

Review

Progress in the Composite View of the Newton Gravitational Constant and Its Link to the Planck Scale

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Abstract: The Newtonian gravity constant G plays a central role in gravitational theory. Researchers have, since at least the 1980s, tried to see if the Newton gravitational constant can be expressed or replaced with more fundamental units, such as the Planck units. However, it was already pointed out in 1987 that this led to a circular problem; namely, that one must know G to find the Planck units, and that it is therefore of little or no use to express G through the Planck units. This is a view repeated in the literature in recent years, and is held by the physics' community. However, we will claim that the circular problem was solved a few years ago. In addition, when one expresses the mass from the Compton wavelength formula, this leads to the conclusion that the three universal constants of G , h , and c now can be replaced with only l_p and c to predict observable gravitational phenomena. While there have been several review papers on the Newton gravitational constant, for example, about how to measure it, we have not found a single review paper on the composite view of the gravitational constant. This paper will review the history of, as well as recent progress in, the composite view of the gravitational constant. This should hopefully be a useful supplement in the ongoing research for understanding and discussion of Newton's gravitational constant.

Keywords: Newton gravitational constant; Planck units; composite constant; gravity; mass; quantum gravity; cosmology



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1. Short History on the Newton Gravitational Constant and the Planck Units

Newton's gravitational constant plays an important role in almost any gravity calculation. However, Newton actually never introduced or used a gravitational constant [1] in his gravitational force formula. His formula was stated by words in Principia [2] as $F = \frac{M\bar{m}}{R^2}$. This is equivalent to today's gravitational force formula without the gravitational constant. Well, almost so, as we are, on purpose, using the notation \bar{M} and \bar{m} for the two masses, rather than M and m . This is because Newton had a quite different view of mass than the view held today. Even without a gravity constant, Newton was able to do many predictions, such as finding the relative mass between planets and the sun, see also Cohen [3]. He also found the relative density of Earth relative to the sun to be a value very close to what is known today. What he tried but failed to do was to find the density of Earth relative to a known substance, such as water, lead, or gold.

Modern physicists often consider Cavendish [4] in 1798 to be the first to measure the gravitational constant. However, Cavendish, in his paper, did not describe a gravitational constant or use one as also pointed out by Clotfelter [5]. What Cavendish did was to measure the density of Earth relative to the density of a known substance, which Newton had tried but failed to do. Cavendish succeeded by using a torsion balance, also known today as a Cavendish apparatus. The main point of using such an apparatus is to measure the gravity effects from a human-sized object; that is, due to the balls in the Cavendish apparatus, one can easily control what kind of substance the mass (the large spheres in the apparatus) are made of, for example, lead. Then by next comparing the gravitational effect measured in the apparatus with the gravitational effect of Earth, one can find the density of

Earth relative to the density of a known substance. Such an apparatus can also be used to extract the gravitational constant.

The so-called Newton gravitational constant was first introduced in 1873 by the two French physicists Cornu and Baille [6]. In their paper, they gave the formula as:

$$F = f \frac{Mm}{R^2} \quad (1)$$

where f is the gravitational constant. Big G as the notation of the gravity constant was likely first introduced by Boys [7] in 1894. It took many years before the notation G became standard in the international physics' community; for example, Max Planck [8] used f for the gravitational constant as late as 1928, and Einstein used notation k in 1916. Naturally, whether one uses f , k or G as a symbol for the gravitational constant is purely cosmetic. What is important to bear in mind is that the gravitational constant is relatively new (at least compared to Newton's Principia) and that it also came into existence at about the same time that the kilogram became the international standard mass. For a more in-depth historical perspective on the Newton gravitational constant, see [9].

In 1899, a few years after the invention of the Newtonian gravitational constant, Max Planck [10,11] introduced the Planck units. He assumed there were three important universal constants: G , c , and h , and then used dimensional analysis to derive a unique length $l_p = \sqrt{\frac{Gh}{c^3}}$, time $t_p = \sqrt{\frac{Gh}{c^5}}$, mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_b^2}}$. Today these are known as the Planck units. In 1916, Einstein [12] already suggest that the next step forward in gravity would be quantum gravity. Eddington [13] was, in 1918, suggesting that quantum gravity must be linked to the Planck scale or, in his own words:

But it is evident that this length (the Planck length) must be the key to some essential structure. It may not be an unattainable hope that someday a clearer knowledge of the process of gravitation may be reached?

However, Eddington's idea was criticized by Bridgeman [14] in 1931. Bridgeman (who later received the Nobel prize in physics) thought the Planck units were more likely mathematical artifacts coming out of dimensional analysis rather than something fundamental and related to gravity. Today, most researchers working with quantum gravity theory seem to think the Planck units will play an important role in a final unified theory; see, for example, [15–17]. Others are more critical. Meschini [18] pointed out that the "the significance of Planck's natural units in a future physical theory of spacetime is only a plausible, yet by no means certain". The lack of certainty in the significance of the Planck units is because the Planck scale at that time could still only be found very indirectly by dimensional analysis. For example, Unzicker [19] still seems to hold on to the view of Bridgeman, that the Planck units are little more than mathematical artifacts from dimensional analysis and that they are of no use and can basically be seen as undetectable mathematical artifacts. The Planck units are almost rather like the ether; if there are no ways to detect the Planck units then why not simply abandon the idea that they will play a central role in physics?

These opposing views of the Planck units also play a historically important role when it comes to the gravity constant itself. Okun [20] in 1991 pointed out that "The status of G and its derivatives, m_p , l_p , t_p , is at present different from that of c and \hbar , because the quantum theory of gravity is still under construction.". So a better understanding of G can perhaps also bring us closer to understanding the Planck scale and even closer to a unified quantum gravity theory. Thus it is important to keep questioning the real meaning of G ; something we will look at this paper, mainly by reviewing the existing literature on how G can potentially be linked to the Planck scale.

2. History of the Composite View of G and the Circular Problem

The gravity constant has SI units of $\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$. It would be strange if anything physical had units: meters cubed, divided by kilograms, and seconds squared. We can all

imagine something that has length, for example, a cat, or something that has mass in terms of kilograms, for example, a cat, and we all have a sense of time being related to change, and it can also be measured with clocks. So, the output unit of Newton's gravitational constant is perhaps the first hint that it could be a universal composite constant that can actually be represented by some more fundamental constants that we can physically link more directly to something [21]. Still, as we will see, G is a composite constant has been discussed for more than 60 years without a resolution, until perhaps very recently. We will here review much of the important history and progress around how the gravitational constant can be expressed in the form of Planck units.

Thüring [22], in 1961, concluded that G had been introduced somewhat ad hoc and that it cannot be associated with a unique property of nature; see also Gillies [23]. Zee [24] in 1982, in a paper titled "Calculation of Newton's Gravitational Constant in Infrared-Stable Yang-Mills Theories" wrote:

"Is Newton's gravitational constant G a fundamental parameter or is it calculable in terms of other fundamental parameters? In this paper I would like to argue the latter view and to present a calculation of G , unfortunately not in the real world, but in a toy world, just to demonstrate that G is indeed calculable."

Cahill [25,26] in 1984 was likely the first to suggest that instead of calculating the Planck mass from G , \hbar , and c , that perhaps G can be calculated from the Planck mass and suggested that G is given by:

$$G = \frac{\hbar c}{m_p^2} \quad (2)$$

This is nothing more than solving the Planck mass formula, $m_p = \sqrt{\frac{\hbar c}{G}}$, with respect to G . Chaill commented:

"The actual distribution of energy throughout space-time causes the tetrads to assume vacuum expected values of the order of the Planck mass, m_p . Thus the gravitational constant, $G = \frac{\hbar c}{m_p^2}$, may be viewed not as a fundamental constant, but as a mass scale that is dynamically determined by the large-scale structure of the Universe."

Cohen [27] suggested the same formula in 1987, and that he correctly pointed out also can be found from dimensional analysis or, in his own words,

"Dimensional analysis let us write $G = \hbar c / m_{pl}^2$, where m_{pl} is the Planck mass 21.77×10^{-9} kg, but this is of no help of determining G since there are no independent determination of m_{pl} ." (Page 74. Note that we will use notation m_p for the Planck mass, while several papers also use notation m_{pl})

This insight is of great importance and is what we will call the circular problem of the composite view of the Newton gravitational constant. Namely, this is the view that to express G from Planck units is of little or no use if one needs G to find the Planck units in the first place. Independently, a series [20,28–31] of researchers also later on suggested the same formula for G , likely without knowing about the paper of Cahill or Cohen, but none of these solved the circular problem. McCulloch [32] in 2016 again pointed out the circular problem with expressing the gravity constant from the Planck mass with the same formula as introduced by Cahill and Cohen or, in his own words,

"In the above gravitational derivation, the correct value for the gravitational constant G can only be obtained when it is assumed that the gravitational interaction occurs between whole multiples of the Planck mass, but this last part of the derivation involves some circular reasoning, since the Planck mass is defined using the value for G ."

Again, this demonstrates that the circular problem of expressing G in the form of Planck units has existed for a very long time, and this is, in our view, directly linked to the problem that, in quantum gravity, there has been little or no progress in detecting the Planck scale, and therefore limited progress, as we soon will discuss.

Clark [33] in 2003 suggested the gravitational constant is given by:

$$G = \frac{a_g \hbar c}{u^2} \tag{3}$$

where $\frac{a_g}{u^2} = \frac{1}{m_p^2}$, so this is in many ways just an indirect way of writing the Cahill and Cohen formula, as we have $G = \frac{a_g \hbar c}{u^2} = \frac{\hbar c}{m_p^2}$. Independently, Zwiebach [34] and Nastasenکو [35] both, in 2004, described the following formula to express G from the Planck units:

$$G = \frac{l_p^3}{t_p^2 m_p} \tag{4}$$

Zwiebach described this as a “Planckian system of units” but gives no indications that the Planck units can be found independently of G . Bruneton [36] suggested the same formula in 2013, and also his view that instead of G , \hbar , and c being the fundamental universal constants, they are just composites, and the Planck units are much more fundamental; see also [37,38]. This formula for G can, for example, be derived from dimensional analysis of G . The dimensions of G are $[G] = L^3 M^{-1} T^{-2}$; then simply replace L with l_p , and M with m_p , and T with t_p , and we get the formula $G = \frac{l_p^3}{t_p^2 m_p}$. However, if one needs to find G first to find the Planck units then one can naturally question the usefulness of this. The same formula was later used by, for example, Mercier [39] and Humpherys [40]. In 2013, Zivlak [41] suggested the formula of $G = \frac{c^2 l_p}{m_p}$, but without any suggestions for how to find l_p or m_p independent of G , thus leading one back to the circular problem, see also Eldred [42] 2019 that basically suggested the same formula.

In natural units, when first setting $c = \hbar = 1$, we must have $G = 1/m_p^2$, as pointed out by Kiritsis [43] in 1997 as well as by Cerdeno and Munoz [44] in 1998 and later mentioned by, for example, [36,45–50]. We find others like Peebles [51] who in 1989 already pointed out that $m_p = G^{-1/2}$ when $\hbar = c = 1$, so one could claim he then also pointed out $G = 1/m_p^2$, as it is naturally trivial to turn the equation around. Still, writing $G = 1/m_p^2$ rather than $m_p = G^{-1/2}$ gives a hint or even a strong indication that perhaps we should think that the gravity constant is a function of the Planck units, and not only the Planck units can be a function of G as first suggested by Max Planck; this idea is what this paper focuses on. Further, in the natural units system, when $\hbar = c = 1$ we will then have $G = l_p^2$ as pointed out by Schwarzschild [52] in 2000 and also [48,50,53]. In addition, since $t_p = \frac{l_p}{c}$, we must naturally also have $G = t_p^2$ when $\hbar = c = 1$. When only $c = 1$ must we then have $\hbar = m_p l_p$ and we get $G = \frac{l_p}{m_p}$ as pointed out by Casadio [54] in 2009, and also discussed by [36,55–60].

We also have:

$$G = \frac{t_p^2 c^5}{\hbar} = \frac{c^5}{\hbar f_p^2} \tag{5}$$

where v_p is the Planck frequency $f_p = \frac{c}{l_p}$; this was likely first mentioned by Nastasenکو [61] in 2013. Haug, in 2016, refs. [21,62,63] suggested that G is a universal composite gravitational constant of the form:

$$G = \frac{l_p^2 c^3}{\hbar} \tag{6}$$

This he arrived at from dimensional analysis by assuming the more fundamental constants are l_p , \hbar , and c and that the gravitational constant is simply a composite constant.

His argument is that the complex output units of G indicate it is a composite constant and, further, that the gravity constant coming before the Planck length does not mean the gravity constant is more fundamental than the Planck length. It is natural that we first understand the world more from the surface, before we understand the deeper aspects of it. Further, he shows how many of the Planck units can be simplified when one assumes G is such a composite. Still, none of the above-mentioned papers has solved the circular problem, so they are at best hypotheses that perhaps G can be expressed in the form of Planck units, but that there are unsolved problems to do so.

As we have seen, a series of ways to express the G in the form of Planck units have been expressed in the literature. Some authors have done this because they think G is a composite constant and that the Planck units are more real and fundamental, while others have mentioned G as a function of Planck units just so as to use in some calculations they have been done to achieve other results not directly related to the view that G is a composite constant.

Table 1 shows a series of ways to write G from Planck units and that we have found in the literature, and there are also many more additional ways. All these ways are valid mathematically, but again it is assumed one needs to know G to find the Planck units. A series of the formulas are marked as being presented first in this paper; we do not do this to indicate we have made any important new inventions simply by this, but simply to demonstrate that there are many ways to express G from Planck units. Basically, any Planck unit-related formula can be simply solved with respect to G . This is trivial mathematically; the big question is if it can lead to some significant new insight or not?

Table 1. The table shows various ways we can express the gravity constant from Planck units.

From	Gravity Constant Formula	Likely First Described ¹
Planck mass $m_p = \sqrt{\frac{\hbar c}{G}}$	$G = \frac{\hbar c}{m_p^2}$	Cahill [25] 1984 and Cohen ² [27] 1987
Planck time $t_p = \sqrt{\frac{\hbar G}{c^3}}$	$G = \frac{t_p^2 c^3}{\hbar}$	Nastasenko [61] 2013
Planck length $l_p = \sqrt{\frac{\hbar G}{c^3}}$	$G = \frac{l_p^2 c^3}{\hbar}$	Haug [21] 2016
Planck energy $E_p = \sqrt{\frac{\hbar c^5}{G}}$	$G = \frac{\hbar c^5}{E_p^2}$	this paper Haug [64] 2020
Planck temperature $T_p = \sqrt{\frac{\hbar c^5}{G k_b^2}}$	$G = \frac{\hbar c^5}{T_p^2 k_b^2}$	this paper
Planck mass $a_g = \frac{m^2}{m_p^2}$	$G = \frac{a_g \hbar c}{u^2} = \frac{\hbar c}{m_p^2}$	Clark [33] 2003
Planck frequency $f_p = \sqrt{\frac{c^5}{\hbar G}}$	$G = \frac{c^5}{f_p^2 \hbar}$	Nastasenko [61] 2013
Planck acceleration $a_p = \sqrt{\frac{c^7}{\hbar G}}$	$G = \frac{c^7}{a_p^2 \hbar}$	this paper
Planck density $\rho_p = \frac{c^5}{\hbar G^2}$	$G = \sqrt{\frac{c^5}{\rho_p \hbar}}$	this paper
Planck momentum $p_p = \sqrt{\frac{\hbar c^3}{G}}$	$G = \frac{\hbar c^3}{p_p^2}$	this paper
Planck force $F_p = \frac{E_p}{t_p}$	$G = \frac{c^4}{F_p}$	this paper
Planck length, time and mass	$G = \frac{l_p^3}{m_p t_p^2}$	Zwiebach [34] 2004 and Nastasenko [35] 2004
Planck length and Planck time	$G = \frac{l_p c^2}{m_p}$	Zivlak [41] 2013
Planck mass and Planck time	$G = \frac{t_p c^3}{m_p}$	Eldred [42] 2019
Planck length, time and Planck energy	$G = \frac{l_p^3 c^2}{E_p t_p}$	this paper
Planck time and Planck length	$G = \frac{t_p l_p c^4}{\hbar}$	this paper
Planck frequency Planck mass	$G = \frac{c^3}{f_p m_p}$	this paper
Planck acceleration and mass	$G = \frac{c^4}{a_p m_p}$	this paper
Planck charge and Planck length	$G = \frac{l_p^2 c^2 10^7}{q_p^2}$	this paper
Planck charge and Planck mass	$G = \frac{10^7}{m_p q_p^2}$	this paper
Planck charge and Planck time	$G = \frac{t_p^2 c^4 10^7}{q_p^2}$	this paper

Table 2 shows how to write G from Planck units when $h = c = 1$ and when $c = 1$ and $\hbar = m_p l_p$. So these formulas are simplified cases of the formulas in Table 1.

Table 2. The table shows various ways we can express the gravity constant from Planck units.

From	Gravity Constant Formula	Likely First Described
when $\hbar = c = 1$	$G = 1/m_p^2$	Kiritzis 1997 [43] and Cerdeno and Munoz 1998 [44]
when $\hbar = c = 1$	$G = l_p^2$	Schwarzschild 2000 [52]
when $\hbar = c = 1$	$G = t_p^2$	this paper
when $\hbar = c = 1$	$G = 1/a_p^2$	this paper
when $\hbar = c = 1$	$G = 1/E_p^2$	this paper
when $\hbar = c = 1$	$G = 1/p_p^2$	this paper
when $c = 1$	$G = l_p/m_p$	Casadio 2009 [54]
when $c = 1$	$G = t_p/m_p$	this paper
when $c = 1$	$G = l_p/E_p$	this paper
when $c = 1$	$G = t_p/E_p$	this paper
when $c = 1$	$G = l_p/a_p$	this paper
when $c = 1$	$G = t_p/a_p$	this paper

3. The Breakthrough in the Circular Problem

We have just looked at a long series of ways to express G in the form of Planck units. However, as long as one needs to know G to find the Planck units, this just leads to a circular problem as has been pointed out by a series of researchers, so at first glance this does not seem to help us understand G better. Still, we will claim that in recent years there has been a breakthrough in the circular problem. In 2017, Haug [65] showed a reliable way of find the Planck length independent of G , but still dependent on knowledge of \hbar and c . This was done by using a Cavendish apparatus as described in the appendix of that paper. That one needs to use a Cavendish apparatus has nothing to do with one needing to know G . Haug derived the formula:

$$l_p = \sqrt{\frac{\hbar 2\pi^2 L r^2 \theta}{M T^2 c^3}} \tag{7}$$

where r is the distance between the centers of the large and small balls (when the balance is deflected), further, L is the distance between the small balls in the apparatus. M is the kilogram mass of the large ball in the apparatus that can be found, for example, with a standard letter weight as compared to the one-kilogram prototype mass. θ is the angle of deflection measured and T is the measured period of oscillation of the torsion balance. In other words, this way of finding the Planck length is only dependent on \hbar and c , and not on prior knowledge of G . The formula above can be simplified further so we also get rid of the Planck constant, and then only depend on knowledge of c ; this point we will soon return to.

In 2020, Haug [66,67] showed it is possible to find the Planck length and the Planck time without knowledge of both G and \hbar , but that to find the Planck mass (in kilograms) one needs to know \hbar and c . Further, in 2021 Haug [66] showed an approach combined with a long list of gravity phenomena that can be used to find the Planck length independent of G and \hbar . In another paper [68], his main focus was on how to find the Planck time independent of G and \hbar . In 2022, Haug showed a way to find the Planck length and the Planck time without knowledge off G , \hbar , and c ; see [69,70]. If one knows how to find the Planck length independent of G and \hbar , one naturally knows how to find the Planck time independent on G and \hbar as the Planck time is simply the Planck length divided by the speed of light. However, the Planck time is also the Planck length divided by the speed of gravity, so if we can extract the speed of gravity from observable gravity phenomena only with no prior knowledge off c , then this must be the speed of gravity, and we have recently demonstrated that this is practically possible.

That the Planck units can be found without any knowledge of G means the gravity constant can indeed be expressed in the form of Planck units. This alone is a breakthrough, in our view. Still, what does it mean? This we will look more closely at in the next sections.

4. Putting the Pieces Together

We now know that the Planck units can be found without any knowledge of G . Table 3 shows a series of predictions from Newton and Einstein gravity simply re-written when we replace G with $G = \frac{\hbar c}{m_p^2}$. For example, the gravitational acceleration that can be predicted by $g = \frac{GM}{R^2}$ can now be re-written as:

$$g = \frac{GM}{R^2} = \frac{\hbar c}{m_p^2} \frac{M}{R^2} \tag{8}$$

This, in our view, gives little if any new insight or important results; one could even argue the formula is now even less intuitive than before. We can claim that this shows that gravity is related to the Planck mass and that it therefore gives some new insight, but it is not obvious why this should be the case. Still, G is replaced with an expression containing the Planck mass, and the Planck mass can be found independently of G , so this is a big step forward from the view held during the time of Max Planck and up until recently when researchers thought the Planck units could not be found without knowing G first. We could argue that this approach replaces three universal constants G , \hbar , and c with three new ones, namely m_p , \hbar , and c . Still, so far it seems that even after we have solved the circular problem in this composite view of G , this simply means we can replace G with another constant; namely m_p . This could be interesting on its own, as it indeed could indicate G is more of a human construct than something representing directly physical aspects of the depth of reality. The many formulas in Table 3, when re-written $G = \frac{\hbar c}{m_p^2}$, do not seem to make things more intuitive or, we could argue, it looks perhaps even less intuitive. It looks as though we still need three constants, but we have replaced G with m_p .

Table 3. The table shows the standard gravitational prediction formulas re-written when we assume $G = \frac{\hbar c}{m_p^2}$. We can see that the end results are likely even less intuitive than the existing results, and that we basically only have swapped one constant for a new one (G for m_p).

	Gravity with $G = \frac{\hbar c}{m_p^2}$:
Mass	M and m (kg)
Gravity force	$F = G \frac{Mm}{R^2} = \frac{\hbar c}{m_p^2} \frac{Mm}{R^2}$ ($\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$)
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{\hbar c M}{m_p^2 R^2}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = \frac{1}{m_p} \sqrt{\frac{\hbar c M}{R}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R m_p}{\sqrt{\frac{\hbar c M}{R}}}$
Periodicity pendulum ³ (clock)	$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi R \sqrt{\frac{L}{GM}} = 2\pi R m_p \sqrt{\frac{L}{\hbar c M}}$
Frequency Newton spring	$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi R m_p} \sqrt{\frac{\hbar c M}{x}}$
Velocity ball Newton cradle ⁴	$v_{out} = \sqrt{2 \frac{GM}{R^2} H} = \frac{1}{R m_p} \sqrt{2 \hbar c M H}$
Predictions from GR:	
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi \hbar c M}{a(1-e^2)c^2 m_p^2}$
Gravitational redshift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2\hbar M}{R_1 c m_p^2}}}{\sqrt{1 - \frac{2\hbar M}{R_2 c m_p^2}}} - 1$
Time dilation	$T_R = T_f \sqrt{1 - \frac{2GM}{R} / c^2} =$
Deflection	$\delta = \frac{4GM}{c^2 R} = \frac{4\hbar M}{c R m_p^2}$
Microlensing	$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{(d_s - d_L)}{d_s d_L}} = \sqrt{\frac{4\hbar M}{c m_p^2} \frac{(d_s - d_L)}{d_s d_L}}$

We could choose any of the other ways to express G from Planck units as shown in Table 1 or Table 2; for example, we could choose Haug’s formula $G = \frac{l_p^2 c^3}{\hbar}$. This would, at first glance, seem to merely lead to G , \hbar , and c being replaced with l_p , \hbar , and c . In other words, after we know that the Planck units can be found without G , we can replace the three universal constants G , \hbar , and c with a chosen Planck unit plus c and \hbar . So then, one can question whether this is just a change of unit systems. This alone is interesting, but obviously not a big break through; we could even claim it is trivial information.

Another important step is needed before we can discover the great utility of the composite view of the gravitational constant. The mass in kilograms of any mass can be described as:

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{9}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. This expression for mass we simply get by solving the Compton [72] wavelength formula $\bar{\lambda} = \frac{\hbar}{mc}$ with respect to mass. One could claim that only elementary particles have Compton wavelength and that composite masses do not, or at least not such large objects as planets or suns. Only elementary particles likely have a “physical” Compton wavelength can be measured by Compton scattering, but larger masses consist of elementary particles and the aggregated Compton wavelength in the composite mass is given by [1,67]:

$$\bar{\lambda} = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \dots + \frac{1}{\lambda_n}} \tag{10}$$

This aggregation is fully consistent with standard mass aggregation $m = m_1 + m_2 + m_3 + \dots + m_n$, and can even be derived from it. It is also important to understand that one can find the Compton wavelength of any mass without knowing \hbar and G . This mass addition formula is known to over-estimate the mass slightly, mostly due to nuclear binding energy, see [73,74]. However, one can easily adjust for this by treating the binding energy as mass equivalent and subtracting it.

Let us start with an electron. The Compton wavelength can be found by shooting a photon at an electron, and by measuring the photon wavelength before and after the impact with the electron, and also the angle between the incoming and outgoing photon, and thus we have:

$$\lambda_e = \frac{\lambda_2 - \lambda_1}{1 - \cos \theta} \tag{11}$$

The reduced Compton wavelength of the electron is simply thus divided by 2π as is well known. Next, we can find the reduced Compton wavelength of the proton by utilizing the fact that the cyclotron frequency ratio is proportional to the Compton wavelength ratio. This is because the charge on the electron and proton is the same, and the cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \tag{12}$$

So, we must have

$$\frac{f_e}{f_p} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_p}} = \frac{\bar{\lambda}_p}{\bar{\lambda}_e} \approx \frac{1}{1836.15} \tag{13}$$

So, if we know the electron Compton wavelength, we know the proton Compton wavelength as it is arrived at by taking the electron Compton wavelength and dividing it by 1836.15. Next, we can find the Compton wavelength of any larger mass by “simply” counting the number of atoms in the object of interest and then dividing the Compton wavelength of the proton by this count. To count atoms in a clump of matter is not easy, but fully possible. One way is to construct a precise silicon (^{28}Si) sphere. As one knows the crystal structure here very well and since it is very uniform, one can accurately calculate the number of atoms in such a sphere. This way of counting atoms has even been one of the

recently suggested methods to re-define the kilogram; see [75–77]. There also exist other methods to count atoms [78,79], so this is fully possibly in practice, even if it takes some effort.

Based on that, we can write the formula of a mass as $M = \frac{\hbar}{\lambda} \frac{1}{c}$ and we can replace the mass in Equation (7) with this mass and this gives us:

$$\begin{aligned}
 l_p &= \sqrt{\frac{\hbar 2\pi^2 L r^2 \theta}{M T^2 c^3}} \\
 l_p &= \sqrt{\frac{\hbar 2\pi^2 L r^2 \theta}{\frac{\hbar}{\lambda} \frac{1}{c} T^2 c^3}} \\
 l_p &= \sqrt{\frac{2\pi^2 L r^2 \theta \lambda}{T^2 c^2}} \tag{14}
 \end{aligned}$$

That is, the two Planck constants cancel each other out to find the Planck length. In other words, we do not need to know \hbar or G to find the Planck length. All the other parameters in the formula we can easily find without knowledge of G or \hbar using a Cavendish apparatus. The reason we use a Cavendish apparatus is because we can deal with sizes of matter where we can count the number of atoms, but similar methods for even much larger masses can be used [1].

Haug [1,67] has recently shown a practically feasible way to find the Compton wavelength independent of G and \hbar for planets, stars, galaxies and even of the whole mass of the observable universe, see [80]. The main point here is that any mass in terms of kilograms can be expressed by the Formula (9). Next, let us multiply the composite $G = \frac{l_p^2 c^3}{\hbar}$ with the composite mass $M = \frac{\hbar}{\lambda} \frac{1}{c}$ and we get:

$$GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\lambda} \frac{1}{c} = c^2 \frac{l_p^2}{\lambda} \tag{15}$$

What is important to pay attention to here is that the two Planck constants actually cancel each other out, and we are left with two constants, c and l_p , and both these can be found without knowledge of G and \hbar . Table 4 shows a series of predicted gravitational phenomena that can actually be observed. As we see, in all the observable phenomena, we have GM and not GMm . The small mass m in the Newton gravitational force formula is only used in derivations of observable gravitational phenomena and then one of the two masses always cancels out. In real two mass gravity phenomena we have the gravity parameter $\mu = G(M_1 + M_2) = GM_1 + GM_2 = c^2 \frac{l_p^2}{\lambda_1} + c^2 \frac{l_p^2}{\lambda_2}$ so also in real two body gravitational phenomena, the Planck constant cancels out.

It is evident from Table 4 that a long series of observable gravity phenomena can be predicted by knowing only two constants, namely l_p and c and naturally a variable which is linked to the mass size, namely the reduced Compton wavelength of the gravitational object. As seen from the table, some observable gravity phenomena only needs one constant, namely the Planck length. Again, it has, in recent years, been demonstrated how to find the Planck length independently off G so this is a fully practical way to do gravity predictions, and is not just a hypothesis.

Table 4. The table shows that any observable gravity phenomena contains GM and not GMm and, further, that when assuming G is a composite, then we end up being able to predict all observable gravity phenomena only from l_p and c .

Mass	$M = \frac{\hbar}{\lambda_M} \frac{1}{c}$ (kg)
Non observable (contains GMm)	
Gravitational constant	$G, \left(G = \frac{l_p^2 c^3}{\hbar} \right)$
Gravity force	$F = G \frac{Mm}{R^2}$ (kg · m · s ⁻²)
Observable predictions: (contains only GM)	
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2}{R^2} \frac{l_p^2}{\lambda_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = cl_p \sqrt{\frac{1}{R\lambda_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi \sqrt{\lambda_M R^3}}{cl_p}$
Periodicity pendulum ⁵ (clock)	$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi R \sqrt{\frac{l}{GM}} = \frac{2\pi R}{cl_p} \sqrt{L\lambda_M}$
Frequency Newton spring	$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi R} \sqrt{\frac{GM}{x}} = \frac{cl_p}{2\pi R} \sqrt{\frac{1}{\lambda_M x}}$
Velocity ball Newton cradle ⁶	$v_{out} = \sqrt{2 \frac{GM}{R^2} H} = \frac{cl_p}{R} \sqrt{\frac{2H}{\lambda_M}}$
Observable predictions (from GR): (contain only GM)	
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)} \frac{l_p^2}{\lambda_M}$
Gravitational redshift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p^2}{R_1 \lambda_M}}}{\sqrt{1 - \frac{2l_p^2}{R_2 \lambda_M}}} - 1$
Time dilation	$T_R = T_f \sqrt{1 - \sqrt{\frac{2GM}{R}}^2 / c^2} = T_f \sqrt{1 - \frac{2l_p^2}{R\lambda_M}}$
Deflection	$\delta = \frac{4GM}{c^2 R} = \frac{4}{R} \frac{l_p^2}{\lambda_M}$
Microlensing	$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{(d_s - d_L)}{d_s d_L}} = 2l_p \sqrt{\frac{d_s - d_L}{\lambda_M (d_s d_L)}}$

As all predictions of observable gravity phenomena contain GM , this leads to $GM = c^3 \frac{l_p}{c} \frac{l_p}{\lambda}$. Haug has, in a series of papers, suggested that c^3 can be used as a gravity constant and that real gravity mass should be re-defined as $\bar{m} = \frac{l_p}{c} \frac{l_p}{\lambda}$; something we will soon get back to. This view is shown in Table 5, which gives all the exact same predictions as the standard gravity formulas, but without any G and also without any need for \hbar . One exception is for the gravity force itself, but the gravity force cannot be measured directly; we can only observe the consequences from it. Our new way of representing the gravity force formula gives the same predictions for observable phenomena, and is only linked to the Planck length and the speed of light. The speed of light is, in this context, the same as the speed of gravity (“gravitons”?). For example, Abbot et al. [81] in 2017 constrained “the difference between the speed of gravity and the speed of light to be between -3×10^{-15} and $+7 \times 10^{-16}$ times the speed of light.”.

Table 5. The table shows that we can write the gravitational constant as c^3 when using, in our view, a more complete mass definition, $\bar{m} = \frac{l_p}{c} \frac{l_p}{\lambda}$. That is, mass is related to time, or what Haug has called collision-time. Different mass sizes then only differ in different Compton wavelengths. Writing the gravitational force formula this way yields the same predictions as standard Newton gravity except we only rely on two constants, l_p and c , to describe mass and any observable gravity phenomena. In addition, in general relativity predictions, we can replace the mass with this mass definition if we replace G with c^3 . The reason we can do this is that $c^3 \bar{M} = GM$. This is clear when we understand that G is a composite constant and, in addition, understand that the kilogram mass can be written by simply solving the Compton wavelength formula with respect to m .

Mass	$M = \frac{\hbar}{\lambda_M} \frac{1}{c}$ (kg)
Non observable :	
Gravitational constant	c^3
Gravity force	$F = c^3 \frac{\bar{M}\bar{m}}{R^2}$ (kg · m · s ⁻²)
Observable predictions:	
Gravity acceleration	$g = \frac{c^3 \bar{M}}{R^2} = \frac{c^2}{R^2} \frac{l_p^2}{\lambda_M}$
Orbital velocity	$v_o = \sqrt{\frac{c^3 \bar{M}}{R}} = c l_p \sqrt{\frac{1}{R \lambda_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{c^3 \bar{M}}{R}}} = \frac{2\pi \sqrt{\lambda_M R^3}}{c l_p}$
Periodicity pendulum ⁷ (clock)	$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi R \sqrt{\frac{L}{c^3 \bar{M}}} = \frac{2\pi R}{c l_p} \sqrt{L \lambda_M}$
Frequency Newton spring	$f = \frac{1}{2\pi} \sqrt{\frac{k}{M}} = \frac{1}{2\pi R} \sqrt{\frac{c^3 \bar{M}}{x}} = \frac{c l_p}{2\pi R} \sqrt{\frac{1}{\lambda_M x}}$
Velocity ball Newton cradle ⁸	$v_{out} = \sqrt{2 \frac{c^3 \bar{M}}{R^2} H} = \frac{c l_p}{R} \sqrt{\frac{2H}{\lambda_M}}$
Observable predictions (from GR):	
Advance of perihelion	$\sigma = \frac{6\pi c^3 \bar{M}}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)} \frac{l_p^2}{\lambda_M}$
Gravitational redshift	$z = \frac{\sqrt{1 - \frac{2c^3 \bar{M}}{R_1 c^2}}}{\sqrt{1 - \frac{2c^3 \bar{M}}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p^2}{R_1 \lambda_M}}}{\sqrt{1 - \frac{2l_p^2}{R_2 \lambda_M}}} - 1$
Time dilation	$T_R = T_f \sqrt{1 - \sqrt{\frac{2c^3 \bar{M}}{R}} / c^2} = T_f \sqrt{1 - \frac{2l_p^2}{R \lambda_M}}$
Deflection	$\delta = \frac{4c^3 \bar{M}}{c^2 R} = \frac{4}{R} \frac{l_p^2}{\lambda_M}$
Microlensing	$\theta_E = \sqrt{\frac{4c^3 \bar{M}}{c^2} \frac{(d_s - d_L)}{d_s d_L}} = 2l_p \sqrt{\frac{d_s - d_L}{\lambda_M (d_s d_L)}}$

5. Is the Inertial Mass Really Identical to the Gravitational Mass?

As shown for all the observational gravitational phenomena reported in Tables 4 and 5, we have GM and not Gmm . Again, $GM = \frac{l_p^2 c^3}{\hbar} \frac{\hbar}{\lambda} \frac{1}{c} = c^3 t_p \frac{l_p}{\lambda}$, where Haug has claimed in a series of papers [67,82] that c^3 can then be seen as a gravitational constant and $t_p \frac{l_p}{\lambda}$ as a more complete mass definition. This mass definition he has coined collision-time, which again can be seen as the gravitational mass. Yes, this mass definition has dimensions of simply time. This mass is already indirectly embedded in standard gravity theory since one is multiplying the kilogram mass with the gravitational constant. But here the traditional view is that this is a gravity constant multiplied by a mass, and that they are two separate things. No one has yet figured out exactly, from a deeper perspective, why this must be done. Well, what is a gravitational mass and what is an inertial mass? A gravitational mass is linked to the mass caused by, and acted on, by a body by the force of gravity, so it has always been assumed that both the masses in Newton’s formula represent gravitational masses. However, we will challenge that view here. This is because if the small mass m has insignificant impact on M then it cancels out in all derivations of direct observable gravitational phenomena, so we could even write:

$$ya = G \frac{My}{R^2} \tag{16}$$

and we would still get the correct predictions about measurable gravitational phenomena from this equation; that is, y on both sides of the equation could be replaced with basically anything. We could even define y as money. Money has naturally nothing to do with gravity, but since y is on both sides of the equation we can divide by y on both sides and we get $a = GM/R^2$, and we can measure both a and the gravitational acceleration. Our point is that even when putting in a completely wrong mass definition for m on both sides of $ma = G \frac{Mm}{R^2}$, these two m masses will cancel out in the derivation of anything observable. This is the case for derivation of any observable gravity phenomena, except for real two-body problems where one has $GM_1 + GM_2$, not GMm . The reminding kilogram mass M is always multiplied by the gravity constant. We will claim this is done (unknowingly) to correct an incomplete mass definition (the kilogram mass) into a more complete mass definition, so the real gravitational mass is $\frac{G}{c^3}M = \frac{l_p}{c} \frac{l_p}{\lambda} = \bar{M}$. This was discussed in detail by Haug [67,82]. Newton naturally did not have this in mind when he developed his gravity theory and, as we have pointed out, he also never used a gravity constant. The gravity constant is a missing value constant simply found by calibration to observable gravitational phenomena when one has decided upon using the kilogram definition of mass. This is also at least part of the reason why the gravity constant came into existence about the same time as the kilogram mass became popular in Europe.

The mass linked to non-gravitational acceleration is often thought of as the inertial mass, and since it has been shown experimentally that the following relation seems to hold

$$m_i a = G \frac{Mm}{R^2} \tag{17}$$

we assume that the inertial mass m_i must be equal to the gravitational mass m since it seems to be an equivalence between standard acceleration (for example, in an elevator) and in a gravitational acceleration field, and we do not doubt this, we simply claim the mass m is not used directly for any predictions of any observable gravitational phenomena. That is, one is not measuring ma or $G \frac{Mm}{R^2}$, but is observing a and $g = a = G \frac{M}{R^2}$; in other words, after the two small masses have cancelled each other out in derivations for predictions of observable phenomena. In our view, there is only one type of mass and we have just defined it as $\bar{M} = \frac{G}{c^3}M$. Inputting this mass definition in all parts in the Newton formula would lead to:

$$\frac{G}{c^3}ma = c^3 \frac{\frac{G}{c^3}M \frac{G}{c^3}m}{R^2} \tag{18}$$

and we would also now end up with $a = \frac{GM}{R^2}$, since $\frac{G}{c^3}m$ is on both sides and cancels out. And since $\bar{M} = \frac{G}{c^3}M$ and $\bar{m} = \frac{G}{c^3}m$ we can write this as:

$$\bar{m}a = c^3 \frac{\bar{M}\bar{m}}{R^2} \tag{19}$$

Our point is that in the standard Newton gravity formula, in its modern form invented in 1873, one is likely unknowingly using two different masses: the standard kilogram mass multiplied by G ; that is, GM which combined can be seen as the gravitational mass (collision-time mass) multiplied by c^3 ; that is, $GM = c^3\bar{M}$ and the other non-gravitational mass m is an incomplete kilogram mass that says nothing about gravity. When we want to just know, for example, the relation between mass and energy, then the standard kilogram mass will have enough information to do so. So, we can still use m , that is the kilogram mass definition without adjustments, in relations such as $E = mc^2$ or in $E = mc^2\gamma$, but the same mass definition cannot be used for calculating gravity effects from that mass without multiplying it with G or, better, by understanding that GM actually represents the real gravitational mass (divided by c^3). There is only one mass, but to describe gravity requires additional information that is lacking in the kilogram mass definition. The kilogram mass is incomplete, but good enough for calculations related to just energy and mass (the kilogram definition), but it is incomplete when also taking into account gravity. We can then either

fix this mass ad hoc by using G or we can understand that G is a composite constant and, when combined with M , it gives a deeper insight into mass also related to gravity.

So the real gravitational mass even of the small mass is $\bar{m} = \frac{G}{c^3}m = t_p \frac{l_p}{\lambda} = \frac{l_p}{c} \frac{l_p}{\lambda}$, while the kilogram mass is given by $m = \frac{\hbar}{\lambda} \frac{1}{c}$. Since the Planck length, the speed of light, and the Planck constants are constants, and the only thing changing is the Compton wavelength (mass size) in both the gravitational mass (collision-time mass) and in the kilogram mass, they are proportional. So, the weak equivalence principle holds also under this view. This view does not change the output from predictions of observable phenomena, but it shows us how the Planck scale is already directly linked to gravity. Detection of gravity is, in our view, detection of the Planck scale. This view is new and controversial, but we think it should be taken seriously enough to also be carefully investigated by other researchers before being rejected prematurely.

At the same time, one is incorporating important aspects of mass linked to the Planck length. The same can easily be done with energy, as has been demonstrated in the papers referred to. We naturally do not ask other researchers to take this for granted, but simply suggest that, in our view, this looks like a promising path that we think requires further investigation.

We have, in this paper, claimed that it looks as though all observable gravity phenomena can be predicted by using the two constants: the speed of light and the Planck length, without dependence on G or \hbar . One could think such a view is wrong, as c and l_p only contain length and time, so how could this ever represent energy, which is normally presented as joule? This at first seems inconsistent. However, we have recently shown how energy can indeed be represented as a length (collision-length) and that mass can be represented as time (collision-time); see [82,83]. Energy is most often expressed as joule which again, in SI units, represents $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$. That is, energy contains kilogram. Kilogram is an arbitrary amount of matter that came into use in science around 1870, a few years before the introduction of the gravitational constant in 1873. Our recent research indicates this is an unnecessarily complex way to express energy and that it can be expressed as collision-length. One can easily move from the collision-length energy and back to the joule energy, simply by multiplying the collision-length energy with $\frac{c^4}{G} = \frac{\hbar}{l_p} c$ or, the other

way around, multiplying the joule energy with $\frac{G}{c^4} = \frac{l_p}{\hbar c}$. This is discussed in great detail in [67,82]. The main point is that kilogram and joule both contain information about the kilogram, and the kilogram is an arbitrary, human-chosen clump of matter that gravity does not care about. Our new suggested way to look at mass and energy, that is already embedded in standard gravity theory through the use of G , strips the standard mass of unnecessary information about the kilogram.

When going from the collision-length energy, \bar{E} , to the joule energy, E , or from the joule energy to the collision-length energy, it seems (from the paragraph above) that we need to know the Planck constant. However, one can easily find the collision-length energy more directly; for example, from the gravitational acceleration field of the Earth that simply given by:

$$\bar{E} = \frac{gR^2}{c^2} = l_p \frac{l_p}{\lambda} \tag{20}$$

So the only constant one needs to know to find the gravitational energy⁹ is the speed of light (in $\frac{gR^2}{c^2}$). Neither the gravity constant nor the Planck constant are needed. To estimate the gravitational acceleration, one has $g = \frac{GM}{R^2}$, so one could mistakenly think one needs to know G to find \bar{E} . This is only if one wants to predict g from standard theory. One can also measure g without any knowledge of G ; for example, by dropping a ball. The gravitational acceleration is then simply given by $g = \frac{2H}{T^2}$, where H is the height from which the ball is dropped, and T is the time it took for the ball to fall to the ground.

To find this energy for smaller masses, we can use a Cavendish apparatus; see [67,84] that provides a detailed discussion of the topic. Or alternatively, we can find the gravita-

tional energy by first finding the Planck length and the reduced Compton wavelength of the mass in question without knowledge of G and \hbar , as we have demonstrated, in this and other papers, is fully possible.

The Planck constant is normally associated with the concept that energy comes in quanta, something that has been clearly demonstrated in experimental research. One might therefore think that a theory without the Planck constant must be incomplete with respect to describing energy. To our own surprise, even without \hbar in the gravitational energy (collision-length), this is actually a quantum energy that comes in quanta. The collision-length energy comes in quanta of the Planck length. The quantity are given by $\frac{l_p}{\lambda}$. This is the quanta of the gravitational energy in a Planck time observational window that we easily can find indirectly. For a Planck mass it is one, for a mass larger than a Planck mass it is a quantity larger than one. Here, the integer part will be the number of full Planck events, and the reminding fractional part is the probability for an additional one such event. For particles smaller than the Planck mass, this factor is always smaller than one and then represents the probability for the particle to be in a particular state that we, in previous papers, have called collision-state inside the Planck time window see [67,82]. It is not necessary to measure anything at the Planck time to measure this indirectly, as we have demonstrated in the papers just mentioned. The term $\frac{l_p}{\lambda}$ embedded in the collision-length energy (quantum gravitational energy) can simply be seen as an aggregate of probabilities for Planck events, as is discussed in-depth in the papers just referred to.

6. The Gravity Constant Calculated from Cosmological Entities

Another line of thought in relation to the composite view of G has been that the Newtonian gravitational constant can perhaps be calculated from cosmological entities or constants. Already, in 1936, Milne [85] explicitly suggested that:

$$G = \frac{c^3 T_H}{M_u} \quad (21)$$

where T_H is one divided by the Hubble constant, and M_u is the mass of the universe or what Milne called the fictional mass of the universe, as he thought the universe was infinite, but that this was the mass inside the Hubble sphere. Unzicker [86] pointed out that such a relation indirectly and ad hoc was already hinted at by Einstein [87] in 1917 but at best obliquely so, and further that Schrödinger [88] in 1925, according to Unzicker, must have suspected this relation based on numerical calculations. Bleksley [89] suggested that the gravitational constant can be expressed as:

$$G = \frac{R_u c^2}{M_u} \quad (22)$$

where R_u is the radius of the observable universe and M_u is the mass of the observable universe. So this is basically the same as the Milne formula since: $R_u = \frac{c}{H_0} = cT_H$.

As the mass of the universe from the Friedmann equation published in 1922 (general relativity theory) requires knowledge of G , Bleksley could not use this mass to find G , but instead came up with his own way to calculate the universe mass, a way we think looks a bit like numerology or at least is very speculative. For example, he suggested that the number of protons in the universe must be $R_u^2 / (4\rho^2)$ where ρ is the diameter of the proton. It is far from clear how he got to this formula, so we are questioning the validity of this approach. But $G = \frac{R_u c^2}{M_u}$ is still, at best, only a rough approximation for the Friedmann model. The correct formula in the Friedmann model is $M_c = \frac{c^3}{2GH_0}$, where M_c is the critical mass of the universe, so this leads to $G = \frac{R_u c^2}{2M_c}$ and not $G = \frac{R_u c^2}{M_c}$, as suggested by Milne and Bleksley. Mercier [39] in 2020 basically gave the same formula for G as Milne and Bleksley. He used a universe mass rooted in a paper by Carvalho [90]. Carvalho started

with a relation between mass density and the Hubble constant that he claimed is given by Weinberg [91]

$$G\rho_0 = H_0^2 \tag{23}$$

And, from this, got

$$M_u \approx \frac{c^3}{GH} \tag{24}$$

Carvalho, in addition, derived a universe mass to be the same as given by this formula, but independently from G by some assumptions of π mesons. However, his derivation here seems quite speculative. Carvalho further claimed “*This is identical to expression derived in the context of Friedmann’s cosmological model.*”. This claim is not fully correct or, at least, not precise enough. Both the formula he presented, the one he claimed is from the Friedman model and the other universe mass he derived from π mesons, are both actually twice what one gets from the Friedmann model. The universe mass one gets from the Friedmann model is:

$$M_c = \frac{1}{2} \frac{c^3}{H_0 G} \tag{25}$$

as one also finds indirectly in the book of Weinberg as well as in a series of other independent sources (see, for example, [92,93]). Several authors (for example, Cook in 2011 [94] and Mercier [39]) have suggested that G can be expressed as:

$$G = \frac{c^3}{HM_u} = \frac{T_H c^2}{M_u} \tag{26}$$

This is identical to the Milne 1936 formula and is naturally equal to $G = \frac{R_H c^2}{M_u}$, as the Hubble radius is given by $R_H = \frac{c}{H_0}$, and $T_H = \frac{R_H}{c} = \frac{1}{H_0}$. Again, this formula is not fully consistent with the Friedmann model, as that would require $G = \frac{c^3}{2HM_u}$ and, since the Hubble time is given by $T_H = \frac{1}{H_0}$, this is naturally the same as $G = \frac{T_H c^2}{2M_u}$, but the Equation (26) is consistent when using the Haug [95] universe mass, which is predicted to be twice that of the Friedmann critical mass of the universe.

None of these authors have shown how to find the universe mass without already knowing G , except from what we would call relatively speculative approaches which we think lack a solid foundation, even if this naturally can be discussed further. Still, there is, as we will see, a way to find this critical mass of the universe without knowing G . First, if we solve the Friedmann critical mass equation (Equation (25)) with respect to H_0 , then this gives the formula:

$$H_0 = \frac{1}{2} \frac{c^3}{M_c G} \tag{27}$$

And since any mass in kilogram can be written as $m = \frac{\hbar}{\lambda} \frac{1}{c}$, and also because G can be written as $G = \frac{l_p^2 c^3}{\hbar}$, this means we have:

$$H_0 = \frac{1}{2} \frac{c^3}{\frac{\hbar}{\lambda_c} \frac{1}{c} \frac{l_p^2 c^3}{\hbar}} = \frac{\bar{\lambda}_c}{2l_p^2} \tag{28}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical mass in the Friedmann universe (the critical universe). This also means we must have:

$$G = \frac{\bar{\lambda}_c c^4}{2H_0 \hbar} \tag{29}$$

The Hubble constant can be found with no knowledge of G as also the Compton wavelength of the universe mass can be found without this knowledge of G as we [80]

recently demonstrated. Further, as $H_0 = \frac{c\bar{\lambda}_c}{2l_p^2}$, this means the Equation (29) can be simplified further to:

$$G = \frac{\bar{\lambda}_c c^4}{2H_0 \hbar} = \frac{l_p^2 c^3}{\hbar} \tag{30}$$

which is the same composite formula for G that one gets by solving Max Planck’s Planck length formula; $l_p = \sqrt{\frac{G\hbar}{c^3}}$, with respect to G .

For G times the critical mass of the universe (the gravitational parameter of the universe), we must have:

$$\mu_c = GM_c = \frac{l_p^2 c^4}{\hbar} \frac{\hbar}{\bar{\lambda}_c} \frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}_c} \tag{31}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the mass in the critical universe. To predict gravitational phenomena related to the mass of the critical universe, all we need is the Planck length and the speed of light; that is, two constants, and the reduced Compton wavelength of the critical mass of the universe. All these can be found with no knowledge off G or even \hbar ; see [80]. Actually, the Hubble constant is given by:

$$H_0 = \frac{1}{\frac{l_p}{c} \frac{l_p}{\bar{\lambda}_u}} = \frac{1}{t_p \frac{l_p}{\bar{\lambda}_u}} \tag{32}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the Haug universe. Pay attention to how $t_p \frac{l_p}{\bar{\lambda}_c}$ is identical to what we call the collision-time mass. So, the Hubble constant, in this view, is nothing more than one divided by the collision-time mass of the observable universe [83].

If one knows the collision-time mass of the universe, then there is no need to multiply it with G to do gravitational predictions. This is why cosmological red-shift can be predicted simply by:

$$Z = \frac{Hd}{c} = \frac{d}{c\bar{M}_c} \tag{33}$$

where $\bar{M}_c = t_p \frac{l_p}{\bar{\lambda}_c}$ is the collision-time of the observable universe. If we use the critical mass of the universe in terms of kilogram, then we need to multiply it with G divided by c^3 to convert it into the real gravitational mass, so we have:

$$Z = \frac{d}{\frac{G}{c^3} \bar{M}_c} = \frac{d}{c\bar{M}_c} = \frac{H_0 d}{c} \tag{34}$$

This means we can also predict cosmological phenomena from the Planck length and the speed of light. This strongly indicates there is a link between the largest and the smallest scales of the universe. This is not a very big surprise as the largest scales are built from the smallest. The rules of the smallest (quantum) somehow give us the rules for even the cosmic scales. Our view is that the Planck scale is actually indirectly detected in any (or at least most) gravitational observation, including also cosmological red-shift.

Table 6 shows some ways to express the gravity constant in the form of cosmological entities. All these are at the deepest level nothing else than $G = \frac{l_p^2 c^3}{\hbar}$. In addition, pay attention to how closely the formulas linked to the Hubble scale are linked to the formulas presented that are linked to the Schwarzschild radius and Haug radius. The reason for this is that the Hubble radius is identical to the Schwarzschild radius for the observable universe, and this is why in papers it has been considered whether the Hubble sphere is actually a gigantic black hole; see, for example, [96,97].

Table 6. The table shows various ways we can express the gravity constant from cosmological units, as well as from units related to black holes.

From	Gravity Formula	Comments
From universe mass and Hubble time	$G = \frac{c^3 T_H}{M_u}$	Milne 1936 [85]
From universe mass and Hubble radius	$G = \frac{R_H c^2}{M_u}$	Bleksley 1951 [89]
From universe mass and universe radius	$G = \frac{R_u c^2}{6 M_u}$	Unzicker 2020 [19]
Hubble constant, Friedmann critical mass	$G = \frac{c^3}{2 H_0 M_c}$	
Hubble radius, Friedmann critical mass	$G = \frac{R_H c^2}{2 M_c}$	
Hubble constant, Friedmann critical mass	$G = \frac{T_H c^3}{2 M_c}$	
Hubble time, Friedmann critical mass	$G = \frac{c^3}{2 H_0 M_c}$	
Hubble radius, Hubble time, and Friedmann critical mass	$G = \frac{R_H^3}{2 M_c T_H^2}$	
Hubble constant, Haug universe mass,	$G = \frac{c^3}{H_0 M_u}$	
Hubble radius Hubble time and Haug universe mass	$G = \frac{R_H c^2}{M_u}$	
Hubble radius, Haug universe mass,	$G = \frac{T_H c^3}{M_u}$	
Hubble time, Hubble time, and Haug universe mass	$G = \frac{T_H c^3}{M_u T_H^2}$	
Hubble constant, Friedmann critical mass,	$G = \frac{c^3}{2 H_0 M_c}$	
Hubble time and Haug universe mass	$G = \frac{T_H c^2}{M_u}$	
Schwarzschild radius, mass,	$G = \frac{R_s c^2}{2 M}$	$R_s = \frac{2GM}{c^2}$
Schwarzschild time, mass,	$G = \frac{T_s c^2}{2 M}$	$T_s = \frac{R_s}{c}$
Haug escape velocity radius, mass,	$G = \frac{R_h c^2}{M}$	$R_h = \frac{GM}{c^2}$
Haug radius time, mass,	$G = \frac{T_h c^3}{M}$	$T_h = \frac{R_h}{c}$

7. The Composite View of G with Respect to Alternative Gravitational Theories

An interesting question is naturally how well or not the composite view of G also fits into alternative gravitational theories. There exist a long series of alternative or modified gravitational theories so it is outside the scope of this paper to discuss all of them or go in depth about them. Still, we can say something about a few of these theories. One set of modified gravitational theories are minimum acceleration theories. They try to explain galaxy rotation without dark matter by incorporating a minimum acceleration. The most well-known of these is modified Newton dynamics (MOND) introduced by Milgrom [98] in 1983. Here, we suspect it would just be to replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and M with $M = \frac{\hbar}{\lambda c}$ without any issues. Whether this can give deeper insight into the Milgrom model would need to be investigated. Another minimum acceleration modified gravitational theory is the so-called quantized inertia [30,32,99]. Here the composite view of G falls nicely together with this model, as it has even been suggested by McCulloch that G is a composite and that the Planck units are perhaps a deeper and more important level. However, back then he was then not able to find the Planck units without knowledge of G, as pointed out earlier in his paper. To be able to find the Planck units without first knowing G has been solved in recent years and strengthens the view of McCulloch that one can indeed write G as a composite constant from Planck units. This alone naturally does not give credit to his model. In most alternative gravity models, where G is considered constant, then the composite view of G should in general cause no problems, but likely only helps us to get deeper insight into these models. This naturally must be carefully investigated for each alternative gravitational model before any conclusions are made, and to do so is beyond this paper’s scope. The recent collision space-time gravitational theory [67,83] is another alternative gravitational theory we already have mentioned and that fits very well with the composite view off G.

Several researchers have discussed and suggested gravity theories in which the gravity constant can vary. The suggestion that G is not a constant goes back at least to Milne [85]

in 1936. In modern times, for example, Zee [100], Adler [101], and Smolin [102] discuss symmetry breaking in relation to gravity where it seems G needs to vary. Davis [103], Pollock [104], Raychaudhuri and Bagch [105], and Masood-ul-Alam [106] discusses gravity theories where G has to vary. If G is simply a composite constant that can be expressed on the form $G = \frac{l_p^2 c^3}{\hbar}$, then either the Planck length, or the speed of light, or the Planck constant has to vary for G to vary, or alternatively there are other, still- missing parts in gravity that our models do not take into account, and which these models have tried to express through varying G . Could c , or l_p , or \hbar vary? The constancy and isotropic futures of the speed of light is one of the corner stones in special relativity theory. The round-trip speed of light has, through a long series of experiments, been tested to be constant and invariant (in vacuum). However, there are, to this day, discussions about how to measure the one-way speed of light and if it truly is invariant. It was Poincaré [107] that first pointed out that to measure the one-way speed of light, one has to have two clocks synchronized over distance, and to synchronize these clocks one has to know the one-way speed of light, so it led to a circular problem that seemed impossible to solve. Poincaré therefore suggested that, for continuity, one could also assume the one-way speed of light was constant for synchronization purposes.

Einstein went one step further and abandoned the ether, as detecting the ether is directly related to detecting anisotropy in the one-way speed of light, and since it could not be done, why not simply abandon something that adds complexity to the theory? However, if the one-way speed of light can somehow be measured, and whether it is invariant, is actually still an ongoing discussion, as is clear from recent publications; see, for example, Spavieri [108,109] and Kipreos and Balachandran [110,111].

There are also researchers discussing whether the Planck constant even could vary; see, for example, Masood-ul-Alam [106]. However, as we have shown, the Planck constant seems to cancel out between the embedded \hbar in the gravity constant and in the mass, so this would likely not be helpful for gravitational theories needing variable gravitational constant.

The Planck length has been less investigated since until recently we have only been able to derive it from dimensional analysis. The possibility cannot be excluded that the Planck length can be variable; this could be due to relativistic effects such as length contraction, but several researchers think the Planck scale (Planck length and Planck time in particular) will break with Lorentz symmetry and that the Planck length is invariant. This despite Lorentz symmetry being a foundational principle in general relativity and the standard model; however, many expect new physics to be discovered at the Planck scale, which again is often linked to quantum gravity; see for example, Tasson [112], so there are questions clearly still open for debate. A possible explanation for why the Planck length can be invariant was suggested by Haug [69,82]. All we can say is that our composite view of the gravitational constant does not yet exclude the view that the gravitational constant could be variable and that further research is needed before any “final” conclusion is made.

8. Conclusions

The idea that the gravitational constant can be a composite constant, which is related to more fundamental Planck units, goes back to at least 1984. However, in 1987 it was already pointed out that expressing the gravitational constant through Planck units led to a circular problem; namely, that one had to know the gravity constant to find the Planck units. This view has been repeated by researchers as recently as 2016. However, in more recent years, a series of papers have shown how one can clearly find the Planck units without knowledge of G , and even without knowledge of G and \hbar , so the circular problem regarding G and the Planck units has been solved. An in-depth study showed that this leads to a reduction in universal constants from G , \hbar , and c to only c and l_p and, in addition, one needs other constants like the fine structure constants when describing electromagnetic phenomena, but the traditional three universal constants that Max Planck used can be reduced from three to two. To predict all observable gravity phenomena, one only needs knowledge of l_p and c ,

and both can be found without knowledge of G and \hbar . This paper has given an overview of much of what has been done in relation to the composite view of the gravitational constant, but we have also tried to tie it nicely together. The implications of this should be worth studying further as this seems to open doors of insight into relationships between macroscopic gravity phenomena and the Planck scale.

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Notes

- 1 It is impossible for anyone today to know the full literature on physics, so there could be other authors already publishing these formulas; however, we have made a very serious attempt to search and find anyone who might have published these results first.
- 2 See also McCulloch [30,32].
- 3 The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for full circle, see [71].
- 4 Where H is the height of the ball drop.
- 5 The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for a full circle; see [71].
- 6 Where H is the height of the ball drop.
- 7 The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for a full circle, see [71].
- 8 Where H is the height of the ball drop.
- 9 That not should be confused with gravitational potential energy.

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