Relativistic Compton Wavelength

Espen Gaarder Haug

Abstract

In 1923, Arthur Holly Compton introduced what today is known as the Compton wavelength. Even if the Compton scattering derivation by Compton is relativistic in the sense that it takes into account the momentum of photons traveling at the speed of light, the original Compton derivation indirectly assumes that the electron is stationary at the moment it is scattered by electrons, but not after it has been hit by photons. Here, we extend this to derive Compton scattering for the case when the electron is initially moving at a velocity v.

Index Terms

Compton scattering; Compton wavelength; moving electron.

I. INTRODUCTION

In 1923, Arthur Holly Compton introduced Compton scattering [1] and, indirectly in his formulation, the Compton wavelength of the electron. The Compton wavelength of the electron plays a central part in several areas of physics. It can be used to find the rest mass of an electron as described by Prasannakumar et al. [2]. It can indirectly be found in the relativistic wave equation of Klein and Gordon and in the Dirac equation [3] as well as in the non-relativistic Schödinger equation [4]. Haug has recently suggested that the Compton wavelength is the true matter wavelength, and that the de Broglie [5], [6] wavelength is merely a mathematical derivative of the Compton wavelength. We will not discuss or make any conclusion about that suggestion here, but it is worth noticing that the de Broglie wavelength is always identical to the Compton wavelength multiplied by $\frac{c}{v}$, and naturally the Compton wavelength is then always equal to the de Broglie wavelength times $\frac{v}{c}$.

Here we will shortly repeat how to find the Compton wavelength from a rest-mass electron as Compton did in 1923, and next extend our derivation to also take into account Compton scattering done with an initially moving electron.

II. COMPTON SCATTERING AND THE COMPTON WAVELENGTH

We have the following two equations:

$$p_1 c + mc^2 = p_2 c + mc^2 \gamma_a \tag{1}$$

where p_1 and p_2 are the photon momentum in the incoming and outgoing wave, respectively, m is the electron rest mass, and $\gamma_a = \frac{1}{\sqrt{1-\frac{v_a^2}{c^2}}}$, where v_a is the velocity of the electron after the impact of the photon. Further, we have

$$\left(mc^2 \gamma_a\right)^2 = p_1^2 c^2 + p_2^2 c^2 - 2p_1 p_2 c \cos\theta \tag{2}$$

where θ is the angle between the incoming and outgoing photon Be aware that here we, like Compton, assume the electron is at rest immediately before impact. This gives

$$p_1 c + mc^2 = p_2 c + mc^2 \gamma_a$$
$$(p_1 - p_2 + mc)^2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta$$
$$p_1^2 - 2p_1 p_2 + p_2^2 + 2mcp_1 - 2mcp_2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta$$
$$mcp_2 - mcp_1 = p_1 p_2 (1 - \cos\theta)$$
$$\frac{1}{p_1} - \frac{1}{p_2} = \frac{h}{mc}(1 - \cos\theta)$$
$$\frac{h}{p_1} - \frac{h}{p_2} = \frac{h}{mc}(1 - \cos\theta)$$
$$\lambda_1 - \lambda_2 = \frac{h}{mc}(1 - \cos\theta). \tag{3}$$

This is the well-known Compton scattering formula, where $\frac{\hbar}{mc}$ is what today is known as the Compton wavelength. This means the rest wavelength of the Compton wavelength can be found directly from the incoming and outgoing wavelength of the photon plus a measurement given the angle θ, so we have

$$\begin{aligned}
\lambda_1 - \lambda_2 &= \lambda(1 - \cos\theta) \\
\lambda &= \frac{\lambda_1 - \lambda_2}{1 - \cos\theta}
\end{aligned} \tag{4}$$

where λ is the Compton wavelength of the particle at rest (the electron). The Compton wavelength is therefore indirectly measured by watching the change in wavelength in the photon used to scatter the electrons. However, Compton's formula, even if relativistic, assumes the electron that is scattered by the photon is itself at rest before the scattering.

Compton scattering, if the electron is also moving initially, at velocity v relative to the laboratory frame (and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$), is given by

$$\begin{aligned}
p_1 c + mc^2\gamma &= p_2 c + mc^2\gamma_a \\
(p_1 - p_2 + mc\gamma)^2 &= p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta \\
p_1^2 - 2p_1 p_2 + p_2^2 + 2mc\gamma p_1 - 2mc\gamma p_2 &= p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta \\
mc\gamma p_2 - mc\gamma p_1 &= p_1 p_2 (1 - \cos\theta) \\
\frac{1}{p_1} - \frac{1}{p_2} &= \frac{h}{mc\gamma}(1 - \cos\theta) \\
\frac{\hbar}{p_1} - \frac{\hbar}{p_2} &= \frac{h}{mc\gamma}(1 - \cos\theta) \\
\lambda_1 - \lambda_2 &= \frac{h}{mc\gamma}(1 - \cos\theta) \\
\lambda_1 - \lambda_2 &= \lambda\sqrt{1 - \frac{v^2}{c^2}}(1 - \cos\theta).
\end{aligned} \tag{5}$$

The relativistic Compton wavelength is therefore given by

$$\lambda\sqrt{1 - \frac{v^2}{c^2}} = \frac{\lambda_1 - \lambda_2}{(1 - \cos\theta)}. \tag{6}$$

In other words, we only need to measure the photon wavelength before or after the scattering and the angle θ to know the relativistic Compton wavelength. This means the relativistic Compton wavelength can also be written as

$$\lambda_r = \frac{h}{mc\gamma} = \lambda\sqrt{1 - \frac{v^2}{c^2}}. \tag{7}$$

Since the relativistic de Broglie wavelength is given by $\lambda_b = \frac{h}{mv\gamma}$, this means the de Broglie wavelength always is equal to the Compton wavelength multiplied by $\frac{c}{v}$, as also shown by Haisch, Rueda, and Dobyns [7], and discussed in more detail by Haug [8], [9]. Interestingly, the de Broglie wavelength is not defined or is infinite for a rest-mass particle since this gives $\lambda_b = \frac{h}{m \times 0}$, and is mathematically undefined. Or we could claim it is infinite, as several other researchers have done since we can make v as close to 0 as we want, and we then see the de Broglie wavelength converge to infinite, see [11], [12]. It seems almost absurd to assume the matter wavelength of a particle at rest is infinite or not defined. The Compton wavelength, on the other hand, is always well defined. One can question why there should be two matter waves, and not only one. We [8] have recently suggested it is the Compton wavelength that is the true matter wavelength and that the de Broglie wavelength is a mathematical derivative of this wavelength, as the relation shown here also indicates. However, this discussion is not the main focus here, but worth thinking about. The main purpose here was to show that we can derive a fully relativistic Compton wavelength based on the assumption that also the electron is initially moving also before it is hit by photons.

III. WHY HAS THIS NOT BEEN DERIVED BEFORE?

As demonstrated in the section above, to derive a relativistic Compton wavelength that also takes into consideration the fact that the electron is initially moving is easy. So, an interesting question is why has it then not been done before? After all, the Compton wavelength has been well known since 1923. One of the reasons for the lack of interest in deriving and investigating the consequences of a full relativistic Compton wavelength is likely related to the physics' community's negativity towards the relativistic mass concept.

In 1899, Lorentz [13] already suggested relativistic mass as we know it today: $m\gamma$. Einstein, at the end of his famous 1905 paper on special relativity, also suggested relativistic mass formulas; namely $m\gamma^2$ and $m\gamma^3$, though neither of them is used today. We must remember that Lorentz's relativity theory was, for many years, considered a competing theory to Einstein's special relativity theory. Perhaps because Einstein likely got the relativistic mass wrong and did not want to rely on the competing theory, he decided to ignore relativistic mass later on. Many well-known physicists have followed in Einstein's footsteps in regard to this point and have been very negative towards relativistic mass; for example, Okun [14], Adler [15], Hetch [16], and Taylor and Wheeler [17], while a minority have been quite positive towards the concept of relativistic mass, such as Rindler [18] and Jammer [19]. A series of researchers, see for example Milne [20], Grøn [21], and Haug [22], have, however, also mistakenly thought (or given the impression) that relativistic mass as known today, i.e., $m\gamma$, was introduced by Einstein. More importantly, the consequences of incorporating relativistic mass were never fully investigated before it was rejected.

In 1906, Max Planck introduced relativistic momentum, i.e., $p = mv\gamma$, which is identical to how we know it today, and this was later incorporated into fourth momentum. According to Adler, Okun, Hetch, and Taylor and Wheeler, the mass is always the rest mass as there is no such thing as relativistic mass in their view. The only other thing the Lorentz factor can act on instead of the mass in the relativistic momentum formula is the velocity itself. However, as recently pointed out by Haug, [23] if $v > \frac{c}{\sqrt{2}} \approx 0.71c$ then the relativistic velocity, $v\gamma$ is above c which is contradictory to the assumptions in the theory. We could even claim such a view of relativistic velocity above the speed of light would be absurd. The only thing that, in our view, makes logical sense is that the Lorentz factor acts on the mass in the relativistic momentum formula. We personally think one of the biggest mistakes in modern physics was to reject relativistic mass before full investigations into its implications, both in respect to prediction results and logic.

The standard, non-relativistic Compton wavelength is given by $\lambda = \frac{h}{mc}$, so to implement a Lorenz factor makes little sense as long as one has the view that there can be no relativistic mass, because then the Lorentz factor must, in such a view, act on either the Planck constant or the speed of light, which are both considered constants. So this could be the reason why no one has derived a full relativistic Compton wavelength before[1]. It is trivial to incorporate relativistic effects in relation to the Compton wavelength if we just accept the concept of relativistic mass. The relativistic Compton wavelength we can derive by simply multiplying the mass m with γ to incorporate relativistic mass; this gives a relativistic Compton wavelength off $\lambda_r = \frac{h}{m\gamma c}$, which is the same formula we derived more formally in the sections above.

IV. CONCLUSION

We have shown how Compton scattering for the case where an electron is moving initially can be derived. This gives us the relativistic Compton wavelength rather than the Compton wavelength for an electron at rest, as was achieved by Compton himself. The relativistic Compton wavelength, for example, plays a central role in the recent quantum gravity theory presented by Haug [8], [10]. Moreover, when we also have a relativistic Compton wavelength formula, then we see that the de Broglie wavelength is always the Compton wavelength multiplied by $\frac{c}{v}$. While the de Broglie wavelength is infinite (or undefined, but it is anyway convergent towards infinity, as there are no limitations in standard physics of how close v can be to zero.) for a rest-mass particle, the Compton wavelength is more well-defined and has been measured for rest-mass particles. Even if the derivation in this paper is trivial, we think it could be important for the physics community to be aware of the difference between the Compton wavelength and the relativistic Compton wavelength.

REFERENCES

[1] A. H. Compton. A quantum theory of the scattering of x-rays by light elements. *Physical Review*, 21(5):483, 1923. URL https://doi.org/10.1103/PhysRev.21.483.
[2] S. Prasannakumar, S. Krishnaveni, and T. K. Umesh. Determination of rest mass energy of the electron by a Compton scattering experiment. *European Journal of Physics*, 33(1):215, 2012. URL https://doi.org/10.1088/0143-0807/33/1/005.
[3] P. Dirac. The quantum theory of the electron. *Proc. Roy. Soc. London*, (117):610, 1928. URL https://doi.org/10.1098/rspa.1928.0023.
[4] E. Schrödinger. Experimental test of Heisenberg's measurement uncertainty relation based on statistical distances. *Physical Review*, 28(6):104–1070, 1928.
[5] de. L. Broglie. Waves and quanta. *Nature*, 112:540, 1923. URL https://doi.org/10.1038/112540a0.
[6] de. L. Broglie. *An introduction to the Study of Wave Mechanics*. Metheum & Co., Essex, 1930.
[7] B. Haisch, A. Rueda, and Y. Dobyns. Inertial mass and the quantum vacuum fields. *Annalen der Physik*, 10(5):393, 2001.
[8] E. G. Haug. Collision space-time: Unified quantum gravity. *Physics Essays*, 33(1):46, 2020. URL https://doi.org/10.4006/0836-1398-33.1.46.
[9] E. G. Haug. *Quantum Gravity Hidden In Newton Gravity And How To Unify It With Quantum Mechanics*. in the book: The Origin of Gravity from the First Principles, Editor Volodymyr Krasnoholovets, NOVA Publishing, New York, 2021.
[10] E. G. Haug. Unified quantum gravity field equation describing the universe from the smallest to the cosmological scales. *Physics Essays*, 35:61, 2022. URL https://doi.org/10.4006/0836-1398-35.1.61.
[11] H. Chauhan, S. Rawal, and R. K. Sinha. Wave-particle duality revitalized: Consequences, applications and relativistic quantum mechanics. *https://arxiv.org/pdf/1110.4263.pdf*, 2011.
[12] A. I. Lvovsky. *Quantum Physics: An Introduction Based on Photons*. Springer, 2018.
[13] H. A. Lorentz. Simplified theory of electrical and optical phenomena in moving systems. *Proc. Acad. Scientific, Amsterdam*, 1, 1899.
[14] L. B. Okun. The concept of mass. *Physics Today*, 42, 1989. URL https://doi.org/10.1063/1.881171.

---

[1]This paper is a strongly improved version of a working paper first put on viXra 26 February 2020.

[15] C. G. Adler. Dose mass really depends on velocity dad? *American Journal of Physics*, 55:739, 1987. URL https://doi.org/10.1119/1.15314.

[16] E. Hecht. Einstein never approved the relativistic mass formula. *The Physics Teacher*, 47:336, 2009. URL https://doi.org/10.1119/1.3204111.

[17] E. F. Taylor and J. A. Wheeler. *Spacetime Physics, Introduction To Special Relativity*. W. H. Freeman and Company, New York, 1992.

[18] Wolfgang Rindler. *Relativity, Special, General and Cosmology, Second Edition*. Oxford University Press, 2006.

[19] M. Jammer. *Concepts of Mass in Contemporary Physics and Philosophy*. Princeton University Press, 2000.

[20] E. A. Milne. On the foundations of dynamics. *Proceedings of the Royal Society A*, 154:22, 1936. URL https://doi.org/10.1098/rspa.1936.0034.

[21] Ø Grøn. *Introduction to Einstein's Theory of Relativity, From Newton's Attractive Gravity to the Repulsive Gravity of Vacuum Energy, Second Edition*. Springer Verlag, 2020.

[22] E. G. Haug. Modern physics incomplete absurd relativistic mass interpretation. and the simple solution that saves Einstein's formula. *Journal of Modern Physics*, 9(14), 2018. URL https://doi.org/110.4236/jmp.2018.914163.

[23] E. G. Haug. Planck speed: the missing speed of physics? Absolute still without breaking Lorentz symmetry! *European Journal of Applied Physics*, 4 (1):15, 2022a. URL https://www.ej-physics.org/index.php/ejphysics/article/view/144.