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Assessment of vibrational properties of laboratory tested timber joist floors

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Abstract

Five timber joist floors were deflection-tested with and without added boarding layers, in order to determine how much transversal stiffness can be considered for vibration design of the different floor constructions. The transversal stiffnesses were found to be higher than simplified analytical hand calculations would predict. A correlation between low joist stiffness and increased transversal stiffness was seen.

The thesis involves literature study, laboratory testing, verifications of the floors based on the test results, and analytical considerations. Improvements of current vibration design formulas are suggested.

The tested floors are high-frequency and will not suffer resonance problems from walking. The transient velocity response due to heel drop impulses can however be a concern, and the floors do not satisfy high vibration demands. The floors are generally deemed as satisfactory by half of the verification methods.

Sammendrag

Fem trebjelkelagsgulv ble nedbøyningstestet med og uten ekstra platelag, for å finne ut hvor mye tverrstivhet som kan regnes med i vibrasjonsdimensjonering av de forskjellige gulvkonstruksjonene. Tverrstivheten ble funnet til å være høyere enn det forenklede analytiske håndberegninger anslår. En korrelasjon mellom lav bjelkestivhet og økt tverrstivhet ble observert.

Denne masteroppgaven omfatter litteraturstudium, laboratorietesting, vibrasjonsberegninger basert på testresultatene, og analytiske betraktninger. Forbedringer av eksisterende dimensjoneringsformler for gulvvibrasjoner er foreslått.

De testede gulvene har høy egenfrekvens og vil ikke være utsatt for resonansproblemer ved gåing. Hastighetsresponsen fra hælstøt kan imidlertid være et problem, og gulvene tilfredsstiller ikke høye vibrasjonskrav. Generelt bedømmes gulvene som tilfredsstillende av halvparten av verifikasjonsmetodene.

Acknowledgements

Thanks to prof. Roberto Tomasi for introducing me to this topic and pointing me in the right direction throughout the semester.

Thanks also to everyone else who assisted in various ways with the laboratory testing. Especially Roar Økseter helped out a lot.

Thanks to Støren Treindustri AS for supplying the floors, which provided me with a very interesting topic for my master's thesis.

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δ	Deformation/displacement/deflection
Δ	Deformation/displacement/deflection
ζ	Modal damping ratio (ζ is the Greek letter zeta)
a_{rms}	Root-mean-square acceleration
b_{ef}	Effective static width
c	Center distance between floor joists
D	Modal damping ratio
D_{x}	Longitudinal stiffness per unit length (same as $(EI)_{\mathrm{L}}$)
D_{y}	Transversal stiffness per unit length (same as $(EI)_{\mathrm{b}}$)
E	Modulus of elasticity / Young's modulus
EC5	Eurocode 5
e. g.	exempli gratia (Latin phrase meaning "for example")
$(EI)_{\mathrm{b}}$	Transversal stiffness per unit length (same as D_{y})
$(EI)_{\mathrm{L}}$	Longitudinal stiffness per unit length (same as D_{x})
et al.	et alia (Latin phrase meaning "and others")
f_1	Fundamental frequency
G	Shear modulus
i. e.	id est (Latin phrase meaning "that is" or "that is to say")
L	Span length
m	Distributed mass
M^*	Modal mass (sometimes referred to as M_{gen})
NA	National Annex (to Eurocode 5)
v	Velocity
w	Deformation/displacement/deflection

1 Introduction

Norway has a long-standing tradition for timber construction. And with the increasing focus on reducing global warming, this is likely to continue in the years to come, as timber has a much better carbon footprint than concrete, steel and aluminium. [1] [2]

The design of timber floors is very often governed by the serviceability requirements. In part due to the light weight, they are quite sensitive to walking-induced vibrations that feel uncomfortable to the users, as opposed to the much heavier concrete floors that are not associated with such problems. In order for timber to be more competitive economically, there is a desire to find ways to increase the floor spans without reducing the floor performance. A lot of research has been done in the last few decades in order to better understand the vibrational behavior and how it relates to human perception. There is however still no clear consensus on exactly how timber floors should be considered for vibration design.

The new Eurocode 5 is under development, and it is a source of debate how walking-induced vibrations should be calculated and how strict the requirements should be. While these kinds of floor vibrations are not dangerous, people do not want to feel uncomfortable and unsafe when walking. On the other hand, if the vibration requirements are made too strict then timber floors will become too expensive.

This thesis focuses on the verification of vibrational properties of five timber joist floors from Støren Treindustri AS that were laboratory tested. The floors were subjected to static loads and deflections were measured, from which the transversal stiffnesses could be calculated. This input is necessary for all of the verification methods, and analytical calculations tend to underestimate it. We were curious to find out how much the different modifications of the reference floor would improve the transversal stiffness and thereby also the vibrational properties.

The thesis is structured as follows:

Chapter 1: Introduction

Chapter 2: Floor vibration theory

Chapter 3: Verification methods

Chapter 4: Laboratory testing of floors

Chapter 5: Verification of the floors based on the test results

Chapter 6: Analytical considerations

Chapter 7: Conclusion

2 Floor vibration theory

This chapter presents the theoretical background needed to understand the verification methods and the vibrational behavior of the tested floors. Except where otherwise is stated, the main source for chapter 2 is the publication "Design of floor structures against human-induced vibrations" by Hicks and Smith [3]. It is for steel-framed floors, but the basic theoretical foundation is the same as for timber joist floors. All formulas involving sine or cosine use radians and not degrees.

2.1 Continuous and discrete systems

Vibration is related to the movement of mass. Each vibration problem can be classified into either a continuous system or a discrete system. In a continuous system, the mass is directly linked together, such as in a beam in bending. In a discrete system, the masses involved are independent. An example of a discrete system is a multi-storey building subjected to horizontal vibration (e. g. from an earthquake), where the floors are taken as the masses and the columns as the springs. Continuous system vibration problems generally involve solving a differential equation where a continuous function is integrated. Discrete systems are easier to solve, through the help of matrix equations.

To avoid solving a complicated differential equation, a continuous system can instead be solved through numerical methods by transforming it into a discrete system. The most well-known way of doing this is the finite element method (FEM). It is an approximate method, but very accurate as long as the chosen mesh size isn't too coarse.

2.1.1 Continuous systems

The response (i. e. displacement, velocity and acceleration) of a continuous system at a given position and time depends on the mass and stiffness of the system as well as the initial force. For example, a beam in bending will behave in accordance with Formula 2.1. To calculate the natural frequencies of the beam, the forcing function should be set to zero while the proper boundary conditions are applied.

$$m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = F(x,t)$$

Formula 2.1 - Governing equation for beam in bending

where:

m	is the distributed mass
w	is the beam's displacement, as a function of x and t
t	is the time
EI	is the flexural rigidity / bending stiffness
x	is the position along the beam
$F(x, t)$	is the forcing function

2.1.2 Discrete systems

In general, discrete systems are modelled from three components: concentrated masses, springs and dampers. Discrete system problems are solved by considering the forces applied on each mass by the other components and thereby finding and solving matrix equations that link the displacement, velocity and acceleration to the external forces.

A discrete problem is either categorized as a single-degree-of-freedom (SDOF) system or as a multi-degree-of-freedom (MDOF) system. SDOF systems only have one mass, and so they are easy to solve. MDOF systems feature two or more masses and can be coupled in many ways.

A typical SDOF system is depicted in Figure 2.1. There is a simple mass on a spring, connected to a dashpot (viscous damper). This model is useful as it can be used for each mode of a continuous system, with different parameters for each mode, to find the response at each natural frequency.

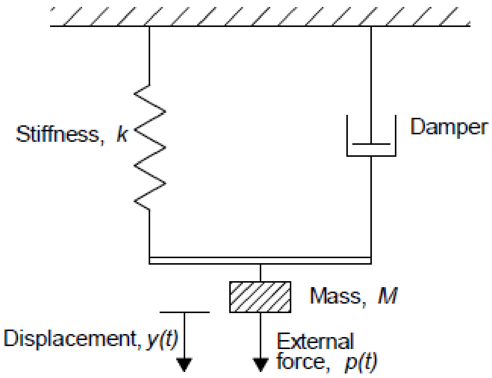


Figure 2.1 - Model of SDOF system with damper

2.2 Frequency

The natural frequencies of a system, with units of either Hz (cycles per second) or radians per second, are a measure of the rate of system vibration. They are essential parameters because the effects of any external forces on a system cannot be predicted before the natural frequencies have been determined. For a given dynamic load to cause a large reaction in a system, its frequency must be within a certain range, not too low and not too high. If the system's frequency is too low for a given dynamic load then the system will not have enough time to react to the load before it is gone, and if the natural frequency is too high then it will be like applying and removing a static load.

2.2.1 Frequency calculation

For free elastic vibration of a beam with uniform cross section (i. e. constant bending stiffness / flexural rigidity), the frequency of the *n*th mode of vibration is found by solving Formula 2.1, which gives this result:

$$f_n = \frac{\kappa_n}{2\pi}\sqrt{\frac{EI}{mL^4}} = \frac{\kappa_n}{2\pi L^2}\sqrt{\frac{EI}{m}}$$

Formula 2.2 - Beam natural frequencies

where:

EI	is the dynamic flexural rigidity [Nm^2]
m	is the effective mass [kg/m]
L	is the span of the beam [m]
κ_n	is a constant dependent on the support conditions for the *n*th vibration mode

f_n will then have the unit of Hertz (Hz). The radial frequency ω_n is obtained by multiplying with 2π and will have the unit of radians per second; $\omega_n = 2\pi f_n$.

Some values for κ_n for various boundary conditions are given in Table 2.1. One can note that for a simply-supported beam, the value of κ_n for mode n is given by $(n\pi)^2$.

Support Conditions	κ_n for mode n		
	$n = 1$	$n = 2$	$n = 3$
pinned/pinned ('simply-supported')	π^2	$4\pi^2$	$9\pi^2$
fixed both ends	22.4	61.7	121
fixed/free (cantilever)	3.52	22	61.7

Table 2.1 - κ_n coefficients for uniform beams

One way of finding a beam's fundamental (i. e. the lowest) natural frequency, denoted f_1 (or f_0 in some literature), is to use the maximum deflection δ caused by the weight of a uniform mass per unit length m. For a simply-supported beam ($\kappa_1 = \pi^2$) with a uniformly distributed load, this expression (where g is the gravitational acceleration of 9.81 m/s^2) is to be used:

$$\delta = \frac{5mgL^4}{384EI}$$

Formula 2.3 - Deflection of simply-supported beam from uniformly distributed load

Only loads considered to be permanent should be included in the calculation of δ. By rearranging Formula 2.3 and substituting into Formula 2.2 along with the value for κ_1, while changing the unit of δ from m to mm, this relation between the fundamental frequency and the maximum deflection is obtained:

$$f_1 = \frac{17.75}{\sqrt{\delta}} \approx \frac{18}{\sqrt{\delta}}$$

Formula 2.4 - Relation between fundamental frequency and maximum deflection

A numerator of approximately 18 is also found even for different boundary conditions if the same procedure is performed, with the applicable equation for δ and the appropriate κ_n value. Formula 2.4 therefore applies generally and can be used directly for design purposes to determine the natural frequency of individual members with different boundary conditions, as long as δ is determined correctly.

To find the fundamental natural frequency (f_1) of a floor system, Dunkerly's approximation can be used. The f_1 of each individual member (e. g. primary (f_p) and secondary beams (f_b), and slab (f_s)) should then be calculated from Formula 2.4 as described above, and then be input into Formula 2.5 to calculate the f_1 of the whole system. Alternatively, using Formula 2.4 directly will also yield the same result when δ is taken as the sum of the deflections of the individual structural components.

$$\frac{1}{f_1^2} = \frac{1}{f_s^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}$$

Formula 2.5 - Dunkerly's approximation

2.2.2 Mode shapes

Continuous systems have multiple natural frequencies, each with its own associated mode shape. A mode shape shows the shape of the system at maximum deflection. The fundamental frequency corresponds to the first mode shape, which will always be the simplest mode shape. The first three mode shapes of a uniform simply-supported beam are as shown in Figure 2.2. The first mode shape is like that of half a sine wave, the second is like one full sine wave, and the third is like one and a half sine wave. The modes of single span floors have the same shape [4]:

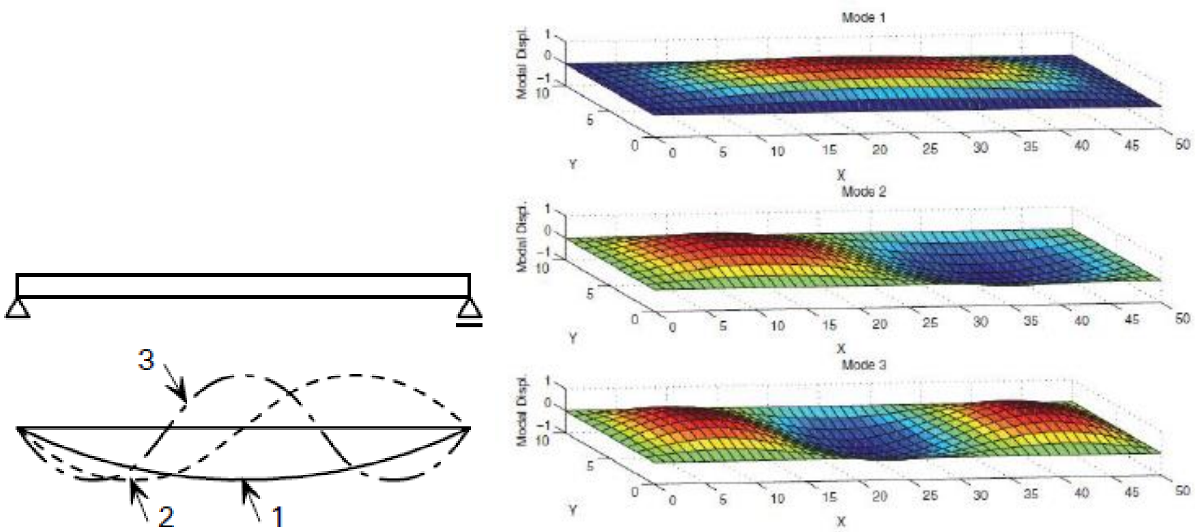


Figure 2.2 - Mode shapes of uniform simply-supported beam and single span floor

In Figure 2.2, the max deflection (the amplitude) is presented as equal for all of the mode shapes even though this won't be the case in practice. For some purposes it is however common to unity-normalize them, i. e. set all of the mode shape amplitudes equal to 1 (unitless). The beam in Figure 2.2 can be expressed mathematically like this (where $\mu_n(x)$ is positive downward):

$$\mu_n(x) = \sin\left(\frac{n\pi x}{L}\right)$$

Formula 2.6 - Mode shapes of uniform simply-supported beam

where:

$\mu_n(x)$	is the unity normalized amplitude at position x
n	is the mode number (a positive integer; $n = 1, 2, 3\ldots$)
x	is the position along the beam; $0 \leq x \leq L$
L	is the span of the beam

To get the displacement of any point at any given time, the shape function $\mu_n(x)$ can be multiplied by an amplitude function $g_n(t)$ varying with the time t, dependent on the frequency of motion f:

$$g_n(t) = \sin(2\pi f t)$$

Formula 2.7 - Time-varying amplitude function

2.2.3 Modal superposition

To find the actual displacement of a system at any given time, the principle of superposition needs to be applied; the contributions from all of the modes should be added together. In the case of one sinusoidal forcing function of frequency f, the total response $w_n(x, t)$ is:

$$w_n(x,t) = \sum_{n=1}^{\infty} u_n \sin(2\pi f t + \phi_n) \sin\left(\frac{n\pi x}{L}\right)$$

Formula 2.8 - Total modal response, displacement of simply-supported beam

where:

$w_n(x,t)$	is the displacement of the beam at time t and position x
t	is the time
f	is the frequency of the forcing function
u_n	is the maximum amplitude of mode n
ϕ_n	is the phase lag of mode n

u_n and ϕ_n are determined from the forcing function or the initial excitation.

2.2.4 Modal mass

A system's modal mass is a measure of how much mass is involved in the mode shape, and thereby how much kinetic energy there is in the system. For a continuous system, a modal mass is determined for each mode so that the system can be expressed as a series of SDOF discrete systems. The modal mass is found via the well-known equation for kinetic energy (where the velocity is a function of time); $E_k = \frac{1}{2}mv^2$. But for this purpose, the equation becomes much more complex than that. A double integral has to be solved, and the modal mass is then found via the unity normalized kinetic energy. These equations are not presented here.

The modal mass for each mode indicates how much the mode contributes to the overall response of the system, for an equal modal force. A large modal mass means that a lot of energy is required to excite the mode, which makes its contribution to the total response less significant than that of a small modal mass.

For numerical analysis with vibration dose values (VDV), the modal mass can be determined by performing a modal analysis with a finite element software. In many of the code-based verification methods however, the modal mass is instead approximated via simple equations.

2.3 Excitation

2.3.1 Continuous forcing function

A system's response to a continuous excitation is found via u_n (max amplitude of mode *n*) and ϕ_n (phase lag of mode *n*) from Formula 2.8. This formula is however only applicable for the case of one sinusoidal forcing frequency, while for most practical purposes the forcing function will be more complex than such. Luckily, mathematicians have discovered that a more complicated continuous forcing function can be split up into a series of sine waves, each of which will have a frequency at an integer multiple (or harmonic) of the forcing frequency. By doing this, the overall response can be established. This set of harmonics are known as a Fourier series, where each harmonic will have its own amplitude and phase shift.

An example of such a set is shown below in Figure 2.3, with the four first harmonics (of the Fourier series) for the excitation force due to low impact aerobics. The total force is the sum of the harmonics (plus the static), and we can also observe that all of the harmonics have an extremum in sync with the total force, hence the word harmonic, due to the integer multiple frequencies. We can also see from the decreasing max amplitudes that the higher harmonics are less significant in terms of force and energy.

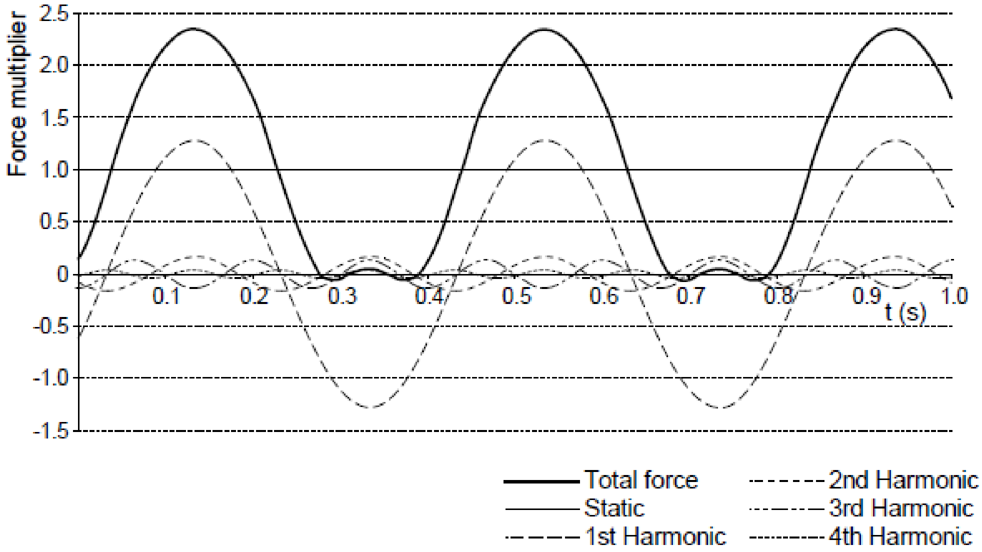


Figure 2.3 - Fourier series for light aerobic activities

2.3.2 Impulsive force

In a high frequency floor (commonly defined as having f_1 higher than the fourth harmonic of walking), the response from one footstep will dissipate before the next, and then the forcing function will resemble a series of separate events rather than a continuous function. The modelling can then be simplified by using impulses to describe the footfall forces. Mathematically, a unit impulse is an infinite force over an infinitesimal time, with the multiple of force and time equal to 1. While that's not physically possible, it's useful as a model.

2.4 Response

2.4.1 Acceleration

In many code-based methods, vibration requirements are verified through threshold values for acceleration rather than displacement.

As known from basic physics courses, acceleration is the second derivative of the displacement with respect to the time t; a(t) = v'(t) = s''(t). By deriving the first derivative of Formula 2.8, this expression for the acceleration of a simply-supported beam (as a function of the position *x* along the beam and the time *t*) is found:

$$a(x,t) = \sum_{n=1}^{\infty} -4\pi^2 \, f_n^2 \, u_n \sin(2\pi f_n t + \phi_n) \sin(\frac{n\pi x}{L})$$

Formula 2.9 - Acceleration of simply-supported beam

The acceleration of a system can be presented in many ways. It is often natural to measure the peak acceleration (a_{peak}), however this provides no information about the timeframe of high-level acceleration. Therefore, the root-mean-square (rms) acceleration is often used as a measure instead:

$$a_{rms} = \sqrt{\frac{1}{T}\int_{0}^{T} a(t)^2 \, dt}$$

Formula 2.10 - rms acceleration

where:

T	is the period under consideration
$a(t)$	is the acceleration function
t	is the time

The period T needs to be chosen as a time period that will cover at least one complete cycle of the acceleration. Walking has a mean frequency of 2 Hz (i. e. two steps per second), and the recommendation of ISO 2631-1:1997 is to then use a period T of 1 second when calculating the response.

For vibration dose values (VDV), the root-mean-quad (rmq) acceleration is used. It is calculated in a similar way (different exponents), and it gives more emphasis to the higher values of acceleration.

To better understand the physical meaning of these quantities, Table 2.2 is helpful. For example, a square wave instantly shifts between the extrema, and so the rms (as well as the rmq) acceleration is then equal to the peak acceleration. A sine wave with the same peak acceleration will however have a lower rms as it varies between high and low values.

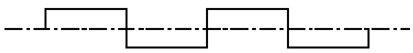
Waveform		a_{peak}	a_{rms}
	Sine	1	$1/\sqrt{2}$
	Triangular	1	$1/\sqrt{3}$
	Square	1	1

Table 2.2 - Acceleration measures for various waveforms

2.4.2 Damping

Damping is an influence within or upon a system that reduces, restricts or prevents its vibrations, to an eventual stop. The associated energy is either dispersed or dissipated from the system. Structural damping is provided from friction between components and slip at joints, and from the contents of a room (the furniture will remove vibrational energy by moving or vibrating themselves). The material damping of timber is quite low and contributes less to the total damping.

The damping ratio ζ is a unitless measure describing how quickly system vibrations decay with each bounce after a disturbance, as shown in the figure below. The general cases are undamped (ζ=0), underdamped (ζ<1), critically damped (ζ=1) and overdamped (ζ>1). Timber floors are very much underdamped, and appropriate damping ratios (generally in the range of 0.01 to 0.05), are presented for various floor types in chapter 3.

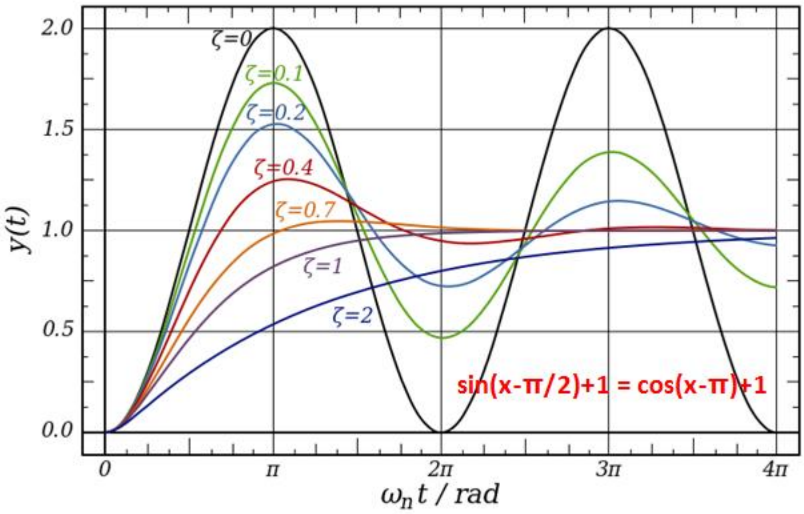


Figure 2.4 - The effect of various damping ratios on oscillations

It can be difficult to estimate the level of damping, and so it is necessary to base the design on damping values that have been appropriate for similar cases in the past. The recommendations in the table below (for steel-framed floors, but also applicable in principle for timber floors) show the effect of furniture and partition walls on the damping ratio:

ζ	Floor finishes
1.1%	for completely bare floors or floors where only a small amount of furnishings are present.
3.0%	for fully fitted out and furnished floors in normal use.
4.5%	for a floor where the designer is confident that partitions will be appropriately located to interrupt the relevant mode(s) of vibration (i.e. the partition lines are perpendicular to the main vibrating elements of the critical mode shape).

Table 2.3 - The effect of furniture etc. on the damping ratio

While partitions are not dampers, they are usually modelled as such, as a simplification. The weight of people also provides some damping; however, this is usually ignored in design in order to be conservative.

2.4.3 Transient and steady state

The response of a system to a regular excitation can be split into two parts; the transient response and the steady-state response. As shown in Figure 2.5 for the displacement $u(t)$ for a harmonic force, the steady-state part of the response is the (total) response when the waveform has settled down. The transient response, which is the difference between the total and the steady-state response, is only significant in the beginning before it is gradually nullified due to damping. [5]

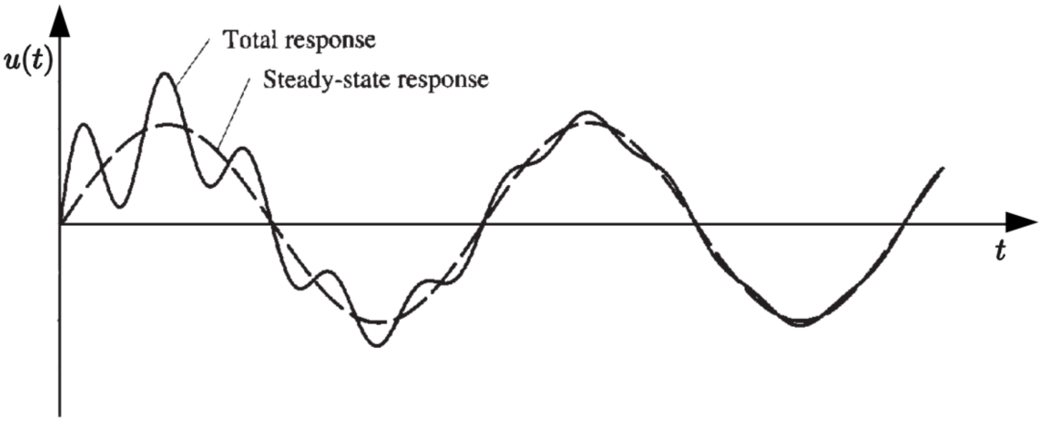


Figure 2.5 - Response of damped system to harmonic force

The acceleration response will take the shape of (a) or (b) in Figure 2.6, where the steady-state response is the same in both graphs. The difference is that for (b), the transient response is also significant and gives a high total acceleration initially.

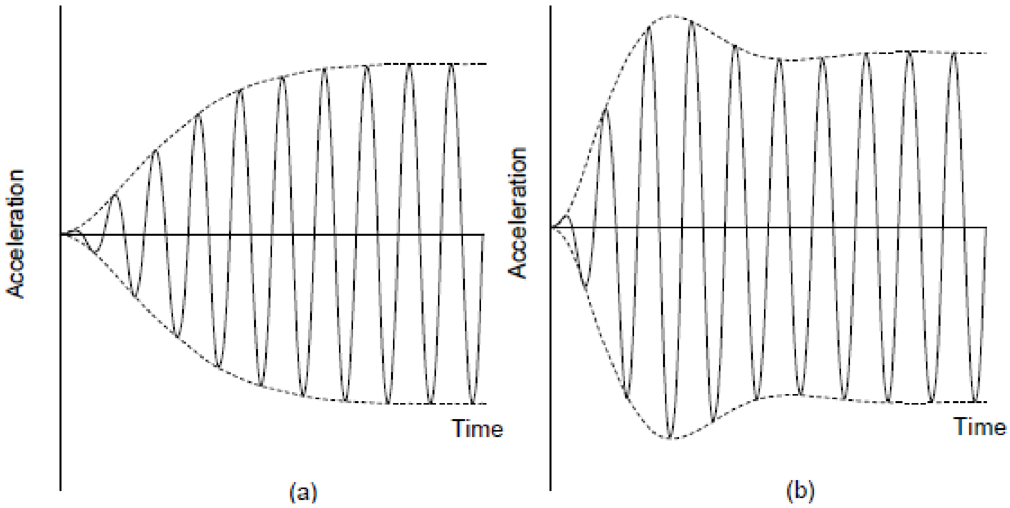


Figure 2.6 - Total acceleration responses

If the floor's frequency is high in relation to that of the forcing frequency (for example higher than 10 Hz, with walking activities below 2.5 Hz, respectively), then the transient part of the (total) response will be much more significant than the steady-state. The applied force can then instead be modelled as a series of impulses rather than as a continuous forcing function. The impulsive acceleration response will then be as in Figure 2.7:

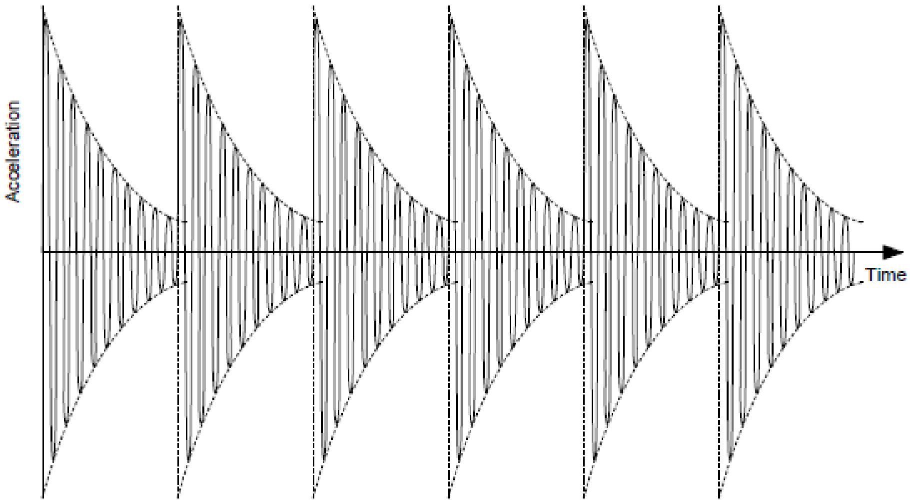


Figure 2.7 - Impulsive acceleration response

For design, it is common to make the conservative assumption that the force is applied at the most responsive location of the floor. In reality, the walking path will only pass across this point for a moment, but an analysis based on this becomes more complex.

For low frequency floors (where f_1 is lower than the appropriate value from Table 2.4), both the transient and the steady-state response need to be checked, because the transient response could be larger than the steady-state due to the higher frequencies of the floor. The steady-state response is large when one or more of the harmonics of the walking are close to one of the floor's natural frequencies. All vibration modes with natural frequencies up to 2 Hz higher than the relevant value from Table 2.4 should then be considered, to account for off-resonant vibration of the activity's highest harmonic.

Floor type	Low to high frequency cut-off
General floors, open plan offices etc.	10Hz
Enclosed spaces, e.g. operating theatre, residential	8Hz
Staircases	12Hz
Floors subject to rhythmic activities	24Hz

Table 2.4 - Cut-off limit between high frequency floors and low frequency floors

For high frequency floors it is sufficient to only check the transient response. For transient analysis, the response is dominated by a number of impulses, corresponding to the heel impacts from a person. Natural frequencies higher than twice the f_1 will contribute very little to the response and can therefore be neglected. (In the Eurocode 5 approach, the line is drawn at 40 Hz.)

2.4.4 Resonance

The response to a dynamic force is higher when the excitation frequency is close to a natural frequency of the system. As an example, a steady-state rms acceleration response due to a constant cyclic force applied at various excitation frequencies is shown in Figure 2.8. The graph peaks when the excitation frequency equals one of the natural frequencies, and this is known as resonance. The response in between peaks is known as off-resonant response, and it is also significant.

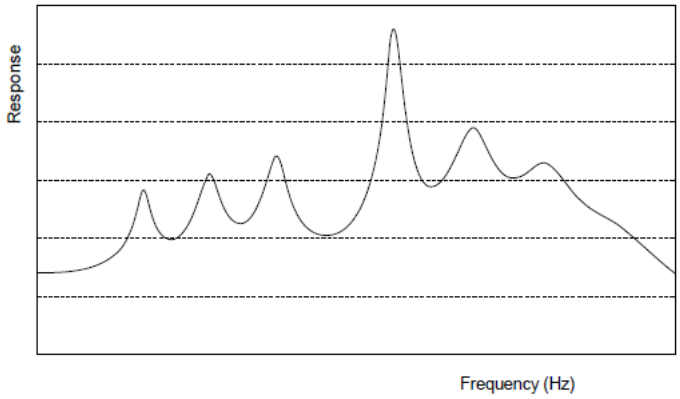


Figure 2.8 - Resonant and off-resonant response

The most commonly cited example of resonance is the famous and spectacular video showing the collapse of the Tacoma Narrows Bridge. However, this is actually an example of aeroelastic flutter, not resonance, as explained in an article by Billah and Scanlan written in 1990. [6]

The dynamic magnification factor $D_{n,h}$ (often abbreviated as DMF, DAF or DIF) for acceleration is a dimensionless number that gives the ratio between the peak amplitude and the static amplitude. It describes how many times larger the amplitude will be from a dynamic load compared to if it had been a static load. It is calculated as follows:

$$D_{n,h} = \frac{h^2\beta_n^2}{\sqrt{(1-h^2\beta_n^2)^2+(2h\zeta\beta_n)^2}}$$

Formula 2.11 - Dynamic magnification factor

where:

h	is the number of the hth harmonic
β_n	is the frequency ratio; f_p/f_n, where f_p is the frequency of the first harmonic of the activity, and f_n is the frequency of the mode under consideration
ζ	is the damping ratio

A large response (resonance) results when the denominator nears zero. From Figure 2.9 we see that this happens when the frequency ratio β_n is close to 1 and the damping ratio is low.

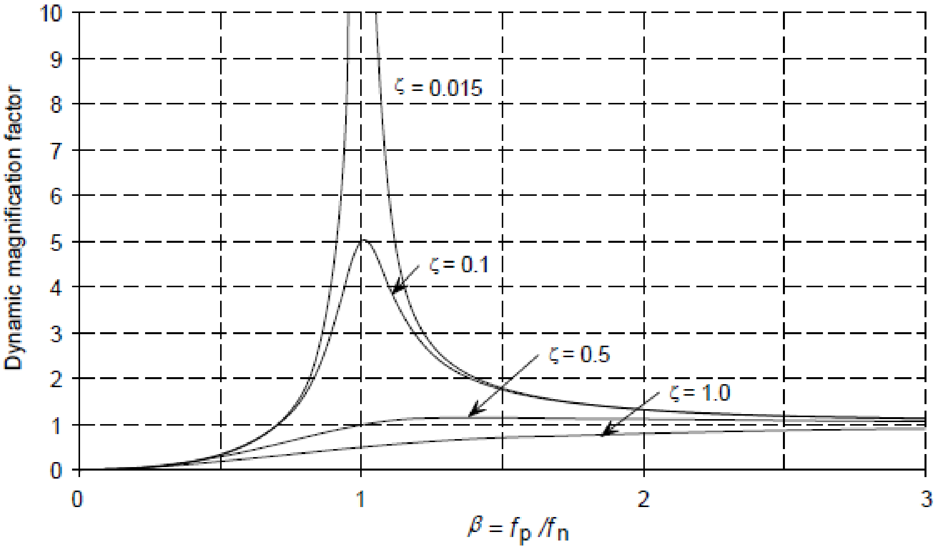


Figure 2.9 - Dynamic magnification factor for acceleration

The damping ratio of a timber floor is usually in the range of 1-5 %, which as shown in Figure 2.9 is far from enough to avoid a large amplification when a natural frequency of the floor closely matches the frequency of its intended activity, be it walking or dancing. Floors should be designed to have a fundamental frequency high enough to at least avoid off-resonant vibration from the first harmonic of walking, which has a much larger amplitude than the other harmonics. That would imply 3 Hz as a minimum for f_1, but most code-based methods for timber floors don't allow anything lower than 4.5 Hz.

2.5 Sources of vibration – dynamic excitation forces

Each harmonic of the loading function can cause resonance with a natural frequency of the floor, and it is the walking pace frequency that gives the worst-case response that should be used for design. The pace frequency (f_p) for walking has a mean value of 2.0 Hz. It can vary between 1.5 and 2.5 Hz, but the f_p range for design can be narrowed down to 1.8 – 2.2 Hz. Within enclosed spaces where slower walking speeds are expected, f_p should be set to 1.8 Hz. The velocity as a function of the pace frequency can be approximated by this empirical formula (applicable for f_p between 1.7 and 2.4 Hz):

$$v = 1.67{f_p}^2 - 4.83f_p + 4.50$$

Formula 2.12 - The velocity of walking with a given frequency

For a pace frequency of 2.0 Hz, the walking velocity is then 1.52 m/s, or 5.47 km/h.

The forcing function from walking is periodic and typically looks like this [4]:

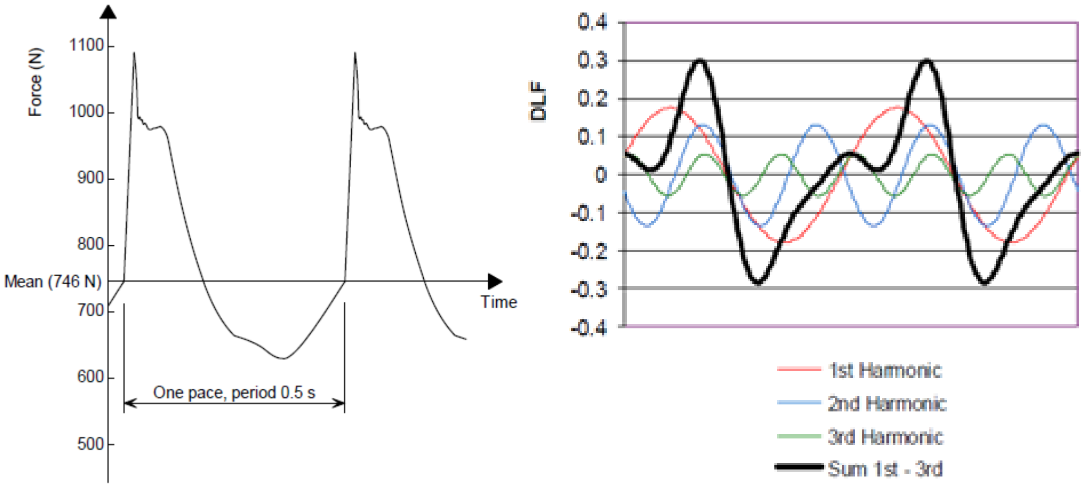


Figure 2.10 - Dynamic load function for continuous excitation from walking

For resonant loading, the forcing function can be idealized by up to four significant harmonics. The amplitude of the harmonic force for the *h*th harmonic, F_h, is a product of the Fourier coefficient α_h for the *h*th harmonic (α_h is named DLF in the figure above) and the static force *Q* caused by an average person (equal to 76 kg = 746 N in the figure above):

$$F_h = \alpha_h * Q$$

Formula 2.13 - Amplitude of harmonic force

The figure below gives the harmonic loading due to footfall in terms of dynamic load factors (abbreviated DLF, equal to α_h), where DLF is the ratio between the harmonic force amplitude and the static weight of the walker. Most of the applied force is in the first harmonic, and the subsequent harmonics are of decreasing significance. The force in a given harmonic increases with the walking frequency. Many of the code-based methods have tried to describe this with a single formula, like the one from the Austrian National Annex [7] to Eurocode 5 that is plotted in the same figure:

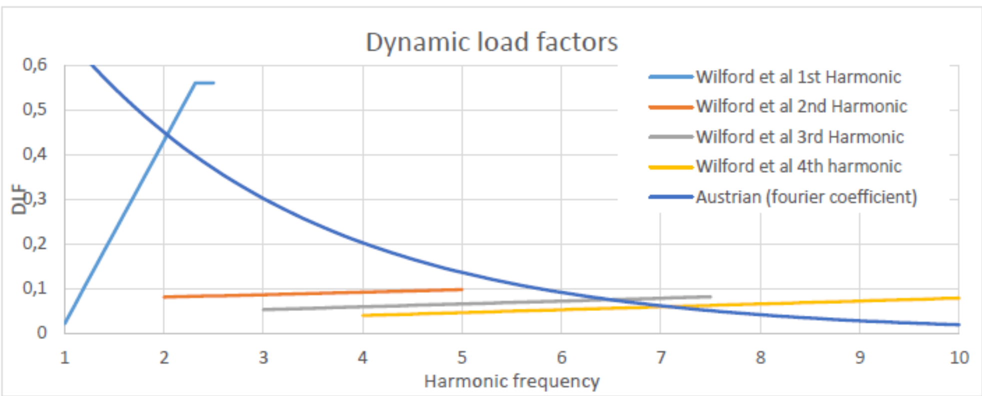


Figure 2.11 - Loading model for when response is expected to be resonant

The root mean square acceleration can be approximated by this formula [4]:

$$a_{rms} = \frac{F_h}{\sqrt{2}\,2\,\zeta\,M^*} = \frac{\alpha_h * Q}{\sqrt{2}\,2\,\zeta\,M^*} = \frac{e^{-0.4f_1} * Q}{\sqrt{2}\,2\,\zeta\,M^*}$$

Formula 2.14 - rms acceleration calculated via Fourier coefficient

Here, M^* is the modal mass and ζ is the damping ratio. Similar formulas are presented in chapter 3.

For staircases, the human loading during ascent and descent is different both in force and frequency versus walking on flat surfaces. The staircase loads are generally larger and more high-frequency, with expected pace frequency range of 3 – 4 Hz, or possibly up to 4.5 Hz. Only the two first harmonics of dynamic loads induced on stairs need to be included for design, as oppose to the first four harmonics from walking on flat surfaces.

For floors that are to be subjected to rhythmic activities, with multiple people in synchronized movements (e. g. dancing or aerobics), the frequency range should be set to 1.5 – 3.5 Hz for individuals and 1.5 – 2.8 Hz for groups. This covers the increased activity due to jumping; the worst-case scenario for crowd loading. Groups have a lower frequency range because it is harder for a big crowd to sustain a high frequency. It is recommended that the crowd density for rhythmic activities should be set to 2.0 persons/m^2 for social dancing activities, and to 0.25 persons/m^2 for aerobic and gymnasium activities.

2.6 Structural considerations

The distributed mass for vibration analysis must be representative of the in-service conditions, as a high mass will reduce a floor’s response at a given frequency. The design mass per unit area should be the dead loads (including ceiling self-weight and possibly some of the service load), with partial factor $\gamma = 1$. If the bare structure is to be analyzed, then only the self-weight of the structure itself should be included. When the designer is confident that a semi-permanent load will be present in the finished structure, it can also be included (not applicable for dance- or aerobic floors). For the quasi-permanent design situation in the Eurocodes, $\Psi_2 = 0.3$ for deflection from imposed loads in the serviceability limit state for offices and residential buildings. However, including as much as 30 % of the imposed load is excessive in most cases. So, only the loads that can reasonably be assumed to be present at all times during service should be included. Either imposed loads can be ignored completely, or the nominal imposed load can be multiplied by 10 % or less.

2.7 Architectural considerations

Some areas of a floor will have a higher response than others due to the vibration mode shapes. Areas close to beams and columns tend to be less responsive than areas in the middle of the floor. So, if walking paths are moved closer to the less responsive areas in the design phase, vibration problems can be mitigated. The length of corridors also matters, as the walking time increases with the length, which increases the vibration dose. The walking pace frequency will also be higher in open areas than within enclosed areas, which also increases the vibration dose.

Due to the possibility of vibration transfer, offices and residential locations should ideally be separated from floors where rhythmic activities take place. And due to possible force transfer, the vertical placement should also be considered. Areas used for rhythmic activities should ideally be as low in a building as possible.

2.8 Human perception of vibrations

Vibration of floors is generally a serviceability issue related to discomfort. Other potential problems can be related to disturbance of sensitive equipment or crack growth leading to fatigue, but the latter is generally only a concern for bridges, offshore structures, airplanes etc. that are subject to more severe dynamic loading from traffic, waves and wind, respectively.

Discomfort and human perception of vibrations is highly individual and cannot be directly quantified, and as such there is no way to guarantee that 100 % of the users will be satisfied with a floor’s vibration response. As a rule of thumb in the construction industry (ref. Povl Ole Fanger), 5 % of people will be dissatisfied no matter how good any measured values are, whether related to air quality, thermal comfort or floor vibrational properties. And as such, the goal of the design standards is simply to reduce the probability of complaints.

Humans are quite perceptible to relatively low levels of vibrations. At the same time, a relatively large change in vibration amplitude corresponds to a relatively small change in perception. If a person is asked to evaluate the level of vibration in two different rooms on separate occasions, it is unlikely he/she will notice a difference unless the quantitative difference is at least a factor of 2. This means that to improve the subjective evaluations of a floor, it has to be modified quite a lot.

Subjective evaluations also differ a lot between different people, and there can be some cultural differences. Personal discomfort also depends on the situation; for example, a surgeon focusing on an operation will be more perceptible to lower levels of floor vibrations than a spectator at a football match, and this along with the level of importance is accounted for in the acceptance criteria in various design standards.

The level of acceleration that can be perceived depends on how the body is positioned in relation to the vibration. Figure 2.12 shows the commonly used basicentric coordinate system where the z-axis follows the spine. The human body doesn’t perceive z-axis vibrations as easily as it does for x- and y-direction.

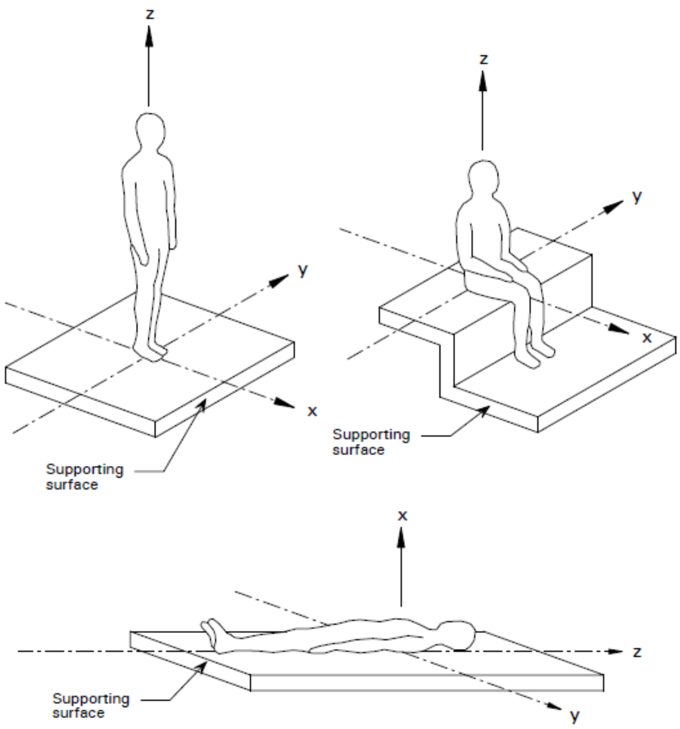


Figure 2.12 - Basicentric coordinate system for vibration directions for the human body

The perception of vibration also depends on the frequency, because the body's sensitivity to a given vibration amplitude changes with the vibration frequency. Much in the same way as humans can't hear dog whistles or see ultraviolet light because the output frequencies are outside of the perceptible range, the human body has a variable range of maximum sensitivity to vibration frequencies. This can be accounted for in the design through frequency weighting. An example of that is shown in Figure 2.13, where the relevant curve (based on vibration direction and the activity) gives the weighting factor to be used in design for a given vibration frequency. If for example the W_g curve is to be applied, a sine wave of 5 Hz will be equivalent to a 17 Hz sine wave with double the amplitude because the weighting factors are 1 and 0.5, respectively.

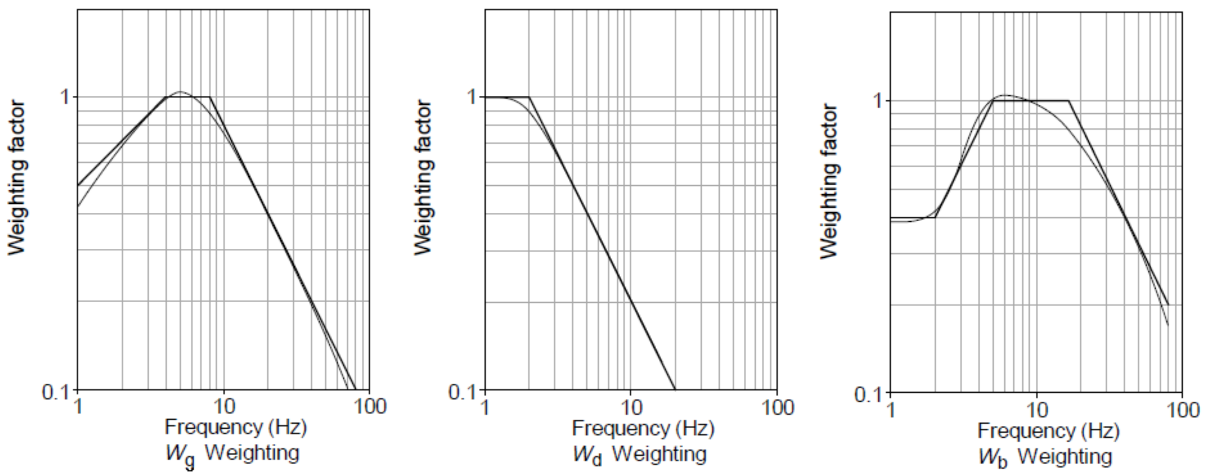


Figure 2.13 - Frequency weighting curves

3 Verification methods

There is still some unexplored territory in the research of floor vibrations, and to this date there is no clear consensus on how the vibrational properties should be assessed and verified, particularly due to the dynamic aspects of it.

A floor's vibration performance is determined by its stiffness, mass and damping. Stiffness and mass determine its natural frequencies, and damping reduces the duration of induced vibrations. Current verification methods focus on some of the key parameters that have been proven to correlate well with subjective assessments; namely frequency, deflection/stiffness, velocity and acceleration. Stiffness is important to avoid noticeable vertical displacements when walking, and the floor's natural frequencies determine whether the response to a given dynamic excitation will be transient or resonant. An acceleration criterion is generally only proposed for low frequency floors because it relates to resonance problems. Velocity is more directly linked to the energy involved in the structural movement, and high frequency floors subjected to impulses from heel impacts require verification of the transient velocity response by many of the code-based methods. Damping is a sensitive parameter in some of the dynamic verifications and it is very difficult to estimate accurately.

This chapter presents the verification methods most commonly used for timber floors, and some of the research behind them. Verifications of the tested floors (presented in chapter 4) based on these methods is done in chapters 5 and 6.

3.1 Hamm/Richter/Winter

A research project at the Technical University of Munich by Patricia Hamm, Antje Richter and Stefan Winter investigated the vibrational properties of timber floors. Their findings were presented at the 2010 World Conference on Timber Engineering [8]. Measurements of about 100 floors from 50 buildings were carried out, in addition to laboratory tests. The results (frequency, deflection, velocity and acceleration) were coupled with the subjective evaluations of the floors, and this resulted in rules and suggestions on how to design timber floors; for higher demands and for lower demands.

The measured floor values were: velocity and damping after heel drop, natural frequency from heel drop or jump, and the acceleration due to walking (if possible, walking with a step frequency of 1/2 or 1/3 of the natural frequency).

These values were calculated for the floors: natural frequency, velocity due to heel drop, static deflection from single load, and the acceleration due to walking in resonance with the second or third harmonic of the Fourier series.

The subjective evaluations were done by Hamm and Richter themselves (which opens up for confirmation bias if the floor configuration is known in advance), and also by the users if the floor was already in use. The floors were graded from 1 to 4, in the same way as previously done by Kreuzinger/Mohr [9], where a grade of 1 means no vibration problem and 4 means heavy vibration problem. Afterwards, the subjective assessments were paired with all of the measured and calculated values individually to see if a correlation could be established.

Generally, timber-concrete composite systems (1.2) had the best scores, followed by "special constructions" (1.7) and timber floors with heavy screed (2.0). The numbers in parentheses are my own (roughly) estimated average grades from studying the figures. The worst performers in general were floors with light screed (3.2) or no floor finish (3.1), and "elastic bearing" (3.1).

Neither the calculated natural frequency nor the measured one correlated sufficiently with the subjective evaluations. That is not to say that the natural frequency is not important as a criterion; it rather shows that other criteria are needed in addition.

In nearly all cases, the measured natural frequencies were a lot higher than the calculated ones. This was because the assumptions for the calculations were too conservative. For example, supports calculated as pinned were in reality influenced by a torsional spring due to the walls in the above storeys or the roof loads. Another source of error was that partition walls that were calculated as not load bearing also contribute with some stiffness to help reduce vibrations. To eliminate such variables, laboratory tests were also performed.

For the laboratory measurements, the square test floor had a width and span of 5.0 m. Different floor finishes and support conditions were tested. The fundamental frequency and the damping ratio was measured for 12 different floor configurations. The f_1 ranged from 9.5 to 15.0 Hz while the damping ratio varied between 2.2 and 4.6 %. Afterwards, similar testing was also done for CLT (cross laminated timber) floors.

Based on all of the research done, a verification method was suggested. The design rules and demands are summarized in Table 3.1, from [8]:

demands regarding the vibrations	floors with higher demands	floors with lower demands	floors without demands
evaluation	1,0 to 1,5	1,5 to 2,5	2,5 to 4,0
installation position	floors between different units of use	floors between one unit of use	
during the research project examined type of use	e.g. corridors with low spans, e.g. floors between different users, floors in apartment buildings or floors in office buildings	e.g. floors within a single-family house, floor in existing buildings or with agreement of the owner	e.g. floors under not used rooms or in not developed attic storeys
description of perception of vibrations	Vibrations are not perceptible or only perceptible when concentrating on them. Vibrations are not annoying.	Vibrations are perceptible but not annoying.	Vibrations are clearly perceptible and sometimes annoying.
frequency criterion $f_e \geq f_{limit}$	$f_{limit} = 8\ Hz$	$f_{limit} = 6\ Hz$	-
stiffness criterion / deflection due to single load $w(2kN) \leq w_{limit}$	$w_{limit} = 0{,}5\ mm$	$w_{limit} = 1{,}0\ mm$	-
additional examination / acceleration, if $f_e < f_{limit}$	$f_{min} \leq f_e < f_{grenz}$ where $f_{min} = 4{,}5\ Hz$ and $a_{limit} = 0{,}05\ m/s^2$	$f_{min} \leq f_e < f_{grenz}$ where $f_{min} = 4{,}5\ Hz$ and $a_{limit} = 0{,}10\ m/s^2$	-
demands on construction	set up of floating screed, heavy screed or light screed on grit fill or not, see table		-

Table 3.1 - Hamm/Richter/Winter design rules summary

The various construction demands are shown in Table 3.2, from [8]. Floating heavy screeds are better than floating light screeds due to the higher mass and stiffness. “Heavy fill” in the table means 60 kg/m^2 or above, and this extra mass also improves the vibration behavior.

	floors between different units of use	floors between one unit of use
Type of construction	evaluation 1,0 to 1,5	evaluation 1,5 to 2,5
Timber concrete composite systems	no more demands	no more demands
Massive timber floors, e.g. cross laminated timber or nail laminated timber floors	heavy floating screed on a light or heavy fill	heavy floating screed (fill not necessary)
	light floating screed on a heavy fill	light floating screed on a heavy fill
timber beam floors	heavy floating screed on a heavy fill	heavy floating screed (fill not necessary)
	probably not possible	light floating screed on a heavy fill

Table 3.2 - Hamm/Richter/Winter demands on construction

3.1.1 Frequency criterion

The natural frequency (measured or calculated) of floors should be higher than limit frequencies (dependent on the demands) to avoid resonance from walking persons. If lower than the limit frequency, an acceleration criterion must be met while still staying above a given minimum frequency.

The live load mass can be neglected when calculating the natural frequency, despite what Eurocode 5 says. The stiffness of screed can be added to the stiffness of the construction. The support conditions should be considered; if the floor is supported on more than two sides or if it is like a continuous beam then this can be regarded, and if it has elastic bearings (e. g. a beam below) then that must be regarded.

The fundamental natural frequency of single spanning simply supported floors can be calculated from Formula 3.15. If there are supports on four sides, the frequency of a plate can be calculated from this formula:

$$f_{\text{plate}} = f_{beam} \sqrt{1 + 1/\alpha^4}$$

Formula 3.1 - Fundamental frequency of plate

where:

f_{beam} is the single-span floor fundamental frequency, calculated from Formula 3.15

$$\alpha = \frac{b}{L}\sqrt[4]{\frac{(EI)_L}{(EI)_b}}$$

b	is the width of the floor
$(EI)_L$	is the effective longitudinal stiffness (screed included)
$(EI)_b$	is the effective transversal stiffness (screed included)

3.1.2 Deflection/stiffness criterion

The Hamm/Richter/Winter research project as well as others show that a criterion for deflection or stiffness is at least as important as a frequency criterion [9]. The deflection is calculated from Formula 3.2 (with a 2 kN concentrated load chosen as default, and limit value according to demands):

$$w(2\ \mathrm{kN}) = \frac{2L^3}{48(EI)_\mathrm{L}\, b_\mathrm{ef}} \leq w_\mathrm{limit}$$

Formula 3.2 - Hamm/Richter/Winter deflection criterion

$$b_{ef} = \frac{L}{1.1}\sqrt[4]{\frac{(EI)_b}{(EI)_L}} = \frac{b}{1.1\,\alpha}$$

Formula 3.3 - Effective width

The effective width b_{ef} cannot be larger than the width of the floor. (It could also be argued that b_{ef} must be at least as large as the joist spacing because a single joist has to at least carry the load in its vicinity.)

Screed stiffness and elastic bearings are considered in the same way as for the frequency calculation. If the floor is supported on four sides it can be calculated as a beam grid. Floors supported on two sides are calculated according to Formula 3.2, where the original system is transferred to a substitute system of a simply-supported beam, and the transversal stiffness is factored in through b_{ef}.

While the correlation between frequency and subjective evaluation was not good, it was easier to see a relation between stiffness (via the deflection calculated with b_{ef}) and the ratings. In Figure 3.1 from [8] this is plotted along with the verification demands (outlined in green):

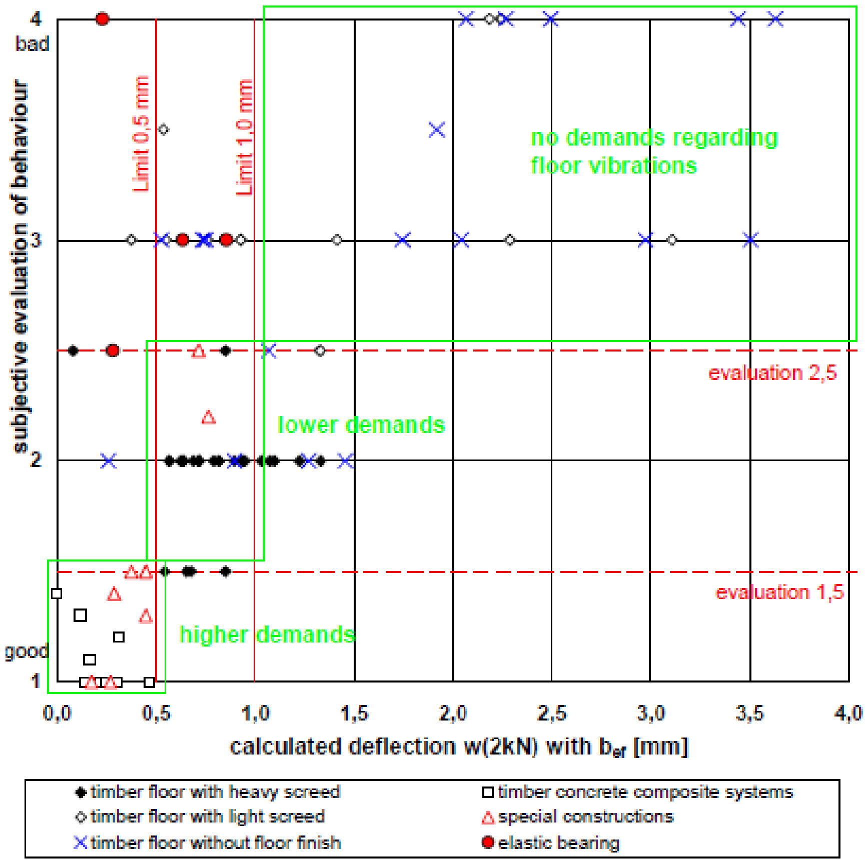


Figure 3.1 - Hamm/Richter/Winter deflection vs. evaluation

3.1.3 Acceleration criterion

The frequency criterion is especially limiting for long-span floors as f_1 is inversely proportional to L^2. However, the results showed that long-span floors can still function well even if below f_{limit}; the two conditions being that the natural frequency must be above f_{min} = 4.5 Hz while the acceleration due to walking in resonance with 1/2 or 1/3 of the natural frequency is less than a_{limit} from Table 3.1. For the acceleration to be below the limit value, the floor generally has to be quite heavy or have a long span. The acceleration is verified through this criterion:

$$a = \frac{F_{\mathrm{dyn}}}{M^{*}\ 2D} = \frac{0.4\ F(\mathrm{t})}{m\ 0.5L\ 0.5b\ 2D} \leq \mathrm{a}_{\mathrm{limit}}$$

Formula 3.4 - Hamm/Richter/Winter acceleration criterion

where:

$M*$	is the modal mass of the floor
L	is the span of the floor
b	is the width of the floor, limited by: $b \leq 1.5\ L$
m	is the distributed mass (per floor area)
D	is the damping of the structure, taken from Table 3.3
F_{dyn}	is the total dynamic force
$F(t)$	are the harmonic parts of the force on the floor, taken from Figure 3.2

SI units are used for all formula parameters. The factor 0.4 accounts for that the force is acting during a limited time and not always in the middle of the span. [9] The harmonics of *F*(t) depend on the natural frequency of the floor and can be taken from the figure below. For most of the natural frequency range that's relevant for design, *F*(t) will simply be equal to 70 N.

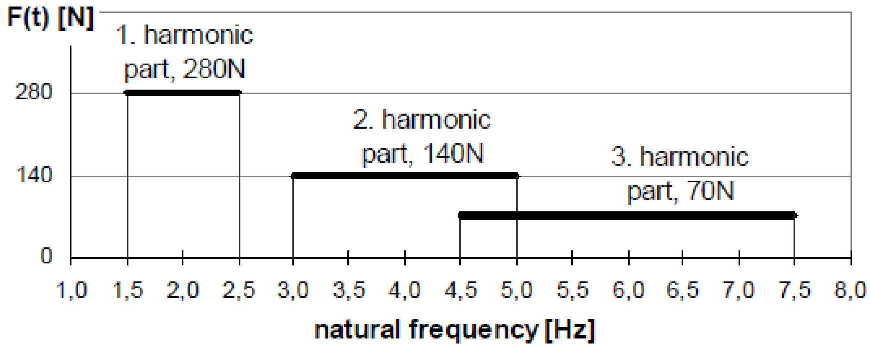


Figure 3.2 - The harmonic parts of the floor force depending on the floor's f_1

Type of floor	**Modal damping ratio *D***
Timber floors without any floor finish	0.01
Plain glued laminated timber floors with floating screed	0.02
Girder floors and nail laminated timber floors with floating screed	0.03

Table 3.3 - Hamm/Richter/Winter recommended damping ratios

3.1.4 Summary

A summary of the verifications is shown in this table:

	Design equations	**Limit values**
Frequency	$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}$ $f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}\sqrt{1+\left(\frac{L}{b}\right)^4\frac{(EI)_b}{(EI)_L}}$	High demands: $f_1 \geq 8$ Hz Low demands: $f_1 \geq 6$ Hz No demands: None
Deflection	$w = \frac{FL^3}{48(EI)_L\, b_{ef}}$ $b_{ef} = \frac{L}{1.1}\sqrt[4]{\frac{(EI)_b}{(EI)_L}}$	High demands: $w \leq 0.25$ mm/kN Low demands: $w \leq 0.50$ mm/kN No demands: None

Velocity	None	None
Acceleration (4.5 Hz $\leq f_1 < f_{\text{limit}}$)	$a = \frac{F_{\text{dyn}}}{M^* \, 2D} = \frac{0.4 \, F(\text{t})}{m \, 0.5L \, 0.5b \, 2D}$	High demands: $a \leq 0.05$ m/s^2 Low demands: $a \leq 0.10$m/s^2 No demands: None

Table 3.4 - Hamm/Richter/Winter summary

3.2 Mohr

Bernhard Mohr of the Technical University of Munich published a paper [10] in 1999 with recommendations on how floor vibrations should be verified, largely based on the research project done together with Heinrich Kreuzinger [9]. The proposal was based on how subjective evaluations from in situ tests correlated with calculated vibration parameters, and limit values could then be suggested from this.

For the subjective evaluations, 20 floors in Switzerland and Germany were rated (by user and examiner) from 1 to 4, where a rating of 1 meant “no vibration problem” and 4 meant “heavy vibration problem”.

The human perceptibility to vibration

- depends on the vibration acceleration for frequencies lower than about 8 Hz
- depends on the vibration velocity for frequencies higher than about 8 Hz
- increases with the duration of the vibration
- increases with an increasing number of impulses
- decreases with the relationship to and the awareness of the vibration cause
- decreases with human activity
- decreases by increasing damping
- has logarithmic character like the sensibility to sound
- is strongly subjective.

To ensure that a floor is comfortable to the users, the design standards tend to focus on some main points:

- Resonance is a problem for low frequency floors subjected to cyclic loading. The fundamental frequency should therefore be above a minimum value, or the resulting acceleration should be limited.
- With regards to footfall (impulses of longer duration), the stiffness of the floor should be sufficiently high.
- For impulses of shorter duration, like a heel drop, there should be a requirement for the mass of the floor.

The following values for the modal damping ratio are recommended (based on literature and in situ tests):

Type of floor construction	**Modal damping ratio *D***
Timber floors without any additional boarding for sound insulation	0.01
Plain glued laminated timber floors with additional boarding for sound insulation	0.02
Girder floors and nail laminated timber floors with additional boarding for sound insulation	0.03

Table 3.5 - Modal damping ratios recommended by Mohr

3.2.1 Frequency requirement

As an example, the figure below from [10] illustrates clearly how sensitive the vibration acceleration is to the fundamental frequency. It is calculated with the given dynamic parameters, and the resulting acceleration is high when the fundamental frequency f_1 of the floor coincides with the forcing frequency f_F, due to resonance.

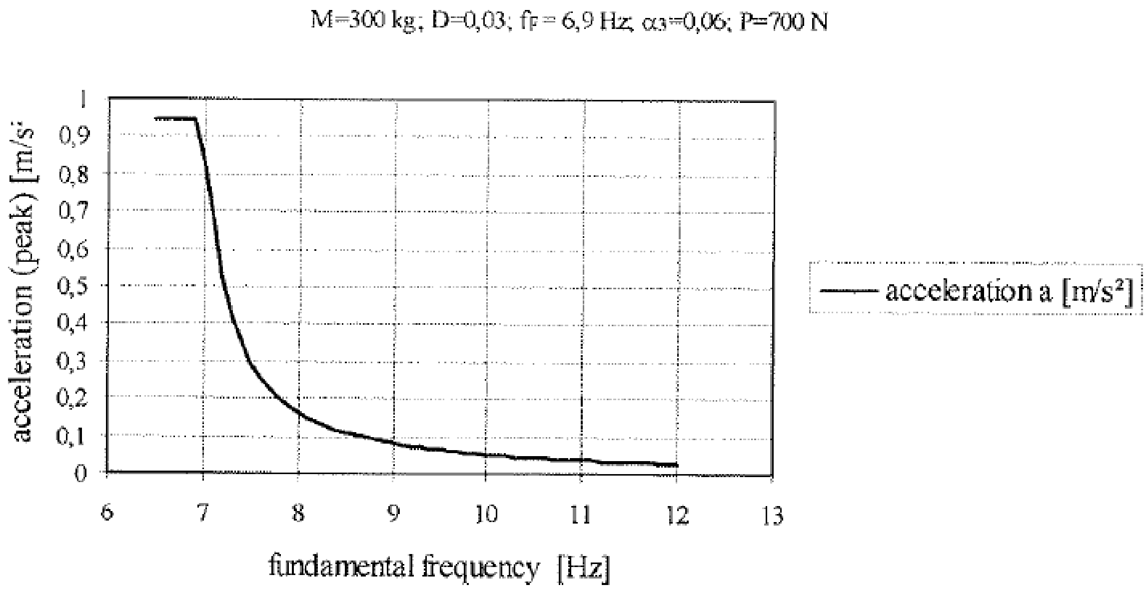


Figure 3.3 - Acceleration response depending on frequency

Because of this, Mohr's verification method has a limit value of $f_1 \geq 8$ Hz for residential timber floors.

Contrary to e. g. Hamm/Richter/Winter, but in line with the current Eurocode 5, Mohr calculates the mass to be used in the formulas with 30 % of the live load mass on top of the self-weight mass, through this EC5 quasi-permanent load combination:

$m = m_g + \Psi_2 * m_q = m_g + 0.3 * m_q$

Formula 3.5 - Mohr mass calculation

Floors with fundamental frequencies below 8 Hz can still be verified if this expression for acceleration (due to cyclic loading) gives a value below a_{limit} = 0.10 m/s^2:

$$a \approx 0.4 \frac{P_0\, \alpha_i(f_1)}{M_{gen}} * \frac{1}{\sqrt{\left[\left(\frac{f_1}{f_F}\right)^2 - 1\right]^2 + \left(2D\frac{f_1}{f_F}\right)^2}}$$

Formula 3.6 - Acceleration due to repeated actions

where:

P_0 is the weight force of an average person, taken as 700 N
f_F is the forcing frequency from Table 3.6 (taken from [10])
$\alpha_i(f_1)$ is the Fourier coefficient, dependent on the floor's fundamental frequency
$M_{gen} = m\frac{L}{2}b_{ef}$
b_{ef} is as defined in Formula 3.3

fundamental frequency	Fourier coefficient	Forcing frequency f_F	Comments
$3{,}4 < f_1 \leq 4{,}6$	$\alpha_2 = 0{,}2$	$F_F = f_1$	/ISO 10137/
$4{,}6 < f_1 \leq 5{,}1$ Hz	$\alpha_2 = 0{,}2$	$F_F = f_1$	Simplification
$5{,}1 < f_1 \leq 6{,}9$ Hz	$\alpha_3 = 0{,}06$	$F_F = f_1$	/ISO 10137/
$f_1 > 6{,}9$ Hz	$\alpha_3 = 0{,}06$	6,9 Hz	/ISO 10137/

Table 3.6 - Fourier coefficient and forcing frequency based on fundamental frequency

3.2.2 Stiffness/deflection requirement

For the stiffness requirement, many timber floors with different parameters (spans, widths, masses and stiffnesses) were considered by finite element modelling. A centric concentrated load, as well as a force-time analysis of a footfall in the middle of the floor was modelled. By then doing regression analysis from all the computations, the formula below was found:

$$w_{stat} = \frac{1}{47.36} * \frac{P\, L^{2.07}}{(EI)_L{}^{0.75}\, (EI)_b{}^{0.21}}$$

Formula 3.7 - Deflection formula derived numerically

The coherence between this formula and the finite element calculations was very good, as indicated by the statistical measure R^2 = 0.962 being very close to 1. The largest differences were found in the cases of low-width floors and floors with high transversal stiffness $(EI)_b$.

A timber floor was also modelled analytically, as a girder on elastic bedding:

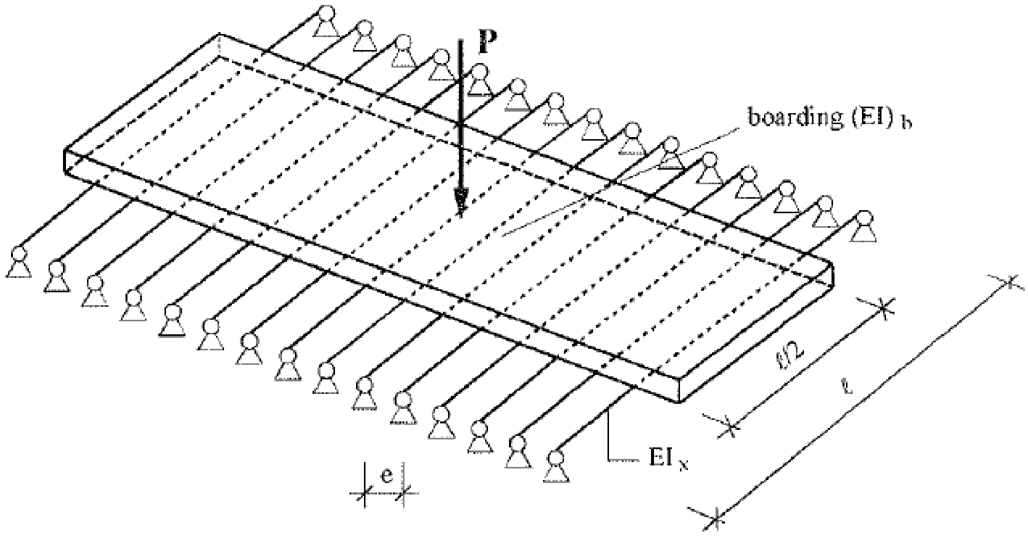


Figure 3.4 - Timber floor model

The deflection from the centric load was then computed to be as shown in the formula below, which doesn't differ much from the one obtained through the numerical analyses.

$$w = \frac{1}{43.37} * \frac{P\,L^2}{{(EI)_L}^{0.75}\,{(EI)_b}^{0.25}}$$

Formula 3.8 - Deflection formula derived analytically

The effective static width b_{ef}, as presented in Formula 3.3, was also originally computed by Mohr.

A moderate correlation between calculated deflections and subjective evaluations was found. In general, 1.0 mm/kN resulted in a vibration problem, 0.5 mm/kN resulted in a small vibration problem, and 0.25 mm/kN resulted in no vibration problem.

For design, Mohr proposes to use Formula 3.8 with a limit value of 1.0 mm when the load P is 1 kN, placed in the least favorable position of the floor. For higher demands, limits of 0.5 mm/kN and 0.25 mm/kN are suggested. For floors with lower or no demands, deflections higher than 1.0 mm/kN can be permitted.

The deflection verification assumes a damping ratio of 0.01. For floors with D = 0.02 or D = 0.03, this can be accounted for by modification factors of k_D = 1.15 or k_D = 1.25, respectively. This dynamic parameter is introduced into this verification of a static property because there was a very good correlation between deflection and an acceleration verification of a footfall as a function of f_1 and damping.

For floors with more than one span, the possibility of vibration transfer must be considered.

3.2.3 Mass/velocity requirement

There were two proposals for the mass requirement; verification of velocity response from either heel drop or unit impulse.

Figure 3.5 - Heel drop

A similar FEM and regression procedure as for Formula 3.7 was performed for the short-term action of a heel drop (i. e. a person is standing on his toes and lets himself fall to the heels as shown in Figure 3.5, from [8]), and this was the result after rounding (where the respective units are m/s, kg/m^2 and MNm^2/m):

$$v_{heel\ drop} = \frac{0.6}{m^{0.5}\ (EI)_L{}^{0.25}\ (EI)_b{}^{0.25}}$$

Formula 3.9 - Heel drop velocity formula derived numerically

By modifying the Eurocode 5 formula and neglecting all additive terms, this approximation formula for the vibration velocity was found:

$$v = \frac{0.4}{(EI)_b{}^{0.25}\ L\ m^{0.75}}$$

Formula 3.10 - Unit impulse velocity approximation

Here, the velocity is in m/s per 1 Ns, and the other units are MNm^2/m, m and kg/m^2, respectively.

For the mass/velocity requirement, the correlation between the subjective evaluations and the velocity verification was good when using the limit value expression below (a modified version of the expression from Eurocode 5, making the verification stricter):

$$v_{limit} = \frac{100^{(f_1 D - 1)}}{3}$$

Formula 3.11 - EC5 velocity limit modified by Mohr

The correlation with subjective evaluations was better for the heel drop velocity verification, with limit value calculated from the formula below.

$$v_{heel\ drop\ limit} = 6 * 100^{(f_1 D - 1)}$$

Formula 3.12 - Limit value for heel drop velocity

3.3 Eurocode 5

The verification criteria for vibrations in Eurocode 5 [11] are largely based on the work of Ohlsson from 1988 [12]. A combination of natural frequencies, damping and amplitude (in the form of maximum impulse velocity) is embedded in Ohlsson's method; arguably the three most important dynamic parameters [13]. Eurocode 5 (EC5) additionally has a deflection criterion.

The topic of vibrations is covered in section 7.3 of Eurocode 5, and it is mainly a serviceability concern. It is stated that vibrations shall be limited so as to not cause discomfort to people. A modal damping ratio of 1 % (ζ = 0.01) is to be assumed for floors unless other values are proven to be more appropriate. Mean values should be used for the elastic modulus and other stiffness moduli. Section 7.3.2 about vibrations from machinery is a separate topic.

There is a lot of uncertainty about damping as a parameter. A damping ratio of 1 % is generally quite conservative, as measurements tend to be in the range of 2-4 %. Damping is however hard to estimate. Measurements of damping are often characterized by a lot of scatter due to the measurement situation, method, and boundary conditions of the construction. At the same time, the damping ratio is a very sensitive parameter in the unit impulse velocity verification; Formula 3.14 implies that the fundamental frequency f_1 can be reduced by 50 % if the damping is doubled. (However, Formula 3.17 also involves f_1.) Humans tolerate higher impulse velocities if the oscillations are quickly damped, and damping is especially important for low frequency floors. [13]

For residential floors with a fundamental frequency lower than 8 Hz ($f_1 \leq 8$ Hz), a special examination should be performed. For residential floors with $f_1 > 8$ Hz, it is sufficient to verify that both of these requirements are satisfied:

$$\frac{w}{F} \leq a$$

Formula 3.13 - EC5 deflection criterion

$$v \leq b^{(f_1 \zeta - 1)}$$

Formula 3.14 - EC5 velocity criterion

where:

- *w* — is the maximum instantaneous vertical deflection (in mm) caused by a vertical concentrated static force *F* (in kN) applied at any point on the floor, taking load distribution into account

v is the unit impulse velocity response (in m/(Ns²)), i. e. the maximum initial value of the vertical floor vibration velocity (in m/s) caused by an ideal unit impulse (1 Ns) applied at the point of the floor giving maximum response. Components above 40 Hz may be disregarded

ζ is the modal damping ratio (unitless)

The recommended range of limiting values of *a* and *b*, as well as the recommended relation between them is as shown below in Figure 3.6, where "1" is key for better performance and "2" means poorer performance. Information on the national choice can be found in the National Annex.

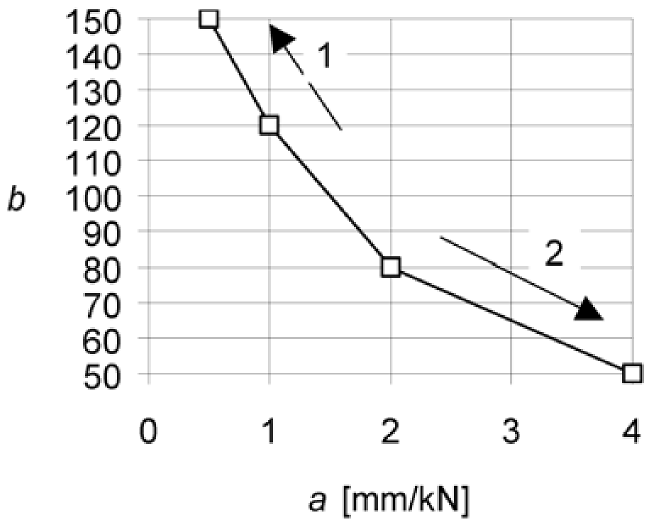


Figure 3.6 - EC5 recommended range of and relation between a and b

These calculations, for residential floors with $f_1 > 8$ Hz as per EC5 7.3.3(2), should be done under the assumption that the floor is unloaded, i. e. only the floor's self-weight and other actions considered to be permanent should be included in the calculation of the mass *m*.

For a rectangular floor with overall dimensions *L* and *B*, simply-supported along all four edges and with timber beams of span *L*, the fundamental frequency f_1 can be approximated according to Formula 2.2 (which is correct for a simply-supported beam), with $\kappa_n = \kappa_1 = \pi^2$ from Table 2.1, which gives this expression for the floor:

$$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}$$

Formula 3.15 - EC5 simply-supported rectangular floor fundamental frequency

where:

m is the mass per unit area [kg/m²]

L is the floor span [m]

$(EI)_L$ is the equivalent plate bending stiffness of the floor about an axis perpendicular to the beam direction, i. e. the longitudinal bending stiffness of the floor [Nm²/m]

For a rectangular floor with overall dimensions *L* and *B*, simply-supported along all four edges, the value *v* can be approximated as:

$$v = \frac{4(0.4 + 0.6\, n_{40})}{mbL + 200}$$

Formula 3.16 - EC5 unit impulse velocity response

where:

v	is the unit impulse velocity response [m/(Ns^2)]
n_{40}	is the number of first-order modes with natural frequencies below 40 Hz
b	is the floor width [m]

L and *m* are the same as in Formula 3.15. The value n_{40} can be calculated from this expression:

$$n_{40} = \left\{ \left(\left(\frac{40}{f_1} \right)^2 - 1 \right) \left(\frac{b}{L} \right)^4 \frac{(EI)_L}{(EI)_b} \right\}^{0.25}$$

Formula 3.17 - Amount of first-order modes with f_n < 40 Hz

The parameter $(EI)_b$ is the equivalent plate bending stiffness (in Nm^2/m) of the floor about an axis parallel to the beams, i. e. the floor's transversal bending stiffness, where $(EI)_b < (EI)_L$.

3.3.1 The Norwegian National Annex

For beams with a span of up to 4.5 m, the value *a* from Formula 3.13 should be set to:

- 0.9 mm/kN for normal stiffness requirements
- 0.6 mm/kN for high stiffness requirements

No national limitations exist with regards to Formula 3.14, but the natural frequency must be considered for long spans. If the recommendations of Figure 3.6 are to be followed, the *b* value should, by linear interpolation, be set to 126 for normal stiffness requirements and 144 for high stiffness requirements.

3.3.2 The Austrian National Annex

The Austrian National Annex [7] of Eurocode 5 is included here because of its more advanced verification method for floor vibrations. It is largely based on what Hamm/Richter/Winter suggested and is very similar to the new proposal for Eurocode 5.

The regulations are applicable to floors of usage categories A, B, C1, C3.1 and D as described in Table 6.1 in the national annex of EN 1991-1-1, with a distributed mass of at least 50 kg/m^2. The floors must fit into one of the categories of Table 3.7 below and meet the design requirements. For floors with a lower distributed mass and/or special usage (e. g.

gymnasiums, gymnastics halls, dance studios or laboratories), a special examination is required. This table and the ones that follow have been translated into English:

Category of use according to EN 1991-1-1	**Floor class 1**	**Floor class 2**	**Floor class 3**
	A1, B, C1, C3.1, D		**A2**
Typical usage	- Floors between different units of use (also continuous floors) - Floors between apartments - Office floors for meetings or use of computers - Corridors with short spans	- Floors within one unit of use - Floors in single-family houses of regular use	- Floors without residential use or attic floors not in regular use - Floors without vibration requirements

Table 3.7 - ÖNORM floor vibration classes

Constructive requirements		**Floor class 1**	**Floor class 2**	**Floor class 3**
Timber joist floors	with wet screed	Floating composition, at least 60 kg/m^2	Floating composition (also without filling)	None
	with dry screed	Special documentation required	Floating composition, at least 60 kg/m^2	None
Two-dimensional timber floors (e. g. CLT or "Brettstapel floors")	with wet screed	Floating composition, heavy and light	Floating composition (also without filling)	None
	with dry screed	Floating composition, at least 60 kg/m^2	Floating composition, at least 60 kg/m^2	None

Table 3.8 - ÖNORM constructive requirements

The characteristic permanent actions should be used in the vibration calculations. For the stiffness properties, the mean values should be used. The bending strength of screed may be taken into account, provided that the screed meets the technical requirements related to its production etc. The elasticity/springiness of beams etc. must also be considered.

For rectangular pin-supported floors without transverse load distribution, the fundamental frequency is calculated from Formula 3.15 in the same way as usual.

For rectangular pin-supported floors with transverse load distribution, this formula (which is just a more convenient rewriting of Formula 3.1, from Hamm/Richter/Winter) is used instead:

$$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}\sqrt{1 + \left(\frac{L}{b}\right)^4 \frac{(EI)_b}{(EI)_L}}$$

Formula 3.18 - Fundamental frequency of four-side supported floor

It can be noted that the transverse distribution effect is generally only significant (with regards to f_1) if $(EI)_b$ is at least 5 % of $(EI)_L$.

If the support conditions are different than described above for Formula 3.15 and Formula 3.18, the fundamental frequency can be approximated by multiplying with the appropriate factor from this table:

Coefficients for considering different support conditions (without transversal load distribution)	$k_{e,1}$
pinned – pinned	1.000
clamped – pinned	1.562
clamped – clamped	2.268
clamped – free (cantilever)	0.356

Table 3.9 - Support condition coefficients for the fundamental frequency

For a two-span floor (supported on all four sides), the fundamental frequency can be approximated according to Formula 3.15 or Formula 3.18 and multiplied with the appropriate $k_{e,2}$ coefficient according to the table below, where L is the largest span and L_2 the smallest. Intermediate values can be found through linear interpolation.

L_2/L	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$k_{e,2}$	1.000	1.090	1.157	1.206	1.245	1.282	1.318	1.359	1.410	1.474	1.562

Table 3.10 - Fundamental frequency coefficients for two-span floors

The stiffness/deflection criterion is the same as for Hamm/Richter/Winter (Formula 3.2), with the same limit values in mm/kN. For continuous floor systems, this criterion can as a simplification be applied to the largest span which should then be calculated as a simply-supported one-span floor.

The limit values for the fundamental frequency are also the same as suggested by Hamm/Richter/Winter. They are 8 Hz and 6 Hz for floor classes 1 and 2 respectively, while there for floor class 3 is no limitation (same as for the deflection criterion). If both of those criteria are met, then the floor is verified with regards to vibrations.

If f_1 is lower than the limit values (but higher than 4.5 Hz, for floor classes 1 and 2), then the floor will still be verified if an acceleration criterion is met (in addition to the other criteria). This additional check is very similar to Formula 3.4 as suggested by Hamm/Richter/Winter, but calculated somewhat differently. The effective value of the vibration acceleration for single-span floors pinned along all edges may be approximated as follows:

$$a_{rms} = \frac{0.4\ \alpha\ F_0}{2\ \zeta\ M^*}$$

Formula 3.19 - Root-mean-square acceleration

where:

a_{rms}	is the root-mean-square acceleration, in m/s^2
α	is the Fourier coefficient as a function of f_1, with $\alpha = e^{-0.4 f_1}$
F_0	is the weight of a person walking on the floor, usually F_0 = 700 N
ζ	is the modal damping ratio, from Table 3.11
M^*	is the modal mass, in kg

Another difference from Hamm/Richter/Winter is that the modal mass is here calculated from the effective width b_{ef} rather than the floor width b:

$$M^* = m\ \frac{L}{2}\ b_{ef}$$

Formula 3.20 - ÖNORM modal mass

Here, m is still the distributed mass in kg/m^2, and L is the span (in m, same as for b_{ef}).

These values are to be assumed for the modal damping ratio ζ:

Type of floor construction	**Modal damping ratio ζ**
Floor constructions without or with a light floor construction	0.01
Floor constructions with floating screed	0.02
Cross laminated timber floors with or without a light floor construction	0.025
Timber joist floors and mechanically connected “Brettstapel floors” with floating screed	0.03
Cross laminated timber floors with floating screed and heavy floor construction	0.04

Table 3.11 - Standard modal damping ratio values for various timber floor constructions

The limit values for a_{rms} are as for Hamm/Richter/Winter; $\leq$ 0.05 m/s^2 for floor class 1 and $\leq$ 0.10 m/s^2 for floor class 2 (and N/A for floor class 3).

As an alternative to the calculations, the vibrational properties of timber floors can also be verified through measurements. Information about this can for example be taken from ISO 10137. For the root-mean-square acceleration, ÖNORM ISO 2631-2 is referenced.

A summary of the calculations is given in Table 3.12. According to Figure 3.6, the value of b (for the limit value of the unit impulse velocity response) should be 150 for floor classes 1 and 2.

	Design equations	Limit values
Frequency	$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}$ $f_1 = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}}\sqrt{1+\left(\frac{L}{b}\right)^4\frac{(EI)_b}{(EI)_L}}k_{e,1}k_{e,2}$	Class 1: $f_1 \geq 8$ Hz Class 2: $f_1 \geq 6$ Hz Class 3: None
Deflection	$w = \frac{FL^3}{48(EI)_\mathrm{L}\, b_\mathrm{ef}}$ $b_{ef} = \frac{L}{1.1}\sqrt[4]{\frac{(EI)_b}{(EI)_L}}$	Class 1: $w \leq 0.25$ mm/kN Class 2: $w \leq 0.50$ mm/kN Class 3: None
Velocity	$v = \frac{4(0.4 + 0.6\, n_{40})}{mbL + 200}$ $n_{40} = \left\{\left(\left(\frac{40}{f_1}\right)^2 - 1\right)\left(\frac{b}{L}\right)^4\frac{(EI)_L}{(EI)_b}\right\}^{0.25}$	Class 1: $v \leq 150^{(f_1\zeta-1)}$ Class 2: $v \leq 150^{(f_1\zeta-1)}$ Class 3: None
Acceleration (4.5 Hz $\leq f_1 < f_{\mathrm{limit}}$)	$a_{rms} = \frac{0.4\, e^{-0.4 f_1}\, F_0}{2\,\zeta\, M^*}$ $M^* = m\,\frac{L}{2}\, b_{ef}$	Class 1: $a_{\mathrm{rms}} \leq 0.05$ m/s^2 Class 2: $a_{\mathrm{rms}} \leq 0.10$m/s^2 Class 3: None

Table 3.12 - Austrian National Annex summary

3.3.3 New Eurocode 5 proposal

The section about floor vibrations from footfalls in the newest Eurocode 5 proposal (dated October 2018, [14]) is largely based on the methods given by Wilford et al. [15] It should be noted that these loading models are only applicable for floors with a mass of at least ten times the weight of the walker.

The hand calculation methods are applicable to timber floors on rigid supports (walls), but not yet for flexible/elastic supports (beams). A reduction factor to account for that the walker and the vibration perceiver cannot be in the same location at midspan is under discussion. Multiplication factors for multi-span floors (for frequency and modal mass) are also being evaluated. The possibility of including more than just the floor's self-weight in the mass calculation is being debated as well. [4]

The minimum modal damping ratio (ζ) for floors is no longer 1 %, as many studies has found this to be overly conservative [4]. The recommend ζ values, unless other values are proven to be more appropriate, are as presented in the table below. (For determination of ζ through on-site testing, prEN 16929 is referenced.)

Type of floor construction	**Modal damping ratio ζ**
Joisted floors	0.02
Timber-concrete and massive floors	0.025
Joisted floors with a floating layer	0.03
Timber-concrete and massive floors with a floating layer	0.035
All floors with a floating layer and supported on 4 sides	0.04

Table 3.13 - New EC5 modal damping ratios

A floating layer is defined as not being connected to the layer underneath.

The sum of the floor's characteristic self-weight mass, including all supported or suspended horizontal layers, should be used for the vibrational calculations.

The bending stiffness of a floating floor layer or screed may be included in the calculations, but no composite action should be considered for a floating layer.

The flexibility of supporting members such as beams should be considered.

The design rules are applicable for human induced footfall vibrations of timber floors in categories of use A, B, C1, C3 and D as defined in EN 1991-1-1. This includes residential floors, office floors and areas with moderate amounts of people, like museums, exposition rooms etc., as well as access areas in offices. It excludes areas where crowd loading can be expected, like theatres, gymnastic and dance halls, and areas with sensitive equipment.

The table below gives the recommended floor performance levels for office and residential categories. Depending on which aspect has the highest emphasis, the choice can be based on quality or economy, or it can be a middle ground between the two. The choice of performance levels, also for other categories than A and B, can be specified in the National Annex.

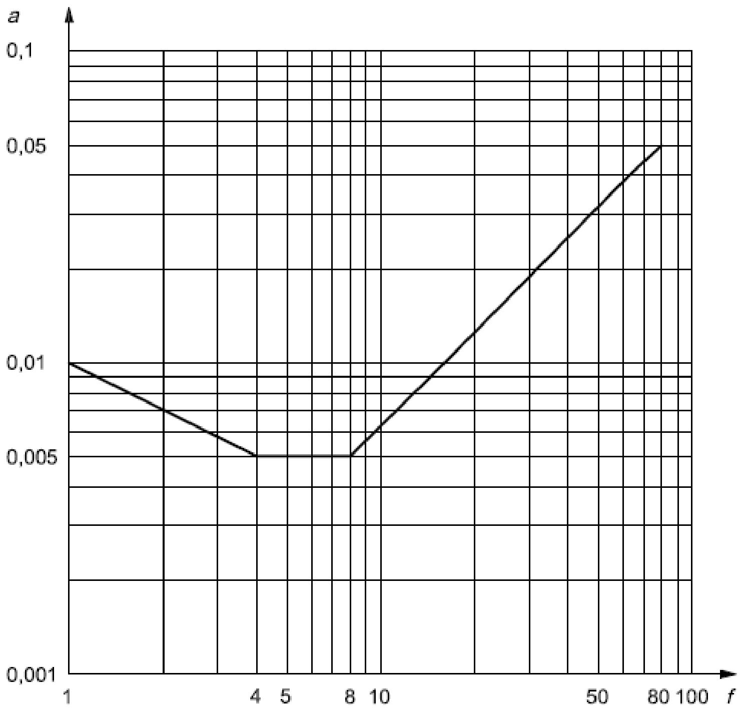
Usage category	**Quality choice**	**Base choice**	**Economy choice**
A (residential)	level III	level IV	level V
B (office)	level II	level III	level IV

Table 3.14 - Recommended floor performance levels for categories of use A and B

People are more likely to complain about vibrations they cannot see the source of, so e. g. multi-family buildings where floors cross from one apartment to another will generally have higher requirements.

A floor should satisfy the vibration criteria according to its performance level. For floor performance levels I through VI, no further investigations are necessary if the requirements for fundamental frequency, acceleration or velocity, and stiffness from Table 3.15 are met.

When verifying the acceleration or velocity caused by vibrations from a walking person, it is the root-mean-square values that should be considered. The rms acceleration or velocity responses are compared to the vibration perception base curve (in figure C1 of appendix C in ISO 10137), which is shown below in Figure 3.7. Human perception of vibration is frequency dependent, being most sensitive to vibrations in the range of 4-8 Hz, where the minimum perceptible a_{rms} is constantly equal to 0.005 m/s^2 in the figure. Above 8 Hz, the curve has a constant slope, and when this part of the curve is integrated, the minimum perceptible v_{rms} is found to be constantly equal to 0.0001 m/s. By utilizing this, the limit values for the verifications can be reliably related to the real human disturbance levels for vibrations. The acceleration and velocity criteria are expressed as a multiple of these base values, and this multiple is termed as the response factor R in Table 3.15. R is equal to the response (a_{rms} or v_{rms}) divided by the minimum perceptible value (0.005 or 0.0001), depending on whether the fundamental frequency of the floor is below 8 Hz or above.



Key

a acceleration (r.m.s.), m/s^2

f frequency, Hz

Figure 3.7 - Vibration perception base curve, frequency vs. acceleration

For resonant vibration design situations, i. e. for low frequency floors (f_1 < 8 Hz), the criteria for minimum fundamental frequency, maximum rms acceleration and deflection of Table 3.15 should be fulfilled. For the transient vibration design situations, i. e. for high frequency floors ($f_1 \geq 8$ Hz), the criteria for maximum deflection and rms velocity of Table 3.15 should be fulfilled. In both cases, floors will be assumed to be single spanning and simply-supported on all edges.

	Floor performance levels					
Criteria	**level I**	**level II**	**level III**	**level IV**	**level V**	**level VI**
Frequency criteria for all floors f_1 [Hz] $\geq$	4.5					
Stiffness criteria for all floors $w_{1\,kN}$ [mm] $\leq$	0.25		0.5	0.8	1.2	1.6
Response factor R	4	8	12	16	20	24
Acceleration criteria for resonant vibration design situations a_{rms} [m/s^2] $\leq$	R x 0.005					
Velocity criteria for transient vibration design situations v_{rms} [m/s] $\leq$	R x 0.0001					

Table 3.15 - Floor vibration criteria according to performance level

For example; for floor performance level I when f_1 is between 4.5 and 8 Hz, the response factor gives a limit value for a_{rms} of 0.02, which is much stricter than the lowest limit value of 0.05 from the Austrian National Annex.

The figure below shows the resonant and transient vibration design situations for walking pace frequencies of 2 Hz. [4]

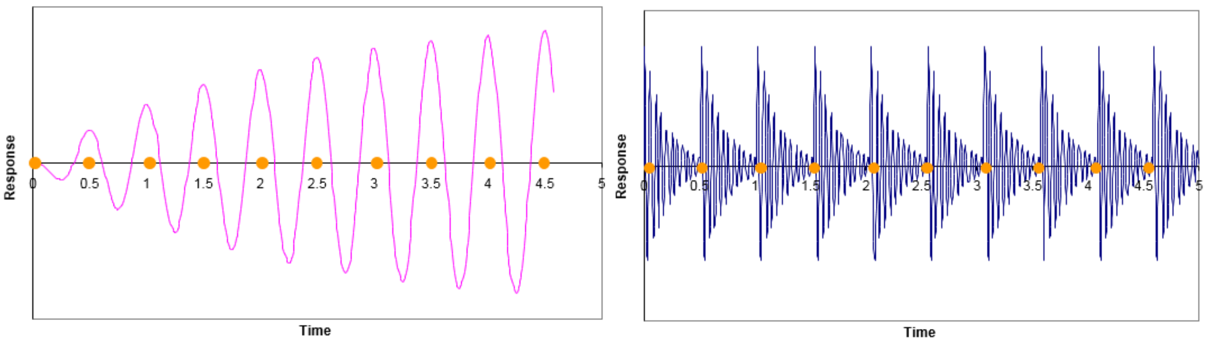


Figure 3.8 - Resonant vs. transient response

3.3.3.1 Frequency

The fundamental frequency for rectangular one-span or two-span floors, supported by walls or beams, can be calculated according to Formula 3.21, which is a convenient combination of Formula 2.4 and Formula 3.1 (from Hamm/Richter/Winter, similar to Formula 3.18 from the Austrian NA), with factor taken from Table 3.10 (as in the Austrian NA, but here only applicable for $L_2/L_1 \geq 0.2$):

$$f_1 = k_{e,1}\, k_{e,2}\, \frac{18}{\sqrt{\delta_{sys}}}$$

Formula 3.21 - New EC5 fundamental frequency formula

where:

$k_{e,1}$	is the frequency multiplier from Table 3.10, applicable for two-span floors
$k_{e,2}$	is a frequency multiplier for two-span floors, $k_{e,2} = \sqrt{1+\left(\frac{L}{b}\right)^4 \frac{(EI)_b}{(EI)_L}}$
δ_{sys}	is the one-way spanning deflection of the floor system (including the effect of any support beams, shear and connections in composite structures) under the self-weight load, in mm.

The multiplication factors are then 1 for single-span floors. The above formula considers all factors that affect deflection, but a simplified approach that just factors in the bending stiffness can also be used:

$$f_1 = k_{e,1}\, k_{e,2}\, \frac{\pi}{2L^2} \sqrt{\frac{(EI)_L}{m}}$$

Formula 3.22 - New EC5 simplified fundamental frequency formula

3.3.3.2 Acceleration

When the fundamental frequency of a floor is lower than 8 Hz, the floor vibration is assumed to be resonant, and an acceleration criterion should be met. The root-mean-square acceleration is approximated from the formula below (which assumes a single-span floor), which is the same as in the Austrian National Annex, except that here the a_{rms} is divided by $\sqrt{2}$:

$$a_{rms} = \frac{0.4\, \alpha\, F_0}{\sqrt{2}\, 2\, \zeta\, M^*}$$

Formula 3.23 - New EC5 proposal, rms-acceleration

where:

a_{rms}	is the root-mean-square acceleration, in m/s^2
α	is the Fourier coefficient as a function of f_1, with $\alpha = e^{-0.4 f_1}$
F_0	is the weight of a person walking on the floor, usually F_0 = 700 N
ζ	is the modal damping ratio, from Table 3.13
M^*	is the modal mass, in kg

While Mohr and the Austrian NA calculates the modal mass from the effective width b_{ef}, the latest EC5 proposal rather utilizes the width b, in the same way as for Formula 3.4 from Hamm/Richter/Winter, with the limitation of $b \leq 1.5\, L$. The ratio of modal mass to total mass of the first mode of a plate is known to be 25 %:

$$M^* = m\,\frac{L}{4}\,b$$

Formula 3.24 - New EC5 proposal, modal mass

The response factor R is calculated as such:

$$R = \frac{a_{rms}}{0.005}$$

Formula 3.25 - Response factor for acceleration

3.3.3.3 Velocity

If f_1 is 8 Hz or above, the floor vibration is assumed to be transient, and a velocity criterion should be met. The rms velocity response is verified according to Formula 3.26 through Formula 3.31, which assumes a one-span floor.

$$I = \frac{42{f_w}^{1.43}}{{f_1}^{1.3}}$$

Formula 3.26 - Mean modal impulse

where:

I	is the mean modal impulse [Ns]
f_w	is the walking frequency, taken as 2 Hz
f_1	is the floor's fundamental frequency [Hz]

$$V_{1,peak} = \frac{I}{M^*}$$

Formula 3.27 - Peak velocity response of the fundamental mode

In order to also account for the contribution from the higher modes of vibration to the floor response, the multiplication factor K_{imp} is introduced:

$$K_{imp} = 0.48\,\frac{b}{L}\left(\frac{(EI)_b}{(EI)_L}\right)^{-0.25} \geq 1.0$$

Formula 3.28 - Impulsive multiplier accounting for higher modes

$$V_{tot,peak} = K_{imp}\,V_{1,peak}$$

Formula 3.29 - Total velocity peak response

$$V_{rms} = \beta\, V_{tot,peak}$$

Formula 3.30 - Root-mean-square velocity response

where:

β = (0.65 - 0,01 f_1) * (1.22 - 0.11 ζ) η

η = 1.52 - 0.55 K_{imp} , when 1.0 ≤ K_{imp} ≤ 1.5. Else: η = 0.69 .

$$R = \frac{V_{rms}}{0.0001}$$

Formula 3.31 - Response factor for rms velocity

3.3.3.4 Stiffness/deflection

The stiffness criterion, with limit values from Table 3.15, is calculated from the same expression as known from Hamm/Richter/Winter and the Austrian National Annex (with effective width b_{ef} as defined in Formula 3.3), where *F* is 1 kN placed in the most unfavorable position:

$$w_{1\ kN} = \frac{FL^3}{48(EI)_{\mathrm{L}}\, b_{\mathrm{ef}}}$$

Formula 3.32 - New EC5 deflection formula

This formula applies for single-span floors. If continuous over two or more spans, the formula can still be used as an approximation, by considering the maximum span of the continuous beam system.

3.3.3.5 Alternative verification

As an alternative to all of these simplified calculations, the floor vibrational properties can be verified by thorough dynamic analysis, for example by using the finite element method. The same criteria for floor performance levels still apply. Another alternative verification method is in-situ measurements of test floors, as described in prEN 16929. Furthermore, ISO 10137, ISO 2631-1 and ISO 2631-2 are referenced for alternative verification.

3.4 Deflection and frequency criteria

Vibrational properties of timber joist floors have been verified in many different ways around the world over the years. In Norway there was previously just one criterion; for the deflection caused by a concentrated load. Other verification methods focused on for example the fundamental frequency f_1, maximum unit impulse velocity response, or a combination of parameters. In recent years, a criterion proposed by Hu and Choi [16] which combines f_1 and deflection has been used a lot in Norway, in part due to the difficulty of accurately estimating damping. [13]

3.4.1 Deflection criterion

Studies at Byggforsk as well as internationally show that deflection as a sole criterion is insufficient to ensure acceptable floor vibrational performance [13]. Stiffness is very important, but the floor vibrations will still be problematic if e. g. resonance or significant off-resonant response can occur.

The recommendations of Canadian Wood Council from 1997 in Table 3.16 [17], as presented in this table excerpt from Homb (2007) [13], are interesting as an example. It shows the maximum recommended deflection of timber joist floors, calculated from a centric concentrated load of 1 kN for various spans:

Nasjon / metode	Maksimal nedbøyning Λ 1) (mm)
Kanada, CWC (1997): Teoretisk beregnet	
– Spennvidde, L < 3 m	2.0
– Spennvidde, L = 3 m til 5.5 m	$\frac{8}{L^{1.3}}$
– Spennvidde, L = 5.5 m til 9.9 m	$\frac{2.55}{L^{0.63}}$
– Spennvidde, L > 9.9 m	0.6

Table 3.16 - Max recommended deflection from 1 kN point load for timber joist floors

We can see that these deflection requirements are stricter with increasing span. Why is that? As known from statics, the deflection from a concentrated load mid-span is proportional to L^3/EI. And as seen in Formula 2.2, the fundamental frequency is inversely proportional to $L^2/(EI)^{0,5}$. So if for example the span is tripled, EI has to be increased $3^3 = 27$ times to get the same deflection, but to get the same fundamental frequency f_1, EI would need to be $(3^2)^2 = 81 = 27*3$ times larger. Generally, if the span is increased X-fold, then f_1 is affected X times more than the deflection is. With this sensitivity analysis by the involved stiffness parameters in mind, it is then interesting to note that both the ratio between the maximum and minimum span limit values (9.9 / 3), as well as between the given maximum allowed deflections (2.0 / 0.6), equals 3.3. So, although this is a sole deflection criterion, resonance is still accounted for indirectly. By having the limit values so that f_1 doesn't change with the span, this particular deflection criterion is actually more of a fundamental frequency criterion. For low-span floors, this verification method allows such a high displacement amplitude under the foot that it would be very noticeable when walking.

It can be shown, by rearranging both Formula 3.15 and Formula 3.32 with respect to the parameters L and $(EI)_L$ so that the formulas can be combined, that $f_1 = \frac{100}{\sqrt{m\, b_{ef}}}$ is a good approximation with this verification method. Here, the effective width b_{ef} is as defined in Formula 3.3, the concentrated load F is 1000 N as defined, and δL was approximated/simplified as constantly equal to 0.005 m^2 based on Table 3.16. So, if for example a floor has a distributed mass m of 60 kg/m^2 and b_{ef} equal to 1.0 m, then the fundamental frequency of the floor will be 12.9 Hz with this verification method if designed to deflect as much as the limit values allow. (If designed to only deflect half as much, then f_1 would have to be multiplied by $\sqrt{2}$ because δL was assumed two times as high when written into the approximation formula as a constant.)

Nevertheless, it can be understood from the formulas for deflection and f_1, and from the limit values in Table 3.16, that long-span timber joist floors are especially sensitive to vibrations, and it highlights why this has been a topic of interest for researchers, in order to find ways to achieve longer spans.

3.4.2 Fundamental frequency criterion

As a sole criterion, focusing on minimum values of the fundamental frequency f_1 has its limitations. As known from Formula 2.2 and Formula 3.15, f_1 depends on the mass, the longitudinal bending stiffness, the span, and the boundary conditions of the floor; and f_1 is roughly inversely proportional to the square root of the deflection from permanent loads, as seen in Formula 2.4 for a beam. And so, floors with increased transversal stiffness will be calculated too conservatively from just a limitation of f_1, as then just the longitudinal stiffness matters (for single-span floors). Also, heavy floors will have a lower fundamental frequency, especially if also required to be calculated with a higher load, as instructed by Dolan et al. [18] However, heavy floors tend to perform better than lightweight floors when subjectively assessed in walking tests. So in short, a fundamental frequency criterion doesn't do justice to floors with high transversal stiffness and/or high (modal) mass. As long as there is no resonance or significant off-resonant response, then f_1 alone is far from sufficient as an indicator of floor vibration response. [13]

This is evident in the test results below, from Hamm/Richter/Winter [8]. No correlation between f_1 and subjective evaluations could be found. Many of the best performers were low-frequency (according to hand calculations), heavy timber-concrete floors (with high transversal stiffness). Timber floors without floor finish, although quite high-frequency due to the light weight, were much more negatively evaluated.

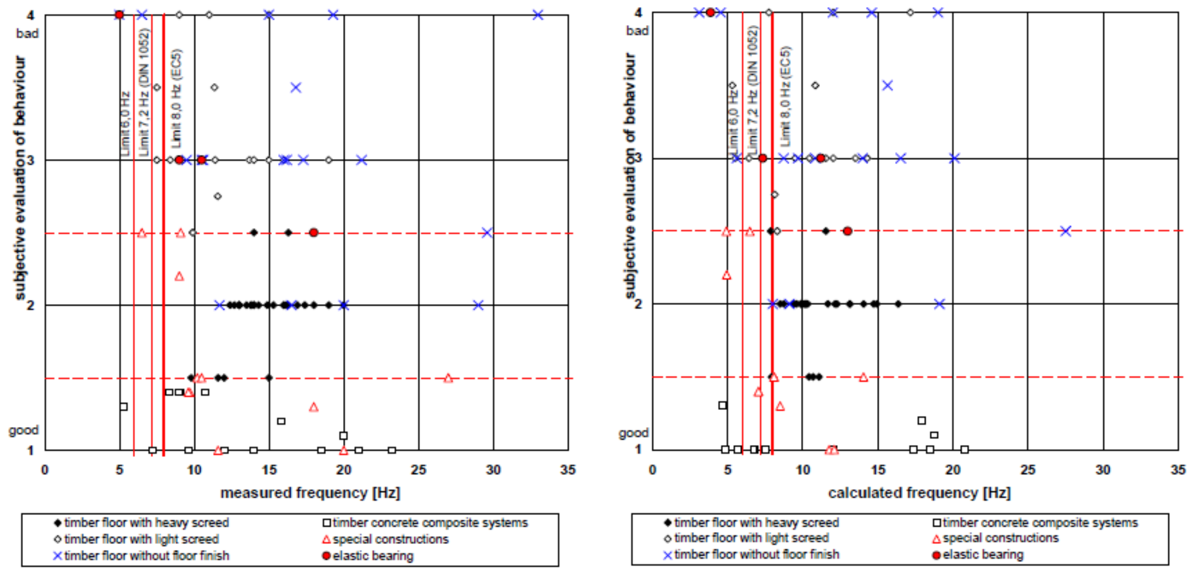


Figure 3.9 - Subjective evaluations vs. f_1 (measured and calculated)

While the fundamental frequency is still important as a criterion, others are needed in addition.

There was quite a large discrepancy between calculated and measured values of f_1. This is discussed in section 3.4.3.3.

3.4.3 Combined deflection and fundamental frequency criterion

There has been a lot of research looking into vibrational properties of timber floors in Canada. Hu et al. [16] [19] paired different combinations of parameters (e. g. frequency and deflection) against subjective evaluations of in-use floors based on standardized questionnaires in order to find the most reliable correlation. The parameters were calculated as well as verified through field or laboratory testing. The basis of the research was a database of 112 floors in buildings and some laboratory floors. The verification methods existing at the time were evaluated, and then five tentative models were fitted based on regression analysis of the subjective evaluations versus combinations of physical parameters. All of the models combined the fundamental frequency with another parameter, namely these: acceleration, impulse velocity, deflection from concentrated load, span, and distributed weight. [13]

The conclusion of the reports was that a criterion combining the fundamental frequency and the deflection from a concentrated load correlated well with the subjective evaluations, while also being easy to calculate fairly accurately. Based on this report as well as later work done by Hu, Chui and others, this criterion was suggested:

$$\frac{f_1}{\Delta^{0.44}} > 18.7$$

Formula 3.33 - Combined deflection and fundamental frequency criterion

Here, f_1 is the fundamental frequency in Hz and Δ is the deflection in mm from a 1 kN concentrated load in the middle of the span. This formula was later also adopted by SINTEF Byggforsk for their "comfort criterion", chosen ahead of other verification methods considered, such as Ohlsson's approach which is adopted by the current Eurocode 5. This was in large part due to the difficulties in estimating damping, which is a sensitive parameter. [13] Anders Homb maintained the same conclusion in an article published in 2018. [20]

The figure below shows how subjectively evaluated floors were calculated according to Hu's criterion. The blue squares represent floors deemed as acceptable while the purple were unacceptable. The formula predicted this quite well, placing most of the acceptable floors to the right of the design curve and most of the unacceptable floors to the left of it. The design curve is somewhat conservative in that more acceptable floors are calculated as non-approved compared to how many unacceptable floors are approved by the formula.

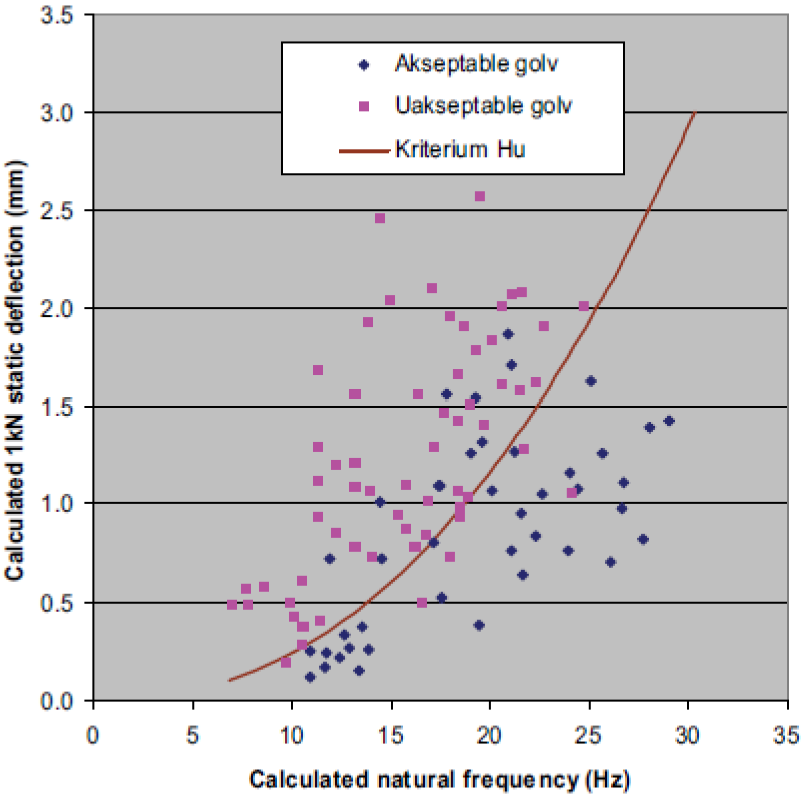


Figure 3.10 - Subjective evaluations vs. Hu's formula

None of the tested floors were mass-timber floors. They were all timber joist floors of various build-ups, but without screed. Hu stated that the criterion is applicable for floors with $f_1 > 7$ Hz, but all of the acceptable floors had a fundamental frequency of at least 10 Hz.

Because of this, Homb set a limit value of 10 Hz for the "comfort criterion" – along with a deflection limit of 1.3 mm/kN for a centric concentrated load. Floors with f_1 as low as 8 Hz could also still be verified if an additional criterion that considers damping is met. Fundamental frequencies lower than 8 Hz are discouraged due to possible resonance and off-resonant response from the 2nd, 3rd and 4th harmonics of walking.

For timber floors with high transversal stiffness, such as e. g. mass-timber floors or joisted floors with screed, a higher limit value of 12.5 Hz was proposed for the "comfort criterion", due to lack of available data at the time. [13]

For the "comfort criterion", the fundamental frequency of one-span floors has not been calculated according to Formula 3.15. Due to better correlation with measurements, an orthotropic plate model has been used instead. This is looked into in section 3.4.3.3.

To calculate deflection for the "comfort criterion", SINTEF Byggforsk has used the calculation program BTAB. This was deemed as more accurate than KAN, a calculation model based on ribbed-plate theory that was developed along with the design formula.

Analytical hand calculation of the transversal stiffness is typically very conservative for timber joist floors, which in turn overestimates the deflection. Since the laboratory tested Støren floors were verified according to the "comfort criterion", it was appropriate to investigate how BTAB works, to better understand also the measurements. BTAB is described in detail in a report by Kolstad and Homb [21], and it is the source used for sections 3.4.3.1 and 3.4.3.2.

3.4.3.1 BTAB description

The calculation program "BTAB" is based on the method described in a report by Norges Byggforskningsinstitutt (NBI) ref. Megård & Hansteen. [22]

The program calculates deflections of timber joist floors. The deflections are calculated in the middle of each joist for a timber joist floor with a concentrated load in the middle of a chosen joist. Alternatively, the maximum allowed span can be calculated based on deflection demands.

The method is based on a shell and plate theory, where the plate (the subfloor) is calculated as a shell supported by beams. Between the plate and the beams there is assumed to be a spring connection that resists relative movements in the longitudinal direction of the beam.

Simply supported beams are used in the timber joist floor model. The lateral bending stiffness of the beams is neglected, and torsion of the beams is not considered either. It is assumed that the timber joist floor is only supported on two of four sides. BTAB assumes that the theoretical span is 0.10 m larger than the free distance between the supports.

The y direction is defined as parallel to the beam/joist direction, the x direction is transversal to their span direction, and the z direction is vertical.

In BTAB, beams can be defined by spacing, height, axial stiffness and bending stiffness. Rectangular beams can alternatively be defined by spacing, thickness, height and modulus of elasticity. Up to seven different beams can be defined and modelled in the program.

The plate/subfloor is defined by specifying thickness, diaphragm stiffnesses and plate stiffnesses. The diaphragm and plate stiffnesses are added after doing hand calculations based on the plate thickness, elastic moduli (for axial loading and bending longitudinally and transversally), shear modulus (in-plane) and Poisson's ratio.

Diaphragm stiffnesses (denoted by *C*) and plate stiffnesses (denoted by *D*) of subfloors of *k* layers are calculated as such:

$$C_x = \sum_{i=1}^{k} E'_{xi} \int_A dz \qquad D_x = \sum_{i=1}^{k} E'_{xi} \int_A z^2 dz$$

$$C_y = \sum_{i=1}^{k} E'_{yi} \int_A dz \qquad D_y = \sum_{i=1}^{k} E'_{yi} \int_A z^2 dz$$

$$C_1 = \sum_{i=1}^{k} E''_{i} \int_A dz \qquad D_1 = \sum_{i=1}^{k} E''_{i} \int_A z^2 dz$$

$$C_{xy} = \sum_{i=1}^{k} G \int_A dz \qquad D_{xy} = \sum_{i=1}^{k} G \int_A z^2 dz$$

Formula 3.34 - Subfloor stiffnesses in-plane and out of plane

Here, *G* is the shear modulus and ν_n is Poisson's ratio in n direction. E_n is the modulus of elasticity in n direction:

$$E'_x = \frac{E_x}{1-\nu_x\nu_y}$$

$$E'_y = \frac{E_y}{1-\nu_x\nu_y}$$

$$E'' = \frac{\nu_y E_x}{1-\nu_x\nu_y} = \frac{\nu_x E_y}{1-\nu_x\nu_y}$$

Formula 3.35 - Elastic moduli of subfloor

The connection between plate and beam is calculated as a linear elastic spring connection (in the beam's longitudinal direction), and the spring stiffness can be defined by the user. Based on testing of nailed connection (with nail distance 20 cm), the average spring stiffness was found to be 200 N/mm [22]. For glued and nailed floor, the same source suggests using a spring stiffness of 10^6 N/mm, which in practice means close to full static interaction (transversally) between joists and plate.

BTAB doesn't automatically factor in transversal stiffeners (blocking), but it can be done manually by increasing the diaphragm and plate stiffnesses of the plate. The spring stiffness between the transversal stiffening and the beams will then be equal to the spring stiffness between subfloor and beams.

For stiffeners/blocking mounted perpendicular to the joists, their transversal contribution can be considered by increasing the stiffnesses C_x and D_x. This contribution will then be uniformly distributed over the timber joist floor's length, like an extra plate layer perpendicular to the joists.

The contribution from rectangular blocking, where C_x is the diaphragm stiffness and D_x the plate stiffness in x direction, can be calculated by these formulas:

$$C_x = \frac{E_{tv} \cdot b_{tv} \cdot h_{tv}}{cc_{tv}}$$

$$D_x = \frac{E_{tv} \cdot b_{tv} \cdot h_{tv}^3}{cc_{tv}}$$

Formula 3.36 - Stiffnesses of rectangular blocking

Here, E_{tv} is the blocking's elastic modulus in the globally defined x direction, b_{tv} is the width and h_{tv} the height of the blocking, and cc_{tv} is the spacing of the blocking.

The contribution of the ceiling, if applicable, can also be considered by increasing relevant stiffness values. Due to the ceiling's discontinuity at splices, not all stiffness parameters should be included.

BTAB only calculates one-span timber joist floors that are simply supported, and not floors with clamped/fixed supports.

3.4.3.2 BTAB calculations versus measured deflections

The table below shows measured deflections of various timber joist floors along with theoretically calculated deflections using BTAB. Two of the six floors are with ceiling, and the percentage difference is also included:

Betegnelse	Nedbøyning [mm]		Avvik	Merknad
	Målt	Beregnet		
BDB	1,33 1)	1,34	+1 %	
TL1	1,16	1,34	+16 %	
TL2	0,98	1,27	+30 %	Med himling
IL6	2,32	2,32	0 %	
IL8	1,32	1,52	+16 %	
IL9	1,03	1,43	+39 %	Med himling

Table 3.17 - Measured vs. BTAB calculated deflections of timber joist floors

The calculated deflection is generally higher than what was measured. The calculations assumed a spring constant of 200 N/mm for the connection between the subfloor and the joists, which is appropriate for a nailed connection. However, the tested floors had a screwed connection, which in reality probably has a higher stiffness. If the spring stiffness constant is increased to 550 N/mm for the floor "TL1", then the calculated deflection is reduced so that it equals what was measured.

The two floors with ceiling had the highest difference between measured and calculated deflections. Laboratory tests showed that the ceiling reduced deflection by 16-22 %, while the calculated reduction was only 5-6 %, so that is part of the explanation.

To show the effect of parameter variation, the figure below shows what the theoretically calculated deflection of floor "TL1" would be with various spring constants. (Its measured deflection of 1.16 mm would give a spring constant of 550 N/mm if that is the only corrected flaw in the calculations, but that is not shown in the figure.) If the subfloor is just floating (i. e. unconnected) on top of the joists, then the spring constant of 0 gives a high deflection. Increasing the spring constant to 200 or 550 N/mm reduces the deflection a lot. For a glued and nailed connection between joists and subfloor, the spring constant is taken as the very high number 10^6 N/mm. That gives a further deflection decrease, but we see that the deflection is much more sensitive to variations of the spring constant when it is low:

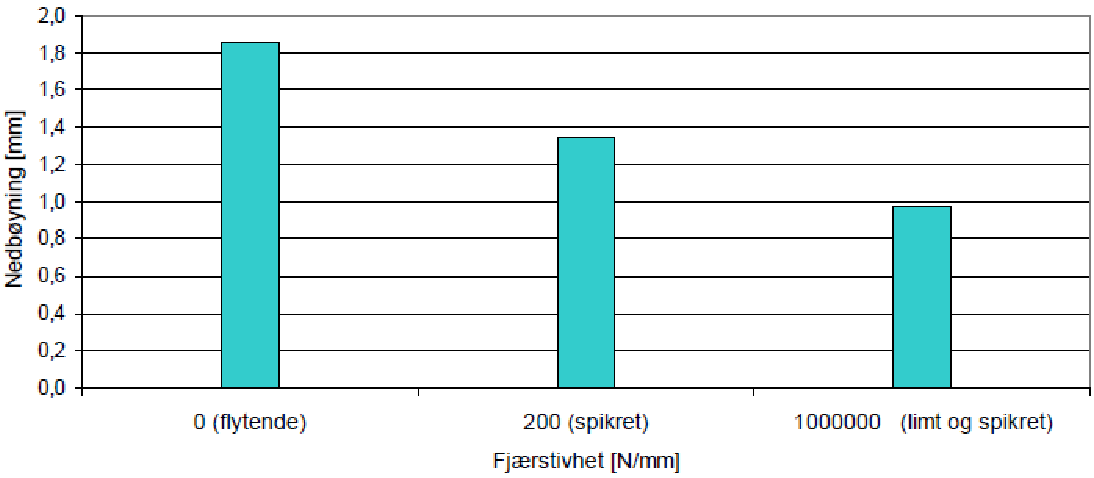


Figure 3.11 - Effect of connection between subfloor and joists, calculated with BTAB

Transversal load distribution depends on stiffness and geometry. For one-span floors subjected to a concentrated load in the center of the floor, the end joists will deflect nothing (or very little) if the floor is very wide. This also means that the end joists will contribute nothing towards reducing the middle joist deflection.

A one-span example floor with a joist spacing of 600 mm calculated with BTAB in the Kolstad/Homb report [21] showed that the width/span ratio had to be below about 0.5 for the middle joist deflection to increase. For a span of 4.8 m, it didn't matter much if the width was 2.4 m or 3.6 m, but a width of 1.2 m gave a much higher maximum deflection. Similarly, BTAB calculations also showed that supporting the floor on all four sides only helped reduce the deflection if the width/span ratio was below 0.5.

It should be stressed that this ratio of 0.5 pertains to these specific examples, and that floors with higher transversal stiffness will be better at distributing the load along the floor width, i. e. a higher width/span ratio would then help reduce the mid-joist deflection. The calculated examples had low transversal stiffness.

If a timber joist floor has low width and the end joists are supported along their length, these supports can be modelled by defining a very high bending stiffness of the end joists.

In BTAB, the stiffness of blocking has to be added as plate stiffness evenly distributed over the width of the floor. It is conservative to assume that the stiffness is evenly distributed over the floor's span. By doing this, one calculated example found that adding blocking to a given timber joist floor reduced the maximum deflection by 53 %. Measurements of the same floor showed the reduction to be 60 %. Floors that have higher transversal stiffness to begin with will however not see such a high percentage reduction from adding blocking. By doubling the evenly distributed stiffness from the blocking in BTAB, the calculated reduction in deflection was 59 %. This is more in line with the measurement, and the small difference between 53 % and 59 % shows that the returns are severely diminished when the transversal stiffness is already high.

As mentioned above, the ceiling also helps to reduce the deflection. This is contrary to what one may assume initially, because it's on the bottom side of the joists. In the example calculations given in the latest EC5 vibration committee document [4], the ceiling is not considered towards the transversal stiffness $(EI)_b$. But the two examples in the Kolstad/Homb report [21] showed deflection reductions of 16 % and 22 % when measured with 12 mm chipboard added as ceiling. (Once again, the percentages pertain to the specific examples and are by no means generally applicable.)

The laths also provide some transversal stiffness. Their height is more important than their width. In a given BTAB calculated example floor with 48 mm wide laths spaced 600 mm apart, heights of 23, 36 and 48 mm reduced deflections by 4 %, 11 % and 19 %, respectively.

The figure below shows measured deflections vs. deflections calculated with BTAB and KAN for six timber joist floors. Both programs are conservative, and BTAB was more accurate (17 % vs. 35 % error, on average):

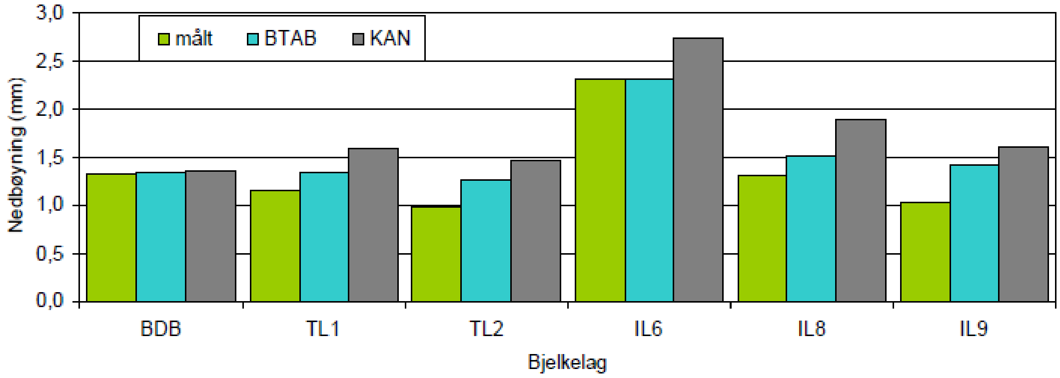


Figure 3.12 - Deflections measured vs. calculated with BTAB and KAN

In summary: Boarding, blocking, ceiling, laths, sufficient floor width, and a rigid connection between joists and sheathing increases the transversal stiffness and thereby decreases the deflection of timber joist floors subjected to a concentrated load. Supporting the end joists may also help, depending on geometry and stiffness properties.

3.4.3.3 Frequency calculation

Timber joist floors have much higher stiffness in the joist direction than transversally and are therefore only supported on two of four sides in most cases. For the Støren floors, that means that the basic f_1 formula for one-span floors (Formula 3.15) is applicable. This is the formula obtained when the anisotropic plate model of Leissa [23], which takes different boundary/support conditions into account, is simplified accordingly. However, Homb [20] has found better correlation with measurements from using an orthotropic plate model for four-side supported floors (the same as in Ohlsson [12] and Leissa [23]). Orthotropic is an abbreviation of orthogonal and anisotropic, and it means here that the plate is modelled as having different stiffness properties in the two main, perpendicular directions ($D_x > D_y$). This describes timber floors quite well. Resonance frequencies for a rectangular, orthotropic plate (simply supported on all four sides), can be calculated from this formula (where e. g. f_{21} means mode 2, natural frequency 1):

$$f_{mn} = \frac{\pi}{2L^2}\sqrt{\frac{1}{g}}\sqrt{D_x\, m^4 + 2\, D_{xy}\, m^2\, n^2 \left(\frac{L}{b}\right)^2 + D_y\, n^4 \left(\frac{L}{b}\right)^4}$$

Formula 3.37 - Resonance frequencies for two-span, rectangular, orthotropic plate

where:

L	is the floor span [m]
b	is the width of the floor [m]
g	is the unit weight [kg/m^2]
m	is an integer
n	is an integer

$$D_x = (EI)_L = E_{beam}\, I_x \,/\, c$$

$$D_y = (EI)_b = \Sigma(E_{boarding}\, I_y \,/\, L) = \Sigma(E_{boarding}\, t^3/12)$$

$$D_{xy} = \nu\, D_x + 2\, D_k$$

$$D_k = \frac{G\, h^3}{12}$$

h is the beam height [m]
v is Poisson's ratio

D_{xy} can normally be taken as equal to D_y. [20] [24] [25] [12] To find the fundamental frequency f_1, both integers *m* (not the mass) and *n* are set equal to 1. The difference between Formula 3.15 and Formula 3.37 can then be simplified to this multiplication factor:

$$\sqrt{1 + \frac{D_y}{D_x}\left[2\left(\frac{L}{b}\right)^2 + \left(\frac{L}{b}\right)^4\right]} = \sqrt{1 + \frac{(EI)_b}{(EI)_L}\left[2\left(\frac{L}{b}\right)^2 + \left(\frac{L}{b}\right)^4\right]}$$

Formula 3.38 - Fundamental frequency multiplication factor

This is similar to the f_1 multiplication factors for two-span floors given by Hamm/Richter/Winter, the Austrian National Annex to EC5, and the new EC5 proposal. The only difference is that those code-based methods have set D_{xy} equal to zero rather than equal to D_y, thus nullifying the second-order term.

The formula is warranted because, as seen in the figure below from Hamm/Richter/Winter, the fundamental frequency tends to be higher in real life than in the theoretical calculations. In these cases it was due to conservatively assumed boundary conditions and stiffness from partition walls. [8]

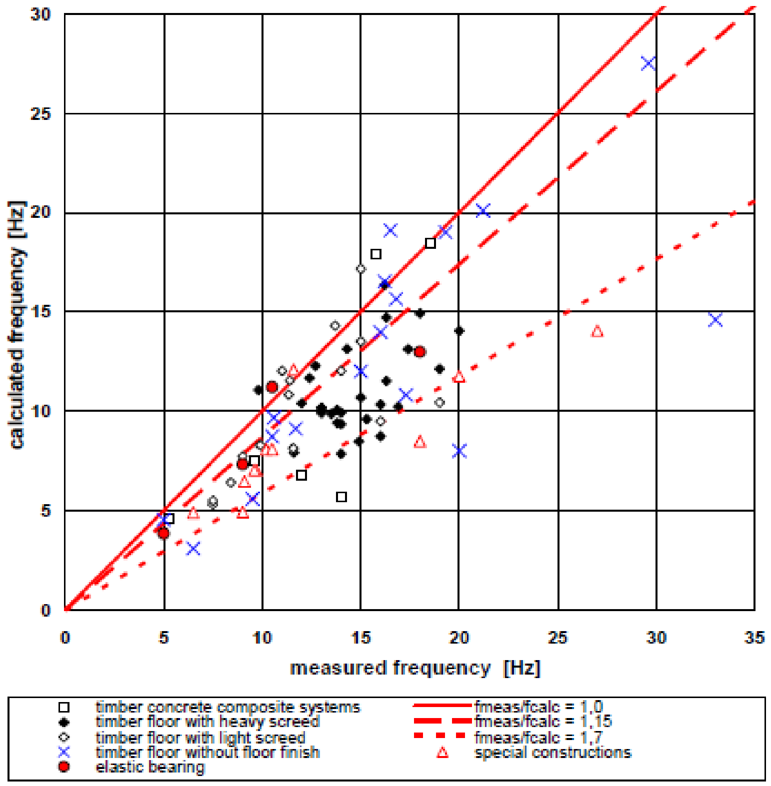


Figure 3.13 - Calculated vs. measured fundamental frequencies

4 Laboratory testing of floors

In June of 2018, one unmounted floor and five mounted floors were delivered from Støren Treindustri to NMBU for laboratory testing of vibrational properties. Static deflection tests were done shortly after delivery for five floors, from which stiffness properties necessary for vibration verifications could be calculated. The main purpose of the tests was to see how much transversal stiffness can be achieved from different modifications, and thereby how the vibrational properties can be improved the most efficiently.

At the end of the year, the beams of the unmounted floor were bending tested in the wood laboratory in order to determine the elastic modulus. One of the beams was also loaded until failure. Dynamic tests, with a walking person as the load, will be done in the future for all six floors.

This chapter describes the testing of the floors and presents the results. Most of the calculations based on the test results are shown in chapters 5 and 6.

4.1 Description of the floors

The properties of the Støren floors are summarized in the table below. Differences from the reference floor (floor number 1) are highlighted in bold.

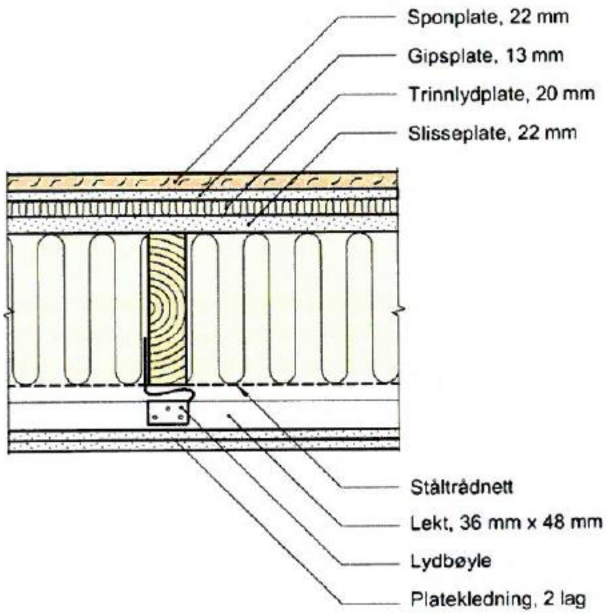
Floor	Joist	Joist spacing	Layout	Sheathing panel	Floor span	Floor length	Number of joists	Floor width
1	48x300 K-plus	600 mm	Complete: with non-structural components	Perforated particleboard	4,7 m	4,9 m	5	2,4 m
2	48x300 K-plus	600 mm	Complete: with non-structural components	**Not perforated particleboard**	4,7 m	4,9 m	5	2,4 m
3	48x300 K-plus	**300 mm**	Complete: with non-structural components	Perforated particleboard	**5,35 m**	**5,55 m**	**9**	2,4 m
4	**48x250 K-bjelke**	600 mm	Complete: with non-structural components	Perforated particleboard	**4,00 m**	**4,20 m**	5	2,4 m
5	48x300 K-plus	600 mm	Complete: with non-structural components	**Not perforated particleboard**	4,7 m	4,9 m	**9**	**4,8 m**
6	48x300 K-plus	600 mm	Complete + **blocking**	Perforated particleboard	4,7 m	4,9 m	5	2,4 m

Table 4.1 - Description of the six Støren floors

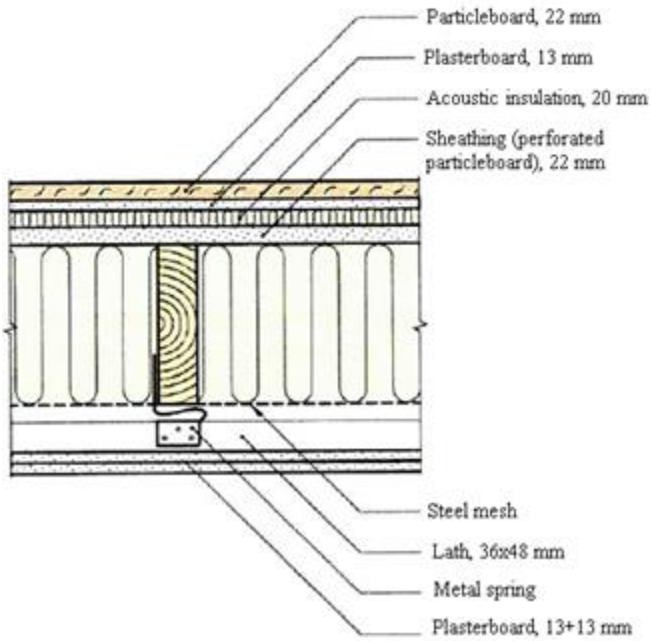
The only difference between floors 1 and 2 is the perforation. Floors 1 and 6 are identical except that the latter has blocking. The static test results for floors 2 and 6 provide a direct comparison of the added transversal stiffness from adding blocking versus switching to sheathing without holes. Floors 2 and 5 are identical other than the floor width (and the number of joists), but the joist spacing is the same; however, static tests were not done for floor 5. Other than that, there are at least two variables that differ between the floors. The transversal stiffness can however be compared between all of them through the calculation of b_{ef} (the effective width) from the test results.

Floor 4 has a different type of joist, which has a lower modulus of elasticity (as shown in Table 4.3) as well as a lower height, but with a lower span. Floor 3 is the only one with a lower joist spacing, and thus it also has a longer span.

The various floors were tested with and without non-structural parts. The bare structure was simply the joists and the sheathing (and blocking for floor 6). Then the same tests were performed after adding 20 mm acoustic insulation, 13 mm plasterboard and 22 mm particleboard on top of the sheathing, to see how much that helped distribute the load to the other joists. The cross section of the Støren floors (with perforated sheathing) are as shown in Figure 4.1, from “SINTEF Certification Nr. 2232” [26]. The non-structural components on the bottom side of the floor were not mounted. How this affects the test results and calculations is discussed later.



Layout from Norwegian certificate



English translation

Figure 4.1 - Cross section of the floors

Floor 1 is the reference floor. It has good acoustic properties, but vibration problems can occur as it is lightweight with low transversal stiffness. The reference floor and the modified versions of it were to be tested to see which changes could improve the vibrational properties the most.

The Støren floors were delivered with the maximum allowed spans, with the exception of floor number 4 which is 0.2 m too long-span to be approved. These values are summarized in Table 4.2 below, taken from “SINTEF Certification Nr. 2365”, the technical approval document for the joists [27]. The relevant values for the tested floors are outlined in red. Floor 4 would have been approved if the joists were K-Bjelke Plus rather than K-Bjelke.

For the technical certification, the vibrational properties were verified according to the “comfort criterion”, as suggested by Homb and adopted by SINTEF Byggforsk [13] [28]. It requires a deflection of less than 1.3 mm from a 1 kN concentrated load, the fundamental frequency must be higher than 10 Hz, and Formula 3.33 (Hu's combined deflection and fundamental frequency criterion) must be satisfied.

According to Table NA.6.2 in the Norwegian National Annex of NS-EN 1991-1-1, the maximum live load (in addition to the self-weight of movable partitions) for floors in residential buildings is 2.0 kN/m² as oppose to 3.0 kN/m² for office buildings [29]. But as seen in Table 4.2, the maximum allowed span is the same for both load cases. So it is the vibration requirements that are the strictest, thus highlighting the purpose of our research project.

Bjelketype	Maksimal lysåpning i meter 1)											
	Nyttelast 2,0 kN/m² og tilleggslast fra lette skillevegger (boliger o.l.)						Nyttelast 3,0 kN/m² og tilleggslast fra lette skillevegger (kontorer ol.)1)					
	Bjelker over ett felt			Kontinuerlige bjelker over to like felt			Bjelker over ett felt			Kontinuerlige bjelker over to like felt		
	Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm		
	300	400	600	300	400	600	300	400	600	300	400	600
K-Bjelke												
36 x 200	3,45	3,25	2,95	3,60	3,40	3,10	3,45	3,25	2,85	3,60	3,30	2,85
36 x 250	4,15	3,90	3,55	4,35	4,10	3,75	4,15	3,90	3,55	4,35	4,10	3,60
36 x 300	4,80	4,50	4,15	5,00	4,75	4,35	4,80	4,50	4,15	5,00	4,75	4,35
48 x 200	3,65	3,45	3,10	3,80	3,60	3,30	3,65	3,45	3,10	3,80	3,60	3,15
48 x 250	4,35	4,15	3,80	4,60	4,35	3,95	4,35	4,15	3,80	4,60	4,35	3,95
48 x 300	5,05	4,80	4,40	5,30	5,05	4,65	5,05	4,80	4,40	5,30	5,05	4,65
70 x 200	3,90	3,70	3,40	4,10	3,90	3,55	3,90	3,70	3,40	4,10	3,90	3,55
70 x 250	4,70	4,45	4,10	4,90	4,65	4,30	4,70	4,45	4,10	4,90	4,65	4,30
70 x 300	5,40	5,15	4,75	5,70	5,40	5,00	5,40	5,15	4,75	5,70	5,40	5,00
K-Bjelke Plus												
36 x 200	3,65	3,45	3,10	3,85	3,60	3,25	3,65	3,45	3,10	3,85	3,55	3,10
36 x 250	4,40	4,15	3,75	4,60	4,35	3,95	4,40	4,15	3,75	4,60	4,35	3,90
36 x 300	5,05	4,80	4,40	5,30	5,05	4,65	5,05	4,80	4,40	5,30	5,05	4,65
48 x 200	3,85	3,65	3,30	4,05	3,80	3,50	3,85	3,65	3,35	4,05	3,80	3,40
48 x 250	4,65	4,40	4,00	4,85	4,60	4,20	4,65	4,40	4,00	4,85	4,65	4,20
48 x 300	5,35	5,10	4,70	5,65	5,35	4,90	5,35	5,10	4,70	5,65	5,35	4,90
70 x 200	4,15	3,95	3,60	4,35	4,15	3,80	4,15	3,95	3,60	4,35	4,15	3,80
70 x 250	5,00	4,75	4,35	5,25	4,95	4,55	5,00	4,75	4,35	5,25	4,95	4,55
70 x 300	5,75	5,50	5,05	6,05	5,75	5,30	5,75	5,50	5,05	6,05	5,75	5,30

Table 4.2 - Maximum spans of "K beam" floors

The "K-Bjelke" and "K-Bjelke Plus" glulam joists used in the Støren floors are of timber quality C24, except for the outer lamellas of the latter which are C40, increasing the quality of the product to C33. The material properties, with standard Eurocode abbreviations and units of N/mm², are as shown in Table 4.3, taken from the technical approval document. [27]

	K-Bjelke	K-Bjelke Plus
$f_{m,k}$	24,0	33,0
$f_{t,0,k}$	14,0	14,0
$f_{t,90,k}$	0,4	0,4
$f_{c,0,k}$	21,0	21,0
$f_{c,90,k}$	5,3 1)	5,3 1)
$f_{v,k}$	3,5	3,5
$E_{0,05}$	7400	9400
$E_{0,m}$	11000	14000
$E_{90,m}$	370	370
$G_{0,m}$	690	690

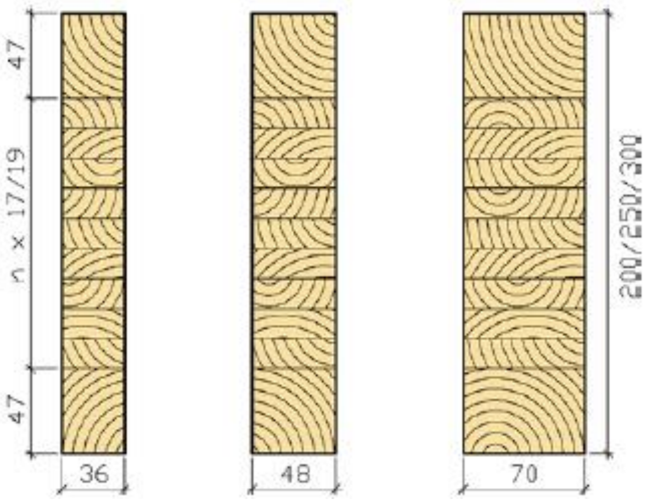


Table 4.3 - Material properties of the "K beams"

Figure 4.2 - Standard "K beam" cross section build-ups

The 22 mm perforated particleboards ("slisseplater" in Norwegian) are connected to the joists by screws and glue. They are mounted so that their longitudinal direction is in the floor's transversal direction, and they are glued along all edges.

The holes (7*20 = 140 in total per element) comprise 15 % of the net area (and volume), and so this is accounted for in the mass calculations. The elastic modulus in the product's longitudinal direction is 2550 N/mm^2. The density is 685-700 kg/m^3. [30]

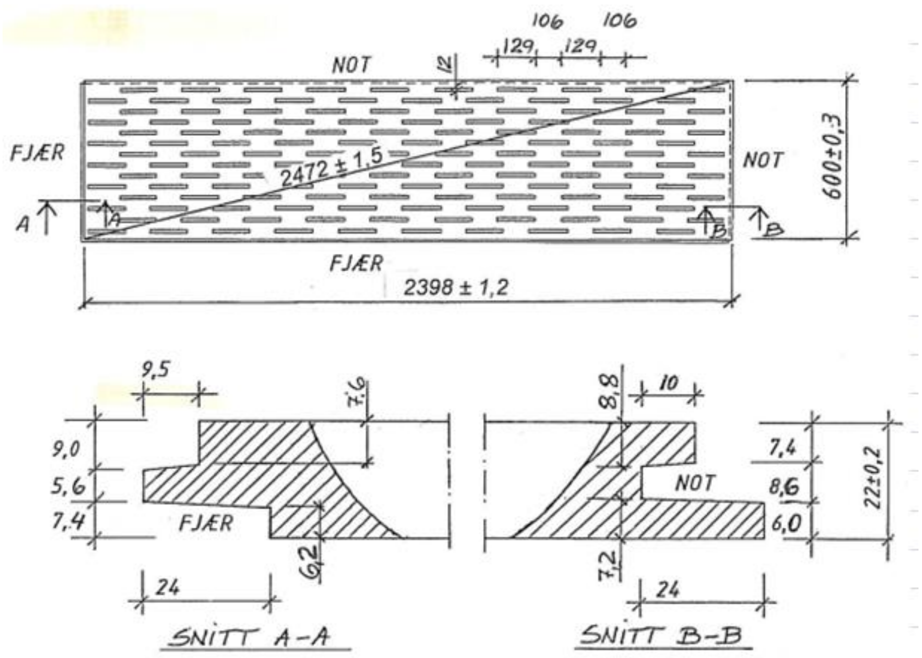


Figure 4.3 - Technical specification drawings of the perforated particleboard

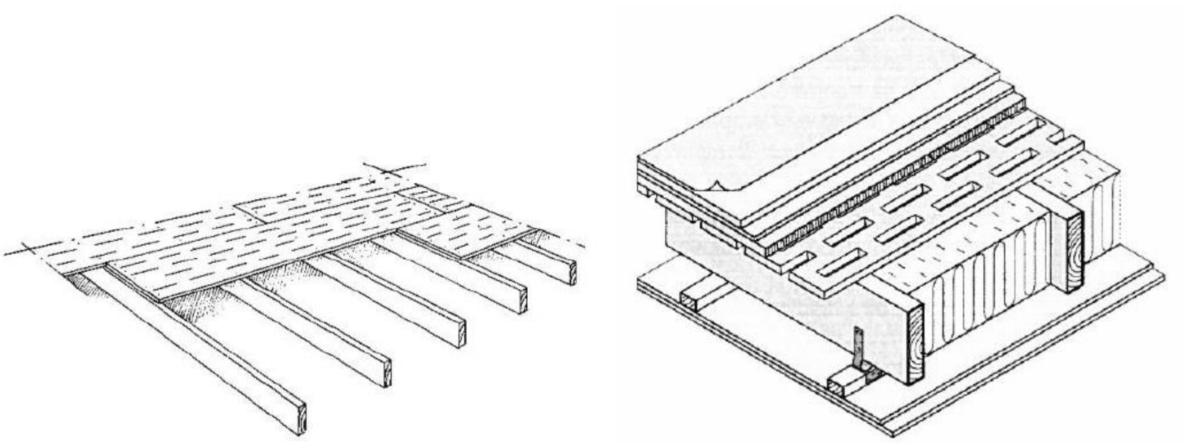


Figure 4.4 - Floor build-up, structural and complete

The blocking of floor 6 was as pictured in Figure 4.5. Such transversal stiffeners were only mounted mid-span. The supports are also (barely) visible in the back. They are steel cylinders, making for perfect roller supports. All of the joists are actually supported by rollers on both sides, but for deflection that doesn't matter; the result will be the same as for a simply-supported floor. But if there were horizontal constraints on both sides however, it would introduce a tensile force in the longitudinal direction that would reduce the deflection. For safety reasons, it was necessary to restrict the possibility of very large horizontal movements (by nailing laths to the outer side of the timber supports), but the supports were still rollers on both sides. The visible gap between the cylinders is irrelevant as all of the joists were supported:

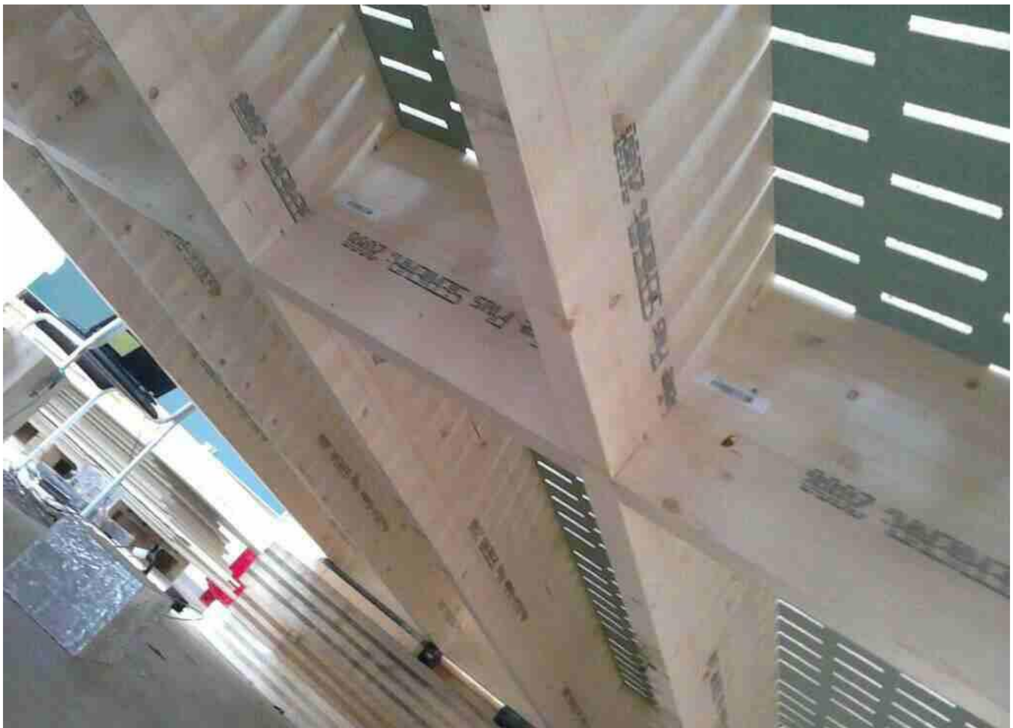


Figure 4.5 - Floor 6, mid-span blocking

4.2 Method

The static testing was done together with an exchange student from the University of Trento who was also looking into vibrations of timber joist floors for her master's thesis.

Two load configurations were tested for all floors; a concentrated load mid-span on the middle joist, and concentrated load mid-span on all joists simultaneously. The displacements of the joists were measured with displacement transducers connected to the computer software. They are shown in the pictures below, taken from Bevilacqua's thesis [31]:

Figure 4.6 - Displacement transducers

All of the weights at our disposal were weighed. Floor 1 was initially tested with a centric concentrated load of 100.0 kg (43.65 + 43.00 + 3.30 + 10.05). Then we decided to test the floor with 200.0 kg also (by adding 41.95 + 22.95 + 31.80 + 3.20 + 0.10 = 100.0 kg), in order to be more certain about the accuracy of the measurements. As a general example, 2.0 mm $\pm$ 0.1 mm is more reliable than 1.0 mm $\pm$ 0.1 mm in terms of percentage error from the measuring equipment. So more weight is better, and for the subsequent floors, only 200.0 kg = 1.96 kN was used for the single concentrated load.

We also had some "background noise" (for the lack of a better term) that made the measured displacement value jump a bit up and down from second to second (possibly due to vibrations of the tested floor caused by surrounding technical applications in the laboratory?). And so to get sufficient accuracy, we took 10 measurements for each test with the plan to use the calculated average in the further calculations. Part of the reason for the magnitude of the fluctuations seemed to be because of the "factor" value in the software linked to the displacement transducers. If for example a 3.0 mm plate placed over the spring gave a reading of 0.3 mm (after offset value was set equal to raw value, with factor equal to one), setting a factor of 10.0 would calibrate the device properly to give an output of 3.0 mm. However, one of the factors was about 35 (as oppose to 10-15 for the other displacement transducers), and so its raw measurement was accordingly multiplied by a number about three times larger, resulting in three times higher inaccuracy compared to the others. The results from this transducer were therefore not trustworthy and could not be used. Also, due to time constraints we could not do measurements for all joists for all floors as we didn't have enough devices that functioned properly at all times. Some results that were obviously wrong were excluded.

For the loading of all joists simultaneously, 43.65 kg was used on each joist. More weight could have been used for the five-joist floors, but for those with nine joists it would then have been difficult to combine the available weights in a such a way that all joists were loaded equally with the same higher load. Five of the combinations used for all of the floors were exactly 43.65 kg, and the four others were 43.00, 43.60, 43.80, and 44.00. Here the loading is shown for floor 3 which has nine joists:

Figure 4.7 - Loading of nine joists

The 200 kg concentrated load in the middle of the span of the middle joist was done as shown in Figure 4.8. Here we can also see the different layers of the floors when complete; with particleboard on top, followed by gypsum plasterboard, acoustic insulation, and sheathing above the joists. The components on the bottom side in Figure 4.1, such as thermal insulation and the ceiling, were not mounted.

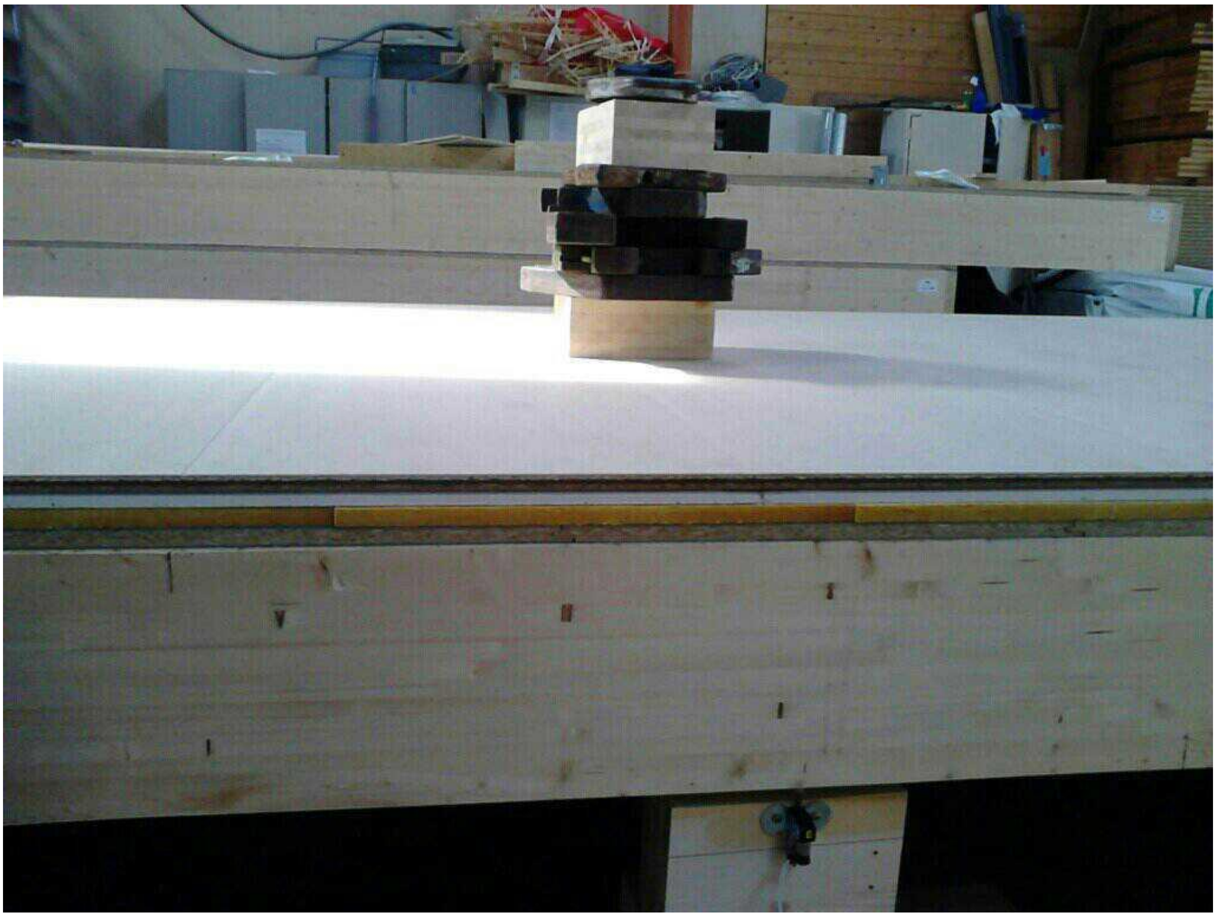


Figure 4.8 - Centric 200 kg concentrated load on floor, complete with non-structural parts

4.3 Hypothesis

4.3.1 Load on all joists

For the equal loading of all joists simultaneously, the deflection should be (more or less) the same for all joists, given that their modulus of elasticity is the same. Obviously there will be some variation here as the construction material is produced by the forest, and sorted into strength classes by visual grading rather than accurate laboratory testing.

The deflection for this load case should also be virtually the same with and without non-structural components, for three reasons. First of all, the self-weight of the non-structural components was irrelevant for these tests, because the equipment was calibrated (with the initial displacement set to zero) after the components were applied. Secondly, the transversal stiffness becomes irrelevant when the joists are loaded equally; the plate will distribute as much of the joist load to the neighbor joists as is distributed back, making the net effect zero. (Possibly a small difference for the edge joists as they only have one neighbor joist.) And thirdly, there is obviously no composite action from the non-structural parts as they are just floating on top, not connected to the rest of the floor (and the elements are also not rigidly connected in the floor's longitudinal direction), and so any added longitudinal bending stiffness is orders of magnitude lower than that from the joists.

And if there is also no composite action (with regards to floor longitudinal stiffness) from the sheathing which is glued and screwed to the joists, then the joist deformation from the equal loading of all joists simultaneously should be predicted quite accurately by the bending deformation $FL^3/(48\ EI_{joist})$ plus the shear deformation, in accordance with Timoshenko beam theory.

The longitudinal stiffness $(EI)_L$ can anyway be calculated based on these measurements, and that is necessary to estimate the transversal stiffness more accurately, as the calculation of b_{ef} (via the other load case measurements) involves the parameter $(EI)_L$. So these measurements are important for the calibration of the model, as the stiffness of timber joists can vary a lot.

4.3.2 Load on middle joist

For the loading of only the middle joist with a concentrated load (of 200 kg), the transversal stiffness is to be investigated. It seems obvious that floor 2 with the non-perforated particleboard, as well as floor 6 with the blocking, both should have higher transversal stiffness than floor 1 (the reference floor). The question is how much these separate modifications of the reference floor contribute to the load redistribution. For floors 3 and 4 it is less apparent how the altered properties would affect the transversal stiffness.

The effect of adding the non-structural components is investigated for all of the tested floors. Since the exact same components are added each time, it would be reasonable to assume that the same additive increase of $(EI)_b$ should be found for all of the floors.

The tests should provide information about how much transversal stiffness that can be considered for analytical calculations, both for the added layers and for the sheathing that is glued and screwed to the joists. As shown in Figure 3.11, it is expected that the more rigid connection of the sheathing should give a higher benefit per mm of particleboard thickness, but the tests performed by us do not measure the benefit of the unconnected particleboard alone, as the plasterboard (and acoustic insulation) is added at the same time.

The floors (with the exception of floor number 4) are approved according to the "comfort criterion" by BTAB calculations, so it would be expected that our measurements also verify the floors. Other verification methods will give different answers.

4.4 Expected deformation from bending and shear

The basic deflection expression $FL^3/(48\ EI)$ from Euler/Bernoulli only considers bending deformation. In order to better understand the measurements, the importance of shear deformation was investigated. A finite element model of a beam was created in SAP2000. It was modelled as simply supported and with the same basic properties as the reference floor joists: b = 48 mm, h = 300 mm, L = 4700 mm, E = 14 000 N/mm^2. The shear modulus G was initially set to 690 N/mm^2, as reported by the producer. Then, to get only the bending deformation, the shear modulus was set to infinite. The selfweight was modelled as zero, just like in our measurements. The mid-span concentrated load F was arbitrarily chosen as 1000 N. The percentage difference between the bending deformation and the total (shear+bending) deformation could then be found. Mesh density was irrelevant:

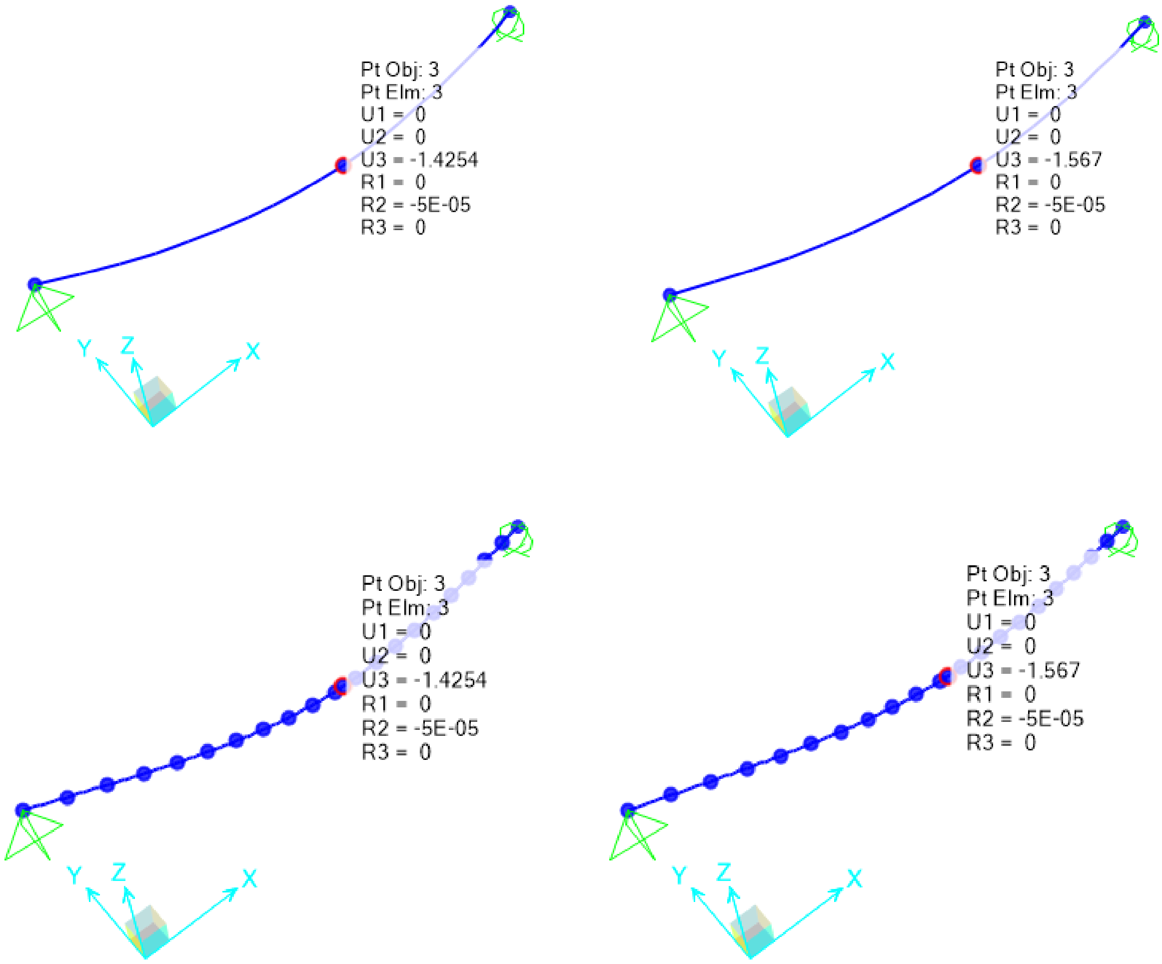


Figure 4.9 - Euler/Bernoulli vs. Timoshenko beam models

So, the Timoshenko beam deflection is 10 % higher (1.567 mm / 1.4254 mm = 1.0993) than for Euler/Bernoulli, for the reference floor joists. The shear deformation is therefore significant and should not be neglected in the further calculations, as our measured deflections of course are due to both bending and shear.

The contribution of shear to the total deflection can also be hand-calculated from the formula below, applicable for a concentrated load mid-span on a beam with rectangular cross section. [32]

$$\delta_{shear} = \frac{3\,FL}{10\,GA}$$

Formula 4.1 - Shear deflection for rectangular section beam point-loaded at mid-span

Here, *A* is the cross-sectional area *bh*.

The finite element model (FEM) found a shear deflection of 1.567 mm - 1.4254 mm = 0.1416 mm, which corresponds very well with the 0.14198 mm obtained by using the above formula.

The FEM modelled Euler/Bernoulli beam showed a bending deflection of 1.4254 mm, also basically the same as $FL^3/(48\ EI)$ = 1.4305 mm.

It can be shown that the ratio of shear to bending deflection is given by the formula below, which shows that shear is more significant the larger the $(h/L)^2$ ratio is (for constant E/G):

$$\frac{\delta_{shear}}{\delta_{bending}} = \frac{6h^2}{5L^2}\frac{E}{G}$$

Formula 4.2 - Ratio of shear to bending deflection for the tested floor joists

To obtain this, the Formula 4.1 expression (with A substituted for $12\ I/h^2$) was simply divided by $FL^3/(48EI)$.

These are the values for the tested Støren floor joists:

	Floor 1	**Floor 2**	**Floor 3**	**Floor 4**	**Floor 6**
h	300	300	300	250	300
L	4700	4740	5380	4050	4720
E	14 000	14 000	14 000	11 000	14 000
G	690	690	690	690	690
$\delta_{shear}/\delta_{bending}$	9.92 %	9.75 %	7.57 %	7.29 %	9.84 %

Table 4.4 - Theoretically expected contribution of shear deflection for the Støren floors

The total deformation can be predicted from this:

$$\delta = \delta_{bending} + \delta_{shear} = \frac{FL^3}{48\ EI} + \frac{3\ FL}{10\ GA} = \frac{FL^3}{48\ EI} * \left(1 + \frac{6h^2}{5L^2}\frac{E}{G}\right) \approx 1.1\frac{FL^3}{48\ EI}$$

Formula 4.3 - Deflection for rectangular Timoshenko beam point-loaded at mid-span

If instead used as cantilevers, with much shorter spans as needed to meet various design rules, the ratio of shear to bending deflection can become much higher than 10 %. [33]

4.5 Results

The test results are summarized in this subchapter. The test results in their entirety are found in "Appendix A – Floor test data".

4.5.1 Load on all joists

Table 4.5 shows the measured deflections (in mm) for the load case with 43.65 kg above each joist. The middle joist is indicated as "1200", while the end joists were at positions 0 mm and 2400 mm, and so on. Floor 5 with the width of 4800 mm is not included in the table as no static tests were done for it. As previously explained, not all joist displacements were measured, and they are indicated as empty cells of light blue color.

				LOAD ON EVERY JOIST - 43.65 kg									
		0	300	600	900	1200	1500	1800	2100	2400	Avg.	δ_{Timo}	Ratio
FLOOR 1	Structural			0,754		0,622		0,812		0,689	0,72	0,67	1,07
	Complete			0,690		0,577		0,567		0,628	0,62	0,67	0,91
FLOOR 2	Structural	0,622		0,643		0,667					0,64	0,69	0,93
	Complete	0,591		0,645		0,659					0,63	0,69	0,92
FLOOR 3	Structural					1,008		1,207		1,091	1,10	0,99	1,12
	Complete					1,128		1,055		1,067	1,08	0,99	1,10
FLOOR 4	Structural	0,990		1,057		0,976					1,01	0,92	1,09
	Complete	0,913		1,001		0,921					0,94	0,92	1,02
FLOOR 6	Structural	0,781		0,756		0,773					0,77	0,68	1,13
	Complete	0,809		0,840		0,773					0,81	0,68	1,18
													1,047

Table 4.5 - The measured deflections for the loading of all joists simultaneously

The column "δ_{Timo}" shows the deflection predicted by Timoshenko beam theory (Formula 4.3) when only the joist EI (with values as reported by the producer) is considered, while the sheathing's stiffness contribution is assumed negligible and calculated as zero. The average joist deflection ("Avg.") was then, on average for all tested floors, 4.7 % higher than theoretically calculated, as per the "Ratio" column. (13.9 % higher than for Euler/Bernoulli, not presented in the table.) So, according to these measurements, there is no composite action (with regards to longitudinal stiffness) between the joists and the sheathing that would warrant the use of Steiner's parallel axis theorem formula for analytical calculation of $(EI)_L$ for such floors. That implies that the decreased deflection with more rigid joist-sheathing connections, as shown in Figure 3.11, is explained purely by increased transversal stiffness $(EI)_b$, which is irrelevant for this load case.

Floor 1 has the most suspicious values. It was the first floor we did measurements for, and without established routines it was less likely that mistakes and anomalies would be discovered. The average joist deflection for floor 1 is 17 % higher without the non-structural components, whereas for the other floors there is little or no difference.

For joist 4 of floor 1, the measured deflection was 43 % higher for the bare structure. In theory it should be virtually the same as for the complete structure, so this cannot be correct. The second largest difference for a joist with and without non-structural parts (for joist 7 of floor 3) is "only" 14.4 %, while there are many cases of 5-10 % increase or decrease in measured deflection from adding the three layers. The expectation was that e. g. these two lines for floor 1 (the most extreme example) would be much better aligned, i. e. straighter and more equal to each other:

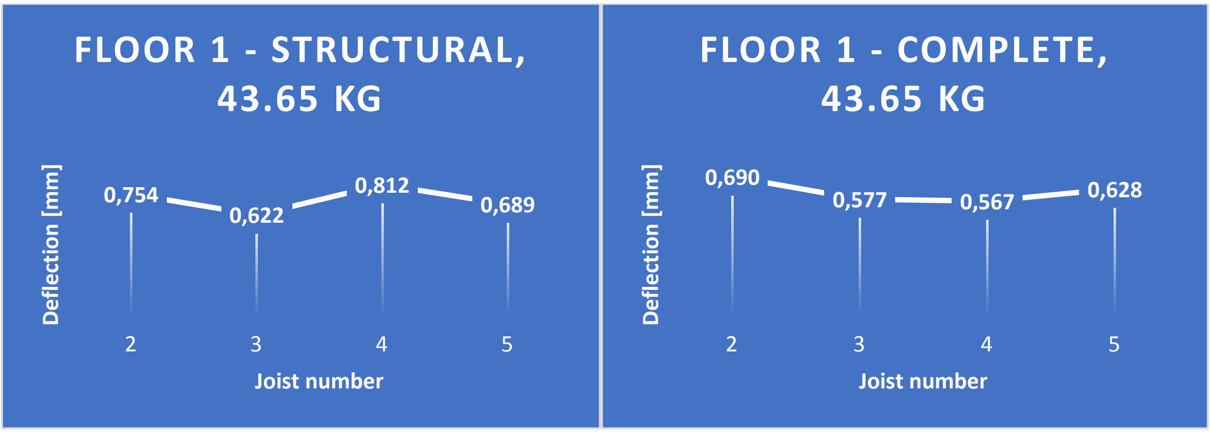


Figure 4.10 - Joist deflections from equal loading of all joists for floor 1, bare and complete

Since the longitudinal stiffness of the non-structural parts is orders of magnitude lower than for the joists, the $(EI)_L$ is the same for both floor configurations, and the deflection should be the same. So to make the values more reliable for further considerations, the joist deflections for “structural” and “complete” have been averaged:

				LOAD ON EVERY JOIST - 43.65 kg									
		0	300	600	900	1200	1500	1800	2100	2400	Avg.	δ_{Timo}	Ratio
FLOOR 1	Structural Complete			0,722		0,599		0,689		0,658	0,67	0,67	0,991
FLOOR 2	Structural Complete	0,607		0,644		0,663					0,64	0,69	0,925
FLOOR 3	Structural Complete					1,068		1,131		1,079	1,09	0,99	1,106
FLOOR 4	Structural Complete	0,951		1,029		0,948					0,98	0,92	1,055
FLOOR 6	Structural Complete	0,795		0,798		0,773					0,79	0,68	1,157
													1,047

Table 4.6 - Averaged joist displacements

It is interesting to note that there was much more variation between floors than between the joists of a floor.

The “Ratio” column implies that the floor 2 joists on average have a 25 % higher modulus of elasticity than those of floor 6 (because 1.157 / 0.925 = 1.25).

Not counting the uncertain measurements for floor 1, the highest difference in deflection between joists of a single floor is only 9 %, for floor 2 (where 0.663 / 0.607 = 1.09).

Even floor 1 with a 20.5 % difference between joists 2 and 3 is lower than the 25 %.

Statistically, it should of course be the opposite (with respect to stiffness); with more variation between joists than floors. And the testing of the joists (presented in section 4.7) showed variations of the Young's modulus (E) far greater than 9 %, as expected.

The logical explanation seems to be that, since the joists are connected to the sheathing, the floor acts somewhat like one unit. So when one joist is loaded, the neighbor joists are also pushed down a bit along with the sheathing, leading to relatively equal joist displacements. So the individual joist displacements cannot be used to calculate the E of the respective joists with good accuracy. The floor's longitudinal stiffness $(EI)_L$ can however be calculated from this, and that is also necessary to estimate the transversal stiffness more accurately. If for example the $(EI)_L$ is assumed 7 % too high, then the b_{ef} will be too low by a factor of 1.07 when calculated from measurements of the other load case. These calculations are done in chapters 5 and 6.

4.5.2 Load on middle joist

As seen in Table 4.5, only three of nine joist displacements were measured for floor 3 for that load case. But as they in theory should give roughly the same deflection, it was prioritized to spend more time on the other load case. The joist displacements for the 200 kg concentrated load on the middle joist are shown here:

				CONCENTRATED LOAD - 200 kg						
		0	300	600	900	1200	1500	1800	2100	2400
FLOOR 1	Structural			0,714		1,826		0,624		0,094
	Complete			0,797		1,410		0,787		0,077
FLOOR 2	Structural	0,266		0,691		1,575				
	Complete	0,210		0,608		1,323				
FLOOR 3	Structural	0,016	0,157	0,502	0,892	1,600	1,002	0,435	0,248	0,167
	Complete	0,105	0,115		0,861	1,044	0,876	0,611	0,101	0,085
FLOOR 4	Structural	0,344		1,071		2,404				
	Complete	0,348		1,110		1,814				
FLOOR 6	Structural	0,164		0,830		1,641				
	Complete	0,170		0,781		1,471				

Table 4.7 - The measured deflections for the 200 kg centric concentrated load

The relative joist deflection compared to that of the joist with the highest deflection is shown in this table:

				CONCENTRATED LOAD - 200 kg						
		0	300	600	900	1200	1500	1800	2100	2400
FLOOR 1	Structural			39 %		100 %		34 %		5 %
	Complete			57 %		100 %		56 %		5 %
FLOOR 2	Structural	17 %		44 %		100 %				
	Complete	16 %		46 %		100 %				
FLOOR 3	Structural	1 %	10 %	31 %	56 %	100 %	63 %	27 %	15 %	10 %
	Complete	10 %	11 %		82 %	100 %	84 %	59 %	10 %	8 %
FLOOR 4	Structural	14 %		45 %		100 %				
	Complete	19 %		61 %		100 %				
FLOOR 6	Structural	10 %		51 %		100 %				
	Complete	12 %		53 %		100 %				

Table 4.8 - Relative displacements

Below, the deflections of floor 3 with and without the non-structural components are shown graphically. Since the complete floor has higher transversal stiffness, the deflection of the mid-joist is considerably lower. The floor distributes the load better to the neighbor joists, and so the curve is less steep than for the bare structure.

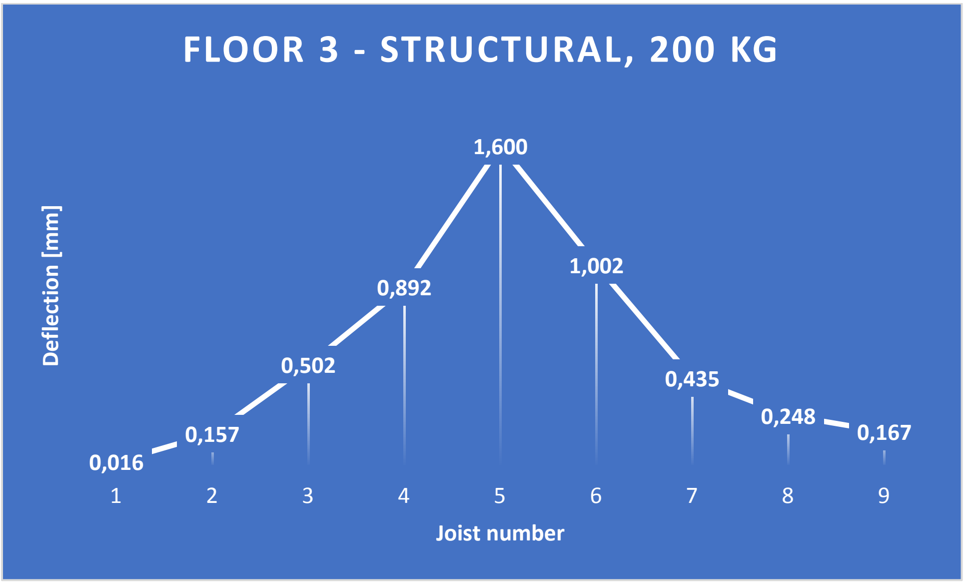


Figure 4.11 - Joist deflections from concentrated load for floor 3, bare and complete

Calculations based on the test results are done in chapters 5 and 6.

4.6 Testing of the displacement transducers

The accuracy of the measuring equipment used was investigated in the laboratory some months later, in order to determine or rule out a source of error. The measurements of the displacement transducers were compared against Instron, which is known to be much more accurate. The transducers were placed under the machine and calibrated in the same manner as during the summer. The Instron machine was then given a chosen vertical displacement, which could be compared to that of the other software. One or two transducers were tested at the time, as shown in the picture:

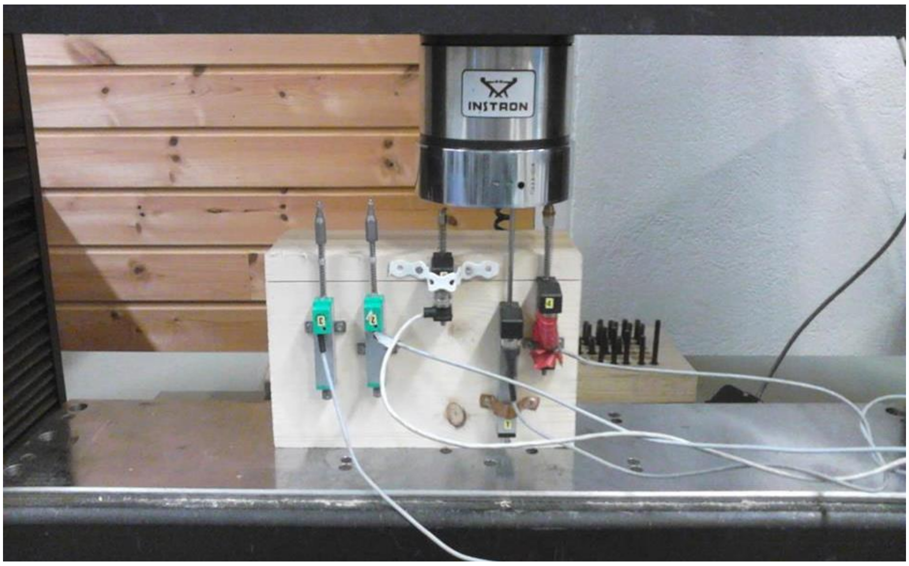


Figure 4.12 - Displacement transducers tested against Instron

The chosen Instron deflections were 0.65 mm and 1.64 mm, values similar in magnitude to those measured for the floors for the two load cases. And just like in the summer, ten displacement values were used to calculate the average.

Instron displ.	0,6524		1,645		0,6523		1,642		0,6504		0,6527	1,643	0,6509	0,6506	
Transducer	2	3	2	3	1	4	1	4	1	4	5	5	5	2	
1	0,714	0,617	1,637	1,651	0,539	0,734	1,575	1,694	0,663	0,593	0,639	1,649	0,680	0,589	
2	0,630	0,673	1,609	1,595	0,663	0,649	1,699	1,638	0,705	0,621	0,625	1,677	0,653	0,646	
3	0,630	0,673	1,609	1,651	0,497	0,706	1,699	1,638	0,746	0,621	0,625	1,607	0,653	0,730	
4	0,630	0,729	1,525	1,707	0,663	0,621	1,782	1,553	0,705	0,649	0,625	1,690	0,653	0,730	
5	0,630	0,617	1,693	1,651	0,663	0,649	1,782	1,581	0,580	0,734	0,625	1,649	0,611	0,674	
6	0,658	0,673	1,693	1,651	0,663	0,649	1,658	1,638	0,663	0,706	0,653	1,649	0,680	0,646	
7	0,574	0,673	1,637	1,679	0,746	0,593	1,575	1,722	0,663	0,621	0,694	1,621	0,625	0,589	
8	0,630	0,589	1,609	1,679	0,663	0,621	1,699	1,638	0,788	0,621	0,680	1,677	0,639	0,589	
9	0,602	0,673	1,637	1,735	0,580	0,706	1,575	1,694	0,788	0,565	0,611	1,677	0,653	0,730	
10	0,686	0,645	1,693	1,651	0,580	0,706	1,658	1,694	0,580	0,649	0,611	1,649	0,653	0,674	
Average displ.	**0,638**	**0,656**	**1,634**	**1,665**	**0,626**	**0,663**	**1,670**	**1,649**	**0,688**	**0,638**	**0,639**	**1,655**	**0,650**	**0,660**	
Difference	**-2,1 %**	**0,6 %**	**-0,7 %**	**1,2 %**	**-4,1 %**	**1,7 %**	**1,7 %**	**0,4 %**	**5,8 %**	**-1,9 %**	**-2,1 %**	**0,7 %**	**-0,1 %**	**1,4 %**	**0,2 %**
Max - min	0,140	0,140	0,168	0,140	0,249	0,141	0,207	0,169	0,208	0,169	0,083	0,083	0,069	0,141	
Total variation	21 %	21 %	10 %	9 %	38 %	22 %	13 %	10 %	32 %	26 %	13 %	5 %	11 %	22 %	

Table 4.9 - Comparison between displacement transducers and Instron

The maximum minus the minimum measured value is generally the same for 0.65 mm and 1.645 mm. The 0.65 mm measurements then give a higher error in terms of percentage. This is exemplified by the "total variation", calculated here as "max - min" divided by the Instron displacement.

The average difference between Instron and the transducers is only 0.2 %, which means that the transducers give correct values, given that the calibration is done accurately and that enough values are used to calculate the average. However, averaging based on ten values is not sufficient for transducer number 1, which gave the most inaccurate results. With values varying from 0.580 to 0.788 mm, one has to be a bit lucky to get as many extreme values on both sides of the correct, true displacement to get a representative average, and the highest difference of 5.8 % would have been even higher (7.6 %) without the tenth value.

This is explained by the calibration factor. The steel plate used for calibration in this case was measured as 4.97 mm thick. In the software, the initial output of 0.306 mm required a calibration factor of 4.97 / 0.306 = 16.242 to return the true displacement of 4.97 mm. For transducers 2, 3 and 4, the factors were all close to 11.0. So, a 50 % higher calibration factor for number 1 means that any error is amplified accordingly. Transducer number 5, with a calibration factor of about 5.4, had the best accuracy. It was about three times as accurate as number 1, and two times as accurate as the three other transducers when considering the "max – min" values, just as predicted by the difference in calibration factors.

The spring of transducer number 5 is actually not working properly over its entire length, but as long as it is only used within the range where it does work, it provides higher accuracy than the others. In the summer we also had a transducer with a required calibration factor of about 35, which was not acceptable.

The conclusion is that the transducers give reliable answers as long as they are calibrated correctly and enough values are used to calculate the average measured deflection. Since the average of 20 values could be used to calculate the longitudinal stiffness, because it is virtually the same with and without the three added layers, any large inaccuracies will have resulted from human error rather than the transducers themselves. No model updating is needed since the measurements only differed from Instron by 0.2 % on average.

4.7 EN 408 laboratory testing of beams

In the middle of December 2018, bending tests according to EN 408 [34] were performed for the joists of the unmounted floor 5, i. e. the only floor that we did not do static tests on in June. The objective was to determine whether the mean elastic modulus (E) parallel to the grain given in the technical certification [27] is reliable, both in terms of average value and how much variation there is between the tested joists.

Chapter 9 of EN 408 gives the procedure for determining the local E, and chapter 10 describes how the global E should be found. The formula for the global E involves the shear modulus G, which should be found by tests according to chapter 11. This was not done, but the shear modulus can be estimated by requiring that the global E should be equal to the local E. One of the beams was also loaded until failure.

4.7.1 Procedure for finding the elastic modulus parallel to the grain in bending

EN 408 specifies a lot of standardized distances that should be used for the testing. Some of these were difficult to satisfy, because the cross-sectional height (h = 300 mm) of the beams is quite large in relation to the beam length. Also, for the determination of the local E, the distance between the nails in the neutral axis for the steel gauge in the middle of the beam (see Figure 4.13) was too short in relation to h. The standardized distances are written in black and the distances as tested are written in red:

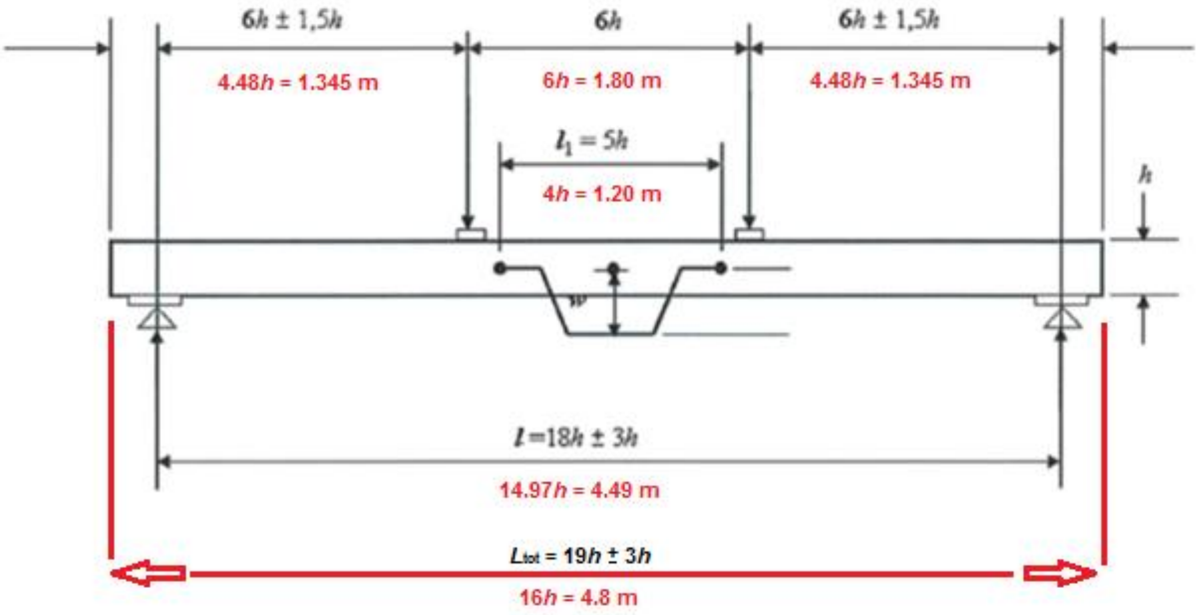


Figure 4.13 - Distances for determination of the local Young's modulus

The arrangement for finding the global E is very similar, the difference is just the displacement measurement:

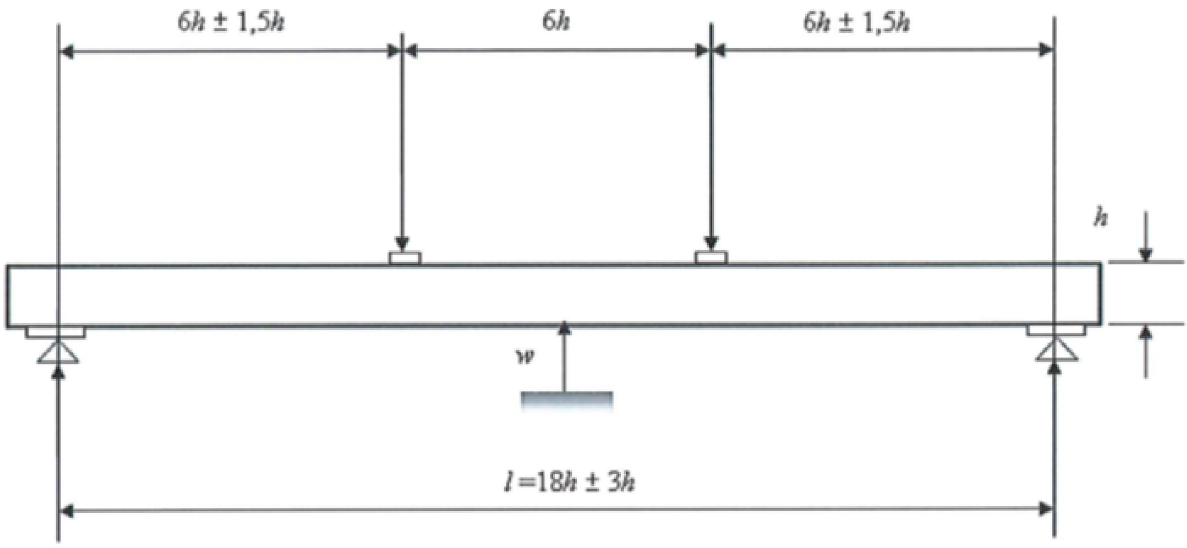


Figure 4.14 - Distances for determination of the global Young's modulus

The local modulus of elasticity in bending is calculated from this formula:

$$E_{m,l} = \frac{a\, {l_1}^2 (F_2 - F_1)}{16\, I\, (w_2 - w_1)}$$

Formula 4.4 - Local modulus of elasticity in bending where:

a	is the distance [mm] between the support and the load
l_1	is the distance [mm] between the nails supporting the steel gauge
I	is the second moment of area [mm⁴], equal to $bh^3/12$ for rectangular beams
$F_2–F_1$	is the increase in load [N] on the regression line with a correlation coefficient of at least 0.99
$w_2–w_1$	is the increase in deformation [mm] corresponding to $F_2–F_1$ in Figure 4.15

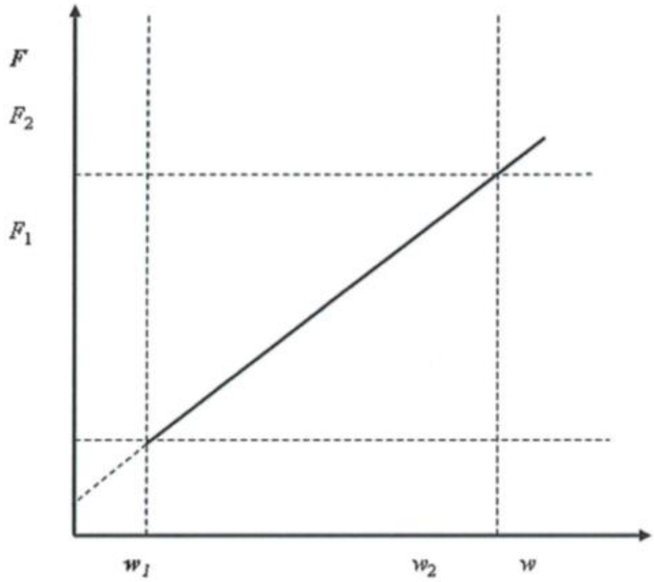


Figure 4.15 - Load/deformation graph within the range of elastic deformation

The global modulus of elasticity in bending is calculated from this formula (where G is the shear modulus):

$$E_{m,g} = \frac{3al^2 - 4a^3}{2bh^3(2\frac{w_2 - w_1}{F_2 - F_1} - \frac{6a}{5Gbh})}$$

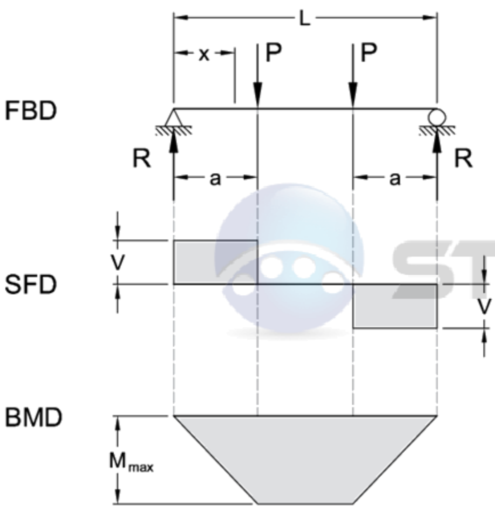
Formula 4.5 - Global modulus of elasticity in bending

Since the procedures are the same, the measurements for the local E and the global E were done simultaneously. (Instron also gave the displacement of the loading head, which isn't relevant for the formulas.) As required by EN 408, the test station for the beams was set up as simply supported with low friction lateral restraints to prevent lateral torsional buckling. Steel plates (which should not be longer than *h*/2 = 150 mm) were placed between the timber beam and the concentrated loads, to minimize local indentation from compression perpendicular to the grain, when loaded symmetrically in bending at the two points. In hindsight, the ones we used under the loads were probably a bit longer than *h*/2, but they were however quite thin (which the standard doesn't say anything about).

Figure 4.16 - Test station

The load was applied at a constant rate of 15 mm/min, lower than the maximum limit of (0.003 h) mm/s = 0.9 mm/s = 54 mm/min. Instron satisfies the standard's accuracy requirements.

The moment diagram, shear force diagram and formulas for this load case is as shown in the figure below, from StructX.com. [35]



$$\mathrm{R} = \mathrm{V} \ \ldots\ldots\ldots = P$$

$$\mathrm{M_{max}}\ \text{(between loads)} \ \ldots\ldots\ldots = Pa$$

$$\mathrm{M_x}\ (\mathrm{x} < a) \ \ldots\ldots\ldots = Px$$

$$\Delta_{\max}\ \text{(at centre)} \ \ldots\ldots\ldots = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$\Delta_{\mathrm{x}}\ (\mathrm{x} < \mathrm{a}) \ \ldots\ldots\ldots = \frac{Px}{6EI}(3La - 3a^2 - x^2)$$

$$\Delta_{\mathrm{x}}\ \left(\mathrm{a} < x < (L - a)\right) \ \ldots\ldots\ldots = \frac{Pa}{6EI}(3Lx - 3x^2 - a^2)$$

Figure 4.17 - Data for the EN 408 load case

So, the local Young's modulus in bending is measured in the middle of the beam where there is no shear force and a constant moment. The global E in bending is measured for the entire beam, and so Formula 4.5 includes the shear modulus G.

The maximum load had to be estimated before the testing. With respect to moment, the maximum load was calculated as such (without partial factors):

$M_{max} = \sigma_{max} * W = f_{m,k} * bh^2/6 = 33\ N/mm^2 * 720\ 000\ mm^3 = 23.76\ kNm$

$P_{max} = M_{max} / a = 23.76 / 1.345 = \underline{17.67\ kN}$

The maximum shear stress is equal to the characteristic shear strength for a load of 22.5 kN, when the width of the cross section is reduced with the factor k_{cr} = 0.67 (to account for cracks) as instructed by Eurocode 5 of 2010:

$\tau_{max} = 1.5\ V/A = 1.5 * \underline{22.5\ kN} / (0.67 * 48\ mm * 300\ mm) = 3.5\ N/mm2 = f_{v,k}$

Compression perpendicular to the grain was also calculated, and deemed a problem without steel plates, but no problem with. Lateral stability was also found to not be a problem.

So, the maximum estimated load P_{max} is 17.67 kN applied at each point, i. e. $F_{max,est}$ = 35.34 kN total for Instron. EN 408 states that the maximum load applied shall not exceed 40 % of $F_{max,est}$. The section of the graph (like in Figure 4.15) between 10 % and 40 % of $F_{max,est}$ should be used for regression analysis. The longest portion of this section that gives a correlation coefficient of at least 0.99 should be used, provided that at least the range from 20 % to 30 % is covered.

According to this, Instron should be instructed to log the results from about 3.5 kN and up to almost 14 kN (with minimum load value below 3.5 and maximum slightly above 14, to avoid "noise" in the results, associated with Instron switching between cycles, see Figure 4.18).

But on the advice from an experienced professor, the choice was made to only go up to 11 kN, which is just above the required 30 % of $F_{max,est}$, to make sure that the plastic range was avoided.

However, there was a problem with the .txt file output by Instron. For cycle 1, results were logged between 3.5 kN and 11.0 kN, as instructed. But strangely enough, for cycles 2 and 3 the results were only logged between 3.5 kN and approximately 7.5 kN, which only covers the range of 10 % to 21 % of $F_{max,est}$, less than required by the standard. Regardless of this, the choice was made to use the results of cycles 2 and 3 rather than cycle 1, again after listening to advice. It is common laboratory practice to throw out the results of the first cycle, because the material needs some time to "settle". The EN 408 standard gives a lot of rules, but it doesn't say anything about which load cycles that should be used. The computed elastic moduli of cycles 2 and 3 were more in agreement with each other than with that of cycle 1.

All of the load histories looked like in the figure below. Instron did go up to above 11 000 N for all cycles, but the results for cycles 2 and 3 were only logged below the straight line at about 7500 N.

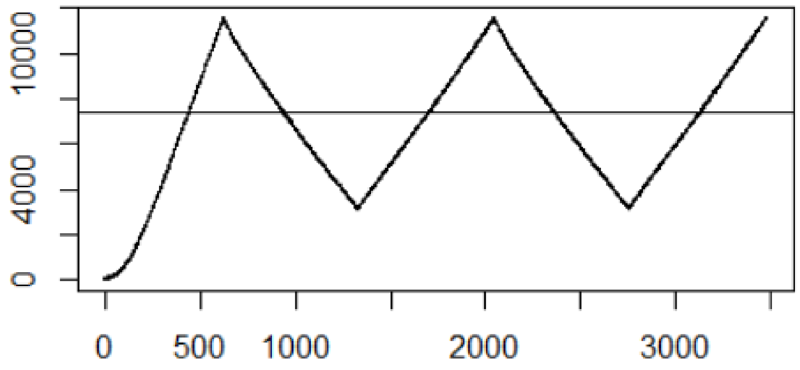


Figure 4.18 - Load cycles for determination of elastic moduli

4.7.2 Results

The full results are found in “Appendix B – Beam test data”. The correlation was well above the required 0.99 for all cycles, and with averages of above 0.999 for both the local E and global E regression lines. The mathematical operations were done by the programming software RStudio.

The table below gives the elastic moduli (the average E from cycles 2 and 3) for the beams. The $E_{m,g}$ assumes a shear modulus of 690 MPa, as stated in the technical certification. The shear modulus G is then estimated by requiring that $E_{m,g}$ should be equal to $E_{m,l}$. (For this, the “Goal Seek” function in Excel was used.)

Beam	$E_{m,l}$	$E_{m,g}$ (G=690)	$G_{estimated}$
1	16 022	14 378	312
2	15 528	15 352	614
3	17 066	14 596	257
4	15 837	14 656	370
5	15 855	14 659	368
6	12 303	12 980	2 507
7	16 365	15 013	358
8	15 064	14 401	454
9	13 510	13 458	660
10	13 098	12 826	542
Average	**15 065**	**14 232**	644
Max - min	4 764	2 525	2 251
Max / Average	113,3 %	107,9 %	389,3 %
Min / Average	81,7 %	90,1 %	39,9 %
Total variation	31,6 %	17,7 %	349,4 %

Table 4.10 - Summary of the elastic modulus test results

The local and global elastic moduli parallel to the grain in bending, $E_{m,l}$ and $E_{m,g}$, should be pretty much equal. This is not the case in the table above, which is surprising. This discrepancy leads to much larger variations in the estimated G. I would have thought that the shear modulus wouldn't vary much more than the elastic modulus in terms of percentage. Especially the computed G of 2507 MPa cannot be correct. It is also surprising that half of the beams are computed to shear modulus values of about 50 % of what is stated in the technical certification.

The moisture content of the beams when tested was a bit lower than the (approximately) 14 % they are to be delivered with, which impacts the elastic modulus.

4.7.3 Moisture content

A moisture meter was used on the beams at the time of testing. Initially, it gave moisture contents of 11-14 %, with an average of 12.7 % (for 9 beams tested). Then, towards the end of the testing, it started giving values of below 8 %, also for a beam that was measured as above 11 % just a couple hours earlier. This raised questions about how trustworthy the numbers were.

For the beam that was loaded until failure, six small samples from the middle part of the beam were used to measure the moisture content in a more accurate way. The pieces were weighed, then put in the oven for a couple days, and then weighed again afterwards. The moisture content (MC) was calculated as the weight difference divided by the dry weight:

Piece	Initial weight	Dry weight	MC
1	217,248	193,910	12,04 %
2	465,097	414,830	12,12 %
3	377,381	336,530	12,14 %
4	274,100	244,850	11,95 %
5	577,682	515,910	11,97 %
6	439,376	392,400	11,97 %
			12,03 %

Table 4.11 - Moisture content of samples taken from beam loaded to failure

So, the moisture content of the beam was measured as 12.0 %. This was a few days after it was tested, meaning that it had probably dried out a little since then.

All things considered, it seems reasonable to assume a moisture content of 12.5 % as an average for all beams at the time of EN 408 testing. The technical certification gives the mean elastic modulus as 14 000 N/mm^2, and the moisture content as 14 $\pm$ 2 % at the time of delivery. [27] This difference in MC means that it is expected that our laboratory testing would find a slightly higher elastic modulus. The figure below shows the effect of moisture content on the elastic modulus parallel to the grain at 20^o C. [36] The relevant values for 12.5 % and 14 % MC have been highlighted:

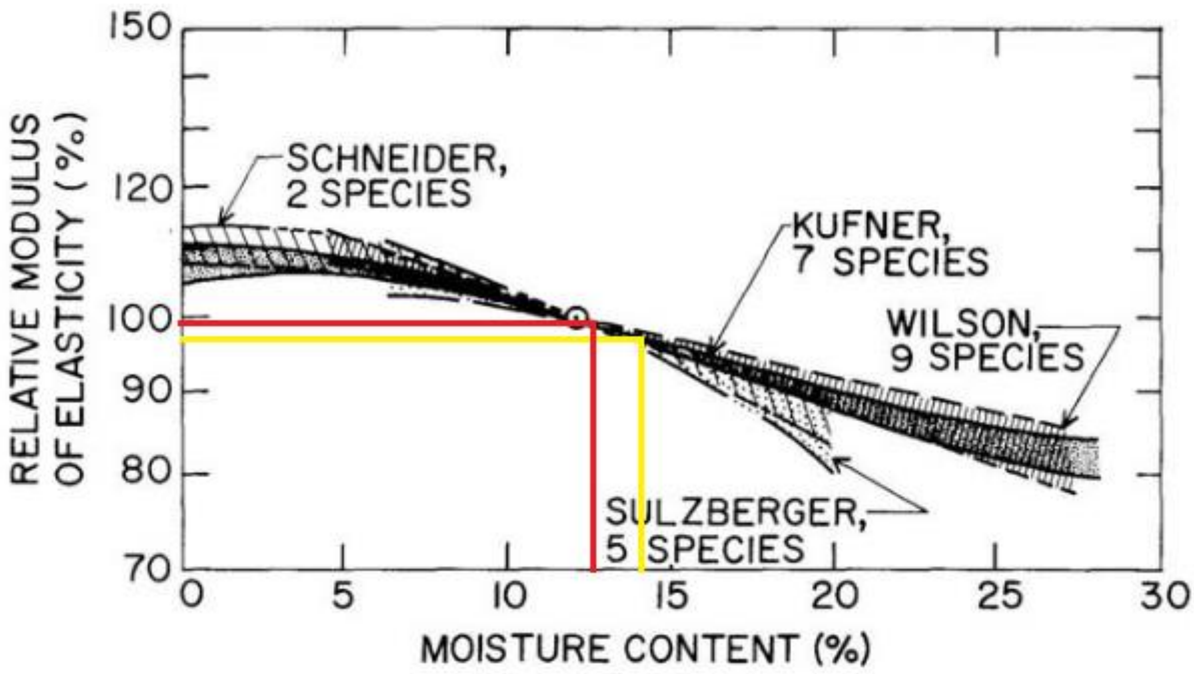


Figure 4.19 - Effect of moisture content on Young's modulus parallel to the grain at 20° C

So, the difference between 12.5 % and 14 % moisture content equates to an expected difference in elastic modulus of roughly 3 %. If adopted for the test results, the average $E_{m,l}$ reduces to 14 600 N/mm^2 while $E_{m,g}$ reduces to 13 800 N/mm^2.

4.7.4 Failure testing

One of the beams was also loaded until failure. The force vs. deformation graph for the test was as shown below:

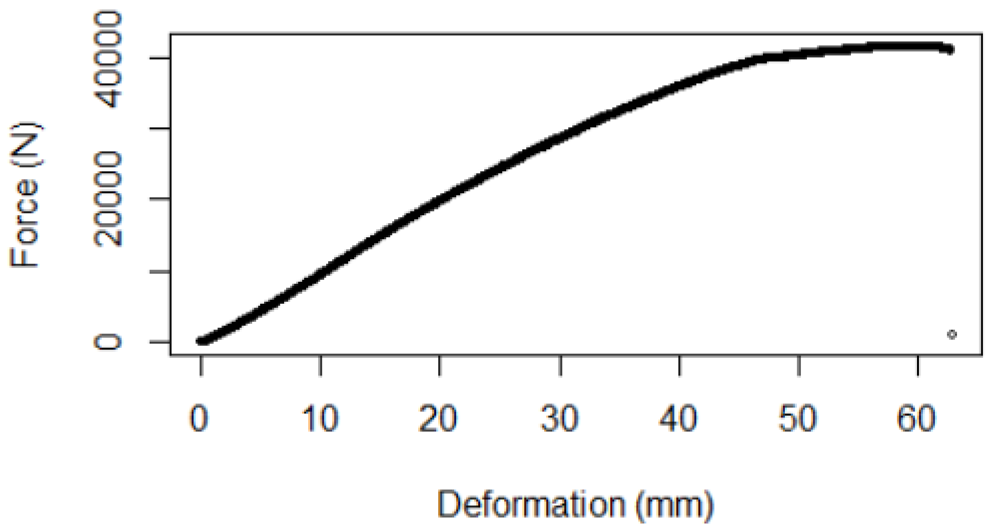


Figure 4.20 - Force vs. deformation graph for beam loaded until failure

The maximum Instron force was 41.57 kN, i. e. two concentrated loads of 20.875 kN. The deformation at this instant was 59 mm. The maximum deformation, at the moment of failure when the force dropped from 40.9 kN to 1 kN, was 63 mm. The predicted bending deformation for such loading is 41 mm (by formula shown in Figure 4.17), and 45 mm in total when shear deformation is included. The last millimeters are explained by plastic deformation, occurring when the graph above starts deviating from the initial straight line that the Euler/Bernoulli and Timoshenko beam theories assume, and the stress distribution over the cross-section is no longer linear.

Unexpectedly, the beam failed due to shear rather than exceeded bending moment capacity. It was anticipated that the beam would break close to mid-span on the bottom side of the cross-section, where the tensile stress is the largest. Rather, it went to shear failure in the neutral axis on one side of the beam:

Figure 4.21 - Shear failure of beam

The failure, for the two concentrated loads of 20.875 kN, occurred at 118 % of estimated bending moment capacity and only 93 % of estimated shear capacity, calculated without partial factors as shown below Figure 4.17. This shows that the k_{cr} factor for shear design in EC5 of 2010, not present in EC5 of 2004, is very much warranted. [11]

It is especially unexpected that it failed below 100 % since the characteristic strength values are supposed to be the lower 5 % fractiles. While there is uncertainty about the elastic modulus results giving quite low estimated shear modulus values, maybe this failure supports the notion that the shear properties of the beams aren't all that good, in which case the percentage contribution of shear deformation will have been higher than previously assumed for the floors. If the shear modulus is lower, it also means that the elastic modulus is higher than the value reported by the producer, and the total expected deformation is unchanged. It would however impact all of the calculations involving the longitudinal stiffness of the floor. No model updating will anyway be done based on the beam testing, due to uncertainty. In tables, the values for both the shear modulus (G_{mean}) and shear strength ($f_{v,k}$) are the same for glulam strength classes 20 through 32 ($f_{m,k}$). [37]

5 Verification of the floors based on the test results

This chapter considers the floors as they were tested. They were tested without ceiling and floor finish, which means that the vibrational properties of the floors will be somewhat different when built with the extra components. The transversal stiffness will be higher (which means lower deflection), and the mass will be higher (which means lower fundamental frequency f_1, but higher modal mass).

It is difficult to accurately predict how also the other verifications (such as e. g. velocity response) will be affected, because the magnitude of added transversal stiffness from the ceiling and floor finish is unknown, and the lower f_1 is never a positive for the verifications.

But adding the ceiling and floor finish should result in better subjective evaluations of the floors. Heavier and more transversally stiff floors perform better, and the fundamental frequencies of the Støren floors will still be above the limit values.

Some of the floors had higher longitudinal stiffness $(EI)_L$ than theoretically expected and some had lower $(EI)_L$. Chapter 5 considers the actual floors that were tested, while in chapter 6 the same verifications are also shown with respect to how the properties would be if $(EI)_L$ was exactly as expected (while the b_{ef} found from the tests is kept constant), so as to make the verifications more generally applicable for the Støren floors (without ceiling and floor finish). Some analytical considerations about the transversal stiffness are also done in chapter 6.

The verifications are done according to chapter 3 with the test results of section 4.5.

5.1 Longitudinal stiffness

The results for the "43.65 kg load case" of Table 4.6 showed that the longitudinal stiffness varies significantly between floors. (The laboratory testing of the joists also showed a lot of variation of the elastic modulus.) Since the calculation of the transversal stiffness via measured deflection is dependent on the longitudinal stiffness, it is important to consider this correctly. If for example $(EI)_L$ is assumed 9 % too high, then b_{ef} will be calculated as too low by a factor of 1.09, and $(EI)_b$ will be too low by a much larger factor (about 1.3 for these floors).

The longitudinal stiffness $(EI)_L$ of each floor can be calculated from the deflections of Table 4.6 by rearranging this formula (which is basically the same as Formula 4.3):

$$w = \frac{FL^3}{48\,EI} * \left(1 + \frac{\delta_{shear}}{\delta_{bending}}\right) = \frac{FL^3}{48\,(EI)_L} * \frac{1}{c} * \left(1 + \frac{\delta_{shear}}{\delta_{bending}}\right) = \frac{FL^3}{48\,(EI)_L} * \frac{1}{c} * \left(1 + \frac{6h^2\,E}{5L^2\,G}\right)$$

Formula 5.1 - Deflection when the transversal stiffness is not a factor

where:

- *w* is the mid-span deflection
- *F* is the mid-span load

L is the span

EI is the (unknown) longitudinal bending stiffness

$(EI)_L$ is the (unknown) longitudinal bending stiffness per unit length, i. e. EI/c

c is the joist spacing

$\frac{\delta_{shear}}{\delta_{bending}}$ is the ratio of shear to bending deformation (like in Formula 4.2)

When calculating the transversal stiffness via the “200 kg load case” measurements, it is the longitudinal stiffness at the middle joist that should be considered.

By dividing the middle joist deflection by the theoretically predicted deflection from the “43.65 kg load case” (see Table 4.6), these numbers are obtained:

	Floor 1	**Floor 2**	**Floor 3**	**Floor 4**	**Floor 6**	Average
$\delta_{mid\ joist}/\delta_{Timo}$	0.890	0.961	1.081	1.025	1.135	1.018

Table 5.1 - Ratio of measured to expected middle joist deflection

So, the middle joists deflected 1.8 % more than expected, on average. For example; for floor 6, the deflection was 13.5 % higher, meaning that the longitudinal stiffness $(EI)_L$ is lower than expected by a factor of 1.135. The $(EI)_L$ of floor 6 can be found either by using Formula 5.1 (where w is 0.773), or by dividing by 1.135 when calculating $(EI)_L$ analytically.

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
b	mm	48	48	48	48	48	48	48	48	48	48
h	mm	300	300	300	300	300	300	250	250	300	300
I	mm^4	1,08E+08	1,08E+08	1,08E+08	1,08E+08	1,08E+08	1,08E+08	6,25E+07	6,25E+07	1,08E+08	1,08E+08
E	N/mm^2	14 000	14 000	14 000	14 000	14 000	14 000	11 000	11 000	14 000	14 000
c	mm	600	600	600	600	300	300	600	600	600	600
Stiffness cal.		0,890	0,890	0,961	0,961	1,081	1,081	1,025	1,025	1,135	1,135
$(EI)_L$	Nmm	2,83E+09	2,83E+09	2,62E+09	2,62E+09	4,66E+09	4,66E+09	1,12E+09	1,12E+09	2,22E+09	2,22E+09
E_{cal}	N/mm^2	15 735	15 735	14 563	14 563	12 954	12 954	10 729	10 729	12 339	12 339

Table 5.2 - Longitudinal stiffness parameters for the floors

Since the sheathing is negligible for the longitudinal stiffness, the stiffness calibration factor also gives an estimate of the middle joist’s elastic modulus (E_{cal}). However, as mentioned in chapter 4, there was surprisingly little difference in deflection between joists of a single floor, suggesting that the joists deflect more equally when connected via the sheathing, in which case the joists’ actual E may deviate more from the mean value.

Floor 1 had some incongruent results (Table 4.5). However, the middle joist deflection was almost the same both times, and so the choice was made to trust also these results for the stiffness calibration above.

5.2 Transversal stiffness

For the “200 kg load case” of Table 4.7, Formula 5.1 is not applicable because it doesn’t consider the transversal load distribution. The formula given in various verification methods is the one below. By rearranging it, the effective width b_{ef} of the floors can be calculated based on the measured deflection *w* of the middle joist.

$$w = \frac{FL^3}{48\,(EI)_L\,b_{ef}} = \frac{FL^3}{48\,(EI)_{joist}} * \frac{c}{b_{ef}}$$

Formula 5.2 - Deflection when the transversal stiffness matters

It is unclear whether this formula considers the shear deformation. At first glance it seems the answer is no, but it may be hidden in the form of the factor 1.1 for the b_{ef} (defined in Formula 3.3). Bernhard Mohr doesn’t specify how he computed the effective width, and others who adopted it (like Hamm/Richter/Winter) don’t mention anything about it either [10] [38] [8]. If the answer is that shear is not considered, I would suggest the design formula below (adapted from Formula 4.3, for rectangular cross-sections) as a more accurate way of considering deformation:

$$\delta = \delta_{bending} + \delta_{shear} = \frac{FL^3}{48\,(EI)_L\,b_{ef}} + \frac{3\,FL}{10\,GA} = \frac{FL^3}{48\,(EI)_L\,b_{ef}} * \left(1 + \frac{6h^2\,E}{5L^2\,G}\right) \approx 1.1\frac{FL^3}{48\,(EI)_L\,b_{ef}}$$

Formula 5.3 - Possible new deflection design formula

If shear deformation is not accounted for in the current design formula and limit values, then measurements of the real deflection in laboratories will give stricter verifications, and that’s definitely not how it should be. The new Eurocode 5 should specify this.

Because of the uncertainty about this, and to be concordant with the various verification methods, Formula 5.2 is used in the further calculations of the transversal stiffness. This is also the conservative choice, which gives b_{ef} values almost 10 % lower than with Formula 5.3. The floor stiffness parameters based on the test results are summarized here:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
$(EI)_L$	Nmm	2,83E+09	2,83E+09	2,62E+09	2,62E+09	4,66E+09	4,66E+09	1,12E+09	1,12E+09	2,22E+09	2,22E+09
L	mm	4700	4700	4740	4740	5380	5380	4050	4050	4720	4720
F	kN	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96
w	mm	1,83	1,41	1,57	1,32	1,60	1,04	2,40	1,81	1,64	1,47
b_{ef}	mm	820	1063	1055	1255	853	1307	1010	1339	1179	1316
$(EI)_b$	Nmm	3,85E+06	1,08E+07	9,40E+06	1,89E+07	4,32E+06	2,38E+07	6,34E+06	1,96E+07	1,27E+07	1,96E+07
α		0,0532	0,0411	0,0414	0,0348	0,0511	0,0334	0,0432	0,0326	0,0370	0,0332

Table 5.3 - Stiffness properties of the tested Støren floors

The deflection *w* for floor 1 was 30 % higher for the bare structure. That means that the effective width b_{ef} was increased by 30 % when the non-structural components were added. Likewise, the increase of b_{ef} from the non-structural parts for floors 2, 3, 4 and 6 were 19 %, 53 %, 33 % and 12 %, respectively.

The percentage increase of the transversal stiffness $(EI)_b$ is much higher. It could be calculated by rearranging Formula 3.3 as such:

$$(EI)_{\mathrm{b}} = (EI)_{\mathrm{L}} * \left(\frac{1.1\, b_{\mathrm{ef}}}{L}\right)^4 = (EI)_{\mathrm{L}} * \left(\frac{b}{L\,\alpha}\right)^4$$

Formula 5.4 - Transversal stiffness

Here, α is as defined for Formula 3.1, b is the joist width and L is the span. The parameter α could be calculated from either b_{ef} or $(EI)_b$ by rearranging one of the formulas presented.

The exponent of 4 of course means that any error in b_{ef} (from the measurements) will lead to a much higher error in the calculated $(EI)_b$. Further considerations about the transversal stiffness are discussed in chapter 6.

5.3 Verifications

5.3.1 Deflection

The table below shows how the tested floors performed against the deflection/stiffness criteria of the various verification methods. It gives the ratio between measured deflection (from Table 4.7) and the limit value, both shown below in mm/kN. For the ratios highlighted in red, the requirement is not satisfied. The deflection requirements according to Hamm/Richter/Winter, Mohr, Eurocode 5 (the newest proposal, as well as Norwegian and Austrian national annexes), Canadian Wood Council, as well as SINTEF Byggforsk's "comfort criterion" are considered.

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Requirements		Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
	w/F	0,93	0,72	0,80	0,67	0,82	0,53	1,23	0,92	0,84	0,75
EC5 NO low	*0.9*	103 %	80 %	89 %	75 %	91 %	59 %	136 %	103 %	93 %	83 %
EC5 NO high	*0.6*	155 %	120 %	134 %	112 %	136 %	89 %	204 %	154 %	139 %	125 %
"Comfort criterion"	*1.3*	72 %	55 %	62 %	52 %	63 %	41 %	94 %	71 %	64 %	58 %
HRW none *EC5 AT class 3*	*None*	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
HRW high *EC5 AT class 1* *New EC5 level 1 & 2*	*0.25*	372 %	287 %	321 %	270 %	326 %	213 %	490 %	370 %	335 %	300 %
HRW low *EC5 AT class 2* *New EC5 level 3*	*0.5*	186 %	144 %	161 %	135 %	163 %	106 %	245 %	185 %	167 %	150 %
New EC5 level 4	*0.8*	116 %	90 %	100 %	84 %	102 %	67 %	153 %	116 %	105 %	94 %
New EC5 level 5	*1.2*	78 %	60 %	67 %	56 %	68 %	44 %	102 %	77 %	70 %	62 %
New EC5 level 6	*1.6*	58 %	45 %	50 %	42 %	51 %	33 %	77 %	58 %	52 %	47 %
Mohr	*1.0*	93 %	72 %	80 %	67 %	82 %	53 %	123 %	92 %	84 %	75 %
CWC	$8/L^{1,3}$	87 %	67 %	76 %	64 %	91 %	59 %	94 %	71 %	79 %	70 %

Table 5.4 - Verification of deflection for the tested Støren floors

The deflection part of the “comfort criterion” is not the limiting factor.

For the Norwegian National Annex of EC5, only the complete floor 3 satisfies high stiffness requirements, seven of the ten floor configurations satisfy normal requirements, while three of them don’t meet either of the demands.

All of the complete floors deflected less than 0.8 mm/kN, except floor 4 which is the only one not approved according to Table 4.2, so it was expected to perform the worst.

The deflections would however have been lower if the floors had been tested with ceiling and floor finish.

5.3.2 Frequency

To calculate the fundamental frequency of the floors, their distributed mass is needed. As an example, the mass calculation for floor 6 (the one with blocking) is shown in the table below. The mass of some secondary components such as nails and screws was neglected. The mass of the end beams (at the supports, parallel to the blocking) was also neglected as it isn’t evenly distributed and is taken directly by the supports. A floor finish of 14 mm thick parquet was assumed. Laths for the ceiling are assumed to be spaced 600 mm apart.

Floor 6	**Density [kg/m³]**	**Height [m]**	**Width [m]**	**Length [m]**	**Number of elements**	**Mass [kg]**	**Distributed mass [kg/m²]**
K-Bjelke Plus	460	0.300	0.048	4.9	5	162.3	13.8
Perforated particleboard	685	0.022	2.4	4.9	0.85	150.6	12.8
Blocking	460	0.300	0.048	2.4	0.9	14.3	1.2
						Bare structure	**Σ 27.8**
Particleboard	685	0.022	2.4	4.9	1	177.2	15.1
Plasterboard	516	0.013	2.4	4.9	1	78.9	6.7
Acoustic insulation	16.5	0.020	2.4	4.9	1	3.9	0.3
						Tested “complete” floor	**Σ 49.9**
Plasterboard	516	0.013	2.4	4.9	2	157.8	13.4
Lath	380	0.036	0.048	4.9	9	29.0	2.5
Insulation	16.5	0.300	0.552	4.9	4	53.6	4.6
Parquet	550	0.014	2.4	4.9	1	90.6	7.7
						The floor with all components	**Σ 78.2**

Table 5.5 - The distributed mass of floor 6

By repeating the same type of calculations from Table 5.5 for all the floors, the values shown in Table 5.6 are obtained. Here, the values are also calculated for when 10 %, 20 % or 30 % of the live load mass q_k for residential building floors is included. From Table NA.6.2 in Eurocode 1, this is 2.0 kN/m², as oppose to 3.0 kN/m² for office building floors. [29]

Distributed mass *m* [kg/m²]	Floor 1	Floor 2	Floor 3	Floor 4	Floor 6
Bare structure	26.6	28.9	37.6	24.3	27.8
Tested "complete" floor	48.7	51.0	59.8	46.4	49.9
The floor with all components	76.9	79.2	87.6	73.9	78.2
The floor with all components + 0.2 kN/m²	97.3	99.6	108.0	94.3	98.5
The floor with all components + 0.4 kN/m²	117.7	120.0	128.3	114.6	118.9
The floor with all components + 0.6 kN/m²	138.1	140.4	148.7	135.0	139.3

Table 5.6 - Distributed mass for all of the floors

In the table below, the fundamental frequency of the floors is calculated from Formula 3.15, based on the distributed mass *m* from Table 5.6 and the calibrated $(EI)_L$ of Table 5.2.

Fundamental frequency f_1 [Hz]	Floor 1	Floor 2	Floor 3	Floor 4	Floor 6
Bare structure	23.2	21.1	19.1	20.5	19.9
Tested "complete" floor	17.1	15.9	15.2	14.9	14.9
The floor with all components	13.6	12.7	12.5	11.8	11.9
The floor with all components + 0.2 kN/m²	12.1	11.3	11.3	10.4	10.6
The floor with all components + 0.4 kN/m²	11.0	10.3	10.3	9.5	9.6
The floor with all components + 0.6 kN/m²	10.2	9.6	9.6	8.7	8.9

Table 5.7 - Fundamental frequencies of the floors

It is the f_1 values for the *Bare structure* and the *Tested "complete" floor* in the table above that will be considered for the various verifications. To use the f_1 values for *The floor with all components* would not result in a fair assessment of the floors. As explained in section 3.4.3.23.4.3.2, the ceiling (here: two gypsum plasterboards, as well as the laths) provides considerable transversal stiffness $(EI)_b$ that would have reduced the deflection if it had been mounted for the testing. Increased $(EI)_b$ also helps for the verifications of velocity response, while the lowered f_1 makes the verifications stricter. A floor finish such as parquet (which should be laid out in the floor's transversal direction) will also improve the floor's vibrational behavior. So to consider just the added mass for the reduction of f_1, without including the benefit for the transversal stiffness in the calculations, would be a much too conservative and wrong way of verifying the floors.

In accordance with the normal recommendations (e. g. Hamm/Richter/Winter and the new EC5 proposal), the live load mass will be taken as zero, i. e. $\Psi_2 * q_k = 0 * q_k$. When in use, the mass will of course be higher than this due to furniture and people, but the added damping from this is not accounted for, and in practice a heavier floor will perform better. And also, while the values above are calculated for single-span floors where the transversal stiffness isn't considered, Homb [20] has found that the f_1 formula for two-span orthotropic floors (Formula 3.37), which gives higher f_1 values, correlates better with measurements. In real life, things are often a bit different from what is assumed in the early, conceptual design stage. Partition walls placed so that the main modes of vibration are interrupted will increase the stiffness and thus f_1, and the support conditions can also be stiffer than assumed. So in practice it doesn't necessarily make sense to compare f_1 values calculated with a live load against the limit values, making the verification stricter.

The fundamental frequency criterion is not a problem for the Støren floors. A simplified summary of the f_1 limit values of the various verification methods is presented in the table below.

<table>
<tr><th></th><th>$f_{1,limit}$ high demands</th><th>$f_{1,limit}$ low demands</th><th>$f_{1,min}$ (with extra requirements)</th></tr>
<tr><td>EC5 Austrian NA</td><td>8</td><td>6</td><td>4.5</td></tr>
<tr><td>Hamm/Richter/Winter</td><td>8</td><td>6</td><td>4.5</td></tr>
<tr><td>New EC5 proposal</td><td colspan="2">8</td><td>4.5</td></tr>
<tr><td>EC5</td><td colspan="2">8</td><td>None</td></tr>
<tr><td>Mohr</td><td colspan="2">8</td><td>None</td></tr>
<tr><td>"Comfort criterion"</td><td colspan="2">10</td><td>8</td></tr>
</table>

Table 5.8 - Summary of the limit values for the fundamental frequency

The tested Støren floors have design fundamental frequencies well above the typical limit value of 8 Hz that is generally seen as the cut-of limit between high frequency and low frequency floors, for walking activities. Resonance won't be a problem for the floors (unless they are used for rhythmic activities etc.), and no verification of vibration acceleration is required by any of the code-based methods. The floor vibration will rather be transient, and some of the methods require a verification of the velocity response.

5.3.3 Deflection and frequency combined

Hu's combined deflection and f_1 criterion (Formula 3.33) forms the basis of the "comfort criterion". The requirement is that $f_1/\Delta^{0.44}$ should be higher than 18.7 for the floor to be verified. The deflections measured for the "200 kg load case" (of Table 5.4, in mm/kN) are paired with the f_1 values for the same floor configuration (from Table 5.7), and compared against the limit value.

Because of the better correlation with f_1 measurements from using the two-span orthotropic plate model, the fundamental frequency was rather calculated according to Formula 3.37 when the floors were verified according to the "comfort criterion" [27] [28]. In the table below, the tested floors were also considered in this way against the requirement, using the f_1 multiplication factor of Formula 3.38.

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
m	kg/m^2	26,6	48,7	28,9	51,0	37,6	59,8	24,3	46,4	27,8	49,9
f_1	Hz	23,2	17,1	21,1	15,9	19,1	15,2	20,5	14,9	19,9	14,9
Δ	mm/kN	0,93	0,72	0,80	0,67	0,82	0,53	1,23	0,92	0,84	0,75
$f_1/\Delta^{0,44}$		23,9	19,8	23,2	18,9	20,9	20,0	18,8	15,4	21,5	16,9
Verification		78 %	94 %	81 %	99 %	89 %	93 %	100 %	122 %	87 %	111 %
Factor		1,015	1,042	1,040	1,080	1,016	1,086	1,038	1,114	1,063	1,096
Verification		77 %	91 %	77 %	92 %	88 %	86 %	96 %	109 %	82 %	101 %

Table 5.9 - Combined deflection and fundamental frequency criterion verification

Floor 4 was expected to perform the worst, so that is no surprise. The red numbers for floor 6 can be understood from it having the lowest longitudinal stiffness relative to expectations, as shown in Table 5.1, which negatively affects both f_1 and Δ. (In chapter 6 this will be accounted for and adjusted so that the verifications become more generally applicable for the floor configurations.)

What is surprising is that these verifications imply that the floors are better without the non-structural components. This is not the case, and a critique of the method is that it may be more accurate for floors that are not as high-frequency. And of course, such simple floor structures will not be built in practice anyway. Another critique of the method is that it only sees mass as a bad thing and doesn't account for damping. It would be interesting to see how the floors are considered by the method with the missing components added, but the transversal stiffness and deflection is difficult to estimate accurately without test results or BTAB available.

5.3.4 Velocity

The recommended damping ratios of the various code-based methods are shown in the table below for the different floor configurations.

Recommended damping ratios ζ	Bare structure	Tested "complete" floor	The floor with all components
Eurocode 5	0.01	0.01	0.01
EC5 Austrian NA	0.01	0.01	0.01
Hamm/Richter/Winter	0.01	0.01	0.01
Mohr	0.01	0.02	0.02
New EC5 proposal	0.02	0.03	0.03

Table 5.10 - Recommended damping ratios

Based on this (as well as other findings [20]), it seems the most appropriate to assume damping ratios of 1 % and 2 %, respectively, for the floors without and with the added layers. These are the values that will be used for all verification methods.

The floors are calculated with the mass and frequency from Table 5.6 and Table 5.7, and the stiffness parameters from Table 5.3.

The Norwegian National Annex of EC5 doesn't give directions with regards to limit value for the unit impulse velocity verification, only for the deflection criterion. However, if the Eurocode 5 recommendations of Figure 3.6 are to be followed, the *b* value should be 126 for normal requirements and 144 for high requirements. If the same is done for the Austrian National Annex, this limit value is 150 both for floor classes 1 and 2. In the table below, the floors are verified according to all of these demands.

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
n_{40}		3,15	2,98	2,63	2,65	3,47	2,61	2,79	2,58	2,44	2,62
v	m/(Ns^2)	0,0183	0,0117	0,0150	0,0102	0,0145	0,0081	0,0190	0,0120	0,0145	0,0103
ζ		0,01	0,02	0,01	0,02	0,01	0,02	0,01	0,02	0,01	0,02
b_{normal}		126	126	126	126	126	126	126	126	126	126
$v_{limit,normal}$	m/(Ns^2)	0,0244	0,0417	0,0220	0,0368	0,0200	0,0344	0,0214	0,0334	0,0208	0,0334
Verification		75 %	28 %	68 %	28 %	72 %	24 %	89 %	36 %	70 %	31 %
b_{high}		144	144	144	144	144	144	144	144	144	144
$v_{limit,high}$	m/(Ns^2)	0,0220	0,0382	0,0198	0,0336	0,0179	0,0313	0,0193	0,0304	0,0187	0,0304
Verification		83 %	31 %	76 %	30 %	81 %	26 %	99 %	39 %	77 %	34 %
$b_{Austria}$		150	150	150	150	150	150	150	150	150	150
v_{limit}	m/(Ns^2)	0,0213	0,0372	0,0192	0,0327	0,0174	0,0305	0,0187	0,0296	0,0181	0,0296
Verification		86 %	31 %	78 %	31 %	83 %	27 %	102 %	40 %	80 %	35 %

Table 5.11 - EC5 unit impulse velocity response verification with ζ of 1 % and 2 %

So, the unit impulse velocity response is not a problem for the Støren floors according to the current Eurocode 5. The damping ratio is a sensitive parameter; the differences between the verification percentages for "structural" and "complete" are mostly due to the difference of 0.01 versus 0.02.

Mohr's velocity/mass requirement requirements (presented in section 3.2.3) are however stricter. Only one of these verifications (velocities by heel drop or modified EC5 unit impulse) needs to be done, but both of them are considered here:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
$v_{heel\ drop}$	m/s	0,360	0,205	0,282	0,178	0,260	0,135	0,419	0,229	0,278	0,186
$v_{heel\ drop,limit}$	m/s	0,175	0,291	0,158	0,258	0,145	0,242	0,154	0,236	0,150	0,236
Verification		206 %	71 %	178 %	69 %	179 %	55 %	272 %	97 %	185 %	79 %
$v_{EC5\ mod}$	m/(Ns^2)	0,0292	0,0143	0,0218	0,0119	0,0191	0,0088	0,0320	0,0149	0,0208	0,0121
$v_{EC5\ mod,limit}$	m/(Ns^2)	0,0097	0,0162	0,0088	0,0144	0,0080	0,0135	0,0086	0,0131	0,0083	0,0131
Verification		301 %	88 %	247 %	83 %	238 %	65 %	373 %	113 %	250 %	92 %

Table 5.12 - Mohr's velocity/mass requirement verifications with ζ of 1 % and 2 %

As for EC5/Ohlsson, the damping ratio is a sensitive parameter for the verifications, responsible for the majority of the difference between "structural" and "complete".

The proposed velocity verification method for the new Eurocode 5 (presented in section 3.3.3.3) is very strict. The least strict limit value for the response factor is 24, and the strictest is 4. Neither requirement can be fulfilled without making big structural changes to the floors:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
f_w	Hz	2	2	2	2	2	2	2	2	2	2
I	Ns	1,90	2,81	2,15	3,12	2,45	3,30	2,23	3,39	2,32	3,39
M*	kg	75,0	137,4	82,1	145,0	121,5	192,9	59,1	112,8	78,8	141,4
$V_{1,peak}$	m/s	0,0253	0,0205	0,0262	0,0215	0,0201	0,0171	0,0377	0,0300	0,0294	0,0239
K_{imp}		1,277	1,0	1,0	1,0	1,227	1,0	1,036	1,0	1,0	1,0
$V_{tot,peak}$	m/s	0,0323	0,0205	0,0262	0,0215	0,0247	0,0171	0,0391	0,0300	0,0294	0,0239
η		0,82	0,97	0,97	0,97	0,84	0,97	0,95	0,97	0,97	0,97
β		0,417	0,565	0,519	0,581	0,473	0,589	0,515	0,592	0,533	0,592
V_{rms}	m/s	0,0135	0,0116	0,0136	0,0125	0,0117	0,0101	0,0201	0,0178	0,0157	0,0142
R		134,7	115,8	136,2	124,8	116,8	100,8	201,1	178,0	156,6	141,8
R_{limit}		24	24	24	24	24	24	24	24	24	24
$V_{rms,limit}$	m/s	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024
Verification		561 %	482 %	568 %	520 %	486 %	420 %	838 %	742 %	653 %	591 %
R_{limit}		4	4	4	4	4	4	4	4	4	4
$V_{rms,limit}$	m/s	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004
Verification		3366 %	2895 %	3405 %	3119 %	2919 %	2520 %	5027 %	4449 %	3915 %	3545 %

Table 5.13 - Proposed new EC5 velocity verification with ζ of 1 % and 2 %

It should be noted that the loading models for velocity and accelerations of the new EC5 proposal are only applicable for floors with a mass of at least ten times the weight of the walker, i. e. generally not applicable for these floors until the mass from the ceiling etc. is added. [4]

An analysis of how much the various input parameters affect the verifications is done in section 6.1.

5.3.5 Acceleration

Verification of acceleration is not warranted because the Støren floors are high frequency floors according to all of the methods (given that they won't be used for dance studios etc.). The calculations are anyway shown in the table below. On top is the Austrian National Annex to EC5 (limit values 0.05 or 0.10, or none), in the middle is Mohr (limit value 0.10), and on the bottom is the new EC5 proposal (limit values ranging from 0.02 to 0.12 m/s^2). Accelerations that satisfy all demands are written in black, red indicates that the strictest (lowest) limit value is not satisfied, while numbers written in red and bold font indicate that the highest limit value is not satisfied. Hamm/Richter/Winter has a similar formula for acceleration as the other methods, and the same limits as the Austrian NA, but it is omitted from the table because the input parameter of Figure 3.2 is not shown for high frequency floors.

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
M*	kg	51,3	121,7	72,2	151,6	86,4	210,1	49,7	125,9	77,5	155,1
a_{rms}	m/s^2	0,03	0,06	0,04	0,08	0,08	0,08	0,08	**0,15**	0,06	**0,12**
M_{gen}	kg	51,3	121,7	72,2	151,6	86,4	210,1	49,7	125,9	77,5	155,1
a	m/s^2	0,03	0,03	0,03	0,03	0,03	0,02	0,04	0,04	0,03	0,03
M*	kg	75,0	137,4	82,1	145,0	121,5	192,9	59,1	112,8	78,8	141,4
a_{rms}	m/s^2	0,01	0,04	0,03	0,06	0,04	0,06	0,05	0,12	0,04	0,09

Table 5.14 - Accelerations with ζ of 1 % and 2 %

The accelerations are higher when the non-structural components are added, even though damping is very important (except in Mohr's method). This is because, although a high modal mass in itself also helps the verification, an increased mass reduces f_1 which increases the acceleration response a lot.

The best performer was floor 1, because of its unexpectedly high longitudinal stiffness (at the middle joist) found in the tests, but this will look different when adjusted in chapter 6.

While Mohr and the Austrian NA calculates the modal mass via the effective width b_{ef}, the new EC5 proposal (like Hamm/Richter/Winter) rather uses the width *b*. The transversal stiffness not being a factor means that all input parameters are known also for "the floor with all components" (from Table 5.6 and Table 5.7), and it is possible to do this verification. Because the new EC5 proposal states that a damping ratio of 3 % is appropriate for "joisted floors with a floating layer", the calculations are shown for both 2 % and 3 %:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**		Full		Full		Full		Full		Full
M*	kg		217,0		225,2		282,7		179,5		221,3
a_{rms}	m/s^2		0,10		**0,14**		0,12		**0,25**		**0,19**

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**		Full		Full		Full		Full		Full
M*	kg		217,0		225,2		282,7		179,5		221,3
a_{rms}	m/s^2		0,06		0,09		0,08		**0,17**		**0,13**

Table 5.15 - Accelerations for the floors with ceiling and parquet, with ζ of 2 % and 3 %

The accelerations are now higher, even with the increased damping ratio. None of the floors satisfy the strictest requirement of 0.02 m/s^2, but some of them satisfy the least strict of 0.12. Verification of acceleration response is anyway not required for walking activities for these high frequency Støren floors.

The new EC5 proposal states that the design rules are not applicable for e. g. gymnastic halls and dance halls, so the above calculations cannot be used to verify the floors for rhythmic activities.

5.3.6 Constructive requirements and summary

The Hamm/Richter/Winter method and the Austrian National Annex both have constructive requirements (as shown in Table 3.2 and Table 3.8). For the floors to be approved (given that there are vibration requirements), a layer of screed is mandatory, and in some cases also filling to increase the mass. All of the floors are then by default not approved according to both methods. None of the tested floors fulfilled their least strict deflection limit value of 0.5 mm/kN. Floor 3 was however close at 0.53 mm/kN, and if tested with ceiling and a floor finish then it would be below the limit.

All of the floors are approved according to Mohr's criteria, except floor 4.

All of the floors (except floor 4) satisfy the least strict demands of the Norwegian NA to EC5, while only floor 3 satisfies the high demands.

According to the new EC5 proposal, the velocity response of the floors is way too high. (But not according to the Ohlsson or Mohr methods.)

The floors (except number 4) are OK according to the "comfort criterion".

The floors are high-frequency, and so there will be no resonance from walking activities, and no verification of acceleration response is needed.

A summary of selected verifications is shown below. Based on this, the performance of the ten tested floor constructions has been rated in order from #1 (best) to #10 (worst):

	Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Verifications	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
Deflection [mm/kN]	0,93	0,72	0,80	0,67	0,82	0,53	1,23	0,92	0,84	0,75
Comfort criterion	77 %	91 %	77 %	92 %	88 %	86 %	96 %	109 %	82 %	101 %
EC5 velocity, b=144	83 %	31 %	76 %	30 %	81 %	26 %	99 %	39 %	77 %	34 %
Mohr heel drop vel.	206 %	71 %	178 %	69 %	179 %	55 %	272 %	97 %	185 %	79 %
New EC5, velocity, VI	561 %	482 %	568 %	520 %	486 %	420 %	838 %	742 %	653 %	591 %
New EC5, velocity, I	3366 %	2895 %	3405 %	3119 %	2919 %	2520 %	5027 %	4449 %	3915 %	3545 %
Rating	#7	#2	#5	#2	#5	#1	#10	#7	#7	#4

Table 5.16 - Summary of selected verifications

The ratings look different in section 6.1 when the verifications are made more generally applicable for these kinds of floor structures.

6 Analytical considerations

This chapter makes some further analytical considerations about the test results and verification methods. In section 6.1, the verifications of chapter 5 are made more generally applicable with regards to the longitudinal stiffness. In section 6.2, the transversal stiffnesses found are investigated in-depth. Section 6.3 suggests an improvement of a design formula.

6.1 Verifications

The verifications in chapter 5 considered the actual floors that were tested. For some of them the longitudinal stiffness $(EI)_L$ was higher than expected and for some it was lower. To make the verifications more generally applicable for these floor structures, the elastic modulus E is changed to the average value reported by the producer, and a new deflection w is calculated based on the assumption that the effective width b_{ef} found in chapter 5 is the same. (The calculated deflections will be different by at most $\pm$ 3 % if the $(EI)_b$ is kept constant instead – because $(EI)_b$ and b_{ef} are related via $(EI)_L$, which is now different.)

None of the other input parameters have been changed from chapter 5, only the longitudinal stiffness. The discussion is therefore, for the most part, much more brief here, and the basic explanations for the same tables are found in chapter 5. The verifications will now be worse for floors 1 and 2, while they are improved for floors 3, 4 and 6.

New deflections w are calculated based on the same load F that was used in the measurements:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
b	mm	48	48	48	48	48	48	48	48	48	48
h	mm	300	300	300	300	300	300	250	250	300	300
I	mm^4	1,08E+08	1,08E+08	1,08E+08	1,08E+08	1,08E+08	1,08E+08	6,25E+07	6,25E+07	1,08E+08	1,08E+08
E	N/mm^2	14 000	14 000	14 000	14 000	14 000	14 000	11 000	11 000	14 000	14 000
c	mm	600	600	600	600	300	300	600	600	600	600
$(EI)_L$	Nmm	2,52E+09	2,52E+09	2,52E+09	2,52E+09	5,04E+09	5,04E+09	1,15E+09	1,15E+09	2,52E+09	2,52E+09
L	mm	4700	4700	4740	4740	5380	5380	4050	4050	4720	4720
F	kN	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96	1,96
b_{ef}	mm	820	1063	1055	1255	853	1307	1010	1339	1179	1316
w	mm	2,05	1,58	1,64	1,38	1,48	0,97	2,35	1,77	1,45	1,30
$(EI)_b$	Nmm	3,43E+06	9,65E+06	9,04E+06	1,81E+07	4,67E+06	2,57E+07	6,50E+06	2,00E+07	1,44E+07	2,23E+07
α		0,0532	0,0411	0,0414	0,0348	0,0511	0,0334	0,0432	0,0326	0,0370	0,0332

Table 6.1 - Input parameters for the verifications

The deflection verifications are shown below. With the three added layers the floors were tested with, they satisfy requirements of 0.81, 0.70, 0.49, 0.90 and 0.66 mm/kN, respectively.

Requirements	w/F	Floor 1 Structural	Floor 1 Complete	Floor 2 Structural	Floor 2 Complete	Floor 3 Structural	Floor 3 Complete	Floor 4 Structural	Floor 4 Complete	Floor 6 Structural	Floor 6 Complete
		1,05	0,81	0,83	0,70	0,75	0,49	1,20	0,90	0,74	0,66
EC5 NO low	*0.9*	116 %	90 %	93 %	78 %	84 %	55 %	133 %	100 %	82 %	73 %
EC5 NO high	*0.6*	174 %	135 %	139 %	117 %	126 %	82 %	199 %	150 %	123 %	110 %
"Comfort criterion"	*1.3*	80 %	62 %	64 %	54 %	58 %	38 %	92 %	69 %	57 %	51 %
HRW none *EC5 AT class 3*	*None*	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
HRW high *EC5 AT class 1* *New EC5 level 1 & 2*	*0.25*	418 %	323 %	334 %	281 %	302 %	197 %	478 %	361 %	295 %	264 %
HRW low *EC5 AT class 2* *New EC5 level 3*	*0.5*	209 %	162 %	167 %	140 %	151 %	99 %	239 %	180 %	147 %	132 %
New EC5 level 4	*0.8*	131 %	101 %	104 %	88 %	94 %	62 %	149 %	113 %	92 %	83 %
New EC5 level 5	*1.2*	87 %	67 %	70 %	58 %	63 %	41 %	100 %	75 %	61 %	55 %
New EC5 level 6	*1.6*	65 %	50 %	52 %	44 %	47 %	31 %	75 %	56 %	46 %	41 %
Mohr	*1.0*	105 %	81 %	83 %	70 %	75 %	49 %	120 %	90 %	74 %	66 %
CWC	*8/L*^{1,3}	98 %	75 %	79 %	66 %	84 %	55 %	92 %	69 %	69 %	62 %

Table 6.2 - Verification of deflection for the Støren floors

The fundamental frequencies are well above the typical limit value of 8 Hz:

Fundamental frequency f_1 [Hz]	**Floor 1**	**Floor 2**	**Floor 3**	**Floor 4**	**Floor 6**
Bare structure	21.9	20.7	19.9	20.8	21.2
Tested "complete" floor	16.2	15.5	15.8	15.0	15.8
The floor with all components	12.9	12.5	13.0	11.9	12.7
The floor with all components + 0.2 kN/m^2	11.4	11.1	11.7	10.6	11.3
The floor with all components + 0.4 kN/m^2	10.4	10.1	10.8	9.6	10.3
The floor with all components + 0.6 kN/m^2	9.6	9.4	10.0	8.8	9.5

Table 6.3 - Fundamental frequencies of the floors

Opposite to chapter 5, floor 6 is now verified by the "comfort criterion" and floor 1 is not.

Parameter	**Unit**	Floor 1 Structural	Floor 1 Complete	Floor 2 Structural	Floor 2 Complete	Floor 3 Structural	Floor 3 Complete	Floor 4 Structural	Floor 4 Complete	Floor 6 Structural	Floor 6 Complete
m	kg/m^2	26,6	48,7	28,9	51,0	37,6	59,8	24,3	46,4	27,8	49,9
f_1	Hz	21,9	16,2	20,7	15,5	19,9	15,8	20,8	15,0	21,2	15,8
Δ	mm/kN	1,05	0,81	0,83	0,70	0,75	0,49	1,20	0,90	0,74	0,66
$f_1/\Delta^{0,44}$		21,5	17,8	22,4	18,2	22,5	21,5	19,2	15,7	24,3	19,0
Verification		87 %	105 %	84 %	103 %	83 %	87 %	97 %	119 %	77 %	98 %
Factor		1,015	1,042	1,040	1,080	1,016	1,086	1,038	1,114	1,063	1,096
Verification		86 %	101 %	80 %	95 %	82 %	80 %	94 %	107 %	73 %	90 %

Table 6.4 - Combined deflection and fundamental frequency criterion verification

The velocity verifications according to the Ohlsson method of the current Eurocode 5 are not a problem:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
n_{40}		3,29	3,09	2,66	2,68	3,38	2,55	2,77	2,56	2,34	2,53
v	m/(Ns^2)	0,0190	0,0120	0,0151	0,0103	0,0142	0,0079	0,0189	0,0119	0,0140	0,0100
ζ		0,01	0,02	0,01	0,02	0,01	0,02	0,01	0,02	0,01	0,02
b_{normal}		126	126	126	126	126	126	126	126	126	126
$v_{limit,normal}$	m/(Ns^2)	0,0229	0,0379	0,0216	0,0357	0,0207	0,0364	0,0217	0,0340	0,0221	0,0367
Verification		83 %	32 %	70 %	29 %	68 %	22 %	87 %	35 %	63 %	27 %
b_{high}		144	144	144	144	144	144	144	144	144	144
$v_{limit,high}$	m/(Ns^2)	0,0206	0,0347	0,0194	0,0326	0,0186	0,0333	0,0195	0,0310	0,0199	0,0335
Verification		92 %	35 %	78 %	32 %	76 %	24 %	97 %	38 %	70 %	30 %
$b_{Austria}$		150	150	150	150	150	150	150	150	150	150
v_{limit}	m/(Ns^2)	0,0200	0,0337	0,0188	0,0317	0,0180	0,0323	0,0189	0,0301	0,0193	0,0326
Verification		95 %	36 %	81 %	33 %	79 %	25 %	100 %	39 %	73 %	31 %

Table 6.5 - EC5 unit impulse velocity response verification with ζ of 1 % and 2 %

The new Eurocode 5 proposal paints a very different picture of the velocity response:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
f_w	Hz	2	2	2	2	2	2	2	2	2	2
I	Ns	2,05	3,04	2,21	3,20	2,33	3,14	2,19	3,33	2,13	3,12
M*	kg	75,0	137,4	82,1	145,0	121,5	192,9	59,1	112,8	78,8	141,4
$V_{1,peak}$	m/s	0,0273	0,0221	0,0269	0,0220	0,0191	0,0163	0,0371	0,0296	0,0271	0,0221
K_{imp}		1,277	1,0	1,0	1,0	1,227	1,0	1,036	1,0	1,0	1,0
$V_{tot,peak}$	m/s	0,0349	0,0221	0,0269	0,0220	0,0235	0,0163	0,0384	0,0296	0,0271	0,0221
η		0,82	0,97	0,97	0,97	0,84	0,97	0,95	0,97	0,97	0,97
β		0,430	0,577	0,524	0,584	0,465	0,582	0,512	0,590	0,518	0,581
V_{rms}	m/s	0,0150	0,0127	0,0141	0,0129	0,0109	0,0095	0,0197	0,0174	0,0140	0,0128
R		149,8	127,5	141,0	128,8	109,2	94,7	196,7	174,5	140,1	128,1
R_{limit}		24	24	24	24	24	24	24	24	24	24
$V_{rms,limit}$	m/s	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024	0,0024
Verification		624 %	531 %	588 %	537 %	455 %	394 %	820 %	727 %	584 %	534 %
R_{limit}		4	4	4	4	4	4	4	4	4	4
$V_{rms,limit}$	m/s	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004	0,0004
Verification		3746 %	3187 %	3526 %	3220 %	2729 %	2367 %	4918 %	4361 %	3503 %	3202 %

Table 6.6 - Proposed new EC5 velocity verification with ζ of 1 % and 2 %

For sensitivity analysis of input parameters, "Floor 1 – Complete" will be considered, with the least strict response factor limit of 24, changing one parameter at the time:

If the damping ratio is changed from 2 % to 100 %, the verification only reduces from 531 % to 484 %. This is despite it being known that vibration velocities are tolerated better when quickly damped [20], which the Ohlsson and Mohr methods account for. So the effect of damping is basically just ignored, like in the "comfort criterion" / Hu's formula.

Changing the walking frequency from 2 Hz (suggested for offices) to 1.5 Hz (suggested for residential floors) lowers the verification to 352 %.

To get the verification down from 531 % to 100 %, the span could either be lowered by 40 % (from 4.7 m to 2.8 m) or the longitudinal stiffness could be increased by a factor of 5.5.

Increasing the (modal) mass doesn't help much because it is counteracted by the simultaneous lowering of f_1. A four times higher mass only lowers the verification to 381 %.

The modal mass is here calculated via the floor width (like for Hamm/Richter/Winter), while both the Austrian NA and Mohr instead used b_{ef} to approximate the modal mass.

Increasing the floor width from 2.4 m to 3.7 m lowers the verification to 374 %, but increasing the width further than that does not help.

Increasing b_{ef} or $(EI)_b$ has zero impact in this case. (As instructed, the f_1 formula for one-span floors is used, which only factors in the longitudinal stiffness.) But the transversal stiffness does matter if the floor width is increased at the same time. Doubling b_{ef} while also doubling the floor width reduces the verification to 267 %. A further increase of the width to 7.4 m gives 187 %, while increasing the width more than that does not help.

Overall it seems rather hopeless to get such timber joist floors verified according to this method for even the least strict response factor limit. The example floor calculated in the EC5 committee document [4] had 40 mm of screed and a much larger width.

The floors (except number 4) are approved by both of Mohr's velocity criteria:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
$v_{heel\ drop}$	m/s	0,382	0,218	0,287	0,182	0,250	0,129	0,414	0,226	0,261	0,174
$v_{heel\ drop,limit}$	m/s	0,164	0,266	0,155	0,251	0,150	0,256	0,156	0,240	0,159	0,258
Verification		232 %	82 %	185 %	72 %	167 %	51 %	265 %	94 %	164 %	68 %
$v_{EC5\ mod}$	m/(Ns^2)	0,0300	0,0147	0,0220	0,0121	0,0187	0,0086	0,0318	0,0148	0,0202	0,0117
$v_{EC5\ mod,limit}$	m/(Ns^2)	0,0091	0,0148	0,0086	0,0140	0,0083	0,0142	0,0087	0,0133	0,0089	0,0143
Verification		329 %	100 %	255 %	86 %	225 %	61 %	366 %	111 %	228 %	81 %

Table 6.7 - Mohr's velocity/mass requirement verifications with ζ of 1 % and 2 %

Verification of acceleration response is not needed for high-frequency floors like these, but they are anyway shown below, with respective damping ratios of 1 % and 2 % without and with the added layers. On top is the Austrian National Annex to EC5 (limit values 0.05 or 0.10, or none), in the middle is Mohr (limit value 0.10), and on the bottom is the new EC5 proposal (limit values ranging from 0.02 to 0.12 m/s^2).

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
M*	kg	51,3	121,7	72,2	151,6	86,4	210,1	49,7	125,9	77,5	155,1
a_{rms}	m/s^2	0,04	0,09	0,05	0,09	0,06	0,06	0,07	**0,14**	0,04	0,08
M_{gen}	kg	51,3	121,7	72,2	151,6	86,4	210,1	49,7	125,9	77,5	155,1
a	m/s^2	0,04	0,03	0,03	0,03	0,03	0,02	0,04	0,04	0,03	0,03
M*	kg	75,0	137,4	82,1	145,0	121,5	192,9	59,1	112,8	78,8	141,4
a_{rms}	m/s^2	0,02	0,06	0,03	0,07	0,03	0,05	0,04	0,11	0,03	0,06

Table 6.8 - Accelerations with ζ of 1 % and 2 %

For the new EC5 proposal, all input parameters for the acceleration are also known for "the floor with all components" (from Table 5.6 and Table 6.3). They are shown for damping ratios of 2 % and 3 %:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**		Full		Full		Full		Full		Full
M*	kg		217,0		225,2		282,7		179,5		221,3
a_{rms}	m/s^2		**0,13**		**0,15**		0,10		**0,23**		**0,14**

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**		Full		Full		Full		Full		Full
M*	kg		217,0		225,2		282,7		179,5		221,3
a_{rms}	m/s^2		0,09		0,10		0,06		**0,16**		0,09

Table 6.9 - Accelerations for the floors with ceiling and parquet, with ζ of 2 % and 3 %

The floors are high-frequency with regards to walking activities, so the acceleration response is then not relevant, just the velocity response. The floors' transient vibration behavior is assessed very differently by Ohlsson, Mohr and the new EC5 proposal.

Hamm/Richter/Winter and the Austrian NA to EC5 require a layer of screed (and in some cases filling to increase the mass), so the floors are by default not approved according to these two methods. Only floor 3 with 0.49 mm/kN satisfies their least strict deflection limit value of 0.5 mm/kN.

The floors satisfy the least strict demands of the Norwegian NA to the current EC5, but only floor 3 is approved by the higher demands.

The floors are mostly deemed as OK by the "comfort criterion".

A summary of selected verifications is shown below. Based on this, the performance of the ten floor constructions has been rated in order from #1 (best) to #10 (worst):

Verifications	Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
Deflection [mm/kN]	1,05	0,81	0,83	0,70	0,75	0,49	1,20	0,90	0,74	0,66
Comfort criterion	86 %	101 %	80 %	95 %	82 %	80 %	94 %	107 %	73 %	90 %
EC5 velocity, b=144	92 %	35 %	78 %	32 %	76 %	24 %	97 %	38 %	70 %	30 %
Mohr heel drop vel.	232 %	82 %	185 %	72 %	167 %	51 %	265 %	94 %	164 %	68 %
New EC5, velocity, VI	624 %	531 %	588 %	537 %	455 %	394 %	820 %	727 %	584 %	534 %
New EC5, velocity, I	3746 %	3187 %	3526 %	3220 %	2729 %	2367 %	4918 %	4361 %	3503 %	3202 %
Rating	#9	#4	#8	#3	#5	#1	#10	#5	#5	#2

Table 6.10 - Summary of selected verifications

The vibration behavior should be somewhat better than the calculations imply when the floors are built with a floor finish and ceiling, which were not mounted for the testing.

The “comfort criterion” is the only verification method that implies that the floors are better without the three added layers. This is addressed in section 6.3.

6.2 Transversal stiffness

First of all, it should be said that $(EI)_b$ is a very sensitive parameter when calculated from the test results. Its percentage error will be much larger than that of b_{ef}, because the relation between them (Formula 5.4) involves an exponent of 4. So the hand calculations presented here will by default just be approximations.

The transversal stiffness properties calculated from the test results are shown in Table 5.3, and are reiterated here:

Parameter	Unit	Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
		Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
$(EI)_b$	Nmm	3,85E+06	1,08E+07	9,40E+06	1,89E+07	4,32E+06	2,38E+07	6,34E+06	1,96E+07	1,27E+07	1,96E+07
b_{ef}	mm	820	1063	1055	1255	853	1307	1010	1339	1179	1316

Table 6.11 - Transversal stiffness properties of the tested floors

A breakdown of the $(EI)_b$ values is shown in section 6.2.2.

The transversal stiffness of the bare structure will be looked into first, and then how much $(EI)_b$ that was gained from adding the three layers.

6.2.1 Stiffness of sheathing and blocking

Floor 6 had a transversal stiffener at the mid-span. The sheathings of all tested floors were perforated, except the one of floor 2 which didn’t have any holes.

As expected, floor 2 does indeed have a higher $(EI)_b$ than floors 1, 3 and 4. For simplified analytical hand calculations, the sheathing of floor 2 (with E = 2550 N/mm^2 and t = 22 mm) would normally be considered as such:

$(EI)_b = 2550 * 22^3 / 12 =$ $\underline{2.26 * 10^6 \text{ Nmm}}$

This is 9.40 / 2.26 = $\underline{4.16}$ times lower than found from the test results. But it was to be expected that this ratio would be much higher than 1, because the glued and screwed connection between sheathing and joists adds stiffness and reduces deflection compared to if it had been floating/unconnected, as seen in Figure 3.11.

The average $(EI)_b$ found for floors 1, 3 and 4 is:

$(EI)_{b,avg} = (3.85 + 4.32 + 6.34) * 10^6 / 3 =$ $\underline{4.84 * 10^6 \text{ Nmm}}$

If the perforations are ignored when doing hand calculations, the multiplication factor should then be 4.84 / 2.26 = $\underline{2.14}$, which is slightly more than half of the 4.16 factor found for the non-perforated particleboard of floor 2.

The reduction in area (and volume) due to the holes is only 15 %. But the reduction in second moment of area will be larger as this is a 4th order term ($bh^3/12$) rather than 2nd order (bh). As a simplification, a multiplication factor to account for the holes could be assumed as follows: $0.85^{(0.5*4)} =$ $\underline{0.7225}$. This is however not conservative according to the test results, which found it to be 4.84 / 9.40 = $\underline{0.515}$.

But of course, testing just one floor with a non-perforated particleboard isn't a large enough sample size for this purpose, and the difference in calculated transversal stiffness between floors 1, 3 and 4 also varied a lot. By standard hand calculations, the transversal stiffness $(EI)_b$ should be the same for these three floors, both with and without the added layers. But according to the measurements there are large differences, and even more so when the three layers are added, as will be shown in section 6.2.2.

The transversal stiffness contribution from the blocking can be approximated by subtracting the average $(EI)_b$ found for the sheathings of floor 1, 3 and 4 from the total $(EI)_b$ of floor 6:

$(EI)_{b,blocking} = 1.27 * 10^7 – 4.84 * 10^6 =$ $\underline{7.86 * 10^6 \text{ Nmm}}$

6.2.2 Stiffness of the added layers

A breakdown of how the components contribute to the transversal stiffness is shown below:

$(EI)_b$ [Nmm]	Floor 1	Floor 2	Floor 3	Floor 4	Floor 6
Blocking					*$7.86 * 10^6$*
Sheathing	$3.85 * 10^6$	$\underline{9.40 * 10^6}$	$4.32 * 10^6$	$6.34 * 10^6$	*$4.84 * 10^6$*
Σ Structural	**$3.85 * 10^6$**	**$9.40 * 10^6$**	**$4.32 * 10^6$**	**$6.34 * 10^6$**	**$1.27 * 10^7$**
The three added layers	$6.99 * 10^6$	$9.47 * 10^6$	$1.95 * 10^7$	$1.32 * 10^7$	$6.96 * 10^6$
Σ Complete	**$1.08 * 10^7$**	**$1.89 * 10^7$**	**$2.38 * 10^7$**	**$1.96 * 10^7$**	**$1.96 * 10^7$**

Table 6.12 - Breakdown of transversal stiffness added from components

Since the exact same layers were added each time, one would expect to find the same added transversal stiffness for all floors, which should be predicted quite well by this formula:

$\Sigma\ (EI)_b = \Sigma\ (E * t^3 / 12)$

This was however not the case. For example, the added $(EI)_b$ for floor 3 was 2.8 times higher than for floor 6.

The benefit of the added layers on average for all floors was $1.29 * 10^7$ Nmm.

The acoustic insulation has no stiffness. The unconnected particleboard has roughly the same basic properties as the sheathing, so $(EI)_b \approx 2.26 * 10^6$ Nmm according to hand calculations.

The elastic modulus of the 12.5 mm gypsum plasterboard (from Knauf) is not available. By requiring that it must account for the rest of the average added $(EI)_b$, the computed elastic modulus of the plasterboard is 65 000 N/mm^2. Clearly this is not the case, so the test results show that the transversal stiffness is increased by more than what the hand calculations would predict, also for the layers that are unconnected.

It is difficult to explain all of these results. Floor 3 having the largest stiffness increase from the added layers could be due to it being the only floor with a joist spacing of 300 mm. But it is then odd that such an effect was not seen for the bare structure.

The transversal stiffness of floor 4 was large for both floor configurations. This is the floor with the worst stiffness properties longitudinally, so the high calculated $(EI)_b$ might be explained by load distribution according to stiffness.

Floor 6 had the highest $(EI)_b$ of all floors without the added layers, and also the lowest increase when they were added. This may be a case of diminished returns.

The calculated $(EI)_b$ values for floor 1 are quite low. This is however the floor that had the most uncertainty regarding the measurements for $(EI)_L$. If $(EI)_L$ is 5 % lower than assumed in the calculations, then the $(EI)_b$ values should be 18 % higher, and b_{ef} 6 % higher.

It would be harder to spot any measurement errors for the “200 kg load case” than for the other load case, because of not knowing as much what to expect, and because the middle joist displacement can’t be compared with the others. The load of 200 kg is however high enough so that the percentage error from the measuring equipment is very low, as long as no errors were made.

As seen in Table 6.11, the effective width b_{ef} maxed out at close to 1300 mm for all floors except number 1. This makes sense mathematically; an increase of $(EI)_b$ by a factor of 2 only results in a 19 % increase of b_{ef}, due to the exponent of 0.25 in the relation between them, as can be seen from Formula 5.4 or Formula 3.3.

There seemed to be an inverse correlation between EI/L^3 and the transversal stiffness $(EI)_b$ gained from adding the non-structural components:

	Floor 1	Floor 2	Floor 3	Floor 4	Floor 6
EI/L^3	16.37	14.77	8.89	10.09	12.67
$(EI)_b$ [$*10^6$] from the added layers	6.99	9.47	19.46	13.22	6.96

Table 6.13 - Joist stiffness vs. transversal stiffness from added layers

It seems that more added transversal stiffness can be expected for floors with low joist stiffness EI/L^3. The relatively low stiffness in the joist's longitudinal direction then means that a larger portion of the load is rather transferred in the orthogonal direction, and maybe more so when the joist center distance is low. Floor 3, the one with the longest span and the only one with a joist spacing of 300 mm, was the one that performed the best in the verifications. The low increase for floor 6 could be due to it having the highest transversal stiffness to begin with, i. e. diminished returns.

So, a given $(EI)_L$ results in higher transversal stiffness if the joist spacing c is low than if the joist EI is high (and c is high). But of course, since $I = bh^3/12$, it will in most cases still be more rational to increase h than to lower the spacing c. Lowering c by a factor of 2 has the same effect on $(EI)_L$ as increasing the joist height by 26 %. Storey height could be a limitation, though. And it is anyway good news that more transversal stiffness may be achieved for long-span floors.

If the joist center distance is halved (while the joist EI is also halved), a person walking between two joists in the floor's longitudinal direction will then cause a $2^3 = 8$ times lower local deformation of the boarding. The effect on the global deformation is of course far lower, but not insignificant. A quick estimate showed that the difference between a joist spacing of 300 mm and 600 mm for the reference floor may be about 0.2 mm/kN of local deformation of the sheathing, and this has to be added to the joist deformation to get the total deformation. For strict demands such as 0.5 mm/kN in total, the local deformation would account for a high percentage if the joist spacing is high. Our tests did however only place the load above the joists and not between them.

6.3 Suggested design formula

The main weaknesses of Hu's formula are that it doesn't account for damping, and mass is only seen as a negative. The problem is of course that the damping ratio ζ is difficult to estimate accurately, while it is a sensitive parameter in other verification methods. But since it is very important, it shouldn't be discarded completely.

A higher modal mass means that it takes more energy to excite a mode. The Austrian National Annex and Hamm/Richter/Winter have constructive requirements regarding mass (from screed and fill). And it is in reality also a requirement for the proposed new EC5 velocity verification, because it is so strict. Even if a higher floor mass doesn't equate to a higher damping ratio [8], it could still be modelled as such (as a simplification, like done for partition walls in Table 2.3) to account for the positive influence of mass.

As an improvement of Hu's formula, I will suggest the following:

$$\frac{f_1}{\Delta^{0.44}} * \left(\frac{\zeta}{0.03}\right)^{0.8} > 18.7$$

Formula 6.1 - Combined frequency, deflection and damping criterion

For a damping ratio of 0.03, the criterion is unchanged. In Table 3.13, the recommended damping ratios ζ for the new EC5 proposal are shown. They range from 0.02 to 0.04 for various floor types, which is more realistic in situ than the overly conservative 0.01 recommended by many of the code-based methods. The exponent of 0.8 is to limit the sensitivity of ζ in the verifications.

On the basis of Table 3.13 being implemented for the new EC5, the in-situ damping ratios of the tested floors should be 0.02 without the non-structural parts and 0.03 with. (ζ will be somewhat lower in laboratory test conditions.) If adopted, it makes these "comfort criterion" verifications more logical:

		Floor 1		Floor 2		Floor 3		Floor 4		Floor 6	
Parameter	**Unit**	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete	Structural	Complete
m	kg/m^2	26,6	48,7	28,9	51,0	37,6	59,8	24,3	46,4	27,8	49,9
f_1	Hz	21,9	16,2	20,7	15,5	19,9	15,8	20,8	15,0	21,2	15,8
Δ	mm/kN	1,05	0,81	0,83	0,70	0,75	0,49	1,20	0,90	0,74	0,66
$f_1/\Delta^{0,44}$		21,5	17,8	22,4	18,2	22,5	21,5	19,2	15,7	24,3	19,0
Verification		87 %	105 %	84 %	103 %	83 %	87 %	97 %	119 %	77 %	98 %
Factor		1,015	1,042	1,040	1,080	1,016	1,086	1,038	1,114	1,063	1,096
Verification		86 %	101 %	80 %	95 %	82 %	80 %	94 %	107 %	73 %	90 %
ζ		0,02	0,03	0,02	0,03	0,02	0,03	0,02	0,03	0,02	0,03
Verification		119 %	101 %	111 %	95 %	113 %	80 %	130 %	107 %	100 %	90 %

Table 6.14 - Modified comfort criterion verifications

Table 3.13 only includes four basic floor types, and it would be warranted to expand on it. One could e. g. be able to assume some more "damping" for floors with additional mass. Tabulated values would regulate this for the use of the formula. For example, floors heavier than X kg/m^2 could be calculated with a ζ value that is 0.01 higher than for more lightweight floors, and so on.

Partition walls are often modelled as extra damping as a simplification (although it is the f_1 that is increased through added stiffness). The problem with accounting for partition walls (perpendicular to the main vibrating elements of the critical mode shape) in the conceptual design stage is that they may be removed some years later if the owner wants to change the floor plan/layout. It cannot always be seen as a constant like the floor's self-weight mass, so it may be advised against considering partition walls towards ζ. And because the "comfort criterion" uses the orthotropic plate model for two-span floors to calculate f_1, it is already accounted for that f_1 tends to be somewhat higher in real life than theoretically assumed (due to stiffer support conditions and partitions).

The suggested design formula is very simple and has potential, in my opinion. In addition to accounting for frequency and stiffness, this modification would make it also consider damping as well as modal mass (indirectly), so that all important parameters are accounted for in one small formula with associated table values.

And if it is the case that Formula 5.2 does not consider shear deformation, then I will suggest that Formula 5.3 is instead adopted for the new Eurocode 5, as explained in section 5.2.

7 Conclusion

The Støren floors are high frequency floors characterized by transient vibration behavior when subjected to walking activities. Resonance is not a problem for the floors (unless they are to be used for rhythmic activities), and it is the vibration velocity rather than vibration acceleration that may give rise to negative feedback from people. Whether the velocity response is problematic for the Støren floors depends which verification method is applied. According to the Ohlsson (current EC5) and Mohr methods, the floors are satisfactory. But according to the latest proposal for the new EC5, the velocity response is way too high.

This large discrepancy between methods is due to a couple of things. As explained in section 2.8, it is necessary to modify a floor construction quite a lot to achieve a perceptible difference of the vibrational behavior. It is also highly subjective. Subjective evaluations vary (and the criteria may be interpreted differently) between people, and also between different cultures. This combination explains why the different code-based methods give such different answers.

It is good that the new Eurocode 5 proposal involves six floor performance levels, to allow for different cultures to prioritize between economy and walking comfort in the national annexes. But while the highest deflection limit of 1.6 mm/kN is very lenient, even the least strict velocity requirement cannot be reasonably fulfilled for the Støren floors without using screed and increasing the floor width. If implemented for the new EC5, people will be more likely to simply choose concrete instead of timber.

With the three added layers that the Støren floors were tested with, they satisfy deflection requirements of 0.81, 0.70, 0.49, 0.90 and 0.66 mm/kN, respectively. These numbers would be better if tested with a floor finish and ceiling, due to the added transversal stiffness.

The floors are in general approved by the "comfort criterion", Mohr and the current Eurocode 5. They are not approved according to the new EC5 proposal, Hamm/Richter/Winter and the Austrian National Annex to EC5.

The best performer was floor 3; the most long-span floor with the lowest joist spacing. A correlation was seen: Floors with joists of low EI/L^3 had a higher increase of the transversal stiffness from the added floor layers. This suggests that, for the same $(EI)_L$, it is better with a low joist distance than a high joist *EI*. (This also reduces the local deformation of the boarding when walking between two joists.) But in practice it is more efficient to increase *EI* as it has the cross-sectional height as a 3rd order term. It is anyway good news that the results suggest that the transversal stiffness is higher for long-span floors. Load distribution according to stiffness would be a logical explanation for these results.

The transversal stiffness was in general higher than what simplified analytical hand calculations would predict, as shown in sections 6.2.1 and 6.2.2. To enable structural engineers to make realistic estimates of the deflection etc., a reliable calculation program like BTAB should be made user-friendly and publicly available.

The effective width b_{ef} maxed out at close to 1300 mm for all floors except number 1, as seen in Table 6.11. Diminished returns were to be expected, as an increase of $(EI)_b$ by 100 % only results in a 19 % increase of b_{ef}, (due to the exponent of 0.25 in the relation between them, as can be seen from Formula 5.4,) and then the deflection can only be divided by a factor of 1.19.

As expected, floor 4 has the worst vibrational properties (as shown in Table 6.10). The best performer was clearly floor 3. Floor 6 was found to be the second best, closely followed by floor 2. Floor 1 was the fourth best.

Without the three added floor layers, floor 6 (with blocking) was found to be just as good as floor 3. But floor 3 saw a much higher increase of the transversal stiffness from the added layers. Floor 6 had the highest transversal stiffness among all of the floors before the layers were added, and this could be a reason for the diminished returns.

Improvements of existing vibration design formulas were suggested. (Formula 6.1, and possibly also Formula 5.3.)

A floor's vibration performance is determined by its mass, stiffness and damping. The simplest way to increase all three would be to add a layer of screed to the floors, but this may not be desirable for a company wanting to construct the floors in the factory rather than on the building site.

When the dynamic walking tests will be performed for the Støren floors, deflection tests should also be performed with the ceiling and a floor finish added, to see how much transversal stiffness the floors have in as-built condition.

The floors do not satisfy high vibration demands, but they are not bad either. In Norway we are used to the live feel of timber joist floors, and so for residential purposes where only normal/medium vibration performance is required then they should be satisfactory for most people.

Appendix A – Floor test data

FLOOR 1

SECTION	48x300	mm
SPAN	4,7	m
JOIST SPACING	0,6	m

STRUCTURAL

CONCENTRATED LOAD - 100 kg

1	2	3	4	5
	0,322	0,888	0,319	0,083
	0,456	0,747	0,377	0,055
	0,402	0,934	0,290	0,083
	0,456	0,934	0,348	0,055
	0,402	0,888	0,319	0,096
	0,483	0,794	0,377	0,055
	0,349	0,934	0,319	0,028
	0,429	0,747	0,348	0,055
	0,456	0,841	0,406	0,055
	0,442	0,794	0,348	0,069
	0,420	0,850	0,345	0,063

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
	0,724	1,868	0,609	0,083
	0,778	1,728	0,696	0,083
	0,698	1,822	0,609	0,124
	0,644	1,822	0,551	0,110
	0,751	1,868	0,696	0,069
	0,698	1,822	0,609	0,083
	0,698	1,822	0,609	0,110
	0,698	1,915	0,551	0,083
	0,724	1,775	0,696	0,069
	0,724	1,822	0,609	0,124
	0,714	1,826	0,624	0,094

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
	0,698	0,644	0,841	0,715
	0,698	0,682	0,754	0,688
	0,671	0,720	0,754	0,715
	0,885	0,531	0,841	0,660
	0,751	0,531	0,841	0,674
	0,724	0,644	0,754	0,701
	0,805	0,531	0,899	0,701
	0,724	0,682	0,754	0,688
	0,805	0,644	0,841	0,674
	0,778	0,606	0,841	0,674
	0,754	0,622	0,812	0,689

Average: 0,719

COMPLETE STRUCTURE

CONCENTRATED LOAD - 100 kg

1	2	3	4	5
	0,469	0,664	0,407	0,000
	0,391	0,664	0,436	0,027
	0,391	0,664	0,378	0,068
	0,339	0,746	0,349	0,054
	0,313	0,664	0,349	0,054
	0,313	0,788	0,349	0,027
	0,391	0,664	0,378	0,054
	0,417	0,705	0,436	0,000
	0,339	0,788	0,349	0,054
	0,339	0,788	0,349	0,041
	0,370	0,714	0,378	0,038

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
	0,677	1,534	0,668	0,095
	0,833	1,286	0,842	0,095
	0,755	1,451	0,755	0,095
	0,859	1,369	0,813	0,068
	0,755	1,286	0,813	0,095
	0,807	1,451	0,842	0,068
	0,833	1,451	0,755	0,068
	0,833	1,410	0,842	0,068
	0,833	1,410	0,784	0,054
	0,781	1,451	0,755	0,068
	0,797	1,410	0,787	0,077

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
	0,729	0,539	0,552	0,613
	0,677	0,622	0,552	0,640
	0,677	0,581	0,581	0,585
	0,755	0,581	0,581	0,613
	0,651	0,498	0,581	0,653
	0,677	0,581	0,581	0,599
	0,677	0,456	0,639	0,640
	0,729	0,581	0,581	0,653
	0,677	0,664	0,523	0,640
	0,651	0,664	0,494	0,640
	0,690	0,577	0,567	0,628

Average: 0,615

FLOOR 2

SECTION	48x300 mm
SPAN (THEORY)	4,7 m
SPAN (REAL)	4,74 m
JOIST SPACING	0,6 m

STRUCTURAL

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,208	0,702	1,669		
0,374	0,810	1,586		
0,291	0,675	1,586		
0,291	0,729	1,586		
0,291	0,648	1,586		
0,250	0,621	1,530		
0,250	0,675	1,530		
0,208	0,621	1,558		
0,208	0,702	1,613		
0,291	0,729	1,502		

0,266	0,691	1,575		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
0,515	0,594	0,716		
0,674	0,675	0,573		
0,555	0,621	0,716		
0,634	0,648	0,716		
0,674	0,729	0,687		
0,674	0,675	0,630		
0,555	0,621	0,687		
0,634	0,621	0,716		
0,714	0,621	0,601		
0,595	0,621	0,630		

0,622	0,643	0,667		

Average: 0,644

COMPLETE STRUCTURE

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,238	0,594	1,317		
0,198	0,675	1,289		
0,159	0,594	1,317		
0,278	0,648	1,375		
0,198	0,594	1,317		
0,198	0,567	1,289		
0,198	0,540	1,317		
0,238	0,621	1,403		
0,238	0,567	1,289		
0,159	0,675	1,317		

0,210	0,608	1,323		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
0,674	0,729	0,659		
0,634	0,675	0,773		
0,595	0,621	0,716		
0,555	0,621	0,687		
0,595	0,648	0,630		
0,555	0,621	0,659		
0,555	0,594	0,659		
0,595	0,675	0,544		
0,595	0,648	0,601		
0,555	0,621	0,659		

0,591	0,645	0,659		

Average: 0,632

FLOOR 3

SECTION	48x300 mm
SPAN (THEORY)	5,35 m
SPAN (REAL)	5,38 m
JOIST SPACING	0,3 m

STRUCTURAL

CONCENTRATED LOAD - 200 kg

1	2	3	4	5	6	7	8	9
0,120	0,083	0,598	0,799	1,606	1,006	0,438	0,275	0,154
0,040	0,165	0,478	0,881	1,634	0,978	0,518	0,248	0,196
0,000	0,083	0,638	0,799	1,593	1,034	0,478	0,248	0,154
0,000	0,165	0,478	0,964	1,620	0,978	0,399	0,303	0,154
0,000	0,165	0,399	0,964	1,606	1,020	0,359	0,275	0,154
0,000	0,193	0,478	0,909	1,634	1,020	0,438	0,248	0,182
0,000	0,220	0,558	0,881	1,565	1,006	0,399	0,220	0,140
0,000	0,193	0,478	0,881	1,565	1,020	0,518	0,165	0,196
0,000	0,165	0,399	0,964	1,593	0,978	0,399	0,248	0,154
0,000	0,138	0,518	0,881	1,579	0,978	0,399	0,248	0,182

0,016	0,157	0,502	0,892	1,600	1,002	0,435	0,248	0,167

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5	6	7	8	9
				0,996		1,212		1,118
				0,957		1,212		1,076
				0,996		1,212		1,090
				1,076		1,157		1,076
				1,036		1,212		1,118
				1,036		1,212		1,090
				0,996		1,212		1,118
				0,837		1,267		1,090
				1,116		1,157		1,090
				1,036		1,212		1,048

				1,008		1,207		1,091

Average: 1,102

COMPLETE STRUCTURE

CONCENTRATED LOAD - 200 kg

1	2	3	4	5	6	7	8	9
0,000	0,084		0,882	1,116	0,826	0,568	0,085	0,093
0,117	0,140		0,714	1,156	0,964	0,568	0,085	0,093
0,155	0,112		0,840	1,076	0,881	0,622	0,071	0,053
0,000	0,084		0,840	1,076	0,799	0,622	0,100	0,120
0,117	0,140		0,840	0,957	0,826	0,676	0,085	0,080
0,155	0,084		0,925	0,917	0,909	0,622	0,142	0,107
0,117	0,140		0,882	0,917	0,826	0,622	0,100	0,080
0,194	0,112		0,967	1,116	0,936	0,541	0,085	0,080
0,039	0,168		0,840	1,036	0,936	0,676	0,114	0,093
0,155	0,084		0,882	1,076	0,854	0,595	0,142	0,053

0,105	0,115		0,861	1,044	0,876	0,611	0,101	0,085

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5	6	7	8	9
				1,156		1,055		1,094
				0,996		1,109		1,081
				1,156		1,109		1,027
				1,156		1,001		1,067
				1,116		1,109		1,054
				1,156		0,974		1,067
				1,196		0,974		1,067
				1,076		1,109		1,054
				1,076		1,055		1,094
				1,196		1,055		1,067

				1,128		1,055		1,067

Average: 1,083

FLOOR 4

SECTION	48x250 mm	K-Bjelke!
SPAN (THEORY)	4,00 m	
SPAN (REAL)	4,05 m	
JOIST SPACING	0,6 m	

STRUCTURAL

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,271	1,087	2,377		
0,271	1,060	2,404		
0,425	1,115	2,404		
0,387	1,143	2,323		
0,387	1,004	2,350		
0,348	1,115	2,323		
0,425	1,115	2,486		
0,309	1,060	2,514		
0,387	0,976	2,459		
0,232	1,032	2,404		

0,344	1,071	2,404		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
1,005	1,060	0,929		
0,928	1,032	0,984		
1,044	1,032	0,956		
0,928	0,976	0,929		
0,967	1,087	1,011		
1,121	1,115	0,984		
0,928	1,060	1,038		
1,083	1,087	1,066		
0,967	1,060	0,956		
0,928	1,060	0,902		

0,990	1,057	0,976		

Average: 1,007

COMPLETE STRUCTURE

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,387	1,143	1,803		
0,271	1,115	1,858		
0,348	1,143	1,776		
0,387	1,143	1,858		
0,425	1,115	1,749		
0,309	1,032	1,913		
0,271	0,976	1,776		
0,348	1,143	1,803		
0,425	1,171	1,831		
0,309	1,115	1,776		

0,348	1,110	1,814		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
0,889	1,004	0,902		
0,851	0,976	0,984		
0,928	0,976	0,984		
0,812	1,004	0,984		
0,889	1,032	0,874		
0,851	1,004	0,956		
1,044	1,087	0,874		
0,967	0,976	0,874		
0,967	1,004	0,902		
0,928	0,948	0,874		

0,913	1,001	0,921		

Average: 0,945

FLOOR 6

SECTION	48x300 mm
SPAN (THEORY)	4,7 m
SPAN (REAL)	4,72 m
JOIST SPACING	0,6 m

STRUCTURAL

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,192	0,925	1,460		
0,152	0,898	1,676		
0,192	0,898	1,703		
0,192	0,789	1,649		
0,113	0,789	1,676		
0,192	0,843	1,622		
0,113	0,734	1,649		
0,152	0,789	1,568		
0,152	0,816	1,649		
0,192	0,816	1,757		

0,164	0,830	1,641		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
0,750	0,682	0,757		
0,710	0,710	0,784		
0,750	0,791	0,838		
0,868	0,819	0,811		
0,789	0,764	0,703		
0,750	0,764	0,811		
0,868	0,791	0,784		
0,789	0,710	0,784		
0,710	0,737	0,730		
0,828	0,791	0,730		

0,781	0,756	0,773		

Average: 0,770

COMPLETE STRUCTURE

CONCENTRATED LOAD - 200 kg

1	2	3	4	5
0,118	0,764	1,379		
0,197	0,846	1,460		
0,197	0,819	1,460		
0,197	0,764	1,487		
0,197	0,791	1,487		
0,158	0,737	1,541		
0,197	0,737	1,541		
0,158	0,764	1,352		
0,158	0,737	1,541		
0,118	0,846	1,460		

0,170	0,781	1,471		

LOAD ON EVERY JOIST - 43.65 kg

1	2	3	4	5
0,828	0,901	0,784		
0,868	0,791	0,838		
0,710	0,791	0,838		
0,828	0,873	0,811		
0,789	0,873	0,703		
0,868	0,846	0,784		
0,868	0,901	0,730		
0,710	0,846	0,784		
0,828	0,791	0,757		
0,789	0,791	0,703		

0,809	0,840	0,773		

Average: 0,807

Appendix B – Beam test data

Beam test file	Cycle	F1	F2	w1_L	w2_L	Emodul_L	Cor_L	w1_G	w2_G	Emodul_G	Cor_G
plank1_emodul.txt	1	3541,374	10973,990	5,536	6,071	15571,45	0,9999	3,690	11,705	13854,01	0,9999
plank1_emodul.txt	2	3525,567	7121,873	5,803	6,053	16123,44	0,9993	4,030	7,782	14358,91	0,9998
plank1_emodul.txt	3	3526,783	7282,340	5,791	6,052	16127,79	0,9996	4,089	7,989	14431,39	0,9999
plank1_nr2_emodul.txt	1	3542,912	10991,729	5,423	5,956	15663,94	0,9999	3,769	11,715	14017,20	1,0000
plank1_nr2_emodul.txt	2	3527,158	7337,880	5,677	5,945	15937,25	0,9998	3,992	7,967	14361,65	0,9999
plank1_nr2_emodul.txt	3	3539,783	7299,346	5,678	5,943	15901,30	0,9998	4,053	7,975	14360,20	0,9999
plank10_emodul.txt	1	3545,737	10991,890	8,619	9,267	12879,47	0,9999	2,176	10,928	12625,59	0,9999
plank10_emodul.txt	2	3525,585	7253,927	8,932	9,253	13018,22	0,9997	2,398	6,728	12789,21	0,9999
plank10_emodul.txt	3	3528,857	7326,668	8,925	9,248	13178,68	0,9998	2,454	6,841	12863,46	0,9999
plank2_emodul.txt	1	3539,604	10927,552	6,143	6,680	15420,22	0,9998	2,553	9,932	15055,35	1,0000
plank2_emodul.txt	2	3547,186	7631,725	6,372	6,665	15624,87	0,9993	2,757	6,754	15394,60	0,9999
plank2_emodul.txt	3	3534,079	7292,748	6,394	6,667	15431,65	0,9991	2,798	6,495	15308,88	0,9999
plank3_emodul.txt	1	3520,542	10997,826	6,404	6,954	15237,80	0,9999	3,289	11,100	14338,97	0,9999
plank3_emodul.txt	2	3524,655	7403,004	6,660	6,912	17249,93	0,9970	3,562	7,505	14767,57	0,9999
plank3_emodul.txt	3	3523,529	7349,378	6,659	6,913	16882,44	0,9973	3,508	7,483	14423,50	0,9999
plank4_emodul.txt	1	3522,474	10992,462	5,196	5,756	14951,09	0,9999	1,813	9,588	14395,74	0,9999
plank4_emodul.txt	2	3528,303	7378,632	5,457	5,726	16043,04	0,9993	2,073	5,980	14798,44	0,9998
plank4_emodul.txt	3	3549,600	7398,587	5,444	5,720	15630,70	0,9994	2,110	6,086	14514,21	0,9998
plank5_emodul.txt	1	3524,280	10952,210	8,923	9,448	15858,04	0,9998	2,154	9,882	14402,26	0,9999
plank5_emodul.txt	2	3545,719	7297,808	9,162	9,428	15810,02	0,9993	2,409	6,258	14624,39	0,9999
plank5_emodul.txt	3	3543,717	7373,697	9,155	9,425	15899,15	0,9995	2,460	6,372	14693,03	0,9999
plank6_emodul.txt	1	3522,903	10992,050	6,026	6,717	12115,30	0,9999	2,481	11,033	12986,35	1,0000
plank6_emodul.txt	2	3533,471	7282,913	6,358	6,698	12360,29	0,9998	2,738	7,023	13012,53	0,9999
plank6_emodul.txt	3	3540,409	7353,222	6,347	6,696	12245,07	0,9998	2,762	7,140	12946,71	0,9999
plank7_emodul.txt	1	3550,959	10981,769	7,213	7,738	15864,19	0,9999	2,123	9,675	14773,27	1,0000
plank7_emodul.txt	2	3523,278	7252,783	7,466	7,722	16328,72	0,9992	2,359	6,067	15130,46	0,9999
plank7_emodul.txt	3	3522,027	7282,555	7,462	7,719	16400,49	0,9992	2,360	6,153	14895,51	0,9998
plank8_emodul.txt	1	3533,757	10986,257	5,291	5,849	14969,55	0,9999	1,500	9,340	14230,18	1,0000
plank8_emodul.txt	2	3520,256	7297,879	5,559	5,838	15175,93	0,9997	1,724	5,636	14475,08	0,9999
plank8_emodul.txt	3	3526,157	7328,313	5,555	5,840	14952,92	0,9998	1,736	5,711	14326,65	0,9999
plank9_emodul.txt	1	3537,708	10967,964	6,000	6,635	13115,08	0,9998	2,278	10,528	13423,62	0,9999
plank9_emodul.txt	2	3541,803	7257,056	6,304	6,613	13476,31	0,9996	2,463	6,570	13487,61	0,9999
plank9_emodul.txt	3	3524,923	7234,794	6,302	6,609	13544,45	0,9995	2,477	6,595	13427,71	0,9999

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