



Preface

In the last part of our university studies, we have written this thesis focusing on power production in Norwegian hydropower plants. The road has been long and winding, but exciting and educational along the way. After several meetings with section head of renewable energy in Multiconsult, Ryan Glenn Anderson, the focus of the study adapted from an overall energy analysis to a precise study of production within the plants. Along the road, with the guidance from associate professor Petter H Heyerdahl, we have discussed the different pathways and approaches to solving our problem. Many frustrating dead-ends and troubles has occurred along the way, but together, we have supported each other, resulting in the paper you are now reading.

In the beginning, the work was focused on retrieving production data from hydropower plants. The focus shifted from multivariate analysis with Knut Kvaal, to creating a revenue-optimizing model in Microsoft Excel. With the help from the hydrologists Svein Taksdal from NVE and Arne Koksæter from Multiconsult, we learned a lot about retrieving and processing the hydrological data. Energy economics advisor at Multiconsult, Marko Viiding, assisted us with the necessary information about the Norwegian power market and retrieving the production data in a professional way. In the end of March, the revenue-optimizing model in Excel was finished and ready to run.

We could like to thank Petter H. Heyerdahl and Ryan Glenn Anderson. This work would not be possible without them. They both supplied us with knowledge and input that proved invaluable to our thesis. Petter H. Heyerdahl shared his experience concerning both general setup of a thesis and his technical know-how. With this information, we were able to create a more realistic model, and our thesis gained a better structure. Ryan Glenn Anderson gave us valuable input on model appliance and setup in an industrial setting. We wanted to create a tool that has further use beyond this thesis, and Ryan gave us advice on how this could be achieved. We would also like to thank Marko Viiding. His contribution in the economical part of the thesis was valuable, and he also gave us important input on different critical parts of our thesis.

Ås – 15.05.2015

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Summary

The interest in Norwegian hydropower is increasing, and in 2014, there were multiple acquisitions in the Norwegian hydropower industry. Hydropower is a renewable energy source known for its high efficiency and long lifetime. Estimating the potential within hydroelectric power plants can be of great value for both current owners and investors. This thesis focuses on finding the maximum yearly revenue for Norwegian hydropower plants. The goals in this study are:

- Constructing a retrospective analysis tool that optimizes the revenue by changing the production, based on historical production data, market price and reservoir measurements
- Comparing the revenue-optimized production with the actual production in order to find the potential for improvement
- Examine the potential in the model as a production planner

The model was constructed in Microsoft Excel with the Frontline Solver add-in. Four Norwegian hydropower plants were analyzed, finding an increase in revenue between six and sixteen percent. Two of the plants were able to increase revenue by holding back production in the fall, waiting for the price to increase in the winter. One plant had no clear trends due to a complex production pattern and miscalculations. The last plant failed to take advantage of sudden price changes in the market, and therefore failed in maximizing its revenue. Using this model as a production planner was also possible. A test of the planner led to an eight percent increase in revenue, but crossed the lower water limit when tested.

Sammendrag

Interessen for Norsk vannkraft er økende og i 2014 var det flere oppkjøp i vannkraftindustrien. Denne fornybare energikilden er kjent for sin høye virkningsgrad og lange levetid. Å estimere potensialet i Norske vannkraftverk kan ha høy verdi for både investorer og eiere. Denne masteroppgaven fokuserer på å finne maksimal årlig driftsinntekt for norske vannkraftverk. Modellen som er beskrevet in denne masteroppgaven skal være et verktøy hvor hovedoppgaven er å optimalisere driftsinntektene ved å endre på vannkraftverket sin produksjon. Målene med denne oppgaven er:

- Konstruering av en retrospektiv modell som optimaliserer driftsinntekter ved å endre på produksjon, basert på historisk produksjonsdata, markedspris og magasinmålinger
- Sammenligning av optimaliserte driftsinntekter mot faktiske driftsinntekter for å finne forbedringspotensialet
- Utforske potensialet av modellen som produksjonsplanlegger

Modellen er konstruert i Microsoft Excel med Frontline Solver som tilleggsprogramvare. Fire vannkraftverk har blitt analysert hvor vi fant en økning i driftsinntekten på seks til seksten prosent. To av kraftverkene kunne økt driftsinntekter ved å holde tilbake produksjonen om høsten, og vente på at prisen økte om vinteren. Et kraftverk hadde ingen klarere trender på grunn av kompleks produksjonsmønster og feilkalkulasjoner. Det siste vannkraftverket klarte ikke å ta hensyn til raske endringer i markedet, og feilet derfor med å maksimere sine driftsinntekter. Det er også mulig å bruke denne modellen som produksjonsplanlegger. En test av planleggeren ledet til åtte prosent økning i driftsinntekter, men den krysset den nedre vanngrensen.

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Nomenclature

R	MWh	Reservoir inventory/Energy available in the water
R^*	MWh	Theoretical reservoir inventory from revenue optimizing plan
\bar{R}	MWh	Upper reservoir limit
ε	MWh	Total energy entering the system starting from actual reservoir value in day one of the season.
E	MWh	Daily production
E^*	MWh	Theoretical daily production from the revenue optimizing model
\bar{E}	MWh	Maximum daily production
I	MWh	Daily inflow of water
S	MWh	Daily loss of water
M	MWh	Minimum flow of water
H_a	m	Head of the hydropower plant
\bar{P}	MW	Hydropower plant performance
η_{tot}		System efficiency
η_{tur}		Turbine efficiency
η_{gen}		Generator efficiency
q	m^3/s	Absorption capacity
δ	NOK/MWh	The daily average price in the day-ahead market

1 Introduction

1.1 History and hydropower today

The conversion of energy from moving water into mechanical work is based on ancient technology. Thousands of years ago, Greek and Roman civilizations used hydropower for grinding grain for the use of making bread and in sawmills (Energy.gov). Today, the use of hydropower is mainly directed towards production of electricity powering the industry and households. Hydropower plants convert energy from water dammed up in storage reservoirs and from water flowing in rivers. Norway, Iceland, Austria, Canada, Switzerland and New Zealand get more than 50% of their energy from hydropower (International Energy Agency 2012). In fact, Norway gets 96.6% of its total energy from hydroelectric power. Already in 1885, Skien in Norway was partially supplied by electricity from hydropower (Hveding 1992). Oslo was the next place to benefit from hydropower technology. The first hydroelectric power station was built in Maridalen with a capacity of 2.5 MW. From the beginning, there was an international interest in the Norwegian hydropower resources.



Figure 1: Sarpsfossen 1910. The picture shows Sarpsfossen and the west side of the river where Borregård is located (Wilse 1910).

Borregård, a British company bought the rights to the western side of Sarpsfossen in Sarpsborg. Figure 1 shows Sarpsfossen and the western side of the water fall. Production started in 1889 with two aggregates amounting to 800 kW each, evolving to 9 MW in 1920. A German company bought the rights for the east side of the waterfall for production of calcium carbide, with a capacity of 5.3 MW. In 2013, the total electricity production in Norway was 134 TWh, whereas 129 TWh comes from hydroelectricity power (Olje- og energidepartementet 2015). Out of 1476 plants, 1141 had a production under 10 MW, and only 80 have a capacity over 100 MW. These 80 hydropower plants still produce over 79.5 TWh annually. The ten largest producers in Norway have 75 % (21 000 MW) of the total production capacity in Norway (Multiconsult 2013). Statkraft Energy AS is by far the largest owner with 35 % (10 000 MW) of the total production capacity.

1.2 Background

Hydropower technology has exceptional efficiency and long lifetime, making it a valuable resource. Today there is an increasing interest in the Norwegian hydropower industry, and in 2014, there was several acquisitions in the hydropower sector. Aquila Capital, an asset management section of Aquila Group localized in Germany, bought 100% of Norsk Grønnkraft, with a yearly production of 200 GWh (Kurschinski 2015b). They also bought 33.35% of Eramet's ownership in Tinfos, producing 320 GWh (Kurschinski 2015a). Clemens Kraft AS bought several licenses and projects from Nordkraft in 2014 (Nordkraft 2015). KKB AG, a Swiss independent producer stated that they want to own a renewable energy portfolio consisting of 300 MW by the end of 2020 (KKB renewable energy 2015). Sognekraft are also investing, aiming to double their production from 515 GWh within 2020 (Sognekraft 2013).

When an investor is considering certain power plants, they can calculate an estimated revenue, based on historical data from the plant and the market price. However, this does not tell the investor anything concerning the potential within the plant. With these recent acquisitions, a research question was raised in cooperation with Multiconsult. The idea was to explore the potential in Norwegian hydropower plants and compare it to actual operation. This comparison could lead to an overview over the possible increase in revenue within the plant. The final problem was raised for this study: How can historical data be used to calculate revenue-optimizing production in a hydroelectric power plant? Could this information be useful in production planning?

1.3 Goals and assumptions

The main goal for this thesis was to make a model that obtains a hydropower plant's daily production, maximizing the yearly revenue. This model would have two separate versions. One version looking back at a prior year and one version looking to the future, both calculating a revenue-optimizing production. Several limitations had to be made concerning both the energy market and the plant in order to keep within the time limit and the model capacity.

The market used by the owner for selling the production is unknown, and the owner may use multiple markets in its production strategy. The owner could sell the energy production in different markets. Multiple types of markets is available when producing energy. Elspot, or the day-ahead market, is the main market for energy trading . This thesis will assume that Elspot is the only available market for the owner. Elspot modifies price every hour depending on the supply and demand. With only total daily production available, this thesis assumes that all the power production is sold at the daily average price in the price zone where the plant is located. When the owners of the plants delivers production offers to Nord Pool, it is not guaranteed that they win all the contracts. If the owner demand a higher price than the market equilibrium, they will lose the contract. In this

thesis, all of their planned production is sold to the market. This thesis will assume that all of the plant's daily production is turned into revenue according to the daily average price in the Elspot market. This paper also assumes that the plant is a price taker. When the model changes the plant's production in a retrospective view, it assumes that the price is unchanged in the market. This is not necessarily true as the change in production would have increased or decreased the total supply in the market, possibly affecting the market price.

The model optimizes the revenue by adjusting the production to the market price, but the market price varies, leading to an unstable production. There is assumed no limit on rapid changes in the production or the reservoir. Multiple factors affect the total efficiency of the plant. Considering all of these factors is beyond the scope of this thesis. This study will assume that the only factors that decided the total system efficiency is the turbine and generator. These efficiencies were collected from the owners of the plant. If no value was given, both efficiencies was assumed 0.95, not varying with water flow. In some systems, a minimum water flow is required from the reservoir or the plant. Whether it is required from the plant or the reservoir may depend on the setting between the two. In this study, the minimum water flow will be running through the plant, meaning production includes minimum water flow. The head to the reservoir is used when calculating the energy available in the reservoir water. This is a constant value, but in reality, the value changes with the water elevation. This thesis will assume that water at the minimum level of the reservoir has the same potential energy value as the water at the maximum level of the reservoir. When supplying the power lines, some congestion may arise when lines gets close to their capacity. This model does not include this as a factor, and assumes that there is always available capacity in the grid for the plant's production.

2 Theory

2.1 Hydroelectric power

Hydropower is the principle of extracting energy from water in motion. The water is guided through steep, large pipes from the reservoir to the turbines. The potential energy in the water is used to force the turbines to propel, converting the energy in the water into mechanical power. The mechanical power from the turbine is then used in the generator, converting the mechanical power into electric power. Various types of hydroelectric projects with unique designs are used worldwide, such as in-stream and pumped-storage plants (NVE, E., Norwegian research council og Innovation Norway 2015).

Hydroelectric power plants are often categorized by size. Power plants with installed production of less than 10MW are known as small hydropower plants. These are divided into three, small (1–10MW), mini (0.1–1MW) and micro (0–100W) (Olje- og energidepartementet 2015). Smaller hydropower plants are often located in creeks and rivers without reservoirs or storage capacity. Hydropower plants may also be classified by the height difference in intake and output, purpose and storage capacity. Categorized by height difference (head), there are two types of power plants, high and low pressure power plants. High pressure plants use reservoirs with water at higher heights, while low pressure plants use water as it flows in rivers. These low head plants are often located in streams or rivers, and are known as run-of-the-river or low pressure power plants (NVE, E., Norwegian research council og Innovation Norway 2015).

2.1.1 Energy conversion process

Water stored in a reservoir can be compared to energy in a battery, it is stored potential energy, ready to be released. By opening the valve, the water flows through the pipes creating an almost instant production as shown in Figure 2, page 7. Potential energy stored in the reservoir is written as:

$$E_p = mgh \quad (1)$$

Where E_p is the potential energy, m is the mass of the water, g is the gravitational constant set to $9.81 \frac{m}{s^2}$ and h is the head. The potential energy of the water is gradually transferred into kinetic energy when it is released from the reservoir. The force of the water hitting the turbine, creates electric power when the energy is transferred through the turbine and the generator. This electrical power output is calculated by Equation (2) (Weir 2006).

$$P_{pot} = \eta \rho Qgh \quad (2)$$

Where P_{pot} is the electrical power, η is the plant efficiency, ρ is the density of water and Q is the flow of water. The efficiency η includes the turbine and generator efficiency resulting in a plant efficiency factor. The efficiency of the turbine varies with design. The most used turbines in hydroelectric power plants with high heads are Pelton and Francis turbines. Turbine efficiency also varies with the water flow through the pipes and generator efficiency varies with performance (Koksæter 2015). The head, h , used in Equation (1) and (2) above is the height from the surface of the water to the power plant, see Figure 2 below. With a varying reservoir curve follows varying water level and head, therefore the potential energy varies.

2.1.2 Power plant and reservoir

Hydroelectric power plants extract potential energy from water by using the height difference. Water flows from the reservoir through pipes into turbines as illustrated below in Figure 2. The angle and length of the pipes varies with plant design and geographic as well as the topographic setting. A conventional setup of a hydroelectric power plant is shown below. The illustration below shows water flowing from the reservoir towards the power station. It includes the steps of the power conversion of energy, through the turbine, generator and delivery to the grid. Upper and lower water level is shown by HRV and LRV.

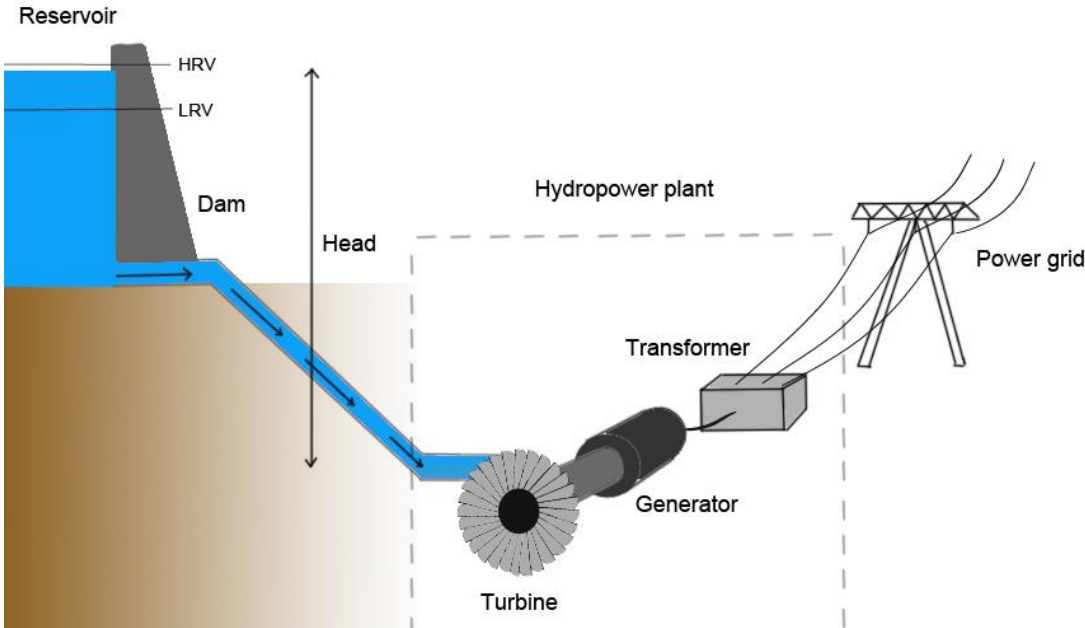


Figure 2: Illustration of a conventional hydroelectric power plant. The dam is shown in the left of the figure with upper and lower water levels (HRV & LRV). The height down to the turbine is shown by head, and the figure includes generator and transformer in the power plant. Wiring to the grid from the power plant is shown in the right of the figure.

The HRV and LRV are Norwegian abbreviations for the upper and lower reservoir limit, given in meters above sea level. These values are given in the license application of the hydropower plant and

is given as an upper and lower limit for our model. These restrictions exist in accordance with the plant's licensing concerning safety and its surroundings. The consequences of exceeding the upper water level are possible loss of water, flooding and fining. Figure 3 below shows an example of a reservoir with unusual low water levels, a result of drought or excess production (Kallestad 2006).



Figure 3: Reservoir measurements in Osvatnet in Sunndalsøra 2006. The picture shows unusual low water levels (Kallestad 2006).

The transportation of water through pipes from the reservoir to the power plant should be as efficient as possible with minimum loss. The power plant should be placed as close to the reservoir as possible with maximum height difference to the storage pool, shortening the length of the pipe (Koksæter 2015). The losses in the pipes, due to friction usually falls between three to ten percent in Norwegian conventional plants. The friction in the pipes increase with length and decrease with increasing diameter. This makes shorter pipes with a large diameter the preferred type. Reservoirs may consist of multiple drainage basins connected through rivers and creeks, varying from a simple structure to complex systems. Measurements of the primary reservoir will contain possible inflow from the rivers and the unregulated pools, in addition to subsurface flow, groundwater and precipitation. An illustration of the inflow to a stream, or reservoir, is shown below in Figure 4.

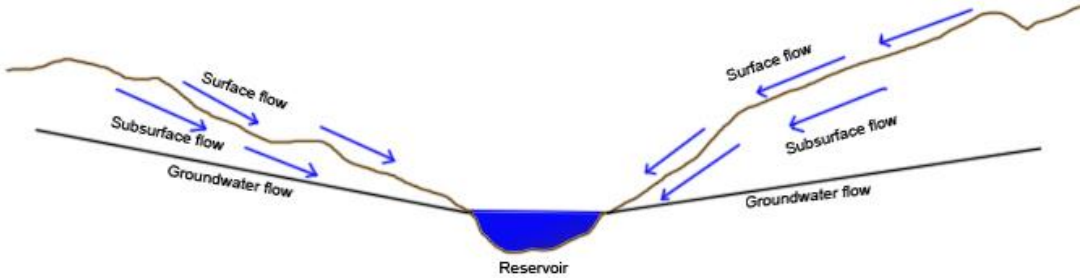


Figure 4: Hydrologic illustration of inflow. Direct runoff as surface flow, infiltrated water as subsurface flow and groundwater enters the reservoir/stream.

The input comes from different forms of precipitation. Precipitation can be snow, rain and hail, and depending on temperature, the fluid is evaporated or infiltrated into the soil. When rainfall hits the ground in the catchment area around the reservoir, some of the water infiltrates the ground. The infiltration leads to subsurface flow and groundwater, the rest runs off as direct runoff (Gribbin 2006).

Some hydropower stations have requirements regarding the minimum water flow (Koksæter 2015). This flow could either be required to exit the reservoir or the hydropower station. The geographical and topographical setting between the two takes part in determining where the minimum water flow should exit. If the hydropower station is located close to the reservoir, the minimum water flow could exit the power station only. If the water exits the hydropower station, it is possible to use the water for production purposes.

2.2 The Market – Nord Pool ASA

Nord Pool is the Nordic and Baltic market for energy trade where Norway, Sweden, Denmark, Finland, Estonia, Latvia and Lithuania all have the same deregulated trading system (Nord Pool Spot AS 2015a). Statnett SF, Svenska kraftnät, Fingrid Oyj, Energinet.dk, Elergin, Litgrid and Agustsprieguma own the system. Norwegian Water Resource and Energy Directory (NVE) has licensed Nord Pool Spot AS to be the regulator of the market. The physical trading markets are Elspot and Elbas (Nord Pool Spot AS 2015d; Nord Pool Spot AS 2015i). Elspot is the day-ahead market because the hourly production plan for the next day is set the day before. Elbas is the intraday market. In this market, contracts may be traded up until one hour before the delivery. It exists due to the uncertainty in the wind and run-of-river hydropower.

2.2.1 Elspot – the day-ahead market

In 2013, 84% of the Nordic and Baltic power production went through Elspot (Olje- og energidepartementet 2015). In the day-ahead market, members have to inform Nord Pool how much they are willing to buy or sell in MWh at different price levels (EUR/MWh) for every hour of the next day (Nord Pool Spot AS 2015j). The deadline for posts of power production closes at 12:00 CET, and at 12:45 CET the aggregated demand and supply for every hour of the following day is published as seen in Figure 5. This plan starts 00:00 CET the following day.

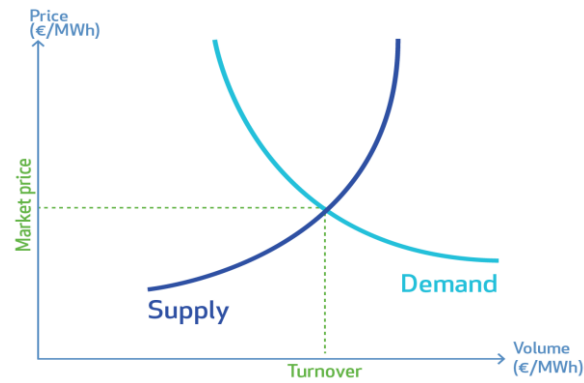


Figure 5: Aggregated supply and demand curve. Created by Nord Pool Spot AS during the 45 minutes from 12:00 CET to 12:45 CET (Nord Pool Spot AS 2015d).

There are multiple ways of posting orders in Elspot. Single hourly order is an option where the member at Nord Pool decides an independent price order for every hour of the following day (Nord Pool Spot AS 2015k). A block order is a deliverance lasting for multiple hours, with a minimum of three hours (Nord Pool Spot AS 2015c). This type of order could be used when there is high startup cost for production. Flexible hourly order is an order from a member where the price and quantity is set but the hour is not specified (Nord Pool Spot AS 2015g).

2.2.2 Congestion and bidding areas

There could be congestion in the power grid when large volumes of energy has to be transferred between different areas (Nord Pool Spot AS 2015d). This is why there are different zones. By introducing different price zones, the price difference can relief the congested areas. Norway has five zones, Sweden has four zones and Denmark, Estonia, Lithuania, Latvia and Estonia each have one zone as seen in Figure 6.



Figure 6: The trading zones in Nord Pool Spot AS. Norway is divided into five zones. UK has recently been added (Nord Pool Spot AS 2015b).

2.3 Different approaches for production planning and analysis

Owners of Norwegian hydropower plants use different approaches for production planning and production analyzes (Bye 2015). Some develop their own model, while other hire in professionals to develop a model. Powel is a company who specializes in production planning and prognosis in both the long and short term (Powel 2015). Textbooks has also been written about this topic (Wangensteen 2012) (Førsund 2007). Premium Solver could be used when developing a model for production planning and retrospective analysis (FrontlineSolvers).

The Premium Solver in Microsoft Excel is an extended version of the basic solver add-in, which is an easy-accessible feature in Excel (FrontlineSolvers). The program is commonly used in optimization problems, often within economy. By optimizing problem via the Simplex Method, the Solver obtains the optimal solution for the problem with the following constraints. When there is a small number of constraints, the Lagrange multiplier is often used, but Simplex is able to handle larger problems with less processing power. The Basic solver has an upper variable limit at 200, with one-sided bounds on the variables. Improving this Simplex method with two bounds per variable, the premium version used is limited by 2000 variables. Furthermore, the method varies with the complexity of the problem. For further details about the Lagrange and Simplex approach, see Appendix B.

3 Model construction

This model finds the daily production values for a hydropower station that maximizes the revenue over the duration of one year. The daily revenue for the stations is a product of the daily produced energy and the price in the day-ahead price zone where the plant is located. Most of the model work is done in Microsoft Excel. In order to recreate the model, one must have the Excel the add-in Analytic Platform Solver from FrontlineSolvers, or a similar program (FrontlineSolvers). The add-in is necessary for the optimization part of the thesis. In this case, the add-in finds the production value for each day that maximize the revenue for a hydropower stations over the duration of 365 days. In order to make the model, some input data is necessary from both the hydropower station, the reservoir and the market. Heading 5.1.6 contains information about our plant selection.

Setting constraints is a critical part of this process. The number of limits determines how realistic the model is. The model needs a reservoir limit, both minimum and maximum, a maximum daily production, amount of water/energy available each day and the price in the market. To find these values, multiple datasets is necessary from different sources. Our model is based on data received from the owners of the hydropower stations, NVE and Nord Pool. Technical information concerning the hydropower station is critical in determining the maximum daily production of the plant. When determining the maximum production per day, several different factors is considered. Heading 3.1.2 contains more information about this topic. In order to make the model work, all of the limits has to be in the same unit. Our model therefor calculates the energy equivalent to all of the constraints. Even though the reservoir measurements from NVE is in cubic meters, this model calculates how much energy the plant could produce from the water in the reservoir. Rapid changes in the reservoir and production, possibly damaging the components, is discussed in 5.4.1 and 0.

3.1 Mathematical function and constraints applied in the model

The function that is being optimized is shown in (1).

$$\text{Max} \sum_{t=1}^{365} E_t^* \cdot \delta_t \quad (3)$$

Where E_t^* is the daily energy production in MWh and δ_t is the average daily price in the day-ahead market in NOK/MWh. This function is subject to several constraints as seen below in Table 1. The setup in Excel is shown in Appendix C.

Table 1: List of constraints applied to the model. The asterisk (*) symbol implies a theoretical value calculated by the model for the revenue optimizing plan.

Variables	Constraint	Description
E_t^* : Theoretical daily production \bar{E} : Maximum daily production	$E_t^* \leq \bar{E}$ for $t = 1, \dots, 365$	The daily production cannot surpass the plant's daily production limit. Heading: 3.1.2
R_t^* : Theoretical reservoir inventory \bar{R} : Reservoir capacity	$R_t^* \leq \bar{R}$ for $t = 1, \dots, 365$	The reservoir inventory cannot be greater than the reservoir HRV-capacity. Heading: 3.1.7
E_t^* : Theoretical daily production ε_t : Total energy entering the system, day t	$E_t^* \leq \varepsilon_t$ for $t = 1, \dots, 365$	The daily production cannot be exceed the total energy. Heading: 3.1.4
E_t^* : Theoretical daily production R_t^* : Theoretical reservoir inventory on day t	$E_t^* \leq R_t^*$ for $t = 1, \dots, 365$	The daily production cannot be greater than the reservoir inventory. Heading: 3.1.5
E_t^* : Theoretical daily production M_t : Minimum flow of water on day t	$E_t^* \geq M_t$ for $t = 1, \dots, 365$	The daily production must include the minimum flow of water. Heading: 3.1.7
R_1^* : Theoretical reservoir inventory, day one R_1 : Actual reservoir inventory, day one	$R_1^* = R_1$	The reservoir inventory for the first day must equal the measured inventory on day one. Heading: 3.1.5
R_{365}^* : Theoretical reservoir inventory, day 365 R_{365} : Actual reservoir inventory, day 365	$R_{365}^* \geq R_{365}$	The reservoir inventory for the last day must be equal or greater than the measured inventory. Heading: 3.1.6
E_t^* : Theoretical daily production	$E_t^* \geq 0$ for $t = 1, \dots, 365$	The daily production value must be greater or equal to zero. Heading: 3.1.7
R_t^* : Theoretical reservoir inventory	$R_t^* \geq 0$ for $t = 1, \dots, 365$	The daily reservoir inventory must be greater or equal to zero: Heading: 3.1.7

3.1.1 Revenue from the day-ahead market

When calculating the daily revenue, the market price is necessary. The price used in this model is retrieved from Nord Pool's day-ahead market/Elspot. The day-ahead market changes price every hour in all the different price zones, or bidding areas, but our model calculates production on a daily basis. A daily average price in the zone where the plant is located will therefore be the price used in this model. Heading 0 contains information about daily price variation. In order to find the daily revenue the model calculates the product between the daily average energy price and the total energy produced that day. The model then sums all of the daily revenues acquired over one year,

finding the total yearly revenue. This process can find both an estimate on the actual yearly revenue acquired, and the potential revenue that the station could have acquired.

3.1.2 Finding the maximum daily production value

The model considers three variables when deciding the daily production capacity. The given performance of the turbines as seen in Formula (4), the calculated maximum daily performance found in Formula (5), and the maximum daily production measured during the season, as seen below in Formula (6). It considers all three values for the daily production capacity and selects the lowest value as this is considered the maximum daily production value. The first limitation on the daily production value is found in Formula (4).

$$\bar{E} \leq \bar{P} \cdot t \quad (4)$$

\bar{E} is the maximum daily production value in MWh, \bar{P} is the given performance of the hydropower plant in MW and t is hours per day. A calculated maximum daily production is found by using Formula (5). The data received from the station owners such as head, absorption capacity, turbine- and generator performance decides this value.

$$\bar{E} \leq \frac{q \cdot H_a \cdot g \cdot \eta_{tot} \cdot t}{1000} \quad (5)$$

\bar{E} is the maximum daily production in MWh, q is the absorption capacity in m^3/s , H_a is the given head of the hydropower plant in m , g is the gravitational acceleration in m/s^2 , η_{tot} is the efficiency of the turbine and the generator and t is hours per day. The last value the model considers is the maximum daily production measured during the season for the hydropower station. Available data from the plants is given in MWh on a daily basis. The highest daily production value of these 365 days is also considered as the daily maximum production value.

$$\bar{E} \leq \text{Max}(E_t) \text{ for } t=1, \dots, 365 \quad (6)$$

\bar{E} is the maximum daily production value in MWh, E_t is the daily production value measured from the power plant in MWh.

3.1.3 Inflow and loss

No inflow data was available when making this model. Some of the data available in this study was production and reservoir data. The unknown variables are the inflow into the reservoir and the water lost in the reservoir. Formula (7) gives the relation between the change in reservoir volume and the water going in and out of the system.

$$R_{t+1} - R_t = I_t - E_t - S_t \quad (7)$$

R_{t+1} and R_t is the available reservoir energy on day $t+1$ and t in MWh, I_t is the total amount of energy going into the reservoir on day t in MWh, E is the day t 's total production in MWh and S is the lost energy on day t in MWh. The loss is all the water that is not used for production, but leaves the reservoir in some other way. There is not data available on inflow and loss. By using Formula (7) and moving the production to the other side of the equation, one can find the difference between the inflow and the loss. If this value is positive, the inflow is larger than the loss, and if the value is negative the loss is larger than the inflow. If the value is zero, the inflow and the loss is equal, and will therefore not affect the change in volume for that day.

3.1.4 Total energy entering the reservoir

If there is no limit on the water/energy available each day, the model assumes that all the water entering the reservoir over the season is available from day one. In order to remove this option from the model, the daily increase in total energy entering the reservoir over the season had to be calculated. Energy available tells the model how much more or less water becoming available every day. The total energy available, ϵ , will increase or decrease over the season depending on the amount of water going into the system, and the amount of water that is lost to its surroundings. Total energy, ϵ , does not take into account the daily production happening during the season.

3.1.5 Reservoir supply for the revenue optimizing production

The reservoir level is a result of all the production, inflow and loss taking place during the season. The reservoir-data received from NVE is therefor also a consequence of the actual production from the hydropower plant, as well as the inflow and the lost water during that season. This model requires to calculate a new reservoir inventory where the real production is replaced by the revenue optimizing production that our model makes. This new value, called the theoretical reservoir inventory is a constraint for the model. Production cannot exceed theoretical reservoir supply.

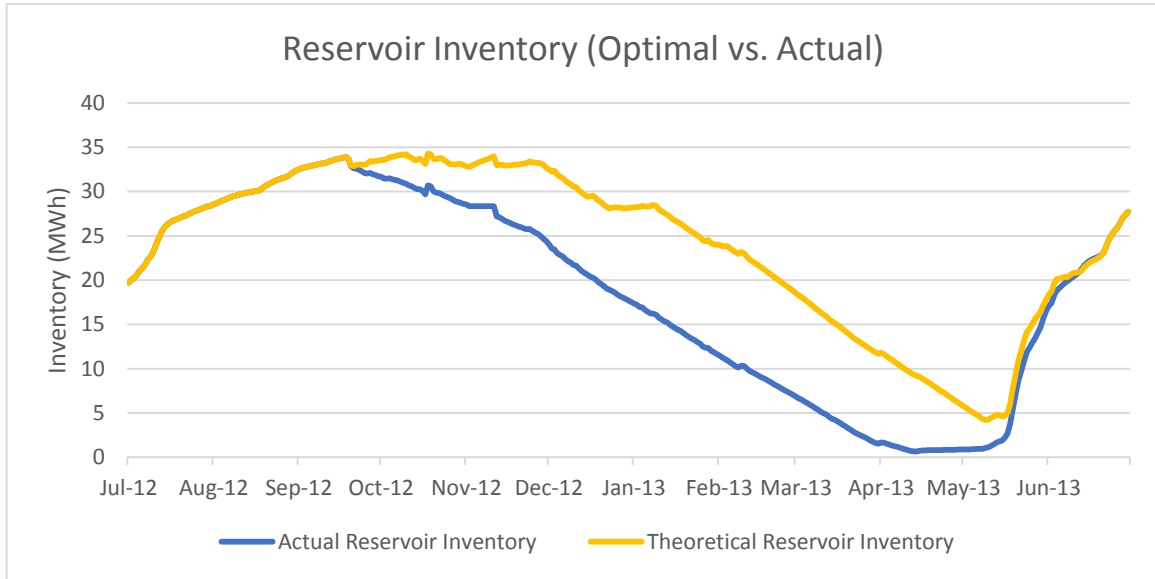


Figure 7: Example of an actual and theoretical reservoir curve. The figure shows an example of the reservoir supply with both the actual reservoir inventory and the theoretical reservoir supply made by the revenue optimizing function.

Figure 7 shows both the actual reservoir inventory (blue) and the theoretical reservoir inventory (yellow). The blue line is constructed based on real measurements from NVE, while the yellow line is constructed based on production values decided by the model, and the real inflow and loss. The theoretical reservoir inventory for the first day in the season is set to equal the real reservoir measurement for the first day as seen in Table 1. The remaining days in the season is given by Formula (8).

$$R_{t+1}^* = R_t^* - E_t^* + (I_t - S_t) \quad (8)$$

R_{t+1}^* is the theoretical reservoir inventory for day $t+1$ in MWh, R_t^* is the theoretical reservoir inventory for day t in MWh, E_t^* is the total production from the revenue optimizing production for day t in MWh, I_t is the total inflow of water on day t in MWh and S_t is the total loss in MWh for the day t .

3.1.6 Water supply on the last day

The model is constructed to maximize the value in the water. If some water is left in the reservoir on the last day, it is wasted because it does not produce any revenue. If there is no reservoir limit on the last day, the model will try to make a production plan that ends up with an empty reservoir. Multiple options could be considered as a reservoir inventory for the last day. One possibility would be to consider the value in saving the water for the following season. To simplify the model, the theoretical reservoir inventory for the last day must be equal or larger to the measured reservoir supply, as seen Table 1 and in Figure 7.

3.1.7 Reservoir regulations and nonnegative values

NVE provided the measurements from the reservoirs. These values uses LRV limit as its reference point. When the measurements are zero, the water level is at the lower reservoir limit, even though there is still more water in the pool. The model's production values and the reservoir inventory is limited to positive integers, which means that the model never breaks the LRV limit. The HRV-constraint in energy is calculated with Equation (1) being equal to the reservoir capacity given in cubic meters, where water is assumed to have a density of 1000 kg/m^3 . The model uses all the water running through the plant for production purposes including the minimum flow of water required in the regulations. By assuming that the water exits the reservoir through the plant, the water could be used in production. Minimum flow of water is required from Plant B and C. Topic 5.3.2 and 5.3.3 contains further discussion of minimum flow of water. The production value calculated by the model must be equal or higher than the production value in the minimum water flow.

4 Results

The timespan for this model is one year, referred to as a season, starting July 1. The hydropower stations, from now on referred to as Plant A, Plant B, Plant C and Plant D, will be analyzed two or three times depending on the data available. Multiple analyses were done for each plant in order to get a better understanding on how much revenue that is being captured for each plant.

4.1 Plant A

Plant A has one reservoir and one turbine, all of the water available can be considered as inventory for the plant. The turbine has a 10 MW capacity and in the period 2009-2013 it had an average yearly production value of 37 GWh. The average yearly revenue in the same period is estimated to be 12.7 mill. NOK. As seen in Figure 8, Plant A has the highest average production from October until March. The average price in the day-ahead market is highest in February (389 NOK/MWh) and lowest price in the trading zone is in September (231 NOK/MWh).

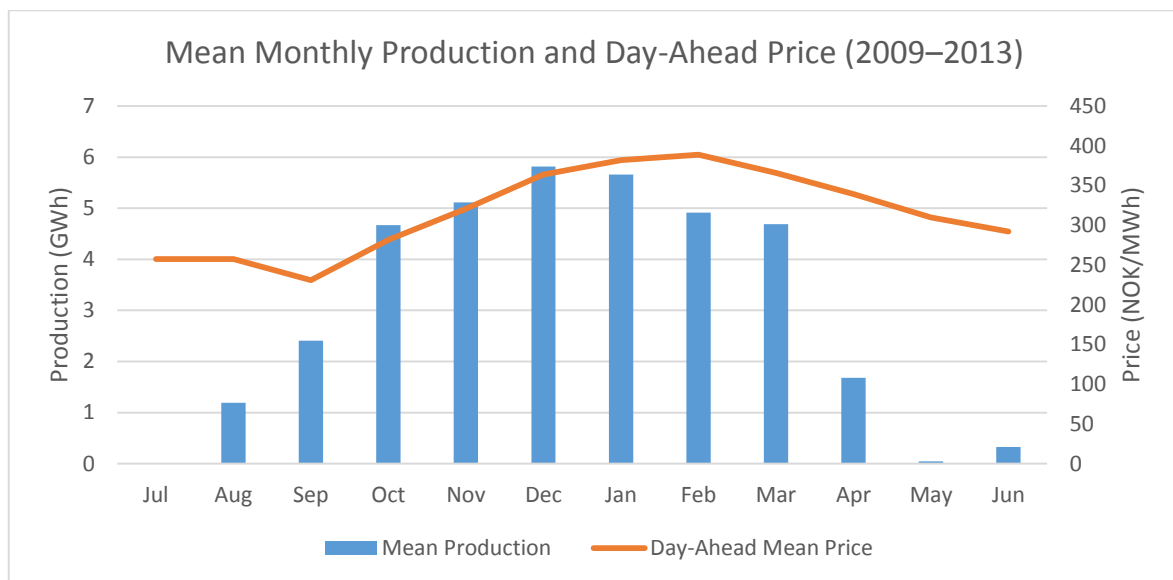


Figure 8: Mean monthly production and mean monthly price for Plant A. The production data is retrieved from the owner of the station and the price information is retrieved from Nord pool. Mean production is the left y-axis and the average price in the day-ahead market is on the right y-axis.

Results A

4.1.1 Analysis – A1 (1/7/2012–30/6/2013)

During this season an estimated revenue of NOK 10.2 million was calculated for Plant A. With an optimal production plan, our model shows revenue of NOK 11.4 million that could be obtained.

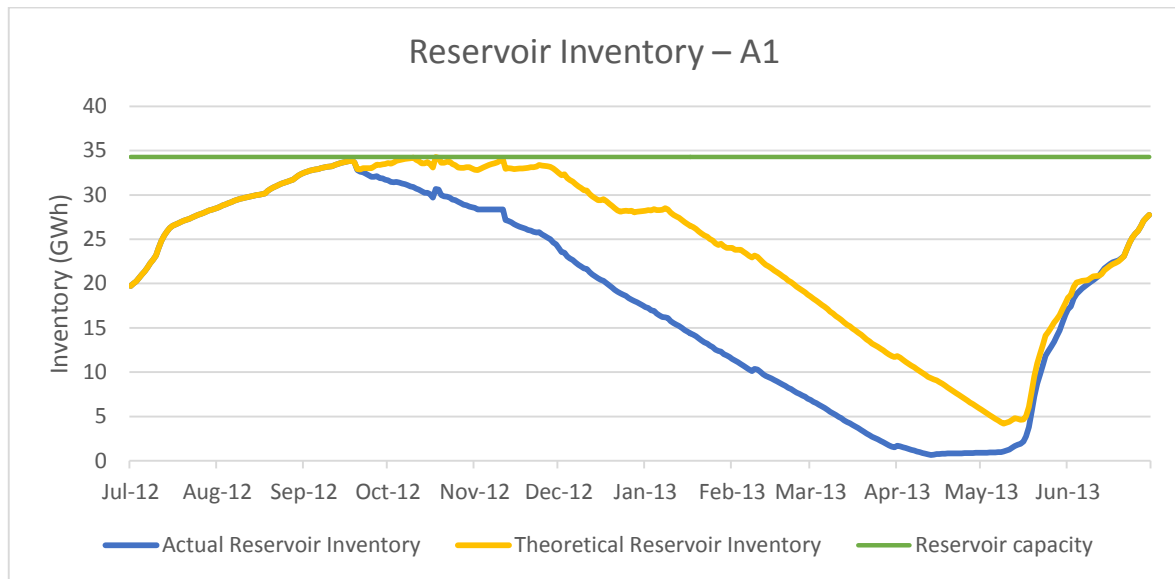


Figure 9: Actual and theoretical reservoir inventory for plant A, season 1. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

Reservoir supply for Plant A in Figure 9 shows that both the theoretical and actual inventory is identical until mid-September. Figure 10 and Figure 11 shows the difference in the production plant. The revenue optimized production plan produced more in the spring. At this point the actual reservoir inventory starts decreasing while the theoretical inventory stabilize until the end of December when it starts decreasing. Actual production for Plant A is shown in Figure 10. A total of 90% of the seasonal production happens during October–March. During this season, the price is lowest in September and peaking in December, and the price gradually increase from September to April.

Results A

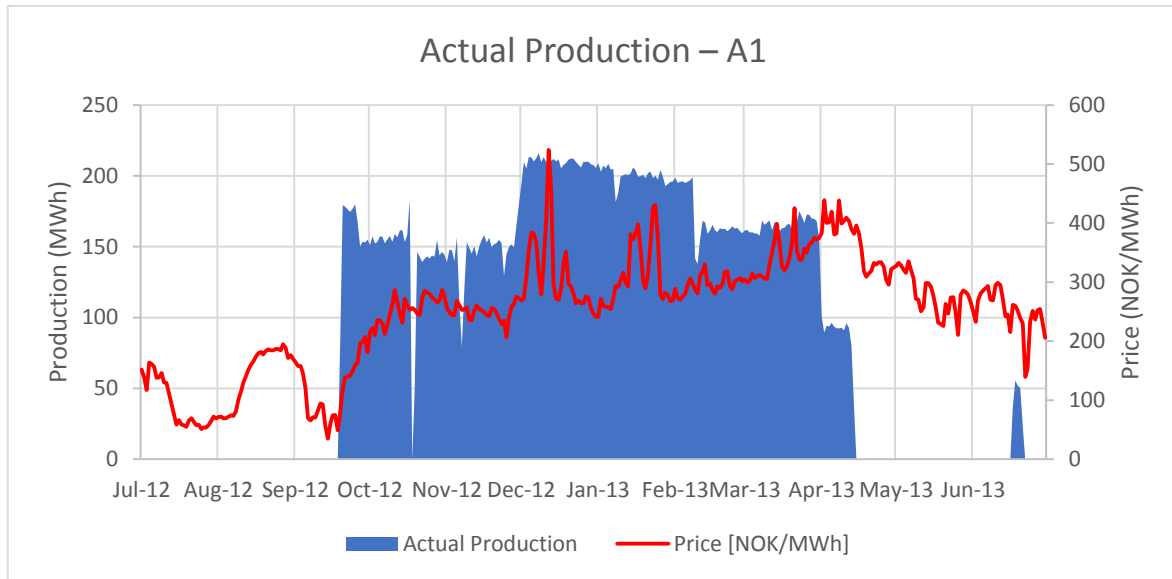


Figure 10: Actual production graph, Plant A, season 1. This figure shows the revenue-optimized production in MWh for Plant A in season 1. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Figure 11 below shows the production accumulating in the months of spring where the price is at its highest. The model avoids production during July, August and September due to low prices.

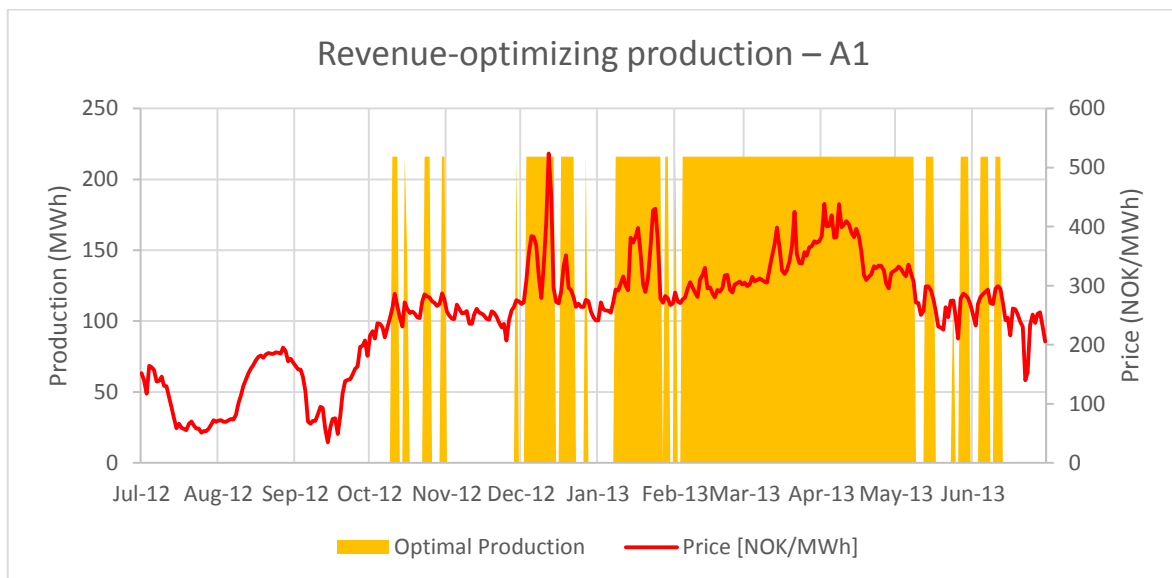


Figure 11: Revenue-optimized production graph, plant A, season 1. This figure shows the revenue-optimized production in MWh for Plant A in season 1. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

With the revenue-optimizing production plan, revenue increases with NOK 1.2 million. This equals an 12 percent increase in revenue. The theoretical reservoir inventory starts and ends at the same point as the actual reservoir inventory.

4.1.2 Analysis – A2 (1/7/2011–29/6/2012)

During July 2011–June 2012, the calculated revenue was NOK 11.6 million. The model calculates that a revenue optimizing production plan could have made NOK 13.2 million. When optimizing the revenue for Plant A from July 2011 until June 2012 by changing the production plan, the revenue increased with NOK 1.7 million. This is a 14% increase in revenue from the actual plan.

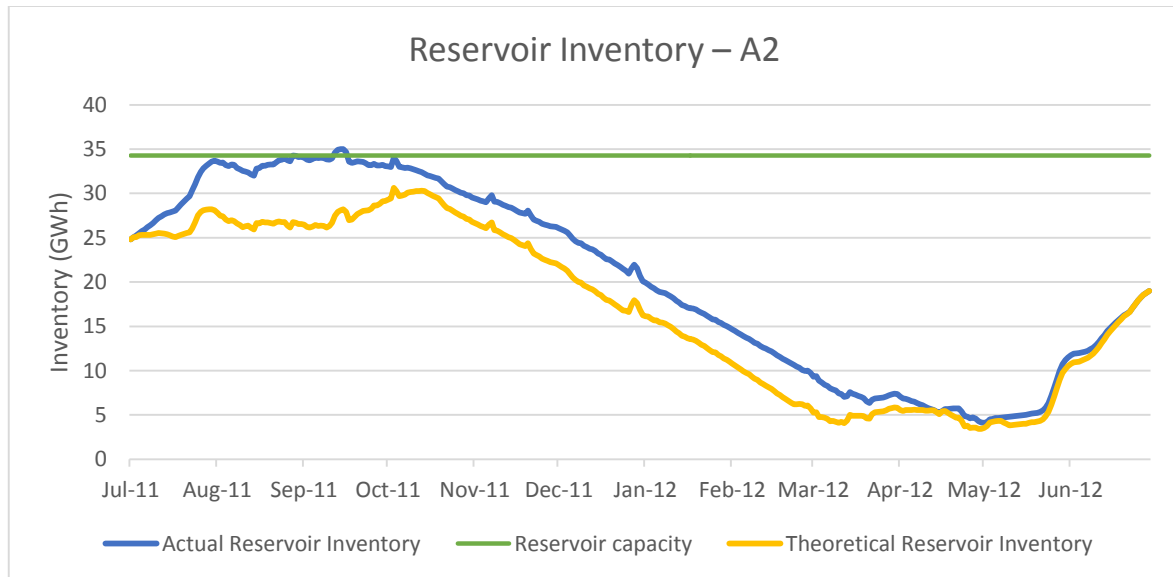


Figure 12: Actual and theoretical reservoir inventory for plant A, season 2. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

As seen in Figure 12, the reservoir starts and ends in the same point. Actual reservoir inventory is larger than the theoretical inventory until mid-April, but exceeds the upper water level in September. Figure 13 shows the actual production for Plant A from July 2011 until June 2012. A total of 31 GWh (69%) of the production was within October–March.

Results A

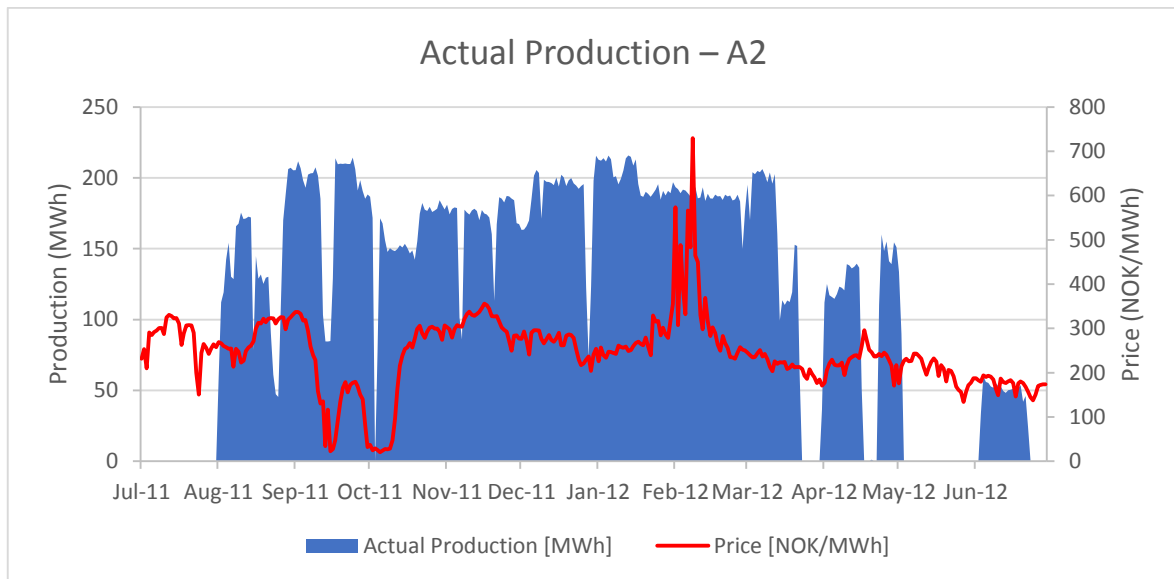


Figure 13: Actual production graph, Plant A, season 2. This figure shows the revenue-optimized production in MWh for Plant A in season 2. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Figure 14 shows the revenue-optimizing production plan. The production for this season shows that 26% of the production is done in July and August. A total of 29 GWh (64%) of the production is happens in October–March.

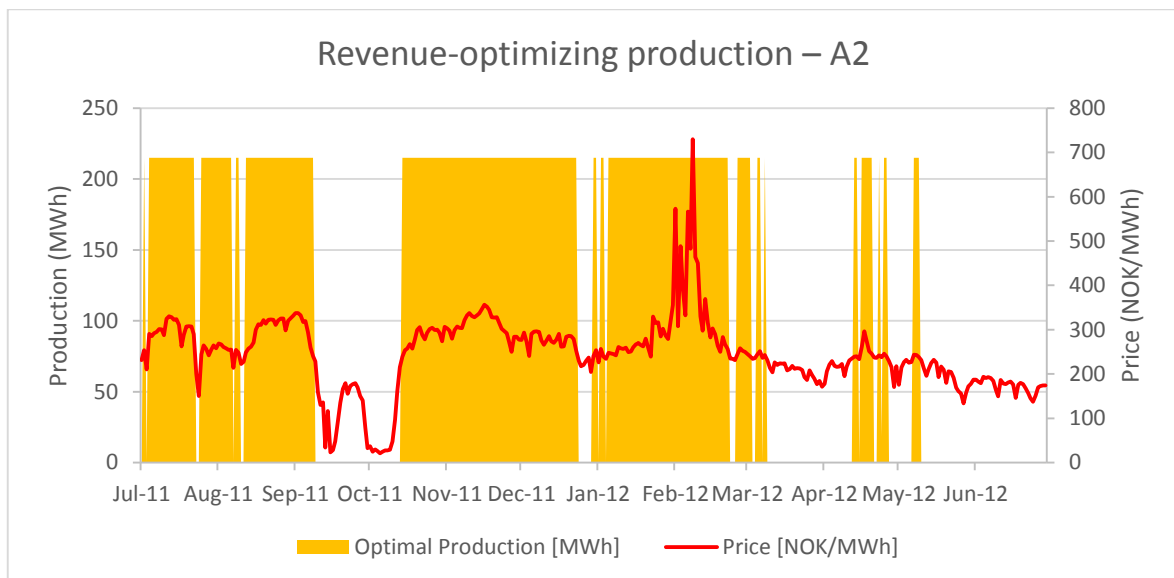


Figure 14: Revenue-optimized production graph, plant A, season 2. This figure shows the revenue-optimized production in MWh for Plant A in season 2. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.1.3 Analysis – A3 (1/7/2010–30/6/2011)

The revenue estimated for Plant A from July 2010 until June 2011 was NOK 16.4 million. When performing a revenue-optimizing function that changes the production plan for the season, a new revenue of NOK 17.4 million is calculated.

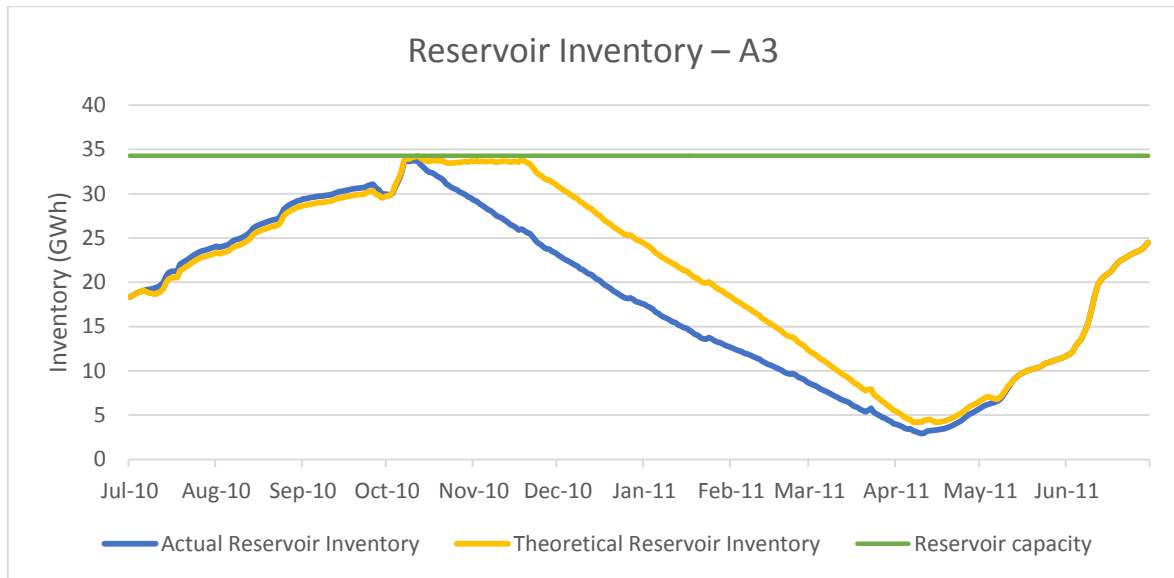


Figure 15: Actual and theoretical reservoir inventory for plant A, season 3. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

Figure 15 seen above shows how the actual reservoir inventory is different from the revenue optimizing production plan. The real production plan, represented by the blue line, shows that Plant A's water supply starts falling in early October, while the theoretical reservoir inventory stays around 33 GWh until late November. From that point it has a steeper slope than the real plan until April, where the two lines meet again.

Results A

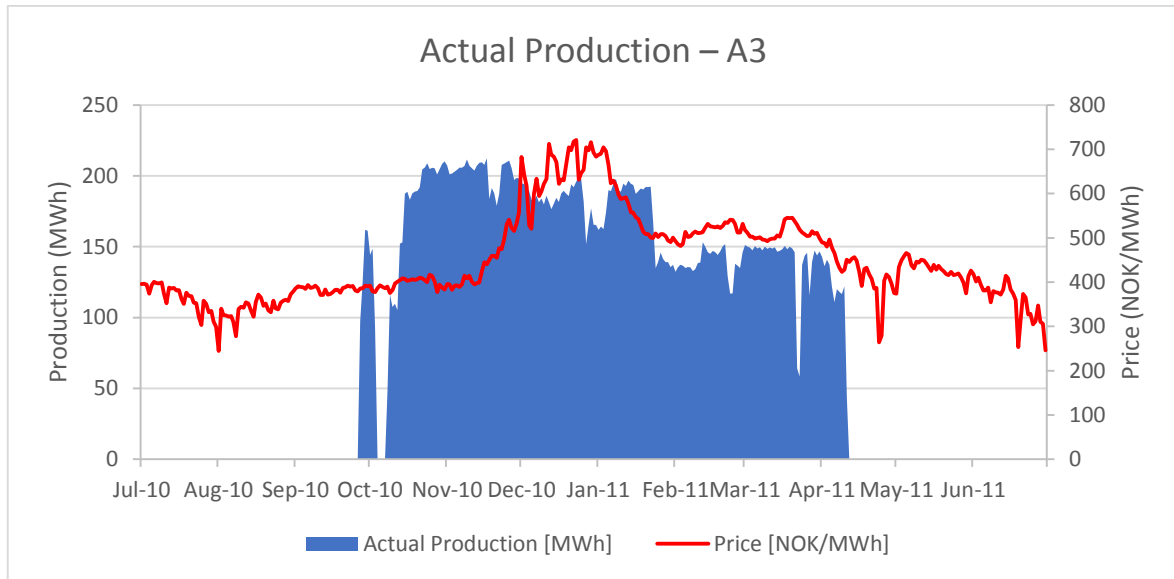


Figure 16: Actual production graph, Plant A, season 3. This figure shows the revenue-optimized production in MWh for Plant A in season 3. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The actual production for Plant A in this season seen in Figure 16. The majority of the production being 30 GWh, 94%, happened in October–March.

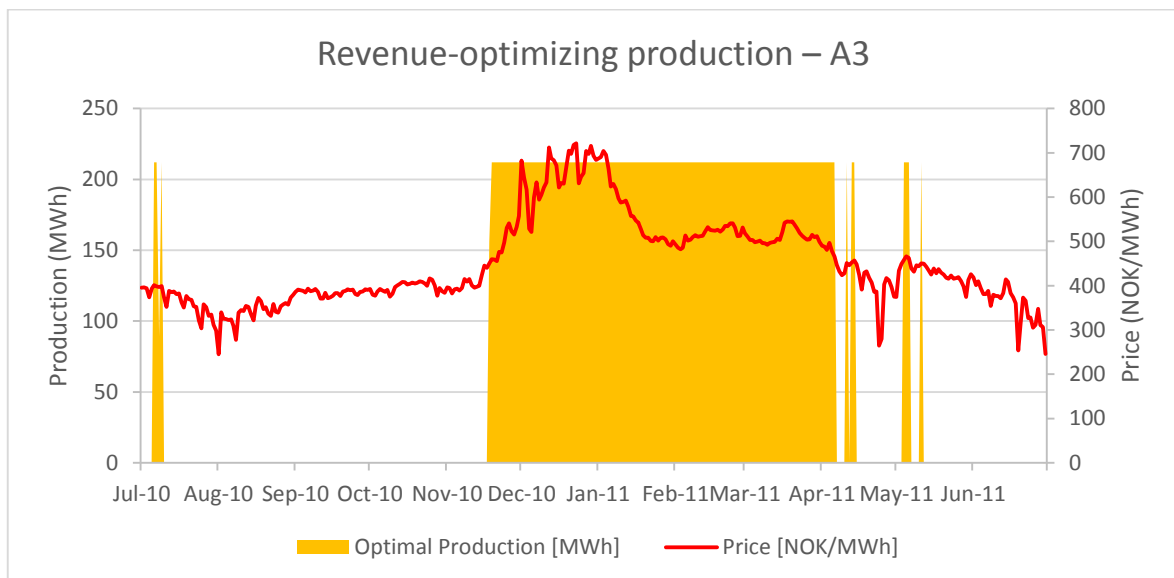


Figure 17: Revenue-optimized production graph, plant A, season 3. This figure shows the revenue-optimized production in MWh for Plant A in season 3. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The graph above seen in Figure 17 shows the revenue-optimizing plan for Plant A during the season. A total of 89% of the production was done in October–March. When optimizing the revenue for this season the revenue increase with NOK 1 million. This is equals a 6% increase in the revenue.

Results A

4.1.4 Summary – Plant A

Plant A was analyzed over 3 seasons. The revenue optimizing function adjusted the actual production plan and found a better revenue in all three cases. Table 2 shows the revenue, both estimated and for the revenue optimizing plan. A2 had the highest change in revenue with NOK 1.7 million in increased revenue. This equals a 14% increase from the estimated revenue. A3 had the lowest change in revenue with 6 % increase in revenue. A3 was the season with the highest estimated real revenue, being NOK 4.8 million higher than A2 and NOK 6.2 million higher than A1.

Table 2: Estimated versus optimal revenue table, summary of plant A. This table shows the estimated revenue (Est. Rev. [mill. NOK]) and the revenue for the revenue optimizing plan (Opt. Rev. [mill. NOK]) for A1, A2 and A3. It also shows the difference between optimal and real revenue (Diff. [mill. NOK]) and the relative change in revenue.

Analysis	Est. Rev. [mill. NOK]	Opt. Rev. [mill. NOK]	Diff. [mill. NOK]	Relative change
A1	10.2	11.4	1.2	12 %
A2	11.6	13.2	1.7	14 %
A3	16.4	17.4	1.0	6 %
Sum	38.1	42.0	3.9	10 %

The periods analyzed were in some cases varying significantly, while in other scenarios similar. By graphing the average monthly production value, both actual and the values in the case when optimizing revenue. These values, are shown in Figure 18 below, where the actual production is show by blue columns, and optimized revenue production by yellow.

Results A

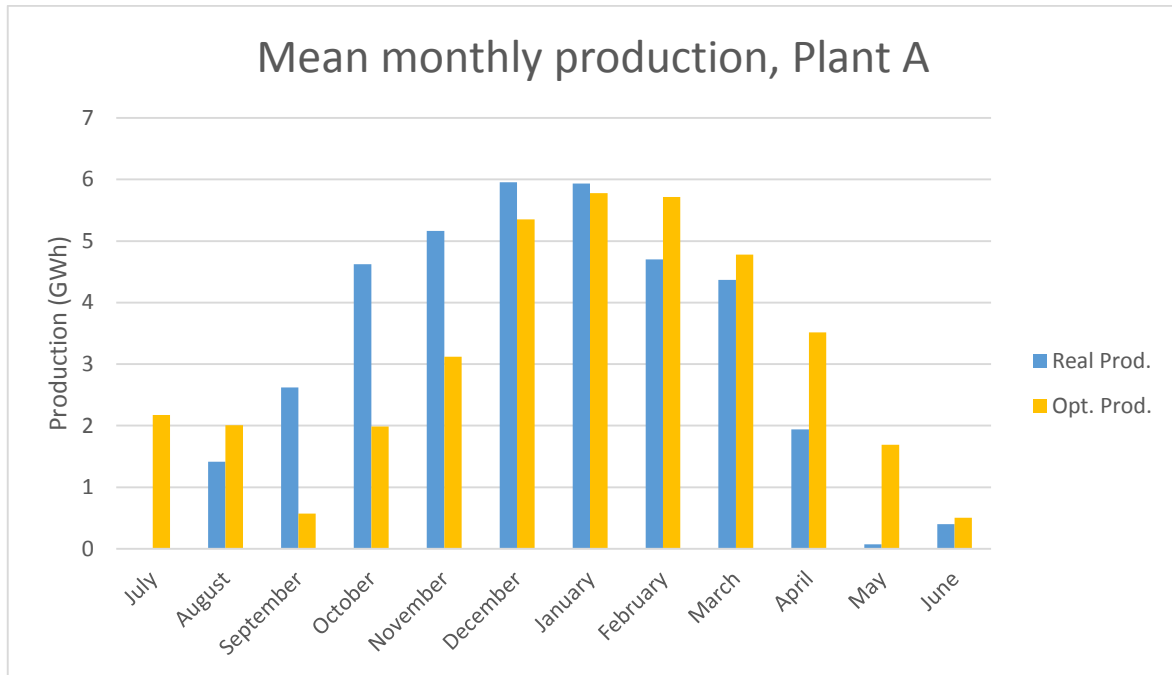


Figure 18: Mean monthly production graph, plant A. The graph shows actual production (blue) and revenue optimizing production (yellow) by average GWh per month.

These values are the mean production per month, and the production pattern is displayed. The results are based on data in the period 1/7/2010–30/6/2013. In Figure 18, the actual production in September is much higher than the production suggested by the model, leaving this remaining production to fall within another month with more obtainable revenue. The majority of high optimal production columns (yellow) falls within in the later months of December through April. Where the blue (actual production) columns exceeds the yellow, the owner has produced more than the model and vice versa. The later winter months shows yellow columns exceeding the blue, creating a gap that resembles the difference in obtained revenue. The mean reservoir graph is shown in Figure 19 below where the mean actual reservoir inventory is shown by the blue columns. The mean values of the optimized revenue reservoir inventory is shown by yellow and the resolution is per month.

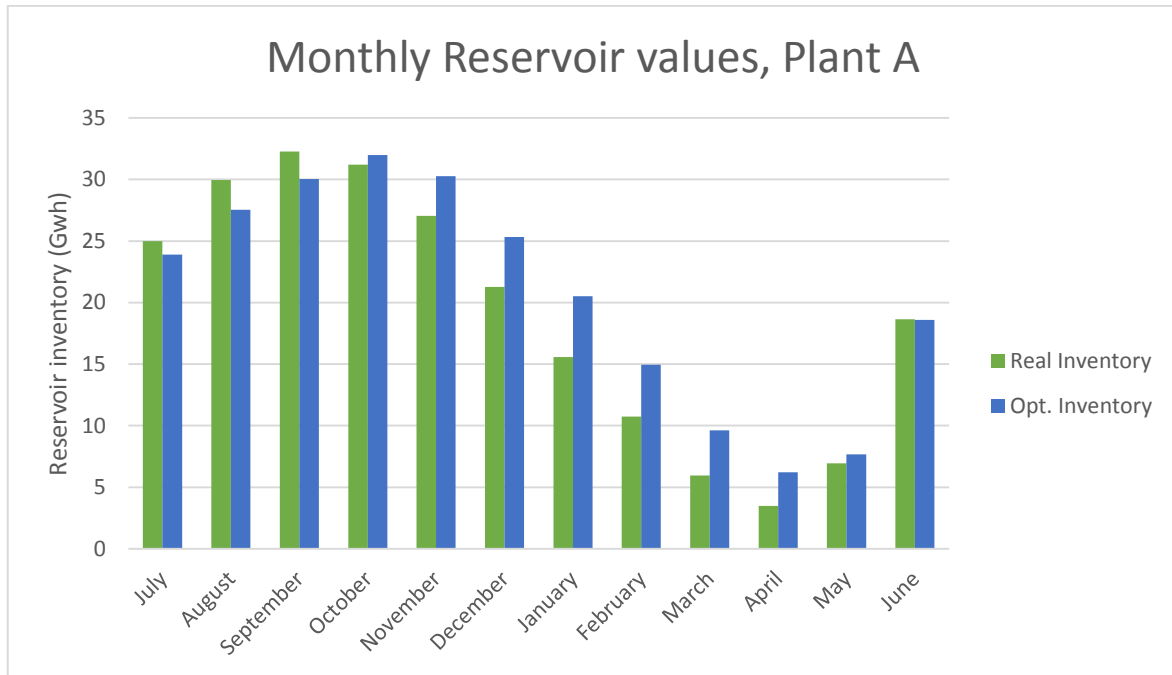


Figure 19: Monthly reservoir graph, plant A. The figure shows the actual mean reservoir inventory (blue) and revenue optimized reservoir inventory (yellow) per month.

The mean actual reservoir columns exceeds the optimized value from July to October, while the mean optimized reservoir amount exceeds the actual the rest of the season. The model saves water instead of producing in September–October. In the months of winter and spring, the optimal reservoir columns exceeds the actual reservoir measurements.

4.2 Plant B

Plant B, located in the south western part of Norway, has one reservoir and a total performance of 25.5MW. Based on data received from the contributor, the plant has an average yearly production of 114 GWh. The data gives an average estimated yearly revenue of 29.2 mill. NOK. Plant B does not share its reservoir with any other plants, so all the water in the reservoir was considered as inventory for the plant. It is an older plant with some restoration done to the electrical equipment. Data available for the plant is only sufficient for two seasonal analysis, B1 and B2. As seen in Figure 20 below, plant B's monthly production peaks in the winter months. The average price, peaks is also highest in the winter months. The average price in the day-ahead market is highest in February (295 NOK/MWh) and lowest in July (196 NOK/MWh). This power plant has restriction of minimum water flow of 350L/s.

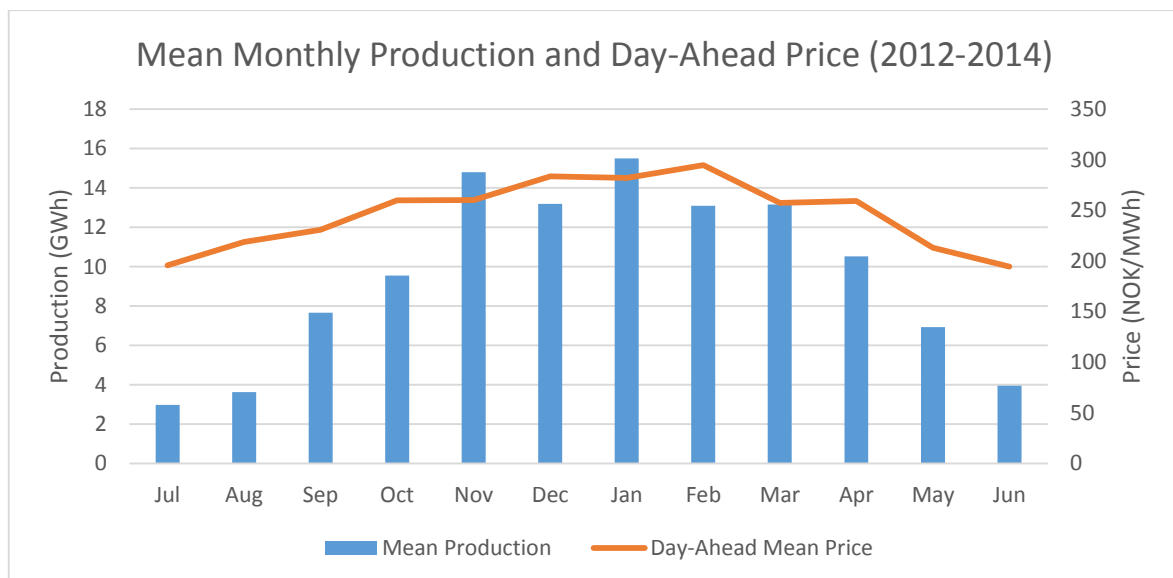


Figure 20: Mean monthly production and mean price for plant B. The graph shows the mean value of the production and the price over the two seasons analyzed. The production data is retrieved from the owner of the plant and the price information is retrieved from Nord Pool.

4.2.1 Analysis – B1 (1/7/2013–30/6/2014)

For the first season analyzed in plant B the estimated revenue was NOK 29.1 million. The theoretical revenue optimized by our model showed a total of NOK 31.4 million.

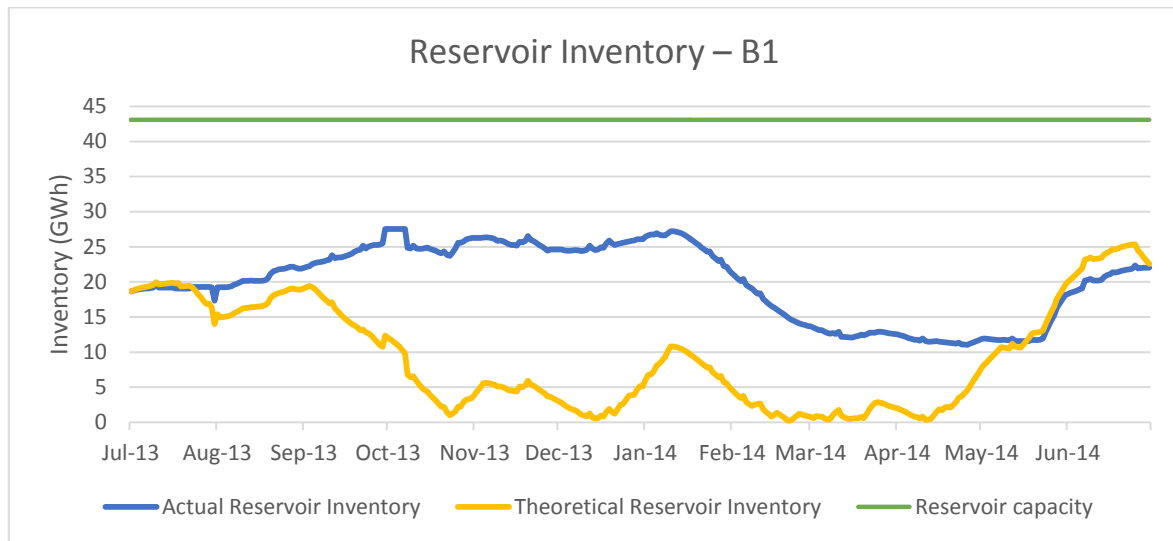


Figure 21: Actual and theoretical reservoir inventory for plant B, season 1. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

The daily optimal reservoir inventory constructed by our model versus the actual reservoir curve is shown above in Figure 21. A clear gap is shown as the optimal reservoir curve differs from the actual curve from the beginning of August. In Figure 21, a small jump in reservoir capacity is observed in early August. Investigating this value led to the explanation that this was a measuring error done by the owner of the plant. NVE confirmed this assumption suggesting to use linear interpolation to achieve a more realistic value (Taksdal 2015). The graph neglecting this measuring error is shown below in Figure 22.

Results B

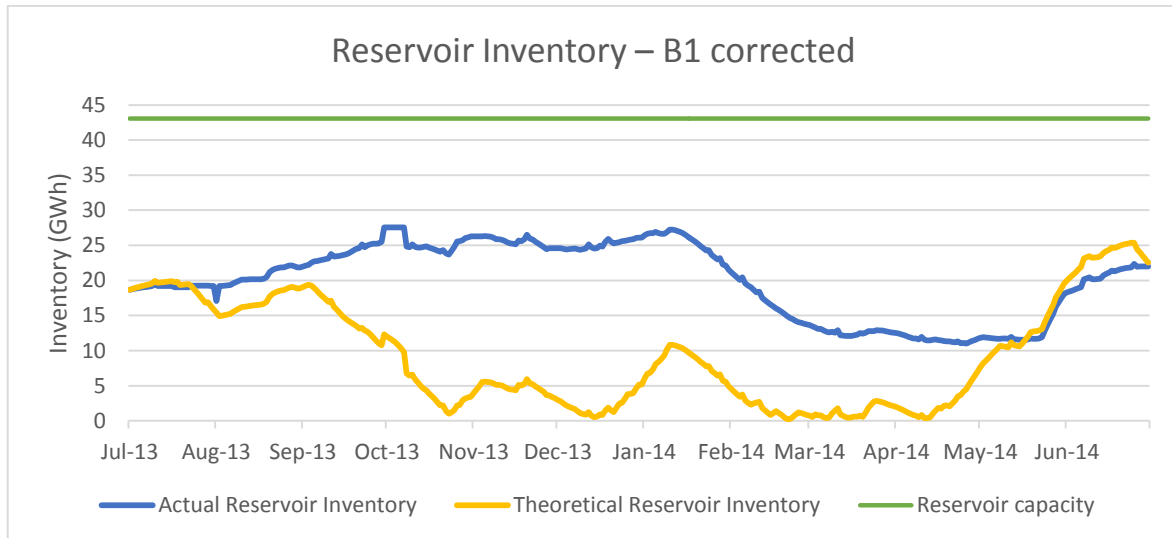


Figure 22: Corrected reservoir graph, plant B, season 1. This figure shows Plant B’s reservoir curve neglecting measuring error from 1st of August. Measurements and calculations are per day. Upper water level is shown by green curve.

The removal of the measurement error resulted in a smoother graph, but did not change the revenue-optimized production. The actual production data received from this period, is shown below in Figure 23. A total of 72% of the production is done in October–March. The graph peaks at a value of 543 MWh/day and continues at this performance for longer periods of time.

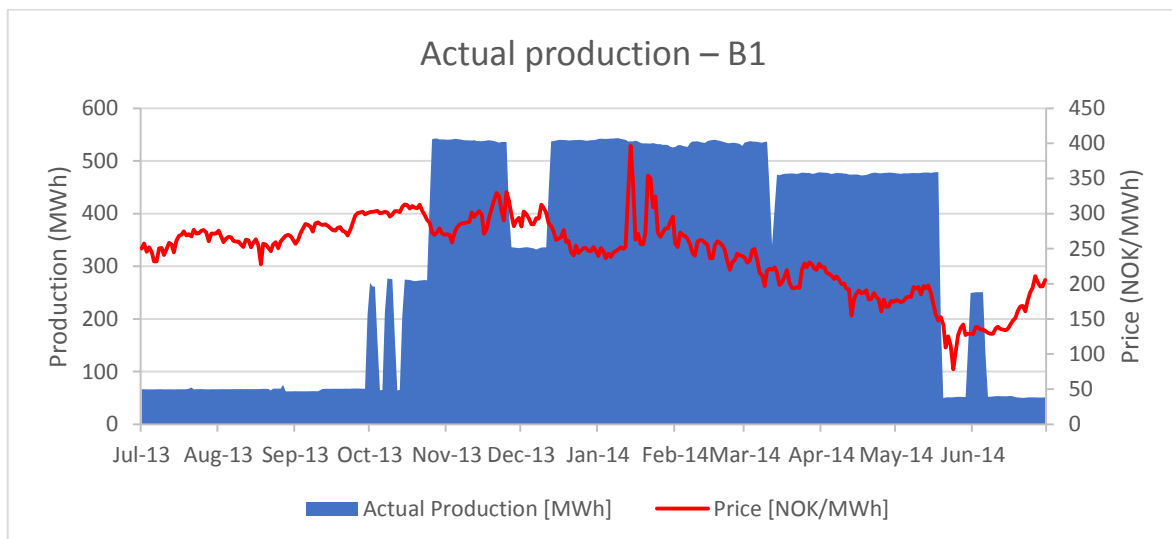


Figure 23: Actual production graph, Plant B, season 1. This figure shows the revenue-optimized production in MWh for Plant B in season 1. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The revenue-optimizing production on a daily basis is shown below in Figure 24. The setup is the same as in Figure 23 replacing the production throughout the year. The model set 49% of the season’s production for the period September–December and 38% of the production from January to April.

Results B

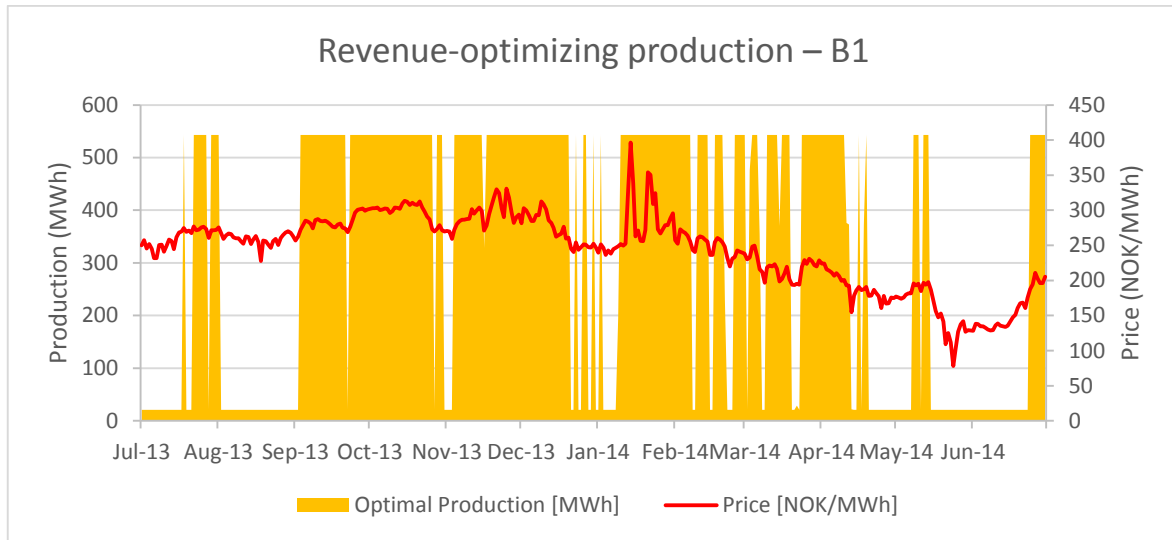


Figure 24: Revenue-optimized production graph, plant B, season 1. This figure shows the revenue-optimized production in MWh for Plant B in season 1. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The total increase in revenue for Plant B from July 2013 until June 2014 was NOK 2.3 million equaling an 8% increase in revenue.

4.2.2 Analysis – B2 (1/7/2012–29/6/2013)

The B2 analysis calculates a revenue of NOK 27.1 million for the season. The revenue optimizing plan calculated a revenue of NOK 31.3 million. This 16% increase in revenue gives a different reservoir curve shown below in Figure 25.

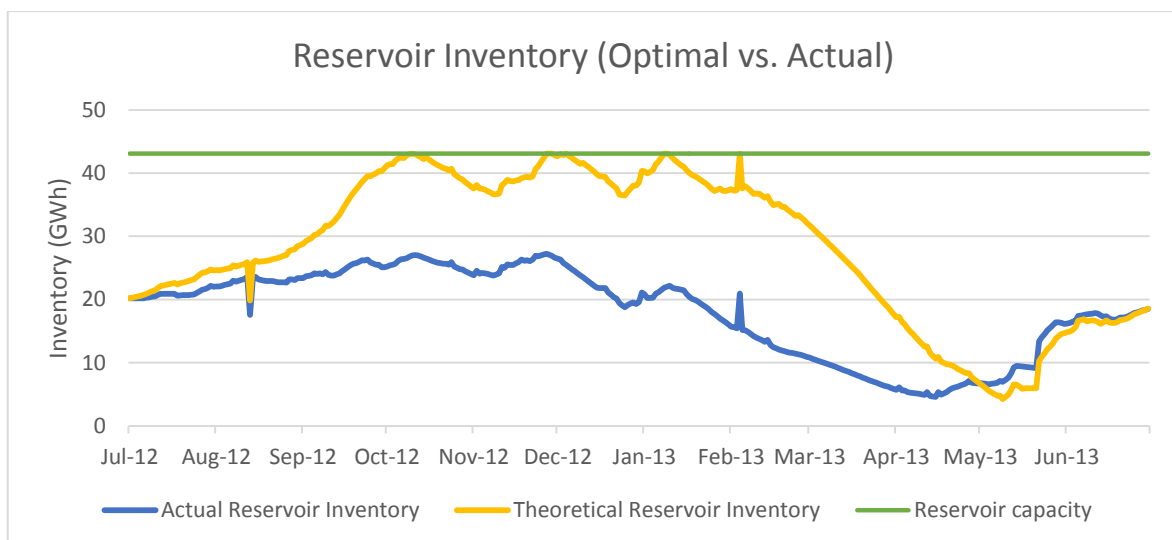


Figure 25: Actual and theoretical reservoir inventory for plant B, season 2. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

Results B

In Figure 25 one can observe two peaks in both the actual and optimized reservoir curve. These curves originate from the reservoir data NVE. A peak occurs in both August 2012 and February 2013 and is, by NVE, confirmed as a measuring error (Taksdal 2015). This error affects the optimized curve as it depends on the actual reservoir curve to calculate inflow. By neglecting this error, using linear interpolation, a more realistic optimization can be obtained. Figure 26 shows the optimized reservoir curve disregarding the two error peaks. This correction resulted in a decrease in revenue of NOK 15 thousands with same amount of water used.

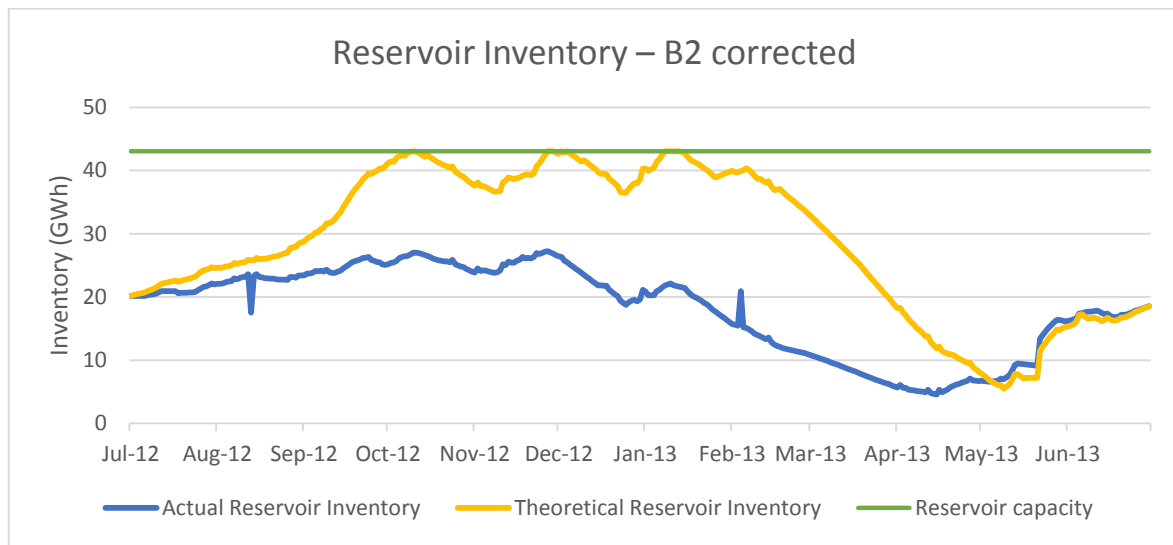


Figure 26: Corrected reservoir graph, plant B, season 2. This figure shows Plant B's reservoir curve neglecting measuring errors from August and February. Measurements and calculations are per day. Upper water level is shown by green curve.

The original production from the plant in this season is shown below in Figure 27. Start of the production is set to the beginning of July lasting a full year. This graph shows an earlier production start in the fall compared to B1. Production from October until March equals 69% of the yearly production.

Results B

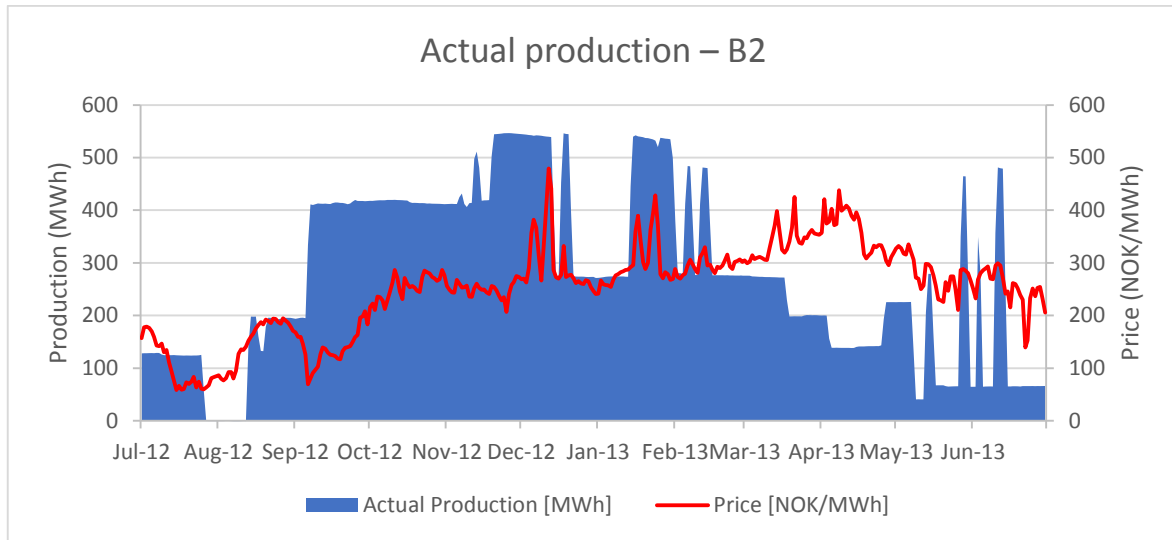


Figure 27: Actual production graph, Plant B, season 2. This figure shows the revenue-optimized production in MWh for Plant B in season 2. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The revenue-optimizing production for B2 is seen in Figure 28. During the summer months, there is almost no production. Only the minimum water flow generating production during this period. The optimized production focus is shifted towards the later winter months. A total of 72% of the total production is done in October–March.

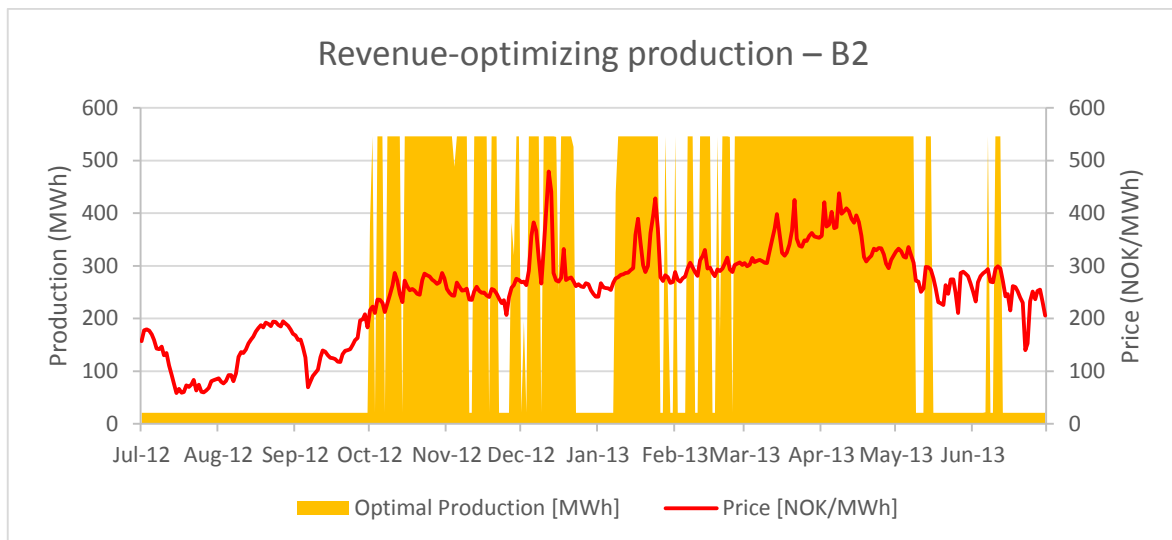


Figure 28: Revenue-optimized production graph, plant B, season 2. This figure shows the revenue-optimized production in MWh for Plant B in season 2. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The estimated total revenue for Plant B is NOK 27.1 million. When neglecting the measurement error the revenue optimizing plan gives a revenue of NOK 31.3 million. This is an increase in revenue of NOK 4.2 million which equals a 16% increase.

4.2.3 Summary – Plant B

The results from analyses of Plant B over the two seasons is shown below in Table 3. The optimization in season B1 resulted in an 8% increase in revenue listed on the right side of the table. Season B2 gave a 16% increase in revenue. It almost doubled the difference from the prior season.

Table 3: Estimated versus optimal revenue table, summary of plant B. The table shows the estimated revenue (Est. Rev. [mill. NOK]) and optimized revenue (Opt. Rev. [mill. NOK]) over the available seasons. The values has been summed in the bottom of the table and the difference, as well as relative change, is shown to the right.

Analysis	Est. Rev. [mill. NOK]	Opt. Rev. [mill. NOK]	Diff. [mill. NOK]	Relative change
B1	29.1	31.4	2.3	8 %
B2	27.1	31.3	4.2	16 %
Sum	56.2	62.7	6.6	12 %

Season B1 resulted in a difference of NOK 2.3 million with 51 MWh difference in production. This production is unused energy by the model, increasing revenue using less water. For Plant B over the two seasons, the total difference in revenue is NOK 6.6 million, an increase of 12%. Graphed mean monthly values of the production for Plant B is displayed below in Figure 29, showing the plant’s mean production plan. The model’s average optimized revenue production is also shown by yellow columns.

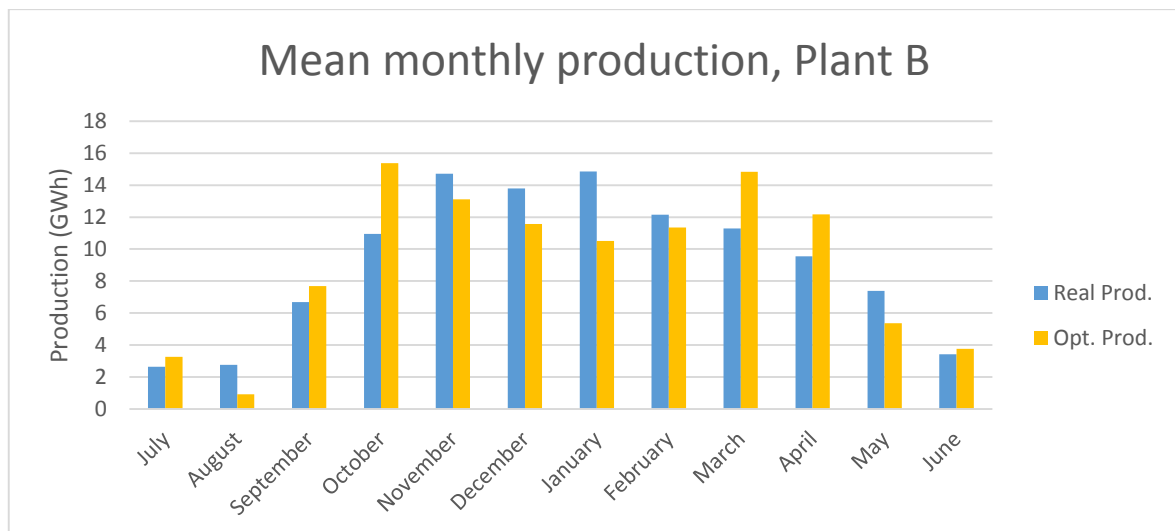


Figure 29: Mean monthly production graph, plant B. The graph shows actual production (blue) and revenue optimizing production (yellow) by average GWh per month. The results is based on data in the period 1/7/2012–30/6/2014

Results B

The months where the actual production exceeds the production for the model is where the actual revenue exceeds the model. The model's mean production exceeds the actual production during months where prices are higher. Figure 29 shows that in order to maximize the revenue the highest mean production value should be in October, March and April. The figure also shows that the mean production for Plant B is highest in November, December and January.

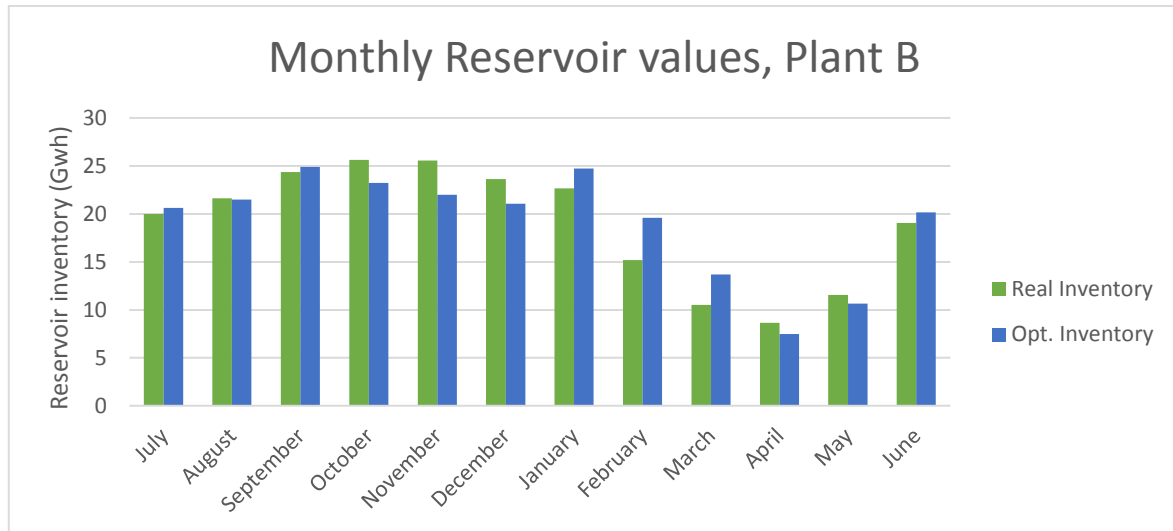


Figure 30: Monthly reservoir graph, plant B. The figure shows the actual mean reservoir inventory (blue) and revenue optimized reservoir inventory (yellow) per month.

Graphing the mean reservoir curve for Plant B gives an impression of the usage of the water basin, both the actual and the one constructed by the model. During October, November and December the mean water supply is higher than the one for the model. In January, February and March the reservoir for the model contains more water on average than the mean value for the measurements.

4.3 Plant C

Plant C is a hydropower station with one reservoir and four turbines with a total capacity of 90 MW. In the period 2009–2013, Plant C had a mean yearly production of 308 GWh. The regulation during the summer limits the reservoir capacity. With a max storage capacity of 133 GWh in the reservoir, Plant C has the possibility to store 43% of its mean yearly production.

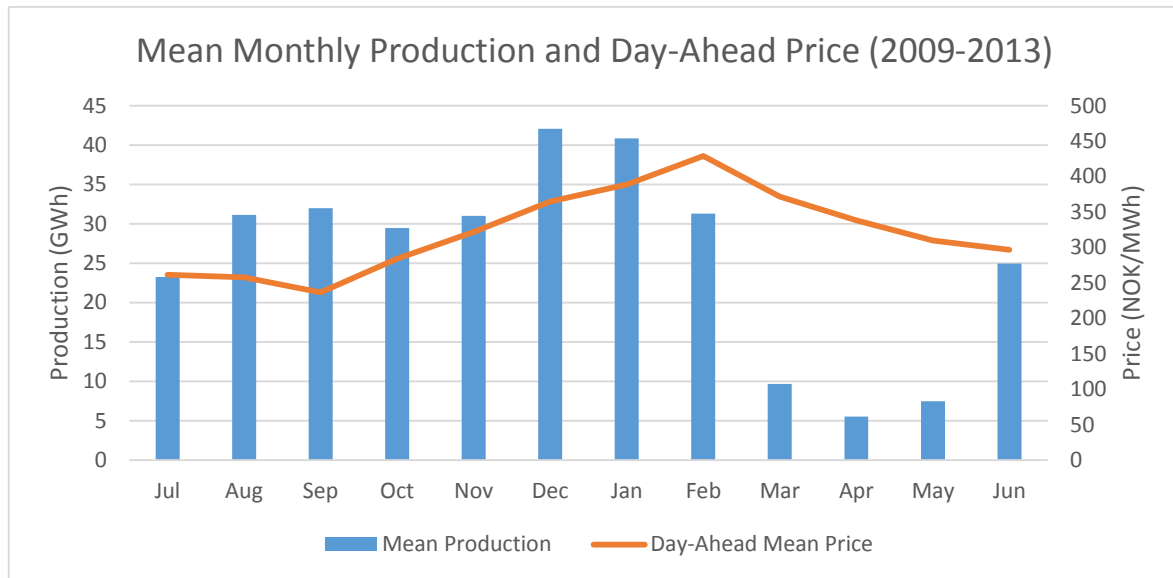


Figure 31: Mean monthly production and mean price for plant C. The graph shows the mean value of the production and the price over the seasons analyzed. The production data is retrieved from the owner of the plant and the price information is retrieved from Nord Pool.

Figure 31 shows the mean monthly production from 2009 until 2013. It also shows the average price for each month during the same period. The mean production is highest in December (42 GWh) and January (41 GWh). The production for Plant C is smallest during March (10 GWh), April (6 GWh) and May (7 GWh). The price in Figure 31 is the average price for that month from the day-ahead market in Nord Pool. The mean price is lowest in September 236 (NOK/MWh) and highest in February 429 (NOK/MWh).

4.3.1 Analysis – C1 (1/7/2012–30/6/2013)

During 1/1/2012–30/6/2013, Plant C had an estimated revenue of NOK 86.6 million. When optimizing the function, a revenue of NOK 99.6 million was calculated. The results from C1 shows that the revenue optimizing plan increase the revenue with NOK 13.0 million. This is a 15% increase in revenue from July 2012 until June 2013. Figure 32 shows the reservoir inventory for both the real and the optimal production plan. The green line is the upper limit for the reservoir and the red is the lower limit. During the period 15. June – 15. August, there is an increased regulation for the reservoir as seen with the green and red line.

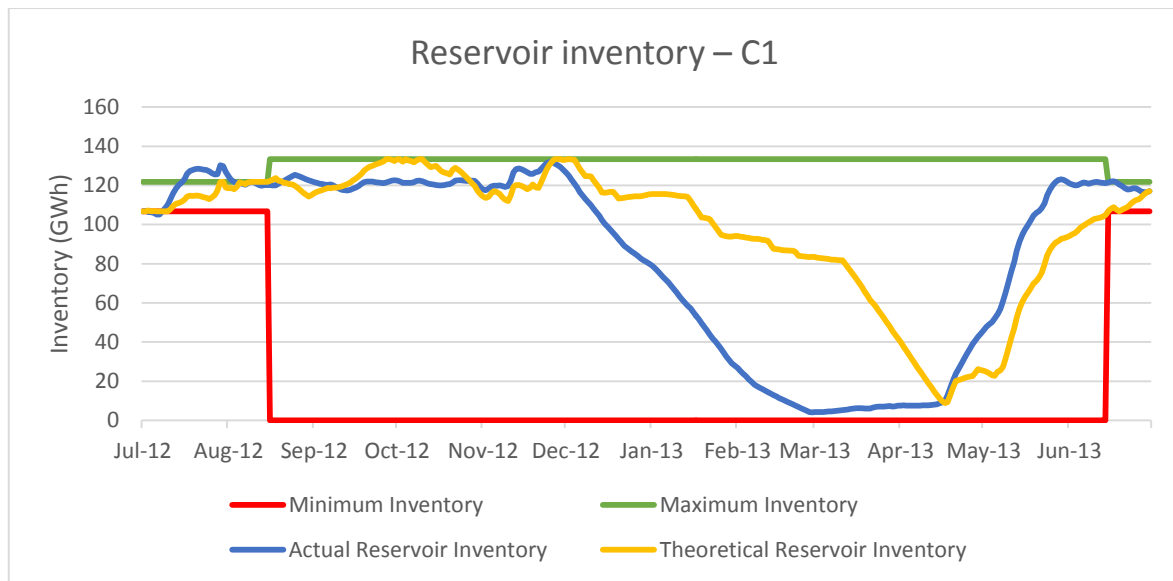


Figure 32: Actual and theoretical reservoir inventory for plant C, season 1. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the upper restrictions and capacity, and the red line shows the lower restrictions.

The reservoir inventory for both the real and the optimized plan is shown in Figure 32. They both start at the same reservoir inventory. The line for the actual inventory shows that Plant C almost empties the reservoir in March and keeps it at under 10 GWh until mid-April. This is at the same time where the revenue-optimizing plan reach the minimum inventory for the reservoir. During the summer when the reservoir is closer regulated the inventory has to be between 107 GWh and 122 GWh. In July 2012, the actual reservoir inventory is both over and under the regulated limits while the optimized plan never break the regulation. Figure 33 shows the production for Plant C during the season, with no production from March until mid-June. Almost half of the season's production (43 %) happened in November, December and January.

Results C

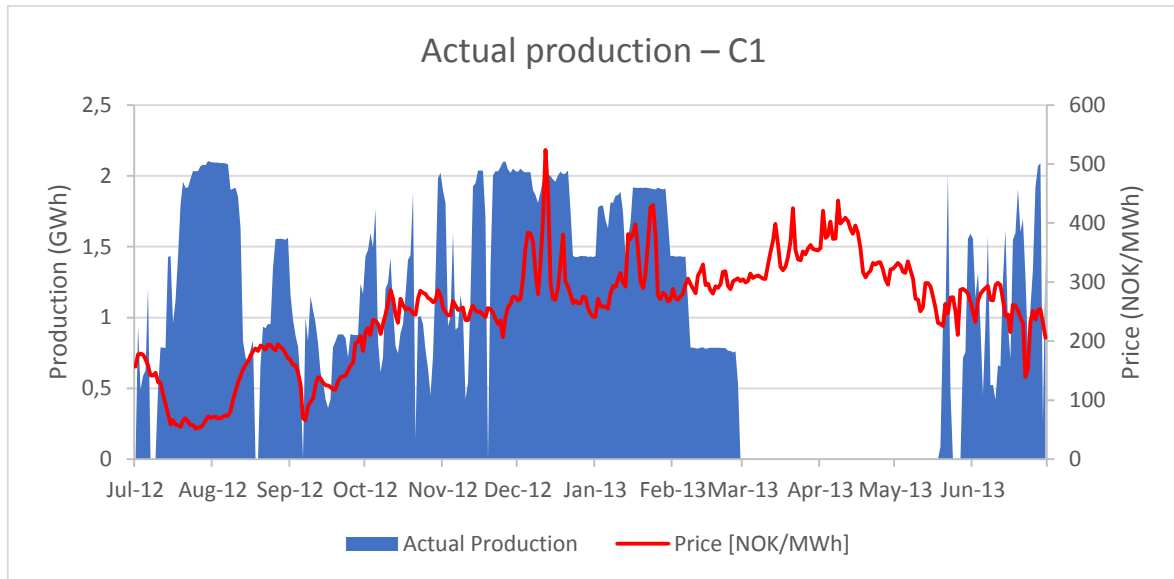


Figure 33: Actual production graph, Plant C, season 1. This figure shows the revenue-optimized production in MWh for Plant C in season 1. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Figure 34 shows the revenue-optimizing production for June 2012–July 2013. March and April included 99 GWh (37%) of the yearly production. September is the lowest producing month with 7 GWh, which is only 2% of the season total production. As seen in Figure 31, September is also the month with the lowest average price in the day-ahead market where Plant C is located.

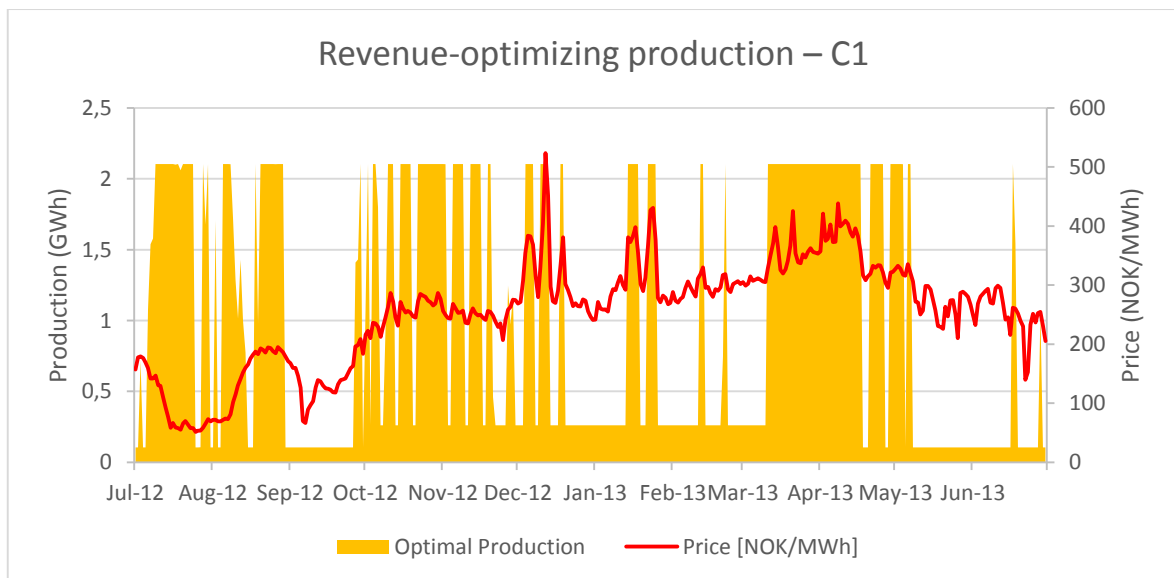


Figure 34: Revenue-optimized production graph, plant C, season 1. This figure shows the revenue-optimized production in MWh for Plant C in season 1. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.3.2 Analysis – C2 (1/7/2011–29/6/2012)

Analysis C2 for Plant C estimated a revenue of NOK 94.1 million from July 2011 until June 2012. By optimizing the revenue in the model, a revenue of NOK 104.3 million could be achieved. The revenue optimizing production plan increased the revenue with NOK 10.2 million. This is an 11% increase in the revenue during this season. Figure 35 shows the actual reservoir inventory and the reservoir inventory for the revenue optimized production plan. It also shows the upper and lower limit for the reservoir.

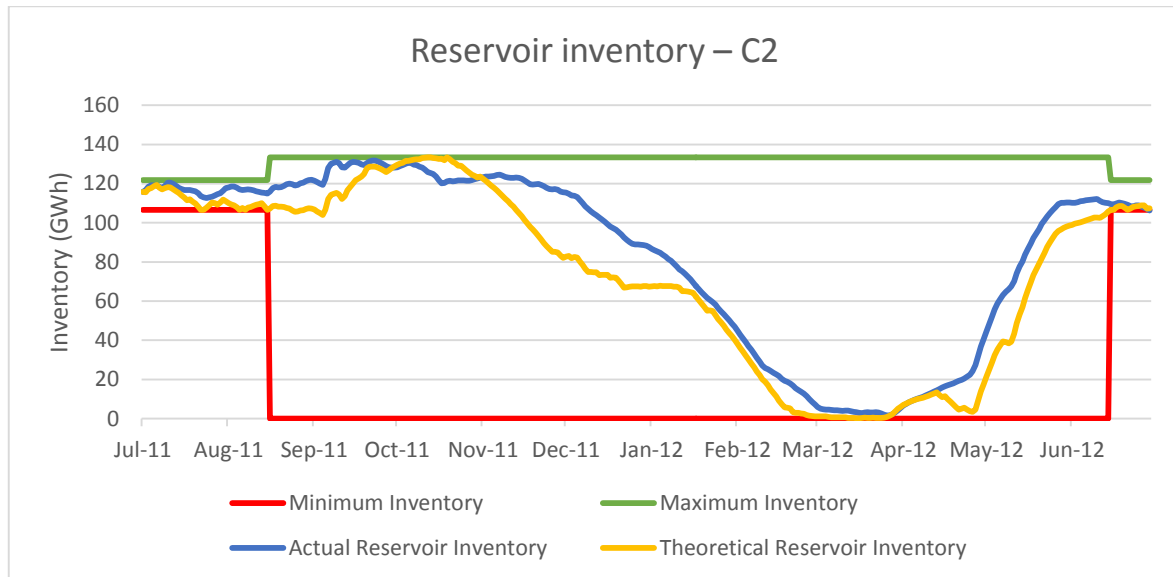


Figure 35: Actual and theoretical reservoir inventory for plant C, season 2. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the upper restrictions and capacity, and the red line shows the lower restrictions.

The actual reservoir and the theoretical inventory seen in Figure 35 stays between the regulated HRV- and LRV-limits during the season. Figure 36 shows the actual production plan for Plant C. During the first 3 months of the season, the plant produces 35% (125 GWh) of the total season's production. Another 35% of the production is done during December, January and February. No production happens in April as seen in Figure 36. This is where the actual reservoir inventory start to increase as seen in Figure 35.

Results C

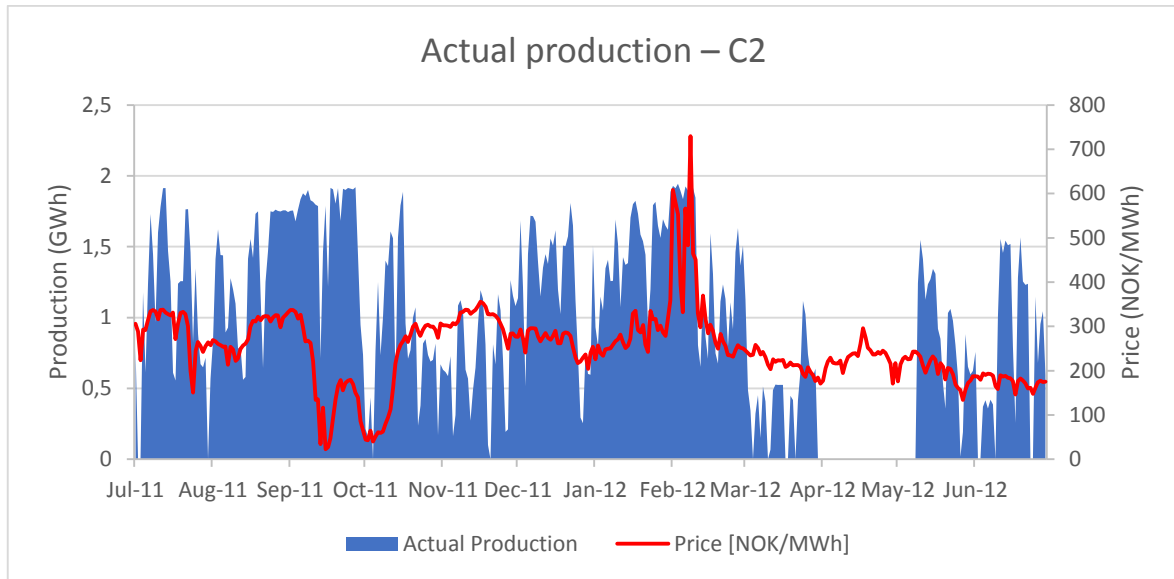


Figure 36: Actual production graph, Plant C, season 2. This figure shows the revenue-optimized production in MWh for Plant C in season 2. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Figure 37 shows the revenue optimized production plan for Plant C. The production is distributed throughout the season. During July, August and September, 35% (124 GWh) of the total production happened.

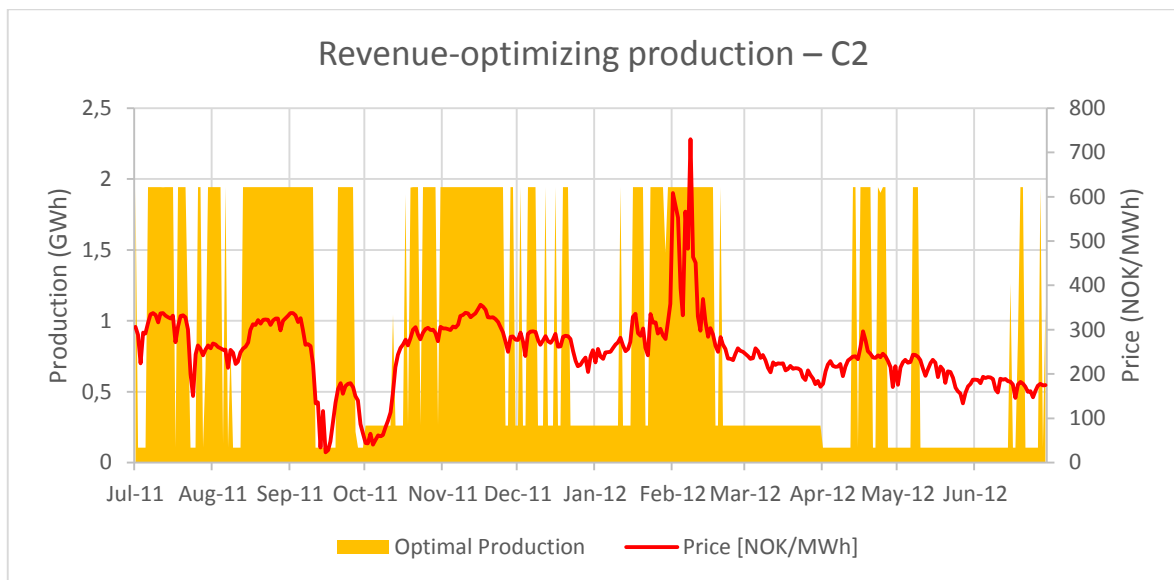


Figure 37: Revenue-optimized production graph, plant C, season 2. This figure shows the revenue-optimized production in MWh for Plant C in season 2. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.3.3 Analysis – C3 (1/7/2010–30/6/2011)

Analysis C3 estimated a real revenue of NOK 127.4 million. When optimizing revenue by changing the production from July 2010 until June 2012, a revenue of NOK 141.3 million was calculated. The function that optimizes the revenue by changing the production plan found a plan that increased the revenue with NOK 13.8 million. Figure 38 shows the real and the theoretical inventory of the reservoir. It also shows the maximum and minimum inventory boundaries during the season.

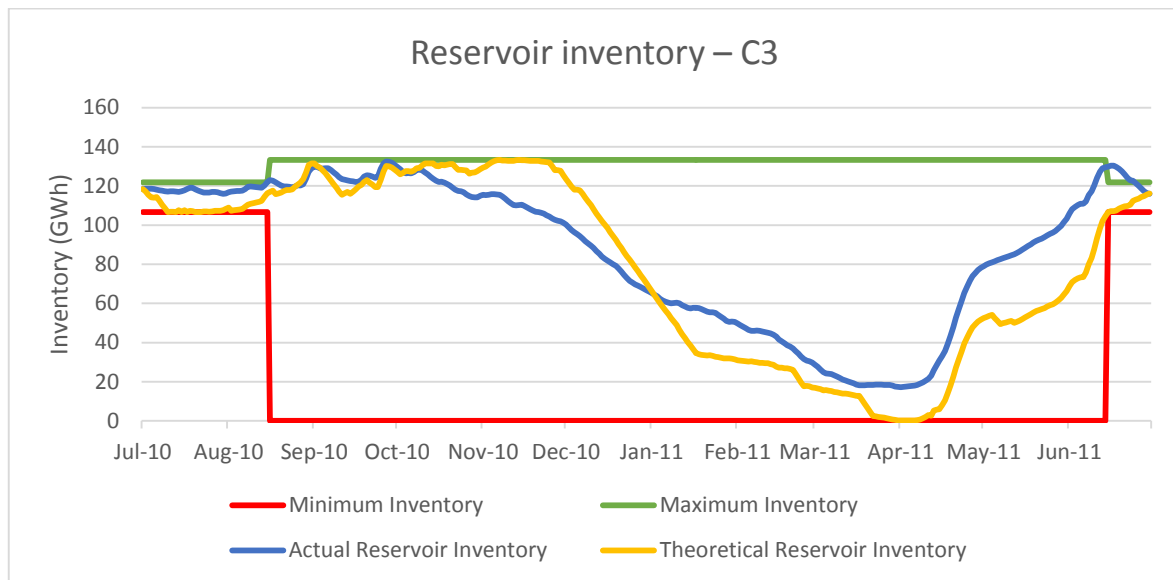


Figure 38: Actual and theoretical reservoir inventory for plant C, season 3. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the upper restrictions and capacity, and the red line shows the lower restrictions.

As seen in Figure 38 the actual reservoir inventory for Plant C breaks the regulation limit in June 2011. The theoretical reservoir curve is touching the limit occasionally. The actual reservoir curve breaks the HRV-limit in June. The optimized plan represented by the yellow line, never breaks the regulation limits.

Results C

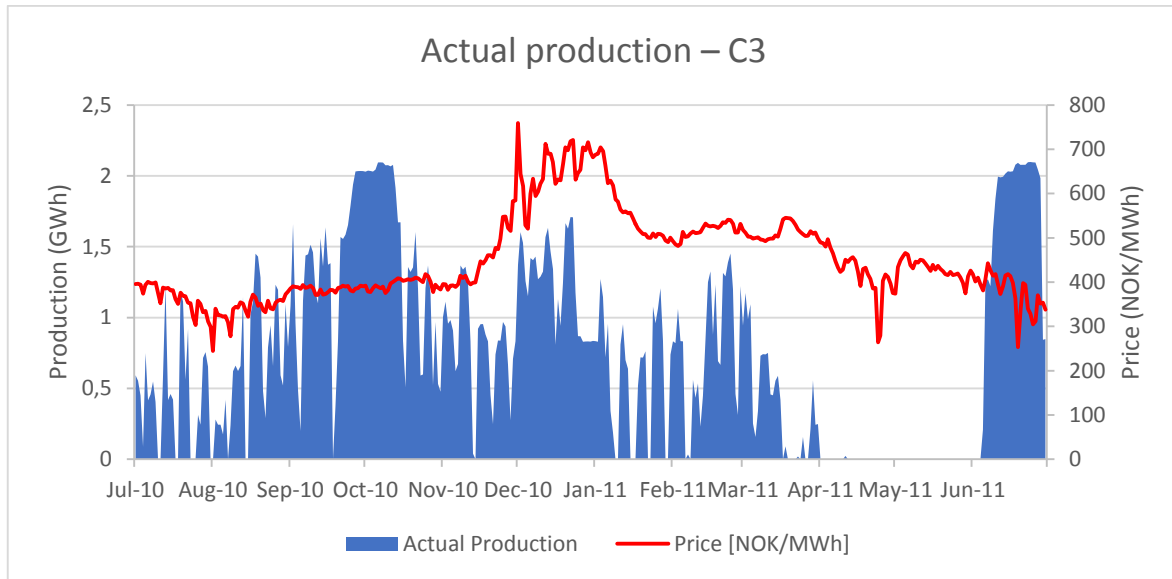


Figure 39: Actual production graph, Plant C, season 3. This figure shows the revenue-optimized production in MWh for Plant C in season 3. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Figure 39 shows the actual production for Plant C. During September and October, 30% of the seasons production is done. Almost no production occurs in April and May. June's production covers 16% of the total. Figure 38 shows where the actual reservoir inventory is breaking the maximum regulated limit. The revenue-optimizing production for plant C from July 2010 until June 2011 is shown in Figure 40. December and January covers 35% (99 GWh) of the season's production and 16% (44 GWh) of the production happens in September.

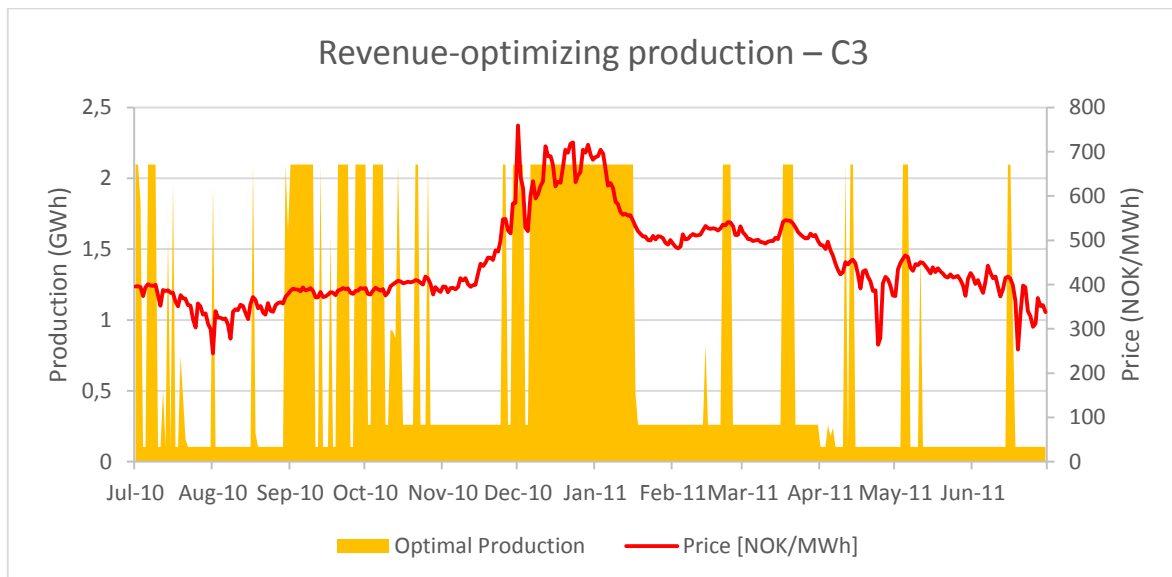


Figure 40: Revenue-optimized production graph, plant C, season 3. This figure shows the revenue-optimized production in MWh for Plant C in season 3. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.3.4 Summary – Plant C

The analyses performed, all showed an increase in revenue. As seen in Table 4 below, the model that optimizes revenue by changing the production plan had an overall increase in the revenue with NOK 37.1 million for Plant B. This is a 12% increase in the total revenue. The biggest change in revenue happened in season C1. The revenue optimizing function increased the revenue with 15%. The largest increase in revenue happened in season C3, where the revenue increased with NOK 13.8 million. Over the 3 seasons a total revenue for Plant C calculated to be NOK 308.1 million, and the new optimized production plan calculated the total revenue to be NOK 345 million.

Table 4: Estimated versus optimal revenue table, summary of plant C. This table shows the estimated revenue (Est. Rev. [mill. NOK]) and the optimized revenue (Opt. Rev. [mill. NOK]) for C1, C2 and C3. It also shows the difference between optimal and real revenue (Diff. [mill. NOK]) and the relative change according to the real revenue.

Analysis	Est. Rev. [mill. NOK]	Opt. Rev. [mill. NOK]	Diff. [mill. NOK]	Relative change
C1	86.6	99.6	13.0	15 %
C2	94.1	104.3	10.2	11 %
C3	127.4	141.3	13.8	11 %
Sum	308.1	345.2	37.1	12 %

Visualized mean production in Plant C with the average optimal revenue production is shown in GWh in Figure 41 below. It shows where the average actual production exceeds the optimized, and vice versa.

Results C

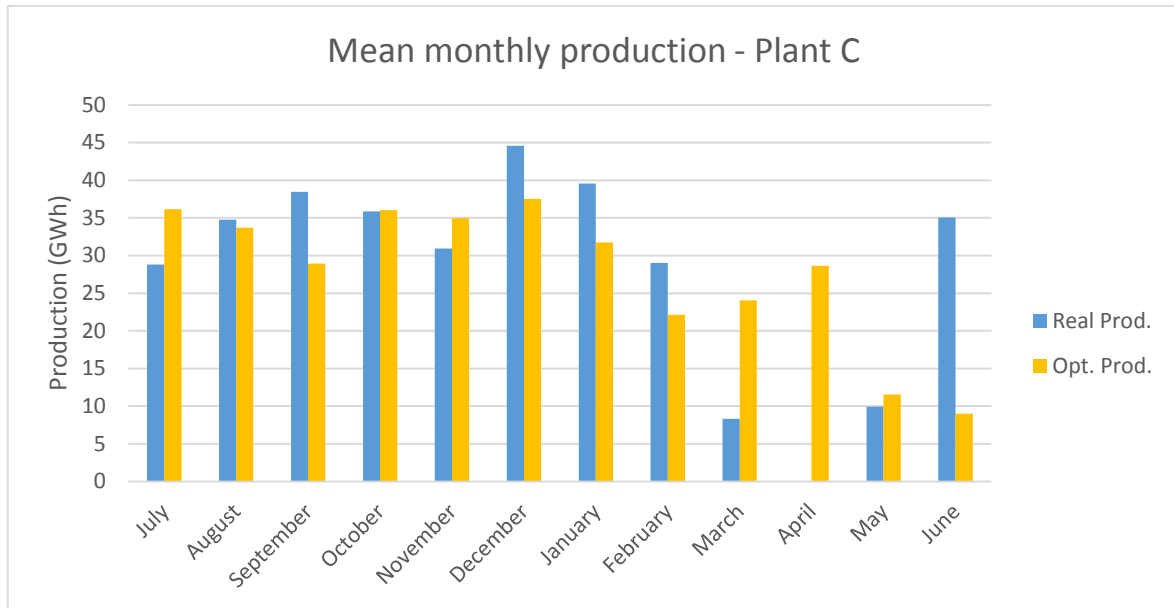


Figure 41: Mean monthly production graph, plant C. The graph shows actual production (blue) and revenue optimizing production (yellow) by average GWh per month. The results is based on data in the period 1/7/2010–30/6/2013

The average production run by the plant (blue bars), seen in Figure 41, exceeds the model's production (yellow bars) in September, from December to February and especially in June. The models revenue optimizing production exceeds actual production in March and April, as well as July.

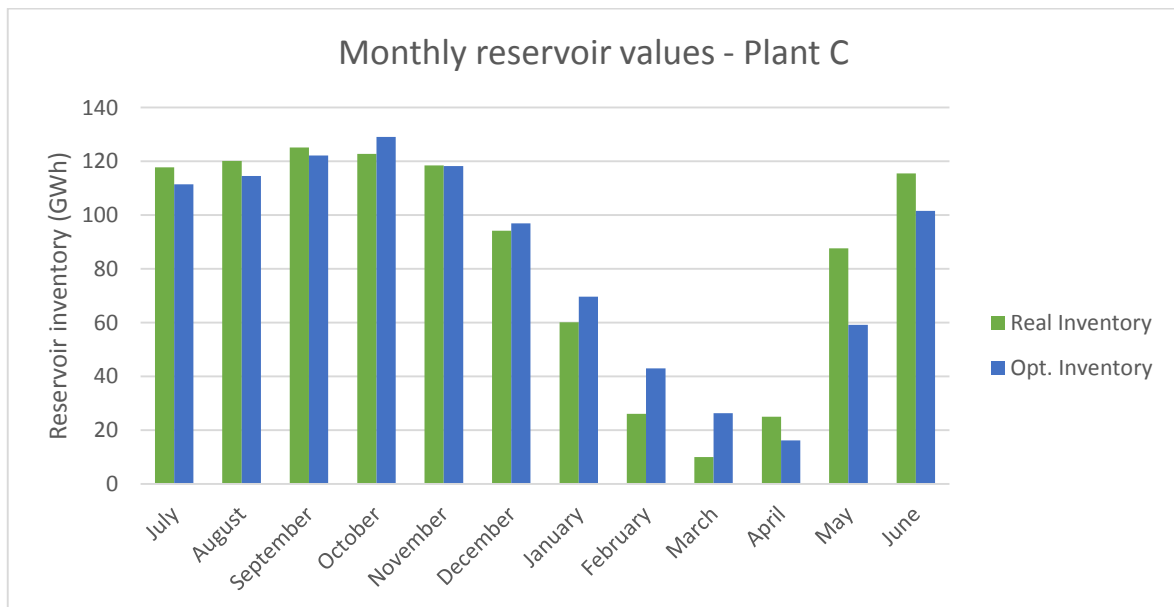


Figure 42: Monthly reservoir graph, plant C. The figure shows the actual mean reservoir inventory (blue) and revenue optimized reservoir inventory (yellow) per month.

The mean monthly reservoir supply for revenue optimizing model exceeds the average actual reservoir curve in the winter months. The mean inventory as seen in Figure 42 is higher for the real measurements than for the revenue-optimizing model in April, May and June.

4.4 Plant D

Plant D is located further north in Norway and is the most northern plant analyzed. Its location sets the plant within prize zone NO4. Plant D has one primary reservoir and one turbine. The plant has a capacity of 55 MW and average yearly production, based on the data received, is 249 GWh. The average monthly production from the data included is shown in Figure 43 below. Average yearly revenue from this data set is NOK 81.3 million. Figure 43 shows that the mean monthly production is highest in June, July and August ranging between 23 and 30 GWh. The mean price in the day-ahead market for NO4 peaks in February at 453 NOK/MWh.

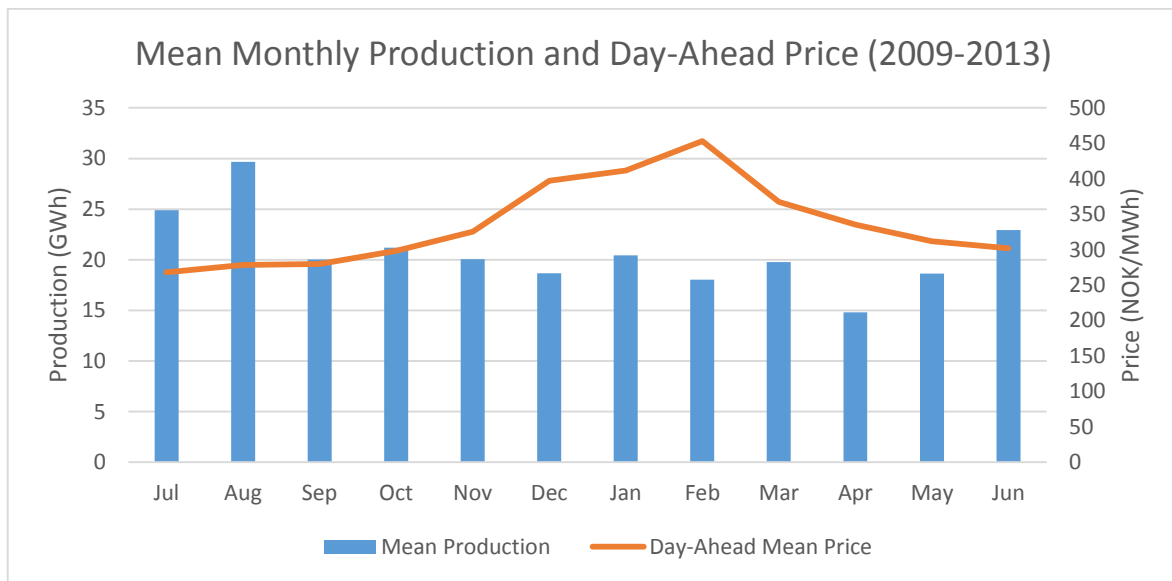


Figure 43: Mean monthly production and mean monthly price for Plant D. The production data is retrieved from the owner of the station and the price information is retrieved from Nord pool. Mean production is the left y-axis and the average price in the day-ahead market is on the right y-axis.

4.4.1 Analysis – D1 (1/7/2012–30/6/2013)

The model optimizes the revenue by changing the production for Plant D from July 2012 until June 2012 getting a revenue of NOK 65.5 million. This value is higher than the estimated actual revenue, calculated from actual production, giving a total of NOK 57.9 million. This total difference in revenue was NOK 7.6 million, a 13% increase.

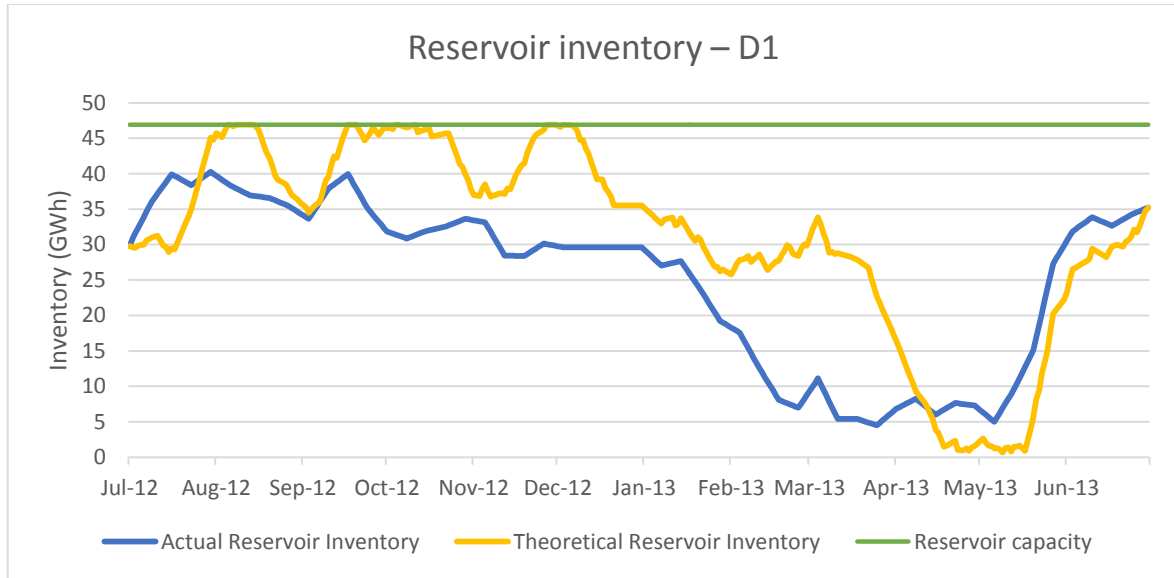


Figure 44: Actual and theoretical reservoir inventory for plant D, season 1. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

The reservoir curve shown in Figure 44 shows the revenue optimizing reservoir curve and real reservoir inventory. The theoretical reservoir awaits the depleting in the spring. The production data received was graphed below in Figure 45, showing the production over the season D1. Over the season, the production was spread in different periods. July through October produced 83 GWh equaling 36% of the total production.

Results D

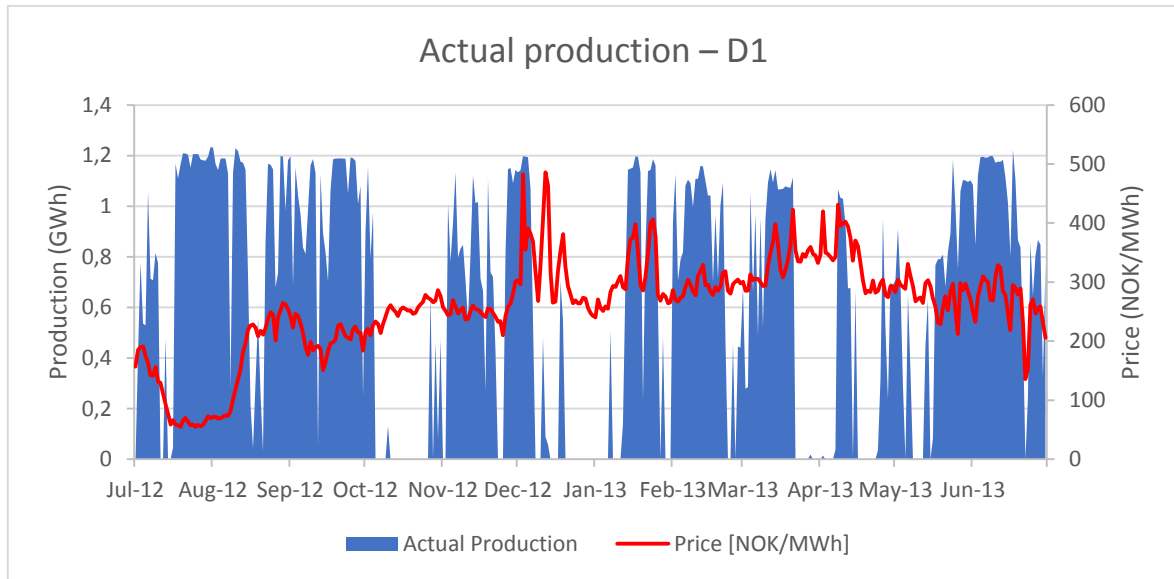


Figure 45: Actual production graph, Plant D, season 1. This figure shows the revenue-optimized production in MWh for Plant D in season 1. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The revenue-optimizing production for Plant D is shown below in Figure 46. The production resulting in the increase in revenue was distributed throughout the year, clustering in the early months of fall and late spring. Total optimized production was 231 GWh with the heaviest production of 47 GWh, 20% of the total, from 11th of March to 18th of April.

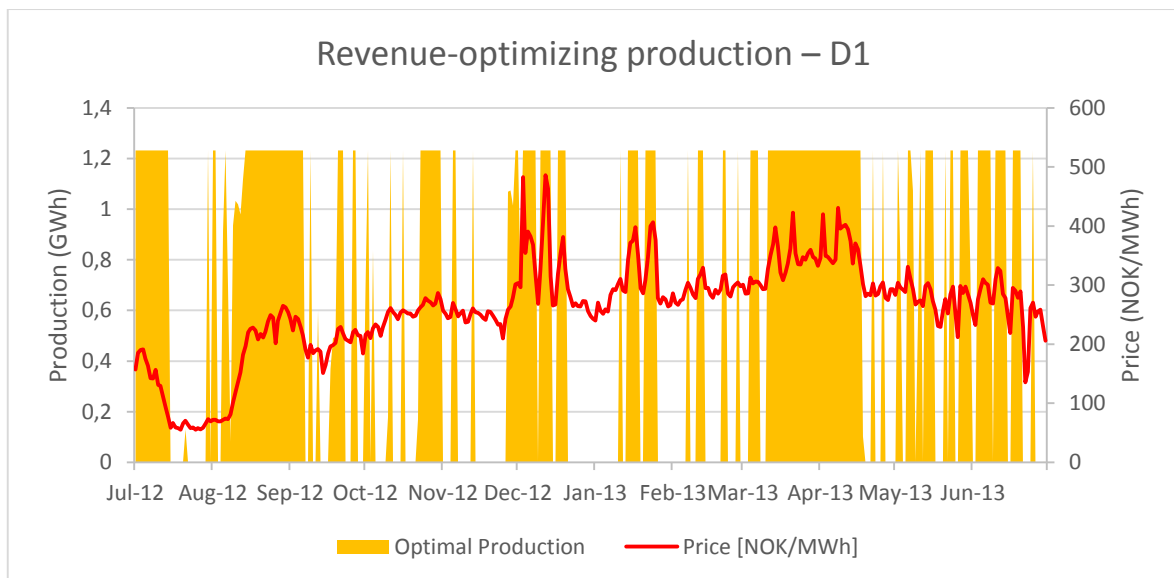


Figure 46: Revenue-optimized production graph, plant D, season 1. This figure shows the revenue-optimized production in MWh for Plant D in season 1. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Results D

4.4.2 Analysis – D2 (1/7/2011–29/6/2012)

Calculating total revenue of season D2 gave an estimated revenue of NOK 82.2 million based on production data received. Using the reservoir data from NVE, the optimized revenue was an 8% increase, a total of NOK 88.4 million. The production optimized for maximum revenue matched the actual production for this season, a total of 295 GWh. This difference is a total of NOK 6.2 million (8%) increase in revenue based on the solver optimization. In Figure 47 the optimized and actual reservoir curve is graphed, showing clear resemblance with some deviations.

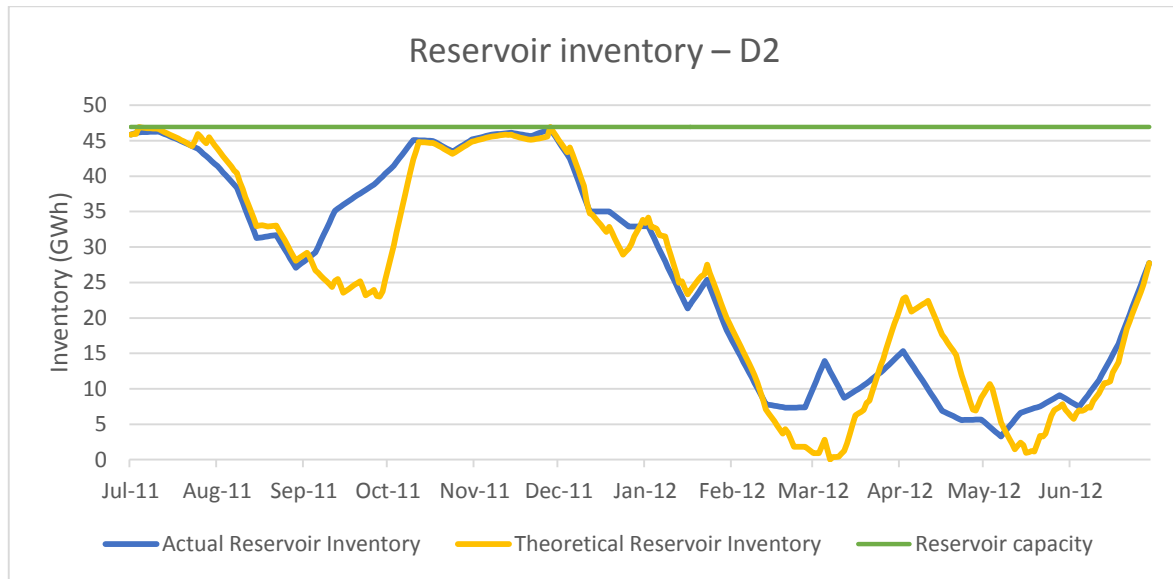


Figure 47: Actual and theoretical reservoir inventory for plant D, season 2. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

Below in Figure 48, actual production over the season D2 is graphed showing even production over longer periods of time. The region price is graphed as the red line, with actual production as daily columns. Production over several days as below is shown by larger clusters as from October to March. 51.4% of the production is within this time period, a total of 151 GWh.

Results D

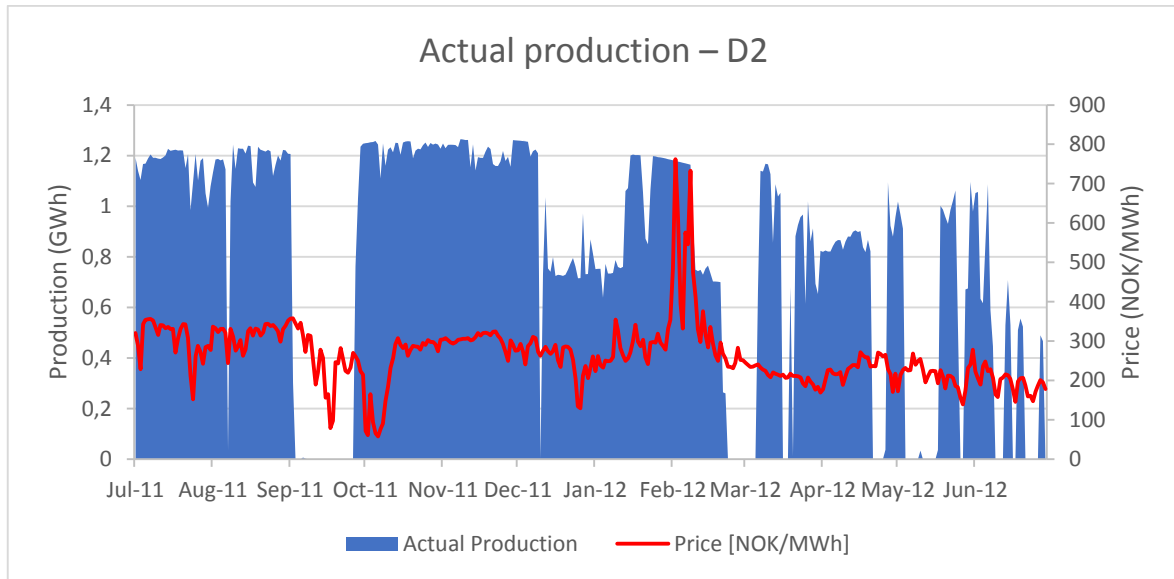


Figure 48: Actual production graph, Plant D, season 2. This figure shows the revenue-optimized production in MWh for Plant D in season 2. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

Optimized revenue is shown in Figure 49 below, showing similarity to the actual production timing. By the reservoir curves in Figure 47, this is as expected. A total of 50% of the total production occurred during October to March, amounting to 147 GWh.

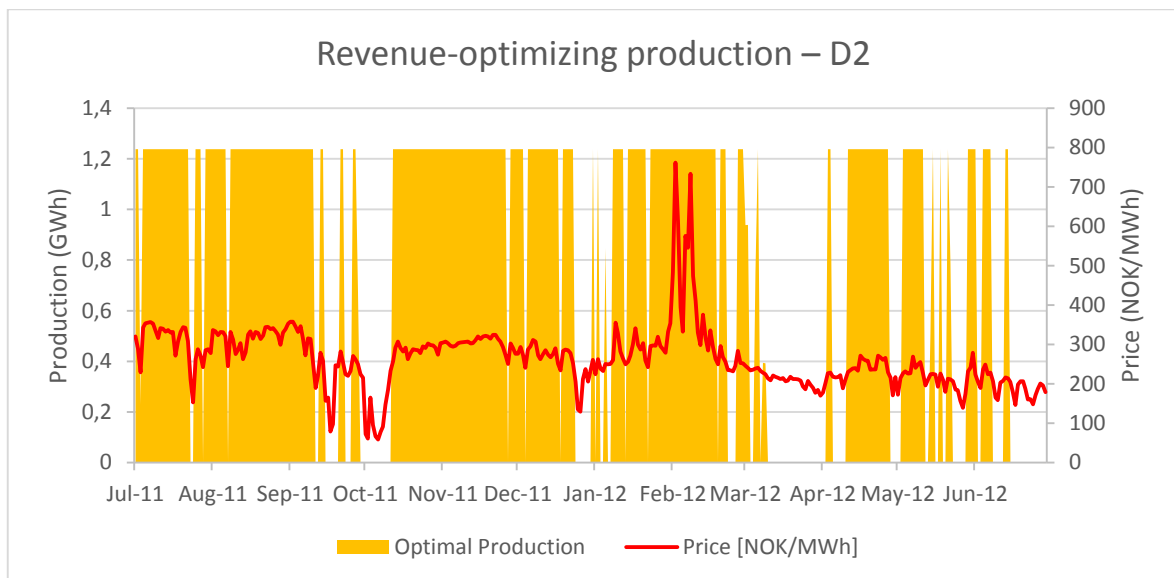


Figure 49: Revenue-optimized production graph, plant D, season 2. This figure shows the revenue-optimized production in MWh for Plant D in season 2. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.4.3 Analysis – D3 (1/7/2010–29/6/2011)

Optimized revenue gave a total of NOK 113.2 million from July 2010 to June 2011. The production data received from the owner resulted in an estimated revenue of NOK 100.2 million. This difference implies a 13% increase in revenue when optimized. The total production was 230 GWh for this season, while the production for optimizing revenue equaled 229 GWh. The reservoir curve of this actual production and the revenue-optimized production by the model is shown below in Figure 50. The yellow optimized curve has higher values the first half of the period, but drops below the actual reservoir line (blue) in December.

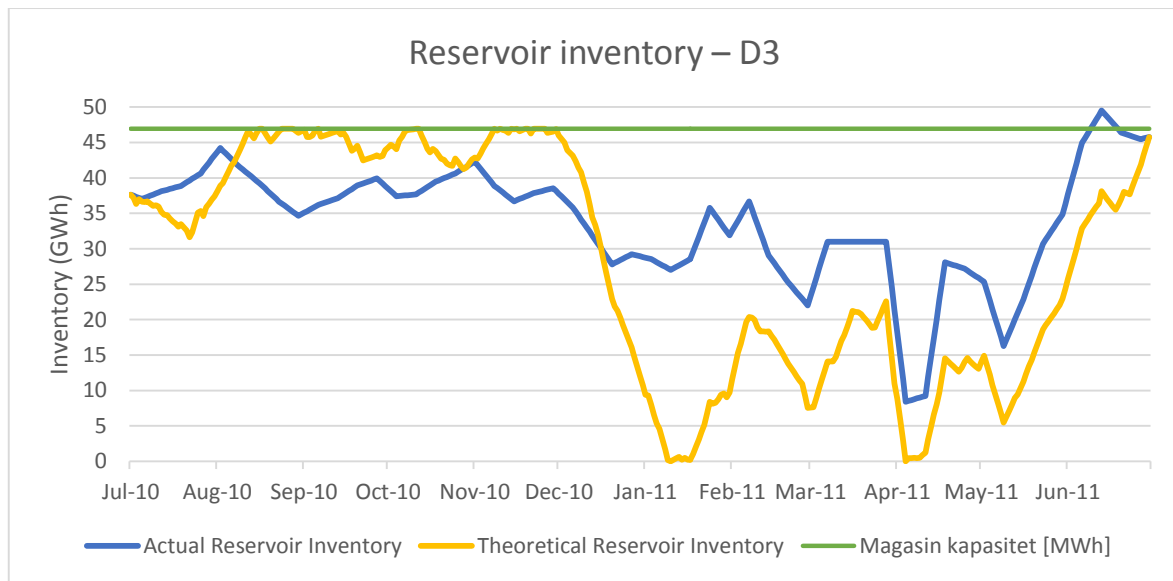


Figure 50: Actual and theoretical reservoir inventory for plant D, season 3. The blue line is the actual reservoir curve and the theoretical curve is shown by yellow in GWh. The green line represents the reservoir capacity.

In Figure 51 below, the actual production for season D3 is graphed by blue columns. The red line is the price per MWh in the region. The price does not vary much throughout the period, and the production seems rather unaffected by the changes. The main clustered production is in the end of the season, from the end of April, a total of 87 GWh. The price peaks December 14th with a staggering 1155 NOK/MWh.

Results D

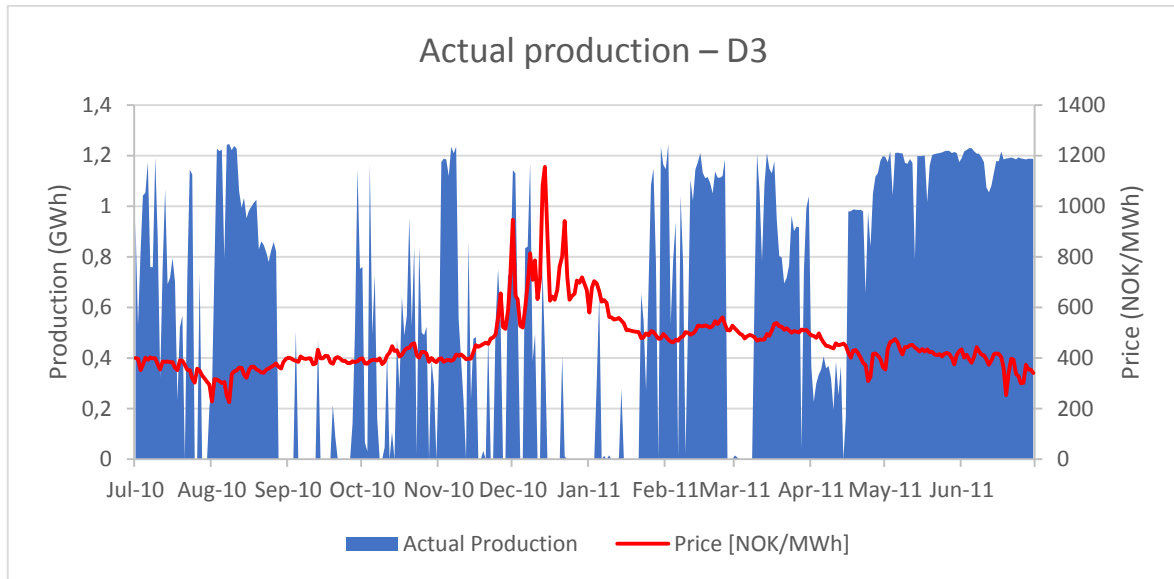


Figure 51: Actual production graph, Plant D, season 3. This figure shows the revenue-optimized production in MWh for Plant D in season 3. The production values are on a daily basis and the red line represents the price in NOK/MWh in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

The revenue-optimizing plan for Plant D during season D3 is shown in Figure 52. 37% of the production happens in May, June and July. This equals 84 GWh. 16 % of the production done from July 2010 to June 2011 is done in December, the month where the average daily price in the day-ahead market is highest.

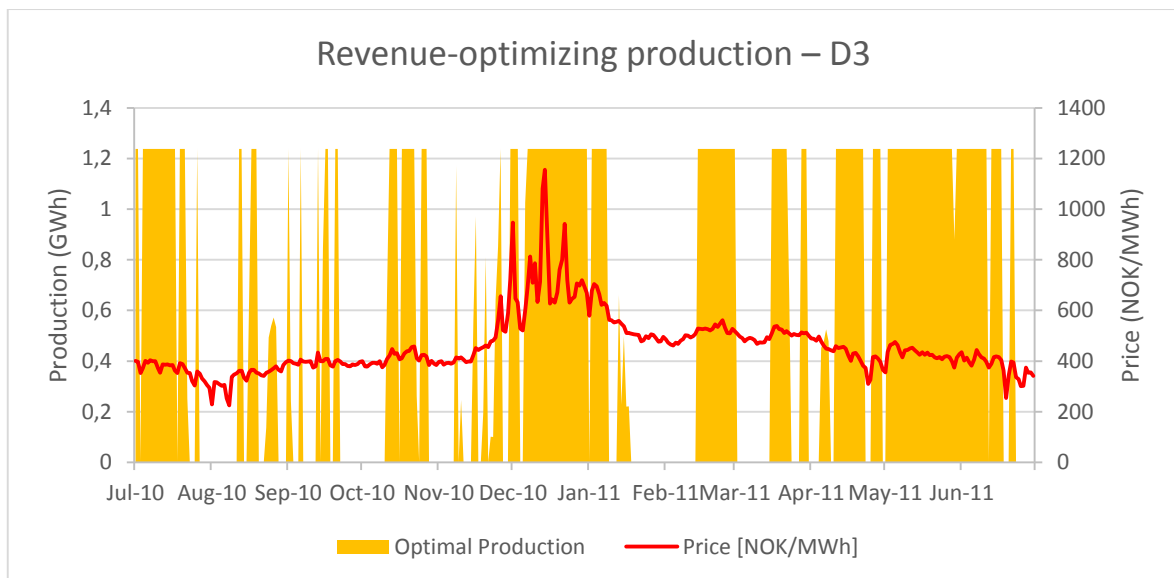


Figure 52: Revenue-optimized production graph, plant D, season 3. This figure shows the revenue-optimized production in MWh for Plant D in season 3. The production values are on a daily basis and the red line represents the price in the corresponding bidding area. The production is on the left y-axis and the price is on the right y-axis.

4.4.4 Summary – Plant D

With data acquired from the owner, Nord Pool and NVE, plant D was analyzed over three seasons. All of the analyses showed a higher revenue when optimizing revenue with the model. Table 5 contains the revenue of every season including the difference between estimated actual profit and optimized profit.

Table 5: Estimated versus optimal revenue table, summary of plant D. This table shows the estimated revenue (Est. Rev. [mill. NOK]) and the optimized revenue (Opt. Rev. [mill. NOK]) for D1, D2 and D3. It also shows the difference between optimal and real revenue (Diff. [mill. NOK]) and the relative change according to the estimated actual revenue.

Analysis	Est. Rev. [mill. NOK]	Opt. Rev. [mill. NOK]	Diff. [mill. NOK]	Relative change
D1	57.9	65.5	7.6	13%
D2	82.2	88.4	6.2	8%
D3	100.2	113.2	13.1	13%
Sum	240.3	267.2	26.9	11%

The percentage increase in each case is listed to the right in the table, and a final sum in the last row. The season with the largest increase in revenue was D3, with NOK 13.1 million. Over the three seasons, the optimization leaves a total difference of NOK 26.9 million, a total 11% increase in revenue. In Figure 53 below, average monthly production, both actual and revenue-optimized, is displayed. The blue curve representing the mean actual production for each month exceeds the production by the model (yellow) in the late summer months of June–August. In November the mean real production is higher than the mean revenue optimizing production, and in December it's the opposite.

Results D

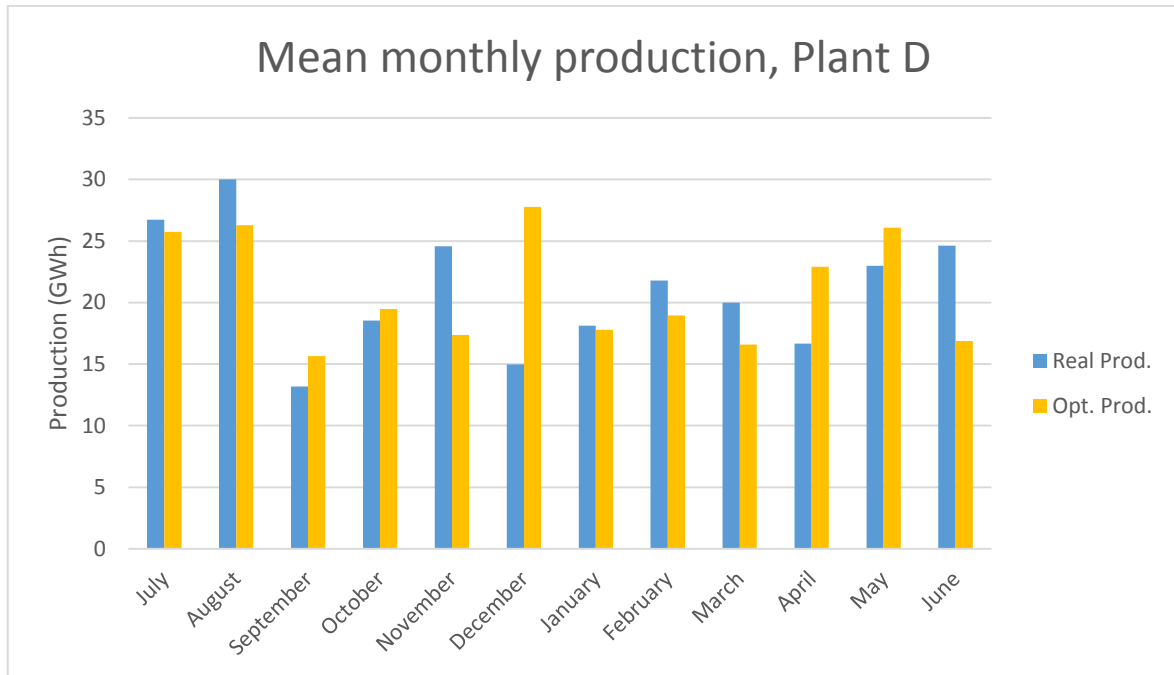


Figure 53: Mean monthly production graph, plant D. The graph shows actual production (blue) and revenue optimizing production (yellow) by average GWh per month. The results is based on data in the period 1/7/2010–30/6/2013.

The mean reservoir curve is shown in Figure 54. Both the actual plant and our model has similarities when distributing the water, but has some deviations found in the other plants. Our model saves water for a longer period of time in the fall, and awaits the water saving until May.

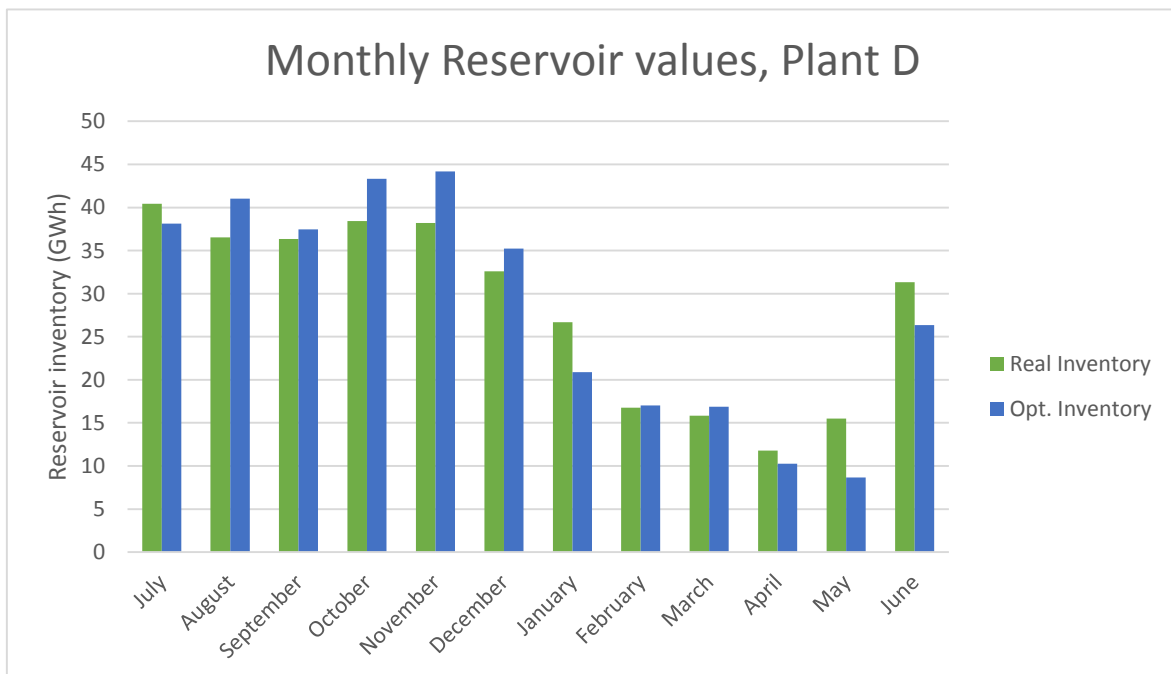


Figure 54: Monthly reservoir graph, plant D. The figure shows the actual mean reservoir inventory (blue) and revenue optimized reservoir inventory (yellow) per month

5 Discussion

5.1 Thesis setup, changes and choice of path

Since the beginning in early January, there has been some adjustments to the initial problem. All the changes made along the way seemed necessary in order to complete the thesis before the time limit in May.

5.1.1 Startup expectations

The beginning idea for this study was to divide the thesis in two parts: production comparison and prediction. This setup is similar to our final product, but there were some differences. We decided to focus our thesis on the run-of-river plants. The idea of dealing with the choice of storing water over longer periods, and the complexity of a drainage basin was agreed to be too much. We wanted to find how much of the energy in the water that was used for production purposes. Plants would be graded based on how well the performance was compared to the potential. The number of plants to be investigated were set to as high as possible, with a time period of 5 years. The second part was meant to include a prediction model for the plants, predicting production based on hydrological and meteorological data. The nearby weather and measuring stations were mapped and data availability was examined.

5.1.2 Collecting metrological and hydrological data

How the hydrological and meteorological data could be acquired, was uncertain in the beginning, but after some communication with NVE and internet research, we found NVE Atlas (NVE 2015) and the Norwegian Meteorological digital database (eKlima 2015). We quickly learned that public data varied with quality and accessibility.

Later, we got in contact with hydrologist Svein Taksdal at NVE. The hydrological measuring stations in Norway collected only water flow, and in some cases, air temperature (Taksdal 2015). This was a good way to limit the hydrological data to water flow only, and we got access to this data by email on demand. The accessibility of meteorological data was tougher than expected, but after a few days of research, we found eKlima, a digital database for Norway's meteorological data (eKlima 2015). By registration, we got access to almost all data measured by weather stations, though once again, the data quality varied. We learned that there are several types of weather stations, and they collect different types of data. Most Norwegian weather stations only collect precipitation, but some also collect more info, such as temperature and snow dept. Since most weather stations only measured precipitation, we decided to exclude the other metrological factors.

5.1.3 The software choice for data processing

When expecting large files with hydrological and meteorological data, we needed a particular software, which could identify correlations between multiple variables. Multivariate Analysis tools were something we were unfamiliar with, and with guidance with Knut Kvaal, we decided to use the PLS toolbox in Matlab (Eigenvector Research Incorporated 2015). This program in Matlab made it possible to identify the correlation between inflow in the run-of-river plants and the production.

5.1.4 Example data and new software

When receiving data from a run-of-river hydropower plant, we discovered some factors we had not thought of thoroughly. By graphing a year of production and water flow through the plant, we could notice great correlation between the two, but not with precipitation. We found that the correlation between daily flow data through the plant and the daily production data had an R-squared value near 0.95, but when comparing daily precipitation at nearby weather station with daily production data, we got nowhere near the same correlation and an R-squared value around 0.25. This made sense as the precipitation takes time getting to the plant, depending on the distance and topography. This problem forced us to include weighted time delay in the different variables included in the prediction model. When including precipitation, snow levels, temperature, wind, etc. from multiple weather stations nearby, the complexity of the assignment increased.

In the end of January, we realized that the second part of the study was too complex. With the prediction model based on metrological and hydrological data, the task was far too time consuming, and we decided to focus on the production analysis part. Excluding the prediction part implied that metrological data was no longer needed, simplifying the model. With counseling from Knut Kvaal and the president of PLS toolbox, we came to an agreement that PLS toolbox and PCA was not recommended for our use. The choice of program to be used fell upon Microsoft Excel.

5.1.5 Collecting data from the owners

Early in the process of collecting data, we discovered that getting hands on marked sensitive data could be challenging. In conversation with Multiconsult, we were warned about companies not always being willing to share information, and we agreed to construct a formal letter with a methodology attached to send to possible contributors, see Appendix A. During February, we contacted 35 companies informing about our project and asking for data. We learned that collecting sensitive data from several producers at once was not as easy as expected, and email conversation was surprisingly time consuming. The formal email with attached methodology constructed with Multiconsult was made to ensure professional approach and to inform about the thesis in a precise way, see Appendix A.

At this time in February, the focus of the thesis was still about comparing the production, the email therefore asked about production data over 6 years as well as general technical data of the plant. This was to ensure a complete 5-year production database, even if some contributors excluded the production from 2014. The methodology went into further detail regarding how data was to be collected and how theoretical production was to be calculated.

When we started to receive data from the contributors, we got varying sets of data. Some of the data was highly detailed and with a lot of information about the plant and reservoir, while others lacked information. The information received regarding the system efficiency varied from a value assumed by the staff, to a highly detailed turbine and generator efficiency varying with water flow and power output.

5.1.6 Plant selection and anonymous participants

All of the stations analyzed has different owners. In the beginning, we wanted a comparison of four plants with similar performance and different owners. Finding four plants with these requirements was though due to lack of data. The lack of reservoir measurements narrowed down our choices. This lead to a bigger difference in performance between the plants than what we originally expected.

The stations selected for the analysis should have an easy setup. An ideal setup is one reservoir and one hydropower station. If there were multiple stations and/or multiple reservoirs, this would complicate the situation. Multiple plants down the same stream could use an optimizing strategy combined for all the plants. This could involve letting water through the station without producing. If there were multiple plants using the same reservoir, the available inventory would be dependent on the production of the other stations.

All of the stations analyzed and run through the model is anonymous according to our agreement with the owners. In order to create a basic understanding of the plant, we chose to share some general information about the facility.

5.2 Data assessment

5.2.1 Technical specifications concerning plant and reservoir

All the technical data necessary for the thesis were retrieved from either the stations owners or NVE. The technical specifications were required to describe the power plant, and to set the proper limitations for the station. Performance, head and absorption capacity was both available through the owners and NVE. The performance value specified was assumed to include loss. The turbine and generator efficiency however had to be retrieved from the owners. With dataset including varying turbine efficiencies with water flow, we used the highest turbine efficiency and maximum absorption

capacity. Generator efficiencies were used as mean values of information received. Some of the owners did not supply us with this information, either because they did not want to share it, or because of the specifications was not available. If no turbine and generator efficiencies were given, both values was assumed 0.95. The turbine and generator represents the total system in this model. Other factors could be included in this system efficiency. One of these values is further discussed in 5.4.3.

5.2.2 Power production data and measurement certainty

The production data is not public information. To access this data we had to contact the owners of the hydropower plants. All the production data received is either in daily or hourly energy produced in megawatt hours and is collected in Appendix H. These values are the assumed energy supplied to the grid, and is therefore the value that reflects how much revenue the station acquires every day. Every dataset received had the highest accuracy of 0.01 MWh, which we assumed was the accuracy of every measurement. Hourly and daily datasets would therefore have different uncertainties when errors were added (Taylor 1997). Maximum error in the hourly dataset per year was 469 kWh, less than an hourly production in a power plant of size B. If the accuracy of the values were 0.1, the error would have been ten times larger. The error in daily production was summed up to 214 kWh, a significantly smaller value. These errors are small enough to be neglected in our calculations.

5.2.3 Elspot market price

The historical data was retrieved from the Nord Pool's data server (Nord Pool Spot AS 2015h). The day-ahead market in Nord Pool or Elspot, changes the market price every hour. Our model did not have the capacity to run the hourly price change for a full year and this made us settled with the daily average price seen in Appendix H. The variation in the daily price is further discussed more in 0. The accuracy of the price dataset was to the second decimal (0.01). Adding the error of the third decimal gives a total error of 0.469 NOK/MWh per year (Taylor 1997). The errors in production in price gives an error in revenue. With hourly numbers from the owner and Nord Pool, the uncertainty in the product is around 110 NOK, an insignificant value.

5.2.4 Reservoir measurements

The reservoir data was available through NVE in both meters above sea level and cubic meters. For the purpose of this model the daily measurements in cubic meters was the most relevant. The data was retrieved by email from Svein Taksdal, hydrologist at NVE and is found in Appendix H. Multiple reservoirs originally selected for this analysis did not have any measuring data available. The lack of reservoir measurements lead to be one of the biggest limitations in selecting the hydropower stations for this thesis. The reservoir measurements made by the owner are also often wrong, and

sometimes not detected by NVE (Taksdal 2015). The errors in plant B are easy detectable when plotted due to the rapid changes in reservoir inventory. The errors found did not have any major effect on the result; see discussion in 5.3.2. The accuracy of the daily measurements received from NVE was to the third decimal, giving an error of 500 cubic meters, which is insignificant in our calculations.

5.2.5 Inflow and losses

In order to find inflow and loss in the reservoir, the production data and reservoir volume measurements were required. The difference in the pool volume including actual production resulted in the difference inflow and loss as seen in Formula (7) on page 15. This value varied with plant location and time of year, often increasing positively during the snow-melting season. When the loss in the system was greater than the inflow the value turned negative. The reason for this value being less than zero several times a year, even in the winter could have multiple reasons. Sometimes the hydropower station gets really close to the upper limit of the reservoir. When this happens, the stations could let out water from the reservoir without necessarily producing power in order to keep within the reservoir's regulated limits (Koksæter 2015). This water would fall within the category of lost water. During days with higher temperatures, the losses in the reservoir could come from evaporation and infiltration into the soil to groundwater flow (Viessman & Lewis 1996).

5.3 Discussing the results

Some of the graphs in the results are hard to interpret, due to the complexity in the production pattern. The reservoir curves, the actual production and the revenue optimizing production is available in Appendix D, E, F and G. By folding them out they can be viewed while reading the discussion. The introduction to the results of each plant start with an overview of the plant and a graph displaying mean monthly production and average monthly price in the day-ahead market price zone where the plant is located. As seen in Figure 8 on page 18 the graph shows an overview of the distribution of actual production, in average values. Using average values to display the production pattern eliminates some of the yearly deviations in production. In Plant A's analysis, there is a small change in the actual daily production pattern (Figure 10 and Figure 13), but the main production is focused in October to May, which follows in the average production in Figure 8. In Plant B and C, the pattern in each season is well transferred and displayed in the mean value graphs (Figure 20 page 28 and Figure 31 on page 36), while in Plant D the production is more varying. With production distributed throughout the whole year with varying accumulation, the mean value of the production does not resemble the pattern well. The largest accumulation of production is observable in the mean value graph, as August in Figure 43, giving a small sense of the production distribution. In all cases, the increase in revenue was between 6-16%.

5.3.1 Discussion – Plant A

Due to the plant's specifications and the fact that the production is more accumulated than the other hydropower plants, the results including graphs are simple and clear. Several figures from Plant A's results can be found in Appendix D. The three reservoir graphs Figure 9, Figure 12 and Figure 15, have similar forms and are showing a common trend of saving water from May until October and producing in the remaining part of the year. In analysis A1 and A3, the start of spring production is moved from October to November/December. In season A2, the trend is the same, but the model produced in July-September due to the high price. This early production led to less water available in the winter months, shown in Figure 12, with less inventory than the actual curve. The model's choice of producing in the late summer months leads to increased revenue because the model knows the price will not vary much later in the season. When the reservoir curve gets close to the upper limit, and the price begins to increase, in September, the owner starts production, this may be because of price increase, but also because it is a high risk of losing water.

The value of the sudden reservoir drop in September is 780 MWh, a much higher value than possible daily production capacity. This implies loss in the power plant, possibly by the owner letting water through the plant to avoid breaking the HRV-limit or by routine by startup of the plant. The startup of the plant may require great amounts of water for a smooth run of the turbines, depending on plant design. November 11th 2012 in analysis A1, another sudden drop occurs in the reservoir curve exceeding production value. Leading to this drop, the inventory stays constant for four days straight, leaving questions of measurement error. This was a measurement error made by the owner, not detected by NVE (Taksdal 2015). Errors like these would be solved by linear interpolation. The error was not corrected and a new optimization was not run due to the late discovery of this error.

An example of the reservoir curve crossing the upper water level is seen in season A2 in Figure 12. In September, the actual reservoir curve closes up to HRV gradually, then exceeding. The owner then seems to release water in order to stay beneath the upper level. A similar curve is seen in the next season, A3. In Figure 15, the model-based reservoir curve hovers close to HRV for several weeks before descending. Being this close to the upper is risky and demands close attention to inflow. If the inflow continues without the prices being high enough to produce, the water will exceed the upper water limit (HRV) and the owner may lose water. This is where the model has the advantage. The perfect information of inflow during the season gives the model more time to halt production until the price peaks, and then start production leading to an increase in revenue. This halt in production is too risky for any realistic plant to follow, so for the model to be more realistic, one could introduce a risk percentage reducing the upper water limit of the reservoir. This risk factor could also be introduced when approaching the lower water limit (LRV), see discussion in 5.3.4.

Discussion

When summarizing the results in Plant A, the graph including mean monthly production illustrates the different production patterns. Both the production and the reservoir graph (Figure 18 and Figure 19) indicates that the increase in revenue occurs by halting production in September. The action of holding back production when the prices are high and the reservoir is full is risky, so the operator often start production early to avoid loss of water. In the seasons analyzed in Plant A, the production should have been halted for a brief month, to increase the revenue by 10%.

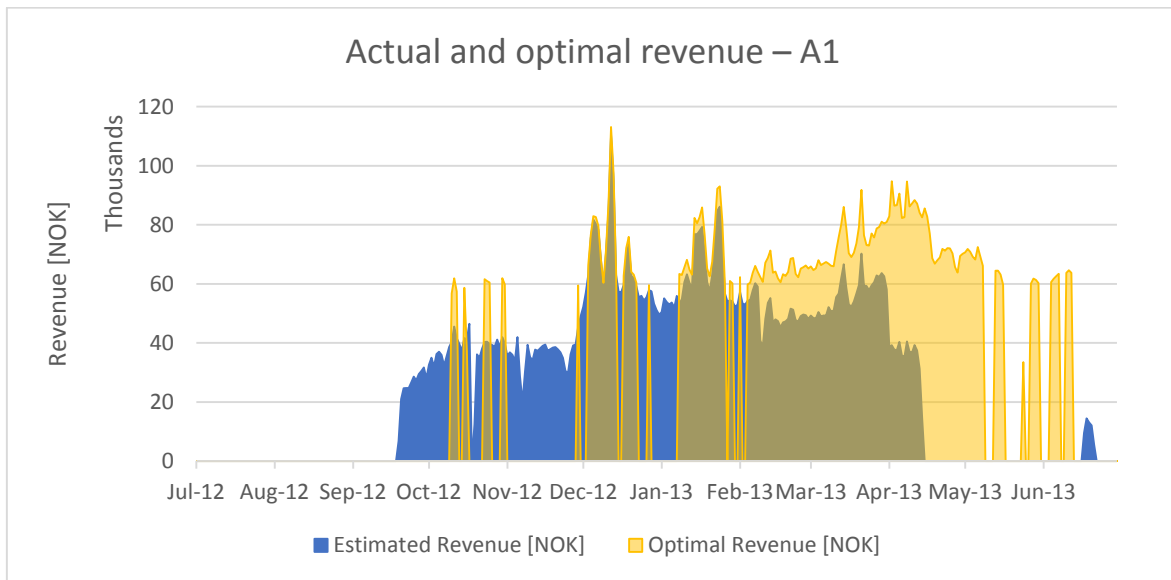


Figure 55: Actual and optimal revenue, Plant A, season 1. This figure show the daily revenue from the estimates and the revenue-optimizing plan in Plant A in season 1.

Figure 55 shows the daily revenue based on our estimates and the revenue-optimizing plan. This graph clearly shows how the difference in revenue between the two. The revenue-optimizing plan retrieves most of its revenue during the spring and the actual production gets most of its revenue during winter.

5.3.2 Discussion – Plant B

The mean production pattern in the introduction of Plant B is similar to the pattern of Plant A. It is weighted around the winter months where the prices are high, but some of the production takes place in the summer months as well. The actual production keeps a constant value for longer periods as seen in Figure 23 and Figure 27, with peaking production in November and January. Data was received in hourly resolution and then summed to daily values. The summation in Excel was later discovered to be incorrect. The daily values included an extra hourly value from the previous day, leading to incorrect production values. This error in daily production lead to further miscalculations in reservoir inventory and estimated revenue.

Discussion

Optimizing the revenue in the model ended in a production plan with great similarity to the original production, accumulating in the winter months and when the prices were high. When observing Figure 29 the actual production in B2, the market price is highest during March and April. In this season it would be best to move some of the production towards spring. In B1, the price decreases during the spring. In this scenario, it would be unwise to save water during winter. The production during the summer could originate from the fact that the power plant has a minimal water flow from the reservoir or minimum production to the grid. Fish and other animals depend on the river to live, as well as certain plants by the riverbanks. This minimum water flow through the power station is applied as minimum production throughout the year as seen in Figure 28. Whether this water flow is suitable for production purposes varies with different plants, but if the flow is large enough, it is possible to use the water for production (Koksæter 2015).

In some cases, there could be requirements to keep a minimum production in order to supply households etc. As seen in the actual production graphs in Figure 23 and Figure 27, the plant has a varying least value production being higher than the energy value of the lower water flow. The minimum water flow may not be enough for the production to be effective, so the plant increases the flow to increase the production value. Since this has to be continuous though the whole year, there may be significant revenue losses if the production is in a period with low prices. The production graph from the plant seem unaffected by the price changes in the periods with lower production in the first season, but there are signs of variations in the second season. In B2, the plant shuts off the production completely leaving no other signs of minimum water flow, except by looking at the inflow and loss values. The owner explained that if we wanted the generator output we should increase the daily production values by one percent. The low and negative production period could be a maintenance shut down.

From the data retrieved from the owner, two seasonal analysis were possible (B1 and B2). The reservoir curves shown in the analysis of Plant B are varying significantly. The first season (Figure 22) gave a reservoir curve with the revenue-optimized reservoir curve, shown in yellow, significantly below the actual curve. In the next season, it is the other way around. In Figure 26, the yellow curve exceeds the actual curve, saving more water until the end of the season. When combining these two reservoir graphs into the average reservoir graph as seen in Figure 30, the values from the two seasons eliminate the large gaps, leaving a smoother curve. This gives a false impression of the similarity of the two curves, cancelling the differences.

The reservoir capacity was used as upper reservoir limit hovering high over the actual reservoir curve. Since the actual reservoir curve never crosses 27.5 GWh, we have reason to believe the upper

water level is incorrect. It seems the high water levels trigger sudden production and possibly overflow. The reservoir capacity may contain parts of the drainage basin not included in the reservoir. With a lower HRV at around 27.5 GWh, the reservoir curve created by the model could be different and maybe more similar to the actual curve. Further data received from NVE later confirmed this error (Taksdal 2015). Upper water level was 40 million cubic meters, around 27.5 GWh. This opens up the opportunity of the remaining upper water levels for the other plants being incorrect.

5.3.3 Discussion – Plant C

Plant C has four turbines and the possibility to store 43% of its average yearly production. This means that the 57% of the plant's production is depending on the inflow during the season. In Figure 31 shows that the mean production is highest during December and January even though the mean price in the day-ahead market is highest during February. Naturally, one might think that production would be highest when the price is highest. In the same figure shows that the mean price is lowest in September. Again, it is natural to ask why the production is so high during this month. Even though the price is higher in March, April and May, this is where the mean production is lowest for Plant C.

The answer to this might be in the reservoir curves. C1, C2 and C3 shows that the reservoir is almost empty during these months. When the reservoir is almost empty, the only available water for production is the inflow. This is not a good situation, as it does not give the power plant any room to exploit sudden increases in the market price during March, April and May. It is probably the owner's interest to extract the entire reservoir inventory during the winter months, as this is usually where the price is highest. This mentality might lead to an early start of the winter production, and an empty reservoir later in the spring. C1's reservoir inventory shown in Figure 32, clearly shows this.

The reservoir starts decreasing in late November and is steadily reduced until March, where it almost hits the LRV-limit. The optimal production plan does not have this drastic reduction in the reservoir inventory. By saving some of the water until mid-March, the price increase seen in Figure 34 is exploited. The question is always when Plant B should start the winter-production in order to maximize the revenue and at the same time keep within the regulated limits.

Increased regulation during summer makes this even harder. At a certain point, the plant has to start saving water in the reservoir to get above the narrowed limit, starting June 15. In C2 and C3, the power plant made much better decisions as they started producing early. Due to no increased price during the spring, there was no reason in holding back the production. This is also reflected in the relative change in increased revenue seen in Table 4 on page 13 where both C2 and C3 got much closer to the maximum revenue possible compared to C1.

Discussion

The mean production is relatively high in June when the price is lower than March, April and May. This is mainly due to the increased regulation in the summer. In all three analyses, the plant is forced to start production in June and July independent of the price, but mainly to keep within the regulated limits.

Plant C had a minimum flow of water. In our model, we have selected that this water could be used for production purposes. In reality, however this need not be the case. The minimum flow of water could be required to exit from the reservoir not the power plant. If this is the case, the water cannot be used in production as the water leaves the reservoir outside the pipes leading to the plant. The real production shows that there is no production happening during some months. This could indicate that the minimum flow of water cannot be used for production. Another possibility is that the facility has gone through renovation of some sort that requires the production to stop.

5.3.4 Discussion – Plant D

The data received from the owner of Plant D was on an hourly basis, being summed to daily production data. Values of the production are the same, but the uncertainty differs from when receiving daily production values directly, as discussed in 5.2.2. The reservoir data received in millions cubic meters were measurements every fifth day. This may be caused by measuring device design or measuring routines. As with the measurement errors in Plant B, the data was linearly interpolated. This interpolation affects the values for inflow and loss used to create the theoretical reservoir curve used by the model. A complete dataset of the reservoir would give a more accurate reservoir curve and results.

Observing the reservoir inventory graphs in the analysis of Plant D in Appendix G, one can see the reservoir curve created by the model approaching the lower water level (LRV). The theoretical reservoir approach of LRV in the reservoir graph is too close for a risk free operation of the plant. By introducing the HRV risk factor as in Discussion – Plant A on page 61, the lower water limit for the model can be increased by a percentage. A 5 % value would make the model more realistic, since a live operation of a hydroelectric power plant will try to avoid crossing LRV. In the graphs, the lower water limit is at zero-value of the y-axis. This is because the reservoir capacity is measured from lower water level, not a fully dry, empty reservoir.

When observing the revenue increase in Plant D, the mean percent increase is 11%. This originates from the 13% increase in revenue in season D1 and D3 and 8% in D2, as seen in Table 5 on page 53. These values resemble a total of 27 million in revenue increase over the three seasons analyzed. When looking at the reservoir curves, the actual reservoir curve is similar to the model's, especially in

Figure 47 on page 49 in season D2. The plant follows the revenue-optimized curve relatively well and this can be seen in the production graphs.

Plant D has more on/off production than the other plants analyzed. As the production distribution in the model is distributed by price peaks, the actual production pattern is similar. It may seem the actual production is run a similar way as our model. By paying more attention to price changes in December, January and February, the plant could get closer to its maximum potential.

5.4 Critical factors excluded from the model

5.4.1 Rapid changes in production and efficiency variation

The production pattern of Plant C and D stands out because of production going from zero to high values in a day, then back to zero multiple times. This production pattern is not seen as much in Plant A and B, leaving questions of how the production is planned. Plant C goes from zero production the 4th of October, to 765 MWh the next day. Starting and shutting down production many times in a season may lead to a significant increase in mechanical tear (Hjorthol 2015). There is increased stress and tear on the metal turbines when starting and stopping the turbines frequently (Koksæter 2015). The turbine and generator efficiency varies with different inputs. If there are frequent changes to production, there should be a varying system efficiency included in the model. Turbines running for longer periods with no rapid changes, leads to a longer lifetime and a steady production. To reduce on/off production in the model, we added a constraint requiring the production to be within a 75% change from the production value the day before. With this constraint included, the model solving time increased significantly. The model used multiple hours to optimize the problem with the new constraint; therefore, the constraint was dropped. A different approach to reduce on/off production was to use weekly price instead of daily. This change would lead to a constant weekly price, removing the models option to follow rapid change between days. Optimizing Plant B's 2012-2013 season with weekly prices, some of the peaks were eliminated, leaving only the wider peaks in February–March 2014 as seen in Figure 24 on page 31. Removing the thinner peaks resulted in a NOK 0.2 million loss in revenue compared to the results in 4.2.1.

5.4.2 Variation in reservoir head

Potential energy calculations as in Equation (1) on page 6 depends on elevation, in this case, reservoir water level. When constructing the model, the variation in water level and its impact on energy extracted, was neglected. The model was designed to calculate energy levels from a constant head, H_a , but in reality, the water level is usually changing. There are regulations to the rate this water level is allowed to change (Koksæter 2015). The total potential energy in the reservoir-water varies with height. When the height difference within the reservoir is small, compared to the head, the effect of varying head is low. For larger effect, the height difference within must be large compared to the head. How much the level variation and head affect the energy equation are shown in Table 6 below. The table is a linear approximation between the lower and upper water level. The water close to the upper water level has really more value than the lower water level water, increasing the effect. The plants A – D are included in the table, highlighted by a solid border.

Table 6: Linearly approximated table showing the head variation effect on the potential energy at various heads.

Head [m]	Reservoir difference, from LRV to HRV [m]							
	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5
1000	0.5 %	0.8 %	1.0 %	1.3 %	1.5 %	1.8 %	2.0 %	2.3 %
800	0.6 %	0.9 %	1.3 %	1.6 %	1.9 %	2.2 %	2.5 %	2.8 %
600	0.8 %	1.3 %	1.7 %	2.1 %	2.5 %	2.9 %	3.3 %	3.8 %
500	1.0 %	1.5 %	2.0 %	2.5 %	3.0 %	3.5 %	4.0 %	4.5 %
400	1.3 %	1.9 %	2.5 %	3.1 %	3.8 %	4.4 %	5.0 %	5.6 %
300	1.7 %	2.5 %	3.3 %	4.2 %	5.0 %	5.8 %	6.7 %	7.5 %
200	2.5 %	3.8 %	5.0 %	6.3 %	7.5 %	8.8 %	10.0 %	11.3 %
100	5.0 %	7.5 %	10.0 %	12.5 %	15.0 %	17.5 %	20.0 %	22.5 %

Since the reservoir floor and surrounding tend to flat out by height, more water is stored per meter elevated. Higher water storage level will therefore lead to increased potential energy. By saving water close to the upper limit, the plants could therefore increase revenue even further due to the increase potential energy in the water stored. Figure 56 below, illustrates the increase in water volume by height. Depending on the reservoir walls, the water volume increase varies. With steep walls, the volume increases more linearly, but with gradual walls, the volume increases significantly.

Discussion

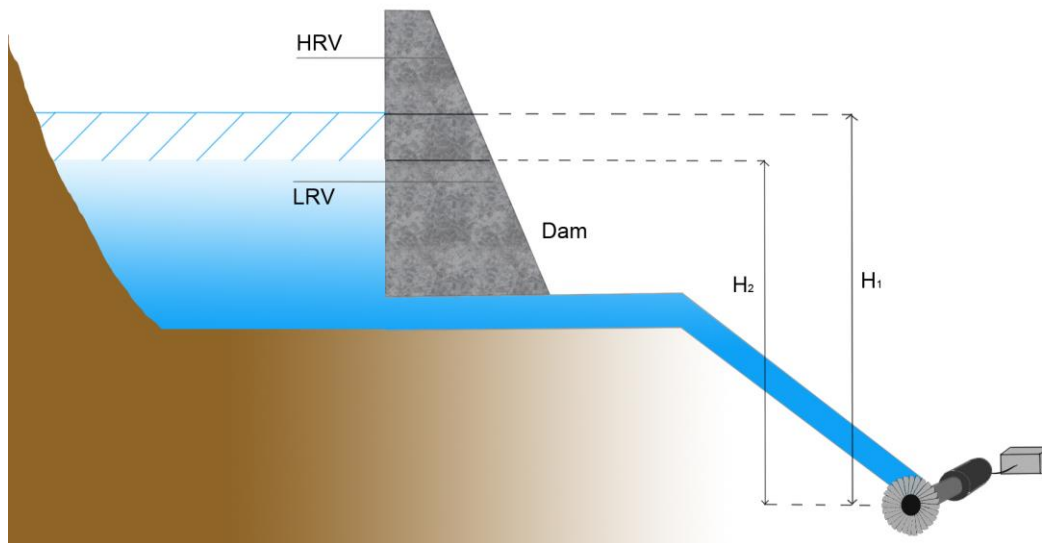


Figure 56: Illustration of varying height in a reservoir. Height differences is shown in the figure, storage volume increases with height depending on reservoir walls.

In Figure 56 above, the difference between the heads H_1 and H_2 resembles a volume increasing with height. The volume difference is larger per meter elevated in the dam. By taking an example in Riskallvatn in Årdal, we could investigate the relationship between cubic meter water stored and height above sea level.



Figure 57: Photo of Riskallvatnet 2009. The figure shows an image of Riskallvatnet and the reservoir walls in 2009 (Skrede 2015).

Discussion

The correlation is shown in the graph in Figure 58 below. Within the upper and lower reservoir levels, information was requested from NVE from 2009 – 2013, showing a smooth increase in volume per elevation height.

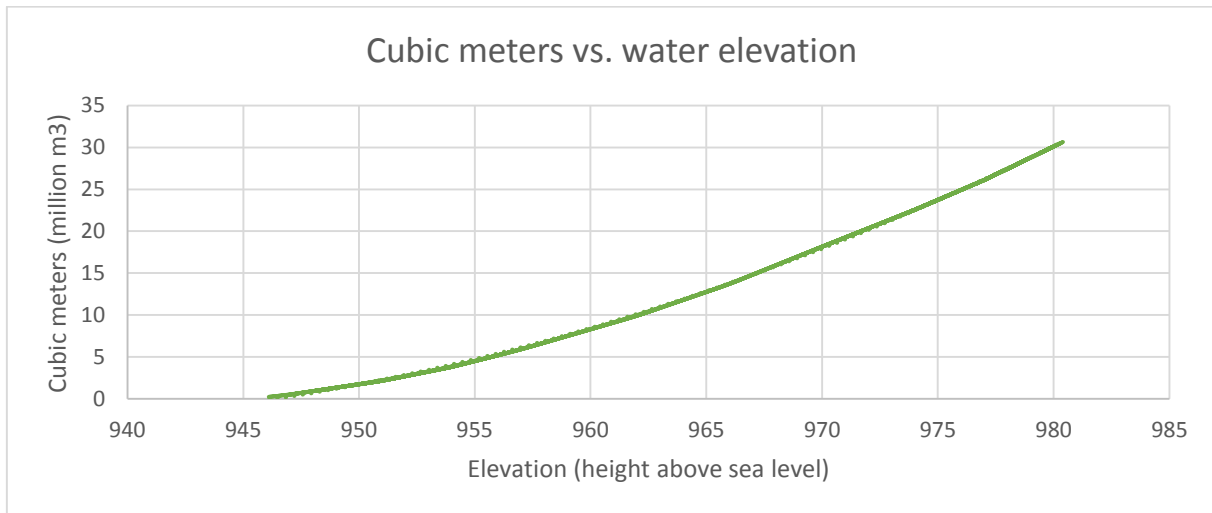


Figure 58: Cubic meter vs water elevation graph. The figure shows the correlation between elevation and cubic meters in millions. The elevation is above mean sea level

5.4.3 Pipe friction

As introduced in the theory section, the losses in the pipes, due to friction usually falls between 3–10% in Norwegian conventional plants (Koksæter 2015). When the head varies, the flow velocity varies with it. The friction is proportional to the length of the pipe, and square of the speed of water (Franzini & B. 2001). The friction working on the fluid is illustrated below in Figure 59, showing fluid closer to the pipe walls being slowed down. Fluid further away from the pipe walls have larger velocity, resulting in the average velocity, V_{avg} , shown below.

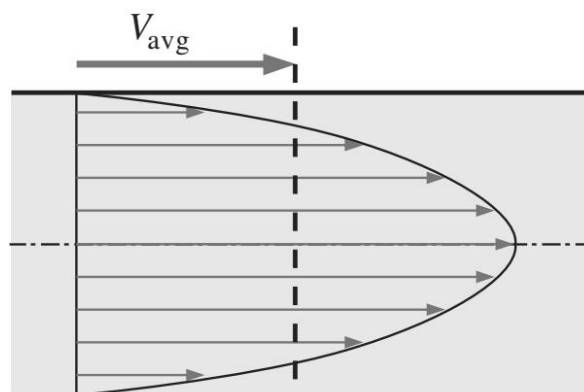


Figure 59: Pipe friction illustration. The figure shows the fluid velocity varying with the radius from the center. The maximum velocity is in the center of the pipe, and the closer to the pipe walls, the slower the fluid. (Çengel & Cimbala 2006)

Figure 59 illustrates ideal flow in a pipe, a laminar flow. In real pipes with varying roughness, turbulent flow can occur. Whether the flow is laminar or turbulent is pipe roughness dependent, and friction is higher in rougher pipes. Hydropower plants use different kinds of piping where roughness is given in the Moody diagram (Franzini & B. 2001). These factors of varying pipe friction and head are affecting the actual power production, and should be included in the model for more precise results. A constant factor in head variation and a factor of 5% in pipe friction is enough for a simple, fast model calculating a realistic potential (Koksæter 2015).

5.4.4 Selecting the right market

The revenue estimates is based solemnly on the prices Elspot market. This is probably not a true estimate as the owner could sell its power in several markets. Owners could for example sell 50% of their production in the future market in order to guarantee a certain profit. Future contracts ranges from daily, until annually, where the maximum is 6 years. The future price starts out with a lower price than the expected Elspot price, because of the risk involved with carrying these types of contracts. However, these expected prices could be higher or lower than the actual price in Elspot, due to unexpected changes in the market. The owners could also sell its power in Elbas. Elbas is the market where power is traded up until one hour before delivery. A market like this is good for leftover capacity, and for unpredictable power production like run-of-river plants and windmills. If the power plant has leftover capacity and wish to produce more than the agreed contract in Elspot, it could sell more power in Elbas. This is only an option when there is an increased demand. There is different types of contracts in Elspot ranging from selected hours and daily production. When the production data received is the total daily production there is no way of saying where the power is sold and in which market. The odds for the owner to sell all of its production at the average price in the day-ahead market is very low, but it is probably a good estimate as Elspot is the main market for energy trade, and according to the ministry of petroleum and energy, most of traded volumes goes through this market (Olje- og energidepartementet 2015).

5.4.5 Hourly price and production variation

In the model and its optimization, the production, price and reservoir curve are on a daily basis, but these values does not take into account changes happening during the day. The price data is the average hourly price per day, eliminating any variation in price happening during the day. Total production during the day is summed up to midnight, giving no information of the hourly distribution. The reservoir measurements received were the value measured at noon, even though the reservoir curve is measured hourly or with higher resolution (Taksdal 2015). Increasing the resolution would require more time for the model to process the data sets. We decided to investigate the hourly changes in price and production to get a better understanding and the results is found in Figure 60 and Figure 61. This was possible because we received hourly data from some of the plants, and the data from Nord Pool included hourly prices. The graph below is from Plant B in late May.

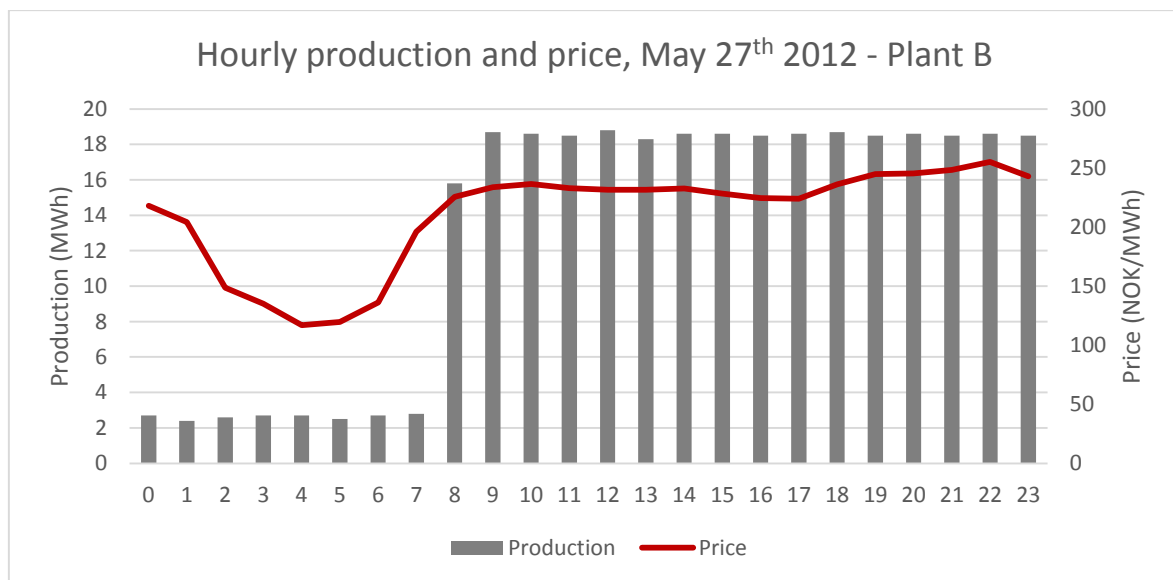


Figure 60: Hourly production and price, Plant B, May 27th 2012 Production on the left y-axis as grey and price on the right y-axis as the red line indicates the variation in production and price within a day.

By observing the figure above, there is a clear difference between production during the day and night. During the night, the hourly production stays between 2 and 3 MWh, and during the day it is around 18.5 and 19 MWh. The production clearly follows the price on May 27. Figure 61 shows the production for Plant B in October 2012, earlier in the same season. The graphs shows how the production varies by the hour, and is one of the reasons why the actual daily production is below its maximum capacity. With low production during the night, and higher values during the day results in a total daily production below maximum. As in Figure 60 the daily production summed is 350 GWh, being less than maximum. If all daily runs are like May 27th, the model assumed 350 GWh to be maximum. This error is eliminated if run the plant with full production for one day or more. This daily

Discussion

production value may explain the daily values in Plant A being less than maximum.

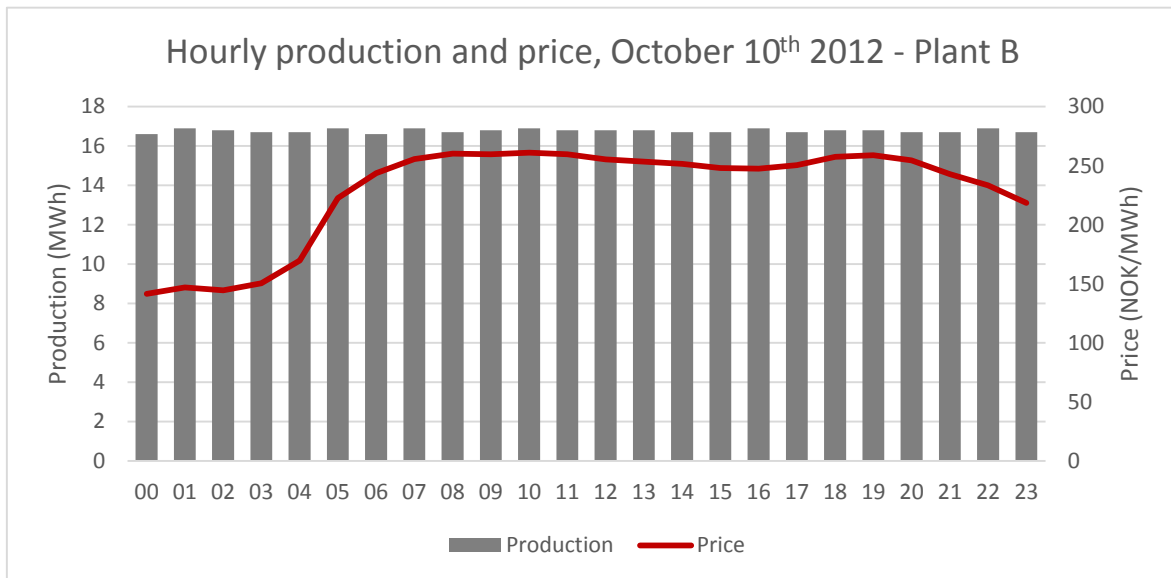


Figure 61: Hourly production and price, Plant B, October 10th 2012. Production on the left y-axis as grey and price on the right y-axis as the red line indicates the variation in production and price within a day.

In Figure 61, there is no clear indications of the plant following the price, but the production stays stable through the entire day. The plants do not necessarily run 80% through the whole day, but sometimes lower or higher. The introduction of a risk factor as mentioned in the discussion of Plant A, could contribute to a more realistic optimization by running at for example 90% the whole day. It would include hours of low production versus the hours with full power, resulting in a percentage of the maximum production.

6 Using the model for production planning

6.1 Retrospective analysis and production planning

Solver, the add-in used in excel, knows the actual limitations of every scenario with its perfect knowledge about the past seasons. It will therefore be able to find a better way to distribute production in a retrospective view. The information obtained from running the optimizations and analysis is a tool to find possible improvements in production during the season. Hydropower plant owners will gain valuable knowledge of how their plant, and could increase revenue by avoid similar mistakes in the future. The more optimizations and analysis done, the more detailed knowledge of possible ways for increase profit. The months and days with the biggest revenue losses can be discovered and possibly avoided in the future. Periods with the highest increase in revenue would also be easier to predict. The retrospective analysis is valuable for future production, but making a production plan is a different matter. The prediction of the next season's revenue and production could also be of great value to the owner.

6.2 Creating a production planner

In order to test the production plan, an example was made for Plant A, during the 2012-2013 season. The model was constructed to make a complete production plan for the next year's season. By pretending the date to be July 1, 2012, the model had no information about the future inflow, losses and price. In order to make a production plan that maximize the revenue, the model required a daily market price and the daily change in the reservoir due to inflow and loss happening during the 24 hours. Information from seasons prior to 2012–2013 were the only available data. The estimated prices used were the daily average price from the last three seasons (2009–2011). From the same prior years, the average inflow and loss were calculated. These calculations gave an estimate on how much water that would enter and leave the system each day. Figure 62 shows the estimates of monthly inflow and loss (blue line) and the actual monthly inflow and loss (orange) during the season.

As seen in Figure 62 on the next page, the predicted inflow and loss seems to follow the same pattern of the season as the actual inflow and loss. May is the month with the biggest difference. The estimated value only expected less than half of the actual inflow and loss. With both price, inflow and loss projection a revenue-optimizing production plan was possible. No other changes were made when turning the retrospective model into a model for production planning. The same constraints mentioned in the Modell construction chapter still applies for this model.

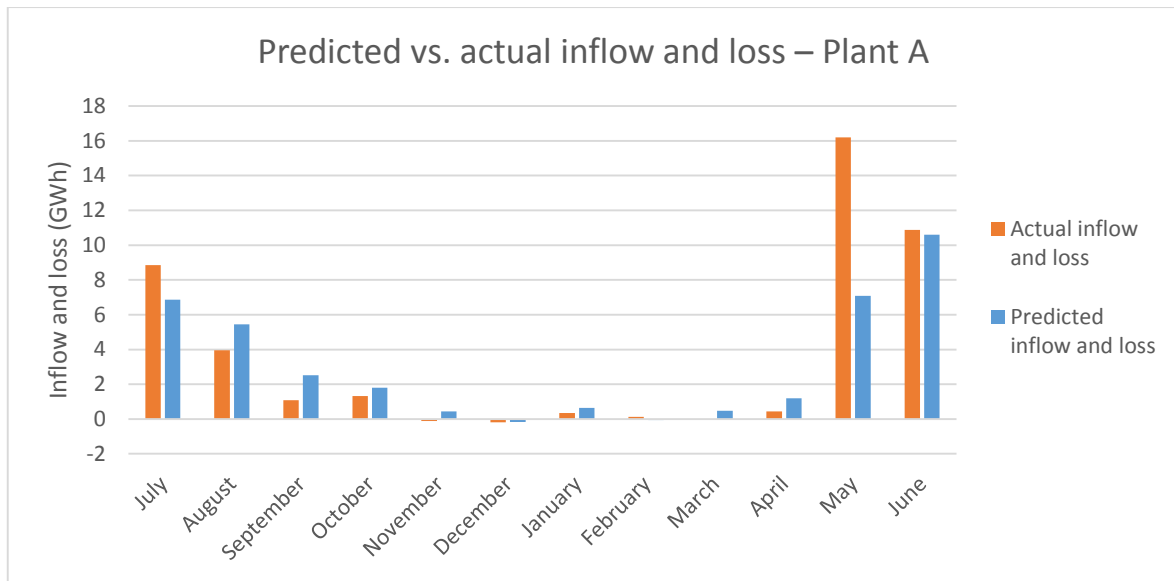


Figure 62: Predicted vs actual inflow and loss on a monthly basis, Plant A, 2012–2013. The graph illustrates both the monthly estimated inflow and loss, and it shows the actual monthly inflow and loss.

6.3 Testing the production planner

Graphed below in Figure 39 is the production plan, the predicted price and the actual price. The red line, display the daily average prices in Elspot during the season. The blue line shows the predicted mean price for the day-ahead market. The production is displayed by the stacked yellow columns.

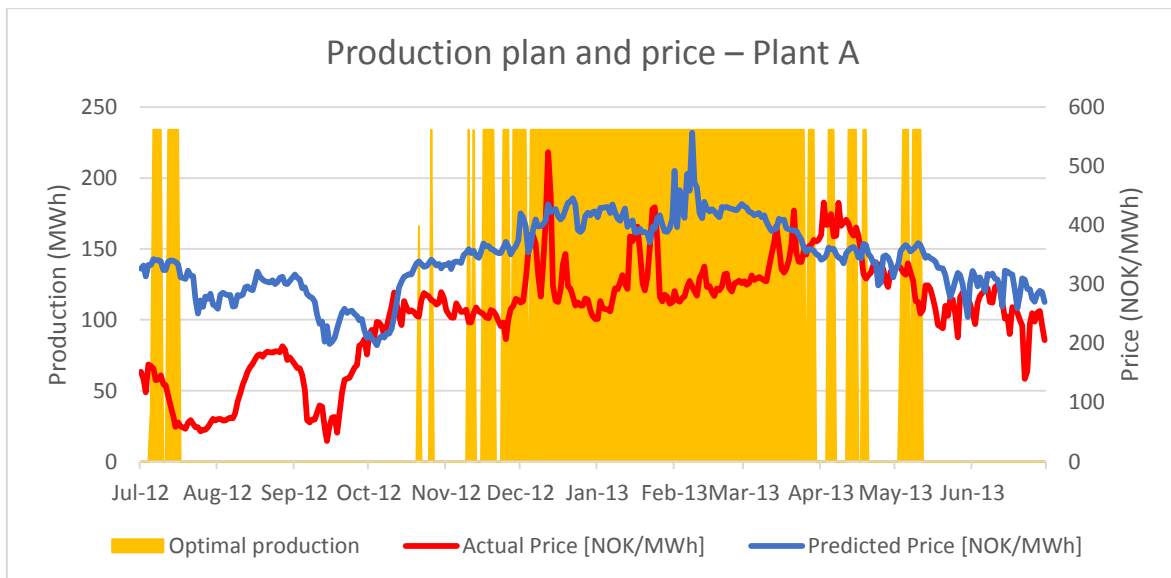


Figure 63: Production plan with predicted and actual prices in the market, Plant A, 2012–2013. The figure shows the production plan for the season (yellow), a price estimation calculated by the daily average Elspot prices from 2009–2011 (blue), and the actual price for 2012 (red). Prices are on the right y-axis and are given in NOK/MWh and the production is on the left y-axis given in MWh.

Production planning

The average price curve peaks in the winter months, gathering the majority of the production in this period. Other peaks throughout the year, as in July, leads to production, if high enough. The actual price for 2012 peaks later than the estimated price and creates a larger gap in the months of July to October. For comparison, the actual production by the plant is shown in Figure 10. The reservoir curve generated by the mean inflow and loss is shown below in Figure 64. When optimizing revenue based on this theoretical reservoir curve, a new curve is generated using the same planned production, but with actual inflow of 2012-2013. This resulting reservoir inventory is graphed in black in the same figure below. It follows the predicted curve until the end of May, where it crosses the lower water level.

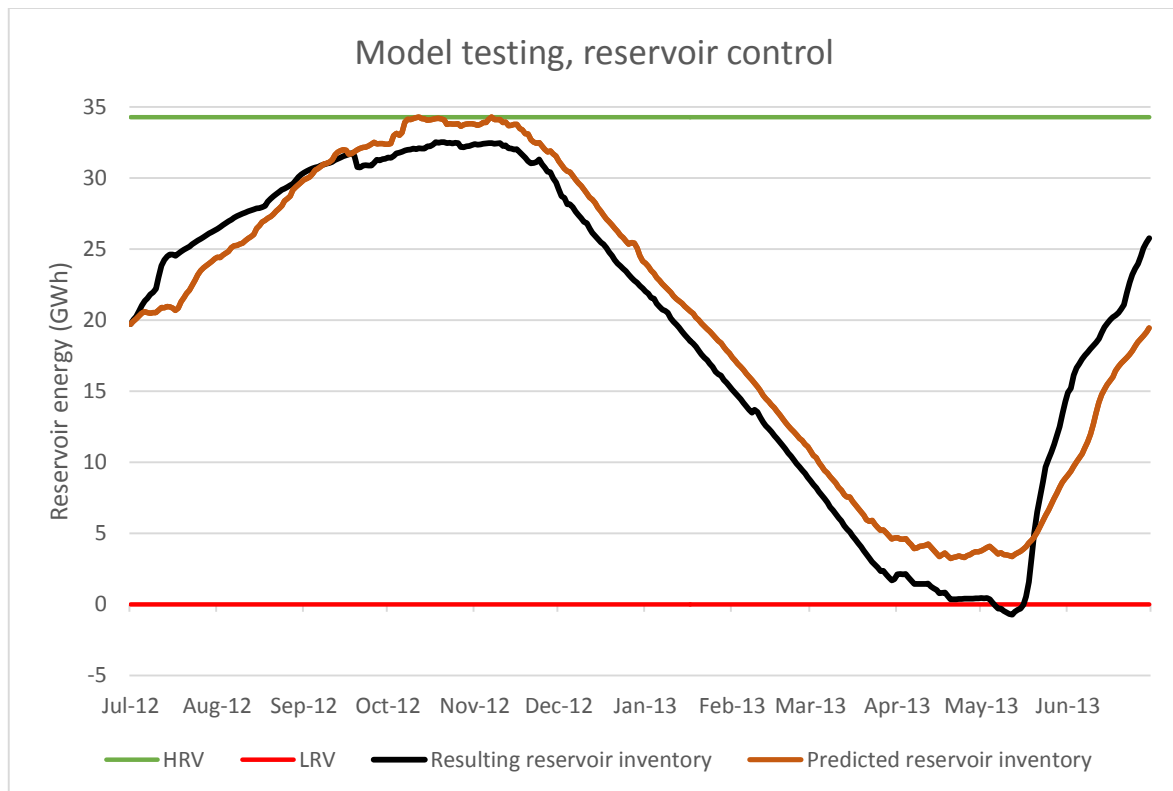


Figure 64: Model testing, reservoir control, Plant A. The graph displays the reservoir curve generated from mean inflow and loss (brown) and the resulting theoretical reservoir (black) based on actual inflow. Upper water level is shown by green line and lower level by red. The unit is in potential energy (GWh) per day.

When following the production planned by the model, the total revenue based on the price in 2012 was NOK 11 million, as shown in Table 7 on the next page. The estimated revenue based on actual production was 10.2, meaning an 8% increase in revenue by using the model.

Production planning

Table 7: Production plan results, plant A, 2012. The table includes values of estimated revenue for plant A in 2012, the revenue based on the production model and the difference between them. The increase in percent is included in the far right column.

Estimated Rev. [mill. NOK]	Model. Rev. [mill. NOK]	Diff. [mill. NOK]	%-change
10.2	11.0	0.8	8%

6.4 Discussing the results

When running a theoretical prediction model, all the future factors are unknown. The predicted reservoir curve may not correspond with the reservoir generated by the production pattern, and the price may vary that particular year. By following the production pattern suggested by the model, situations where the constraints being overrun are made possible. The production is distributed due to the average inflow, leaving room for errors when the deviations are large. As seen in Figure 64 the reservoir is forced underneath the lower water level. This is due to the fact that the average inflow is higher than the actual value (Figure 62), tricking the model in to believing water is available. The production to the far right in Figure 65 is the reason it breaks the constraint.

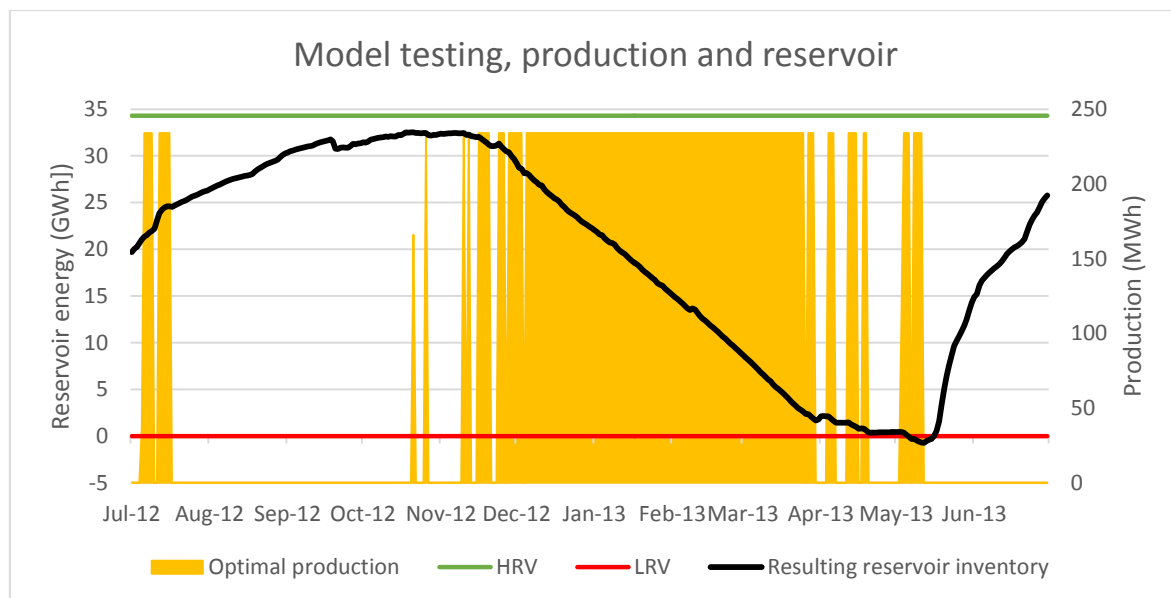


Figure 65: Model testing, production and reservoir, Plant A. The figure shows the revenue-optimized production distribution (yellow) combined with the resulting reservoir curve (black). The lower water level is shown by the red line and upper water level by the green.

The production pushing the curve beneath LRV is from May 5 until 9. It sums up to a value of NOK 0.3 million. Subtracting this value from the total revenue leaves a total of 10.7 million NOK, still a 5% increase.

7 Conclusion

By using historical data concerning the power plant, the reservoir and the market price, a model can be created to analyze production and optimize revenue. The model presented in this paper shows that all the plants analyzed could increase their revenue by distributing production differently. In the first hydropower plant analysis, Plant A had a possible revenue increase of 10%. All three analysis of A indicates an early start in production, emptying the reservoirs too early in the season. The revenue-optimizing model, analyzing Plant B, calculated a possible revenue increase of 11%. No trends in production pattern were found, possibly due to miscalculations and measurement errors. The results of Plant C showed a 12% possible increase in revenue over the three seasons analyzed. Plant C starts filling up the reservoir too early in May, being forced to produce in June in order to satisfy summer restrictions. In plant D, the final plant analyzed, the model showed a possible 11% revenue increase. Plant D produces close to the revenue-optimized production, but needs to take advantage of the sudden price change in the winter months. This model can be used for production planning, but with certain risks. Modifying the model by including risk parameters would increase the chance of a feasible production plan. The model presented in this study could be useful in both production analysis and planning, uncovering the potential in the hydropower plant, which could be of value for both current and future owners.

8 Future work

Our model's ability to optimize revenue and to make a realistic production plan can be increased by some improvements of the model. Mistakes and obvious improvements for the model were found along the way. We were able to adjust the model to some of our findings, but at a certain point we had to stop editing the model, and focus on the results acquired at that point. The changes we wish to see in a future version of the model is mention under this heading.

When calculating the energy stored in the reservoir, we used a constant head. Introducing a varying head, according to the reservoir water level would make a more accurate representation of the energy stored in the reservoir water. The friction in the pipes is a critical factor that we have neglected in our calculations. This is possibly the biggest error in this thesis, because of its magnitude. When including the friction in the pipe this would affect both the system performance and the energy available in the reservoir water. The pipe friction, the turbine efficiency and the generator efficiency are varying with different amounts of water going through the system. By making variable system-efficiency depending on the production could help minimize this error. Due to lack of processing power, our machines could not include this factor. A compromise would be to introduce the friction and efficiency as an approximated value instead of a variable depending on flow velocity.

By introducing additional parameters such as risk factor could improve the production planning model. If the upper and lower limits in the model were narrowed down, it would increase the chances for the reservoir curve to stay between its limits. The model should be a realistic prediction as possible leading to an increased revenue. Other factors that can be included are precipitation and temperature when estimating the inflow. If inflow data are more detailed, the prediction and analysis accuracy could gain increased accuracy. The precipitation parameter would have to be weighted by distance and temperature dependent, since snow is not input the same day it falls.

Our model is only represented with the option of selling at the daily average market price in Elspot, or storing the water for another day. In reality, there is several options, and these could be include in a future version. The option of selling in Elbas is a valid option under the right circumstances, and should be included in the model. In order to find a production plan that reduces risk, it could be of value to include the financial market. Selling future contracts would guarantee a certain income no matter how the prices change in Elbas and Elspot.

A future version should include the daily variation in price, as this is very relevant in power production. Increasing the resolution of the data will increase the data accuracy at the expense of model solving speed. This could be shortened by using computers with better software or power, or

shortening the analysis time span. By analyzing and predicting smaller periods at the time can give results with more accuracy. With wind and solar power introduced to the market, the production in hydropower must respond to this varying production. By linking the model to a database retrieving live data, the model can be updated daily, maybe hourly. Having an updated model can contribute when it comes to decision making within the plant, producing electricity when most needed.

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10 Appendices

10.1 Appendix A

Methodology

By Fredrik Sivertsen & Håkon Johansen Bakke

The purpose of this Master's thesis is to perform a technical analysis of hydroelectric power stations in Norway. The focus will be on the current technical condition of the plant, and historical data from the last 6 years on a daily basis. A database will be created based on the information retrieved. This information will then be used to create a model that performs a comparative analysis of the plants, based on their performance and other contributing factors.

The main focus of the thesis:

- **Creating a database with the most relevant data from the participating plants.**
- **Comparing theoretical production data and actual production data over the last 6 years on a daily basis.**
- **Comparing daily hydrology, weather and technical data with power production data.**
- **Making a model that compares hydroelectric power plants.**

Data will be collected from multiple sources including: NVE, Norwegian Meteorological Institute, Multiconsult, Nord Pool, and several owners of hydroelectric power plants in Norway. The number of participating owners is not yet clarified. Some of the main factors that will be included in the database are: Hydrology data, weather data, technical specifications, and actual power production of the plants.

Theoretical production will be calculated based on hydrological data and technical information from the plants. This calculation includes factors as efficiency, intake flow and head. Actual and theoretical production data within the plant will be compared on a daily basis over the last 6 years. Plant behavior will be analyzed and factors such as price index and seasonal floods/droughts may be included in the process.

The second part will be a comparison of the participating plants. The statistical analysis will be based on the power production. Common trends, operational stability and changes in performance during the time period will be pinpointed and further analyzed. This will be done through PLS_toolbox in Matlab, a statistical analysis tool from Eigenvector.

Formal email sent to power plant owners

Til ...

Refererer til nylig telefonsamtale om vår masteroppgave angående en energianalyse av Norske vannkraftverk. Veiledning av hovedoppgaven drives internt av NMBU og eksternt av Multiconsult.

Del 1: Teoretisk produksjon ved vannkraftverk blir beregnet via hydrologisk, metrologisk og teknisk data. Videre sammenlignes disse verdiene med den faktiske produksjonen til vannkraftverkene.

Del 2: Analyse ved hjelp av MVA (Multivariat Analyse) i Matlab. Trender og variasjon i produksjon samt driftsstabilitet kartlegges ved overordnet database.

For mer informasjon, se vedlagt metodikk.

Databasen og oppgaven forblir konfidensiell i 5 år ved Multiconsult og NMBU. Bidragsyttere holdes fullstendig anonyme og vil ikke refereres til. Resultatene fra oppgaven vil bli tilgjengelig for alle bidragsyttere.

Økt mengde bidrag sørger for en fullstendig og kvalitetsrik database over Norske vannkraftverk, vi setter derfor stor pris på deres bidrag til vår masteroppgave.

Kritisk data for oppgaven:

- Daglig produksjonsdata fra vannkraftverk i perioden 2009-2014.

Annen informasjon nødvendig til oppgaven:

- Teknisk data.
 - Anleggseffektivitet
 - Magasinstørrelse (aktiv m³)
 - Fallhøyde
 - Antall turbiner med tilhørende slukeevne
 - GPS – koordinater
- Driftsrutiner ved flom og tørke
- Kritiske reoveringsdatoer

Ved spørsmål rundt oppgaven, kontakt per telefon eller epost.
Takker på forhånd for bidrag til oppgaven.

Mvh

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Norwegian University of Life Sciences

Faculty of Environmental Science and Technology

Department of Mathematical Sciences and Technology - IMT

10.2 Appendix B

Lagrange and simplex optimization

In order to optimize a function over one or multiple constraints, the Lagrange method is commonly used. This kind of multi variable problem has one main function, which in our case is the profit function. The function is constrained by several factors, such as reservoir capacity or maximum water flow. The systems maximum and minimum points will be in the critical points in Lagrange Multipliers Theorem, assumed differentiable functions. The optimization problem, including multiple variables and constraints follows below (Sydsaeter et al. 2008).

$$\max f(x_1, \dots, x_n) \quad \text{subject to} \quad \begin{cases} g_1(x_1, \dots, x_n) = b_1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ g_m(x_1, \dots, x_n) = b_m \end{cases} \quad (m < n) \quad (9)$$

The function to be maximized is $f(x)$, with $g(x)$ as the constraints being equal to the constant b . The Lagrange multiplier, λ , are introduced with every constraint. The Lagrangian function, L , of the problem is written as seen in Formula (10) below (Bertsekas 1982).

$$L(x_1, \dots, x_n, \lambda_1 \dots \lambda_m) = f(x_1, \dots, x_n) - \lambda_1 g_1(x_1, \dots, x_n) - \dots - \lambda_m g_m(x_1, \dots, x_n) \quad (10)$$

Writing the n and m factors in Equation (9) and (10) as vectors can reduce the expression.

$$\vec{b} = [b_1, \dots, b_m] \quad (11)$$

$$\vec{g} = [g_1(x_1, \dots, x_n), \dots, g_m(x_1, \dots, x_n)] \quad (12)$$

$$\vec{x} = [x_1, \dots, x_n] \quad (13)$$

The vector $\vec{\lambda} = [\lambda_1, \dots, \lambda_m]^T$ is transposed. This is done to match the column number in the first matrix to the row number in the second matrix. The full expression with Equation (11), (12) and (13) can be written as (Li 2008):

$$L(\vec{x}, \vec{\lambda}) = f(\vec{x}) - \vec{\lambda} \cdot \vec{g}(\vec{x}) \quad (14)$$

The Lagrangian function $L(\vec{x}, \vec{\lambda})$ in Equation (14) now includes the vector \vec{x} , and the Lagrangian multiplier, $\vec{\lambda}$. The m constraints are given by $\vec{g}(\vec{x})$. Equation above differentiated gives equation below.

$$\nabla L(\vec{x}, \vec{\lambda}) = \frac{\partial f(\vec{x})}{\partial x_i} - \sum_{m=1}^k \lambda_m \frac{\partial g_m(\vec{x})}{\partial x_i} = 0, i = 1, \dots, n \quad (15)$$

In Formula (15) above, ∇L is the divergent of the Lagrangian and $\frac{\partial f(\vec{x})}{\partial x_i}$ is the partial differentiated of f . The Lagrange multipliers λ_m times the partial derivated g_m are summed up for k entries. The i numbers of x goes from 1 to n .

Inequalities

Some constraints and limitations does not demand the limit value to equal the constraints.

Constraints are often used as a maximum or minimum value, limiting the function range within.

When dealing with inequalities, the same Lagrangian function as seen above is used. The only difference is a nonnegative slack variable, z , being introduced (Weber 2007).

$$\nabla L(\vec{x}, \vec{\lambda}, \vec{z}) = \frac{\partial f(\vec{x})}{\partial x_i} - \sum_{m=1}^k \lambda_m \frac{\partial (g_m(\vec{x}) - z_m)}{\partial x_i} = 0, i = 1, \dots, n \quad (16)$$

The divergent of L now includes the slack variable z included in the summation.

Matrix expression of the Lagrangian and the simplex method

Solving an optimization problem can be done in different ways. Programs can solve the Lagrange optimization problems with the structure shown above in Equation (15) and (16) directly, but methods demanding less processing power, such as the simplex method is more suited for a programming approach. The Lagrangian function above can be shown as a matrix used in linear programming. Letting $f(\vec{x})$ from be:

$$f(\vec{x}) = c^T \vec{x} = c^T_N \vec{x} + c^T_B \vec{z} \quad (17)$$

And the constraint:

$$\vec{g}(\vec{x}) - \vec{z} = A = A_N \vec{x} + A_B \vec{z} = \vec{b} \quad (18)$$

In the simplex approach, the constraint matrix in (18) is divided into A_B containing the basic variables starting with \vec{z} , A_N containing the non-basic variables, x . Basic variables are the variables that includes the leading element in the pivot column. The function matrix in (17) is split into c_N and c_B , c_N containing non-basic variables, and c_B containing the basic variables. The simplex method optimizes

the function based on the constraints in matrix form. By rearranging the directory, it finds the optimal function and values Matrix form. The approach of finding the optimal values varies with complexity of the problem. The pivoting process in the primal simplex scenario is shown below in (19) (Weber 2007).

$$\begin{bmatrix} I & A_B^{-1}A_N & A_B^{-1}b \\ 0 & c_N^T - c_B^T A_B^{-1}A_N & -c_B^T A_B^{-1}b \end{bmatrix} \quad (19)$$

The process in (19) is of varying complexity giving basic feasible solution function as $-c_B^T A_B^{-1}b$ with value $A_B^{-1}b$ in the right of the matrix. The process is more complex and time-consuming when there are more rearrangements necessary to obtain the solution.

10.3 Appendix C

The Excel model setup

1	A	B	C
2	Revenue optimizing model for hydropower stations		
3	<i>Description:</i>	<i>This model optimizes production to maximize the revenue for a hydropower station. The calculations is based on technical information from the plant and the reservoir, daily prices from Nord Pool, reservoir data from NVE and daily production value from the plant. It uses a Simplex optimization process to find the optimal solution.</i>	
4			
5	Input data		
6	Name:		
7	Bidding zone:		
8			
9	Technical data:		
10	Power units:		
11	Performance [MW]:		
12	Efficiency - Turbine:	<i>If no efficiency is entered, the model will assume 0.95</i>	
13	Efficiency - Generator:	<i>If no efficiency is entered, the model will assume 0.95</i>	
14	Absorption capacity [m ³ /s]:	<i>Required</i>	
15	Head [m]:		
16			
17	Reservoir:		
18	HRV [moh]		
19	LRV [moh]		
20	Reservoir capacity [m ³]		
21	Minimum water flow [m ³ /s] winter	<i>If no value is entered, the model will assume 0</i>	
22	Minimum water flow [m ³ /s] summer	<i>If no value is entered, the model will assume 0</i>	
23	Ending reservoir inventory [m3]	<i>If no value is entered, the model will use the actual ending reservoir value</i>	
24			
25	<i>Developers:</i>		
26	<i>Fredrik Sivertsen og Håkon J. Bakke</i>		

The Excel model setup

	A	B	C	D	E	F	G	H
1								
2	Dato	Price [NOK/MWh]	Prod. [MWh]	Optimal Revenue [NOK]	Actual Production [MWh]	Revenue [NOK]		
3	41091	=VLOOKUP(Calculation!A3;inputtable;2;FALSE)		=B3*D3	=VLOOKUP(Calculation!A3;inputtable;3;FALSE)	=B3*G3		
4	41092	=VLOOKUP(Calculation!A4;inputtable;2;FALSE)		=B4*D4	=VLOOKUP(Calculation!A4;inputtable;3;FALSE)	=B4*G4		
365	41453	=VLOOKUP(Calculation!A365;inputtable;2;FALSE)		=B365*D365	=VLOOKUP(Calculation!A365;inputtable;3;FALSE)	=B365*G365		
366	41454	=VLOOKUP(Calculation!A366;inputtable;2;FALSE)		=B366*D366	=VLOOKUP(Calculation!A366;inputtable;3;FALSE)	=B366*G366		
367	41455	=VLOOKUP(Calculation!A367;inputtable;2;FALSE)		=B367*D367	=VLOOKUP(Calculation!A367;inputtable;3;FALSE)	=B367*G367		
368				=SUM(E3:E367)		=SUM(H3:H367)		

	F	J	K	L
1		Reservoir info and calculations		
2	Actual Reservoir Inventory [MWh]		Total energy [MWh]	Inflow and loss [MWh]
3	=VLOOKUP(Calculation!A3;inputtable;4;FALSE)*1000000*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=J3	=K3+L3	=I4+J+G3
4	=VLOOKUP(Calculation!A4;inputtable;4;FALSE)*1000000*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000		=K3+L3	=I5+J+G4
365	=VLOOKUP(Calculation!A365;inputtable;4;FALSE)*1000000*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000		=K364+L364	=I366+J365+G365
366	=VLOOKUP(Calculation!A366;inputtable;4;FALSE)*1000000*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000		=K365+L365	=I367+J366+G366
367	=VLOOKUP(Calculation!A367;inputtable;4;FALSE)*1000000*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000		=K366+L366	

	M	N	O	P
1				Constraints
2	Reservoir capacity [MWh]	Theoretical Reservoir Inventory [MWh]	Max. production /day [MWh]	
3	=Rescap*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=K3	=MIN(Perf*24;Absorp.cap.*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000;MAX(\$G\$3:\$G\$367))	
4	=Rescap*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=O3-D3+L3	=MIN(Perf*24;Absorp.cap.*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000;MAX(\$G\$3:\$G\$367))	
365	=Rescap*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=O364-D364+L364	=MIN(Perf*24;Absorp.cap.*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000;MAX(\$G\$3:\$G\$367))	
366	=Rescap*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=O365-D365+L365	=MIN(Perf*24;Absorp.cap.*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000;MAX(\$G\$3:\$G\$367))	
367	=Rescap*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000	=O366-D366+L366	=MIN(Perf*24;Absorp.cap.*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)*F(Eff.gen;"",0.95;eff.gen)/3600000;MAX(\$G\$3:\$G\$367))	
368		=O367-D367+L367		

	Q
1	
2	Minimum Water flow [MWh]
3	=F(OR(MONTH(A3)=7;MONTH(A3)=8;MONTH(A3)=9;MONTH(A3)=6;MONTH(A3)=5;MONTH(A3)=4);Minflows;Minflow)*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)/3600000
4	=F(OR(MONTH(A4)=7;MONTH(A4)=8;MONTH(A4)=9;MONTH(A4)=6;MONTH(A4)=5;MONTH(A4)=4);Minflows;Minflow)*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)/3600000
365	=F(OR(MONTH(A365)=7;MONTH(A365)=8;MONTH(A365)=9;MONTH(A365)=6;MONTH(A365)=5;MONTH(A365)=4);Minflows;Minflow)*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)/3600000
366	=F(OR(MONTH(A366)=7;MONTH(A366)=8;MONTH(A366)=9;MONTH(A366)=6;MONTH(A366)=5;MONTH(A366)=4);Minflows;Minflow)*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)/3600000
367	=F(OR(MONTH(A367)=7;MONTH(A367)=8;MONTH(A367)=9;MONTH(A367)=6;MONTH(A367)=5;MONTH(A367)=4);Minflows;Minflow)*3600*24*Head*9.81*F(Eff.turb;"",0.95;eff.turb)/3600000

Appendix D

Results – Plant A
A1 (2012–2013)

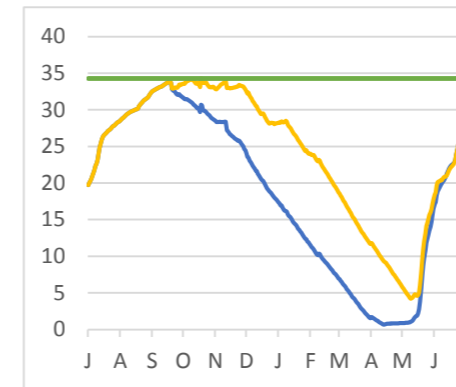


Figure 9

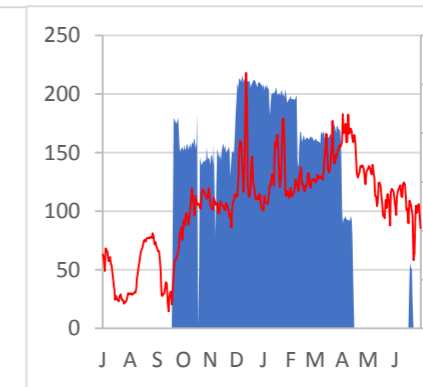


Figure 10

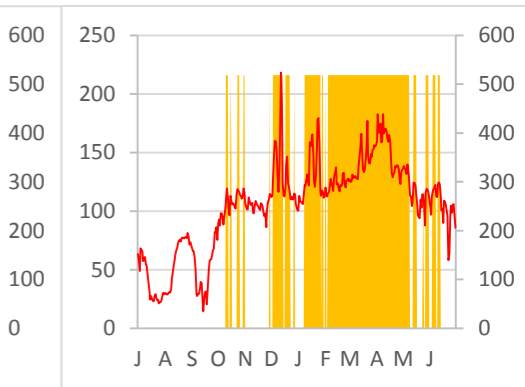


Figure 11

A2 (2011–2012)

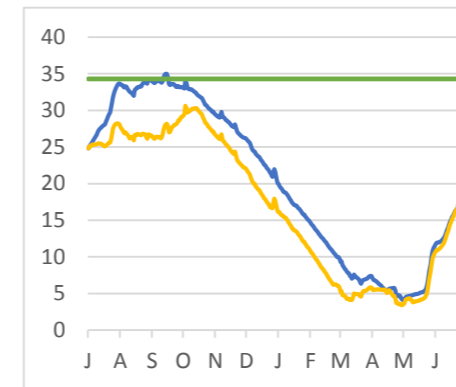


Figure 12

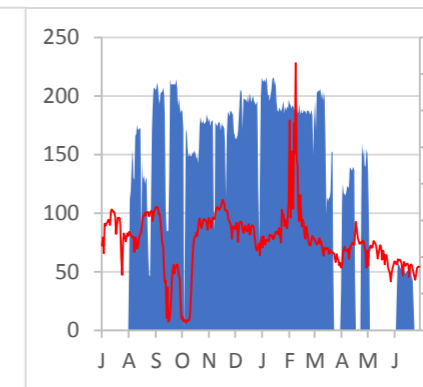


Figure 13

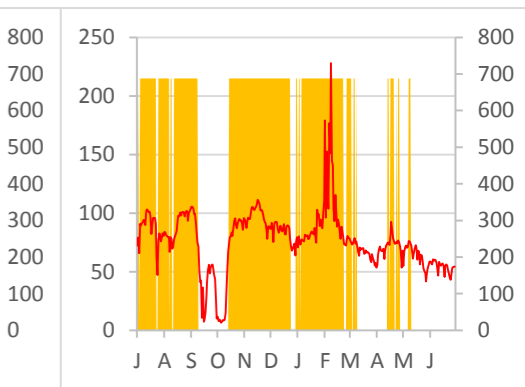


Figure 14

A3 (2010–2011)

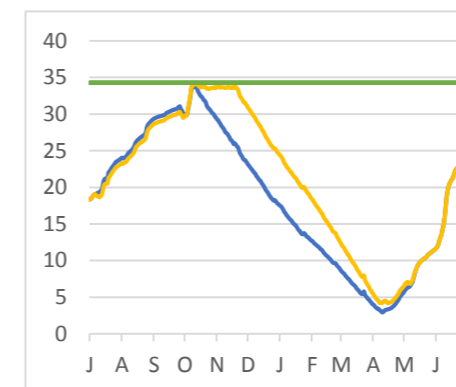


Figure 15

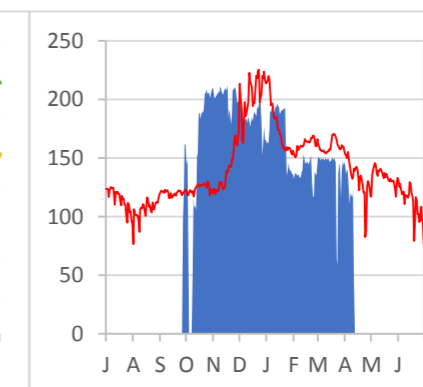


Figure 16

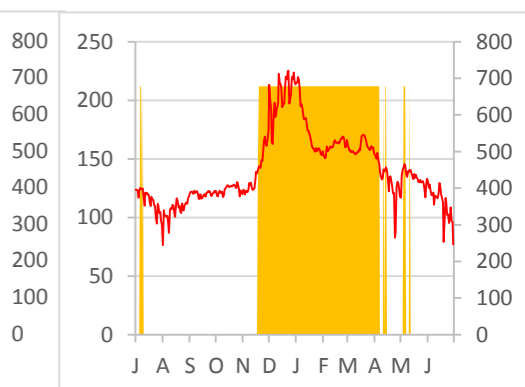


Figure 17

Figure 9, 12, 15: Actual reservoir (blue) and theoretical reservoir (yellow) and HRV (green) in GWh

Figure 10, 13, 16: Actual production (blue) in MWh and price (red) in NOK/MWh

Figure 11, 14, 17: Revenue-optimized production (yellow) in MWh and price (red) in NOK/MWh

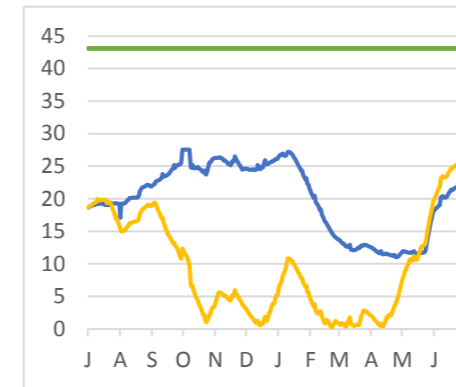


Figure 22

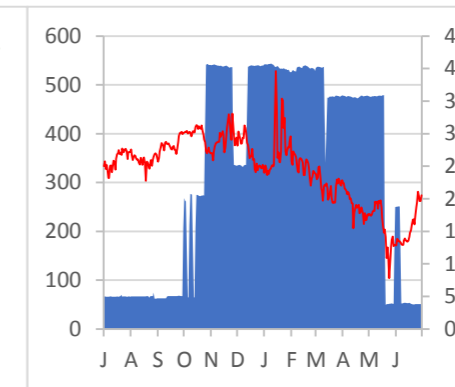


Figure 23

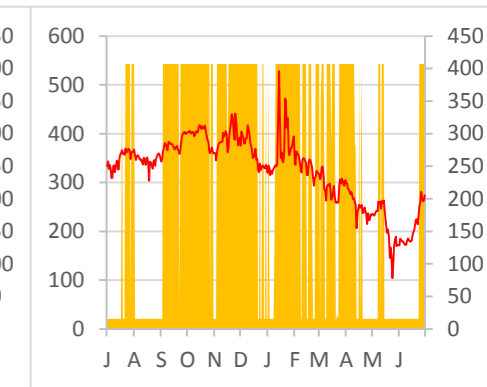


Figure 24

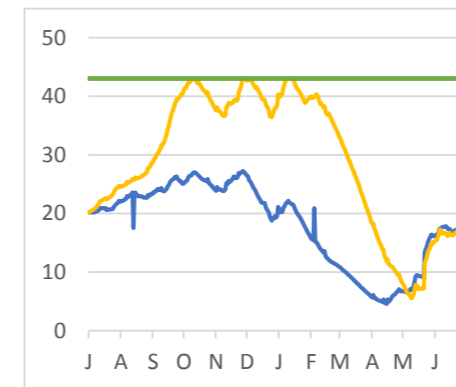


Figure 26

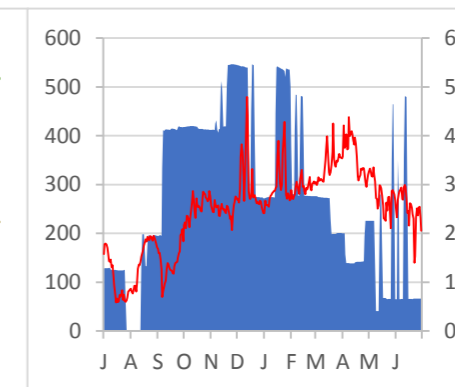


Figure 27

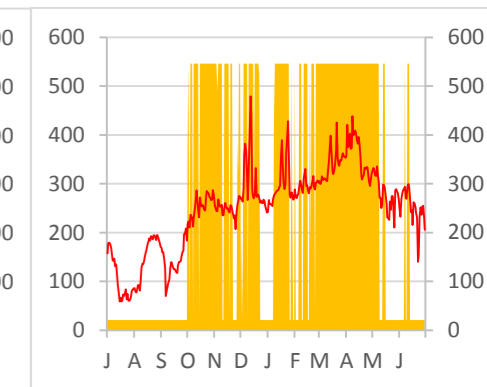


Figure 28

Figure 22, 26: Actual reservoir (blue) and theoretical reservoir (yellow) and HRV (green) in GWh

Figure 23, 27: Actual production (blue) in MWh and price (red) in NOK/MWh

Figure 24, 28: Revenue-optimized production (yellow) in MWh and price (red) in NOK/MWh

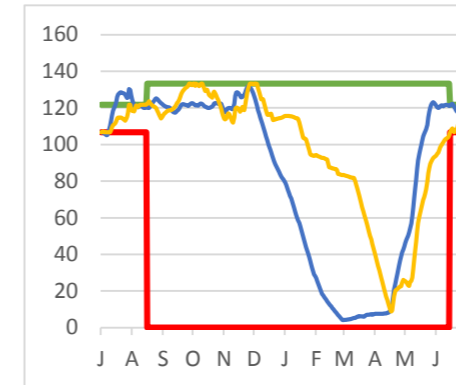


Figure 32

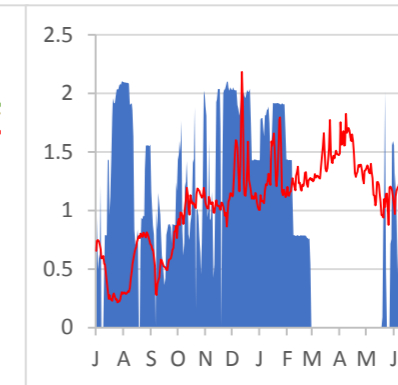


Figure 33

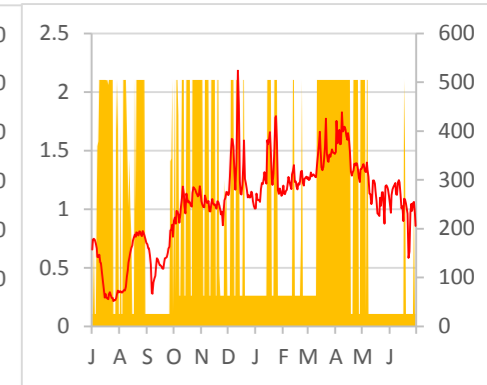


Figure 34

C2 (2011–2012)



Figure 35

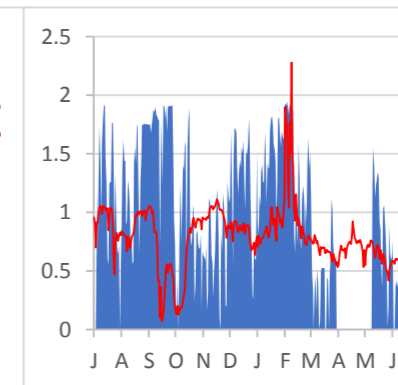


Figure 36

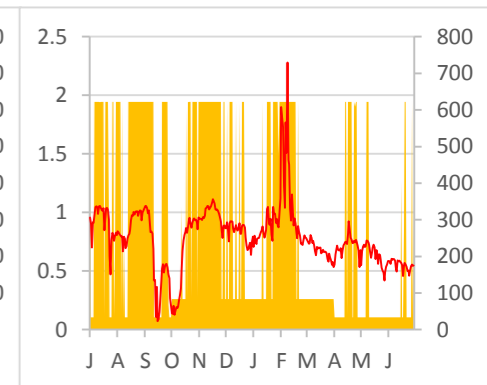


Figure 37

C3 (2010–2011)

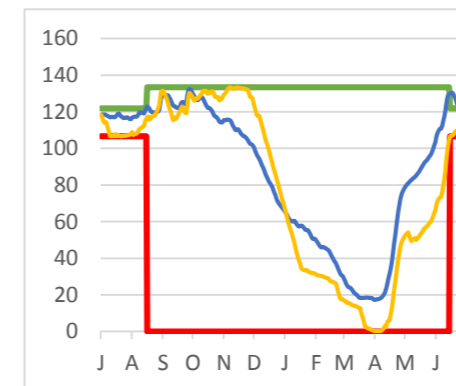


Figure 38

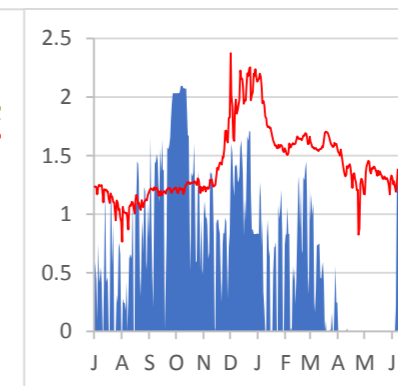


Figure 39

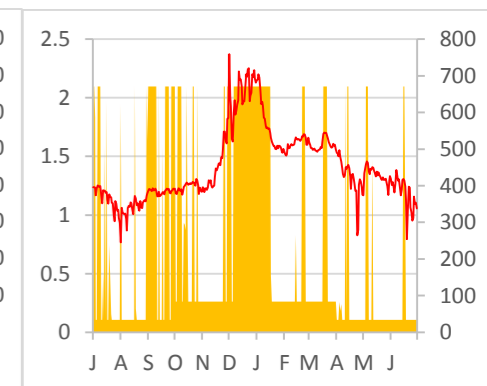


Figure 40

Figure 32, 35, 38: Actual reservoir (blue) and theoretical reservoir (yellow), HRV (green) and LRV (red) in GWh

Figure 33, 36, 39: Actual production (blue) in GWh and price (red) in NOK/MWh

Figure 34, 37, 40: Revenue-optimized production (yellow) in GWh and price (red) in NOK/MWh

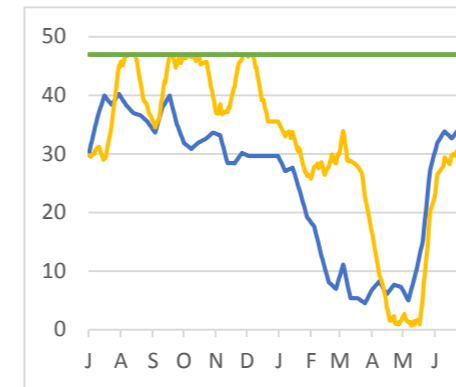


Figure 44

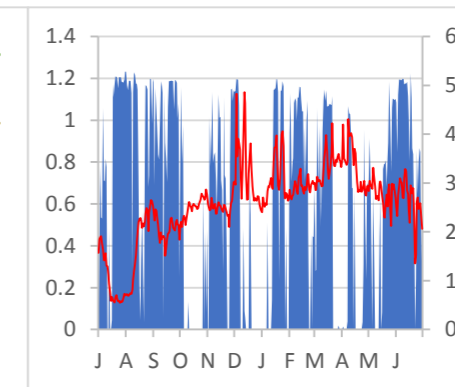


Figure 45

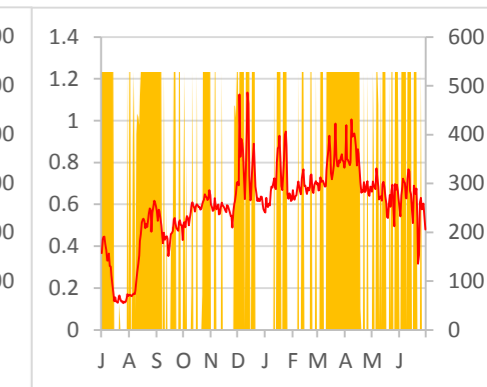


Figure 46

D2 (2011–2012)

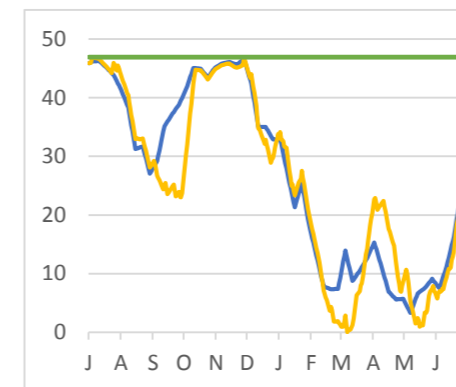


Figure 47

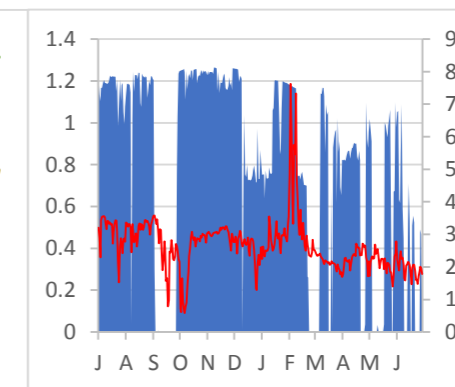


Figure 48

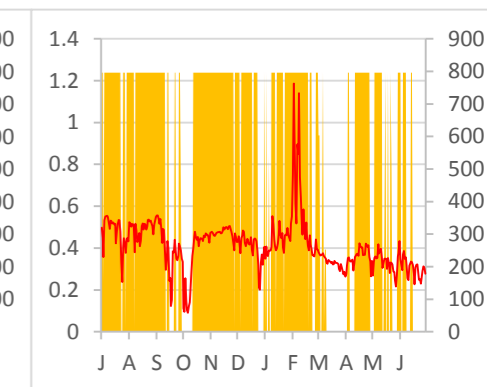


Figure 49

D3 (2010–2011)

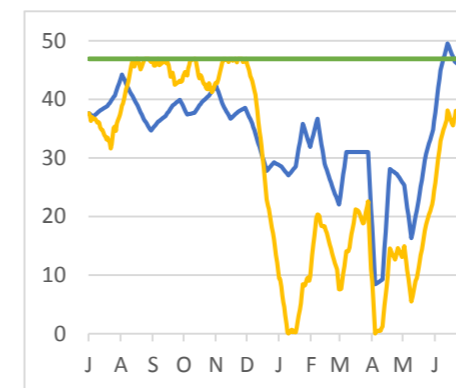


Figure 50

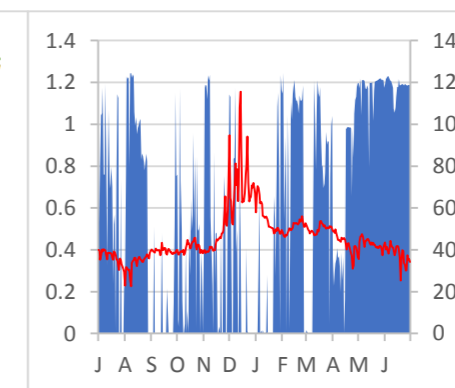


Figure 51

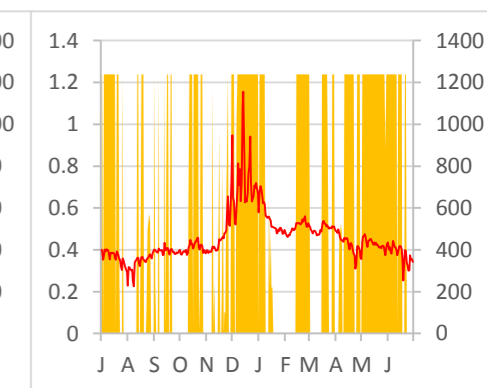


Figure 52

Figure 44, 47, 50: Actual reservoir (blue) and theoretical reservoir (yellow) and HRV (green) in GWh

Figure 45, 48, 51: Actual production (blue) in GWh and price (red) in NOK/MWh

Figure 46, 49, 52: Revenue-optimized production (yellow) in GWh and price (red) in NOK/MWh

A		B		C		D		E		F		G		H		I			
Date	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	Price [\$/MWh]	Production [MWh]	Reservoir [Mm ³]	
1.1.09	899.07	153.91	16.59	899.07	81.90	81.90	899.07	845.4	139.37	404.16	251.64	59.48	11.5.09	289.46	0.00	8.87	289.46	0.00	179.26
2.1.09	428.94	155.56	16.41	428.94	81.76	81.76	428.94	1316.6	138.14	469.49	761.47	59.68	12.5.09	294.06	0.00	8.98	294.06	0.00	182.45
3.1.09	404.48	157.28	16.28	404.48	81.55	81.55	404.48	1308.2	135.27	405.43	147.17	59.87	13.5.09	810.04	0.00	8.08	810.04	0.00	185.12
4.1.09	403.08	160.58	16.11	403.08	81.35	81.35	403.08	1450.6	133.06	404.27	1201.89	60.07	14.5.09	810.04	0.00	8.22	810.04	0.00	187.54
5.1.09	430.82	156.71	16.01	430.82	81.15	81.15	430.82	1863	129.94	426.83	867.67	60.27	15.5.09	810.04	0.00	8.08	810.04	0.00	185.86
6.1.09	454.73	153.15	15.74	454.73	80.98	80.98	454.73	1531.5	126.74	459.13	1069.09	60.27	16.5.09	800.15	0.00	8.58	800.15	0.00	193.71
7.1.09	427.06	154.91	15.68	427.06	80.92	80.92	427.06	1672.6	122.97	454.23	1235.71	59.26	17.5.09	293.40	0.00	8.80	293.40	0.00	191.71
8.1.09	411.13	157.46	15.59	411.13	80.45	80.45	411.13	1743.9	119.69	423.71	1182.53	60.25	18.5.09	324.08	0.00	8.00	324.08	0.00	195.38
9.1.09	395.34	150.82	15.51	395.34	79.70	79.70	395.34	1691.5	116.41	388.45	1196.93	60.25	19.5.09	314.65	0.00	8.85	314.65	0.00	197.12
10.1.09	389.79	155.95	15.29	389.79	79.48	79.48	389.79	1727.2	113.27	365.84	801.39	60.73	20.5.09	809.41	0.00	8.42	809.41	0.00	198.91
11.1.09	362.60	152.26	15.17	362.60	79.48	79.48	362.60	1068.5	110.38	345.57	804.90	60.74	21.5.09	290.92	0.00	8.66	290.92	0.00	201.00
12.1.09	376.49	150.51	15.00	376.49	79.87	79.87	376.49	1308.2	108.67	365.84	801.39	60.73	22.5.09	800.15	0.00	8.58	800.15	0.00	203.00
13.1.09	378.85	155.80	14.90	378.85	81.41	81.41	378.85	1502.2	106.09	407.27	1180.14	60.02	23.5.09	292.22	0.00	8.91	292.22	0.00	203.64
14.1.09	387.23	153.17	14.75	387.23	81.73	81.73	387.23	1696.5	103.52	422.53	1202.56	65.31	24.5.09	299.20	0.00	8.21	299.20	0.00	206.19
15.1.09	408.56	157.99	14.61	408.56	81.61	81.61	408.56	1663.1	100.40	439.50	1215.74	64.59	25.5.09	810.04	0.00	8.61	810.04	0.00	208.37
16.1.09	389.79	157.23	14.47	389.79	81.38	81.38	389.79	1759.3	96.50	417.20	1114.54	63.88	26.5.09	312.00	0.00	7.05	312.00	0.00	213.00
17.1.09	369.02	153.90	14.34	369.02	81.80	81.80	369.02	1708.4	93.38	369.02	1200.31	63.17	27.5.09	810.04	0.00	8.58	810.04	0.00	214.18
18.1.09	348.84	155.40	14.20	348.84	80.78	80.78	348.84	890.4	90.65	348.49	1109.39	62.45	28.5.09	393.71	0.00	8.19	393.71	0.00	218.18
19.1.09	368.34	152.03	14.14	368.34	80.48	80.48	368.34	1320.8	89.09	375.65	1291.60	61.74	29.5.09	317.00	0.00	8.56	317.00	0.00	214.74
20.1.09	361.63	155.79	13.96	361.63	80.12	80.12	361.63	1366.1	86.36	362.27	1201.27	60.74	30.5.09	317.52	0.00	8.29	317.52	0.00	212.42
21.1.09	355.81	153.60	13.89	355.81	79.67	79.67	355.81	1267.8	84.17	362.93	1226.80	60.75	31.5.09	810.04	0.00	8.58	810.04	0.00	213.62
22.1.09	344.93	157.04	13.71	344.93	79.48	79.48	344.93	1383.1	81.52	344.61	960.98	60.75	32.5.09	319.16	0.00	8.29	319.16	0.00	212.63
23.1.09	340.54	154.19	13.65	340.54	79.40	79.40	340.54	894.5	79.26	338.29	666.62	60.76	33.5.09	809.40	0.00	8.29	809.40	0.00	213.23
24.1.09	333.05	157.61	13.49	333.05	79.26	79.26	333.05	1695.5	78.46	330.79	999.47	60.76	34.5.09	326.00	0.00	8.31	326.00	0.00	213.80
25.1.09	354.32	156.99	13.34	354.32	79.90	79.90	354.32	1321.1	76.04	333.45	899.25	60.76	35.5.09	810.04	0.00	8.20	810.04	0.00	215.44
26.1.09	353.95	153.33	13.33	353.95	79.90	79.90	353.95	1324.3	76.62	363.05	1215.31	63.17	36.5.09	327.40	0.00	8.16	327.40	0.00	216.15
27.1.09	362.25	158.93	13.16	362.25	79.82	79.82	362.25	1151.7	74.33	365.19	1001.90	64.37	37.5.09	332.62	0.00	8.22	332.62	0.00	216.23
28.1.09	363.29	154.51	13.01	363.29	79.82	79.82	363.29	925.8	72.59	363.40	1109.39	61.74	38.5.09	318.81	0.00	8.24	318.81	0.00	217.12
29.1.09	369.11	154.30	12.89	369.11	79.90	79.90	369.11	865.5	71.00	367.41	1001.61	63.58	39.5.09	317.52	0.00	8.29	317.52	0.00	217.52
30.1.09	365.38	175.40	12.57	365.38	79.51	79.51	365.38	609	69.63	354.54	517.49	63.18	40.5.09	810.04	0.00	8.58	810.04	0.00	218.43
31.1.09	361.84	197.13	12.14	361.84	79.21	79.21	361.84	814	68.36	354.70	400	62.78	41.5.09	804.17	0.00	8.55	804.17	0.00	215.18
1.2.09	349.48	189.27	12.04	349.48	79.88	79.88	349.48	721.3	67.09	341.69	123.68	62.38	42.5.09	812.21	0.00	8.81	812.21	0.00	212.12
2.2.09	370.28	188.25	11.80	370.28	79.26	79.26	370.28	795.6	65.86	367.70	296.68	61.99	43.5.09	296.75	0.00	8.22	296.75	0.00	213.49
3.2.09	376.59	186.19	11.56	376.59	79.59	79.59	376.59	850.9	64.90	365.99	103.68	61.21	44.5.09	318.20	0.00	8.11	318.20	0.00	213.67
4.2.09	385.38	191.51	11.43	385.38	79.24	79.24	385.38	841.3	63.06	369.58	871.50	60.43	45.5.09	320.40	0.00	8.31	320.40	0.00	215.62
5.2.09	369.76	203.33	11.04	369.76	79.90	79.90	369.76	834.6	61.72	368.50	857.43	60.66	46.5.09	320.32	0.00	8.16	320.32	0.00	215.65
6.2.09	359.18	207.75	10.80	359.18	79.48	79.48	359.18	822.5	60.37	355.87	827.57	60.88	47.5.09	329.53	0.00	8.58	329.53	0.00	216.92
7.2.09	349.83	207.99	10.51	349.83	79.41	79.41	349.83	809.1	59.21	348.75	1005.62	60.80	48.5.09	321.85	0.00	8.21	321.85	0.00	216.10
8.2.09	349.41	203.38	10.41	349.41	79.21	79.21	349.41	759.7	58.66	347.79	978.86	60.73	49.5.09	321.85	0.00	8.43	321.85	0.00	215.18
9.2.09	368.10	204.09	10.21	368.10	79.52	79.52	368.10	1518.2	57.07	378.05	1026.11	60.55	50.5.09	328.09	0.00	8.61	328.09	0.00	216.18
10.2.09	359.93	206.46	9.98	359.93	79.70	79.70	359.93	1372.7	54.42	358.96	991.91	60.60	51.5.09	307.45	0.00	8.55	307.45	0.00	216.23
11.2.09	358.23	210.93	9.69	358.23	79.59	79.59	358.23	1349.1	51.82	359.74	986.00	60.60	52.5.09	342.42	0.00	8.34	342.42	0.00	213.08
12.2.09	355.02	206.68	9.72	355.02	79.27	79.27	355.02	1362.9	49.87	357.42	983.15	63.69	53.5.09	325.18	0.00	8.29	325.18	0.00	212.69
13.2.09	349.79	203.93	9.57	349.79	79.75	79.75	349.79	1283.8	46.79	378.64	899.38	62.73	54.5.09	351.62	0.00	8.18	351.62	0.00	212.69
14.2.09	339.22	207.77	9.45	339.22	79.20	79.20	339.22	806.8	45.61	339.36	369.60	61.78	55.5.09	354.24	0.00	8.24	354.24	0.00	213.83
15.2.09	335.74	195.28	9.01	335.74	79.82	79.82	335.74	766.8	43.00	329.49	460.40	60.82	56.5.09	335.75	0.00	8.19	335.75	0.00	212.76
16.2.09	342.42	172.75	8.99	342.42	79.42	79.42	342.42	1260.3	42.02	377.33	608.42	60.87	57.5.09	337.75	0.00	8.58	337.75	0.00	212.69
17.2.09	341.75	142.49	8.91	341.75	79.97	79.97	341.75	1603.5	39.11	363.80	999.90	61.42	58.5.09	320.52	0.00	8.34	320.52	0.00	212.64
18.2.09	333.92	147.33	8.98	333.92	79.60	79.60	333.92	1567.6	36.07	349.09	998.59	60.76	59.5.09	334.83	0.00	8.72	334.83	0.00	212.30
19.2.09	317.42	155.20	8.83	317.42	79.22	79.22	317.42	165.9	33.37	336.98	711.94	62.21	60.5.09	327.50	0.00	8.80	327.50	0.00	212.61
20.2.09	311.16	166.22	8.58	311.16	78.84	78.84	311.16	1464.7	31.51	318.59	465.39	61.66	61.5.09	325.23	0.00	8.19	325.23	0.00	212.62
21.2.09	305.24	163.60	8.38	305.24	78.54	78.54	305.24	1457.3	29.01	305.90	400	60.71	62.5.09	316.52	0.00	8.68	316.52	0.00	212.26
22.2.09	292.95	162.17	8.28	292.95	78.24	78.24	292.95	1258.1	26.34	291.66	400	60.55	63.5.09	311.14	0.00	8.96	311.14	0.00	212.23
23.2.09	303.88	165.88	8.08	303.88	78.88	78.88	303.88	1387.6	23.88	315.83	891.33	61.73	64.5.09	320.32	0.00	8.20	320.32	0.00	213.18
24.2.09	301.34	164.33	8.01	301.34	78.58	78.58	301.34	1427.2	21.62	313.80	472.87	60.56	65.5.09	325.18	0.00	8.30	325.18	0.00	213.18
25.2.09	294.33	164.86	8.00	294.33	78.28														

Table with 4 columns: numerical values ranging from 26.90 to 10.20, 190.28 to 430.33, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 180.28 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 190.28 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 149.42 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 11.20 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 420.64 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 511.34 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 573.31 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

Table with 4 columns: numerical values ranging from 573.31 to 430.33, 37.66 to 12.88, 37.66 to 12.88, and 37.66 to 12.88.

296.10	380.30	0.00	17.70	580.30	20.32	380.30	118.7	234.90	377.53	288.65	58.21	14.11.10	899.60	209.14	26.30	599.60	37.24	399.60	0.00	219.22	428.23	235.16	56.79
306.10	390.61	0.00	17.97	390.61	20.32	380.84	117.38	234.94	389.84	117.38	57.97	14.11.10	899.60	209.14	26.30	599.60	37.24	399.60	0.00	219.22	428.23	235.16	56.79
1.7.10	395.43	0.00	18.17	395.43	20.32	395.43	59.31	235.37	400.70	59.31	57.74	16.11.10	845.04	207.71	25.96	444.92	37.39	447.18	56.32	218.52	440.88	482.87	56.87
2.7.10	396.24	0.00	18.33	396.24	20.32	396.24	59.42	235.63	397.32	59.42	57.51	17.11.10	840.50	212.37	25.68	439.84	37.21	441.81	56.42	218.56	441.16	482.87	56.87
3.7.10	397.04	0.00	18.51	397.04	20.32	397.04	59.57	235.93	398.48	59.57	57.32	18.11.10	835.96	215.41	25.54	438.98	37.20	440.75	56.42	218.58	440.88	482.87	56.87
4.7.10	397.84	0.00	18.69	397.84	20.32	397.84	59.72	236.23	399.64	59.72	57.13	19.11.10	831.00	218.45	25.60	437.99	37.20	440.75	56.42	218.61	440.88	482.87	56.87
5.7.10	398.64	0.00	18.82	398.64	20.32	398.64	59.87	236.53	400.79	59.87	56.94	20.11.10	826.04	221.59	25.66	437.00	37.19	440.75	56.42	218.64	440.88	482.87	56.87
6.7.10	399.44	0.00	18.91	399.44	20.32	399.44	59.98	236.83	401.94	59.98	56.75	21.11.10	821.18	224.73	25.72	436.01	37.18	440.75	56.42	218.67	440.88	482.87	56.87
7.7.10	400.24	0.00	19.04	400.24	20.32	400.24	60.13	237.13	403.09	60.13	56.56	22.11.10	816.32	227.87	25.78	435.02	37.17	440.75	56.42	218.70	440.88	482.87	56.87
8.7.10	401.04	0.00	19.11	401.04	20.32	401.04	60.28	237.43	404.24	60.28	56.37	23.11.10	811.46	231.01	25.84	434.03	37.16	440.75	56.42	218.73	440.88	482.87	56.87
9.7.10	401.84	0.00	19.19	401.84	20.32	401.84	60.43	237.73	405.39	60.43	56.18	24.11.10	806.60	234.15	25.90	433.04	37.15	440.75	56.42	218.76	440.88	482.87	56.87
10.7.10	402.64	0.00	19.20	402.64	20.32	402.64	60.58	238.03	406.54	60.58	56.00	25.11.10	801.74	237.29	25.96	432.05	37.14	440.75	56.42	218.79	440.88	482.87	56.87
11.7.10	403.44	0.00	19.30	403.44	20.32	403.44	60.73	238.33	407.69	60.73	55.81	26.11.10	796.88	240.43	26.02	431.06	37.13	440.75	56.42	218.82	440.88	482.87	56.87
12.7.10	404.24	0.00	19.48	404.24	20.32	404.24	60.88	238.63	408.84	60.88	55.62	27.11.10	792.02	243.57	26.08	430.07	37.12	440.75	56.42	218.85	440.88	482.87	56.87
13.7.10	405.04	0.00	19.81	405.04	20.32	405.04	61.03	238.93	409.99	61.03	55.43	28.11.10	787.16	246.71	26.14	429.08	37.11	440.75	56.42	218.88	440.88	482.87	56.87
14.7.10	405.84	0.00	20.54	405.84	20.32	405.84	61.18	239.23	411.14	61.18	55.24	29.11.10	782.30	249.85	26.20	428.09	37.10	440.75	56.42	218.91	440.88	482.87	56.87
15.7.10	406.64	0.00	20.93	406.64	20.32	406.64	61.33	239.53	412.29	61.33	55.05	30.11.10	777.44	252.99	26.26	427.10	37.09	440.75	56.42	218.94	440.88	482.87	56.87
16.7.10	407.44	0.00	21.05	407.44	20.32	407.44	61.48	239.83	413.44	61.48	54.86	31.11.10	772.58	256.13	26.32	426.11	37.08	440.75	56.42	218.97	440.88	482.87	56.87
17.7.10	408.24	0.00	21.05	408.24	20.32	408.24	61.63	240.13	414.59	61.63	54.67	32.11.10	767.72	259.27	26.38	425.12	37.07	440.75	56.42	219.00	440.88	482.87	56.87
18.7.10	409.04	0.00	21.09	409.04	20.32	409.04	61.78	240.43	415.74	61.78	54.48	33.11.10	762.86	262.41	26.44	424.13	37.06	440.75	56.42	219.03	440.88	482.87	56.87
19.7.10	409.84	0.00	21.84	409.84	20.32	409.84	61.93	240.73	416.89	61.93	54.29	34.11.10	758.00	265.55	26.50	423.14	37.05	440.75	56.42	219.06	440.88	482.87	56.87
20.7.10	410.64	0.00	22.05	410.64	20.32	410.64	62.08	241.03	418.04	62.08	54.10	35.11.10	753.14	268.69	26.56	422.15	37.04	440.75	56.42	219.09	440.88	482.87	56.87
21.7.10	411.44	0.00	22.26	411.44	20.32	411.44	62.23	241.33	419.19	62.23	53.91	36.11.10	748.28	271.83	26.62	421.16	37.03	440.75	56.42	219.12	440.88	482.87	56.87
22.7.10	412.24	0.00	22.46	412.24	20.32	412.24	62.38	241.63	420.34	62.38	53.72	37.11.10	743.42	275.01	26.68	420.17	37.02	440.75	56.42	219.15	440.88	482.87	56.87
23.7.10	413.04	0.00	22.88	413.04	20.32	413.04	62.53	241.93	421.49	62.53	53.53	38.11.10	738.56	278.19	26.74	419.18	37.01	440.75	56.42	219.18	440.88	482.87	56.87
24.7.10	413.84	0.00	23.09	413.84	20.32	413.84	62.68	242.23	422.64	62.68	53.34	39.11.10	733.70	281.37	26.80	418.19	37.00	440.75	56.42	219.21	440.88	482.87	56.87
25.7.10	414.64	0.00	23.26	414.64	20.32	414.64	62.83	242.53	423.79	62.83	53.15	40.11.10	728.84	284.55	26.86	417.20	36.99	440.75	56.42	219.24	440.88	482.87	56.87
26.7.10	415.44	0.00	23.36	415.44	20.32	415.44	62.98	242.83	424.94	62.98	52.96	41.11.10	723.98	287.73	26.92	416.21	36.98	440.75	56.42	219.27	440.88	482.87	56.87
27.7.10	416.24	0.00	23.43	416.24	20.32	416.24	63.13	243.13	426.09	63.13	52.77	42.11.10	719.12	290.91	26.98	415.22	36.97	440.75	56.42	219.30	440.88	482.87	56.87
28.7.10	417.04	0.00	23.53	417.04	20.32	417.04	63.28	243.43	427.24	63.28	52.58	43.11.10	714.26	294.09	27.04	414.23	36.96	440.75	56.42	219.33	440.88	482.87	56.87
29.7.10	417.84	0.00	23.63	417.84	20.32	417.84	63.43	243.73	428.39	63.43	52.39	44.11.10	709.40	297.27	27.10	413.24	36.95	440.75	56.42	219.36	440.88	482.87	56.87
30.7.10	418.64	0.00	23.63	418.64	20.32	418.64	63.58	244.03	429.54	63.58	52.20	45.11.10	704.54	300.45	27.16	412.25	36.94	440.75	56.42	219.39	440.88	482.87	56.87
31.7.10	419.44	0.00	23.74	419.44	20.32	419.44	63.73	244.33	430.69	63.73	52.01	46.11.10	699.68	303.63	27.22	411.26	36.93	440.75	56.42	219.42	440.88	482.87	56.87
32.7.10	420.24	0.00	23.83	420.24	20.32	420.24	63.88	244.63	431.84	63.88	51.82	47.11.10	694.82	306.81	27.28	410.27	36.92	440.75	56.42	219.45	440.88	482.87	56.87
33.7.10	421.04	0.00	23.92	421.04	20.32	421.04	64.03	244.93	432.99	64.03	51.63	48.11.10	689.96	310.00	27.34	409.28	36.91	440.75	56.42	219.48	440.88	482.87	56.87
34.7.10	421.84	0.00	23.99	421.84	20.32	421.84	64.18	245.23	434.14	64.18	51.44	49.11.10	685.10	313.18	27.40	408.29	36.90	440.75	56.42	219.51	440.88	482.87	56.87
35.7.10	422.64	0.00	24.27	422.64	20.32	422.64	64.33	245.53	435.29	64.33	51.25	50.11.10	680.24	316.36	27.46	407.30	36.89	440.75	56.42	219.54	440.88	482.87	56.87
36.7.10	423.44	0.00	24.46	423.44	20.32	423.44	64.48	245.83	436.44	64.48	51.06	51.11.10	675.38	319.55	27.52	406.31	36.88	440.75	56.42	219.57	440.88	482.87	56.87
37.7.10	424.24	0.00	24.60	424.24	20.32	424.24	64.63	246.13	437.59	64.63	50.87	52.11.10	670.52	322.73	27.58	405.32	36.87	440.75	56.42	219.60	440.88	482.87	56.87
38.7.10	425.04	0.00	24.70	425.04	20.32	425.04	64.78	246.43	438.74	64.78	50.68	53.11.10	665.66	325.92	27.64	404.33	36.86	440.75	56.42	219.63	440.88	482.87	56.87
39.7.10	425.84	0.00	24.83	425.84	20.32	425.84	64.93	246.73	439.89	64.93	50.49	54.11.10	660.80	329.10	27.70	403.34	36.85	440.75	56.42	219.66	440.88	482.87	56.87
40.7.10	426.64	0.00	24.92	426.64	20.32	426.64	65.08	247.03	441.04	65.08	50.30	55.11.10	655.94	332.29	27.76	402.35	36.84	440.75	56.42	219.69	440.88	482.87	56.87
41.7.10	427.44	0.00	25.04	427.44	20.32	427.44	65.23	247.33	442.19	65.23	50.11	56.11.10	651.08	335.47	27.82	401.36	36.83	440.75	56.42	219.72	440.88	482.87	56.87
42.7.10	428.24	0.00	25.19	428.24	20.32	428.24	65.38	247.63	443.34	65.38	49.92	57.11.10	646.22	338.66	27.88	400.37	36.82	440.75	56.42	219.75	440.88	482.87	56.87
43.7.10	429.04	0.00	25.26	429.04	20.32	429.04	65.53	247.93	444.49	65.53	49.73	58.11.10	641.36	341.84	27.94	399.38	36.81	440.75	56.42	219.78	440.88	482.87	56.87
44.7.10	429.84	0.00	25.33	429.84	20.32	429.84	65.68	248.23	445.64	65.68	49.54	59.11.10	636.50	345.03	27.99	398.39	36.80	440.75	56.42	219.81	440.88	482.87	56.87
45.7.10	430.64	0.00	25.44	430.64	20.32	430.64	65.83	248.53	446.79	65.83	49.35	60.11.10	631.64	348.21	28.05	397.40	36.79	440.75	56.42	219.84	440.88	482.87	56.87
46.7.10	431.44	0.00	25.54	431.44	20.32	4																	

1.4.11	489.88	142.963.95	487.87	13.77	489.88	0	34.38	489.55	365.53	37.77	17.81	312.46	128.89	82.85	512.46	38.17	312.46	142.04	233.86	314.22	1093.72	48.15
2.4.11	487.85	136.203.82	487.85	13.73	487.85	0	34.55	487.85	227.51	22.82	18.11	311.03	131.79	82.86	511.03	37.84	311.03	173.17	235.16	318.86	1076.08	48.25
3.4.11	480.38	140.833.67	480.38	13.80	480.38	0	34.87	480.38	298.99	17.87	19.8.11	321.59	125.97	82.97	512.59	37.51	321.59	174.95	234.72	329.49	1234.72	48.34
4.4.11	496.44	137.333.50	496.44	13.85	496.61	0	35.09	497.38	333.88	12.91	20.8.11	313.84	129.53	82.99	513.84	37.15	313.84	176.65	234.93	313.84	1233.18	48.44
5.4.11	477.57	135.733.41	476.72	13.92	477.98	0	35.25	481.37	355.39	13.28	21.8.11	323.29	130.21	83.09	522.29	36.79	322.29	182.32	235.09	322.29	1235.95	48.53
6.4.11	465.37	131.073.43	465.37	14.40	465.37	0	35.45	461.36	407.17	14.40	22.8.11	323.29	130.21	83.24	522.29	36.43	323.29	182.32	235.09	343.65	1177.77	48.63
7.4.11	446.87	120.013.21	446.87	14.23	446.87	0	36.58	448.09	359.77	13.46	23.8.11	329.59	131.21	83.45	529.59	36.11	329.59	146.77	238.57	344.48	1222.17	47.61
8.4.11	432.21	118.133.09	432.21	14.68	432.21	0	37.71	447.22	368.81	13.64	24.8.11	310.93	127.35	83.54	510.93	35.79	310.93	174.94	237.73	348.89	1216.87	46.60
9.4.11	423.42	116.592.99	423.42	15.13	423.42	0	38.99	442.46	318.98	13.82	25.8.11	320.74	125.40	83.68	520.74	35.50	320.74	174.53	236.62	341.27	1194.42	45.50
10.4.11	427.83	122.042.89	427.83	15.51	427.83	0	40.65	438.21	194.94	14.00	26.8.11	325.46	126.09	83.47	525.46	35.15	325.46	176.17	237.30	333.63	1163.40	44.57
11.4.11	450.85	144.162.98	450.85	15.89	450.85	0	42.79	458.31	386.70	14.18	27.8.11	325.13	127.01	83.35	525.13	34.80	325.13	175.21	238.81	325.23	1200.46	43.56
12.4.11	445.77	140.003.19	445.77	16.27	445.77	0	46.06	450.89	252.74	18.32	28.8.11	328.41	128.66	83.40	528.41	34.94	328.41	174.85	239.43	298.92	1180.39	42.54
13.4.11	452.56	140.003.23	452.56	16.62	452.56	0	51.56	453.26	367.93	22.45	29.8.11	319.77	129.06	83.96	519.77	35.90	319.77	175.64	241.11	330.14	1222.24	41.54
14.4.11	456.72	140.003.26	456.72	16.90	456.72	0	56.79	456.72	378.28	26.58	30.8.11	325.67	129.07	83.82	525.67	35.29	325.67	176.64	242.18	339.16	1220.69	42.01
15.4.11	448.34	134.233.23	448.34	17.23	448.34	0	61.26	448.34	348.34	31.26	31.8.11	331.80	129.33	83.76	531.80	35.64	331.80	174.43	243.20	331.80	1210.29	42.48
16.4.11	424.07	130.003.33	424.07	17.48	424.07	0	65.62	424.07	379.02	34.84	32.8.11	337.47	129.39	83.76	537.47	35.96	337.47	175.53	243.54	337.47	1206.32	42.95
17.4.11	391.40	130.003.39	391.40	17.73	391.40	0	70.85	401.00	379.86	38.98	33.8.11	337.45	131.66	83.54	537.45	36.13	337.45	175.53	240.19	337.45	1205.32	43.40
18.4.11	429.48	140.003.44	429.48	17.98	429.48	0	77.74	429.47	387.21	43.11	34.8.11	332.69	129.68	83.46	532.69	36.22	332.69	176.78	238.63	345.24	1204.43	43.93
19.4.11	432.25	140.003.53	432.25	18.24	432.25	0	86.24	432.25	386.04	42.91	35.8.11	317.34	129.22	83.60	517.34	36.31	317.34	176.17	237.32	331.89	1200.00	44.38
20.4.11	418.01	140.003.66	418.01	18.49	418.01	0	95.33	418.01	384.94	42.72	36.8.11	319.20	129.98	83.78	519.20	36.40	319.20	183.54	242.93	345.66	1200.00	44.85
21.4.11	407.03	140.003.80	407.03	18.74	407.03	0	104.82	399.78	385.75	42.52	37.8.11	294.37	129.22	83.69	504.37	36.55	300.06	187.64	254.30	316.39	1200.00	46.14
22.4.11	385.72	140.003.96	385.72	18.99	385.72	0	113.43	381.68	379.75	42.32	38.8.11	259.14	129.13	83.76	506.02	36.61	300.06	188.84	256.62	272.51	1200.00	47.44
23.4.11	386.77	140.012.12	386.77	19.25	386.77	0	121.78	379.92	354.54	42.13	39.8.11	239.54	129.36	83.72	507.72	36.74	300.06	190.02	259.65	215.82	1200.00	48.73
24.4.11	386.41	140.012.30	386.41	19.50	386.41	0	129.76	362.02	376.68	41.93	40.8.11	238.48	129.38	83.56	505.95	36.78	300.06	192.37	260.67	215.82	1200.00	50.02
25.4.11	379.40	140.012.37	379.40	19.76	379.40	0	137.74	349.21	403.13	41.74	41.8.11	238.48	129.38	83.51	505.95	36.78	300.06	192.37	260.67	215.82	1200.00	51.32
26.4.11	402.56	140.012.48	402.56	20.20	402.56	0	142.29	416.20	350.58	41.33	42.8.11	238.48	129.38	83.68	505.95	36.78	300.06	192.37	260.67	189.74	1200.00	52.61
27.4.11	417.28	140.012.50	417.28	20.99	417.28	0	147.04	418.21	316.16	40.72	43.8.11	238.48	129.38	83.40	505.95	36.78	300.06	192.37	260.67	219.57	1200.00	53.90
28.4.11	411.73	140.012.50	411.73	21.31	411.73	0	150.35	410.96	313.95	40.51	44.8.11	238.48	129.38	83.63	505.95	36.78	300.06	192.37	260.67	278.59	1200.00	54.33
29.4.11	396.71	140.012.56	396.71	21.44	396.71	0	153.13	396.71	317.93	40.10	45.8.11	238.48	129.38	83.76	505.95	36.78	300.06	192.37	260.67	257.37	1200.00	54.75
30.4.11	375.46	140.012.53	375.46	21.60	375.46	0	155.24	365.17	319.84	39.69	46.8.11	238.48	129.38	83.47	505.95	36.78	300.06	192.37	260.67	156.74	1200.00	55.17
31.4.11	374.18	140.012.57	374.18	21.65	374.18	0	157.03	354.90	319.16	39.28	47.8.11	238.48	129.38	83.40	505.95	36.78	300.06	192.37	260.67	165.31	1200.00	55.59
32.4.11	333.95	140.012.54	333.95	23.71	333.95	0	158.49	436.04	317.31	38.87	48.8.11	238.48	129.38	83.32	505.95	36.78	300.06	192.37	260.67	78.85	1200.00	56.01
33.4.11	449.10	140.012.60	449.10	23.76	449.10	0	159.83	463.45	327.47	36.89	49.8.11	238.48	129.38	83.17	505.95	36.78	300.06	192.37	260.67	97.90	1200.00	56.44
34.4.11	458.26	140.012.61	458.26	23.76	458.26	0	161.76	463.45	327.47	36.89	50.8.11	238.48	129.38	83.38	505.95	36.78	300.06	192.37	260.67	247.00	1200.00	56.86
35.4.11	466.07	140.012.62	466.07	23.79	466.07	0	162.05	475.45	329.87	32.92	51.8.11	238.48	129.38	83.37	505.95	36.78	300.06	192.37	260.67	243.65	1200.00	57.24
36.4.11	462.29	140.012.63	462.29	23.84	462.29	0	163.25	462.49	321.69	30.93	52.8.11	238.48	129.38	83.33	505.95	36.78	300.06	192.37	260.67	282.36	1200.00	57.63
37.4.11	438.53	140.012.62	438.53	23.87	438.53	0	164.26	433.51	320.21	28.95	53.8.11	238.48	129.38	83.29	505.95	36.78	300.06	192.37	260.67	256.09	1200.00	58.02
38.4.11	431.18	140.012.67	431.18	23.87	431.18	0	165.21	413.97	320.88	26.96	54.8.11	238.48	129.38	83.22	505.95	36.78	300.06	192.37	260.67	224.71	1200.00	58.40
39.4.11	446.28	140.012.66	446.28	23.89	446.28	0	166.24	442.74	317.95	24.98	55.8.11	238.48	129.38	83.24	505.95	36.78	300.06	192.37	260.67	220.00	1200.00	58.79
40.4.11	443.81	140.012.65	443.81	23.92	443.81	0	167.24	443.81	316.68	26.42	56.8.11	238.48	129.38	83.23	505.95	36.78	300.06	192.37	260.67	233.53	1200.00	59.18
41.4.11	443.25	140.012.68	443.25	24.30	443.25	0	168.21	450.28	316.04	28.26	57.8.11	238.48	129.38	83.07	505.95	36.78	300.06	192.37	260.67	270.53	1200.00	59.56
42.4.11	449.33	140.012.63	449.33	24.13	449.33	0	169.15	452.63	317.31	29.30	58.8.11	238.48	129.38	83.27	505.95	36.78	300.06	192.37	260.67	262.53	1200.00	60.14
43.4.11	442.17	140.012.68	442.17	24.21	442.17	0	170.46	442.17	317.98	30.74	59.8.11	238.48	129.38	83.31	505.95	36.78	300.06	192.37	260.67	250.48	1200.00	60.73
44.4.11	434.16	140.012.65	434.16	24.32	434.16	0	172.00	434.16	319.57	32.18	60.8.11	238.48	129.38	83.25	505.95	36.78	300.06	192.37	260.67	224.14	1200.00	61.31
45.4.11	424.96	140.012.68	424.96	24.64	424.96	0	173.74	424.96	319.46	33.62	61.8.11	238.48	129.38	83.22	505.95	36.78	300.06	192.37	260.67	154.34	1200.00	61.89
46.4.11	438.79	140.012.67	438.79	25.46	438.79	0	175.68	435.19	319.81	35.06	62.8.11	238.48	129.38	83.27	505.95	36.78	300.06	192.37	260.67	128.49	1200.00	62.47
47.4.11	427.83	140.012.68	427.83	25.84	427.83	0	177.47	424.94	319.85	36.78	63.8.11	238.48	129.38	83.27	505.95	36.78	300.06	192.37	260.67	61.05	1200.00	63.06
48.4.11	437.14	140.012.64	437.14	26.43	437.14	0	179.09	433.94	310.40	38.49	64.8.11	238.48	129.38	83.40	505.95	36.78	300.06	192.37	260.67	165.08	1200.00	63.64
49.4.11	429.33	140.012.66	429.33	26.84																		

Table with 10 columns and 1000 rows of numerical data. Each row contains 10 values representing model input data. The values are organized in a grid format across the page.

Large table containing numerical data organized in columns and rows, representing model input data.

7.7.13	231.96	0.00	29.08	231.96	66.3	27.90	231.96	0.00	235.09	22.11.13	324.66	159.90	24.78	524.42	534.5	37.75	524.66	128.82	235.00	831.98	550.79	63.35
8.7.13	250.67	0.00	29.20	250.67	66.5	27.95	250.67	419.8	234.21	22.11.13	302.93	162.48	24.63	502.91	536.1	37.33	502.91	128.82	232.26	830.46	0.00	53.39
9.7.13	250.62	0.00	29.30	250.90	66.4	28.10	250.90	420.3	233.58	24.11.13	290.32	164.45	24.43	490.32	535.9	36.97	490.32	128.82	229.91	828.14	0.00	53.39
10.7.13	240.95	0.00	29.42	240.95	66.2	28.60	240.95	427.2	233.95	25.11.13	330.89	161.65	24.36	530.89	536.0	36.98	530.89	165.06	226.96	828.60	798.33	53.41
11.7.13	249.29	0.00	29.47	249.29	66.2	28.10	249.29	420.3	233.95	26.11.13	327.91	161.51	24.21	527.91	536.3	36.97	527.91	165.11	224.44	826.97	1184.45	53.61
12.7.13	256.17	0.00	29.54	256.17	66.4	28.10	256.17	420.3	232.76	27.11.13	297.60	160.27	24.07	497.60	536.3	36.97	497.60	165.27	222.75	827.60	869.3	53.62
13.7.13	254.05	0.00	29.61	255.93	66.3	28.10	255.93	421.3	232.32	28.11.13	289.74	160.58	23.99	482.11	535.5	35.99	482.11	162.11	221.30	826.08	931	54.03
14.7.13	244.63	0.00	29.69	244.63	66.1	28.10	244.63	403.7	232.63	29.11.13	295.43	163.84	23.69	490.35	535.1	35.99	490.35	127.73	219.21	817.07	569.18	54.24
15.7.13	259.96	0.00	29.72	261.59	66.3	28.10	261.59	423.7	232.19	30.11.13	293.96	164.72	23.53	493.96	534.3	35.99	493.96	134.5	217.36	829.96	844.5	54.40
16.7.13	258.03	0.00	29.78	268.20	66.4	28.10	268.20	473.8	231.47	1.12.13	381.49	166.05	23.40	581.46	535.1	35.99	581.49	795.3	216.17	821.55	0.00	54.65
17.7.13	256.03	0.00	29.83	269.60	66.3	27.87	269.60	423.4	230.57	2.12.13	302.47	160.96	23.25	503.14	535.6	35.99	503.14	147.76	214.47	832.11	561.94	54.86
18.7.13	271.02	0.00	29.89	274.62	66.3	27.87	274.62	402.7	230.09	3.12.13	298.29	162.52	23.08	499.56	536.2	35.81	499.56	136.2	212.97	803.11	479.65	55.17
19.7.13	267.70	0.00	29.93	269.60	66.5	27.87	269.60	417.0	229.71	4.12.13	293.23	166.96	22.91	492.89	535.7	35.73	492.89	135.7	213.31	792.76	0.00	55.48
20.7.13	261.46	0.00	29.92	270.86	67.5	27.87	270.86	417.0	229.33	5.12.13	286.61	167.76	22.74	484.79	534.8	35.76	484.79	134.8	213.61	787.53	705.89	55.79
21.7.13	267.44	0.00	29.93	271.44	67.4	27.87	271.44	417.0	229.38	6.12.13	290.21	163.60	22.57	484.03	533.9	35.81	484.03	134.5	213.63	789.85	1096.40	56.10
22.7.13	276.25	0.00	30.02	277.16	66.2	27.87	277.16	417.0	229.46	7.12.13	293.40	186.00	22.35	493.40	533.8	35.90	493.40	134.3	210.84	804.67	1190.23	56.41
23.7.13	271.49	0.00	30.05	271.49	66.8	28.20	271.49	417.0	229.12	8.12.13	294.28	194.71	22.16	492.85	533.8	35.79	492.85	133.8	209.25	816.25	1164.14	56.71
24.7.13	272.21	0.00	30.08	272.21	66.6	28.20	272.21	417.0	229.21	9.12.13	312.01	197.16	21.87	512.42	533.5	35.67	512.42	135.5	208.33	862.04	1166.17	57.02
25.7.13	272.95	0.00	30.11	275.52	66.7	28.20	275.52	417.0	229.99	10.12.13	308.48	189.75	21.65	508.48	535.9	35.76	508.48	135.9	207.93	822.10	1158.12	57.93
26.7.13	276.25	0.00	30.03	276.50	66.6	28.20	276.50	417.0	229.07	11.12.13	310.19	192.64	21.43	501.19	535.8	35.93	501.19	139.1	209.87	801.16	1162.65	58.84
27.7.13	273.30	0.00	29.90	273.30	66.3	28.20	273.30	417.0	228.93	12.12.13	284.28	173.70	21.27	485.82	534.4	36.80	485.82	134.1	197.54	786.17	1166.55	59.75
28.7.13	260.74	0.00	29.99	260.74	66.2	28.20	260.74	417.0	228.88	1.12.13	298.29	161.12	21.18	482.09	536.1	36.19	482.09	130.62	196.02	289.52	1187.68	60.66
29.7.13	271.50	0.00	30.34	271.50	66.4	28.20	271.50	417.0	229.27	14.12.13	275.01	180.37	21.03	475.01	537.8	35.90	475.01	127.79	194.80	275.56	1224.65	61.57
30.7.13	271.54	0.00	30.42	271.64	66.5	28.12	271.64	417.0	229.87	15.12.13	262.38	176.65	20.93	462.38	538.7	35.99	462.38	124.61	193.65	262.38	1215.66	62.48
31.7.13	273.75	0.00	30.54	271.75	66.5	28.24	271.75	417.0	229.80	16.12.13	264.50	180.52	20.75	464.50	538.5	36.50	464.50	124.8	193.88	264.50	1224.89	63.61
1.8.13	273.69	0.00	30.57	275.52	66.6	28.12	275.52	417.0	229.81	17.12.13	266.36	183.77	20.56	466.36	540.0	36.34	466.36	127.11	190.79	272.49	1203.81	62.65
2.8.13	267.03	0.00	30.63	267.03	66.5	28.12	267.03	417.0	229.86	18.12.13	271.22	174.84	20.45	476.68	539.7	37.30	476.68	131.7	188.73	274.86	1218.91	61.91
3.8.13	254.06	0.00	30.74	259.40	66.6	28.12	259.40	417.0	229.92	19.12.13	259.65	178.32	20.22	459.65	539.5	37.00	459.65	133.3	186.98	260.47	1223.86	61.18
4.8.13	258.03	0.00	30.90	264.39	66.4	28.17	264.39	417.0	230.06	20.12.13	260.73	181.56	20.08	460.86	538.8	37.30	460.86	138.2	185.42	260.86	1273.38	60.44
5.8.13	267.03	0.00	31.18	267.03	66.5	28.23	267.03	417.0	229.93	21.12.13	245.13	128.26	20.35	445.13	539.1	36.97	445.13	130.6	183.60	244.39	1208.27	59.70
6.8.13	265.83	0.00	31.47	266.12	66.5	28.34	266.12	418.7	229.84	22.12.13	240.63	133.43	20.27	440.63	539.4	37.12	440.63	130.61	182.93	237.84	851.00	58.97
7.8.13	261.28	0.00	31.73	261.31	66.7	28.67	261.31	417.0	229.97	23.12.13	254.15	137.66	20.05	454.15	539.6	37.27	454.15	117.96	182.30	246.30	832.37	58.23
8.8.13	260.17	0.00	31.90	260.17	66.7	28.90	260.17	417.0	230.22	24.12.13	243.89	138.98	19.92	443.89	540.0	37.42	443.89	117.97	181.95	231.76	719.97	57.95
9.8.13	260.30	0.00	32.15	260.30	68.3	29.17	260.30	418.3	230.17	25.12.13	247.48	138.93	19.87	447.48	539.9	37.54	447.48	118.75	184.75	232.88	583.1	57.67
10.8.13	255.27	0.00	32.14	255.41	66.7	29.45	255.41	417.0	230.23	26.12.13	251.24	137.41	19.71	451.24	539.2	37.66	451.24	107.9	186.82	243.09	705.25	57.39
11.8.13	252.43	0.00	32.26	252.43	66.6	29.45	252.43	417.0	231.27	27.12.13	250.84	139.94	19.61	450.84	538.4	37.78	450.84	107.0	186.76	249.37	1109.22	57.11
12.8.13	262.74	0.00	32.41	262.74	66.6	29.45	262.74	417.0	232.32	28.12.13	247.11	137.26	19.47	447.11	538.7	37.87	447.11	102.8	187.54	220.86	887.93	56.83
13.8.13	262.04	0.00	32.56	262.04	66.7	29.51	262.04	417.0	232.99	29.12.13	246.66	141.77	19.30	446.66	539.4	38.14	446.66	102.8	188.53	246.66	528.8	58.53
14.8.13	251.94	0.00	32.12	251.94	66.6	29.48	251.94	417.0	233.41	30.12.13	252.28	141.02	18.85	452.28	539.4	38.17	452.28	117.85	188.65	244.40	852.03	56.27
15.8.13	258.17	0.00	32.16	258.17	66.7	29.48	258.17	417.0	233.97	31.12.13	247.20	138.93	18.75	447.20	540.1	38.17	447.20	119.23	188.26	240.80	725.49	56.27
16.8.13	263.32	0.00	32.11	263.32	66.6	29.48	263.32	417.0	233.65	1.1.14	239.63	141.8	18.65	439.63	541.8	38.65	439.63	109.53	190.53	245.42	1162.65	57.93
17.8.13	264.39	0.00	32.93	264.39	66.7	29.48	264.39	417.0	234.47	2.1.14	231.14	142.2	18.42	431.14	542.2	38.95	431.14	109.53	192.77	246.65	1242.3	58.24
18.8.13	257.38	0.00	33.15	257.38	66.7	29.48	257.38	417.0	234.93	3.1.14	232.24	141.7	18.33	432.24	541.7	39.14	432.24	109.53	193.88	247.48	1214.7	58.54
19.8.13	256.85	0.00	32.55	256.85	66.7	29.95	256.85	417.0	234.87	4.1.14	236.51	141.7	18.10	436.51	541.7	39.10	436.51	109.53	193.88	248.26	1242.0	58.54
20.8.13	255.94	0.00	32.44	255.94	67.1	30.97	255.94	417.0	234.79	5.1.14	242.96	142.0	18.40	442.96	542.0	39.40	442.96	109.53	194.00	249.09	1202.57	57.39
21.8.13	250.51	0.00	32.24	251.01	66.9	31.44	251.01	417.0	234.31	6.1.14	238.16	142.4	18.10	438.16	542.4	39.15	438.16	109.53	195.07	244.54	1242.5	58.95
22.8.13	246.30	0.00	32.24	246.30	64.2	31.70	246.30	417.0	234.10	7.1.14	244.54	142.5	18.90	444.54	542.5	38.90	444.54	109.53	195.34	246.30	1242.5	62.48
23.8.13	256.10	0.00	32.17	256.10	67.6	31.84	256.10	417.0	234.56	8.1.14	246.31	143.1	18.95	446.31	543.1	38.95	446.31	109.53	195.34	248.13	1242.5	62.48
24.8.13	259.39	0.00	32.12	259.39	67.6	31.96	259.39	417.0	234.56	9.1.14	248.93	143.2	19.34	448.93	543.2	39.34	448.93	109.53	195.34	251.83	1242.5	62.48
25.8.13	250.57	0.00	32.19	250.57	67.6	31.99	250.57															

9.4.14		199,62	476,8	16,97					25.8.14		247,20	458,5	36,70			5,16,681	
10.4.14		200,65	476,0	17,50					26.8.14		266,53	399,4	36,46			1,184,044	
11.4.14		193,07	475,5	16,92					27.8.14		263,84	290,5	36,13			1,425,281	
12.4.14		192,40	474,1	16,77					28.8.14		261,49	264,1	36,16			1,485,139	
13.4.14		154,85	476,0	16,80					29.8.14		256,30	260,2	35,99			747,744	
14.4.14		178,42	474,3	16,90					30.8.14		258,50	257,7	35,87			1,54,901	
15.4.14		185,85	474,3	16,95					31.8.14		262,35	258,5	35,81			882,935	
16.4.14		190,59	473,7	16,82					1.9.14		260,38	385,7	35,76			829,954	
17.4.14		186,30	472,3	16,75					2.9.14		267,95	441,1	35,67			1,308,309	
18.4.14		187,55	472,9	16,67					3.9.14		267,67	440,4	35,26			1,476,244	
19.4.14		190,85	473,6	16,60					4.9.14		269,02	438,1	34,92			837,831	
20.4.14		177,86	474,9	16,54					5.9.14		272,04	437,5	34,63			1,256,924	
21.4.14		178,58	476,7	16,47					6.9.14		269,84	436,6	34,31			2,724	
22.4.14		186,76	477,6	16,39					7.9.14		268,98	436,9	34,20			1,388,936	
23.4.14		181,98	477,5	16,60					8.9.14		265,47	437,1	33,73			1,318,077	
24.4.14		177,43	476,6	16,19					9.9.14		264,65	437,0	33,70			1,488,196	
25.4.14		160,93	476,6	16,22					10.9.14		264,89	436,9	33,52			748,416	
26.4.14		177,84	477,0	16,12					11.9.14		265,39	436,8	33,26			893,537	
27.4.14		167,06	477,4	16,37					12.9.14		266,53	436,2	32,91			831,947	
28.4.14		167,44	477,8	16,62					13.9.14		274,39	435,4	32,65			883,297	
29.4.14		175,74	477,7	16,87					14.9.14		274,11	435,5	32,31			859,967	
30.4.14		174,66	477,2	17,10					15.9.14		277,43	435,7	31,93			1,483,028	
1.5.14		176,94	476,6	17,33					16.9.14		283,28	435,4	31,55			1,652,503	
2.5.14		175,73	476,1	17,45					17.9.14		282,06	434,9	31,18			1,586,056	
3.5.14		173,75	476,5	17,38					18.9.14		282,31	434,6	30,80			1,716,302	
4.5.14		175,95	476,4	17,33					19.9.14		278,63	211,2	30,37			886,066	
5.5.14		179,99	476,5	17,25					20.9.14		276,56	60,2	29,95			372,635	
6.5.14		181,62	476,4	17,18					21.9.14		271,11	60,5	30,30			0	
7.5.14		181,46	476,9	17,13					22.9.14		273,85	61,1	30,12			1,58,36	
8.5.14		195,60	477,1	17,13					23.9.14		278,26	60,9	30,20			594,272	
9.5.14		193,00	476,8	17,18					24.9.14		278,57	158,0	30,17			1,769	
10.5.14		195,37	476,7	17,15					25.9.14		274,90	197,8	30,14			1,26,309	
11.5.14		184,96	477,2	17,10					26.9.14		268,36	197,2	29,84			0	
12.5.14		196,71	477,9	17,50					27.9.14		259,81	196,6	29,56			0	
13.5.14		193,79	478,0	17,00					28.9.14		263,57	197,2	29,53			0	
14.5.14		197,62	478,1	16,95					29.9.14		270,71	197,8	29,51			0	
15.5.14		186,57	477,3	16,92					30.9.14		271,83	197,7	29,51			40,18	
16.5.14		170,54	478,2	16,92					1.10.14		270,46	197,5	29,48			0	
17.5.14		156,82	478,5	16,92					2.10.14		266,30	197,5	29,48			0	
18.5.14		147,71	479,0	16,95					3.10.14		264,14	196,8	29,42			0	
19.5.14		152,73	194,4	17,20					4.10.14		259,94	196,4	29,42			0	
20.5.14		141,69	492,7	17,13					5.10.14		258,47	196,8	29,40			0	
21.5.14		109,17	513,0	17,13					6.10.14		256,67	196,9	29,40			324,457	
22.5.14		125,38	51,1	17,20					7.10.14		256,39	196,4	29,37			1,78,284	
23.5.14		111,76	50,9	17,45					8.10.14		256,32	196,4	29,51			1,45,885	
24.5.14		78,27	51,3	18,72					9.10.14		259,86	196,4	29,51			1,804	
25.5.14		101,95	51,6	20,00					10.10.14		255,85	196,6	29,53			0	
26.5.14		126,91	52,0	21,19					11.10.14		263,40	196,8	30,10			0	
27.5.14		137,23	52,0	22,30					12.10.14		262,21	197,6	30,65			1,08	
28.5.14		142,04	51,7	23,79					13.10.14		261,35	198,3	31,20			803,418	
29.5.14		127,04	51,5	24,71					14.10.14		267,15	198,0	31,38			839,548	
30.5.14		129,07	448,2	25,65					15.10.14		266,74	197,6	31,44			1,737,868	
31.5.14		138,99	448,3	26,54					16.10.14		265,10	197,4	31,44			1,534,25	
1.6.14		128,19	250,0	26,76					17.10.14		269,62	196,6	31,44			1,632,244	
2.6.14		138,29	250,6	26,98					18.10.14		259,41	196,6	31,32			1,432,526	
3.6.14		137,76	250,8	27,20					19.10.14		236,16	198,1	31,32			597,564	
4.6.14		134,61	250,7	27,43					20.10.14		224,36	198,6	31,61			1,289,035	
5.6.14		134,58	250,8	27,65					21.10.14		244,96	198,5	31,90			1,965,804	
6.6.14		132,50	133,2	27,84					22.10.14		241,37	198,8	32,71			2,071,123	
7.6.14		130,04	62,1	29,53					23.10.14		241,02	198,6	33,21			2,070,537	
8.6.14		128,99	62,2	29,70					24.10.14		209,04	199,9	33,61			2,068,617	
9.6.14		129,07	62,8	29,89					25.10.14		179,00	200,8	34,80			1,767,107	
10.6.14		136,22	63,1	29,53					26.10.14		144,13	200,9	34,68			2,131,306	
11.6.14		138,77	63,4	29,53					27.10.14		139,48	170,5	35,29			2,057,992	
12.6.14		135,82	63,3	29,56					28.10.14		51,23	46,6	35,93			2,067,75	
13.6.14		134,73	63,0	29,67					29.10.14		76,45	351,5	37,39			1,652,395	
14.6.14		133,92	63,0	30,31					30.10.14		178,18	608,5	38,95			1,859,363	
15.6.14		135,65	63,2	30,65					31.10.14		194,22	511,1	39,10			2,041,511	
16.6.14		141,52	63,6	30,89					1.11.14		171,07	514,2	39,40			2,058,702	
17.6.14		147,68	63,2	31,30					2.11.14		127,06	514,6	39,19			2,050,003	
18.6.14		151,27	63,6	31,18					3.11.14		154,70	515,5	39,31			1,810,85	
19.6.14		160,75	63,8	31,35					4.11.14		175,16	516,2	39,46			2,030,449	
20.6.14		167,67	60,4	31,55					5.11.14		220,69	515,1	39,61			1,973,099	
21.6.14		168,38	60,0	31,70					6.11.14		224,53	515,0	39,58			1,952,396	
22.6.14		160,60	60,3	31,81					7.11.14		231,00	512,1	39,43			1,902,884	
23.6.14		175,52	60,9	31,90					8.11.14		217,54	512,4	39,16			1,712,823	
24.6.14		187,76	61,0	31,99					9.11.14		205,83	513,3	38,89			1,619,247	
25.6.14		194,50	61,1	32,70					10.11.14		222,20	514,4	39,10			748,673	
26.6.14		210,73	61,0	32,10					11.11.14		232,01	514,2	39,22			1,78,301	
27.6.14		203,25	60,8	32,16					12.11.14		236,11	513,1	39,16			1,258,002	
28.6.14		196,44	60,6	32,19					13.11.14		249,77	512,9	39,40			1,754,437	
29.6.14		196,36	60,7	32,19					14.11.14		250,43	512,0	38,80			1,801,236	
30.6.14		205,25	61,1	32,19					15.11.14		237,83	511,1	38,50			1,873,394	
1.7.14		213,10	60,9	32,22					16.11.14		233,87	511,6	38,20			995,354	
2.7.14		220,45	61,1	32,25					17.11.14		251,40	512,0	37,90			810,522	
3.7.14		219,66	61,0	32,28					18.11.14		261,53	510,6	37,60			861,041	
4.7.14		217,99	60,6	32,31					19.11.14		263,96	510,6	37,24			868,539	
5.7.14		213,24	60,3	32,31					20.11.14		275,68	489,9	36,85			1,523,038	
6.7.14		205,24	60,8	32,31					21.11.14		271,48	468,2	36,43			1,669,407	
7.7.14		212,70	186,3														



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