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Testing light speed invariance by measuring the one-way light speed on Earth

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ABSTRACT

We show that the one-way speed of light, along an open contour AB on the uniformly rotating Earth, is observable. Our approach exploits the property of the Sagnac effect that can measure the Earth's local angular velocity and, correspondingly, the peripheral speed of the contour AB relative to Earth Centered Inertial (ECI) frame. Light speed variations, measured on the rotating Earth, may be related to the velocity **w** of the ECI frame relative to a hypothetical preferred frame. Since it is possible to test Einstein's postulate of a universal light speed, standard special relativity is confirmed to be a viable falsifiable theory.

1. Introduction

In the context of relativistic theories, the one-way speed of light in free space from point A to point B, with fixed distance AB $= L_0$, is considered to be arbitrary and to depend on the procedure adopted to synchronize the spatially separated clocks A and B [1–13]. In standard special relativity based on relative simultaneity, the one-way light speed is assumed to coincide with the average round-trip light speed c of Einstein synchronization. Instead, in relativistic preferred frame theories adopting absolute simultaneity, the one-way light speed is not invariant and differs from the average speed c [3,4]. The aim of our article is to present an experiment suitable for testing light speed invariance by measuring variations of the one-way speed of light on the rotating Earth.

In order to clarify the relevance of our experiment in the context of relativistic theories, in the first sections of our article we review the role of simultaneity in the interpretation of light speed propagation on moving contours. In Section 2 we discuss clock synchronization and show when, in optical experiments, variations of the one-way speed of light are not detectable because they vanish on average. Furthermore, we revise aspects of light propagation in the circular and linear Sagnac [14,15] experiments, as seen in the laboratory frame and in a frame co-moving with a section of the contour. In Section 3 we consider light propagation on an open contour AB on the rotating Earth. Assuming that the Earth Centered Inertial (ECI) frame is in motion, relative to the hypothetical preferred frame of relativistic theories, we show that variations of the one-way speed of light along AB are observable. Our approach is based on the property of the Sagnac effect that can determine the Earth local angular velocity and, correspondingly, the relative speed between the contour AB and the ECI frame.

Epistemologists [16] claim that the basic postulates of a meaningful physical theory must be testable (i.e., falsifiable). Since, from a conceptual perspective, our experiment indicates that the invariance of the speed of light can be tested, we may conclude that standard special relativity is a valid and viable falsifiable theory.

The reader familiar with the role of relative and absolute simultaneity in SR may wish to skip reading Section 2 and the Appendix and go directly to Section 3 where our test of the invariance of c is described.

2. Light speed propagation on a moving contour in the context of relativistic theories

Several authors [1,2,4,9,12,13,17], have emphasized the difficulties that emerge with standard SR – based on Einstein synchronization and the Lorentz transformations (LT) – when applied to light propagation on moving closed contours. These difficulties show up in the diverse interpretations of the Sagnac effect [4,9,10,12,13], widely discussed in the literature [18]. In order to surmount the difficulties, some physicists [1,3,4,9–11], propose adopting coordinate transformations based on conservation of simultaneity, in lieu of the LT based on "relative" simultaneity.

In the case of a closed contour, the root of the problem is the time

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discontinuity in the LT, highlighted by Landau and Lifshitz [17] by stating: "… synchronization of clocks along a closed contour turns out to be impossible in general. In fact, starting out along the contour and returning to the initial point, we would obtain for dx° a value different from zero …". Following Landau and Lifshitz [17], various physicists [1, 2,4,9–13] recognize that the standard synchronization procedure proposed by Einstein fails (or is non-integrable) when applied to the closed contour of the Sagnac effect.

2.1. Describing light propagation on a moving closed contour

Einstein synchronization procedure introduces an indeterminateness in the one-way speed of light, which is related to the conventionality of clock synchronization [3,5–7]. In discussing the speed of light we have to distinguish between one-way speed and average speed. The term Constancy of the Speed of Light refers to the average speed of light $c_{av} = c$ during a round-trip (e.g., the round-trip starting from point A to point B and then back to A), assuming that it is the same in all inertial frames [1, 3,9]. However, when using the LT with standard synchrony the assumption is more stringent because it requires that the one-way speed of light c from A to B is the same as that from B to A.

Relativistic theories assume the existence of an inertial frame S where physical empty space is homogeneous and isotropic and, thus, the one-way speed is c. For theories with coordinate transformations conserving simultaneity, frame S is the preferred frame, while for standard SR, based on the LT and relative simultaneity, the one-way speed is c on S and on any other inertial frame. Within relativistic theories, time depends on an arbitrary synchronization parameter ϵ [3] and the coordinate transformations between two inertial frames of reference, S' and S, in relative motion with velocity $\mathbf{v} = \hat{i}v$, may be written as,

$$t' = t/\gamma - \epsilon x'/c^2; \quad x' = \gamma(x - vt); \quad y' = y; \quad z' = z. \tag{1}$$

Adjusting the value of ϵ, the three most common transformations are:

$$\begin{array}{lll} GT \ \ (\epsilon = 0) & t' = t & x' = x - vt \\ LT \ \ (\epsilon = v) & t' = \gamma(t - vx/c^2) & x' = \gamma(x - vt) \\ LTA \ (\epsilon = 0) & t' = t/\gamma & x' = \gamma(x - vt), \end{array} \tag{2}$$

where in (2) the transformations $y' = y$; $z' = z$ are understood. With $\epsilon = 0$ and $\gamma = 1$, GT stands for the Galileo transformations of Newtonian physics. With the factor $\gamma = (1 - v^2/c^2)^{-1/2}$ depending on v and $\epsilon = v$, LT stands for the Lorentz transformations, based on standard synchrony and relative simultaneity. With $\epsilon = 0$, LTA stands for the Lorentz transformations based on absolute synchrony and simultaneity. The LTA (or ALT in Ref. [13]) are known in the literature as the Tangherlini-Selleri transformations [4,19,20], used by several authors [1,2,9,12,13].

The arbitrariness of the light speed is made apparent by writing the explicit light speed dependence on ϵ,

$$c' = c'(\epsilon) = \frac{dx'}{dt'} = \frac{c}{1 + v/c - \epsilon/c}, \tag{3}$$

which, to first order in v/c, is valid for the GT.

2.2. Clock synchronization and the one-way speed of light

In Fig. 1-a, a light ray (a photon) is sent along a closed stationary circular contour of length $2L = 2\pi r$ from clock A at point A to clock B at point B at the middle distance $L = \pi r = AB$. In the context of preferred frame relativistic theories [3], if space is not isotropic on the frame S where AB is stationary, the outward one-way light speed c_{out} differs from the return one-way speed c_{ret} (either along the upper or lower semicircle). In classical pre-relativistic physics the anisotropy of space can be thought of as due to the effect of a hypothetical ether wind of velocity w that modifies the speed of light. In relativistic theories adopting absolute synchrony with the LTA (2), the one-way speed c is modified to $c_{out} = c/(1 + w/c) = \gamma_w^2(c - w) \simeq c - w$ and $c_{ret} = c/(1 - w/c) = \gamma_w^2(c + w) \simeq c + w$ when AB is moving with velocity $\simeq w$ relative to the preferred frame.

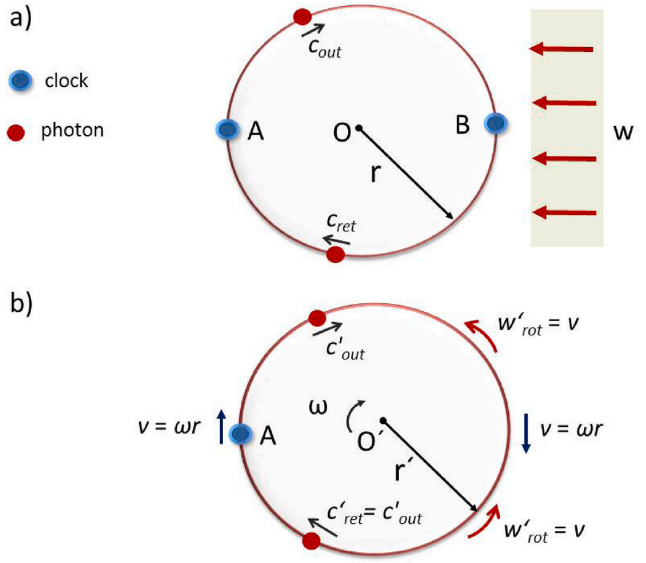

Fig. 1. *a)* On the stationary circular contour a photon is sent from clock A to clock B at the speed c_{out} and returns from B to A at the speed c_{ret}. The speed of light is modified by the ether wind $\mathbf{w}$ but the average speed is not. *b)* If clock and contour are rotating at the angular velocity ω, as seen from the rotating frame there is an ether wind blowing at velocity $w'_{rot} = v = \omega r$ that modifies the speed of light c'_{out}.

Assuming that the two-way average speed is $c_{av} = c$ in frame S, if c_{ret} and c_{out} are evaluated with the LTA, the round-trip time interval T_{round} and the average speed c_{av} are related by,

$$\begin{aligned} T_{round} &= \frac{2L}{c} = \frac{2L}{c_{av}} = \frac{L}{c_{out}} + \frac{L}{c_{ret}} \\ &= \frac{L}{\gamma_w^2(c - w)} + \frac{L}{\gamma_w^2(c + w)} = \frac{2L}{c}. \end{aligned} \tag{4}$$

On average, in (4) the effect of the ether wind cancels for a round trip, as happens in the Michelson-Morley experiment where the LTA foresee the same null result [3] of the LT (with $w = 0$ and $c_{out} = c_{ret} = c$).

Since, on calculating T_{round} with different synchronies, expression (4) provides the same result, some authors assume that the one-way speed of light is arbitrary and different synchronies are physically equivalent [3, 5–8]. However, if the one-way light speed is arbitrary, the fundamental postulate of light speed invariance cannot be tested. In this case, epistemologists [16] could claim that special relativity is not a meaningful physical theory because one of its basic postulates is not testable (falsifiable). Hence, for the standing of the theory it is relevant to determine whether the LTA and the LT are physically equivalent or not.

The effect of space anisotropy on the one-way speed of light may be described in terms of the velocity field $\mathbf{w}$ (as done by Silberstein [21] and Post [18]) which mimics the velocity of the classical ether wind. If $\mathbf{w}$ is uniform, we have $\nabla \times \boldsymbol{w} = 0$. In this case, the procedure (4), which is used to synchronize clock A with clock B, provides the same result when light performs the round trip from A to B and back to A on the upper semicircle, or from A to B on the upper but back on the lower semicircle. Thus, when $\nabla \times \boldsymbol{w} = 0$, the synchronization procedure (4), foresees that, on average, light propagates as if the velocity field is zero ($\mathbf{w} = 0$, no ether wind).

However, in case of rotational motion the velocity field is not curless and $\nabla \times \boldsymbol{w}_{rot} = 2\boldsymbol{\omega} \neq 0$, where $\boldsymbol{\omega}$ is the rotational angular velocity. Consider the contour of Fig. 1-b that is rotating clockwise at the peripheral velocity $v = \omega r$ relative to the isotropic space of the stationary frame S. As observed from the rotating frame S'_{rot} of the contour, the anisotropy of space – produced by the rotation of frame S relative to S'_{rot} – may be interpreted as due to a counter-rotating velocity field $|\mathbf{w}'_{rot}| = |\mathbf{v}|$ (an ether wind) with $\nabla \times \mathbf{w}'_{rot} = 2\boldsymbol{\omega}'$. For light co-propagating outward on the upper and back on the lower semicircle (Fig. 1-b), in agreement with the result of the Sagnac effect, the proper time interval τ_{round} measured by the rotating clock A is,

$$\begin{aligned}\tau_{round} &= \frac{L}{c'_{out}} + \frac{L}{c'_{ret}} = \frac{L}{\gamma^2(c-v)} + \frac{L}{\gamma^2(c-v)} \\ &= \frac{2L}{\gamma^2(c-v)} = \frac{2\pi r}{\gamma^2(c-v)} = \frac{2L}{(c'_{out})_{av}},\end{aligned} \tag{5}$$

where $c'_{out} = c'_{ret} = \gamma^2(c-v) = (c'_{out})_{av} \neq c$ represents the average one-way light speed of a photon co-propagating along the circumference of rest length $2\pi r = 2L$, where c is modified by the wind rotational velocity $\mathbf{v} = \mathbf{w}_{rot}$.

In expression (5) we use the rest length $2\pi r = 2L$ (and not the rest length $\gamma 2\pi r = \gamma 2L$) in agreement with the considerations made by Kipreos and Balachandran in Ref. [13] and its companion paper. In fact, it is known that the inclusion of the γ term is incorrect because the corresponding Sagnac equation implies one-way speeds of light in the rotating frame that will generate an anisotropic two-way speed of light. Moreover, the anisotropic two-way speed of light that is linked to the Sagnac equation with the γ term is invalidated by high-resolution optical data.

As seen from clock A, the ground path of the contour to be covered by the photon is $2L$ and, if $\mathbf{v}$ is uniform along the circumference, due to symmetry, the ground speed, i.e., the local speed along the rotating contour, is uniform and coincides with the average speed. Then, expression (5) shows that Einstein synchronization applied on the rotating contour fails (in the sense that it is not applicable, or integrable) because the average light speed $(c'_{out})_{av} = c'_{out}$ along the closed contour $2\pi r$, differs from the value of the constant speed c postulated with Einstein synchronization (4). Yet, Einstein synchronization can be applied to an open path (arc) AB of the circumference. For example, light can travel from A to a point B on the upper semicircle and back from B to A always on the upper semicircle. Thus, supporters of the LT can still claim that the local light speed is c along AB, a property that we wish to test in Section 3.

A physical example where result (5) applies is given by the Michelson-Gale [22] experiment where the contour is stationary on Earth. The Michelson-Gale experiment is considered to be equivalent to a Sagnac effect and in both cases the failure of Einstein synchronization has been ascribed by some physicists to the fact that the measuring device is not on an inertial frame — " … A state of kinematical acceleration … associated with a state of absolute motion …" Post [18] —. Nevertheless, as considered below, Einstein synchronization and the use of the LT lead to inconsistencies even in the linear Sagnac effect [15] where the measuring device can be on an inertial frame during the interval τ_{round} of the photon round-trip. In fact, the circular contour of Fig. 1 can be stretched to assume an elliptical shape or, at the limit, the shape of the very long contour of Fig. 2, as in the linear Sagnac effect, where the dimension of the pulleys A and B can be assumed to be negligible. It is easy to verify that the same considerations made for the stationary circular contour of Fig. 1-a, apply for the stationary linear contour of Fig. 2-a.

When clock and contour (a flexible optical fiber in the linear Sagnac effect [15]) are in motion (Fig. 2-b) relative to frame S where space is isotropic, we can maintain the analogy of the velocity field acting as an ether wind always blowing against the photon traveling from A to B on the upper section in the out trip, and from B to A on the lower section in the return trip, as in the case of the rotational motion of Fig. 1-b.

Expression (5) applied to the contour of Fig. 2-b indicates again that the one-way ground speed of light is equal to the average speed, $c'_{out} =$

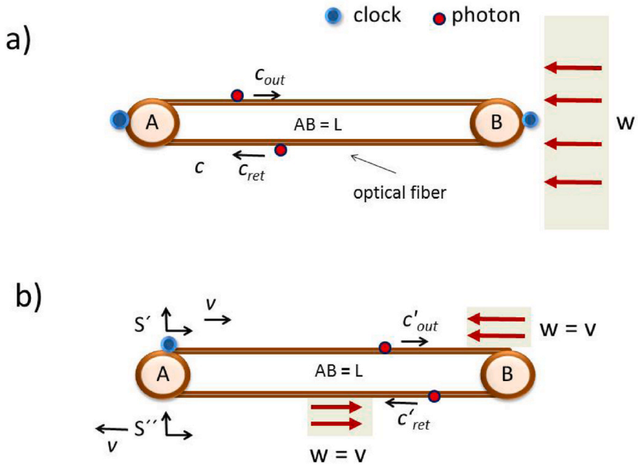

Fig. 2. *a)* Clocks and contour (optical fiber) are stationary, while the photon moves at speed c_{out} from A to B and speed c_{ret} from B to A. *b)* Clock and contour are in motion relative to the preferred frame S where the pulleys A and B are stationary. Relative to the clock, the one-way ground speed of light coincides with the average speed, $c'_{out} = \gamma^2(c-v) = c'_{av} \neq c$. The change of the light speed c can be interpreted as due to an ether wind $w = v$ blowing against the photon in motion.

$c'_{ret} = \gamma^2(c-v) = c'_{av} \neq c$. It should be clear that the failure of Einstein synchronization is not related to the fact that clock A is accelerated (as in the circular Sagnac effect) or not accelerated (as it may be in the linear Sagnac effect), but to the fact that light is bound to propagate along the closed moving contour where the round-trip time interval τ_{round} is measured by a single clock that needs no synchronization and, thus, removes the indeterminacy of the one-way light speed.

In general, curvilinear transformations in one dimension from frame S_σ to the frame $S'_{\sigma'}$ along the whole closed contour and co-moving with it, can be expressed as [9], $\sigma' = \gamma(\sigma - vt)$, where σ is the curvilinear one-dimensional spatial coordinate. The corresponding time transform is, $t' = t/\gamma$ or $t' = \gamma(t - v\sigma/c^2)$, depending on whether we use absolute or relative simultaneity. The important point is that, for describing light propagation on the moving closed contour and interpreting result (5) of the Sagnac effect, no problems arise if absolute simultaneity is adopted [1,3,4,9–13]. In flat spacetime with cylindrical coordinates, is generally used the Langevin–Landau–Lifshitz metric [18,23–25], with $\tau = t' = t/\gamma$ the proper time of the clock on the rotating circumference. However, if relative simultaneity is adopted, we are met with the undesirable consequence mentioned in Refs: [1,4, 9–13]. In fact, the relative time transform $t' = \gamma(t - v\sigma/c^2)$ predicts that a clock at the distance $\sigma = 2L = 2\pi r$ from A (i.e., the same clock A) displays the time gap $\delta t' = -2vL/c^2$ and, thus, is out of synchrony with itself [1,2,4,9,12].

Instead of curvilinear coordinates, for the linear Sagnac effect we may introduce two Cartesian inertial frames, S' co-moving with the upper section, and S'' co-moving with the lower section (Fig. 2-b), moving at velocity v and $-v$, respectively, relative to frame S. In this case, using relative simultaneity, the Lorentz transformations between S' and S'' (derived in Refs: [9]) turn out to be given by, $x' = \gamma_w(x'' - wt'')$, $t' = \gamma_w(t'' - wx''/c^2)$ where $w = 2v/(1 + v^2/c^2)$, $\gamma_w = \gamma^2(1 + v^2/c^2)$. However, as pointed out by Landau and Lifshitz [17], for a closed contour we still have the time gap, which is now at B ($x''_B = L/\gamma$) and is given by $\delta t' = -(\gamma_w/\gamma)wL/c^2 = -2\gamma vL/c^2$. Some of the undesirable consequences emerging by adopting relative simultaneity in describing light propagation along closed moving contours (discussed in Refs. [9,26]) are considered below and in the Appendix.

2.3. Ground path covered by the particle according to relative or absolute simultaneity

We mentioned above that the problems of the LT and relative simultaneity are related, in general, to the failure of Einstein synchronization when applied on a moving closed contour. The point is that with the LT, at the invariant ground speed c, the particle cannot cover the whole closed contour in the measured time interval τ_{round}.

Let us consider the kinematical relation,

$$c_g = \frac{ds}{d\tau}, \tag{6}$$

where c_g is the local ground speed of light along the contour's ground path s_g and the elementary path ds is traversed by light in the proper time interval $d\tau$ measured by a clock fixed to the contour. When measured along s_g, even if the contour is in motion, each section ds of the contour maintains the same ground distance from the clock (at least to the first order). Then, to the first order in v/c, we expect that the following relation be valid,

$$\tau_{round} = \int d\tau = \oint_g \frac{dL_g}{c_g} = \frac{1}{c_g}\oint_g dL_g = \frac{L_g}{c_g}, \tag{7}$$

where the last two terms correspond to the special case $c_g = constant$ and (7) should hold for the rotating contours of Fig. 1-b and Fig. 2-b also. In the circular and linear Sagnac effects, the speed of light is assumed to be c on the stationary frame S. For light counter-propagating in a round trip, the clock on the contour in motion relative to S measures the interval τ_{round} given by (5) (with $v \to -v$ and the ground path given by $2L = 2\pi r$). It follows from (7) that, if the light ground speed is c, the ground path L_{g-eff} effectively covered in the interval τ_{round} is [9,26],

$$\begin{aligned} L_{g-eff} &= \oint_g dL_g = c\tau_{round} = \frac{2L}{\gamma^2(c+v)} \\ &= 2L\left(1-\frac{v}{c}\right) = 2\pi r\left(1-\frac{v}{c}\right) = 2\pi r - c\delta t' < 2L, \end{aligned} \tag{8}$$

where $\delta t'$ is the time gap.

Thus, we may infer from (8) that, by adopting the LT with the invariant light speed $c_g = c$, the path L_{g-eff} effectively covered is open.

Instead, by adopting the LTA with $c_g \neq c$, the path L_{g-eff} is closed and equal to the ground path $2L = 2\pi r$ [9,26]. What is implied by the Sagnac effect is that, if the local light speed is c along one of the two (upper or lower) relatively moving sections of the contour, it cannot be c along the other section. In order to corroborate our claim, for the convenience of the reader in the Appendix we work out in detail two simple examples of light propagation in the linear Sagnac effect (discussed also in Refs. [9, 26]) and verify the length of the path effectively covered by the photon in a round trip. The example 6.1 of the Appendix is original and has not been discussed in previous works. The example 6.2 has been considered in Ref. [9] but is presented with an important change because we use now two clocks for determining the light speed on the upper and lower sections of the contour. Since the two clocks are both in uniform motion, the objection that the observer co-moving with the clock is not an inertial observer does no longer apply.

In conclusion, the Sagnac experiment shows that, with measurements performed with a single clock where no clock synchronization is required, the average one-way $(c'_{out})_{av}$ of (5) differs from the average two-way speed $c = c_{av}$ of Einstein synchronization (4).

3. Experiment for measuring the velocity of the ECI frame relative to the preferred frame

Lorentz invariance experiments based on optical experiments of the Michelson-Morley type have indicated the absence of a detectable external preferred reference frame with precision up to the 10^{-17} level [29]. There are Lorentz invariance experiments that use approaches other than light propagation and deal with properties of matter where Hughes–Drever-type experiments [30,31] test whether the dispersion relation of the kinetic energy of particles are isotropic. Furthermore, searching for violation of Lorentz symmetry for electrons, an electronic analogue of a Michelson–Morley experiment has been performed by splitting an electron wave packet bound inside a calcium ion into two parts with different orientations and recombine them after a time evolution of 95 ms with a precision reaching the 10^{-18} level [32].

We have shown in the previous Sections that, in optical experiments of the Michelson–Morley and Sagnac type, light propagates within a closed contour, performing a round trip starting from the interferometric device and returning back to it. In these experiments that attempt of verifying the isotropy of the speed of light, the average value c of light speed is involved and, thus, these experiments are "round-trip" and not "one-way experiments". As shown also by Mansouri and Sexl [3], this type of experiments are unable to reveal the hypothetical velocity **W** relative to the preferred frame S, if this frame were to exist, because **W** vanishes on average. Thus, in our opinion "round-trip" optical experiments are conceptually unsuitable for detecting light speed anisotropy or testing Lorentz invariance. It is possible that the considerations made above for "round-trip" optical experiments might hold also for properties of matter. For example, in the experiment related to the splitting of an electron wave packet, because of the recombination the change of orientation is "two-way" and the measured quantity is likely reflecting an average two-way property, rather than the one-way orientation of the wave packet.

In any event, for "round-trip" optical experiments using interferometric techniques the limitation for detecting **W** is evident and, for the purpose of testing Lorentz invariance, it becomes necessary to make use of "one-way experiments", such as the one performed on the Earth surface described below, which relies on one-way light propagation along the short open path AB of Fig. 3. The time of flight of one-way light signals along AB are measured with the help of an emitter/detector with a single clock and, in principle, no interferometric techniques are required for our experiment. The essential additional information that leads to determining the one-way light speed in our experiment is the Earth peripheral velocity, which can be determined *independently* by a standard Sagnac experiment that provides the Earth angular velocity without the necessity of synchronizing clocks along the path AB. Although an independent Sagnac experiment is needed for providing the Earth angular velocity, it is worth emphasizing that our experiment is not a "round-trip experiment" of the Sagnac type because the time delay of the light signal propagating along the emitter/detector AB fixed to the Earth covers the "one-way path" AB only.

Let us suppose that the ECI frame S' is moving with uniform velocity

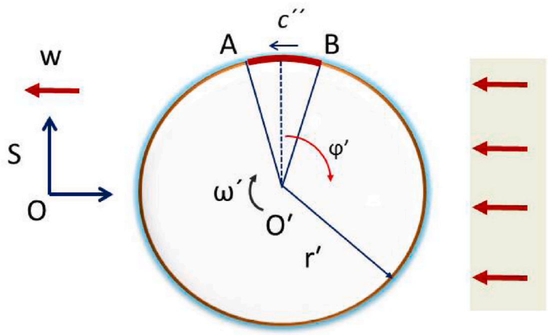

Fig. 3. Relative to the stationary ECI frame S', the preferred frame S is moving with velocity $\simeq -w$, while the circular section of the Earth of radius r' is rotating at angular velocity ω'. Relative to frame S, a photon travels at speed $c'' = c''(\varphi')$ from B to A along the arc AB. Due to the ether wind w, the light speed c'' varies for different angular positions φ' of AB.

$\mathbf{W}'$ relative to the preferred frame S where space is isotropic and the one-way speed of light is c, being $\nabla \times \boldsymbol{W}' = 0$ in this case. As shown in Fig. 3, let w' be the component of $\mathbf{W}'$ on the Earth circular plane section of radius r' perpendicular to the rotation axis. As seen from the ECI frame S', there is a curless uniform ether wind of velocity $\simeq \mathbf{w}$ blowing as indicated in Fig. 1-a. Let us consider the arc (or rod) AB on the Earth surface, rotating with angular velocity ω' about the center O′ of the axis of rotation stationary on the ECI frame S'. One clock is fixed on the rod (of rest length L_0) at the end point A.

For standard SR based on the LT, on the ECI frame the speed of light along the rotating circumference of radius r' is invariant and given by c. However, with absolute simultaneity and the LTA, empty space is not isotropic on S' and the local one-way speed of light $c''(\varphi')$ along the rod AB changes while AB changes direction relative to $\mathbf{w}'$. Due to the motion at speed w', the angular velocity ω' is not uniform and depends on the angular position φ'. We wish to detect the possible variations of the one-way light speed $c''(\varphi')$ along the arc AB that is co-rotating with the Earth at the peripheral speed $v = \omega' r'$ relative to the ECI frame S'.

As shown in Fig. 4, we consider the case when the rod AB is parallel to the velocity $\mathbf{w}'$ and the inertial frame S'' is instantaneously co-moving with the rod AB where clock A performs the following measurements.

a) When B intersects the y' axis of frame S' and coincides with the origin O′ along y', a photon is sent from B to A — in practice, the y' axis may be formed by the radial line ideally joining point O′ to a nonrotating satellite Y′ fixed on the nonrotating ECI frame. Then, the photon is sent when B will intersect the radial Y′O′ line —.

As observed from the preferred frame S, where the one-way speed of light is c, all clocks can be ideally synchronized and initially set at $t = t' = t'' = 0$, when $x_B = 0$, $x_A = -L_0/\gamma_{w''}$ and the rod AB is moving with velocity w''. The photon sent from B toward A travels at speed $-c$ according to the equation $-ct$. The corresponding equation of clock A, moving with velocity w'' from $x_A = -L_0/\gamma_{w''}$, is $-L_0/\gamma_{w''} + w''t$ and, thus, the photon reaches A when $-ct = -L_0/\gamma_{w''} + w''t$. It follows that the photon time of flight from B to A is,

$$t_{BA} = \frac{L_0}{\gamma_{w''}(c+w'')} = \frac{L_0}{c''} \Rightarrow \tau_A = \frac{t_{BA}}{\gamma_{w''}} = \frac{L_0}{\gamma^2_{w''}(c+w'')} = \frac{L_0}{c}\left(1 - \frac{w''}{c}\right) = \frac{L_0}{c''}, \tag{9}$$

where in the second equation of (9) the term $\tau_A = t_{BA}/\gamma_{w''}$ is the corresponding proper time interval measured by clock A in frame S'', in agreement with the time transform (2) from S to S'' given by $t'' = t/\gamma_{w''}$.

b) Working out the calculations from frame S, the y' axis is moving at velocity w' while the rod AB and clock A are moving with velocity w''. Then, we find that clock A reaches the y' axis (or equivalently, the origin O′) when $-L_0/\gamma_{w''} + w''t = w't$ i.e., after the time interval,

$$\Gamma = \frac{L_0}{\gamma_{w''}(w''-w')} \Rightarrow \tau_{A_\Gamma} = \frac{\Gamma}{\gamma_{w''}} = \frac{L_0}{\gamma^2_{w''}(w''-w')} = \frac{L_0}{u''}. \tag{10}$$

The quantity u'' in (10) represents the absolute value of the velocity of O′ and the ECI frame S' relative to S''.

The observable proper time interval measured by clock A — between the arrival of the photon and the crossing of the y' axis — is,

$$\delta\tau_A = \frac{L_0}{u''} - \frac{L_0}{c''} = \frac{L_0}{\gamma^2_{w''}(w''-w')} - \frac{L_0}{\gamma^2_{w''}(c+w'')} = \frac{L_0}{u''} - \frac{L_0}{c}\left(1 - \frac{w''}{c}\right), \tag{11}$$

where u'' and c'' (or w' and w'') are still undetermined.

We see from (11) that, if u'' is known, the one-way light speed c'' may be determined by knowing the measured proper time interval $\delta\tau_A$.

However, in order to evaluate u'' in (10) and (11) we need to measure it by synchronizing clocks on S''. Nevertheless, if we synchronize clocks on S'' by adopting some internal procedure (e.g., clock transport), physicists might claim [3] that such a procedure turns out to be equivalent to Einstein synchronization. In this case, the measured u'' is linked to the average velocity $v = v_{av} = \omega r'$ of standard SR measured on S', as follows. According to the LT, v is related to w'' and w' by the velocity

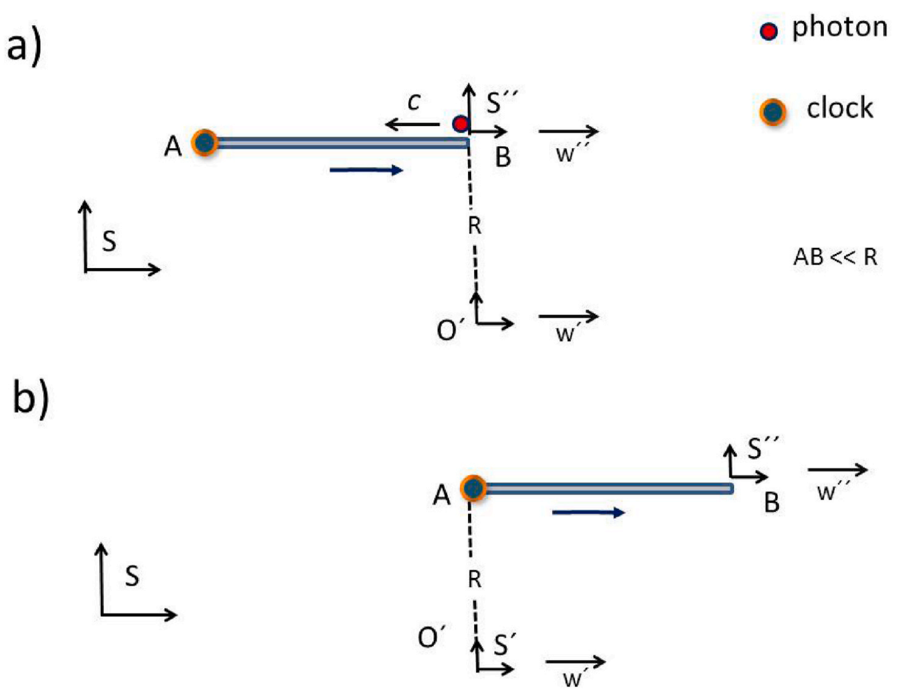

Fig. 4. *a*) Frame S'' is co-moving instantaneously with the rod (or arc) AB with velocity w'' relative to the preferred frame S. A photon is sent from B to A when B coincides with the y' axis of frame S', moving with velocity w' relative to S. *b*) When A coincides with the y' axis of frame S', clock A has measured the proper time interval $\delta\tau_A$ between the arrival of the photon and the crossing of the y' axis.

transform $v = (w'' - w')/(1 - w'w''/c^2)$, which gives $w' = (w'' - v)/(1 - vw''/c^2)$. Then,

$$\begin{aligned} u'' &= \gamma_{w''}^2(w'' - w') = \gamma_{w''}^2\left(w'' - \frac{w'' - v}{1 - vw''/c^2}\right) \\ &= \gamma_{w''}^2\frac{v(1 - w''^2/c^2)}{1 - vw''/c^2} = \frac{v}{1 - vw''/c^2}. \end{aligned} \tag{12}$$

Substituting (12) in (11),

$$[\delta\tau_A]_{rs} = \frac{L_0}{u''} - \frac{L_0}{c''} = \frac{L_0}{v}\left(1 - \frac{vw''}{c^2}\right) - \frac{L_0}{c}\left(1 - \frac{w''}{c}\right) = \frac{L_0}{v} - \frac{L_0}{c}, \tag{13}$$

where the last term represents the observable result predicted by standard SR with relative simultaneity, indicating that w'' (or w') is not observable with the sole measurement of $\delta\tau_A$.

In conclusion, expression (11) (derived from the preferred frame S) cannot be discriminated from expression (13) (based on the LT). Both expressions provide possible equivalent interpretations of the same observable $\delta\tau_A$ and it is not possible to determine w'' and the one-way speed of light c'' from (11) if u'' is measured using an internal synchronization performed on S''. Therefore, in order to determine w'' from (11), the velocity u'' must be measured by means of a procedure that does not depend on synchronization.

Measuring u'' with a Sagnac experiment at the angular position of arc AB.

An independent procedure to measure u'' may be based on a Sagnac experiment, performed locally on the Earth at the angular position of arc AB. Ring lasers Sagnac experiments, which can measure the angular velocity ω_E of the Earth with a precision $\delta\omega/\omega_E < 10^{-8}$, are performed routinely on Earth [27,28]. Regardless of the nature of the interferometer (ring laser or Michelson-Gale interferometer), in the Sagnac experiment the time delay between counter-propagating light signals is measured by a single clock (or interferometer) [18] that needs no synchronization and, thus, the resulting ω_E measured in the experiment is a synchronization-independent quantity.

Let us consider a Sagnac interferometer of radius r_* located on the Earth surface at the position of the rod AB and let the nonrotating inertial frame S'' be co-moving instantaneously with the center O_* of the interferometer. As seen from S'', the single clock C_* — fixed to the rotating interferometer and measuring the time delay Δt_* between the counter-propagating light signals of the Sagnac effect — is rotating at angular velocity $\omega_*'' = \omega_E$ with a peripheral speed $v_* = \omega_*'' r_*$ relative to an observer located at the center O_* of the interferometer. The observed time delay is given by the standard expression [9,18] that can be derived from (5),

$$\Delta t_* = \frac{2\pi r_*}{\gamma_*^2(c - v_*)} - \frac{2\pi r_*}{\gamma_*^2(c + v_*)} = \frac{4\pi r_*}{c}\frac{v_*}{c} = \frac{4\omega_*'' AREA}{c^2}, \tag{14}$$

where in (14) $AREA = \pi r_*^2$. Since the clock C_* of the interferometer is practically moving at the same velocity w'' of frame S'' relative to the preferred frame S, we have $t_* \simeq t'' = t/\gamma_{w''}$. Therefore, it may be claimed that the quantity ω_*'' measured with a Sagnac experiment on the rotating frame S_{rot}' of the Earth corresponds to the angular velocity $\omega'' = \omega_E = \omega_*''$ seen from the nonrotating inertial frame S'' instantaneously co-moving locally (same longitude, or angular position φ') with the point where the experiment is performed. The conclusion is that by means of (14) a Sagnac experiment can provide experimentally the local quantities $\omega'' = \omega_E = \omega_*''$ and $u'' = \omega'' R = \omega_*'' R$, without the need of determining them by means of Einstein or other equivalent internal synchronizations of spatially separated clocks.

As shown in Fig. 4, an observer on S_{rot}' co-rotating with AB sees point O′ fixed on the radial line perpendicular to the arc AB. However, the observer on the nonrotating frame S'' sees the arc AB, together with the radial line and the origin O′ of frame S', rotating instantaneously at the angular velocity $\omega'' = \omega_E$ and, thus, sees point O′ and frame S' moving at the instantaneous speed given by,

$$|-u_{O'}''| = u'' = \omega'' r'' = \omega_E R. \tag{15}$$

Solving for c'' expression (11) we find that the resulting experimental value is,

$$c_{\exp}'' = \frac{L_0}{L_0/u'' - \delta\tau_A} \simeq \frac{L_0}{L_0/(\omega_*'' R) - \delta\tau_A}, \tag{16}$$

where ω_*'' and $\delta\tau_A$ are determined by experiment.

The resulting $c_{\exp}''$ determines the natural clock synchronization parameter ϵ and, referring to the two cases considered here, special relativity based on the LTA ($\epsilon = 0$) predicts the value,

$$(c'')_{LTA} = \gamma_{w''}^2(c + w'') \neq c \tag{17}$$

while special relativity based on the LT ($\epsilon = v$) predicts the value,

$$(c'')_{LT} = c. \tag{18}$$

As seen from frame S'', the light speed c'' along AB depends on the angular position φ' of the arc AB. Therefore, by taking measurements at different values of φ', i.e., different orientations of AB relative to $\mathbf{w}'$, it is possible in principle to evaluate w' and its direction.

When $\varphi' = \pi/2$ and AB is perpendicular to $\mathbf{w}'$ and moving with velocity u_y in the $-y$ direction, we have $(c'')_{LTA} \simeq \gamma_{u_y}^2(c + u_y)$ independent of w' to the first order in w'/c. In this case, the measurement of $(c'')_{LTA}$ leads to the determination of u_y, which is now different from w''. When $\varphi' = \pi$ and AB is parallel to $\mathbf{w}'$ but moving in the opposite direction relative to S', we find an expression similar to (11), where w'' is to be replaced by w_-'', which may be different from w''.

Note that, due to rotation, the kinematical quantities on the rotating frame differ from those on frame S''. For example, after the time interval Γ taken for performing the experiment, the length of arc (or rod) AB seen from frame S'' is smaller than its rest length L_0. With $u'' = \omega'' R$, the length variation ΔL_0 in the time interval Γ is given by,

$$\begin{aligned} \Delta L_0 &= L_0 - L_0\cos(\omega''\Gamma) = L_0\left(1 - \cos\left(\frac{u''}{R}\frac{L_0}{u''}\right)\right) = L_0\left(1 - \cos\left(\frac{L_0}{R}\right)\right) \\ &\simeq L_0\left(1 - \left(1 - \left(\frac{L_0}{R}\right)^2\right)\right) = L_0\left(\frac{L_0}{R}\right)^2 \end{aligned}$$

which is negligible assuming $L_0 \ll R$. Similar conclusions apply to the velocity u''. Therefore, our approach becomes rigorous in the limit $L_0 \to dL = infinitesimal$ and, thus, with our approach the one-way light speed is tested ideally at the differential level.

Determining the one-way speed c'' when $w' = 0$ and the ECI frame S' coincides with the preferred frame.

When $w' = 0$, as seen from the ECI frame S', for both the LTA and LT the peripheral speed of the uniformly rotating circumference is $u = v = \omega R$. According to standard SR with the LT, an observer on frame S'' sees the center O′ of the Earth, fixed on the ECI frame S', moving instantaneously at speed $(u_{O'}'')_{LT} = -v$. However, according to the LTA the velocities transform as

$$u'' = \gamma^2(U - v)$$

when frame S'' moves with velocity v relative to S, as in the case when the ECI frame S' coincides with the preferred frame S. In this case, the velocity of the origin O′ of S' is $U = 0$ and, as seen from frame S'', point O′ and frame S' are moving at the instantaneous speed $(u_{O'}'')_{LTA} = -\gamma^2 v = -u'' = -\omega'' R = -\gamma^2\omega R$ and, therefore, $\omega'' \neq \omega$. With our experiment we can measure the quantity $\delta\tau_A$, while the Earth angular velocity $\omega_*'' = \omega'' = \omega_E$ is measured with a Sagnac experiment by means of (14).

According to the LTA based on absolute simultaneity expression (17) becomes,

$$(c'')_{LTA} = \gamma^2(c+v) = \frac{L_0}{L_0/(\omega''R) - \delta\tau_A}. \qquad (19)$$

Instead, according to the LT based on relative simultaneity expression (18) becomes,

$$(c'')_{LT} = c = \frac{L_0}{L_0/(\omega R) - \delta\tau_A} = \frac{L_0}{L_0/v - \delta\tau_A}. \qquad (20)$$

The difference between (19) and (20) is due to the difference between ω'' and ω, being $\omega'' = \gamma^2\omega$. The result of the experiment will point out the correct synchronization and corresponding one-way light speed.

The important aspect, discussed in the previous Sections, is that the one-way light speed $(c'')_{LTA} = \gamma^2(c+v)$ is consistent with the one-way average speed of light in a Sagnac experiment involving a counter-moving light signal along the Earth circumference. Instead, the light speed $(c'')_{LT} = c$ represents the local light speed on AB (obtained adopting Einstein synchronization), which is said to be non-integrable because it is not consistent with the one-way average speed measured in the Sagnac experiment. Thus, expression (19) with $(c'')_{LTA} = \gamma^2(c+v)$ is the expected result consistent with the one-way average speed revealed by the Sagnac experiment.

Some considerations on the experimental feasibility of our approach.

The purpose of our paper is to show that the LT are not physically equivalent to the LTA and that, if it exists, the preferred reference frame is detectable at least in principle. To show that our approach can be used to detect the preferred reference frame in practice requires to provide experimental designs and details that go beyond the scope of our article. In any case, the realization of an experiment of this type may prove to be quite challenging and expensive.

One of the experimental difficulties to overcome is to realize physically the y' axis of the ECI frame S' that is to be crossed by the emitter and detector. A solution could consists in setting on S' a stationary optical system that emits a steady laser beam. The beam direction can be monitored from frame S' and can be made to go from the laser source (on a satellite stationary on frame S' placed just above the rotating Earth surface) toward the center O′ of the ECI frame (and the Earth), as shown in Fig. 4. Hence, the resulting light beam mimicking approximately the y' axis must be stationary on the ECI frame S' and directed in such a way as to reach the Earth surface being perpendicular to it.

With the beam operating, the emitter at B (sensitive to light) is activated by the beam when crossing it (Fig. 4 a)). Similarly, the receiver at A (equally sensitive to light) is activated by the beam when crossing it (Fig. 4 b)) after the time interval τ_{A_Γ} given by Eq. (10). The laser beam may have some finite spread, or width, but what is important is that the beam and its shape be stable on frame S'. In this case, the result is that the emitter and the receiver will be crossing the border of the beam and be activated by it under the same physical conditions. Thus, if the stability of the beam intensity and shape are assured to a high degree, in practice the emitter and receiver will be activated one after the other at the same physical point of the beam border fixed on S' and close to the y' axis. Then, the subsequent precision in the resulting $\delta\tau_A$ of Eq. (11) will depend mainly on the resolution and response of the emitter-clock system in relation to the light signal sent from emitter to clock.

Concerning the sensitivity of detectors measuring $\delta\tau_A$ and the smallest measurable time interval, there are techniques capable of resolving femtosecond $(10^{-15}(s))$ [33] or even attosecond $(10^{-18}(s))$ [34] pulses of laser light while better limits may be achieved by means of advanced interferometry. The term dependent on w'' in Eq. (11) is

$$\frac{L_0}{c}\frac{w''}{c} \sim \frac{10\,km}{3\times10^8\,m/s}\frac{30\,km/s}{3\times10^8\,m/s} \sim 3.3\times10^{-9}\,s,$$

where, as a typical example, we assume $L_0 = 10$ km and, relative to the preferred frame, an Earth speed $w'' \simeq w' \simeq 30$ km/s, corresponding to the velocity of revolution around the Sun (a possible historical candidate of preferred frame). This term is quite small and, depending on the corresponding experimental errors, quite difficult to detect. Nevertheless, it is well within the range of the sensitivity of available detectors and, thus, not impossible to observe if the experimental errors can be conveniently reduced or taken care of.

Even though in practice our experiment might be not viable with present technology, it shows that, at least in principle, the implementation of the Sagnac effect may play an important role in light speed measurements. This fact, may stimulate further research and advances that could lead to more viable and practical tests of the one-way speed of light.

We conclude this Section by pointing out that, in the context of relativistic theories where transformations with absolute simultaneity can be used, a null result for w' may be supportive of a modern version of the Stokes-Planck theory [9,35], where physical space (the classical medium or ether) is dragged by massive bodies (planets and stars) in their motion in such a way that it can be practically locally at rest with the body. The physical space, which does not share the rotation of the body, can be described in terms of an ideal centered inertial frame (fixed to the center of mass of the body) where the ether wind velocity $\mathbf{w} = 0$. Then, the centered inertial frame assumes the properties of a preferred frame where, locally and within the range of the gravitational field of the massive body, the one-way speed of light is c [9].

4. Conclusions

A Sagnac experiment, performed locally at a point on the surface of the rotating Earth, can determine the angular velocity $\omega''_* = \omega''_E$ of the Earth relative to the nonrotating inertial frame S'' co-moving instantaneously with the point. Then, the speed $u'' = \omega''_* R$ — of the ECI frame relative to S'' — is determined. If follows that the motion of the ECI frame, at velocity $\mathbf{w}'$ relative to the hypothetical preferred frame of reference S of relativistic theories, can be detected by measuring the variations of the one-way speed of light along a rod AB co-rotating with the Earth, while the rod changes its velocity relative to S as the Earth rotates.

In the special case of $\mathbf{w}' = 0$, we may assume that on the ECI frame space is isotropic and the one-way speed of light is c. In this case, our experiment can determine the local one-way speed of light along a rod AB on the rotating Earth surface. In principle, our experiment can test the invariance of the speed of light, showing that — as required by epistemologists [16] — the basic postulates of the theory can be verified experimentally and, thus, special relativity is a viable falsifiable theory.

CRediT authorship contribution statement

Gianfranco Spavieri: Both authors have together done the research behind this study. Both have contributed writing and editing and of quality control of the paper, done the calculations, and developed the figures. **Espen Gaarder Haug:** Both authors have together done the research behind this study. Both have contributed writing and editing and of quality control of the paper, done the calculations, and developed the figures.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Determining the ground path effectively covered by light in the linear Sagnac effect.

Clock C traveling always with uniform velocity on the upper section during the particle round-trip*

As shown in Fig. 5, clock C* is co-moving with the optical fiber sliding on the pulleys A and B at speed v relative to the frame S where the pulleys are stationary. Frame S'' is co-moving with the lower section of the fiber at speed $-v$ relative to S and $-w$ relative to S'. The counter-moving photon starts its round trip from the coinciding origins O, O′, O″ and A and C*, at $t' = t'' = t = 0$. The clocks at the origins O′ (clock C*) and O″ are synchronized at $t' = t'' = 0$, while the remaining clocks along x' or x'' are synchronized adopting either relative or absolute simultaneity. The closed ground path, to be covered by the photon from clock C* and back to C* in a round trip, is $2L$. For the photon out-trip described from C* co-moving with S', the equation $ct' = L/\gamma - vt'$ for the counter-moving photon traveling on the fiber lower section, gives

$$\tau^*_{out} = t^{'}_{out} = \frac{\mathrm{C^*B}}{c} = \frac{L}{\gamma(c+v)} = \gamma\frac{L}{c}(1 - v/c).. \tag{21}$$

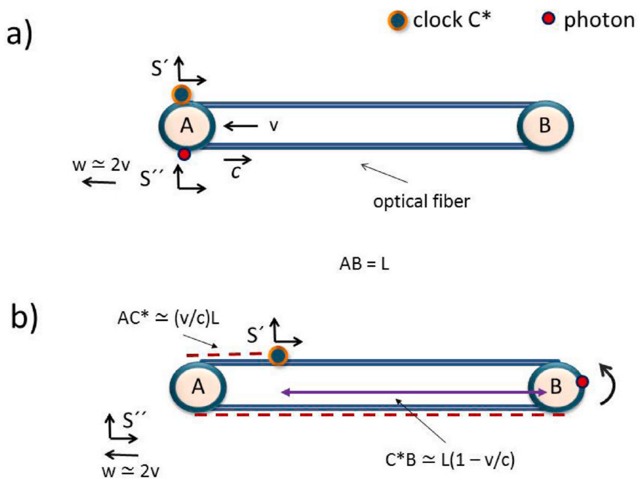

Fig. 5. Relative to the frame of the pulleys, the fiber is moving clockwise. *a)* Clock C* is co-moving with upper section of the optical fiber together with the origin O′ of frame S'. The counter-moving photon leaves clock C* traveling along the lower section of the optical fiber. As seen from S', the speed of the photon is c, the speed of the pulleys and frame S is $-v$, and the speed of the fiber lower section is $-w$. *b)* The curved arrow indicates the direction of motion of the photon. When the photon reaches B, according to S' it has traveled the distance C*B $\simeq L(1 - v/c)$ and A is at the distance AC* $\simeq (v/c)L$. The path C*B effectively covered by the photon at speed c in the interval $t^{'}_{out}$ is less than the fiber ground path AC* + AB $\simeq L(1 + v/c)$..

For the return trip on the upper section the equation $L/\gamma - vt^{'}_{out} - ct^{'} = 0$ gives

$$\begin{aligned} \tau^*_{ret} &= t^{'}_{ret} = \frac{\mathrm{BC^*}}{c} = \frac{L}{\gamma c} - \frac{v}{c}T^{'}_{out} = \frac{\gamma L}{c}(1 - v/c) \\ L_{g-ret} &= ct^{'}_{ret} = \gamma L(1 - v/c) \end{aligned} \tag{22}$$

Fig. 5-b shows that at $t^{'}_{out}$ the photon is at B at $x^{'}_B = ct^{'}_{out} = \gamma L(1 - v/c)$ and point A is at $x^{'}_A = vt^{'}_{out} = (v/c)\gamma L(1 - v/c) \simeq (v/c)L$ to the left of C*. During the interval $t^{'}_{out}$, the ground path supposedly covered by the photon is L_{g-out} = AC* + AB = $(v/c)\gamma L(1 - v/c) + L/\gamma = \gamma L(1 + v/c) > L$. However, according to (21), relative to C* the distance effectively covered at speed c is C*B $= \gamma L(1 - v/c) < L$. If in the time interval $t^{'}_{out}$ the photon covers at speed c the shorter path C*B $= \gamma L(1 - v/c)$, we may expect that, to cover in the same time interval the longer ground path AC* + AB $= \gamma L(1 + v/c)$, the photon ground speed relative to the fiber must be greater than c.

In fact, if we use the LT, we find that from (21) the effective ground path covered in the out trip at speed c in the interval $t^{'}_{out}$ is,

$$L^{eff}_{g-out} = ct^{'}_{out} = \mathrm{C^*B} = \gamma L(1 - v/c),$$

only, and not AC* + AB $= \gamma L(1 + v/c)$. Then, by means of (22), the total ground path effectively covered at speed c is open and given by,

$$\begin{aligned} L^{eff}_{g-tot} &= L^{eff}_{g-out} + L_{g-ret} = 2\gamma L(1 - v/c) \\ &= 2\gamma L - 2(v/c)\gamma L = 2\gamma L - c\delta t^{'} < 2L, \end{aligned}$$

where the missing section $c\delta t'$ is related to the time gap discontinuity $\delta t'$ between S' and S'', as shown in the next example also.

With absolute simultaneity and the LTA, we find that the total ground path effectively covered is closed, but the local ground speed in the out trip is $c' > c$, i.e., not invariant.

Clock C and photon counter-traveling with uniform velocity on the lower section during the out trip and on the upper section during the return trip*

Referring to Fig. 6, our basic assumption is that in the reference frame S'' co-moving with the fiber lower section the one-way light speed is c. Frame S' is moving with speed $w = 2v/(1 + v^2/c^2) \simeq 2v$ relative to S'', while the arm AB is translating at the uniform speed v relative to S''.

The transformations from S'' to S' can be derived as follows (see Ref. [9]): If S is the frame where the pulleys arm AB is stationary, frame S' is moving with velocity v relative to S and the corresponding LT are $t' = \gamma(t - vx/c^2)$, $x' = \gamma(x - vt)$. Frame S'' is moving with velocity $-v$ relative to S and the corresponding LT are $t'' = \gamma(t + vx/c^2)$, $x'' = \gamma(x + vt)$. The inverse LT between S and S' are, $t = \gamma(t' + vx'/c^2)$, $x = \gamma(x' + vt')$, which can be substituted in the LT between S'' and S. The result provides the LT between S'' and S' given by, $t' = \gamma_w(t'' - wx''/c^2)$, $x' = \gamma_w(x'' - wt'')$, where the relative velocity between S'' and S' is now $w = 2v/(1 + v^2/c^2)$ and $\gamma_w = \gamma^2(1 + v^2/c^2)$.

The initial conditions in S'' are that point A is at the distance $(v/c)L/\gamma$ to the left of C*, stationary on S'' at the origin O$''$, and B at $(L/\gamma)(1 - v/c)$ to the right. In the out trip, the counter-moving photon leaves clock C* and, at speed c, reaches point B when $ct'' = (L/\gamma)(1 - v/c) + vt''$, i.e., after the time interval

$$t'' = \tau_{out} = \frac{L/\gamma}{c} = \frac{C^*B}{c}. \tag{23}$$

In the meantime, as seen from frame S'' where C* is stationary, point A is moving at speed v and reaches clock C* after the same time interval. At this instant, A $\equiv$ O$''$ coincide with the origin O$'$ and the clocks at the origin of S, S', and S'' are set at $t = t'' = t' = 0$. Then, at $t'' = 0$ clock C* at A moves to the fiber upper section while, simultaneously in S'', the particle at B does the same.

Measuring time intervals on frame S'' and frame S'.

In changing its velocity while passing from S'' to S' at point A, clock C* takes a finite time interval η. Besides, due to the effect of acceleration, for relativistic theories there may be also a clock time delay, which is of second order in v/c [9]. Nevertheless, we may assume that AB $= L$ is large enough to have $c\eta \ll c\tau_{round} \simeq 2L$ and it is reasonable to neglect η and the time delay due to acceleration, as we shall do for simplicity in the following calculations.

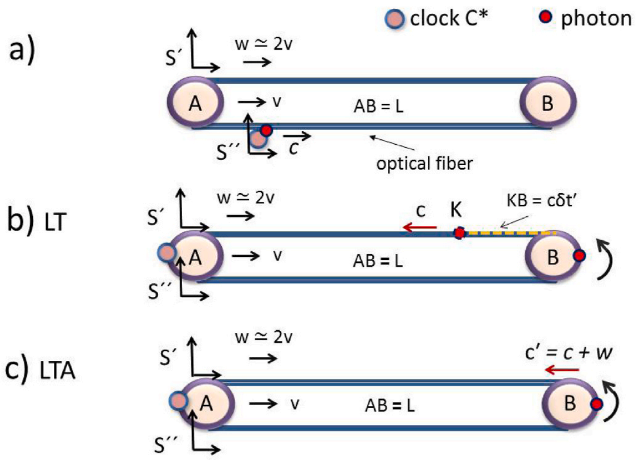

Fig. 6. Relative to the frame of the pulleys, the fiber is moving clockwise. *a*) Clock C* is comoving initially with the fiber lower section and frame S'' and, after leaving clock C*, the photon reaches B when, simultaneously on S'', pulley A reaches clock C*. *b*) The curved arrow indicates the direction of motion of the photon. According to the LT with relative simultaneity, when simultaneously on S'' clock C* is at A and the photon at B, on frame S' clock C* is at A but the photon is already at K, having covered the distance BK in the past ($t' < 0$). *c*) According to the LTA with absolute simultaneity, clock C* is at A and the photon at B simultaneously for both S'' and S'. However, the photon return speed is now $c' \simeq c + w$..

Alternatively, in order to avoid any problem related to the clock acceleration, we may use a second clock (C$'$* $\equiv$O$'$) co-moving with the fiber upper section on the reference frame S', which we synchronize with C* at $t'' = t' = t = 0$. [For simplicity, we do not show the second clock in Fig. 6.] Then, the first clock C* on S'' can measure t''_{out}, while the second clock C$'$* on S' can measure t'_{ret} and both measurements are performed by a device on an inertial frame. Obviously, starting from $t' = 0$, the time readings of C$'$* coincide in practice with those of C* if the latter is co-moving with frame S' at $t' \geqslant 0$. In any event, in the kinematical description of the circular and linear Sagnac effect, the acceleration of C* does not modify the proper time interval measured by the clock.

Thus, we may assume that at $t' \geqslant 0$ clock C* is co-moving with C$'$* $\equiv$ O$'$ on S', while the photon is traveling on the upper section moving toward the clock. As seen from frame S'', in the return trip on the fiber upper section, at time t'' the photon is at $L/\gamma - ct''$, while C$'$* $\equiv$ O$'$, moving at speed w is now at wt''. Hence, photon and clock meet when $L/\gamma - ct'' = wt''$, after the return time interval,

$$t''_{ret} = \frac{L}{\gamma(c + w)}. \tag{24}$$

From the time transform $t'' = \gamma_w(t' + wx'/c^2)$, the proper time of clock C$'$* at $x' = 0$ is $t' = \tau = t''/\gamma_w$ and,

$$\begin{aligned} t'_{ret} &= \tau_{ret} = \frac{t''_{ret}}{\gamma_w} = \frac{L}{\gamma\gamma_w(c + w)} \\ &= \frac{(\gamma_w/\gamma)L(1 - w/c)}{c} = \frac{KC^*}{c} = \frac{L'}{c}, \end{aligned} \tag{25}$$

where KC* is the distance of the photon from C* at $t' = 0$, as shown in Fig. 6-b.

Then, with the help of (23) and (25) the round-trip proper time measured by the clock (or clocks) is,

$$\begin{aligned}\tau_{round} &= \tau_{out} + \tau_{ret} = \frac{\text{C*B}}{c} + \frac{\text{KC*}}{c} \\ &= \frac{L/\gamma}{c} + \frac{(\gamma_w/\gamma)L(1-w/c)}{c} \\ &= \frac{L}{\gamma c} + \frac{(\gamma_w/\gamma)L}{\gamma_w^2(c+w)} = \frac{2\gamma L(1-v/c)}{c} = \frac{2L\gamma}{\gamma^2(c+v)},\end{aligned} \tag{26}$$

where τ_{out} and τ_{ret} are observables measured separately on frame S and S', respectively.

Ground path covered with the LT

The fact that, according to the expression KC*$/c$ in (26), the photon is at $L' = (\gamma_w/\gamma)L(1-w/c)$ at $t' = 0$ (and not at point B at $\gamma_w L/\gamma$) is due to the effect of nonconservation of simultaneity, which places the photon to be at B in the past ($t' < 0$). In fact, from the transform $t' = \gamma_w(t'' - wx''/c^2)$ at $t'' = 0$ and $x''_B = L/\gamma$, we find $t'_B = -\delta t' = -(\gamma_w/\gamma)wL/c^2$ and $x'_B = \gamma_w L/\gamma$. From x'_B the photon then moves by $c\delta t' = (\gamma_w/\gamma)Lw/c = 2\gamma Lv/c$ toward C′* in the time interval $\delta t'$ (time gap) to arrive at KC* $= \gamma_w L/\gamma - c\delta t' = L'$.

Since our aim is to check that, relative to clock C*, every section of the closed contour is being covered by the photon at the local speed consistent with the synchrony adopted, the local speed and corresponding path covered has to be evaluated separately on frames S and S', with the approach adopted above. From inspection of (26), we see that the section BK is missing in the calculation of τ_{round}. Then, because of the missing section covered in the time gap $\delta t'$, the ground path effectively covered at speed c is given by,

$$\begin{aligned}L_{g-out} &= c\tau_{out} = \text{C*B} = L/\gamma \\ L^{eff}_{g-ret} &= c\tau_{ret} = \text{KC*} = (\gamma_w/\gamma)L(1-w/c) \\ L^{eff}_{g-tot} &= L_{g-out} + L^{eff}_{g-ret} = c\tau_{round} = \text{C*B} + \text{KC*} \\ &= L/\gamma + (\gamma_w/\gamma)L(1-w/c) = 2\gamma L - c\delta t' < 2L\end{aligned}$$

Thus, if covered at ground speed c with the LT, the ground path is open because the section BK $= c\delta t'$ has not been covered by the photon during the interval τ_{round}.

Ground path covered with the LTA

The inconsistency found above applies to the LT only, because with the LTA we have to the first order in v/c:

$$\begin{aligned}L_{g-out} &= c\tau_{out} = \text{C*B} \simeq L \\ L_{g-ret} &= \text{BC*} = c'\tau_{ret} = \gamma_w^2(c+w)\tau_{ret} \simeq L \\ L_{g-tot} &= L_{g-out} + L_{g-ret} = c\tau_{round} = \text{C*B} + \text{BC*} \\ &\simeq L + L = 2L,\end{aligned}$$

as expected.

In conclusion, it is physically impossible to cover at the local ground speed c the whole closed path $2L$ in the interval τ_{round} given by (26). Thus, in order to maintain c invariant in both the out and return trips, the standard LT introduce the mechanism of relative simultaneity (with the time gap $\delta t'$, corresponding to the time discontinuity mentioned by Landau and Lifshitz) to shorten the return path by $c\delta t'$ = BK. Thus, the undesirable consequence emerging by imposing the invariance of c with the LT is that the round-trip path covered at speed c is necessarily open.

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