any knowledge of Newton's gravitational constant.
2) The output units of Newton's gravitational constant are given by $m^3 \cdot kg^{-1} \cdot s^{-2}$. It would seem strange if something fundamental existed at the deepest level that is meters cubed, divided by kg and seconds squared. It cannot be excluded, but one should first attempt to find a simpler explanation. We will claim this strongly indicates that Newton's gravitational constant must be a composite universal constant consisting of more fundamental constants.
3) By reformulating G as a composite of the form $G = \frac{l_p^2 c^3}{\hbar}$, a series of Planck units are simplified and become more logical. Take, for example, the Planck time, which is described as $t_p = \sqrt{\frac{G\hbar}{c^5}}$. Such formulas give minimal intuition. We may ask, what is the meaning of c^5 and what is the deeper logic behind the gravitational constant? When replacing G with its composite form, we simply show the Planck time as $t_p = \frac{l_p}{c}$, so the time it takes for light to travel the Planck length, which is naturally well-known.
4) The Planck length and the Planck time can be measured totally independently of any knowledge of Newton's gravitational constant and also independently of knowledge of $\hbar$, as recently shown by Haug [19], [20], [27]. This means all the elements of a composite Newtonian gravitational constant are known. At a fundamental level, it seems more logical that there exists a unique and likely shortest possible length, namely the Planck length, as well as the speed of light (gravity). In addition, embedded the Planck constant is the gravitational constant, which is more complex, but in all observable gravity phenomena, the gravitational constant cancels out. Then we are left with the Schwarzschild radius (or half of this radius in most cases) as the essential thing to know and that can be measured easily. Again, this consists of the number of Planck masses times the Planck length in the gravitational object of interest. In other words, quantized gravity. It is also worth mentioning that the Planck mass is simply given by $m_p = \frac{\hbar}{l_p} \frac{1}{c} \approx 2.17 \times 10^{-8}$ kg, so we also do not need any knowledge of G to find the Planck mass or the Planck time when we know the Planck length. For the Planck mass in kg, in general we also need to know the Planck constant, but this can be found from the Watt balance; see [28], [29], [30]. The Planck constant can also be found and understood in another way, as recently outlined by [31].

In the table, you can also see formulas for the Planck length and Planck time, for which one needs to know the mass size of the large balls in the Cavendish apparatus. This can be done by simply weighing it. Already in 1684, Newton [32] stated that mass and weight are proportional for masses measured in the same gravitational field.

TABLE I
THE TABLE SHOWS THAT THE MOST COMMON GRAVITATIONAL MEASUREMENTS AND PREDICTIONS CAN BE DONE WITHOUT ANY KNOWLEDGE OF NEWTON'S GRAVITATIONAL CONSTANT. ONLY WHEN WE WANT TO SEPARATE OUT THE PLANCK UNITS OR THE GRAVITATIONAL CONSTANT WE NEED TO KNOW THE MASS SIZE OF THE GRAVITATIONAL OBJECT.

What to measure/predict	Formula	How	Is it easy to do?	Knowledge of mass size
Half Schwarzschild radius	$r_h = \frac{gR^2}{c^2}$	From g (9.8 m/s^2 Earth)	Yes	No
Half Schwarzschild radius	$r_h = \frac{v_o^2 R_o}{c^2}$	From orbital velocity	Yes	No
Half Schwarzschild radius	$r_h = R\sqrt{1 - \frac{T_0^2}{T_f^2}}$	From time dilation	Yes	No
	need high precision clocks	need far away clock		
Half Schwarzschild radius	$r_h = \frac{R_h R_L (T_h^2 - T_L^2)}{2(R_L T_h^2 - R_h T_L^2)}$	From time dilation	Yes	No
		need high precision clocks		
Half Schwarzschild radius	$r_h = \frac{R_h R_L (f_h^2 - f_L^2)}{2(R_L f_h^2 - R_h f_L^2)}$	From redshift	Yes	No
Half Schwarzschild radius	$r_h = \frac{\delta R}{4}$	From light-bending	less so "need" eclipse	No
Gravitational acceleration field	$g = \frac{r_h}{R^2} c^2$	Find r_h first	Yes	No
Orbital velocity	$v_o = c\sqrt{\frac{r_h}{R}}$	Find r_h first	Yes	No
Escape velocity	$v_e = c\sqrt{2\frac{r_h}{R}}$	Find r_h first	Yes	No
Time dilation	$t_2 = t_1\sqrt{1 - 2\frac{r_h}{R}}$	Find r_h first	Yes	No
GR bending of light	$\delta = 4\frac{r_h}{R}$	Find r_h first	Yes	No
Gravitational redshift	$\lim_{R\to+\infty} z(R) = \frac{r_h}{R}$	Find r_h first	Yes	No
Bekenstein-Hawking luminosity	$P = \frac{1}{15360\pi} \frac{\hbar c^2}{r_h^2}$	Find r_h first	Yes	No
Schwarzschild radius of the Cavendish sphere	$r_s = \frac{4L\pi^2 R^2 \theta}{c^2 T^2}$	Cavendish apparatus	Yes	No
Planck mass	$m_p = \sqrt{\frac{\hbar c M T^2}{L 2\pi^2 R^2 \theta}}$	Cavendish apparatus	Yes	Yes
Planck length	$l_p = \sqrt{\frac{\hbar L 2\pi^2 R^2 \theta}{M T^2 c^3}}$	Cavendish apparatus	Yes	Yes
Planck time	$t_p = \sqrt{\frac{\hbar L 2\pi^2 R^2 \theta}{M T^2 c^5}}$	Cavendish apparatus	Yes	Yes
Gravitational constant	$G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar} \approx 6.67 \times 10^{-11}$			
	$G = \frac{L 2\pi^2 R^2 \theta}{M T^2}$	Cavendish apparatus	Yes	Yes

IV. CAVENDISH APPARATUS TO FIND THE SCHWARZSCHILD RADIUS

To find Newton's gravitational constant in a Cavendish [33] apparatus, we need to know the mass of the large lead balls first, in relation to their weight.

$$G = \frac{2L\pi^2 r^2 \theta}{MT^2} \tag{19}$$

where θ is the angle and L is the distance between the two small lead balls hanging in a wire, T is the oscillation time period, M is the mass of one of the two identical, large lead balls, and r is the radius from center to center between a small lead ball and a large lead ball. Note that Cavendish himself never used or suggested using such apparatus to measure the so-called Newtonian gravitational constant (see Clotfelter [34] and Hodges [35]), but we can use such an apparatus to deduct it, which is well known.

Next, in order to measure the Schwarzschild radius of the large lead ball, we only need the following formula:

$$r_s = \frac{4L\pi^2 r^2 \theta}{c^2 T^2} \tag{20}$$

In this formula, there is no mass, but instead (compared to the formula for G) we have the speed of light. Also, remember $r_s = 2\frac{M}{m_p} l_p$ and further $r_h = \frac{M}{m_p} l_p$. So, embedded in the Schwarzschild radius is a mass ratio consisting of the mass of the gravitational object divided by the Planck mass multiplied by the Planck length. We also have $r_s = 2\frac{t_c}{t_p} l_p$ and further $r_h = \frac{t_c}{t_p} l_p$, where t_c is the reduced Compton time in the mass M, that is $t_c = \frac{\bar{\lambda}}{c}$. All of the above strongly indicates that gravity is quantized and related to Planck mass events; see [23], [24] for a new suggested quantum gravity theory.

V. COMPARING THE SCHWARZSCHILD RADIUS WITH THE GRAVITATIONAL CONSTANT

To find Newton's gravitational constant in a Cavendish apparatus, we need to know the mass of the large lead balls first, in relation to their weight. In our view, the reason for this is that the embedded gravitational constant also contains the reduced Planck constant, $G = \frac{l_p^2 c^3}{\hbar}$; see [26]. Mass in the form of kilogram is clearly related to the Planck constant. The Schwarzschild radius and also the radius where the escape velocity is c, when taking into account Lorentz relativistic mass, on the other hand, is just a length of the respectively $r_s = 2Nl_p$ and $r_h = Nl_p$, where N is a mass ratio, namely the mass of the gravitational object divided by the Planck mass. Not only that, but N is also the number of Planck events per Planck time; see [24]. The Schwarzschild radius contains essential information about mass and also strongly indicates that gravity is linked to the Planck length and that gravity is quantized.

VI. Conclusion

We have shown how a long series of gravity predictions and measurements are totally independent of knowledge of the Newton gravitational constant, or the (kilogram) size of the mass in question. One component is (half) of the Schwarzschild radius, which is the number of Planck masses in the gravitational object multiplied by the Planck length. However, for most gravitational observations and predictions we do not need to reduce the Schwarzschild radius into these fundamental components.

We also show that we do not need any knowledge of the mass of the gravitational object or Newton's gravitational constant to find the Schwarzschild radius of a cosmological object, or even a small clump of matter on Earth. This strongly supports our view that Newton's gravitational constant is a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$. In understanding this, we may gain a deeper understanding of the link between the quantum world and the macroscopic world in terms of gravity. If our theory is correct, then we are able to perform a series of accurate gravity predictions based on measuring the gravitational acceleration on Earth alone, without any knowledge of G or the mass of the object.

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