Norwegian University of Life Sciences
Faculty of Social Sciences,
School of Economics and Business

Philosophiae Doctor (PhD)
Thesis 2018:63

Four Essays on the Characteristics of the Oil Market

Fire essay om egenskaper ved oljemarkedet

Lin Ma

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Lin Ma

Ås, July 2018

Contents

List of papers

This thesis consists four papers.

Paper 1
OPEC Market Power: An Empirical Dominant Firm Model For the Oil Market
(Rolf Golombek, Alfonso A. Irarrazabal and Lin Ma)

Paper 2
The Effect of Income Shocks on the Oil Price
(Lin Ma and Alfonso A. Irarrazabal)

Paper 3
Importance of Demand and Supply Shocks for Oil Price Variations
(Lin Ma)

Paper 4
Optimal Asset Allocation for Commodity Sovereign Wealth Funds
(Lin Ma and Alfonso A. Irarrazabal)

Summary

The purpose of this thesis is to study the role of oil in the macroeconomy. In particular, this thesis answers four fundamental questions: (1) what factors drive the oil prices in long run; (2) what the effect of permanent income shocks is on the oil prices; (3) what elements explain the dynamic of short-run oil prices; and (4) what the optimal investment strategy should be for a commodity-based sovereign wealth fund (SWF). This thesis consists of an introductory chapter and four papers of empirical studies.

In the first paper, my coauthor and I estimate a dominant firm-competitive fringe model for the crude oil market. We estimate significant elasticities over the sample period 1986–2009. Nonlinear instrumental variable methods are employed that correct for the simultaneity bias. The estimation results reveal that the dominant firm model provides a fair representation of the oil market. Most of the structural parameters have the expected sign and are statistically significant. We also find that OPEC exercised market power during the sample period. Counterfactual experiments indicate that world GDP is the main driving force of long-run oil prices, however, the recent rise in supply (depletion) factors are also responsible for higher prices after 2004.

In the second paper, my coauthor and I discuss the effect of income shocks on the real price of oil. In this paper, we detected the evidence that there is a stationary long-run relationship among the variable of oil prices, the world production of oil, and the global GDP for the period of 1973–2016. Estimating an error correction model, we find evidence that a long-run relationship in the oil market helps to predict oil prices. In contrast, oil production does not correct the disequilibrium from a stable long-run equilibrium. From the impulse responses, we also find that a permanent shock to world GDP leads to a significant hump-shaped response in oil prices. In contrast, price-demand specific shocks are entirely transitory. In fact, we find that most of the variation in oil prices are derived from transitory shocks. The estimation for the period 1900–1973 reveals a structural change in the response of the oil price after 1973. We conjecture that the supply of oil is more elastic in the early period (1900–1973) than the later period (1974–2016)

The third paper studies the importance of demand and supply shocks and explains the formation of short-run oil prices from a speculative point of view. In this paper, I first employ a multivariate method to extract the cyclical component of oil price, and I find a large and positive effect of global GDP shock on the oil price cycles in a VAR model. Later, I estimate a competitive storage model with stochastic features in income and production processes by using cyclical components in the world GDP, world oil quantity, and oil prices. I use methods of simulated moments for estimation, the results of which indicate that the model fits the empirical data. All parameters are estimated to be significant with correct signs. Employing estimates, the extended storage model is able to capture the large volatility and persistence as in the oil price data. Furthermore, I perform a counterfactual analysis and impulse responses of oil prices to exogenous income and production shocks. It shows that the GDP shock generates a much more moderate effect on the oil price cycles in the extended commodity storage model than the empirical evidence from the

VAR analysis, and the production shock plays an important role for the variance of the cyclical component of the oil price.

The final paper solves a dynamic asset allocation problem for a commodity SWF under incomplete markets. An important feature of the commodity SWF is that the commodity income of SWFs is volatile and decreases over time due to the depletion of finite natural resources. My co-author and I calibrate the model using data from three countries: Norway, UAE, and Chile. The solution indicates that the Norwegian SWF should be kept all its financial wealth invested in the risky asset for the first five years, and then start decreasing the investment share on stock gradually to a long-run share. This implication is explained by an initially large oil-wealth to financial-wealth ratio that allows the fund's manager to take large risk positions when the natural resource is far from being depleted. We also find that changes in the volatility of the oil price can substantially affect the size of the hedging demand. Given that UAE has larger levels of oil reserves and similar current financial wealth, the optimal constrained allocation suggests the fund's manager to invest a larger share of the risky asset throughout the investment horizon. The solution for Chile implies that for relatively high risk aversion coefficients the manager should start at a small fraction of her wealth to increase later over the life cycle of the fund.

INTRODUCTION

1 Introduction

Oil has been one of the crucial commodities in the international market. It is referred to as the "master resource" of energy (Bradley and Fulmer, 2004), and in 2016 oil consumption accounted for approximately a third of world energy consumption (BP, 2018). Furthermore, oil is not only a primary source of energy for industrial production but also widely used in transportation as well as the residential and commercial sectors. On account of these uses, the size of the global crude oil market is considerable. According to IEA (2018), the global oil demand in 2017 was approximately 96 million barrels per day, which amounts to almost 2 trillion U.S. dollars per year (at the 2017 crude oil price). Another important feature of the oil market is that the price of crude oil exhibits considerable volatility (with extreme price swings over time) and persistence (Dvir and Rogoff, 2014). There have been many discussions about the factors affecting the dynamics of oil prices (Alhajji and Huettner, 2000, Hamilton, 2009, Almoguera et al., 2011, and so on), including the fast growth of developing countries, the wars in the Middle East, the speculative behavior, and the noncompetitive behavior of the Organization of the Petroleum Exporting Countries (OPEC).

In considering the importance of oil for the global economy, it is necessary to know the role it plays in the macroeconomy and, in particular, what the main drivers are for the oil price in both the short and long run.

Therefore, this thesis answers four fundamental questions within four self-contained papers. (1) The first question pertains to what factors drive the long-run trends in oil prices. This question is mainly discussed in the first paper, "OPEC's Market Power: An Empirical Dominant Firm Model for the Oil Market." In this paper, my co-authors and I hypothesize a structural model for the oil market through considering the noncompetitive behavior of OPEC. An econometric method with instrumental variables is used for the estimation of parameters. All the structural coefficients are estimated with correct signs and are statistically significant. A counterfactual analysis reveals that the global GDP is the main driving force behind long-run oil prices. Through this testing of market power, evidence is provided for the controlling of oil prices by OPEC.

In contrast, there have been several scholarly papers that discuss non-stationary features in the oil market, and there are reasons to believe that the commodity markets are influenced by permanent income shocks. Accordingly, (2) the second question of this thesis concerns the effect of permanent income shocks on the oil price, a topic which is discussed in the second paper of this thesis, "The Effect of Income Shocks on the Oil Price." In this paper, evidence is found for a comovement among the oil price, global income, and world oil production. For the methodology, a time series model with cointegrating features is employed. The findings of this study reveal that the price of oil can be predicted by using a stable long-run relationship and, furthermore, that the world gross domestic product (GDP) permanently effects the oil price and that the shocks to the oil price are almost always transitory.

Since the price of oil is characterized by considerable volatility and occasional large spikes, this thesis examines a further question, pertaining to (3) what elements explain the formation of short-run oil prices. This question is discussed in the third paper, "An Empirical Storage Model for the Oil Market." An extended storage model is hypothesized by using the method of simulated moments. The data for this estimation is detrended by using a multivariate method. The findings reveal that GDP shock generates moderate effects on the oil price in the short run.

Finally, (4) the fourth question of this thesis pertains to what the optimal investment strategy should be for a commodity-based sovereign wealth fund (SWF) with volatile underground wealth, a topic which is discussed in the last paper, "Optimal Asset Allocation for Commodity Sovereign Wealth Funds." In this paper, my co-author and I solve an asset allocation model with incomplete market where income risks cannot be fully hedged in the stock market. We calibrate a model for the Norwegian Sovereign Wealth Fund (SWF) that accumulates fluctuated oil income. The solution of the optimal investment strategy suggests that the Norwegian financial wealth should be invested in stocks at a high level initially and then decreased over time.

In the remainder of this introductory chapter, Section 2 describes the background of the oil market in a broader context. An overview of scholarship on the oil market is provided, on the basis of which the particular contribution of this thesis is explained. Section 3 shows the data sources employed in the thesis. Section 4 summarizes the four papers of the thesis, including the methodologies employed and the final results of each paper. Section 5 provides some concluding observations, and the empirical implications are also discussed.

2 Background

Before introducing the papers, I provide some background information about the oil market. First, I describe the main historical development in the oil market for the last 150 years. Then, I summarize several methodologies that have been used to study the oil market from different aspects.

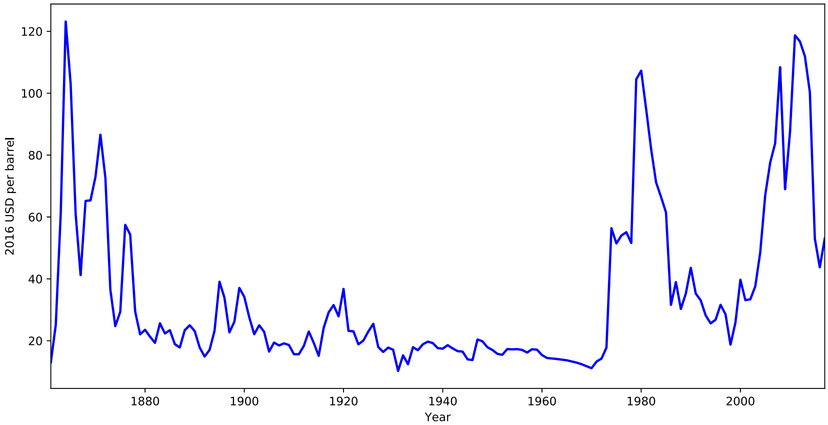

Figure 1: The Real Price of Oil

Note: The figure plots the real price of crude oil measured in 2016 USD. The West Texas Intermediate (WTI) price is used as the measure of crude oil price collected from BP (2018). Nominal crude oil price is deflated using the U.S. CPI obtained from U.S. Bureau of Labor Statistics (2018a).

2.1 Development in the world oil market

Figure 1 plots the real price of crude oil for the period 1861–2017. The figure shows that the price of oil is exceptionally volatile, with occasional large upward spikes. For instance, one can observe the extremely high real oil prices of 123 U.S. dollars per barrel for 1864; 107 U.S. dollars per barrel for 1980; and 108 U.S. dollars per barrel for 2008. Moreover, the volatility of oil prices also changes in long-term samples. There are two high-variance phases in the history of oil. The first phase, for the period 1861–1880, was marked by a standard deviation of 29.0; the second phase, for the period 1974–2016, was marked by a standard deviation of 29.6. This is five times as high as the price volatility in the period 1881–1973.[1] This significant change in the dynamic behavior of the oil price has also been detected in Dvir and Rogoff (2010). They have found two remarkable transition years, in 1878 and 1973 which were marked by changes in oil persistence and volatility. The change in 1878 might be linked to the construction of long-distance pipelines, which weakened the monopoly of oil shipments using railways. In the year 1973, the OPEC cartel rose to prominence when it proclaimed the oil embargo.

Figure 2 shows the production of crude oil in different regions. Due to technological

[1] The standard deviation is 5.8 for the period 1881–1973.

constraints on oil extraction, coal remained the primary source of power after the industrial revolution until the 1950s. As is shown in Figure 2, the production of oil in the U.S. remained at a lower level until 1900 (around 1000 barrels per day). Since 1900, the U.S. production of oil has increased steadily, which is mainly due to improvements in extraction technology for large-scale production (Maugeri, 2006). Over the same period, the world production of crude oil also increased continuously, which may be explained by the common usage of internal combustion engines since the early 1900s. After World War II, the economic boom and lower extraction costs boosted the world oil production, increasing exponentially for the period 1940–1970. In 1960, OPEC, a multinational organization, was founded. Currently, there are 14 members in this organization[2], which controls 1221 billion barrels of oil reserves, or 72 percent of the world's proven reserves (BP, 2018). The objective of OPEC is to "coordinate and unify the petroleum policies of its Member Countries and ensure the stabilization of oil markets in order to secure an efficient, economic and regular supply of petroleum to consumers, a steady income to producers and a fair return on capital for those investing in the petroleum industry" (OPEC, 2018). Although OPEC was founded in 1960, it did not rise to prominence until the early 1970s. Though the oil embargo in 1973 and the outbreak of the Iranian Revolution in 1979 led to a moderate decrease in oil production for OPEC (as shown in Figure 2), the turbulent prices of this period can be observed over the corresponding period in Figure 1. Since 1986, OPEC production has grown steadily. The oil production share of OPEC increased from 35 percent in 1986 to 40 percent in 1997. In 2017, the oil production of OPEC was around 81 million barrels per day, which accounts for 43 percent of the world oil production (EIA, 2018a).

Figure 2 also shows a steady increase in the world oil production since 1986. The rapid increase in world oil production in the last ten years has been fueled by the growth of production in non-OPEC countries; for example, the Canadian oil production increased by one third since 2010. Since 2012, the U.S. production of oil has also increased sharply, which is related to the massive production of shale oil, as shown in Figure 2.

2.2 Literature related to the determinants of oil price

In the following section, I summarize the relevant literature that studies the determinants of oil prices. I also explain how this thesis situates itself in the current scholarship.

[2]Till June 2018, the 14 member countries are Algeria, Angola, Ecuador, Equatorial Guinea, Gabon, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, United Arab Emirates and Venezuela.

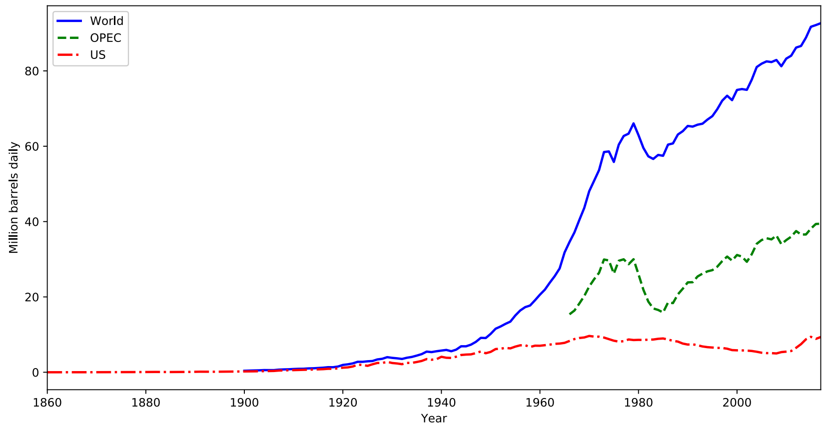

Figure 2: Production of crude oil

Note: The figure plots the production of oil. The solid line denotes the global production of oil collected from Etemad et al. (1998) for the period 1900–1964, BP (2018) for the period 1965–1973 and EIA (2018a) for the period 1974–2017. The dash line denotes the production of oil in OPEC for the period 1961–2017 collected from BP (2018). The red line denotes the production of oil in the U.S. for the period 1900--2016, which is collected from EIA (2018b).

2.2.1 OPEC market power

Since OPEC rose to prominence in the early 1970s, many studies have examined whether OPEC exerts market power in the world oil market. These scholars have recognized a specific model for OPEC's behavior and have estimated the supply and demand functions of the oil market, which have been identified using various supply and demand shifters. However, there has not appeared to be any definite conclusion in these empirical studies as to the extent of OPEC's influence on world oil prices.

Griffin (1985)is a seminal paper in this field, which tests the market power of OPEC under different market structure hypotheses for the period 1971–1983. Models for testing include cartels, competitive, target revenues, and a property rights model. In testing whether OPEC is a cartel, Griffin (1985) considered a single-equation approach for the supply of each OPEC member. He regressed the production of each OPEC country on the oil price and the production of other OPEC nations in a log-log function. The coefficients for the remaining OPEC countries and the price of oil measured the extent of market sharing. Griffin then implemented tests on the correlation coefficients. Griffin (1985) concluded that there is some evidence of partial market sharing in OPEC. In other words, most OPEC countries act as members of a cartel. He also implemented a nested test and found that the partial market-sharing cartel model dominates the competitive model for OPEC countries, and that non-OPEC countries are competitive price takers.

Subsequently, Jones (1990) revisited Griffin's model and estimated for the period 1983–1988, when there was a sharp fall in the oil price. His conclusion was consistent with that of Griffin (1985): most OPEC countries continued to act as the market-sharing cartel, and most non-OPEC countries behaved competitively. Another extension of Griffin (1985) was carried out by Youhanna (1994), who introduced lagged oil reserves into the models. The conclusion of Youhanna (1994) for the period 1983–1989 was the same as that of Griffin (1985).

To extend the earlier studies, Dahl and Yucel (1991) implemented hypothesis tests in different models with dynamic features. They also employed data on the cost of oil production for different countries. The models for testing included dynamic optimization, target revenue, competitive producers, cartels, and swing production. In the target revenue model, they tested whether oil production is a function of investment and net profit. Both strict and weak target revenue behavior was examined. In the dynamic optimization model, Dahl and Yucel (1991) tested whether the user cost (the difference between the oil price and the marginal cost of production) does not change across periods when the market is competitive, and whether the difference between marginal revenue and marginal cost does not change across periods when OPEC exerts market power. In the swing producer model, Dahl and Yucel (1991) tested whether the OPEC member countries have larger proportionate changes in production than the total production of OPEC. This test was carried out through a cointegration test such that production levels of OPEC countries are coordinated. Results showed that there is no evidence of dynamic optimization or for a formal target revenue model. They also did not find evidence of coordination or swing production in OPEC nations. However, they rejected the hypothesis of competitive OPEC members. Therefore, Dahl and Yucel (1991) concluded that loose coordination is the most accurate definition of OPEC's behavior.

Similar conclusions have been drawn by Smith (2005), who employed an alternative production-based approach to test the "compensating" response of OPEC countries when one producer changes production due to exogenous shocks. When the market is perfectly competitive, such "compensating" reactions should not occur. The results of the test showed that OPEC countries exhibit compensating production changes and that OPEC's market behavior lies between a non-cooperative oligopoly and a cartel; the traditional models of OPEC (including competitive, dominant-firm, and Cournot competition) were, however, rejected.

Almoguera et al. (2011)has also hypothesized a cartel model for OPEC when it was faced by a competitive fringe, in the form of non-OPEC oil producers. They estimated a simultaneous equation switching model for oil demand and OPEC supply. Using estimates, they tested whether there was a switch between cooperative and noncooperative behavior for the period 1974–2004. The results showed that OPEC can be characterized as a Cournot player in the oil market with a competitive fringe.

Contrary to previous scholarship, there have been some studies that have found no evidence for OPEC exerting market power. Concerning the problem of simultaneity in the single-equation approach, Alhajji and Huettner (2000) has specified a dominant-firm structural model and tested whether OPEC was the dominant producer in the oil market for the period 1973–1994. In this model, OPEC is viewed as a cartel whose members decide the oil price jointly. Non-OPEC members take the price as given and produce when the price is equal to their marginal costs. OPEC is faced with the residual demand: that is, world demand minus the competitive supply. The empirical dominant firm model is formulated in two equations: world demand and non-OPEC supply, which are estimated using 2SLS. The existence of the dominant firm model is tested by examining the sign of estimates and the condition of profit maximization. The test showed that neither OPEC nor OPEC core acts as the dominant firm, except for the case when Saudi Arabia is assumed as the dominant firm.

Through detecting the non-stationarity and comovement in data on oil price, world demand, and global income, Hansen and Lindholt (2008) have shown that the dominant firm model presented by Alhajji and Huettner (2000) entails an error correction mode (ECM). The total demand and non-OPEC supply functions were estimated separately as single equations in Hansen and Lindholt (2008). They used monthly data for the period 1973–2001. Implementing a similar test of market power as provided by Alhajji and Huettner (2000), they found that OPEC cannot be characterized as the dominant firm, although OPEC core can be characterized as the dominant firm for the period 1994–2001.

Finally, Brémond et al. (2012) have tested whether OPEC members are coordinated and have an impact on the oil price. To answer these two questions, they employed a time series and panel cointegration method to test for a long-run relationship between the production of each individual OPEC member and the production of the rest of that organization, as a whole. Brémond et al. (2012) also implemented Granger causality tests to investigate the impact of OPEC production on oil prices. The results of the tests showed that the impact of OPEC have changed over time. For substantial periods of time after 1973, OPEC have not behaved like a cartel.

The first paper in this thesis is different from previous studies on the market power of OPEC since it employs a dominant firm-competitive model for the oil market. This model is articulated through a simultaneous framework which contains the functions of world demand, non-OPEC supply, and OPEC supply. This model is implemented for an extended period, 1986–2016. By using the estimated parameters, the test of market power shows that OPEC exercised market power during the given period. At the same time, my co-authors and I also find that GDP drives the crude oil price in the long run.

2.2.2 Modeling oil prices using multi-variable time series

The oil price displays a high degree of persistence (Dvir and Rogoff, 2014 and Singleton, 2014), and in some tests it appears to be a non-stationary time series along the time period (Hansen and Lindholt, 2008 and Kaufmann et al., 2008). There has been extensive literature on how to model oil prices by using vector auto-regressions, for which two main types of model have been suggested. First, the stationary VAR model has been used for estimating the short-run effects of shocks on oil prices and quantities. These structural shocks are identified through the Cholesky decomposition method, which assumes exogenous production or income process. Second, the non-stationary auto-regression model has been used for illustrating how variables are cointegrated into the oil market. At the same time, the long-run trends have to be included on the right-hand sides of the regression models.

Kilian (2009)has studied oil prices through a structural VAR model, a stationary time series framework. He has identified supply, aggregate demand, and specific oil demand shocks through using data on world oil production growth, the index of real economic activity (based on dry cargo/bulk freight rate), and the real price of oil. He found that aggregate demand shock drives the long-term variation in the real price of oil and that demand shock that is specific to the oil market creates the sharp rise and decline in oil prices. A similar model has been employed by Kilian and Murphy (2014) and Juvenal and Petrella (2015), who have broadened the model to include the role of speculative oil demand.

There has also been a substantial amount of literature that has discussed the non-stationarity of crude oil prices. Kaufmann et al. (2004), Dées et al. (2007), and Lakuma (2013) have warned about possible cointegration in the crude oil market. New modeling techniques have been introduced. Kaufmann et al. (2004) and Dées et al. (2007) have detected a cointegrated relationship among crude oil price, the number of days of forward consumption of oil stocks, OPEC's production quota, and capacity utilization in OPEC countries. However, these scholars have employed error correction methods for models which utilize a single equation for oil supply and demand. More recently, Lakuma (2013) has studied the cointegrating relationship in the American crude oil industry through using data on the oil prices, the GDP of the U.S., and the production of oil in an error correction framework. Lakuma (2013) found that the American crude oil industry exercised market power during the period of 2000–2012.

The second paper of this thesis provides additional evidence for detecting the comovement among oil price, world production, and global GDP for an extended period, 1974–2016. This paper distinguishes between permanent and transitory components in the price of oil by using a vector error correction (VEC) model.

2.2.3 Speculation

Apart from regression estimation for the oil market, this thesis is also relevant to scholarship that studies the oil market through using structural models that generate important features of oil price data; the oil price has been characterized by sharp spikes, high persistency, and time-varying volatilities. These series of literature have investigated the dynamic behavior of oil price from the speculative point of view, and calibrated the structural parameters by using empirical data.

First, there have been papers that explain speculative behavior in commodity storage models. Deaton and Laroque (1992; 1996), who has written the seminal papers on this topic, employed the competitive commodity storage model to derive the implications for different commodity prices, mostly agricultural crops but also copper, i.e., a non-renewable mineral. Their analysis features a risk-neutral speculator who maximizes profit by holding commodity inventories[3] if, and only if, the expected price is equal to the current price; when the expected price is lower than the current price, there will be no incentives to store. Deaton and Laroque (1992; 1996) assumed the speculator faces a stochastic supply shock through the stationary process. In this scenario, the future commodity price is expected to be lower than the current level due to the mean-reverting features of the shock. Thus, the speculator dampens storage, which, in turn, lowers the rise in price during the current period. Therefore, the inventory adjustments have smoothing effects on the commodity prices, which dampen the volatility. However, by using calibrated parameters, this model cannot explain the high volatility and persistence observed in the commodity prices.

More recently, Dvir and Rogoff (2010) have augmented the commodity storage model of Deaton and Laroque (1992; 1996) so as to derive the implications for oil prices. Dvir and Rogoff (2010) have introduced a stochastic growth dynamics into the income process. Facing a positive shock on the income growth, speculators expect to have stronger future prices compared to the current price. Thus, the arbitrageurs are motivated to build up inventories, which, in turn, amplifies the income shock on prices and increases price volatility. In the latest paper by Dvir and Rogoff (2014), this model has been validated by using the empirical data, and Dvir and Rogoff (2014) assumed different supply regimes: fixed and flexible supply scenarios. They found that for the period after 1973, the correlation between inventory and oil price is positive, which implies that when the supply is restricted, inventory magnifies the effect on price.

Paper 3 also employs an augmented storage model for examining the effect of storage the oil market. In contrast to previous papers, I assume that both production and income processes are stochastic. On account of such an assumption, I am able to evaluate the relative importance of supply and demand shock to variations in the oil price. Moreover, this paper also validates the theoretical model by applying it to the empirical data pertaining to the oil market for the period 1986–2009.

2.2.4 Commodity sovereign wealth funds

Since the price of oil has a large and time-varying volatility (as discussed in the previous sections), a natural problem for a commodity producer is how to manage this volatility over time. The study of how a particular country should manage a fluctuating commodity revenue can involve, at least, two important dimensions. On the one hand, the design of a fiscal policy could consider how a given government can help to stabilize its economy alongside the volatility of commodity prices. Countercyclical fiscal policy has been suggested as one way to isolate a particular economy from commodity volatility. On the other hand, one

[3] It is assumed a free entry into the storage sector.

could take a purely financial approach and enquire as to how a government fund should allocate the commodity revenues in order to maximize social welfare. Both problems are important, and I review them in what follows. Since this thesis only studies the latter topic, in this section, I will thus only briefly review the literature on fiscal policy.

Fiscal policy

Many papers dealing with the fiscal aspect of commodity producers have been based on the permanent income hypothesis. In these models, the representative consumer dislikes fluctuations in revenues and is willing to sacrifice his or her current consumption to avoid these fluctuations. Valdes and Engel (2000) have discussed the possible fiscal policy for countries with stochastic and finite income flow. They have provided a solution to a standard problem of a SWF which is to maximize the utility of consumption of current and future generations with adjustment costs. A variety of policy instruments have been introduced, including savings and debt, taxation, and investment. The solution for the model with uncertain future income suggests that precautionary behavior generates higher saving rate and thus lower consumption. Moreover, Carroll and Jeanne (2009) have used a simple small-open-macroeconomy model to stress the precautionary saving motive. In this paper, Carroll discussed the role of labor income risk in affecting the reaction of capital flows to productivity growth. More recently, Agénor (2016) has employed a DSGE model to study the optimal resource allocation in developing countries with regard to uncertainties in resource prices. This model considers both productivity and the utility effect from private investment; it also addresses the importance of considering a social loss function, which concerns household welfare and fiscal volatility for low-income countries.

Sovereign Wealth Funds

Commodity sovereign wealth fund managers are long-run investors who want to maximize returns over a certain period of time. Dynamic portfolio optimization is a convenient conceptual framework for understanding the allocation problem of SWF. In particular, strategic asset allocation is a suitable framework for understanding the inter-temporal trade that long-run investors encounter. A useful starting point is to consider an investor who has some initial financial wealth but no income (see Merton(1969; 1971)). If one supposes that the representative consumer for this economy has time-separable preferences over consumption and that he or she can invest in time-varying stocks and risk-free bonds, the problem for an investor is to choose a path of consumption and the share to be invested in stocks in order to maximize the expected discounted utility. The optimal portfolio allocation for a risky asset is given through a combination of two factors. First, there is a standard myopic component, in which the share invested in the risky asset is proportional to the risk premium and is inversely related to its volatility. Second, there is a hedging component that is non-zero whenever there are time-varying investment opportunities, which are correlated with the return and affect the marginal utility. Similar results have also been shown in Stoikov and Zariphopoulou (2005) and Chacko and Viceira (2005). Later, Campbell and Viceira (2002) have solved a portfolio investment problem in different models by assuming stochastic expected returns and risk premia. They have shown the link between the myopic static solution and the dynamic solution. The evidence of stochastic expected returns has been discussed in many papers (see Campbell and Shiller (1988), Brandt (1999) and Cochrane (2006)).

Analyses of portfolio problems for consumers have been extended to also include time-varying labor income. Svensson and Werner (1993), Viceira (2001), and Henderson (2005) have investigated asset allocation with labor income and retirement features. Assuming an exogenous income process, they found that the optimal investment on a risky asset is always higher for the employed individual than for the retired investor. Viceira (2001) found that there is a negative hedging demand when labor income is positively correlated with stock-return shocks. Furthermore, Chan and Viceira (2000) have investigated the optimal investment strategy of wealth with stochastic labor income. They provided a solution for the model in an incomplete market where the labor income is not perfectly correlated with stock returns. With endogenous labor income, the decision on the labor/leisure choice of the investor generates large and positive effects on the portfolio decision.

Similarly, one could extend this problem to a country receiving varying income, indicating a problem in the asset allocation of SWFs. However, the research on optimal investment strategies for SWFs is scarcer. Dyck and Morse (2011) have collected a panel dataset of SWF investments in different sectors and investigated the explanatory power of different objectives in the portfolio decisions. Bodie and Brière (2013) and Brière and Bodie (2014) have used the sovereign balance sheet approach to investigate the management of SWFs. They solved the asset-liability management model analytically and obtained an optimal portfolio decision. They found that the optimal portfolio of SWFs includes "a performance-seeking portfolio and three hedging demand terms for the variability of the fiscal surplus and external and domestic debt." In a more recent paper, Xie et al. (2015) have proposed a multiple-goal investment model for SWFs in China. In this paper, they provided a solution through a mean-variance model, which was adapted to include features of return expectations.

In contrast to problems with the general SWF, commodity SWFs receive a stream of non-tradable income from producing an exhaustible natural resource in a finite period of time. Thus, another extension of the asset allocation problem could occur when the source of income is a commodity revenue. Gintschel and Scherer (2008) have built a model for the optimal asset investment of SWFs with oil revenue. The model is a one-period framework. They found that assets that are negatively correlated to the oil price improve the efficiency of investors. Therefore, the asset allocation decisions of SWFs should take both financial wealth and underground wealth into account.

Brown and Petrova (2010) have provided a solution for the optimal strategic asset allocation decision by means of the different objectives of SWFs – either an implicit return objective or a fiscal smoothing objective. They solved an extended Markowitz-model, and found the optimal strategic asset allocation under each objective.

Moreover, Scherer (2011) has formulated a solution to the portfolio allocation problem whereby the fund manager must choose how much to invest in risky assets under a mean-variance framework, where his or her payoff is determined not only by the fund's wealth but also by the entire government's economic wealth. Scherer found that the optimal investment in risky assets requires a function of financial-wealth to oil-wealth ratio (this assumes a constant government budget relative to financial wealth).

The latest research by van den Bremer et al. (2014) have studied the investment implications for SWFs in oil exporter countries in a model that combines theories related to portfolio allocation and optimal extraction rate. Using an approximated solution of the model under the assumption of incomplete markets, they found that a policy maker should consider the below-ground wealth in his optimal investment strategy for an SWF. Although it provides a means for performing different quantitative experiments, their model is not able to match the observed data.

In contrast to the existing literature, the second paper in this thesis considers an asset allocation model for an incomplete market. My co-author and I use empirical data in different countries for calibration, and provide optimal investment strategies for commodity SWFs in Norway, the UAE, and Chile.

3 Data

In this section I explain the data sources that have been used in the four papers of this thesis.

In all four papers, the price of crude oil was ascertained by using the West Texas Intermediate (WTI), which was obtained from the FRED (2018) or EIA (2016). Nominal prices were deflated by the U.S. CPI, which was obtained from U.S. Bureau of Labor Statistics (2018a). Data on oil production and inventories of crude oil in OECD countries was obtained from EIA (2018a). The quarterly world GDP index was collected from Fagan et al. (2001) for the period 1986:Q1–2010:Q4 and from Global Financial Data for the period

2011:Q1–2016:Q4. The GDP series were deflated by the U.S. CPI.

Other than the aforementioned data, each of the four thesis papers use particular data. In Paper 1, the data on OPEC's production costs is a combination of annual data (1986–2000) from Hansen and Lindholt (2008) and quarterly data (2001–2016) from CERA (2000). For non-OPEC production costs, we used U.S. costs of oil production as the non-OPEC production cost; this was obtained from U.S. Bureau of Labor Statistics (2018b). It compiles a producer price index (PPI) for oil and gas field machinery and equipment costs in the United States. According to CERA (2000), the nominal production cost for non-OPEC suppliers was 10 dollars per barrel in 1999:Q2. The data for OPEC's installed extraction capacity was obtained from Kaufmann (2005) for the period 1986:Q1–2007:Q3 and from the IEA (2018) for the period 2007:Q4–2016:Q4.

The fourth paper uses the high-grade copper price and an S&P 500 composite price index, collected from Global Financial Data (2018). Oil production for Norway and the UAE were taken from the U.S. Energy Information Administration (2014), while the reserves of crude oil were obtained from BP (2014). The copper production data was obtained from the Chilean Copper Commission (2014). The data on copper reserves was collected from the U.S. Geological Survey (2014). The data on financial wealth for Norway, UAE, and Chile was obtained, respectively, from Norges Bank Investment Management (2013), SWF Institute (2018), and Chilean Copper Commission (2014). Finally, the GDP data for each country was taken from World Bank (2014).

There is no additional data employed in paper 2 and 3.

4 Summary of the papers

This section summarizes each paper in the thesis. It includes a brief explanation of the methods used and a summary of the main results. The first, third, and fourth paper study the determinants of the oil price from different perspectives; the second paper concerns the implications of crude oil income on SWFs.

4.1 Paper 1. "OPEC Market Power: An Empirical Dominant Firm Model For the Oil Market"

This paper uses the dominant firm and competitive fringe model to study the long-run determinants of oil prices. In this paper, demand is standard, i.e. the quantity of demand is a function of current oil price and global GDP. My co-authors and I assume OPEC, as a group of producers, can exert market power, whereas the non-OPEC oil producers act as a competitive fringe and adopt prices as given – that is to produce when the price is equal to marginal costs. OPEC maximizes total profit by setting the optimal price of oil when marginal revenue equals marginal cost. Once the price of oil is set, total demand and the fringe supply are determined, and OPEC is faced with the residual demand. In this model, the rule of price setting for OPEC's supply is nonlinear.

This dominant firm model entails a simultaneous system of three equations. It includes world demand, non-OPEC supply, and OPEC supply function. This model is estimated using quarterly data for the period 1986–2016. Nonlinear instrumental variable methods are employed for estimation where world GDP, production costs for OPEC, and non-OPEC producers are used as exogenous supply and demand shifters. Due to the non-stationarity of the series, the efficiency of statistical tests is corrected by introducing lagged differences for independent variables.

The estimation results reveal that the dominant firm model provides a fair representation of the oil market. Most of the structural parameters have the expected sign and are statistically significant. The long-run price elasticity of demand is estimated at –0.34, which is higher than the previous estimations in Dahl (1993), Gately and Huntington (2002), and Cooper (2003). The income elasticity is estimated at 1.11, which is higher than the estimates in Gately and Huntington (2002).[4] The higher estimated income elasticity in this paper partly reflects the high GDP growth rates of China and India for a large part of the data period (1986–2016), which is not a feature in most of the previous studies. The non-OPEC supply elasticity is estimated at 0.31. The estimated marginal cost of oil production is lower for OPEC than for non-OPEC countries. Finally, my co-authors and I computed the market power index as a function of the estimated demand and supply elasticities. The market power index has a large value and remains at a high level during the sample period. The computed standard error of the market power index (using the Delta method) provides evidence that OPEC exerted substantial market power for the period 1986–2016.

The results from the estimation suggest that the nonlinearity induced by OPEC's markup is essential in modeling oil prices for the long run. The model is then reestimated under the assumption that OPEC is a price taker and that the market is competitive. From the competitive model, we obtain an insignificant and positive demand elasticity. The income elasticity is low, at 0.5. In addition, the factor price elasticity for OPEC is estimated to be insignificant.

The difference between the results from the competitive model and the dominant firm model reveals the nonlinear response that is induced by OPEC's markup on its residual

[4] In Gately and Huntington (2002), the income elasticity is estimated as of 0.55 for OECD countries and 1.17 for non-OECD countries including China and India.

demand. In the dominant firm model, OPEC's markup is endogenous. It is a function of parameters and endogenous variables. A non-nested statistical test (see Smith (1992)) is used to test the dominant firm model against the competitive model. The test statistic suggests that there is no evidence to reject the dominant firm model against the competitive model.

Finally, a counterfactual analysis is implemented to examine the contribution of world GDP and production costs to long-run trends in oil prices and quantities during the sample period (1986–2016). We generate the in-sample predictions of crude oil prices as well as OPEC and non-OPEC productions, and then we compare the in-sample predictions with a scenario that assumes a fixed GDP or production costs. We find that changes in world GDP explain most of the growth in oil prices and quantities but that the recent rise in production costs is also responsible for higher prices after 2004.

4.2 Paper 2 "The Effect of Income Shocks on the Oil Price"

This paper identifies the effect of GDP shock on the oil price. My co-author and I first observe the cointegrating relationship among the non-stationary series of oil prices, the world production of oil, and the global GDP by using quarterly data for the period 1974–2016. In other words, this analysis indicates that there is a stationary long-run relationship among the variables, and the cointegrating relationship is robust over the sample period.

Given the evidence of cointegration, my co-author and I estimate a vector error correction model and find that this long-run relationship in the oil market helps to predict oil prices – this highlights the importance of including long-run trends in oil market models.

Employing the estimates from the VEC model, we identify structural shocks using a Cholesky scheme and assume world income process to be exogenous and exclusive. The impulse response of world GDP toward a GDP shock suggests that world GDP is almost a random walk and has permanent effects. The response of oil prices to a GDP shock is hump-shaped and significant. Furthermore, the response of oil quantity towards an oil price shock is small and insignificant after four quarters. This is mainly because oil quantity cannot be predicted by using the long-run relationship in the VEC model, and this implies inelastic oil production in the long run. Furthermore, the response of oil price to a price shock is transitory and significant.

We also compare our model to a non-nested stationary VAR model, as depicted in Kilian (2009). We find consistent transitory effects for oil-demand shock; however, there is no significant effect of an oil production shock on the oil price in the VAR model.

Furthermore, my co-author and I reestimate the model for the period 1900–1973, when the oil price had low volatility, by using annual data. In contrast to the benchmark, we obtain significant estimates of rate adjustment in both quantity and price functions, which indicates that oil quantity and price are predictable using long-run relationship. The difference in the impulse responses leads to the conjecture that the supply of oil is more elastic in the early period (1900–1973) than the later period (1974–2016) and that shocks to income are mostly transitory in the early period.

Finally, a similar analysis is implemented for the copper market for the period 1900–2016. The aim of this analysis is to observe whether similar effects can be drawn in other commodity markets. In the copper market, we also find a cointegration relationship. The long-run relationship is the predictor for both copper production and copper price; therefore, the response of the copper price to income shock is mostly transitory.

4.3 Paper 3. "Importance of Demand and Supply Shocks for Oil Price Variations"

This paper studies the importance of demand and supply shocks in the oil market, and explains the formation of short-run oil prices from a speculative point of view. Using quarterly data for the period 1986–2009, I extract the cyclical components of quarterly data on oil price, global oil quantity, and world GDP by using a multivariate Beveridge-Nelson decomposition method. The cyclical components of global oil quantity and of world GDP are positively correlated, while the cyclical component of the oil price is volatile and persistent.

In this paper, I first document the features of oil series in a VAR model on the vector of world GDP, oil quantity and oil price. The impulse responses to orthogonal shocks are computed. It shows that a GDP shock generates significant and positive effects on the oil price cycles.

Second, I extend the competitive storage model, as presented in Deaton and Laroque(1992; 1996), by adding stochastic features in income and production processes (I assume there is no serial correlation in the income and production processes). In this model, the risk-neutral investor seeks to maximize his or her profit. The arbitrage condition suggests that there is no incentive to hold inventories when the current price is larger than the net expected price; on the contrary, when the current price equals the expected price, the inventory is positive. As discussed in Deaton and Laroque(1992; 1996), speculative storage has a smoothing effect on the equilibrium price. When a positive income shock or a negative supply shock occurs, the current price is high. Speculators expect a lower future price as the shock process is assumed to be mean reverting. Thus, they have the incentive to lower storage in the current period. This contributes to a higher price in the next period. In return, the storage "leans against the wind", which mitigates shocks and introduces persistence at equilibrium prices.

After formulating the problem, I discuss the role of income and production shocks in the extended storage model – to simplify the model with either income shock or production shock only. The models are solved numerically using a collocation method. In assuming the same shock volatility, the solution indicates that speculative behavior introduces smoothing effects on oil prices in both cases. Furthermore, the solution also shows that the production shock generates more intense effects in smoothing the equilibrium price with lower variance and higher persistence than the income shock. This is mainly because production shock is an additive shock, whereas the income shock is multiplicative.

Later on in this section, I estimate the extended storage model by using cyclical components in the world GDP, world oil quantity, and oil prices. I use a moment-matching method for estimation, the results of which indicate that the model fits the empirical data. All parameters are estimated to be significant with correct signs. The estimated demand elasticity is –0.2, which is close to the results indicated by Gately and Huntington (2002), Cooper (2003), and Dées et al. (2007), but sightly lower than the estimates in paper 1.[5] In using the estimates, the extended storage model is able to reproduce some crucial features of the oil price cycles: notably, the model captures the large volatility and persistence, as displayed in the oil price data.

Finally, I compute impulse responses of dynamic reactions in oil prices towards exogenous income and production impulses. The results show that, contrary to the evidence from the VAR analysis, the commodity storage model generates moderate responses to income shocks on the oil price cycles. A counterfactual analysis shows that the fluctuation in income only explains a small proportion of the variance in price cycles, and the production shock plays an important role for the variance of the cyclical component of the oil price. A possible explanation lies in small estimated income shock, and the production shock generates stronger smoothing effect.

4.4 Paper 4. "Optimal Asset Allocation for Commodity Sovereign Wealth Funds"

This paper investigates the optimal long-term portfolio decision for a commodity SWF in an incomplete market. The paper focuses on a fund manager's investment strategy of wealth over time. My co-author and I assume that the SWF manager has access to two assets: a risk-free bond and a risky asset (for example, stocks). What distinguishes commodity

[5]The demand elasticity is estimated at -0.35 in paper 1.

SWFs from other funds is their source of income. The commodity income[6] of SWFs is volatile and decreases over time due to the depletion of finite natural resources. In this analysis, we assume that the time to depletion is less than the investment horizon for the investor and that the commodity income is correlated with the stock prices. The SWF manager chooses consumption and asset allocation to maximize utility over a lifetime. We assume a Constant Relative Risk Aversion (CRRA) utility function. To gain intuitions on determinants of optimal investments, we solve the model under complete market in closed form following Munk and Sørensen (2010). In complete markets, income risk is fully spanned by stock prices. Therefore, the manager chooses optimal asset allocation by considering the net present value of commodity wealth. The primary determinant of optimal investment in stocks over time is the commodity wealth to financial wealth ratio. The investment strategies are also sensitive to the value of risk aversion, income volatility, and correlation coefficients.

In practice, markets are incomplete since the income growth is not fully correlated with stock returns; in other words, the income variations cannot be fully hedged in the stock markets. In incomplete markets, the model cannot be solved analytically. Numerical methods are needed. My co-author and I calibrate the model parameters through using data on the Norwegian SWF. Thereafter, we find the optimal investment strategy for an unconstrained problem that allows agents to borrow or lend assets. Due to the lopsided ratio of oil wealth to financial wealth, it is argued that Norway should increase their investment share in risky assets above one initially and then decrease to a long-run share in 40 years. We also solve the model with a constraint of prohibiting short sales, where the stock share should be non-negative and below one. The suggested optimal solution is to invest all wealth in stocks for the first five years and then to start decreasing the investment share in stocks gradually until it reaches a long-run share in approximately 30 years. From the data of oil prices, we observe that the volatility of the oil price changes over time in long-run samples for the studied period: for instance, the volatility of oil price growth is 0.04 for 1942–1973 and 0.34 for 1974–2013. We solve the model for optimal asset allocation with different price volatilities. The results show that the size of a hedging term is sensitive to the changes in the variance – with a difference of up to 30 percent in the stock shares from low to high volatility scenarios.

Sometimes, the investment policy of SWFs is kept fixed in real cases: for example, the stock share is fixed at 60 percent for the Norwegian SWF. My co-author and I quantify the loss of following a constant investment rule versus the optimal time-changing rule by computing the real gross gain in financial wealth of behaving optimally as in the model. We

[6] Here we consider oil and copper.

find substantial gains in financial wealth when following the time-varying asset allocation strategy.

We also conduct a similar analysis for two other commodity SWFs – in Chile and the UAE. Given that the UAE has a larger oil reserve and similar current financial wealth, the optimal constrained solution is to invest a larger share of the risky asset throughout the period. Due to the relatively reasonable high-risk aversion coefficients, the SWF in Chile, a fund based on copper revenue, should be invested in risky assets at a small fraction of wealth initially and then increased over time.

5 Empirical implications, limitations and future research

This section discusses the empirical implications of this thesis, explains methodological limitations, and outlines possible future research.

5.1 Empirical implications

In this section, I discuss the empirical implications of this thesis, which is mainly focused on the determinants of oil prices and the portfolio rule of commodity SWFs.

The thesis sheds light on the determinant of oil prices in both the short and long run. In particular, I have found that the world GDP is the main driving force behind the long-run oil price. This result is useful for anyone who is interested in predicting the long-run oil price; for instance, an oil producer who makes plans for investment strategies. Furthermore, I also show that the long-run relationship is a predictor of oil prices in the short run. The result can be useful for investors in the stock market, who forecast oil prices using high frequency data.

In this thesis, I also present evidence that OPEC speculates and controls prices in the oil market. This finding is important for many oil-producing countries, including Norway, which take the price as given. These countries should give strategic considerations to OPEC's market power and the impact of its speculative behavior on the price of oil.

Finally, this thesis provides a direct implication of modified investment rules for commodity SWFs in different countries. The analysis shows that, in taking underground wealth into consideration, the Norwegian oil fund should invest a higher fraction of its financial wealth into the stock market in the first several years and should then decrease the stock share gradually, whereas the fund in the UAE should be invested into higher share of risky assets throughout the period since the UAE has larger oil reserves.

5.2 Limitations and future research

This subsection discusses the methodological limitations in each of the papers. The results also suggest some possibilities for further research.

First, in Paper 1, my co-authors and I employ a static model. Although the model is augmented by adding lagged differences of exogenous variables and lagged OPEC capacity, the model does not have any dynamic feature for the endogenous variables. The results from the estimations show that the estimated elasticity of OPEC's marginal cost with respect to its lagged capacity is significant. This may reveal the importance of analyzing the role of capacity in a dynamic model.

Second, in Paper 2, my co-author and I estimated a VEC model. The long-run relationship cannot be interpreted as a "demand" or "supply" equation; rather, it reflects the equilibrium prices and quantities across time. Therefore, in the VEC model, we are not able to identify supply or demand shocks, as in the stationary VAR model. Future research may be conducted by formulating a model of oil with supply and demand stochastic processes and may then compare the effects of supply and demand shock on oil prices with those from the VAR model. Meanwhile, taking into account the market power of OPEC (as shown in paper 1) in the structural model can be another extension. Further research may also be possible for studying the effects of an income shock on other commodities.

Third, the limitations in Paper 3 are based on the simplified assumption of a stochastic income process that is stationary and follows an independent and identically distributed (i.i.d.) process with zero persistence. Using world GDP as a measure of world income, I have found that GDP is non-stationary (also shown in Paper 2). Therefore, an assumption of income processes with both transitory and permanent components should be appropriate. According to Dvir and Rogoff (2010), when growth shocks in income processes are assumed, storage no longer acts by "leaning against the wind", but it amplifies the shock's effect on the price. It would, accordingly, be interesting to observe how this setup works in my extended storage model. Through the estimation, I am able to evaluate the fraction of transitory and permanent components in the income process.

Finally, in Paper 4, my co-author and I solve an incomplete market model for a commodity SWF. The limitation of this model is the assumption of a time-separable CRRA utility function over consumption since, in practice, risk aversion and intertemporal substitution are not linked. Therefore, a recursive utility may be introduced: for example, the Epstein-Zin preferences. In addition, in this model, we assume that the oil price has constant volatility; this possibly limits the solution of stock shares with more dynamic features since the volatility in the price varies dramatically over time. Thus, for future research, the model can be extended with a time-varying volatility for the oil price.

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PAPER 1

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OPEC's market power: An empirical dominant firm model for the oil market

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ABSTRACT

We estimate a dominant firm-competitive fringe model for the crude oil market using quarterly data on oil prices for the 1986–2016 period. The estimated structural parameters have the expected signs and are significant. We find that OPEC exercised market power during the sample period. Counterfactual experiments indicate that world GDP is the main driver of long-run oil prices. However, supply (depletion) factors have become more important in recent years.

1. Introduction

Oil prices have changed substantially over the last three decades. Researchers have considered many explanations to account for the long-run behavior of prices, including growing demand from emerging economies, noncompetitive behavior of OPEC, resource depletion, and rising extraction costs. To understand which factors are paramount in driving the oil price, estimation of cost and demand parameters under different market structures is required. Because supply relations and demand function are likely to move simultaneously as a result of exogenous shifters (such as income and technological factors), econometric methods such as instrumental variables should be used to estimate these parameters. Unfortunately, the application of these methods to the oil market has proven difficult, see Hamilton (2009).

We use the dominant firm-competitive fringe textbook model (OPEC versus the group of non-OPEC producers) and estimate significant elasticities over the sample period, 1986–2016. The simultaneity bias is corrected for by using standard instrumental variable (IV) methods. We show that it is critical to correctly specify the market structure to obtain significant elasticities, and document that OPEC exercised market power during the sample period, 1986–2016.

In our model, demand is standard — it depends on the current oil price and world GDP — but we depart from standard supply analysis by assuming that one group of oil producers, OPEC, can exert market power, whereas the non-OPEC oil producers act as a competitive fringe. Once OPEC sets the price of oil, total demand and the fringe's supply are determined, and OPEC is faced with the residual demand: total demand less the competitive supply. OPEC sets the price that maximizes its total profits, taking into account the impact of its pricing decision on the residual demand. This choice leads to a *nonlinear* price-setting rule.

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Our empirical model contains a simultaneous system of three equations and is estimated using nonlinear instrumental variable methods with world GDP and production costs for OPEC and non-OPEC producers as exogenous demand and supply shifters. We use quarterly data from 1986 to 2016, which is a period after the major structural changes in the oil market in the 1960s and the 1970s. Our results suggest that the nonlinearity induced by OPEC's markup is of key importance in modeling oil prices.

We find that the dominant firm model provides a good representation of the oil market: all structural parameters have the expected signs and are statistically significant (except for the marginal cost elasticity of OPEC). We estimate a long-run price elasticity of demand of −0.35, which is somewhat larger than previous estimates reported in the literature (see, for example, Dahl, 1993; Gately and Huntington, 2002; and Cooper, 2003). Our estimate of the income elasticity of demand is 1.15, which is higher than previous estimates, see, for example, the Gately and Huntington (2002) study (0.55 for OECD countries and 1.17 for non-OECD countries including China and India) and Graham and Glaister (2004). We believe our results reflect that China and India, which had high GDP growth rates in the data period, 1986–2016, had high income elasticities in this period.

We find a non-OPEC supply elasticity of 0.32. Because the demand and non-OPEC supply elasticities are statistically significant, we obtain a tight estimate for the degree of OPEC's market power — we find evidence that OPEC exerted substantial market power in the period analyzed.

To gain insight about the role of OPEC's markup for our empirical results, we reestimate the model under the assumption that OPEC is a price taker. With a competitive model we obtain an insignificant (and marginally positive) demand elasticity — a similar result has been obtained in some previous studies, such as Lin (2011). Using the competitive model, we also obtain a lower income elasticity (around 0.5) and find an insignificant factor price elasticity for OPEC. The difference between the results obtained from the competitive model and the dominant firm model reflects the nonlinear response induced by OPEC's markup on its residual demand. In our model, OPEC's markup is *not* a constant; it is a function of parameters (to be estimated) and endogenous variables.

Using our estimates, we examine the contribution of world GDP and production costs to the long-run trend in oil prices and quantities during our sample period from 1986 to 2016. We find that changes in world GDP explain most of the growth in oil prices and quantities, but the recent rise in production costs is also responsible for higher prices after 2005.

We make four contributions to the literature on crude oil prices. First, there is a large literature on estimating the relationship between oil demand and the price of oil, and also the relationship between supply of oil and the price of oil (see, for example, Griffin, 1985, Kaufmann, 2004, Kaufmann et al., 2008 and Brémond et al., 2012). These papers do not account for the simultaneity of supply-and-demand changes. Hamilton (2009) argues that, for some periods, these estimates are probably good approximations, but, in general, they are subject to instabilities. Studies that have taken the simultaneity of supply-and-demand changes into account, as we do, are scarce — some examples are Alhajji and Huettner (2000), Krichene (2002), Almoguera et al. (2011), and Lin (2011). We contribute to this literature by estimating a simultaneous dominant firm-competitive fringe model for the oil market, using the nonlinear instrumental variable method — the nonlinear estimator reflects the nonlinearity of the system of equations to be estimated.We obtain statistically significant demand and fringe (non-OPEC) supply elasticities.

Second, our paper is related to the literature that tests the degree to which OPEC can control prices. Griffin (1985) is a seminal paper in this field. In testing whether OPEC is a cartel, Griffin starts out assuming that OPEC is a dominant firm that sets the price of oil. However, the residual demand function, as well as a first-order condition for OPEC, are *not* part of the empirical model. Alhajji and Huettner (2000) and Hansen and Lindholt (2008) also refer to the dominant firm model, but, again, OPEC's price-setting rule is *not* part of the empirical model in these papers. To the best of our knowledge, the present paper is the first to estimate the simultaneous dominant firm model for the oil market.

Whereas Griffin (1985) concludes that most OPEC countries act as members of a cartel, evidence of OPEC's ability to influence the price of oil is mixed. Papers in the 1980s and 1990s argued in favor of collusive behavior, see, for example, Almoguera et al. (2011), but later studies, using extended data, found mixed evidence of whether OPEC has exerted market power. For example, Spilimbergo (2001) finds no support for the hypothesis that OPEC, except for Saudi Arabia, was a market-sharing cartel during the 1983–1991 period, whereas Smith (2005) finds that OPEC's market behavior lies between a non-cooperative oligopoly and a cartel. Boug et al. (2016) present a model that encompasses several alternative specifications suggested in the literature. They find support for imperfect competition in the oil market, and also that OPEC's behavior has changed significantly over the last years. For other studies, see Jones (1990), Gulen (1996), Brémond et al. (2012), Cairns and Calfucura (2012), Huppmann and Holz (2012), Colgan (2014), Kisswani (2016) and Okullo and Reynès (2016). Smith (2009) and Fattouh and Mahadeva (2013) present reviews of the literature. Our contribution is to *test* whether OPEC had market power by using a non-nested statistical test for competing models: by comparing our dominant firm model with the competitive model, we find no evidence to reject the dominant firm model.

Third, using the model's estimated parameters, we show that growth in world GDP has been the main driving force of oil price increases over the last three decades, but recent rises in production costs have contributed significantly to higher oil prices. To the best of our knowledge, we are among the first to document the relative importance of *demand and supply factors* for the long-run behavior of oil prices, see Section 4.2. In contrast, some studies, like Kilian (2009), assume that supply is fixed, which is reasonable in the short run.

Finally, our paper complements results from the empirical industrial organization literature on measuring the degree of market power, see, for example, Suslow (1986), which finds substantial market power in the aluminum industry in the period between World War I and World War II. Our measure of market power builds on Bresnahan (1982), and, as reported above, we find clear evidence of exertion of market power in the oil market between 1986 and 2016. For a survey of the literature on industries with market power, see Bresnahan (1989).

Our paper is divided into six sections. In Section 2, we provide an overview of the crude oil market, and, in Section 3, we describe the empirical framework used to estimate the model. The main results are presented in Section 4. Here, we compare our estimated elasticities with those reported in the literature and discuss the fit of the model. We also analyze the relative importance of world income and costs of extraction as the driving forces of the oil price. In Section 5, we perform a number of robustness checks. Section 6 concludes.

2. The crude oil market

In this section, we describe the data sources and characterize the crude oil market, focusing on the period that is analyzed in this paper.

2.1. Data

We use quarterly data for the period, 1986:Q1–2016:Q4. The price of crude oil is measured by the West Texas Intermediate (WTI), which we obtained from the Federal Reserve Bank of St. Louis (2017).

Nominal prices are deflated by the US CPI, see U.S. Bureau of Labor Statistics (2017a). Data on oil production and inventory of crude oil in OECD countries were obtained from EIA (2017). World production of crude oil plus the change in the OECD inventory of crude oil is used as a measure for total consumption of (demand for) crude oil.[1]

Our data on OPEC's production costs combine annual data (for the period, 1986–2000) in Hansen and Lindholt (2008) and quarterly data (for the period, 2001–2016) from IHS CERA. Both series cover costs of exploration, development and production. For non-OPEC production costs, we use US costs of oil production, which we believe is a conservative estimate: among the non-OPEC producers, US producers have the highest cost, see Alhajji and Huettner (2000). The source for the non-OPEC cost of production is U.S. Bureau of Labor Statistics (2017b), which compiles a Producer Price Index (PPI) for oil and gas field machinery and equipment costs in the United States. We set the nominal production cost for non-OPEC suppliers to 10 dollars per barrel in 1999:Q2 (IHS CERA, 2000).

Like Kaufmann (2004) and Kaufmann et al. (2008), we also use data for OPEC's installed extraction capacity; these are obtained from Kaufmann (2005) for the period, 1986:Q1–2007:Q3, and from the IEA Oil Market Report for the period, 2007:Q4–2016:Q4.[2] Finally, we used the quarterly world GDP index from Fagan et al. (2001) for the period, 1986:Q1–2010:Q4, and Global Financial Data for the period, 2011:Q1–2016:Q4. The series is deflated by the US CPI.

2.2. Development in the oil market

In this subsection, we describe the main development in the global oil market since 1973, and also relate this to economic development. Panel (a) in Fig. 1 plots the real price of oil (measured in 2010 USD). The figure covers most of the turbulent period between 1973 and 1986, encompassing the huge increase in the oil price that occurred in 1973 when prices rose from 18 to 52 USD per barrel (frequently referred to as OPEC 1). It also includes the sky-high prices around 1979–1980 at roughly 100 USD per barrel (OPEC 2), and the substantial decrease in the oil price during the first half of the 1980s. It is beyond the scope of this paper to discuss this early period — the price path in this period probably reflects structural shocks on the supply side. Rather, our focus centers on the period after 1985, which is characterized by less abrupt changes in the crude oil market.

As seen from panel (a), the real oil price was roughly in the range of 20 to 40 USD per barrel from 1986 to 1998, except for the peak in 1990:Q3–1991:Q1, a rise that can be attributed to supply disruptions stemming from the Gulf War. Beginning in 1999, the oil price increased steadily and peaked at 126 USD per barrel in 2008:Q2, then dropped to around 40 USD due to the financial crisis, but increase again rather rapidly: In 2012–2014, the oil price was close to 100 USD. However, late in 2014, the price dropped; it went down to around 40 USD in 2015–2016.

Panel (b) shows that total production of oil increased steadily after 1985. In this period, non-OPEC production did not change much, but there was a drop in production in the early 1990s, reflecting the contraction of the energy industry in the former Soviet Union. The two plots in panel (b) imply that the OPEC's market share increased from 30% in 1986 to 40% in 1992 (see Fig. 1 panel (c)), where it has remained.

Fig. 2 illustrates the growth in world GDP, and also China and India's combined share of world GDP. As seen from Fig. 2, world GDP increased steadily over the 1986–2016 period, with an average annual growth rate of 2.2%. China and India's share of world GDP (measured by the right vertical axis) increased from 3% in 1987 to 5% in 2000, and then reached 18% in 2016, reflecting China's fast growth.

Fig. 3 plots non-OPEC and OPEC production costs (measured in 2010 USD per barrel). The difference in production cost between these two groups of oil producers narrowed significantly after 1985. The real cost of non-OPEC production decreased steadily after 1983, but increased after 2005. From 2010, the non-OPEC production cost has been around 16 USD per barrel. This development starkly contrasts with OPEC production costs, which increased from 1 USD per barrel in 1986 to 8 USD per barrel in 2008. Then, the OPEC production cost did not change much over the next years, but, in 2016, it dropped to 5 USD per barrel.

We now turn to the relationship between the oil market and GDP. Hamilton (2009), summarizing some studies undertaken between 1991 and 2003, concludes that these suggest an income elasticity near one. He then examines the (partial) relationship between the change in US oil consumption and the growth in US GDP — henceforth termed the income elasticity. He finds income elasticities around 1 for the period, 1949–1973, and around 0.5 for the period, 1985–1997, but a negative income elasticity between 1974 and 1985. We now do the same exercise as Hamilton (2009), but for the entire world (not just the United States).

Fig. 4 provides information about changes in (real) world GDP relative to changes in (real) world oil consumption. As seen from the figure, the 1973–1985 period is characterized by a negative relationship between the change in world GDP and the change in world oil consumption, whereas the opposite is the case for the periods 1986–2000 and 2001–2016.

One simple way to quantify the relationship between global oil consumption and world GDP is to calculate the ordinary least-squares (OLS) estimate for this coefficient. As shown in Fig. 4, the estimate is −0.07 for 1973–1985 (which is a period not included in the data used to estimate our empirical model below), compared with 0.52 for 1986–2000 and 0.64 for 2001–2016. This suggests that the income elasticity of oil did not change significantly over the 1986–2016 period. Therefore, in our empirical model, we impose a constant income elasticity for the period, 1986–2016, but, in Section 5, we estimate the empirical model for subperiods.

3. Empirical models for the crude oil market

In this section, we present two structural models for the crude oil market that differ in the degree to which OPEC exerts market power. We start by describing the common building blocks of the models, such as world demand and the non-OPEC competitive supply. Then, for the competitive model, we assume that OPEC takes the price as given. Finally, we introduce the dominant firm model where OPEC sets the price of oil.

3.1. Theoretical framework

Consider the inverse demand function for oil,

$$P = P(Q^w, Y, V^w), \tag{1}$$

where P is the real price of oil, Q^w is world (w) demand for oil, Y is (real) world GDP and V^w is a measure of other factors that may have an impact on demand for oil.

[1] Ideally, we would have used the change in world inventory of crude oil, but we do not have these data. Because the change in the OECD inventory of crude oil amounts to roughly 1% of world crude oil extraction, we believe our approximation of total demand for crude oil is good. We construct a quarterly data series for world consumption of oil, simply because no such series was previously available.

[2] Because we have data from both sources for 2007:Q2, we can check the extent to which the series differ in this quarter. We find that the difference is very small, but we still use this difference (measured as a percentage) to adjust the Kaufmann et al. (2008) data.

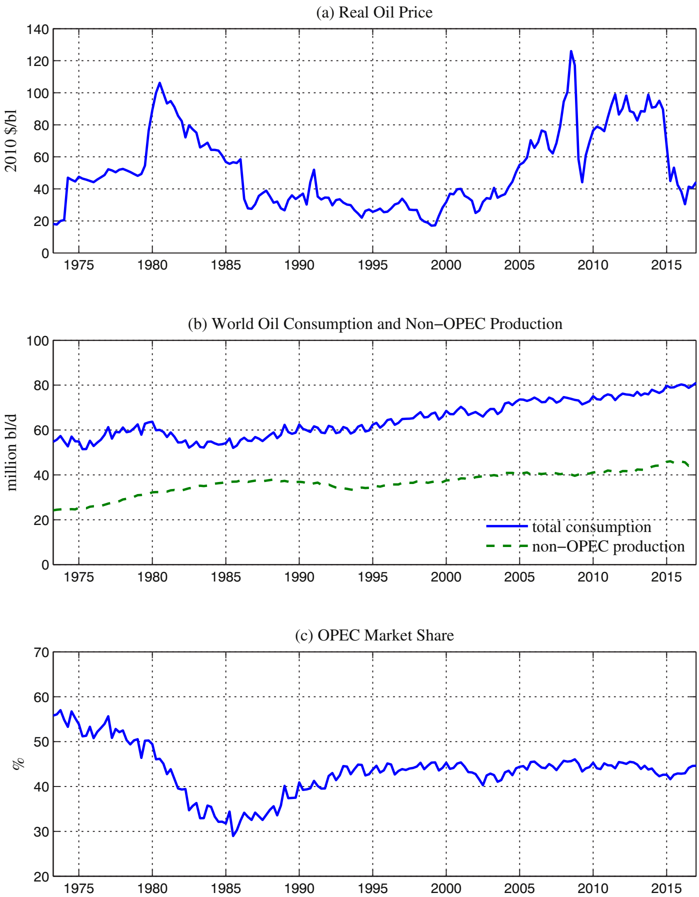

Fig. 1. Oil production and real price of oil (2010 USD).
Notes: Panel (a) plots the real WTI price collected from Federal Reserve Bank of St. Louis (2017). Nominal prices are deflated by the US CPI from the U.S. Bureau of Labor Statistics (2017a). Panel (b) plots world oil consumption and non-OPEC production. World consumption is defined as the sum of world production and the drop in OECD inventory of oil. All quantity series are collected from EIA (2017). Panel (c) plots the OPEC market share.

We assume there are two groups of oil producers, OPEC countries (o) and non-OPEC countries (no). The latter group is assumed to be price takers, and, thus, its first-order condition, derived from profit maximization, requires that the oil price is equal to the marginal cost (MC) of production:

$$P = MC^{no}\left(Q^{no}, W^{no}, V^{no}\right). \tag{2}$$

Here, Q^{no} is non-OPEC production, which we assume has an increasing marginal cost, W^{no} is the input cost for non-OPEC producers, and V^{no} contains other factors that may have an impact on non-OPEC supply of oil.

Below, we consider two alternative hypotheses for OPEC production: (i) OPEC has market power (the benchmark case); and (ii) OPEC is a price taker. In the latter case, the first-order condition for OPEC is, of course, similar to Eq. (2):

$$P = MC^{o}\left(Q^{o}, W^{o}, V^{o}\right), \tag{3}$$

where

$$Q^{o} = Q^{w} - Q^{no} \tag{4}$$

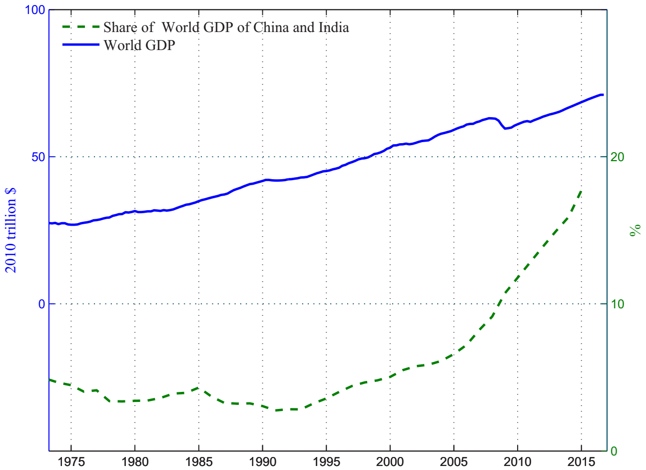

Fig. 2. Real world GDP and China and India's share of world GDP.
Notes: The figure plots real world GDP (measured by the left vertical axis in 2010 USD) and China and India's share of world GDP (measured by the right vertical axis). World GDP is combined using the GDP index from Fagan et al. (2001) for the period 1986–2010 and Global Financial Data (2017) for the period 2011–2016.

is OPEC production $\left(\frac{\partial MC^o}{\partial Q^o} > 0\right)$. Alternatively, OPEC is not a price taker. This hypothesis takes into consideration that OPEC's production has an impact on the price of oil: if OPEC production increases, then, *ceteris paribus*, the price of oil will decrease, and, therefore, non-OPEC extraction will decrease. Formally, Eq. (2) can be rewritten as $P(Q^o + Q^{no}) = MC^{no}(Q^{no}, W^{no}, V^{no})$, which implicitly defines the function $Q^{no} = Q^{no}(Q^o)$ where

$$\frac{dQ^{no}}{dQ^o} = -\frac{\frac{\partial P}{\partial Q^w}}{\frac{\partial P}{\partial Q^w} - \frac{\partial MC^{no}}{\partial Q^{no}}} < 0. \tag{5}$$

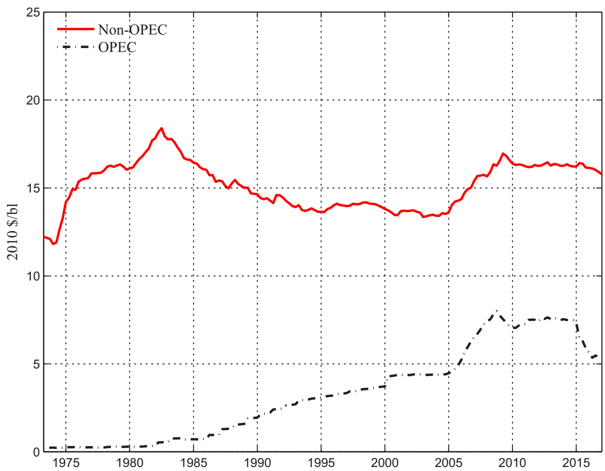

Fig. 3. Real cost of production in OPEC and non-OPEC.
Notes: The OPEC cost series is annual cost of OPEC for 1975–2000 in Hansen and Lindholt (2008) and quarterly observations of costs of exploration, development and production for 2001:Q1–2016:Q4 from IHS CERA. The source for the non-OPEC cost is U.S. Bureau of Labor Statistics (2017b). It is a Producer Price Index for oil and gas field machinery and equipment in the United States. We set the nominal cost for non-OPEC to 10 USD per barrel in 1999:Q2 (IHS CERA, 2000).

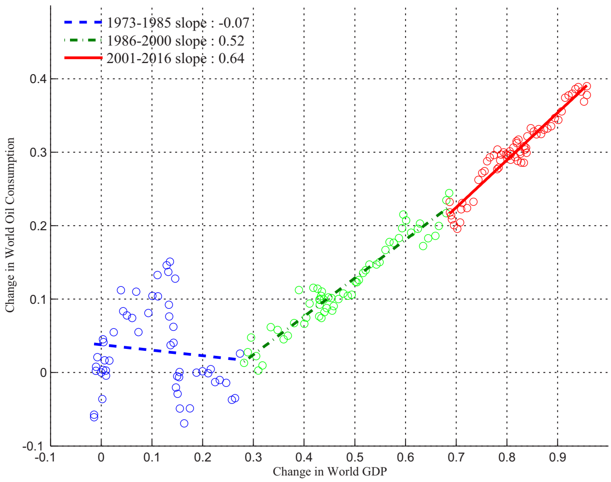

Fig. 4. Changes in real world GDP and world oil consumption.
Notes: The horizontal axis shows cumulative change in (natural logarithm of) real world GDP (measured in 2010 USD) for different periods, that is, $\sum_{s=1}^{s=t}(\ln Y_{s+1} - \ln Y_s)$ where $s = 1$ is the first quarter in the data period, for example, the first quarter in 1986, $s = 2$ is the second quarter in the data period, etc. For the subset of data covering 1973 to 1985, t is a quarter between the second quarter in 1973 and the fourth quarter in 1985. The vertical axis shows cumulative change in (natural logarithm of) total oil consumption Q^w. Each point in the figure represents a pair $\left\{\sum_{s=1}^{s=t}(\ln Y_{s+1} - \ln Y_s), \sum_{s=1}^{s=t}(\ln Q^w_{s+1} - \ln Q^w_s)\right\}$. The slopes are estimated using OLS with a constant.

OPEC maximizes profits, taking Eq. (5) into account, that is, OPEC maximizes $P(Q^o + Q^{no}(Q^o))Q^o - c^o(Q^o, W^o, V^o)$ with respect to Q^o, where $c^o(Q^o, W^o, V^o)$ is the total cost of OPEC production. Under the assumption of an internal solution, that is, positive production from both OPEC and non-OPEC producers, OPEC's first-order condition states that price should be a markup over marginal cost,

$$P = m(\epsilon, \gamma, s^o)\, MC^o(Q^o, W^o, V^o) \tag{6}$$

where the markup m is defined as

$$m(\epsilon, \gamma, s^o) = \frac{\epsilon - (1 - s^o)\gamma}{s^o(1 + \gamma) + \epsilon - \gamma} = \frac{1}{1 + \frac{1}{\epsilon^o}}. \tag{7}$$

Here, $\epsilon = \left(\frac{\partial P}{\partial Q^w}\frac{Q^w}{P}\right)^{-1} = \frac{\partial Q^w}{\partial P}\frac{P}{Q^w} < 0$ is the demand elasticity, $\gamma = \left(\frac{\partial MC^{no}}{\partial Q^{no}}\frac{Q^{no}}{MC^{no}}\right)^{-1} = \frac{\partial Q^{no}}{\partial P}\frac{P}{Q^{no}} > 0$ is the supply elasticity of non-OPEC producers, and $s^o = \frac{Q^o}{Q^w}$ is OPEC's market share of production. The markup's numerator is negative and, hence, the denominator also has to be negative in order to ensure a positive markup. Note that $m(\epsilon, \gamma, s^o) = \left(1 + \frac{1}{\epsilon^o}\right)^{-1}$, where ϵ^o is the elasticity of the residual demand facing OPEC.[3] Because an internal solution of the OPEC optimization problem requires $\epsilon^o < -1$ (in equilibrium), the corresponding requirement of the markup is $m > 1$; our parameter estimates meet this condition, see Section 4.1.1. The markup is, *ceteris paribus*, increasing in s^o and ϵ, but decreasing in γ. Because the markup is nonlinear in the parameters to be estimated, a nonlinear methodology is required.

An alternative representation (see Bresnahan (1982)) of the first-order condition, which we use later, is given by

$$P = MC^o(Q^o, W^o, V^o) - \lambda \frac{\partial P}{\partial Q^w} Q^o \tag{8}$$

where

$$\lambda = 1 + \frac{dQ^{no}}{dQ^o} = \frac{\epsilon}{\epsilon - \gamma(1 - s^o)} > 0. \tag{9}$$

Here, λ is referred to as the market power index. This index embeds several cases: $\lambda = 0$ corresponds to perfect competition, $\lambda = 1$ corresponds to monopoly, and $0 < \lambda < 1$ corresponds to intermediate cases such as Cournot competition and a dominant firm with a competitive fringe (our benchmark case).[4]

3.2. Empirical implementation

Our empirical goal is to estimate parameters for long-run elasticities for supply and demand. Under both market structures (dominant firm and competitive), we have a simultaneous system of equations that determines oil production in OPEC and non-OPEC countries, total oil production and the world price of oil.

[3] The elasticity of the residual demand facing OPEC is $\epsilon^o = \frac{\epsilon - \gamma(1 - s^o)}{s^o}$.

[4] As pointed out in Bresnahan (1982), if both demand and marginal cost are linear in quantity, then estimation of a relation of type Eq. (8) will identify the *gross* effect of increased quantity, which consists of two terms: the unit cost of OPEC production and the factor $\lambda \frac{\partial P}{\partial Q^w}$. Hence, it is not possible to identify λ.

3.2.1. Specification

We assume that world (w) demand for oil is given by a log-linear function:

$$\ln Q_t^w = \alpha_0 + \alpha_1 \ln P_t + \alpha_2 \ln Y_t + \alpha_3 V_t^w + \alpha_4 D_t^w + u_t^w, \tag{10}$$

where t is time, D_t^i is a vector of dummies, $i = w, no, o$,[5] and u_t^i is an error term assumed to be independent and identically distributed with zero mean and variance σ_i^2. Further, $V_t^w = [\Delta \ln Y_t, \ldots, \Delta \ln Y_{t-q}]$ is a vector of shifters. As suggested by Stock and Watson (1993), we augment our empirical model with a vector of lagged differences of independent variables and use dynamic OLS to obtain efficient statistical tests. Demand theory suggests that $\alpha_1 = \epsilon < 0$ and $\alpha_2 > 0$.

The non-OPEC group is a price taker, and they, therefore, set marginal cost equal to price, see Eq. (2). Assuming that marginal cost is log-linear[6], the supply of non-OPEC production is also log-linear:

$$\ln Q_t^{no} = \beta_0 + \beta_1 \ln P_t + \beta_2 \ln W_t^{no} + \beta_3 V_t^{no} + \beta_4 D_t^{no} + u_t^{no}, \tag{11}$$

where $V_t^{no} = \left[\Delta \ln W_t^{no}, \ldots, \Delta \ln W_{t-q}^{no}\right]$ is a vector of shifters. Further, $\beta_1 = \gamma > 0$ and $\beta_2 < 0$ according to standard economic theory.

Also for OPEC we assume that marginal cost is log-linear. We consider two alternative hypotheses for OPEC (see Section 3.1). First, OPEC acts competitively, and, thus, its supply function is given by

$$\ln Q_t^o = \pi_0^c + \pi_1^c \ln P_t + \pi_2^c \ln W_t^o + \pi_3^c V_t^o + \pi_4^c D_t^o + u_t^o, \tag{12}$$

where $V_t^o = \left[\Delta \ln W_t^o, \ldots, \Delta \ln W_{t-q}^o, cap_{t-1}^o, \Delta \ln cap_{t-2}^o, \ldots, \Delta \ln cap_{t-q-1}^o\right]$ is a vector of shifters. Note that V_t^o also contains the capacity of OPEC (cap^o), both the level (lagged to account for the endogeneity of this factor) and lagged differences.

Alternatively, OPEC acts as a dominant firm with a competitive fringe — the non-OPEC suppliers. Then, quantity is set so that price exceeds marginal cost of production. Using Eqs. (6), (7), (10), and (11), we obtain

$$\ln P_t = \pi_0^d + \ln m(\alpha_1, \beta_1, s_t^o) + \pi_1^d \ln Q_t^o + \pi_2^d \ln W_t^o + \pi_3^d V_t^o + \pi_4^d D_t^o + u_t^o, \tag{13}$$

where

$$m(\alpha_1, \beta_1, s_t^o) = \frac{\alpha_1 - (1 - s_t^o)\beta_1}{s_t^o(1 + \beta_1) + \alpha_1 - \beta_1}.$$

It is crucial that the markup is a *nonlinear* function of the parameters α_1 and β_1. The model is, therefore, nonlinear in the parameters to be estimated — this is explored in more detail in the next subsection.

Using the specified functional forms, the market power index becomes (see Eq. (9))

$$\lambda_t = \frac{\alpha_1}{\alpha_1 - \beta_1(1 - s_t^o)} > 0. \tag{14}$$

We use this expression to measure the degree of market power exerted by OPEC.

3.2.2. Estimation methods

In this subsection, we describe how we estimate the parameters under the two alternative market structures. First, in the competitive model where OPEC is a price taker, we estimate the structural parameters $\theta^c = [\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}^c]$ using Eqs. (10), (11) and (12), where $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3, \boldsymbol{\alpha}_4]$, $\boldsymbol{\beta} = [\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\beta}_3, \boldsymbol{\beta}_4]$ and $\boldsymbol{\pi}^c = [\boldsymbol{\pi}_0^c, \boldsymbol{\pi}_1^c, \boldsymbol{\pi}_2^c, \boldsymbol{\pi}_3^c, \boldsymbol{\pi}_4^c]$. Then, for the dominant firm specification, where OPEC charges a markup over marginal cost, we estimate the parameters $\theta^d = [\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}^d]$ using Eqs. (10), (11) and (13), where $\boldsymbol{\pi}^d = \left[\boldsymbol{\pi}_0^d, \boldsymbol{\pi}_1^d, \boldsymbol{\pi}_2^d, \boldsymbol{\pi}_3^d, \boldsymbol{\pi}_4^d\right]$. In both cases, the vector of instrument variables is

$$Z_t = \left[\ln Y_t, \ln W_t^{no}, \ln W_t^o, \ln V_t^w, \ln V_t^{no}, \ln V_t^o\right], \tag{15}$$

and we use the same number of lags in both the dominant firm model and the competitive model ($q = 3$); see Section 5 for a discussion on the importance of lags with respect to the empirical results. When estimating the competitive model, we use the three-stage least-squares (3SLS) method. In contrast, we use system nonlinear instrumental variable (NLIV) method when estimating the dominant firm model, see Appendix A for details.

4. Results

In this section, we present our main results. First, we present the estimated elasticities for the dominant firm model (our benchmark) and compare these with the estimates from the competitive model. Then, we explore the fit of the dominant firm model and identify which factor has been the main driver of the crude oil price. Third, we provide evidence for OPEC's exertion of market power during the 1986–2016 period.

4.1. Elasticities

The second column in Table 1 shows our estimates for the dominant firm model — Eqs. (10), (11) and (13) — using the NLIV method. The third column in Table 1 shows the estimates from the competitive model — Eqs. (10), (11) and (12) — using 3SLS. We use the same instruments and dummy variables as in the estimation of the dominant firm model.

Table 1 also presents an overidentification test for the instruments Z_t, see Eq. (15), for the dominant firm model. To test for the validity of the instruments, that is, the exogeneity of these variables, we use the Sargan-Hansen J-statistic, which equals the value of the GMM objective function evaluated at the estimated parameters. We find that the value of the J-statistic is 1.28. The critical value of the chi-square distribution with 31 degrees of freedom is 29.34 at the 5% significance level. Hence, we cannot reject the null hypothesis that the instruments are exogenous to our system of simultaneous equations.

[5] For the demand function Eq. (10), we use quarterly dummies (one dummy for each of the first three quarters). For the other relations, that is, Eqs. (11), (12) and (13), we use the same quarterly dummies as well as a dummy for wars in the Middle East and another dummy for the former Soviet Union. The war dummy equals 1 for the period of the Iran–Iraq war, 1986:Q1–1988:Q2, and also 1 during the invasion of Kuwait (1990:Q3–Q4). The former Soviet Union dummy equals 1 for the period, 1986:Q1–1990:Q2, reflecting the contraction of the Soviet energy industry in this period.

[6] Implicitly, we allow for a nonhomothetic cost function in factor prices and output. Constant returns-to-scale technology would imply independence of the marginal cost with respect to production.

Table 1
Estimates for the dominant firm and the competitive models.

Models		Dominant firm			Competitive
		NLIV			3SLS
World demand					
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.018)			0.003(0.004)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.154(0.117)			0.543(0.010)
Non-OPEC supply					
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.322(0.034)			0.076(0.018)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−0.758(0.372)			0.392(0.107)
OPEC supply					
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.545(0.904)	$\frac{\partial \ln Q^o}{\partial \ln P}$	π_1^c	0.194(0.083)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.516(0.247)	$\frac{\partial \ln Q^o}{\partial \ln W^o}$	π_2^c	−0.183(0.157)
λ		0.655(0.035)			
Overidentification test $J \sim \chi^2(dof)$					
J-statistic		1.281			1.437
Degrees of freedom		31			31

Notes: We use quarterly data for the period 1986:Q1–2016:Q4; the heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates for elasticities and the market power index λ. The second column shows the results for the dominant firm model, that is, Eqs. (10), (11) and (13), using a nonlinear instrumental variable (NLIV) method. The third column shows the estimates for the competitive model, that is, Eqs. (10), (11) and (12), using three-stage least-squares (3SLS) estimation. The predetermined exogenous variables used in the models are $V_t^w = \Delta \ln Y_t, \ldots, \Delta \ln Y_{t-q}$ in the demand equation, $V_t^{no} = \Delta \ln W_t^{no}, \ldots, \Delta \ln W_{t-q}^{no}$ in the non-OPEC supply equation and $V_t^o = \left[\Delta \ln W_t^o, \ldots, \Delta \ln W_{t-q}^o, cap_{t-1}^o, \Delta \ln cap_{t-2}^o, \ldots, \Delta \ln cap_{t-q-1}^o\right]$ in the OPEC equation, all with $q = 3$. In the dominant firm model, λ is evaluated at the mean of the market share of OPEC, and its standard error is computed using the delta method. The overidentification test of the instruments $Z_t = [\ln Y_t, \ln W_t^{no}, \ln W_t^o, V_t^w, V_t^{no}, V_t^o]$ is shown in the lower table. The critical value of the chi-square distribution with 39 degrees of freedom at the 5% significance level is 54.572.

4.1.1. *OPEC as the dominant firm*

4.1.1.1. Price elasticity of oil demand. The crude oil demand elasticity is estimated to be −0.35 (with standard error of 0.02).[7] It is not easy to compare this estimate with previous studies because these are based on different data, techniques and periods: all these factors may lead to different estimates. Early studies by Cooper (2003), Dahl and Yücel (1991), Gately and Huntington (2002) and Dees et al. (2007), among others, relied on OLS and reported statistically significant long-run price elasticities in the range of −0.3 to −0.6. Gately and Huntington (2002) estimate a single demand equation for the 1971–1997 period, allowing the demand response to a price increase to differ from the demand response to a price decrease. In their most preferred empirical specification, the long-run price elasticity for oil consumption per capita (due to a price increase) was estimated to be −0.64 for the group of OECD countries and −0.18 for non-OECD countries. Among non-OECD countries with a steady growth in per-capita income, the long-run price elasticity was estimated to be −0.12.

Similar to our paper, Alhajji and Huettner (2000) also use instrumental variable techniques to estimate the dominant firm model. In their paper, the OPEC price-setting Eq. (13) is omitted, thereby, de facto treating OPEC's production as exogenous. They obtain an estimate of the demand elasticity of −0.25. The difference between their estimate and our benchmark estimate (−0.35) reflects i) different data, and ii) a different estimation framework. First, Alhajji and Huettner (2000) use OECD demand data (not world demand data like we do), quarterly data for the 1973–1994 period (not 1986–2016 like we do) and their data for cost of production for OPEC and non-OPEC differs from what we use. Second, by omitting the OPEC price-setting equation in their estimation, they do not take into account the effect of the endogenous variables on OPEC's markup. To illustrate the importance of the estimation strategy, we have reestimated our model equation by equation. With OLS, the estimated demand elasticity is 0.01 (0.00), whereas we obtain 0.00 (0.00) with IV (when the same instruments as in the benchmark case are used). These results clearly show the importance of specifying the market structure.

4.1.1.2. Income elasticity. We obtain an income elasticity of 1.15 (the standard error is 0.12). Most previous studies report an income elasticity that is less than one (Dahl and Yücel (1991), Alhajji and Huettner (2000), Brook et al. (2004), Griffin and Schulman (2005), and others). Gately and Huntington (2002) also estimate income elasticities for the 1971–1997 period. Similar to the price elasticity, they allow for asymmetric responses to a change in income, that is, the income elasticity related to a rise in income might differ from the one associated with a decrease in income. They estimate the income elasticity (in the case of higher income) to 0.56 for the group of OECD countries, as opposed to 0.53 for the group of non-OECD countries. For non-OECD countries with a steady growth in per-capita income, the long-run income elasticity was estimated to 0.95. The reasons we obtain a higher estimate of the income elasticity than other studies could be because i) we estimate a simultaneous structural model; ii) our specification of the demand function may differ from other studies, for example, with respect to lag structure (see the discussion in Section 5); and iii) our data period differs from the others.

4.1.1.3. Non-OPEC supply. For non-OPEC producers, we obtain a supply elasticity of 0.32 (the standard error is 0.03), meaning that a 1% increase in the crude oil price will increase extraction from the non-OPEC producers by 0.32%. There are not many estimates of the non-OPEC supply elasticity in the literature. One exception is the Alhajji and Huettner (2000) study that obtained 0.29, which is close to our result. Turning to the factor price supply elasticity of non-OPEC, our estimate is −0.76 (the standard error is 0.39) that is, a 1% increase in the unit cost of extraction leads to a slightly smaller reduction in non-OPEC production.

4.1.1.4. OPEC price-setting equation. We estimate a marginal cost elasticity for OPEC $\left(\frac{\partial \ln MC^o}{\partial \ln Q^o}\right)$ of 1.55; this estimate is insignificant at the 5% level of significance (the standard error is 0.90), but significant at the 10% level. We can examine whether the marginal cost elasticities of OPEC and non-OPEC differ: a simple one-sided *t*-test suggests that at the 5% significance level, the marginal cost elasticity is larger for non-OPEC than for OPEC. This could be due to competitive advantages because reserves are more accessible and cheaper to exploit in OPEC than in non-OPEC countries.

The imprecise estimate of the marginal cost elasticity of OPEC may reflect omitted explanatory variables or poor data. For example, although the data on OPEC cost of production cover exploration, extraction and production, they may not adequately reflect the geology of the oil fields, such as the costs of new fields relative to the costs of fields under extraction. Alternatively, the insignificant marginal cost elasticity may reflect a serious misspecification because the model does not allow for dynamic behavior; for example, a higher OPEC capacity may be taken as a signal by non-OPEC producers of a permanent increase in future OPEC production. Such a signal may trigger a change in the non-OPEC extraction path, which may cause a response by OPEC.

[7] The standard errors are the Newey-West standard errors, which correct for heteroskedasticity and serial correlation in the coefficient covariance matrix. In the estimation, we employ three lags to estimate the Newey-West standard errors.

The OPEC factor price elasticity $\left(\frac{\partial \ln MC^o}{\partial \ln W^o}\right)$ is estimated to 1.52 (the standard error is 0.25). If OPEC production increases, then, *ceteris paribus*, the market price will fall, which would lower non-OPEC production, thereby, modifying the initial price reduction. We call this the *equilibrium* elasticity of OPEC production $\left(\frac{\partial \ln P}{\partial \ln Q^o}\right)$, and it is straightforward to identify it in our framework: our estimate is −0.79 (the standard error is 0.05).[8] The estimate of the market power index λ is 0.66, which is clearly above zero. Moreover, the market power index estimate is sharply estimated — its standard error is only 0.04.[9] These results suggest that OPEC exerts market power; we return to this issue in Section 4.3.

Finally, using our estimated parameters, we find that OPEC's markup, see Eq. (7), varies between 2.3 and 8.1 with a mean of 5.3, that is, far above one.[10]

4.1.2. OPEC as a competitive supplier

We now turn to the estimation of the competitive model: by comparing the benchmark model with the competitive model, we can quantify the misspecification bias induced by not accounting for OPEC taking into consideration that non-OPEC supply depends on OPEC's level of production, see Eq. (5). The competitive model is estimated using 3SLS.

4.1.2.1. Demand. As seen from the last column in Table 1, the demand elasticity has the wrong sign, but it is small and insignificant; 0.003 (0.004) versus −0.35 (0.02) in the benchmark case.[11] In the competitive model, the estimated income elasticity is 0.54 (0.01), which is much smaller than the 1.15 estimate in the benchmark case. This suggests that not accounting for the non-competitive market structure in the specification of the econometric model leads to biases in the estimates of the demand and income elasticities.

4.1.2.2. Non-OPEC supply. The supply elasticity of non-OPEC is estimated to 0.08 (0.02), which is smaller than in the dominant firm model (0.32). The factor price elasticity of non-OPEC is alarming; it has the wrong sign (0.39) and the estimate is significant (the standard error is 0.11).

4.1.2.3. OPEC supply. When OPEC is assumed to act competitively, its estimated supply elasticity is 0.19 (0.08), which is small but somewhat higher than the supply elasticity of non-OPEC (0.08). The factor price elasticity of OPEC is insignificantly different from zero.

In summary, the insignificant factor price elasticity of OPEC, as well as the insignificant demand elasticity, should cast doubt about the empirical relevance of the competitive oil price model. In the remaining part of the paper, we, therefore, focus on the dominant firm model.

4.2. Fit of the dominant firm model

Using the estimated parameters of the dominant firm model, we evaluate the fit of the model using the exogenous variables for the 1986–2016 period. Then, we perform two counterfactual experiments to explore the relative importance of income and cost when explaining the long-run trends of price and quantities.

4.2.1. In-sample prediction

Fig. 5 shows the in-sample prediction of the dominant firm model. In general, the model tracks the main trends in the market reasonably well, but understandably misses some deviations from the trend.

- The dominant firm model is able to predict the decline in world oil consumption in 2009, as well as the recovery from 2010.
- The model also has some success in predicting the trend in non-OPEC supply. In particular, it predicts an increase in non-OPEC supply after 2012. This event coincides with an increase in extraction of light tight oil in the United States.
- However, the model does not capture abrupt changes in the oil price, for example, the sudden fall in 2014. The model is built to capture long-run trends, and, therefore, will have trouble predicting short-term dynamics.

We now examine which exogenous factor — world GDP or cost of oil production — that contributes most to the trends in oil consumption, non-OPEC supply, and the oil price predicted by the benchmark model.[12] Each panel in Fig. 6 shows three curves. The solid curves are the predicted paths of quantities and prices, obtained by using the estimates of the benchmark model and the paths of *all* exogenous variables. The two other curves are derived from counterfactual experiments. First, we set the level of world GDP to be constant over time (equal to the 1997:Q1 level), and use the benchmark model's estimates and the paths of all *other* exogenous variables to predict the evolution of the endogenous variables. Second, we set the cost of oil production to be constant over time (equal to the 1997:Q1 levels), and use the benchmark model's estimates and the paths of all *other* exogenous variables to predict the evolution of the endogenous variables.

As seen from Fig. 6, keeping GDP constant at its 1997:Q1 level has a large impact on all variables. Consumption and non-OPEC production remain roughly constant and even fall after 2005. Remarkably, most of the predicted increase in the oil price during the last part of the data period is due to higher income: if world GDP had stayed at its 1997:Q1 level, then, according to the model, the oil price in 2016 would have been roughly 15% above the 1997 price, whereas the predicted 2016 oil price when world GDP is *not* kept constant is roughly 120% above the 1997 price, see panel (c) in Fig. 6. Finally, from panels (b) and (c), we see that cost of oil production has contributed to a higher oil price in the last six years.

To summarize, the path of world GDP explains most of the increase in the oil price between 1986 and 2016. Increased cost of production has, however, contributed to the increase in the oil price during the last six years. Note that a similar conclusion was found in Smith (2009), who also examined the importance of demand and supply factors for the long-run behavior of the oil price, albeit using a somewhat different method than we do.

[8] Notice that

$$\frac{\partial \ln P}{\partial \ln Q^o} = \frac{1}{\epsilon^o} = \frac{s^o}{\alpha_1 - \beta_1(1 - s^o)}.$$

The equilibrium elasticity is evaluated at the mean of the OPEC market share s^o. The standard error is computed using the delta method. Note that $\hat{\epsilon}^o = -\frac{1}{0.76} < -1$ at equilibrium.

[9] The market power index λ is evaluated at the mean of the OPEC market share s^o. The standard error is computed using the delta method.

[10] Recall that in our estimation, we have imposed that the markup is strictly positive. Our point estimates clearly meet this restriction.

[11] The estimate of the demand elasticity in the competitive model can be compared with Krichene (2006), who estimates a simultaneous equations model for world crude oil demand and competitive oil supply. Krichene applies the two-stage least-square method to estimate short-run elasticities, and error-correction methods to estimate the long-run demand elasticity using annual data from 1970 to 2005. He finds the demand elasticity to vary across countries, ranging from −0.03 to −0.08, which roughly resembles our result for the competitive model; namely, no price effect on demand.

[12] There are clear limitations of the present analysis because we use a partial equilibrium framework. First, GDP and cost of oil production are likely to be dependent on each other. In addition, GDP may be affected by the price of oil, for example, as modeled by Hassler et al. (2012).

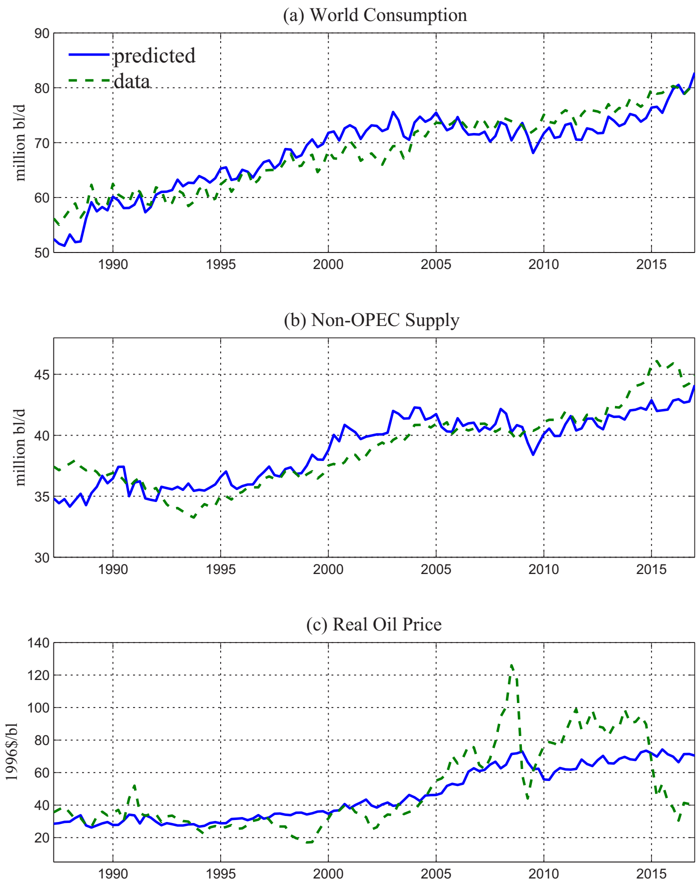

Fig. 5. In-sample prediction for the dominant firm model.
Notes: Panels (a)–(c) plot the in-sample prediction of world consumption, non-OPEC supply, and the real oil price (1996 USD) for the dominant firm model.

4.3. OPEC's market power

We have documented that OPEC's market power index is high — the estimate of λ is 0.66 at the mean value of the market share (the standard error is 0.04), see Table 1. This clearly suggests that OPEC has market power. A simple approach to assess the market power of OPEC is to calculate the standard Lerner index L_t. Using Eq. (8), we find

$$L_t \equiv \frac{P_t - MC^o}{P_t} = \frac{\lambda_t s_t^o}{-\epsilon}.$$

The Lerner index had a positive trend between 1986 (61%) and 1998 (86%). This trend is entirely driven by changes in OPEC's market share because, in our model, the elasticities are constant. Note that $L_t = \frac{1}{-\epsilon^o} = -\frac{\partial \ln P}{\partial \ln Q^o}$, that is, the absolute value of the OPEC production elasticity increased over time in this period. After 1998, the Lerner index varied between 74% and 88%. For the entire 1986–2016 period, the average Lerner index was 79%.

Is it possible to test whether OPEC has market power? Here, there is a fundamental problem because the dominant firm model does not nest the competitive case; we have *assumed* that OPEC is a dominant firm that takes into consideration how the fringe responds to its production decisions, as shown in Eq. (5). Hence, λ = 0 is not defined in our dominant firm model.

We can, however, compute confidence intervals for the market power index, which will give information about OPEC's degree of market power, in particular how far the market power index is from zero. Because the market power index is nonlinear in the parameters and is not defined at zero, we rely on bootstrap methods

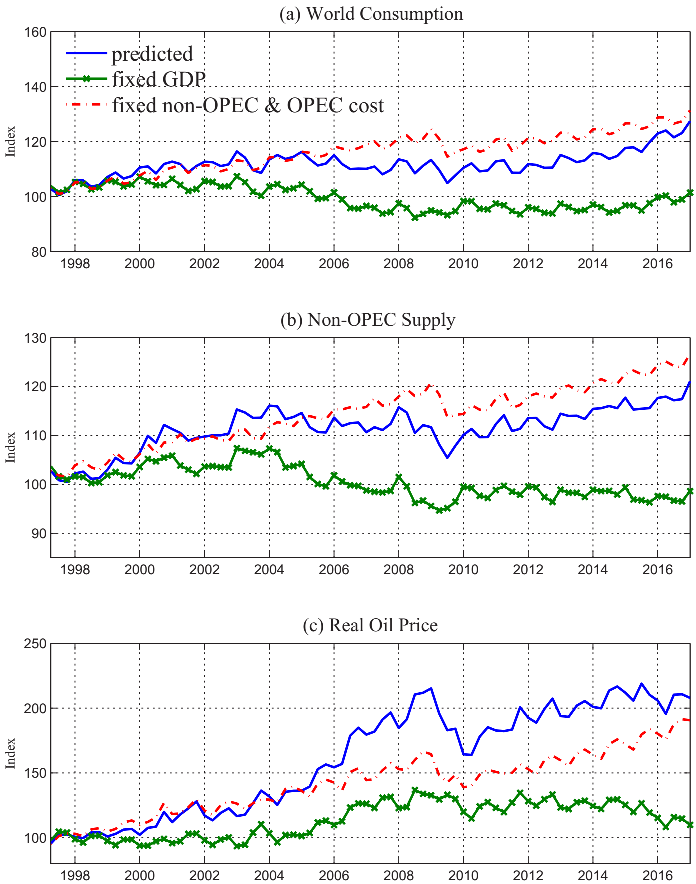

Fig. 6. In-sample prediction for the dominant firm model with constant GDP or constant cost of oil production.
Notes: Panels (a)–(c) show the in-sample prediction for world consumption, non-OPEC supply, and the real oil price (2010 USD) for the dominant firm model in different scenarios. The solid line represents the in-sample prediction with all covariates in the model. The cross-solid line represents the in-sample prediction with fixed world GDP at the 1997:Q1 level. The dot-dash line represents the in-sample prediction with fixed costs of oil extraction in OPEC and non-OPEC at the 1997:Q1 levels. All series are normalized such that their 1997 values are equal to 100. The figure plots the bootstrap 99th percent confidence intervals using percentiles from the empirical sampling distribution of the market power index $\widehat{\lambda}$. We use a resampling method of the residuals to generate bootstrap data. Then, we estimate the dominant firm model and compute λ in each repetition. The number of bootstrap repetitions is 10,000.

to compute its sampling distribution. In particular, we compute confidence intervals using quantiles from the empirical sampling distribution.

First, we use re-sampling methods for the residuals to generate bootstrap data. In each iteration j, $j = 1, \ldots, 10{,}000$, we keep the exogenous variables fixed as in the data, and recompute the endogenous variables $[\ln Q_t^w, \ln Q_t^{no}, \ln P_t]$. Then, for each iteration we estimate the model and use Eq. (14) to compute $\widehat{\lambda}_j^*$ (* denotes the estimate from the bootstrap process); see Appendix B for more details. The set of all $\widehat{\lambda}_j^*$ is the empirical distribution of $\widehat{\lambda}$. Finally, we construct the 99th percentile confidence interval (one for each year) using the bootstrap sampling distribution of $\widehat{\lambda}$. Fig. 7, which shows the confidence intervals for the market power index, reveals a significant degree of OPEC market power. In particular, for the entire sample period, the 99th percentile confidence intervals are well above zero.

To *test* whether OPEC has market power, we compare our benchmark dominant firm model with the alternative competitive model. To this end, we use the non-nested statistical test of Smith (1992) for competing models that are estimated by the generalized method of moments, see Appendix C. Here, we find that there is no evidence to reject the dominant firm model against the competitive model. In addition, we find strong evidence in favor of rejecting the competitive model against the dominant firm model. These results

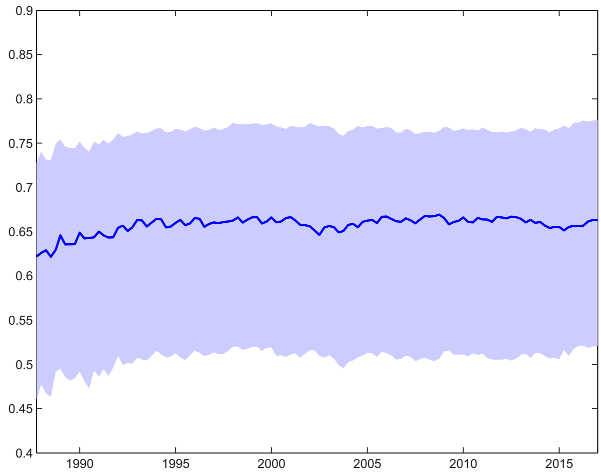

Fig. 7. Bootstrap 99th percent confidence intervals of the market power index.
Notes: The figure plots the bootstrap 99th percent confidence intervals using percentiles from the empirical sampling distribution of the market power index $\widehat{\lambda}$. We use a resampling method of the residuals to generate bootstrap data. Then, we estimate the dominant firm model and compute λ in each repetition. The number of bootstrap repetitions is 10,000.

lend support to the dominant firm model, and, thus, that OPEC exerted market power in the period, 1986–2016.

5. Further analysis

We now examine how different econometric specifications and data may change our estimates. First, we explore the robustness of our estimates when we allow for different numbers of lags. Second, we investigate how the estimates vary between subperiods. This may shed light on parameter shifts due to structural changes in demand and supply. Third, we study the impact of using the consumer price of oil instead of the producer price of oil in the demand function. Fourth, we check whether the estimates change when we use alternative cost data for the non-OPEC countries or an alternative definition of OPEC membership. Finally, we assume that only a few OPEC countries — OPEC core — exert market power, whereas all other OPEC members are de facto price takers.

5.1. Lags

It is standard to include lags in oil market studies; for example, Hansen and Lindholt (2008) use 18 lags (monthly data) whereas Kilian (2009) uses 12 lags in his VAR model (quarterly data). Below we, therefore, discuss the estimates of the dominant firm model under alternative assumptions about the number of lags.

In the benchmark case, we used three lags. Table 2 shows that the estimated coefficients for demand, non-OPEC supply and the

Table 2
Estimates using different numbers of lags: the dominant firm model.

No.lags		0 lag	3 lags	8 lags	12 lags
q		Static	Benchmark		
World demand					
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.295(0.019)	−0.352(0.018)	−0.350(0.018)	−0.367(0.020)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.040(0.082)	1.154(0.117)	1.197(0.128)	1.243(0.150)
Non-OPEC supply					
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.325(0.036)	0.322(0.034)	0.300(0.032)	0.293(0.036)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−0.797(0.361)	−0.758(0.372)	−0.579(0.363)	−0.543(0.375)
OPEC supply					
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	7.681(16.009)	1.545(0.904)	0.063(0.855)	−1.264(0.740)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	0.424(3.291)	1.516(0.247)	1.749(0.228)	1.944(0.225)
λ		0.613(0.041)	0.655(0.035)	0.670(0.035)	0.686(0.038)

Notes: We use quarterly data for the period 1986:Q1–2016:Q4; the heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates of elasticities and the market power index λ for the dominant firm model using different numbers of lags for the lagged differences of (log of) the exogenous variables V_t^w, V_t^{no} and V_t^o in Eqs. (10), (11) and (13).

market power index are robust with respect to the number of lags. For example, the estimate of the demand elasticity varies between −0.30 (no lag) and −0.37 (12 lags), and the non-OPEC supply elasticity varies between 0.33 (no lag) and 0.29 (12 lags). On the other hand, the estimated OPEC parameters are sensitive to the lag specification. For example, the OPEC factor price elasticity varies between 7.68 (no lag) and 0.06 (8 lags), and the marginal cost elasticity of OPEC becomes negative with 12 lags. (This elasticity is not significantly different from zero for any lag specification, except the benchmark case.) The lack of stability may be due to no dynamic behavior in the model; see the discussion above.

5.2. Data period

In our estimations, we assumed constant parameter values over the data period, 1986:Q1–2016:Q4. This may be a strong assumption because of structural changes in demand and supply. For example, over time a higher share of crude oil has been used in the transportation sector, which, according to several studies, has a lower demand elasticity than other oil-consuming sectors, such as the manufacturing industry and power generation. Similarly, rapid growth in some Asian countries has increased this region's share of global oil consumption — these countries may have a different demand structure than OECD countries, and also a higher income elasticity of oil, see the discussion related to Fig. 2. The energy and environmental policy in OECD countries, and also discoveries of unconventional petroleum deposits in non-OPEC countries, may have a powerful impact on OPEC's ability to act as a profit-maximizing cartel, and, thus, on total OPEC production.

To investigate the variation in the parameters across periods, we divide the data period into two subperiods, 1986–2000 and 2001–2016, and estimate the benchmark model separately for each of these subperiods, see Table 3. When splitting the original time period into two subperiods, the two estimates of the demand elasticity do not differ much (−0.42 versus −0.38) and they are close to the benchmark estimate (−0.35). On the other hand, for the income elasticity, the difference in the subperiod estimates is large; 0.18 (1986–2002) versus 1.23 (2001–2016). The 2001–2016 estimate probably mirrors the rapid growth of China and India in this period.

For non-OPEC, the supply elasticity is robust with respect to the estimation period, whereas the factor price elasticity is either insignificant (1986–2000) or has the wrong sign (2001–2016). For OPEC, the factor price elasticity is robust with respect to the sample period, whereas the marginal cost elasticity is insignificant (and has the wrong sign) in each subperiod. The latter result indicates that the model is too simple to represent the dynamics of the oil market, at least within a period of 15 years.

Table 3
Estimates for different time periods: the dominant firm model.

Periods		1986–2016	1986–2000	2001–2016
		Benchmark		
World demand				
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.018)	−0.422(0.046)	−0.379(0.022)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.154(0.117)	0.181(0.256)	1.228(0.656)
Non-OPEC supply				
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.322(0.034)	0.238(0.091)	0.271(0.040)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−0.758(0.372)	0.123(0.692)	−0.558(0.269)
OPEC supply				
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.545(0.904)	−0.523(1.296)	−0.561(0.987)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.516(0.247)	1.402(0.685)	1.413(0.451)
λ		0.655(0.035)	0.751(0.091)	0.714(0.041)

Notes: We use quarterly data; the heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates of elasticities and the market power index λ for the dominant firm model — Eqs. (10), (11) and (13).

5.3. The consumer price of oil

In our analysis, we have used the crude oil price as an explanatory variable for both oil producers and oil consumers; this is standard in the literature. However, consumers in OECD countries typically face a higher price of oil than producers; the difference reflects costs (and profits) of refineries, costs (and profits) of transport of crude oil and oil products, and taxes (value added, energy and environmental taxes, etc.). However, in several non-OECD countries, oil products are subsidized. In this subsection, we first construct a global consumer price for oil products, and then estimate the model using this consumer price as the explanatory variable in the demand function. By using the consumer price of oil products as an explanatory variable, our estimated demand elasticity becomes comparable to demand elasticities that are obtained from other econometric studies.

To calculate a global end-user price of oil, we use data from Energy Prices and Taxes and Energy Balances of non-OECD countries (which also contain OECD data) from the IEA. Because Energy Prices and Taxes has not published detailed end-user prices for oil products after 2010, our series for the consumer price covers the period 1996–2010. Finally, the consumer price data are annual, and we, therefore, transform this series into a quarterly series. To this end, we assume that in each year, the quarterly changes in the consumer price are identical to the quarterly changes in the crude oil price.

Table 4 shows the estimates when we use the world consumer price of oil (instead of the crude oil price — PPI) as the explanatory variable in the demand function. For some elasticities, the change is small; the demand elasticity is now −0.24 (−0.35 if we use the PPI as the explanatory variable in the demand function), whereas, for other parameters, the change is larger; the non-OPEC supply elasticity is 0.53 (0.36 if we use the PPI), and the market power index is 0.44 (0.63 if we use the PPI). To sum up, overall the estimates are moderately affected by using the global consumer price of oil products in the demand function (instead of the crude oil price).

5.4. Alternative cost data for non-OPEC

In the benchmark estimation, we used the US PPI index for oil and gas field machinery and equipment costs as a proxy for cost

Table 4
Estimates using the consumer price of oil: the dominant firm model.

Prices		Crude oil price	Consumer price
		1986–2010	1986–2010
World demand			
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.022)	−0.239(0.033)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.119(0.149)	1.181(0.173)
Non-OPEC supply			
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.357(0.040)	0.530(0.055)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−1.237(0.340)	−2.024(0.498)
OPEC supply			
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.932(1.015)	1.860(1.084)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.373(0.266)	1.356(0.274)
λ		0.631(0.040)	0.438(0.049)

Notes: We use quarterly data for the period 1986:Q1–2009:Q4. The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates of elasticities and the market power index λ for the dominant firm model using either the crude oil price or the world consumer price as an explanatory variable for demand for oil.

Table 5
Estimates using alternative non-OPEC cost data: the dominant firm model.

Prices		PPI	PPI	Oil equipment cost
		1986–2016 benchmark	1986–2009	1986–2009
World demand				
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.022)	−0.367(0.020)	−0.212(0.032)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.119(0.149)	1.077(0.161)	0.847(0.096)
Non-OPEC supply				
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.357(0.040)	0.344(0.035)	0.671(0.068)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−1.237(0.340)	−1.175(0.402)	−1.673(0.323)
OPEC supply				
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.932(1.015)	1.857(1.049)	2.397(0.963)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.373(0.266)	1.332(0.261)	1.242(0.250)
λ		0.631(0.040)	0.648(0.034)	0.353(0.057)

Notes: We use quarterly data for the period 1986:Q1–2009:Q4. The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates of elasticities and the market power index λ for the dominant firm model using either the PPI oil and gas field machinery and equipment or the oil lease equipment cost for non-OPEC cost of production in the periods 1986–2016 and 1986–2009.

Table 6
Estimates using alternative definitions of OPEC: the dominant firm model.

Def. of OPEC		OPEC original	OPEC with entry & exit
		Benchmark	
World demand			
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.018)	−0.344(0.021)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.154(0.117)	1.140(0.116)
Non-OPEC supply			
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.322(0.034)	0.326(0.036)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−0.758(0.372)	−0.864(0.364)
OPEC supply			
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.545(0.904)	0.805(1.217)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.516(0.247)	1.410(0.326)
λ		0.655(0.035)	0.642(0.038)

Notes: We use quarterly data for the period 1986:Q1–2016:Q4. The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. The table reports estimates of elasticities and the market power index λ for the dominant firm model using two alternative definitions of OPEC membership: 1) current status, see EIA (2017); and 2) adjustment for entry and exit, see OPEC (2017) .

of production for the non-OPEC countries. We used this quarterly series because it is available for the whole sample period. The PPI has three major weaknesses. First, while economic theory suggests to use data for *marginal* cost, the PPI measures *average* cost. Second, the PPI is constructed for the US industry, and, hence, it does not cover other countries. Needless to say, costs may vary significantly between non-OPEC countries. Ideally, we would like to use country-specific indices. Third, the PPI may not capture important shifts in the cost structure of non-OPEC producers. To explore the sensitivity of the cost data, we reestimate our model using the US oil lease equipment cost from EIA (2010), which is available for the period 1986–2009. This annual series is transformed into a quarterly series, assuming that, in each year, it has the same quarterly pattern as the PPI.

Note that the cost data in EIA (2010) indicate that the US PPI index has underestimated the increase in average cost of producing oil in the period 2003–2009. According to the EIA report, fluctuations in US oil production equipment costs in 2003–2009 are mainly due to an increase in steel prices, reflecting increased demand from China.

The empirical results are shown in Table 5. The first column shows the benchmark. Here, the PPI is used for non-OPEC cost and the sample period is 1986–2016. The second column shows the estimates when the sample period is 1986–2009 and the PPI is (still) used for non-OPEC cost. Hence, the difference between columns 1 and 2 is the effect of reducing the sample period from 2016 to 2009; this has a negligible effect on most estimates. The last column shows the estimates when the data from EIA (2010) are used for the cost of non-OPEC, and the sample period is 1986–2009. Thus, by comparing columns 2 and 3, we identify the partial effect of replacing the PPI with data from EIA (2010). As seen from Table 5, the price elasticity of demand drops from 0.37 (column 2) to 0.21 (column 3), and the non-OPEC supply elasticity increases from 0.34 to 0.67. These changes imply a lower market power index when the data from EIA (2010) are used; 0.65 vs. 0.35. On the other hand, using 95% confidence intervals, we find that the income elasticity and also the non-OPEC cost elasticity do not differ significantly between the two cases.

5.5. The role of entry and exit from OPEC

In our benchmark, OPEC membership is taken from the EIA database. Here, a country being a member of OPEC in the most current year is considered an OPEC member in *all* previous years. In our benchmark, OPEC members are, therefore, Algeria, Angola, Ecuador, Gabon, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, United Arab Emirates (UAE) and Venezuela (in all years in the period 1986–2016).

Needless to say, over time, some countries enter or exit the group of OPEC.[13] To identify the robustness of our estimates with respect to the definition of OPEC, we have reestimated the model, requiring that, in each year, OPEC consists of those countries that were actually members in that year, see OPEC (2017). We find that demand and non-OPEC parameters are robust to the data adjustment, see Table 6. Hence, the estimate of the market power index hardly changes. For OPEC, the estimate of how higher cost impacts marginal cost is almost unchanged, whereas the estimate of how increased oil extraction impacts marginal cost is decreased.

5.6. OPEC core

So far, we have assumed that OPEC is a coordinated group facing a competitive fringe. However, several papers have pointed out that OPEC countries are heterogeneous and should be analyzed accordingly. Mabro (1998), reviewing the OPEC literature, emphasizes events where the behaviors of OPEC countries have varied. For example, the oil price increase in 1973 was induced by some Arab countries, whereas both Arab Iraq and non-Arab Iran did not join the embargo. The Iraq–Kuwait war caused only a short-run price increase because a few OPEC countries, in particular, Saudi Arabia and UAE, increased their supply to counteract lower production from Iraq and Kuwait.[14]

To explain asymmetric OPEC behavior, Hnyilicza and Pindyck (1976) suggest dividing OPEC countries into two groups — savers and spenders — whereas Eckbo (1976) distinguishes between the price pushers, the expansion fringe, and core OPEC members, the latter group consisting of Saudi Arabia and Kuwait, as well as some other countries. Brémond et al. (2012) use the grouping of Hnyilicza and Pindyck (1976) and conclude that, at least the group of savers, acts as a cartel. Griffin (1985) tests Eckbo's grouping, and concludes that countries within each group do not have similar behavior, but still he

[13] According to OPEC (2017), Angola joined OPEC in 2007; Gabon terminated its membership in 1995, but rejoined in 2016; Ecuador was suspended in 1992, but rejoined in 2007; and Indonesia was suspended in 2009, but rejoined in 2016.

[14] For a study on Saudi Arabia's behavior within OPEC, see Alkhathlan et al. (2014). They conclude that the primary goal of Saudi Arabia is to stabilize OPEC production.

prefers Eckbo's grouping rather than treating OPEC as a monolithic unit. Alhajji and Huettner (2000) estimate a dominant firm model (with the caveats explained above) and conclude that the behavior of the OPEC core countries is not consistent with the dominant firm model. However, Hansen and Lindholt (2008) find evidence that OPEC core countries have acted as a dominant firm after 1994. For comparison with other papers, we now estimate the dominant firm model for the OPEC core countries (Saudi Arabia, Kuwait, Qatar and UAE), assuming that all other oil-producing countries belong to the fringe. Below, this case is referred to as OPEC core.

In Section 2, we explained that we do not have country-specific cost data, only cost of production for an OPEC country and cost of production for a non-OPEC country (both vary over time). We, therefore, assume that cost of production of the new fringe consists of a weighted average of cost of production of a non-OPEC country and cost of production of an (original) OPEC country, with weights equal to the production share of the non-OPEC countries and the production share of the non-core OPEC countries (these shares add up to one).

Table 7 shows the results.[15] For the demand elasticity, the estimate from the OPEC core model with three lags (−0.11) is clearly lower than for the benchmark case (−0.35). Also, the income elasticity and the non-OPEC supply elasticity are lower in the OPEC core model than in the benchmark case. Lower demand and supply elasticities tend to reduce the market power index, see Eq. (14), but, on the other hand, a lower OPEC market share (reflecting fewer OPEC producers) has the opposite effect. As seen from Table 7, the first effect dominates; the market index in the benchmark case (0.66) is almost twice as high as in the OPEC core model (0.35).

To understand why the estimates of the OPEC core model differ from the benchmark estimates, we note that, in the benchmark case, all OPEC members belong to the dominant firm, whereas, in the OPEC core model, the dominant firm consists of four OPEC countries only. Hence, a number of oil-producing countries have changed strategic position from taking into account the response of the fringe to being part of the fringe. This means that, if the dominant firm reduces the oil price, there will be a larger (quantity) response from the fringe in the OPEC core model than in the benchmark case, simply because the fringe is larger in the OPEC core model. For the same reason, the choke price of the residual demand curve is lower in the OPEC core model than in the benchmark case. If, hypothetically, the choke price in the benchmark case is charged by the dominant firm in the OPEC core model, then supply from the fringe would exceed demand. Hence, the residual demand curve shifts downwards (lower choke price) and becomes more price elastic (larger response from the fringe) if the market structure changes to the OPEC core. Standard economic theory then suggests that the degree of market power declines, which is in accordance with our finding.

Table 7 also provides information on whether the estimates are sensitive to the number of lags. When increasing the number of lags from three to eight, most estimates hardly change. This is similar to the results obtained for the benchmark model, see the discussion related to Table 2.

We also provide some information on the sensitivity of the results with respect to the sample period and the price variable in the demand function.[16] For the subperiod, 1986–2000, the demand elasticity is −0.26 (compared with −0.11 for the period 1986–2016), whereas the income elasticity is as low as 0.32 (0.73 for the period 1986–2016). In contrast, for the subperiod, 2001–2016, the demand elasticities, as well as the non-OPEC supply elasticities, are rather similar to the ones for the whole period (1986–2016). Finally, when using the global consumer price of oil products as the explanatory variable in the demand function, demand and non-OPEC supply elasticities do not change much (in absolute terms).

Table 7
Estimates for OPEC and OPEC core as the dominant firm.

Dominant firm		OPEC original definition 3 lags Benchmark	OPEC core 3 lags	OPEC core 8 lags
World demand				
$\frac{\partial \ln Q^w}{\partial \ln P}$	α_1	−0.352(0.018)	−0.107(0.014)	−0.111(0.014)
$\frac{\partial \ln Q^w}{\partial \ln Y}$	α_2	1.154(0.117)	0.733(0.040)	0.756(0.048)
Non-OPEC supply				
$\frac{\partial \ln Q^{no}}{\partial \ln P}$	β_1	0.322(0.034)	0.242(0.021)	0.225(0.019)
$\frac{\partial \ln Q^{no}}{\partial \ln W^{no}}$	β_2	−0.758(0.372)	−0.302(0.180)	−0.218(0.180)
OPEC supply				
$\frac{\partial \ln MC^o}{\partial \ln Q^o}$	π_1^d	1.545(0.904)	−0.106(0.997)	−0.966(0.558)
$\frac{\partial \ln MC^o}{\partial \ln W^o}$	π_2^d	1.516(0.247)	1.625(0.159)	1.788(0.188)
λ		0.655(0.035)	0.354(0.047)	0.381(0.049)

Notes: We use quarterly data for the period 1986:Q1–2016:Q4. In OPEC core, the dominant firm consists of Kuwait, Saudi Arabia, UAE and Qatar. Other countries that are OPEC members are assumed to be price takers. Cost of production of the new fringe is a weighted average of cost of production of non-OPEC members and cost of production of the original OPEC members, with weights equal to production of non-OPEC countries and production of non-core OPEC countries relative to total fringe production. Column 2 uses three lags in the dominant firm model, and column 3 uses eight lags in the dominant firm model. The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis.

6. Conclusions

Oil prices have changed dramatically over the last decade. Since the work of Griffin (1985), different studies have tested a variety of market structures using different econometric techniques, data and models. The results have been mixed, with estimated parameters not being robust to the specification of the model or the sample period, or simply insignificant. In particular, the demand elasticity has proven difficult to estimate reliably.

In this paper, we estimate a parsimonious dominant firm model for the global crude oil market. Non-OPEC countries act as a competitive fringe, whereas OPEC is envisioned to be a dominant firm, setting its price as a markup over marginal cost. The model is estimated using a system of three equations with OPEC's price response being nonlinear (in logs).

We find significant estimates for most of the long-run parameters of the model. In particular, significant demand and non-OPEC supply elasticities allow us to measure the degree of OPEC's market power. We find evidence that OPEC exerted substantial market power between 1986 and 2016, the period analyzed in this paper. Then, using the same data, but assuming that OPEC is a competitive producer, we reestimated the model and compared its fit to the data. We find that the competitive model does not capture the specific characteristics of the global oil market. In particular, significant demand and supply elasticities are not obtained. We conclude that the linear (in logs) competitive system may lead to a misspecification bias. Using the parameters of the dominant firm model, we show that world GDP has been the main driving force of oil prices over the last

[15] As can be seen from Table 7, the results of our benchmark have a slightly better fit than the case referred to as OPEC core. This is, in particular, the case for the marginal cost of OPEC with respect to quantity, thereby, justifying our choice of benchmark. In addition, we are not aware of parameter estimates for OPEC core; it seems reasonable to pick a benchmark that can be compared with other papers.

[16] More detailed results can be obtained from the authors upon request.

three decades. However, rising production costs have contributed to an increase in oil prices after 2004.

The results in this paper suggest some avenues for further research. First, we used a static model augmented by dynamic factors (lag structure) and lagged OPEC capacity. For the dominant firm model, we find that the estimated elasticity of marginal cost of OPEC with respect to lagged OPEC capacity is significant. Therefore, a dynamic approach to understand the role of capacity seems a natural step.

One strand of the literature, which builds on Hotelling (1931), singles out resource depletion as the dynamic factor to explain the path of oil prices. However, attempts to explain long-run prices by focusing on resource scarcity (see, for example, Lin, 2010, Pindyck, 1978; and Jovanovic, 2013) have had limited success, which may reflect the fact that the size of oil reserves has not changed much over the last 30 years — new discoveries have compensated for current extraction (Smith, 2009). An alternative strategy to incorporate resource depletion would be to add dynamics to demand—because of financial or inventory speculation[17]—or dynamics to supply—because of a game between OPEC and non-OPEC where producers (also) choose investment in extraction capacity. This would add persistency and volatility to prices, thereby, providing a foundation for the model to account for the big swings in prices after 2000.[18] Note, however, that a game between OPEC and non-OPEC may require detailed data on costs of production for each OPEC country; we have no access to such data.

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Appendix A. Estimation methods

When OPEC is assumed to be a price taker, the moment condition function $\mathbf{g}(\theta^c)$ is defined as

$$\mathbf{g}(\theta^c) = \begin{bmatrix} Z_t\,(\ln Q_t^w - \mathbf{X}^{w\prime}\boldsymbol{\alpha}) \\ Z_t\,(\ln Q_t^{no} - \mathbf{X}^{no\prime}\boldsymbol{\beta}) \\ Z_t\,(\ln Q_t^o - \mathbf{X}_c^{o\prime}\boldsymbol{\pi}^c) \end{bmatrix},$$

where $\mathbf{X}^w$, $\mathbf{X}^{no}$ and $\mathbf{X}_c^o$ are vectors of the right-hand side variables in Eqs. (10), (11) and (12), respectively. When estimating this system of three equations, we use the three-stage least-squares (3SLS) method. The parameter estimates $\widehat{\boldsymbol{\theta}}^c$ are obtained by solving

$$\widehat{\boldsymbol{\theta}}^c = \arg\min_{\theta^c} \mathbf{g}(\theta^c)'\widehat{W}\mathbf{g}(\theta^c),$$

where the weighting matrix $\widehat{W}$ is evaluated at $(Z'Z)^{-1}$ in the first step and at $(Z'\widehat{\mathbf{u}}\widehat{\mathbf{u}}'Z)^{-1}$ in the second step, and $\widehat{\mathbf{u}} = \left[\widehat{u}_t^w, \widehat{u}_t^{no}, \widehat{u}_t^o\right]$. Because this model is linear, the system general method of moments (GMM) estimation is equivalent to 3SLS estimation.[19]

When estimating the dominant firm model, we use the system nonlinear instrumental variable (NLIV) method with the moment condition function

$$\mathbf{g}(\theta^d) = \begin{bmatrix} Z_t\,(\ln Q_t^w - \mathbf{X}^{w\prime}\boldsymbol{\alpha}) \\ Z_t\,(\ln Q_t^{no} - \mathbf{X}^{no\prime}\boldsymbol{\beta}) \\ Z_t\left(\ln P_t - \ln m\,(\alpha_1, \beta_1, s_t^o) - \mathbf{X}_d^{o\prime}\boldsymbol{\pi}^d\right) \end{bmatrix},$$

where $\mathbf{X}_d^o$ is the vector of the right-hand side variables in Eq. (13) except the markup.[20] The weighting matrix is evaluated at $(Z'Z)^{-1}$.[21]

Appendix B. Construction of confidence intervals

We compute confidence intervals by implementing the following steps:

1. Bootstrap data-generating process
   In this step, we resample the residuals to generate bootstrap data, that is, we hold the exogenous variables fixed, but make the endogenous variables $\left[\ln Q_t^w, \ln Q_t^{no}, \ln P_t\right]$ equal to the expected values $\left[\ln \widehat{Q}_t^w, \ln \widehat{Q}_t^{no}, \ln \widehat{P}_t\right]$ plus a resampled residual $u_j^* = \left[u_{tj}^{w*}, u_{tj}^{no*}, u_{tj}^{o*}\right]$. For the jth repetition, we use the empirical distribution of the predicted errors (Fox, 2008; MacKinnon et al., 2009):

$$\left[\ln Q_t^{w*}, \ln Q_t^{no*}, \ln P_t^*\right] = \left[\ln \widehat{Q}_t^w, \ln \widehat{Q}_t^{no}, \ln \widehat{P}_t\right] + u_j^*, \quad u_j^* \sim EDF\,(\widehat{u}_t)$$

   (* denotes bootstrap data). When we use this method, we rely on the regression model to obtain the correct conditional expectation, but we do not use the empirical distribution of

[17] Kilian and Murphy (2014) develop a structural model for the global crude oil market and estimate demand elasticities when changes in oil inventory are taken into account. Their results suggest that the 2003–2008 oil price surge was not due to speculative trading.

[18] Another potential source of volatility are OPEC announcements about, for example, changes in production. For studies on how OPEC announcements may have impact on oil prices, see, for example, Lin and Tamvakis (2010), Schmidbauer and Rösch (2012) and Loutia et al. (2016).

[19] We have

$$\widehat{\theta}_{3SLS}^c = \left\{X'\left[\widehat{\Omega}^{-1} \otimes Z(Z'Z)^{-1}Z'\right]X\right\}^{-1}\left\{X'\left[\widehat{\Omega}^{-1} \otimes Z(Z'Z)^{-1}Z'\right]Q\right\},$$

where X =diag($[\mathbf{X}^w, \mathbf{X}^{no}, \mathbf{X}^o]$) and $Q = \left[\ln Q_t^w, \ln Q_t^{no}, \ln Q_t^o\right]$. Further, $\widehat{\Omega} = \frac{\mathbf{u}'\mathbf{u}}{N}$ is the covariance matrix of the residuals from 2SLS where N is the number of observations. The variance matrix can be obtained from

$$\text{cov}\quad\left(\widehat{\theta}_{3SLS}^c\right) = \left\{X'\left[\widehat{\Omega}^{-1} \otimes Z(Z'Z)^{-1}Z'\right]X\right\}^{-1}.$$

[20] Although both the markup and the marginal cost of OPEC are functions of OPEC supply, this should not cause a multicollinearity problem because, by using a moment condition-based method to estimate the model (GMM), we handle the endogeneity of OPEC supply adequately.

[21] The variance of this estimator is given by

$$\text{cov}\,\left(\widehat{\theta}^d\right) = \frac{1}{N}\left(\widehat{G}'\widehat{W}\widehat{G}\right)\left(\widehat{G}'\widehat{W}\widehat{S}\widehat{W}\widehat{G}\right)\left(\widehat{G}'\widehat{W}\widehat{G}\right)$$

where $\widehat{G} = \frac{\partial g(\widehat{\theta}^d)}{\partial \theta^d}$ and $\widehat{S} = \frac{1}{N}\sum Z_i\widehat{u}_i\widehat{u}_i'Z_i$.

the errors. As discussed by MacKinnon et al. (2009), in bootstrap hypothesis testing the data should be resampled under the null hypothesis.

2. We estimate the dominant firm model using bootstrap data and use Eq. (14) to compute $\widehat{\lambda}_j^*$.
3. We construct the 99th percentile interval using the quantiles of the bootstrap sampling distribution of $\widehat{\lambda}^* : \widehat{\lambda}^*_{0.5\%} < \lambda < \widehat{\lambda}^*_{99.5\%}$.

Appendix C. Formal test of OPEC market power

Consider two competing hypotheses. Let $H_d : E[g(\theta^d)] = 0$ be the null hypothesis under the dominant firm model. Similarly, let $H_c : E[g(\theta^c)] = 0$ be the hypothesis under the competitive model. Smith (1992) proposes the following Cox-type statistical test to discriminate H_d against H_c. The Cox-type statistic $b_T(H_d|H_c)$ is computed as

$$b_T\left(H_d|H_c\right) = \widehat{g}_T' \widehat{\Sigma}_c^{-1} \widehat{A}_T T^{\frac{1}{2}} \widehat{h}_T \tag{16}$$

where $\widehat{A}_T = T^{-1} Z' Z \widehat{\Sigma}_d^{-1}$, with moment conditions $\widehat{g}_T = T^{-1} Z' \widehat{u}^c$ and $\widehat{h}_T = T^{-1} Z' \widehat{u}^d$, and variances Σ_d and Σ_c.[22] T denotes the number of observations.

Smith (1992) shows that under H_c the test statistic follows a normal distribution with zero mean and variance ω_d^2 given by

$$\omega_d^2 = \text{plim}[\widehat{g}_T]' \Sigma_c^{-1} A_d M_D' A_d' \Sigma_c^{-1} \text{plim}\,[\widehat{g}_T]\,, \tag{17}$$

where $M_D = I - H\left(H' \Sigma_d^{-1} H\right)^{-1} H' \Sigma_d^{-1}$, $H = E\left[\frac{\partial h_T}{\partial \theta^d}\right]$ and $A_d = I$. This result allows us to test whether there is evidence that our dominant firm model (under H_d) can be rejected against the alternative competitive model (H_c).

Using the estimates from Section 4.1.1 (see Table 1), we compute the Cox-type statistic under the hypothesis H_d using Eq. (16). We obtain a value for the statistic of −0.69. We then compute the standard error using Eq. (17); we find $\omega_d/\sqrt{T} = 0.62$. Under normality the 95% confidence interval under the hypothesis H_d is $[-1.90, 0.52]$. The mean 0 is included in the 95% interval under the hypothesis H_d. Thus, we find that there is no evidence to reject H_d (the dominant firm model) against H_c (the competitive model).

We also test if there is sufficient evidence to reject the competitive model in favor of the dominant firm model.[23] We reverse the hypotheses and let H_c be the null hypothesis under the competitive model and H_d the alternative hypothesis under the dominant firm model. We compute the Cox-type statistic for H_c (under the competitive model) against H_d (under the dominant firm model). The value obtained for the Cox statistic is 16,214 and its standard error is 151.65. The 95% confidence interval under the null hypothesis of the competitive model is [15,917, 16,511], which is far above the mean 0. Thus, we can strongly reject the null hypothesis of the competitive model H_c against the dominant firm model H_d.

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[22] Recall that the variances are $\Sigma_c = var\left(T^{-\frac{1}{2}} Z' u_c\right)$ and $\Sigma_d = var\left(T^{-\frac{1}{2}} Z' u_d\right)$.

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PAPER 2

The Effect of Income Shocks on the Oil Price*

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Abstract

This paper identifies the effect of income shocks on the real price of oil. We find that for the period 1973-2016 shocks to world GDP created a response of a permanent rise in the oil price. In contrast, oil production does not correct the disequilibrium from a stable long-run equilibrium. Whereas shocks to GDP are persistent, shocks to the oil price are mostly transitory once we control for changes in world GDP and oil production. We find evidence of a structural change in the response of the oil price after 1973. We conjecture that the response of oil production is key to the differences.

JEL Classification: *C13, C22, Q02, Q43*

Keywords: *Oil Market, Real Oil Price, Commodity Markets, Cointegration*

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1 Introduction

Since 1973 the oil price has shown high levels of volatility. Alternative explanations have resorted to supply and demand factors and the role of speculation as possible causes explaining the variation in prices. Empirical models have used mainly a stationary framework where shocks are deemed to be mean reverting. However, there is evidence that the oil price can be described by a random walk without drift (Hamilton, 2009). More generally, Dvir and Rogoff (2014) have found cointegrating relationships between the oil price, production and inventory levels using annual data from 1900.

There are good reasons to believe that commodity markets have been influenced by permanent demand shocks. For example, Aguiar and Gopinath (2007) have documented that shocks to the growth rate of technology contribute more to the variability of output in emerging economies than in developed economies. It then seems feasible that the strong patterns of growth in emerging economies have contributed significantly to the demand in the commodity market, especially during the recent boom. If shocks are permanent, it raises the question of whether there is a permanent long-run relationship among the price of crude oil, income and oil production. We provide evidence that this is indeed the case for these three variables using quarterly data from the beginning of the 1970s.

We also find evidence that a long-run relationship in the oil market helps to predict oil prices. Indeed, this result is based on our multivariate error correction framework, and highlights the importance of accounting for long-run trends in the oil markets. In contrast, Hamilton (2009) using a simple univariate autoregression finds that common macroeconomic variables lack predictive power, and that the oil price can be approximated by a random walk. We also find that a permanent shock to world GDP leads to a significant hump-shaped response in oil prices. In contrast, price-demand specific shocks are entirely transitory. In fact, and in line with previous studies, we find that most of the variation in oil prices are derived from transitory shocks. We also provide evidence that the long run relationship does not forecast oil production. We conjecture that oil production has become more inelastic from 1973 onward.

We have extended the analysis to study the oil market using annual data from 1900 to 2016. We found that the long run relationship predicts the change in oil prices. Interestingly, in the early period, we find evidence of predictability for oil production. We conjecture that the differential response of production explains why shocks to income are mostly transitory in the early period.

Finally, we have explored whether world GDP also has an impact on another commodity, namely copper prices, on the period 1900-2016. We find indeed that the long run relationship predicts the copper prices and quantity. As a result, the response of the copper price to a shock to income is mostly transitory.

Our article contributes to three branches of the literature on commodity markets. First, our paper is relevant to the research on time series analysis of the oil price. Most papers have used a stationary framework. Kilian (2009) identifies supply, aggregate demand and specific oil demand shocks using data on world oil production growth, the index of real economic activity based on dry cargo bulk freight rate and real price of oil. He finds that aggregate demand shock generates long-term variation in the real price of oil, and the oil-market specific demand shock creates the sharp rise and drop in the oil price. Later Kilian and Murphy (2014) and Knittel and Pindyck (2016) use similar frameworks to study the role of speculation. In contrast, we have studied the oil price using a non-stationary framework. We are also able to link world GDP to commodity prices using annual data from the last century.

Second, our paper is related to a paper in international macroeconomics. Our paper builds on Aguiar and Gopinath (2007), and they find that shocks to growth in the emerging economy tend to be important for understanding consumption patterns. Our contribution is to assess the effect of income shock on the oil market through impulse responses and variance decomposition analysis. We find that income shock has a permanent effect on oil price.

Similarly, our paper is related to the recent theoretical contributions of Dvir and Rogoff(2009; 2014). They employ an extended commodity storage model explaining the persistence and volatility in the oil price. When aggregate demand has a persistent growth component (emerging economies), storage amplifies the shock's effect on the price. However, the magnifying effect of inventory is diminished when supply becomes flexible. Dvir and Rogoff (2014) find that cointegration among oil production, inventories, income and crude oil price can be explained using the storage model with stochastic income growth and different production scenarios. Our paper also uses a cointegration framework. We also show that the impact of income shocks depends on the relative supply elasticity.

Finally, our model is related to the small but growing body of literature on the comovement in commodity markets. Many papers have warned us about possible cointegration in the crude oil market. Kaufmann et al. (2004) and Dées et al. (2007) detect cointegration among crude oil price, days of forward consumption of oil stocks, OPEC production quota and capacity utilization. They employ an error correction model for the world oil market. The model includes three single equations, oil demand, oil supply and the function of price. Lakuma (2013) studies the extent of market power in the American crude oil industry through a vector error correction model for the demand and supply of oil for the period 2000–2012. He also estimates the model using a single-equation method. Our paper differs in that we have used data from more than 100 years. In our model, we estimate the long-run elasticities at equilibriums. We also show that the price is predictable using the long-run equilibrium among oil price, oil production and world GDP.

The remainder of the paper proceeds as follows. In section 2 we provide evidence for stochastic trends in the oil market. We show there is evidence for cointegration among world GDP, oil production and the oil price. We also estimate a stable long-run relationship. In section 3 we quantify the impact of shocks to world GDP on the price of oil. We show that income shocks generate a permanent effect on the oil price.

2 Stochastic trends in the oil market

In this section, we present evidence of cointegration in the oil market. We show that the latest period 1974:Q1–2016:Q4 is critical to the test for stochastic trends in our data. We start by explaining that world GDP, oil production and oil prices are non-stationary time series. We then establish that there is evidence for at least one cointegrating relationship among the series. Finally, we estimate a long-run relationship for the oil market.

Data. We use quarterly data for the period 1974:Q1–2016:Q4. We use West Texas Intermediate (WTI) price as the measure of crude oil price collected from FRED (2018). The crude oil production is collected from EIA (2018). We employ world GDP as the measure of global income. The world GDP series combines quarterly GDP index from Fagan et al. (2001) for the period 1974:Q1–2010:Q4 and Global Financial Data for the period 2011:Q1–2016:Q4. Nominal crude oil price and world GDP are deflated using the U.S. CPI obtained from the U.S. Bureau of Labor Statistics (2018).

Unit root test. We start by performing a standard unit root test for the oil price, oil production and world GDP. Using the augmented Dickey-Fuller (ADF) test (with a constant in the regression) we cannot reject the null hypothesis of a unit root.[1] Hence, no evidence suggests that these variables are stationary. This result is robust to other unit root tests.[2] We also find that this result is robust to different subsample periods:

[1]We use Bayesian information criteria to select the number of lags. We find four lags for the logarithm value of world GDP, 12 lags for the logarithm value of world oil quantity and two lags for the logarithm value of oil price. Similar results are obtained when using the Akaike information criterion. The p-value of the ADF test statistic of unit-root $\log(Y_t)$ is 41.5 percent, 93.5 percent for $\log(Q_t)$, and 30.3 percent for $\log(P_t)$.

[2]In particular, we used the Phillips and Perron test (with intercept). Indeed, we find no evidence to reject the null hypothesis that global GDP, world oil quantity and oil price are unit roots. The p-value of the test statistic for the logarithm value of world GDP is 88.0 percent for nonstationary, 25.3 percent for the logarithm value of world oil quantity, and 59.5 percent for the logarithm value of oil price.

1974:Q1–1994:Q4 and 1995:Q1–2016:Q4. Again, we cannot reject the null hypothesis of unit roots for all variables in different subsample periods.[3]

Cointegration test. Since variables are nonstationary, we can test for an equilibrium relationship among the crude oil price, oil production and world GDP. To this end, we implement a Johansen's unrestricted cointegration rank test (trace test), which is based on the estimates derived from an error correction model. In panel (a) Table 1 we show the results of our cointegration test. In particular, we report the trace statistics for $r = 0, 1, 2$ where r is the number of cointegrating relationships. We find that for the period 1974:Q1–2016:Q4 we reject the null hypothesis for $r = 0$ at both 1 percent and 5 percent significant level, with the trace statistics of 59.5, and critical value of 26.8 at 1 percent significance level and 34.91 at 5 percent significance level. But we cannot reject the null hypothesis of $r \leq 1$ at any significance level, with test statistics of 19.3, critical value of 20.2 at 1 percent significance level and 20.0 at 5 percent level. Thus, we conclude that there is at most one cointegrating relationship among the series. We also implement the trace test for the sub-periods of the sample: 1974:Q1–1994:Q4 and 1995:Q1–2016:Q4. As shown in Table 1, the cointegrating relationship is very robust to the sub-sample period, such that we reject the null hypothesis of $r = 0$, but we can not reject the null hypothesis of $r \leq 1$ at both 1 percent and 5 percent significance level. Hence, we confirm that there is at most one cointegrating relationship in the sub-sample period 1974:Q1–1994:Q4 and 1995:Q1–2016:Q4.

The Long-run relationship in the oil market. Having determined the existence of one cointegrating vector, we can estimate a long-run relationship among the crude oil price, world oil production and global GDP. We estimate this relationship using a standard dynamic OLS regression as in Stock and Watson (1993).[4]

In panel (b) we show the results of the dynamic OLS estimation. We see that the estimates for world GDP and oil production are both significant. It is important to stress that the estimates represent the long-run relationship among the variables. In this sense, there is no claim about causality from any one variable to the other. The long-run relationship cannot be interpreted as a "demand" or "supply" equation, rather it reflects different equilibrium sequences over time, representing the "average" correlation between prices, global GDP and global oil production. In any case, the structure of the oil market can eventually affect the signs of parameters we find.

[3]We use BIC index to select the number of lags using in the test. For the period 1974Q1–1994Q4, we use four lags for the logarithm value of world GDP, one lag for the world oil quantity and oil price. For the period 1995Q1–2016Q4, we use two lags for the world GDP, seven lags for the world oil quantity and three lags for oil price. See Appendix for details about the unit root test.

[4]This method generates asymptotically efficient estimators for cointegrating vectors. To estimate the long run relationship we start by determining the number of lags and leads using AIC and BIC tests. The tests suggest using one lagged and one lead variable.

Table 1: Johansen's Cointegration Test for Different Periods

(a) Johansen's test of cointegration

		Cointegrating rank H0: $rank(\Pi)=r$		
		$r = 0$	$r \leq 1$	$r \leq 2$
Trace statistics:	1974Q1–2016Q4	59.52	19.31	6.89
Trace statistics:	1974Q1–1994Q4	49.66	19.88	9.23
Trace statistics:	1995Q1–2016Q4	36.50	12.72	3.90
Critical value 5%		34.91	19.96	9.24
Critical value 1%		26.81	20.20	12.97

(b) Long run relationship:

$$\log P_t = \alpha_0 + \alpha_y \log Y_t + \alpha_q \log Q_t + controls_t + \epsilon_t$$

	coeff.	se.
α_y	-1.434	(0.292)
α_q	3.914	(0.692)
α_0	-7.505	(3.033)

Note: Table (a) shows the Johansen test of vector cointegration for different cointegration ranks. This vector includes the logarithm value of world GDP, $\log Y_t$, world oil production, $\log Q_t$, and crude oil price, $\log P_t$, for the period 1974:Q1–2016:Q4. Π is the coefficient for the lagged vector in the VEC model. Table (b) shows the dynamic OLS estimates for the long-run relationship among variables for the period 1974:Q1–2016:Q4. The $controls_t$ variable contains one lag and one lead of the differences of the right-hand side variables.

Having in mind that the long-run relationship is neither a "demand" nor "supply" equation, there is a positive relationship between global production and oil prices. In particular, the elasticity of oil price with respect to oil production is 3.9, whereas the elasticity of oil price with respect to global activity is -1.4.

3 Quantifying the Impact of Income Shocks on the Oil Price.

In this section, we use a simple three equation autoregression framework to quantify the effect of income shocks on the oil price. We borrow the identification scheme from Cochrane (1994) and estimate a model with world GDP as an exogenous variable. Consistent with the work of Aguiar and Gopinath (2007), we find that shocks to world GDP are permanent. We find significant predictability of the long-run relationship in the oil price. However, we do not find predictability in quantity production. We show that income shocks generate a persistent response in the oil price. Moreover, shocks to oil prices are mostly transitory in line with many papers in the literature (Hamilton, 2009 and Kilian, 2009). We conjecture that the response of oil production is critical to understand this result. As production becomes more inelastic, the persistent GDP shock creates the high volatility and persistent impact on the oil price.

3.1 Vector error correction model (VECM)

Let $\mathbf{Y}_t = [y_t, q_t, p_t]$ be a vector containing the logarithm values of our three variables world GDP, oil production and oil prices. $\triangle \mathbf{Y}_t$ denotes the vector of differences $[\triangle y_t, \triangle q_t, \triangle p_t]$. Let $\hat{\alpha}$ be the estimated long-run parameters we obtain in the previous section. Then we use this vector of parameters to estimate the following VECM:

$$\triangle \mathbf{Y}_t = c + \gamma \hat{\alpha} \mathbf{Y}_{t-1} + \sum_{j=1}^{J} A_j \triangle \mathbf{Y}_{t-j} + u_t \qquad (1)$$

where c is the vector of constant estimates and γ is the vector of speed of adjustment coefficients $[\gamma^y, \gamma^q, \gamma^p]'$.

In Table 2 panel (a), we show the estimates of our VECM. In the case of the equation for prices, we obtain a significant coefficient of -0.073 for the speed of adjustment (related to the lagged error term). The value of this coefficient implies that each quarter oil prices change in order to correct 7 percent of long-run price misalignments. This value implies

Table 2: The Estimates of VEC Model and Variance Decomposition

(a) Vector autoregression

LHS		const.	LR term	RHS variable $\triangle y_{t-1}$	$\triangle q_{t-1}$	$\triangle p_{t-1}$	R^2
$\triangle y_t$	coeff	0.003	-	0.436	-	-	19%
	s.e.	(0.001)		(0.069)			
$\triangle q_t$	coeff	0.003	0.001	0.743	-0.164	0.015	7.2%
	s.e.	(0.028)	(0.004)	(0.273)	(0.077)	(0.011)	
$\triangle p_t$	coeff	-0.570	-0.073	3.158	-1.266	0.182	11.8%
	s.e.	(0.183)	(0.024)	(1.792)	(0.501)	(0.074)	

(b) Variance decomposition

Due to	$\triangle Y_t$	$\triangle Q_t$	$\triangle P_t$	Variance of $\triangle Y_t - E_{t-1}\triangle Y_t$	$\triangle Q_t - E_{t-1}\triangle Q_t$	$\triangle P_t - E_{t-1}\triangle P_t$
y shock	100	6.37	1.85	100	3.14	0.76
q shock	0	92.71	4.71	0	96.86	0.49
p shock	0	0.92	93.45	0	0	98.76

Note: Table (a) shows the estimates of the VEC model as in equation (1). Table (b) shows the variances of global GDP, world oil production and oil price growth.

that the half-life [5] of past misalignments is 9.4 quarters, for example after 9.4 quarters movements in prices reduce half of the long-term disequilibrium. The significance of γ^p also shows that oil price growth is predictable using the long-run relationship in the oil market. Interestingly, the R^2 is around 12 percent, which is larger than a simple univariate autoregression with four lags (AIC selected) with a R^2 of 5.6 percent. This comparison highlights the predictive effect of the long-run relationship.

Now, in the case of the short-run equation for quantities, we obtain a speed of adjustment coefficient of 0.001. This coefficient is not statistically different from zero. Hence, quantities do not adjust in order to correct past misalignments. Even in the case where we disregard the significance level, the speed of adjustment in oil quantities is only 0.1 percent, which implies a half-life of 300 quarters. Overall, we conclude that prices respond to past long-run misalignments, but quantities do not react or react extremely slow.

Impulse Responses. We now show impulse response functions to a one standard deviation shock of world GDP. We use a Cholesky decomposition scheme with the following ordering $\{y, q, p\}$. Hence, we assume that world GDP does not respond contemporaneously to shocks to neither world production nor oil prices. In Figure (1) we plot the response of the level of the variables to shocks to world GDP. In panel (a) we plot the response of

[5] The half-life is equal to $\ln(2)/\gamma$.

world GDP and the oil price. We observe that the response of world GDP is almost flat, suggesting that the shock has a permanent effect on world GDP. In contrast, the response of oil prices is hump-shaped and significative. One could interpret a shock to GDP as shocks to global demand, and the delay response of the oil price suggests lagged supply adjustment induced by capacity investment and relatively inelastic oil supply. Overall, a one percent shock to world GDP generates a permanent increase in oil prices of 2.4 percent. In panel (b) we see that the impact of world GDP on oil quantities is permanent. In this case a one percent increase in world GDP leads to a permanent increase in oil quantities of 0.9 percent. All in all, we find evidence that shocks to world GDP are permanent and have a permanent impact on both oil prices and quantities, although the impact on oil prices is twice as large as the impact on quantities.

Oil price shock. In Table 2, we found that the speed of adjustment coefficient of oil quantities is small and not statistically different from zero. Thus, quantity is not predictable using the long-run relationship of world GDP, price of oil and world oil quantity in the error correction model, which is consistent with the view that in the long run oil production is relatively inelastic. As a result, most of the predictability comes from the auto-regressive term and lagged differences of GDP growth. In Figure 1 panel (c) we show that an oil price shock of one percent has a small impact on quantities, which is both small (0.2 percent) and not statistically different from zero after four quarters. The response of oil prices to this shock, as seen in Figure 1 panel (d), is statistically different from zero, relatively large (14.6 percent) and transitory. After 30 quarters the effect is no longer statistically different from zero.

Beveridge and Nelson decomposition. We now compare our long-run relationship with a trend component derived using a Beveridge and Nelson decomposition following Beveridge and Nelson (1981). The trend is computed after all transitory shocks have eventually disappeared. Figure 2 shows that the Beveridge and Nelson trend component is very similar to the long-run relationship used in our vector autoregression specification.[6] The decomposition shows a steady increase in the permanent component after 1985. We also can see that most of the run-up in the price starting in 2000 is due to transitory shocks.

3.2 The importance of accounting for the long run behavior

To analyze the importance of accounting for the cointegrating vector we compare our benchmark model with a simple vector autoregression model. In particular, we re-estimate the

[6]The long-run relationship is computed as the predicted oil price in long run. That is $\hat{\alpha}_y \log Y_t + \hat{\alpha}_q \log Q_t$, where $\hat{\alpha}_y$ and $\hat{\alpha}_q$ are the dynamic OLS estimates from Table 1.

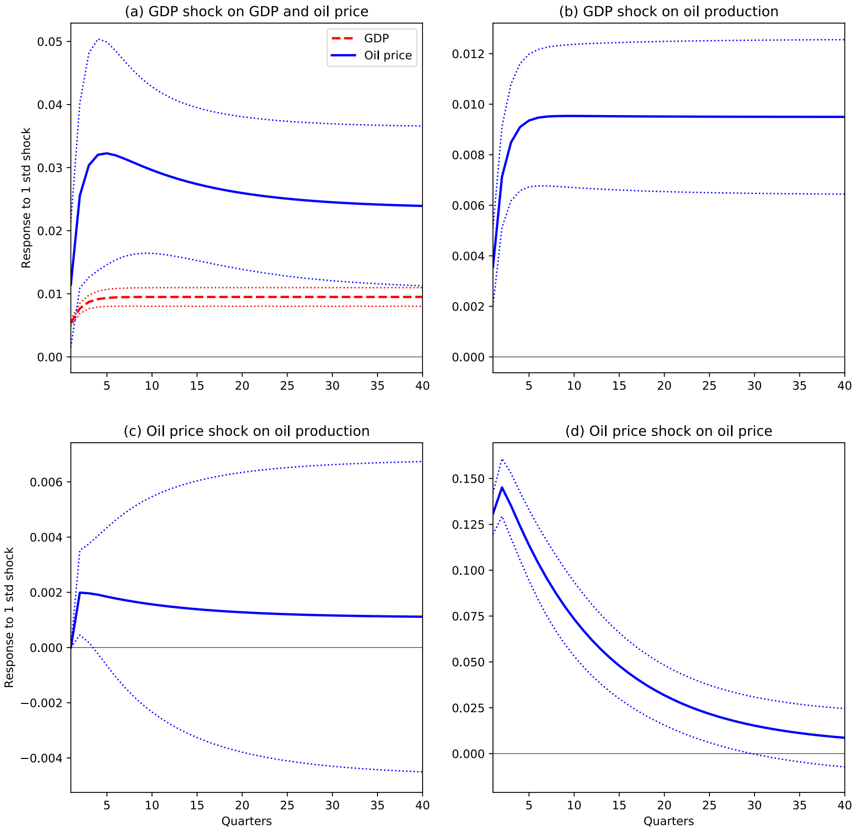

Figure 1: Responses of GDP and Oil Price to Structural Shocks

Note: Panel (a)–(d) show the cumulated responses using estimated parameters in equation (1). Panel (a) shows the response of oil price to one-standard-deviation oil price shock (the solid line) and GDP shock (the dashed line) respectively. Panel (b) shows the response of oil production to one-standard-deviation GDP shock. Panel (c) shows the response of oil production to one-standard-deviation oil price shock. Panel (d) shows the response of oil price to one-standard-deviation oil price shock. The dot lines denote the bootstrap one-standard-error bands.

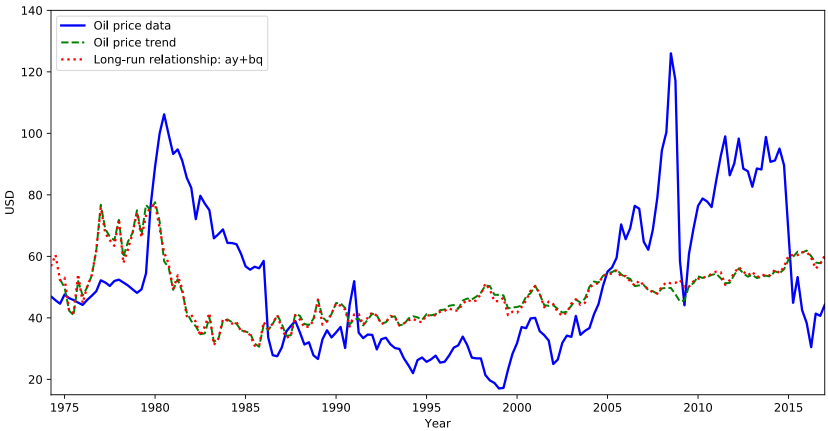

Figure 2: Beveridge-Nelson Oil Price Trend and Long-Run Relationship of Oil

Note: The figure plots the Beveridge-Nelson components of the oil price. The solid line denotes the oil price data. The dashed line represents Beveridge-Nelson trend of oil price, computed using estimates from the VECM in equation (1). The dot line denotes the long-run relationship of $\hat{\alpha}_y \log Y_t + \hat{\alpha}_q \log Q_t$ where $\hat{\alpha}_y$ and $\hat{\alpha}_q$ are the DOLS estimates from panel (b) Table 1, Y_t and Q_t denotes the data on world GDP and global oil production.

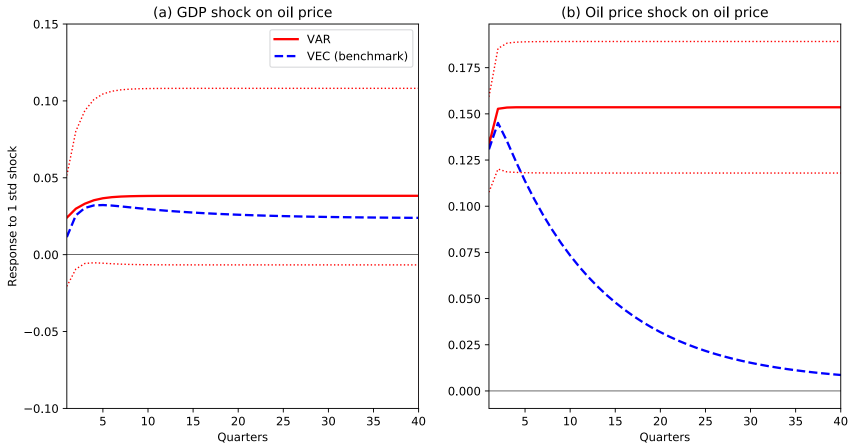

Figure 3: Responses of Oil Price to GDP and Oil Price Shocks: VEC Model and VAR

Note: Panel (a) and (b) show the impulse responses of oil price to one-standard-deviation of GDP and oil price shocks in VEC model and VAR model. The dashed line denotes the impulse responses in the VEC model, the benchmark case, in equation (1). The solid line denotes impulse responses in the VAR model of the vector $[\triangle logY_t, \triangle logQ_t, \triangle logP_t]$. In the VAR model we assume $\triangle Y_t$ is exogenous and exclusive, such that $\triangle logY_t = \sum_{i=1}^{N} a_i \triangle logY_{t-i} + a_0$ where a_i, $i = 0, 1, 2, ...$ are the coefficients. We employ $N = 1$ in the VAR model selected using AIC index. The dot lines denote the bootstrap one-standard-error bands for the impulse responses in the VAR model.

model using a vector autoregression with the same three variables but this time without the error correction term.

In Figure 3 we plot impulse response functions to an oil price shock. The solid line represents the impulse response of our benchmark estimation, whereas the dotted line plots the impulse response in the estimation without the error correction term. We see that the response is almost permanent. Essentially, the model is similar to the behavior of an autoregressive model of the change in the oil price. In contrast, when the error correction mechanism is present, an oil price shock has no permanent effect. In this case, the reduction of the price to the pre-shock level is induced by the long-run relationship among the variables. Basically, comparing these two responses highlights the importance of the adjustment factor in term of the propagation of the price shock.

4 Further Analysis

In this section, we perform three additional analyses that complement our benchmark estimation. First, we compare our estimation with a stationary estimation framework. In particular, we compare with the standard VAR specification as in Kilian (2009). Interestingly, we show that both models capture similar short-run responses of oil prices to oil-specific demand shocks. The main differences are the response of prices to oil supply shocks. Second, we test the robustness of our estimates to a different sample period. Since there is a structural break in the time series in 1973, we re-estimate our model using annual data from 1900 to 1973. We find production responses are key to the response of the model to exogenous income shocks. Finally, we also employ our empirical model to study the effect of income shocks on the copper market. We find similar effects in terms of predictability and the importance of transitory shocks.

4.1 Stationary vector autoregression

Now we compare our estimation from our error correction specification to a three-variable VAR specification similar to Kilian (2009), but on a quarterly frequency.[7] Our version of his estimation includes the change in oil production, an index of real economic activity constructed in Kilian (2009) and the level of the real price of oil. We include only one lag in the estimation which is different from the 24 lags specification in Kilian (2009).[8]

Since both models are non-nested, we compare the effect of oil demand and oil production shocks on oil prices. Figure 4 panel (a) compares the response of oil prices to a one standard deviation oil-demand or price shock. Notice that both impulse responses are strikingly similar. Recall that in our estimation the oil-demand shocks are mostly transitory which is consistent with the VAR framework. In panel (b) we show the response of oil price to an oil production shock. In the Kilian (2009) model, we see there is no significant impact of the production shock on the oil price.

4.2 Yearly Regressions

Our benchmark scenario uses quarterly data from 1973 to 2016, which is a period of high oil price volatility. In Figure 5 we plot world production of crude oil and the real oil price for the period 1900–2016. We observe that production grows consistently through most of the century to abruptly change its trend in 1973. As a result, the price changes follow a

[7]The model in Kilian (2009) is estimated using monthly observations from 1973.1 to 2007.2

[8]We also check that the estimates are robust for a number of lags.

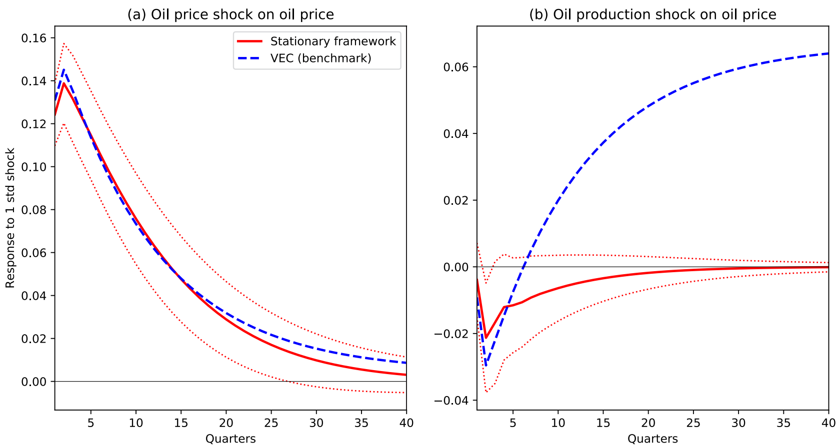

Figure 4: Responses of Oil Price to Oil Price Shock and Oil Production Shock: VEC Model and Stationary Framework

Note: Panel (a) and (b) show the impulse responses of oil price to the one-standard-deviation oil price shock and oil production shock in the VEC model and the stationary framework. The dashed line denotes the impulse responses in the VEC model, the benchmark case, in equation (1). The solid line denotes impulse responses in a stationary framework, the VAR model similar to Kilian (2009), on a quarterly frequency. One lag is employed the VAR model. The dot lines denote the bootstrap one-standard-error bands for the impulse responses in the VAR model.

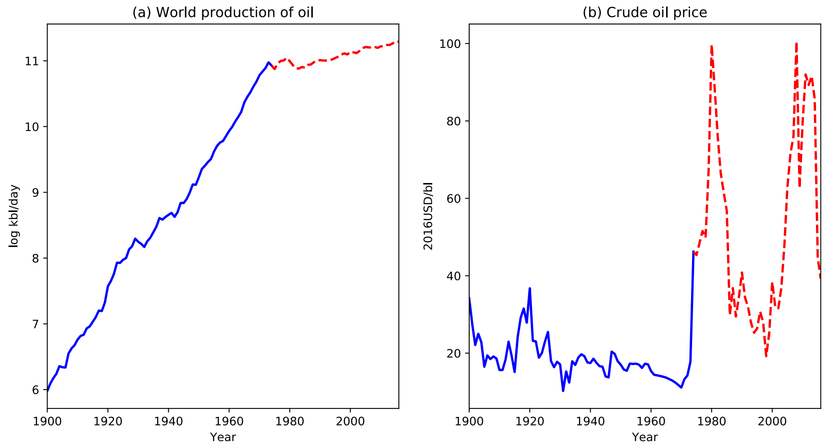

Figure 5: World Production of Oil and Crude Oil Price 1900–2016

Note: Panel (a) and (b) show the annual observation of world oil production and crude oil price for the period 1900–2016. Panel (a) plots the world production of oil. The solid line is the annual series collected from Bouda Etemad and Toutain (1998) for the period 1900–1964 and BP (2018) for the period 1965–1973. The dashed line is annual average computed using quarterly observation of world oil production collected from *EIA (2018)*. Panel (b) plots the crude oil price. The solid line plots the annual series for the period 1900–1973 collected from BP (2018). The dashed line is annual average computed using quarterly observation of world oil production collected from *FRED (2018)* for the period 1974–2016.

regime of high levels and volatility.[9]

We estimate our vector autoregression to check the relative importance of permanent versus transitory shocks in the first period of relative price stability. In particular, we include the annual observations for the period 1900–1973. The test for non-stationarity shows that global GDP and world oil production are both unit roots. Interestingly, the null hypothesis of a unit root for real oil price at 5 percent significance level is rejected. We also find at least one cointegration relationship among the series for the period 1900–1973.

We then estimate equation (1) as in our benchmark estimation. Table 3 shows that as in the benchmark case the long-run relationship forecasts the change in the oil price. The adjustment parameter in the short-run price equation is 0.422. This means that every year 40 percent of the misalignment is corrected, and that the half-life of the error term in the short-run price equation is 1.6 years (six quarters). It is faster than the half-life of ten

[9]The annual observations for the oil price are collected from BP (2018). We use the US CPI to deflate the oil price. World oil production is from Bouda Etemad and Toutain (1998) for the period 1900–1964, and BP (2018) for the period 1965–1973. World GDP is computed using global GDP per capita and world population from Database (2018). There are discontinuous data in 1900–1949. We interpolate the series assuming constant annual growth rate.

Table 3: Estimates of VEC Model and DOLS for Period 1900–1973

(a) Vector autoregression1900–1973 annual data

LHS		RHS variable				
		const.	LR term	$\triangle y_{t-1}$	$\triangle q_{t-1}$	$\triangle p_{t-1}$
$\triangle y_t$	coeff	0.016	-	0.463	-	-
	s.e.	(0.004)		(0.106)		
$\triangle q_t$	coeff	-0.799	0.080	0.201	-0.090	0.013
	s.e.	(0.422)	(0.039)	(0.239)	(0.124)	(0.040)
$\triangle p_t$	coeff	4.557	-0.422	0.357	0.129	-0.052
	s.e.	(1.233)	(0.114)	(0.699)	(0.361)	(0.118)

(b) DOLS estimates for long-run relationship

	coeff.	se.
α_y	-0.419	(0.136)
α_q	0.066	(0.058)
α_0	10.640	(2.287)

Notes: Table (a) shows the estimates of the VEC model using annual data for the period 1900–1973. Table (b) shows the DOLS estimations for the long-run relationship for the same period.

quarters in the estimation that goes from 1973 to 2016.

Now, in the case of the short-run equation for oil quantity, we see that the speed of adjustment is positive and statistically different from zero. The value is 0.08, which implies that the half-life of the error term in this equation is 8.7 years (35 quarters). This speed of adjustment is smaller than the one found in the price equation. Despite this fact, the speed of adjustment in oil quantities is much faster than the one found in the model estimated for the period 1973-2016. In particular, in the period 1900-1973 the half-life of 35 quarters is almost one-tenth of the 300 quarters half-life found in the period 1973 to 2016.

In order to compare the way in which global GDP shocks are transmitted to oil prices and quantities, we compare the responses of oil price and world oil quantity in two different subsamples. As shown in Figure 6 panel (a), in the 1900-1973 period a 1 percent shock to GDP generates an increase in prices of 3 percent. After two years the impact on prices is zero, hence this shock has a transitory effect on oil prices. In the second sample, from 1973 to 2016, this shock has a positive impact on prices, but its effect is permanent. In particular, the price of oil increases permanently by 3 percent. Now, in Figure 6 panel (b), we see that the impact on quantities is, in both samples, permanent. In this case, however, the responses are entirely different: in the 1973-2016 period quantities increase permanently by 0.8 percent. By contrast, in the period 1900-1973 quantities react much more in the face of the same shock. In this case, the quantities increase permanently by 2.5

percent. The collective behavior of prices and quantities in the face of a world GDP shock are coherent with a different market structure in each period. In particular, we conjecture that, in the first period, oil supply was much more elastic than in the second sample.

4.3 Copper

We found that we can identify the effect of world GDP on the price of oil once we account for the long run relationship in the market. In this section, we check whether we can identify a similar effect in the copper market. We use a three variable autoregression estimation again for world GDP, copper production and the real price of copper from 1900 to 2016. We use world copper production from USGS (2018).[10] The copper price is collected from Global Financial Data (2018)[11], and it is deflated using U.S. CPI from the U.S. Bureau of Labor Statistics (2018).

Our unit root test indicates that world copper production is non-stationary, but we do not find evidence to reject the null hypothesis that the real price of copper has a unit root. The Johansen's cointegration test indicates that the time series of world GDP, copper production and the price of copper are cointegrated. We can reject the null hypothesis of at least two cointegrating relationships ($r = 2$) at 1 percent significance level.

In Table 4 we compare the estimation for copper and oil in different periods. In panel (a) we compare the estimations for the period 1900-1973. We observe that for both copper and oil quantities and prices are predicted by the long run relationship. In particular, the estimates of the speed of adjustment rates for quantities for both oil and copper are statistically different from zero. For oil, it is estimated at 0.08 with a standard error of 0.04, and -0.137 with a standard error of 0.035 for copper. These estimates imply a half-life of 8.66 years for oil production and 5.06 years for copper production. Similarly, we find significant speed of adjustment parameters for oil prices. We find a stronger adjustment for oil with an estimate of -0.42 and -0.22 for copper. The half-life for the copper price is 3.11 years, and for oil price it is 1.64 years. In other words, the copper price takes twice as much time as the oil price to reduce half of the long-term disequilibrium.

In panel (b) we compare our estimation using the data for the period 1900–2016. Extending our estimation to include the latest period should shed light on the effect of the period 1974–2016. We observe that the estimate for adjustment parameter for oil price is small at -0.02 and insignificant. For the copper price, the adjustment parameter is estimated at -0.058 (which implies a half-life of about 12 years) which is smaller than for the whole period. In the case of oil, the fact that the speed adjustment parameters turn out to

[10]We employ world mine production.
[11]We employ high-grade copper price.

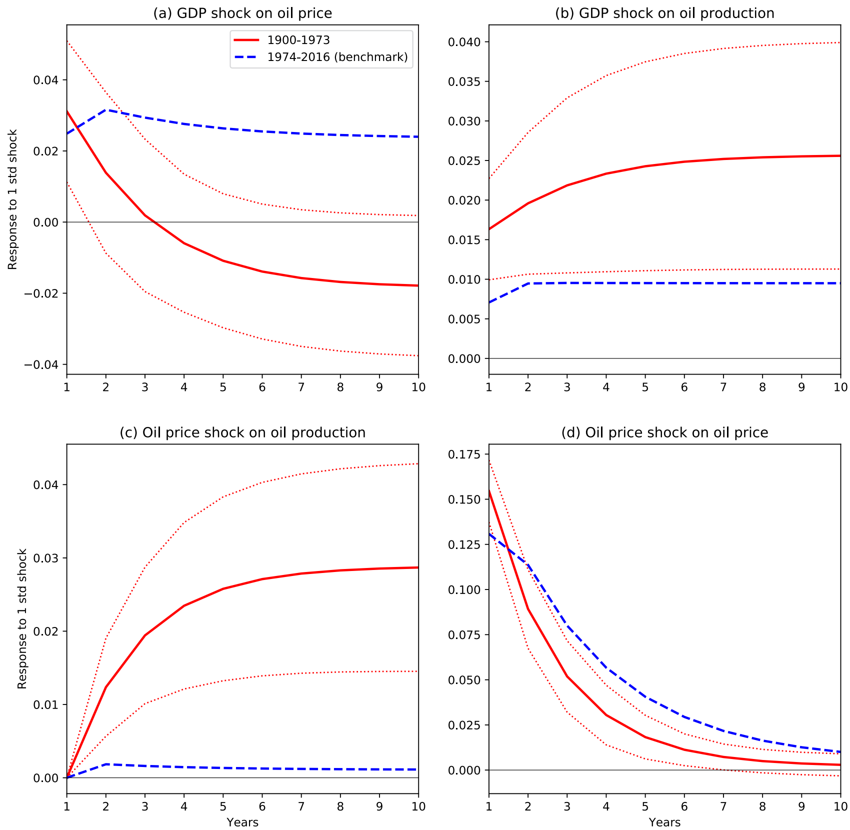

Figure 6: Response of Oil Price and Production to Structural Shocks for Different Periods

Note: Panel (a)–(d) show the impulse response of oil price and oil production to one-standard-deviation of GDP shock and oil price shock. The dashed lines denote the annual average of impulse responses in the benchmark case for the period 1974–2016. The solid lines denote the impulse responses in the model using annual data for the period 1900–2016. The estimates are shown in Table 3. When computing the impulse responses, the estimates of autoregressive terms $\triangle y_{t-i}$, $\triangle q_{t-i}$ and $\triangle p_{t-i}$ are fixed to zero if they are not estimated significantly. The dot lines denote the bootstrap one-standard-error bands for the impulse responses in the case using data for the period 1900–1973.

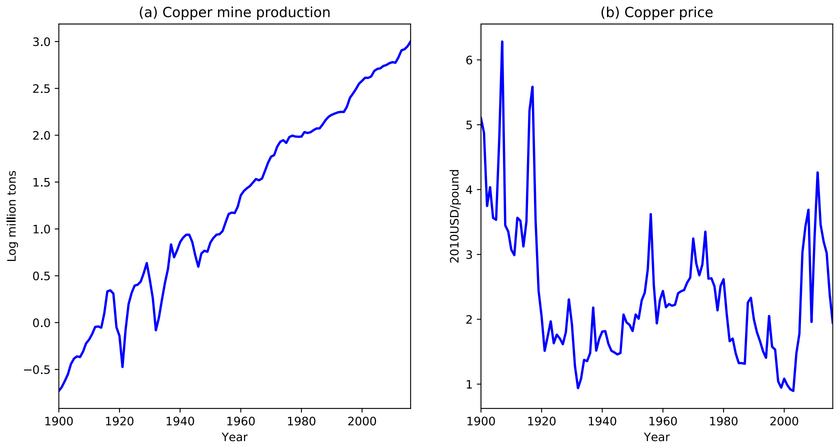

Figure 7: Copper Price and Mine Production

Note: Panel (a) and (b) show the annual data on world copper mine production and copper price for the period 1900–2016. The world copper mine production is collected from USGS (2018). The copper price is from Global Financial Data (2018), deflated using U.S. CPI obtained from the U.S. Bureau of Labor Statistics (2018).

be insignificant is consistent with the previous analyses that it suggests a structural break in the year 1973 and the conjecture of an inelastic oil supply for the period 1974–2016. To the oil market, we do not find a dramatic break in the model for copper in the year 1973, although there is some evidence that the copper supply has adjusted quantities less strongly.

In Figure 8, we compare the impulse response functions for the copper price in the period 1900-2016 with our benchmark estimation for oil price in the period 1974-2016. In panel (a) we observe that the response of copper price to a GDP shock is mostly transitory, mainly explained by the significant response of quantity to the shock. In contrast, the response of oil price is permanent due to a relative inelastic response of supply. In panel (b) we plot the transitory component of the prices to a price shock. Interestingly, both copper and oil prices show a significant transitory component with similar persistence. The response for copper price shows a higher level of the impact over time than for the oil price.

Table 4: Estimates in the VEC Model for Oil and Copper for Different Periods

	(a) 1900–1973		(b) 1900–2016	
	Oil	Copper	Oil	Copper
Speed adj. γ_q	0.080	-0.137	-0.021	-0.058
	(0.039)	(0.035)	(0.013)	(0.023)
$t_{1/2}$ in production (years)	8.66	5.06	33.01	11.95
Speed adj. γ_p	-0.422	-0.223	-0.155	-0.190
	(0.114)	(0.056)	(0.053)	(0.049)
$t_{1/2}$ in price (years)	1.64	3.11	4.47	3.65

Note: Table shows the estimates of the adjustment rates in the VEC model for oil and copper in different time: 1900-1973 and 1900–2016. The half-life values are computed as $ln(2)/\gamma$.

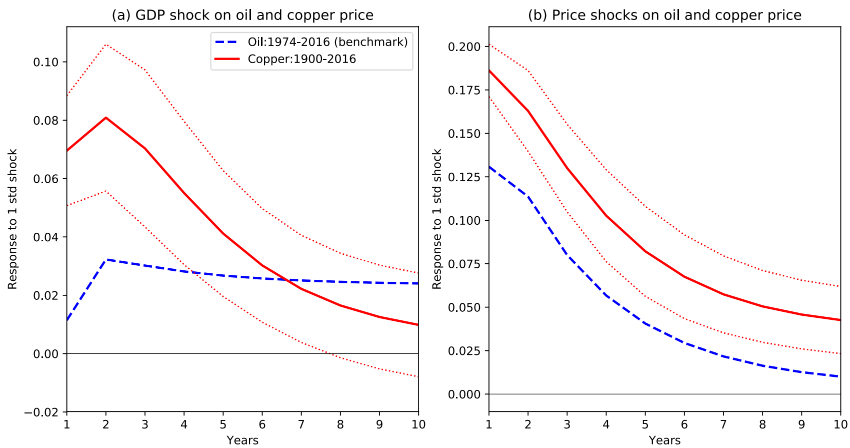

Figure 8: Responses of oil and copper price to GDP and price shock

Note: Panel (a) and (b) plot the impulse responses of oil and copper price to one-standard-deviation GDP and price shock. The dashed lines denote the annual average of impulse responses of oil in the benchmark case for the period 1973–2016. The solid line denotes the impulse responses of copper in the model using annual data for the period 1900–2016. The estimates are shown in Table 4. When computing the impulse responses, the estimates of autoregressive terms $\triangle y_{t-i}$, $\triangle q_{t-i}$ and $\triangle p_{t-i}$ are fixed to zero if they are not estimated significantly. The dot lines denote the bootstrap one-standard-error bands for the impulse responses in the case for copper.

5 Conclusions

In this paper, we have shown how world GDP affects commodity prices. We have focused on explaining the effect on the oil price. We have identified the effect of income shocks on the oil price. We use a simple error correction model and found that for the period of 1973–2016, world GDP has had a permanent effect on the oil price. We also find that the long run relationship predicts the oil price. In contrast, the long run relationship cannot predict quantity production. Consistent with previous studies, most of the variation in oil price is due to transitory shocks. Our conjecture is that since production is relatively inelastic, the permanent shocks to income are transmitted to prices.

We have also extended the analysis to study the oil market using annual data from 1990 to 2016. We found that, still, the long run relationship predicts the change in oil prices. Interestingly, in the early period, we find evidence of predictability for oil production. We conjecture that the differential response of production explains that shocks to income are mostly transitory in the early period.

After that, we explored whether world GDP also has an impact on another commodity, namely copper prices, on the period of 1900–2016. We find indeed that the long run relationship predicts the copper prices and quantity. As a result, the response of the copper price to a shock to income is mostly transitory.

From the discussion in section 4, we find that a permanent shock to global activity is transmitted differently to both oil prices and quantities in different samples, and it depends on the way in which the oil market is structured. Developing a model which can account for this differentiated response goes beyond the scope of this paper, but is a natural extension of the research agenda.

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Appendix

A Unit root tests

This section shows different tests of unit roots for global GDP, world crude oil production and price of oil.

We perform augmented Dicky-Fuller (ADF) test and Phillip & Perron (PP) test for the logarithm values of world GDP, global oil production and crude oil price. The tests are performed for different periods, 1974Q1–2016Q4, 1974Q1–1994Q4 and 1995Q4–2016Q4. The test results are shown in the Table 5. We find that from both tests we can not reject the null hypothesis of the unit root for each series in different test periods.

Table 5: Unit root tests

(a) Augmented Dicky-Fuller test

	1974Q1–2016Q4			1974Q1–1994Q4			1995Q1–2016Q4		
	No.lags	DF stat.	p-value	No.lags	DF stat.	p-value	No.lags	DF stat.	p-value
$\log(Y)$	4	-1.725	0.415	4	-1.070	0.658	2	-1.432	0.525
$\log(Q)$	12	-0.164	0.935	1	-1.787	0.395	1	-1.176	0.620
$\log(P)$	2	-2.028	0.303	1	-1.249	0.593	1	-1.710	0.423

(b) Phillips & Perron test

	1974Q1–2016Q4		1974Q1–1994Q4		1995Q1–2016Q4	
	$Z(\alpha)$ stat.	p-value	$Z(\alpha)$ stat.	p-value	$Z(\alpha)$ stat.	p-value
$\log(Y)$	-4.102	0.880	-7.106	0.700	-5.800	0.778
$\log(Q)$	-15.068	0.253	-9.872	0.537	-16.677	0.138
$\log(P)$	-9.072	0.595	-7.575	0.673	-7.893	0.655

Notes: Table (a) shows the augmented Dicky-Fuller (ADF) test of logarithm values of world GDP, global oil production and crude oil price for different periods. The number of lags used in the tests is selected using BIC. We perform augmented Dicky-Fuller test with constant. The null hypothesis of the ADF test states that the test series is a unit root. Table (b) shows the Phillips & Perron (PP) test of unit roots. The test regression is estimated with intercept. Four truncation lag parameters are used in the test for the period 1974Q1–2016Q4. Three truncation lag parameters are used in the test for the sub-periods 1974Q1–1994Q4 and 1995Q5–2016Q4. The null hypothesis of the PP test states that the test series is a unit root.

PAPER 3

Importance of Demand and Supply Shocks for Oil Price Variations*

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Abstract

This paper studies the importance of demand and supply shocks in the oil market, and tries to explain the formation of the short-run oil price by applying an extended commodity storage model to the cyclical components of the price. First, I employ a multivariate method to extract the cyclical component of the oil price, world oil consumption, and global GDP. Next, I find a large and positive effect of global GDP shock on the oil price cycles in a VAR model. Then, I estimate the commodity storage model using a moment-matching method. All parameters are estimated significantly, and the model shows good capability of reproducing the volatility and persistence of oil price cycles. I find that the GDP shock generates a much more moderate effect on the oil price cycles in the extended commodity storage model than the empirical evidence from the VAR analysis, and the production shock plays an important role for the variance of the cyclical component of the oil price.

JEL Classification: *C15, G1, O13, Q4.*

Keywords: *Oil price, demand shock, supply shock, competitive storage model, Beverage-Nelson decomposition, simulated method of moments.*

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1 Introduction

Crude oil is one of the most frequently traded commodities in the global commodity market. The real oil price features high volatilities and occasionally large spikes. For example, the real WTI price of crude oil increased by more than 400% from 1999 to 2008. What are the main driving forces behind the dynamic behavior of the oil price? Are changes in supply or demand behind behind these large fluctuations? Scholars have studied factors affecting the oil prices from different angles. On one hand, Griffin (1985), Pindyck (1994), Alhajji and Huettner (2000), Hansen and Lindholt (2008), Lawell (2013) and others have proposed theories where OPEC controls the supply of oil. On the other hand, Cooper (2003), Krichene (2006), Hamilton (2009), and many others have proposed demand-driven explanations.

There is, however, a need to use dynamic structural models to evaluate the relative importance of demand and supply factors. The first step is then to build a theoretical model for the oil market and estimate the structural shocks. The aim of this paper is to study what element explains the formation of oil prices in a structural model, and to quantify the relative importance of demand shocks and supply shocks to the variance of oil price.

To this end, I put forward an extended commodity storage model to explain cyclical components of the oil price, taking into account exogenous variation in both global income and crude oil supply. I use a simulated method of moments (SMM) to estimate the structural parameters of the model using data from the period 1986–2009. I show that the extended storage model represents a major improvement over the original traditional commodity storage model in Deaton and Laroque (1992; 1996). I find that GDP shocks generate very moderate effects on the oil price, while production shocks explain a large fraction of the oil price variance.

In this paper, I first extract the cyclical component of quarterly data on oil price, global oil quantity, and world GDP by using a multivariate Beveridge-Nelson (BN) decomposition method for the period from 1986, after the substantial price decrease in the oil market, to the end of the financial crisis in 2009. In the BN decomposition, I show that there is a positive contemporaneous correlation between the cyclical component of global oil quantity and world gross domestic product (GDP). I also document a large and positive effect of world GDP shock on the oil price cycles in a VAR model.

Second, to understand the dynamics of crude oil price cycles, I employ a competitive storage model that is devised with both stochastic income and production process without serial correlation; this is an extended version from the model in Deaton and Laroque (1992, 1996), where only production shock was considered. In this model, the risk-neutral and profit-maximizing speculator or institutional investor holds inventories from one period to the next when expected future prices are equal to the current price. The speculative storage has a smoothing effect on the equilibrium price by reducing the volatility and introducing persistence.

Third, to gain insight into the role of income shock and production shock, I investigate two simplified commodity storage models with single stochastic production or income process. I solve these storage models by using collocation methods that assume the same shock volatility. I find that the production shock introduces a more intensive smoothing effect on the equilibrium price with a lower variance and higher persistence than the income shock.

Subsequently, I apply the extended storage model to the cyclical components of oil price, world oil consumption, and global GDP through using a SMM. I also compare the estimates and simulated moments between the extended storage model and the storage model with only production shocks as in Deaton and Laroque (1992). The extended storage model shows a better fitness to the data. All the coefficients of the extended storage model are estimated to be significant. The estimated short-run demand elasticity is –0.2, which is close to the results garnered by Gately and Huntington (2002), Cooper (2003), and Dées et al. (2007). By employing the estimated parameters, the extended storage model (with both stochastic production and income processes) reproduces some important features of the oil price cycles. In particular, it generates a high price volatility and persistence that are similar to real oil price cycles.[1] On the other hand, one of the estimates from the storage model with only production shocks is statistically insignificant, and the model is not able to match many data moments.

Finally, I perform a counterfactual analysis and compute impulse responses of oil prices to exogenous income and production shocks. I find that the model generates moderate results for the effect of income shocks on oil price cycles. The supply variation explains the largest fraction of the variance in the oil price cycles due to the large estimated production shock volatility.

Increasingly, scholars have been trying to explain the long-run determinants of crude oil prices. The topic in this paper, however, relates more to short-run analyses. For example, Cooper (2003) employed a multiple linear regression analysis and estimated short-run elasticities of crude oil demand in both OECD and non-OECD countries by using ordinary least squares (OLS), over the period 1979–2000. He found that crude oil demand displays high price inelasticity in the short run. Furthermore, Krichene (2006) has also

[1] The extended storage model generates a higher autocorrelation than the canonical model in Deaton and Laroque (1992, 1996).

examined the world oil market in a single equation estimation and has concluded that there is less elastic demand but a high demand income elasticity for crude oil in short run. Contrary to previous scholarship, this paper studies the driving force of the short-run oil price in a nonlinear structural model for the recent period of 1986–2009. Rather than the single estimation method, the structural parameters are estimated using the SMM method.

This paper is relevant to the literature that studies the determinant of oil price in VAR models. The seminal paper of Kilian (2009) has discussed the effects of demand shocks and supply shocks in a three-variable structural vector autoregressive (VAR) model. Kilian (2009) employed the Cholesky decomposition method to identify shocks; it is assumed supply is vertical and does not respond to demand shocks and price shocks simultaneously. Later, Kilian and Murphy (2014) extended the work by Kilian (2009) with speculation using data on oil price, production, global activity, and inventory. The authors have shown that demand shocks are still the main cause of fluctuations in the price of oil for the period 2003–2008. They did not find any strong evidence for speculative shocks during the oil price surge in 2003–2008. However, they found that speculation played an important role in an earlier period, 1986–1990. Juvenal and Petrella (2015), in revising and extending the work of Kilian and Murphy (2014), have assessed the role of both speculative oil demand and supply shocks in a factor-augmented vector autoregressive (FAVAR) model. They have found that speculation has had significant effects on the increase of oil prices since 2004. In addition, the most recent paper by Baumeister and Hamilton (2018) has discussed the relative importance of oil supply and demand shocks using a VAR model incorporating uncertainties to identify shocks using Bayesian method. They have found that the oil supply shock plays an important role in the variation of the oil price. Similar results are also found in Caldara et al. (2016) using an identified structural VAR model.

The empirical model presented in this paper is also connected to scholars that advocates the use of commodity storage models to derive the implications for oil and other commodity prices. The canonical commodity storage model was first developed by Williams and Wright (1991). Deaton and Laroque (1992, 1996) made the first attempts to confront the theoretical model with actual commodity prices. They used annual observations for thirteen commodities, including copper, palm oil, and other agricultural goods, for the period 1900–1987. The model in Deaton and Laroque (1992, 1996) is able to match the volatility, skewness, and kurtosis of commodity prices. However, the model cannot explain the high persistence that has been observed in commodity prices. In their model, the storage acts by "leaning against the wind", which stabilizes the production shock effect on the commodity prices between consecutive periods. Dvir and Rogoff (2010) have augmented the model of Deaton and Laroque (1992, 1996) by introducing stochastic growth dynamics into the income process. The storage, in fact, amplifies the income shock on prices. Nevertheless, Dvir and Rogoff (2010) did not apply their theoretical model to fit the empirical data on oil prices. This paper also employs an augmented storage model for examining the effect of storage on the oil market. In contrast to previous papers, I assume that both production and income processes are stochastic. On account of such an assumption, I am able to evaluate the relative importance of supply and demand shocks to variations in the oil price. Moreover, this paper also validates the theoretical model by applying it to the empirical data pertaining to the oil market for the period 1986–2009.

The remainder of the paper is divided in five sections. In section 2, I first introduce the data sources. Then I explain the multivariate BN decomposition method and discuss some stylized facts of oil cycles based on decomposed data. In section 3, I discuss the effect of a GDP shock on the price of oil in a VAR model using cyclical components of oil series. After that, in section 4, I describe my extended storage model incorporating stochastic production and income shock. I also explain the role of shocks in the simplified models. In section 5, I apply the extended storage model to the cyclical components of the oil price, world oil consumption and global GDP by using the SMM method. On the basis of the counterfactual analysis, I discuss the importance of income and production shocks. Employing estimated parameters, I also compute the impulse response of the oil price to a GDP shock in the extended storage model and compare it to the empirical evidence from the VAR model. Finally, in section 6, I provide some concluding remarks.

2 Oil market cycles

In this section I provide some facts about the oil market cycles through using empirical data. First, I introduce the data sources that have been employed for this analysis. Second, I explain the multivariate decomposition method for extracting cyclical components within the price of oil, word oil consumption, and global GDP.

2.1 Data sources

In this paper, quarterly data spanning the period 1986:Q1–2009:Q4 are used to estimate the model and assess its validity. The price of crude oil is the West Texas Intermediate (WTI) price[2], as taken from EIA (2016). Data on oil production and the total stock of crude oil in

[2]The quarterly observation of Brent oil price does not have large difference from the WTI price for the sample period. It has similar volatility and persistence as the WTI price. The standard deviation of the nominal WTI price is 20.35, and its autocorrelation is 0.96. The Brent price of crude oil has a standard deviation as of 20.37, and a autocorrelation as of 0.97.

OECD countries were obtained from the EIA (2016). World production of crude oil minus the change in the OECD stock of crude oil is used as a measure for total consumption of (demand for) crude oil. The GDP is computed by using the world GDP index, which is collected from Fagan et al. (2001) and transformed into GDP levels by using the annual GDPs from the World Bank (2014).[3] The nominal price of oil and GDP is deflated by using the U.S. CPI, taken from U.S. Bureau of Labor Statistics (2014).[4]

2.2 Extracting the cyclical component

Many studies have confirmed the comovement among oil price, global oil consumption, and world GDP (Hansen and Lindholt, 2008; Golombek et al., 2018). Cointegration tests have suggested that there is a long-run equilibrium among variables for different sample periods and from diverse data sources. Thus, the standard univariate detrending methods, such as the Hodrick-Prescott (HP) filter and the band pass filter, may not be proper and may suffer from misspecification. Furthermore, Harvey and Jaeger (1993), Cogley and Nason (1995) and Canova and Ferroni (2011) have criticized the fact that the Hodoric-Prescott (HP) filter fails to remove the stochastic trend and produces "spurious cycle" phenomena.[5] Cochrane (1988) discusses BN decomposition, introduced by Beveridge and Nelson (1981), gives a sensible definition of the trend component. The trend is the sum of the current variable and all the future expected changes. Therefore, in this paper, I employ the multivariate BN decomposition method, as the data transformation method for oil series. Similar method is also used in Cochrane (1994).

2.2.1 The Beveridge-Nelson decomposition method

The BN decomposition method is a model-based method for isolating series into permanent trend components and cyclical transitory components. Let us first assume a cointegrated vector $\mathbf{X}_t$. Then the Wold representation of $\triangle \mathbf{X}_t$ takes the form:

$$\triangle \mathbf{X}_t = \delta + \mathbf{\Psi}\left(L\right)\epsilon_t, \tag{1}$$

where $\mathbf{\Psi}\left(L\right) = \sum_{k=0}^{\infty} \psi_k L^k$ and $\psi_0 = 1$. δ is the deterministic trend growth rate and $\delta = E(\Delta \mathbf{X})$, $\mathbf{\Psi}$ denotes the coefficient vector, and ϵ_t is the residual in the Wold representation

[3]I first compute a factor as the ratio between the annual GDP level in 2009 and the average GDP index using quarterly observations in that year. The quarterly GDP level is computed as the product of the quarterly index and the constant factor.

[4]The U.S. CPI is commonly used as an measure of world inflation on average. Almoguera et al. (2011), Lin (2011) and Kilian and Murphy (2014) also use the U.S. CPI as the deflator of world GDP and oil prices.

[5]Their discussions are mainly focused on real business cycle models.

of $\triangle \mathbf{X}_t$. Then the trend component follows a unit root process with a drift, such as

$$\mathbf{X}_t^T = \delta + \mathbf{X}_{t-1}^T + \mathbf{\Psi}(1)\,\epsilon_t.$$

Using the recursive substitution for the Wold representation in (1) we can write X_t as a function of all the shocks, such that

$$\mathbf{X}_t = \mathbf{X}_0 + \delta t + \mathbf{\Psi}(1)\sum_{s=1}^{t}\epsilon_s + (1-L)\,\tilde{\Psi}(L)\sum_{j=1}^{t}\epsilon_t, \tag{2}$$

where $\tilde{\Psi}(L) = \sum_{j=0}^{\infty}\tilde{\Psi}_j L^j$ and $\tilde{\Psi}_j = -\sum_{k=j+1}^{\infty}\psi_k$.

By following the example of Beveridge and Nelson (1981), the trend component of a vector $\mathbf{X}_t$ is defined as the limiting forecast as horizon goes to infinity, adjusted for the mean growth rate

$$\mathbf{X}_t^T \equiv \lim_{h\to\infty}\mathbf{X}_{t+h|t} - \delta h = \mathbf{X}_t + \sum_{i=1}^{\infty}\left(E_t\triangle\mathbf{X}_{t+i} - \delta\right). \tag{3}$$

Equation (3) also implies that if the variable $\mathbf{X}_t$ is forecasted to rise, its level is below the trend. Inserting (2) into (3), I obtain the trend component as follow,

$$\mathbf{X}_t^T = \mathbf{X}_0 + \delta t + \mathbf{\Psi}(1)\sum_{s=1}^{t}\epsilon_s. \tag{4}$$

In this equation, $\mathbf{X}_0$ is the initial value of $\mathbf{X}_t$ in period zero, $\mathbf{\Psi}(1)$ measures the long-run impact of forecast error, and δt represents deterministic trend. Furthermore, the cyclical component at time t can be computed by employing the following equation:

$$\mathbf{X}_t^C = \mathbf{X}_t - \mathbf{X}_t^T = (1-L)\,\tilde{\Psi}(L)\sum_{j=1}^{t}\epsilon_t. \tag{5}$$

The term $(1-L)\,\tilde{\Psi}(L)$ is the measure of transitory impact of forecast errors.

According to equation (4) and (5), the implementation of BN decomposition on the crude oil price, world oil consumption and global GDP indicates that oil market growth consists of both deterministic and stochastic trends. The cyclical components of oil occur due to the fluctuations in the structural growth of the oil market.

Table 1: Cointegration Test

	(a) ADF Test of Unit Root				
	No.lag	Test statistic	5% Critical value	p-value	Conclusion
World consumption: lnQ	1	-4.798	-3.459	0.001	I(0)
World GDP: lnY	0	0.777	-3.458	0.999	I(1)
Price of oil: lnP	0	-2.240	-3.458	0.472	I(1)
(b) Johansen Test for the Existence of Cointegration Vectors					
Cointegrating rank	0	1	2		
Trace statistics	68.592	30.142	5.854		
5% critical value	35.193	20.262	9.164		
p-value	0.001	0.002	0.204		
Number of obs.	96				
Differenced lags	1				

Notes: Table (a) shows the ADF statistics with a drift and deterministic trend for the unit root process of each variable. The number of lags used is selected by the AIC and BIC index. The critical value of rejecting the null hypothesis at 5 percent level and the p-value of the statistics are presented. The conclusion of the ADF test at 5 percent significance level is listed. Table (b) shows the Johansen test of vector cointegration for different cointegration ranks. This vector includes $\ln Q$, $\ln Y$, and $\ln P$. is the coefficient for the lagged vector in the VEC model. 96 observations are used in the tests.

2.2.2 Implementation of multivariate BN decomposition for oil data

To evaluate the applicability of multivariate BN decomposition for the oil price, world oil consumption, and global GDP, I employ a state-space approach (see Cochrane (1994) and Morley (2002)). I perform the decomposition in two steps. First, I perform a Johansen's unrestricted cointegration rank test (trace test) among logarithm values of oil consumption, world GDP, and oil price based on a VEC model of vector $\mathbf{X}_t = [\ln GDP_t, \ln OilQuantity, \ln Price_t]$ such that

$$\triangle \mathbf{X}_t = \beta + \Pi \mathbf{X}_{t-1} + A_1 \triangle \mathbf{X}_{t-1} + u_t, \tag{6}$$

where u_t denotes the residual. The augmented Dickey–Fuller test (ADF) tests as shown in Table 1 indicates that world GDP and the crude oil price are I(1). However, world oil quantity is a stationary series. Furthermore, the cointegration test results in Table 1 show that both the null hypothesis of rank(Π)$\leq$0 and rank(Π)$\leq$1 are rejected at 5 percent significant level. But I can not reject the null hypothesis of rank(Π)$\leq$2. It suggests that there are at least one cointegrating relationship within the system. Second, I decompose trend and cyclical components by using estimates from the VEC model in equation (6), with

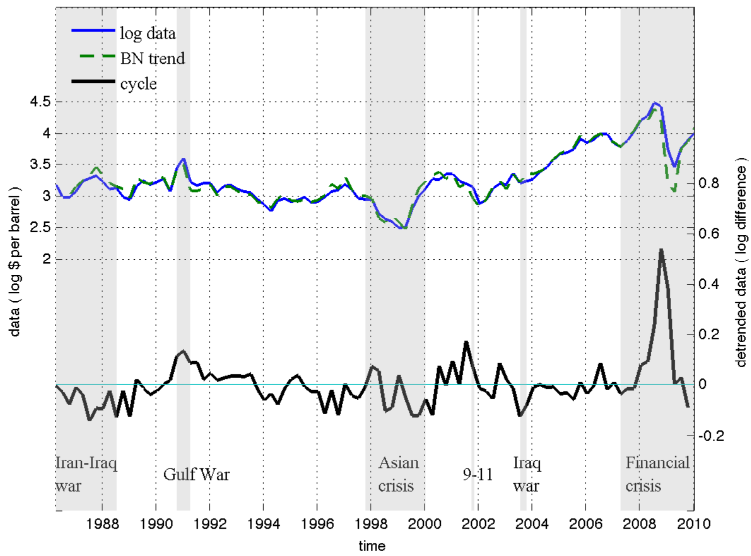

Figure 1: Crude oil prices: $\ln P$

Notes: The figure plots the real crude oil price in logarithm value, the BN trend, and the cyclical component of the real oil price. The real price of oil is measure in 1996 USD. The left scale is for the price data. The right scale is for the deviations from the trend and has units as percent deviations from the trend. The shaded area indicates specific period when important events were taking place in the oil market. These events include the Iran-Iraq war (1986:Q1–1988:Q2), the Gulf War (1990:Q3–1991:Q1), the Asian financial crisis and oil crisis (1997:Q2–1999:Q4), the 9/11 attacks (2001:Q3), the invasion of Iraq (2003:Q2–2003:Q3), and the global financial crisis (2008:Q1–2009: Q4).

detected numbers of cointegration relationship.[6] The approach in details is summarized in Appendix A.

Properties of the transformed data

After steps 1 and 2, I decompose the global GDP, oil consumption, and oil price into trend and cyclical components. Figure 1 plots the trend and cyclical components of the real oil price. The bold solid line in Figure 1 denotes the cyclical component in price. It is measured in terms of its percentage deviation from the trend component. The cyclical component varies around the zero-mean level.

The correlation between the trend and cyclical components of oil price is negative, –0.61.

[6]The VEC in equation (6) can be rewritten into an ARIMA(2,1,0) process, such that

$$\mathbf{X}_t = \beta + \mathbf{X}_{t-1} + (\pi + A_1)\,\mathbf{X}_{t-1} - A_1\mathbf{X}_{t-2} + u_t.$$

Thus an increase in the price trend is associated with a decrease in the cycle component, while, according to Stock and Watson (1988), the variations in the cycle component indicate adjustments towards the shifting trend. This finding suggests that the cycle component will decrease initially with an increase in the price trend and that this results in a lower net increase in the oil price. The negative impact of cyclical innovations is temporary and dissipates over time.

Furthermore, Figure 1 shows that the fluctuations in the cycle component are consistent with the anecdotal evidence on the relative importance and timing of the fluctuations in the global crude oil market, which helps to verify the proposed decomposition. For instance, the price cycle became negative after Saudi Arabia changed its policy from reducing supply to maintain the price to increasing supply to maintain market share from 1986. Then the oil price fell substantially, while the price cycle increased sharply at the beginning of 2007 and dropped dramatically due to the financial crisis in late 2008.

The relationship between cyclical components in the world GDP and the global oil quantity is discussed next. Figure 2 exhibits the evolution of cyclical components in the global oil consumption as measured against world GDP. The global oil consumption presents a procyclical pattern—in other words, the global oil consumption has a positive contemporaneous correlation with world GDP. The correlation is 0.30 for the period 1986:Q1–2009:Q4 and 0.5 for the last part of this period, 1999:Q1–2009:Q1. It is apparent that during the recession period from late 2008 to 2009, there is a tight correlation between the cyclical components of global oil consumption and world GDP.

Furthermore, Table 2 shows the volatility, persistence, and correlation for cyclical components of the variables. Columns 1 and 2 show that the oil price cycle is 9.8 times as volatile as the world GDP cycle, where the cyclical component of world GDP is a 1 percent deviation from its trend, and the price cycle is a 9.8 percent deviation from the trend of the crude oil price. Furthermore, the world oil consumption cycle is more volatile than the world GDP cycle, where the consumption volatility is at 2.4 percent. In addition, column 3 shows the persistence of the series. The first order autocorrelation is 0.58 for the price cycle and 0.40 for the oil consumption cycle. Moreover, the persistence of the world GDP cycle is 0.62. The autocorrelation of world GDP is lower than 1, which confirms the stationary feature of the cyclical components of the world GDP.

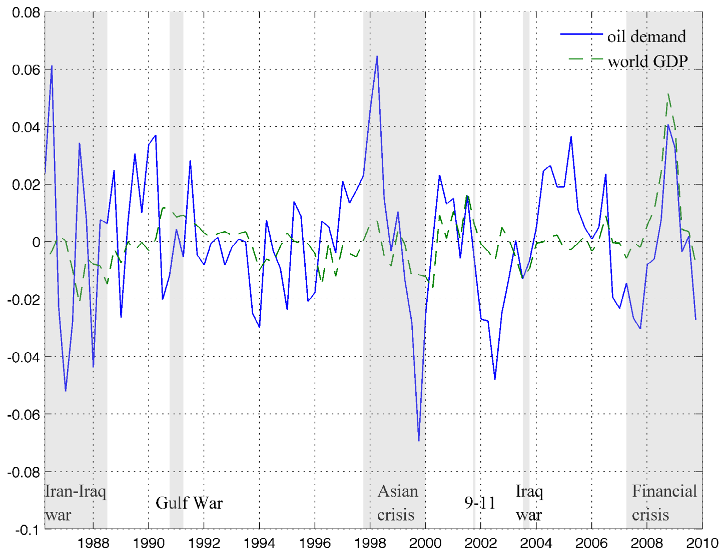

Figure 2: Comovement of cyclical components in the oil consumption and world GDP

Notes: The figure plots the cyclical components of global oil consumption and world GDP. The cyclical components have been estimated through using a multivariate BN decomposition method.

Table 2: Cyclical properties of the global oil market

Log values	Standard deviation	Relative standard deviation	1st-order autocorrelation
World GDP cycle	0.010	1.000	0.624
Oil price cycle	0.098	9.799	0.577
Oil consumption cycle	0.024	2.365	0.404

Notes: This table is generated by using the logarithm value of cyclical components.

3 VAR analysis

In this section, I discuss the features of the oil cycles by using a VAR model. The purpose of this analysis is to investigate the effect of GDP shock to the oil price cycles using the VAR model as in the seminal papers of recent literature in this branch (Kilian, 2009, Baumeister and Peersman, 2013, and Kilian and Murphy, 2014).

Let us consider a structural VAR model of the oil market for $z_t = [\ln Y_t, \ln Q_t, \ln P_t]$. In this formula, $\ln Y_t$, $\ln Q_t$ and $\ln P_t$ denote the logarithm values of cycle components in world GDP, global oil quantity, and oil price (see Figure 1 and 2). In following the design of Kilian (2009), the VAR model takes the following form:

$$z_t = \alpha + \sum_{i=1}^{K} A_i z_{t-i} + e_t, \tag{7}$$

where e denotes the reduced form residuals, A_i is the coefficient matrix of the autoregressive terms, and α is the constant vector. I impose a Cholesky assumption of long-run restrictions in order to identify GDP, supply, and price shocks. I denote ϵ as the structural residuals where $E\epsilon_t\epsilon_t' = I$. Then the Cholesky decomposition states that

$$e_t = \begin{pmatrix} e_t^Y \\ e_t^Q \\ e_t^P \end{pmatrix} = A\epsilon_t = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{pmatrix} \epsilon_t^Y \\ \epsilon_t^Q \\ \epsilon_t^P \end{pmatrix}, \tag{8}$$

and $AA' = Ee_te_t'$.

According to the restriction on A_0^{-1} in (8), I assume an exogenous process for the GDP cycle, where the GDP cycle is not affected by the supply shock ϵ_t^Q and the price shock ϵ_t^P in the same period. I also assume that the current production of oil has no response to instantaneous changes to the oil price.[7]

I estimate the structural VAR model in (7) by using the method of least squares. The estimated values are then employed to construct the impulse response results. Figure 3 plots the response of the oil price cycle to one standard deviation structural shocks on GDP, supply, and the price of oil. The structural shocks are assumed to be orthogonal. The shaded area in Figure 3 denotes the inference generated by using a bootstrap method for 5,000 replications of the simulation.

Figure 3 shows how a GDP shock causes a sharp and significant increase in the oil price. Furthermore, this positive response to the GDP shock lasts at least three quarters. This

[7] These assumptions are different from those of Kilian (2009), who assumed a vertical short-run supply curve, while the shift in the aggregate supply is the result of the simultaneous change in the oil supply.

graphic depiction reveals that GDP is an essential factor that affects the oil price cycles, and this result, moreover, is consistent with that of Golombek et al. (2018), who have concluded that global income is the main driving force behind the oil price.

The supply shock has a positive but lower effect on the oil price cycle when the positive supply shock occurs instantaneously; however, the response of the oil price cycle becomes negative in the ensuing periods as an effect of increasing oil supply, and then converges to zero over the time of simulation. One possible reason for the movement of price back to zero is that the production of oil increases in some countries (OPEC countries, for instance), and the other countries have delayed responses and react to the lower prices by reducing their supply, which in turn contributes to the adjustment back to zero of the oil price in the ensuing period.

The price shock may refer to unanticipated price changes that affect the expectations of oil availability in the market.[8] Figure 3 shows an ambiguous effect of price shock. The price shock has initial positive effect on the oil price cycle by construction, but the effect changes direction and becomes statistically insignificant after four quarters.[9]

To summarize, employing data on cyclical components of world GDP, oil quantity and crude oil price in a VAR model, I find a large and positive effect of world GDP shock on the oil price cycles. In the next two sections, I will discuss the relative importance of GDP shock in an extended storage model, as a comparison to the empirical evidence in the VAR model.

4 An extended commodity storage model

Considering the importance of world GDP shock (discussed in section 3), I introduce an additional shock into the model of Deaton and Laroque (1992, 1996) to capture fluctuations in the income process, while I retain the shock on production. In the following section, I start by describing the extended commodity storage model, including both stochastic income and production processes. Afterwards, I discuss the role of income and production shocks to the oil prices in simplified versions of the model, that is two single-shock models. I solve these two simplified models with fixed parameters and compare the moments of simulated data.

[8]Kilian (2009) has discussed how exogenous political events can be seen as an example of the price shock.

[9]Due to the ambiguous nature of the evidence, I am not focusing on the effect of the price shock in this paper.

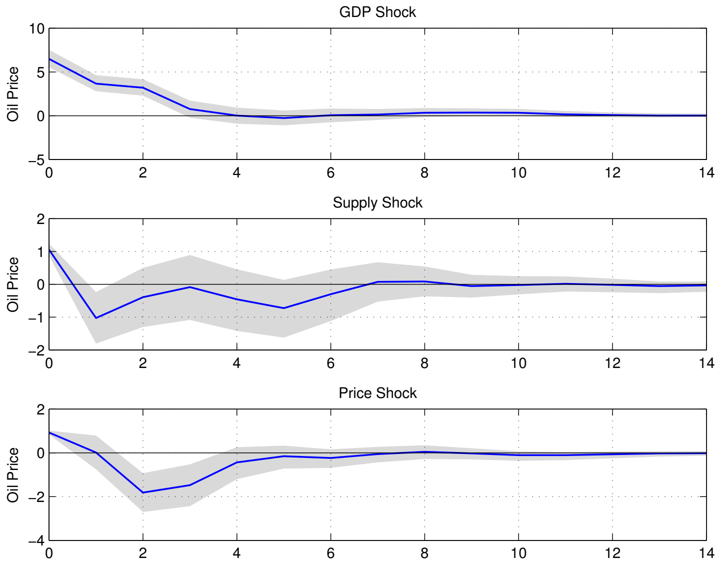

Figure 3: Impulse responses of the oil price cycle to structural innovations from a VAR model

Notes: The figure plots the responses of the oil price cycle to structural shocks of one standard deviation. The inference shown as the shaded area in the figure is constructed through using a bootstrap method for 5,000 replications.

4.1 A storage model with income shocks and production shock

This analysis assumes a risk-neutral speculator who chooses to store crude oil by maximizing his/her aggregate net present value of profit within a discrete time framework. This speculator can be, for instance, seen as the OPEC-core countries who decide the extraction of crude oil or the amount of oil left under ground.[10] Whereas, the rest of the world does not have market power and their supply of oil is assumed to be exogenous in the short run for simplicity.

Availability

I consider A_t as the oil availability, also defined as "amount at hand" in Deaton and Laroque (1992, 1996), which measures the amount of oil available to be consumed in period t. This amount of oil has been produced at time t or an earlier period, and it has not been sold before the current period t . Accordingly, the availability of oil at period t is the sum of any storage carried from the previous period X_{t-1} and current production Z_t, such that the following equation can be formulated:

$$A_t = X_{t-1} + Z_t, \tag{9}$$

where $X \geq 0$. An alternative interpretation of X could be OPEC core's spare capacity. In other words, X can be seen as the measure of the extra production the OPEC core can produce, if the OPEC core group decides to increase supply in the short run. Following the example of Dvir and Rogoff (2010), I assume a zero depreciation rate on carrying storage from previous periods for the sake of simplicity.[11]

At each point of time, the availability of oil, including the storage in earlier period, should equal the current consumption together with the inventory stored for the next period, such that the following equation can be formulated:

$$A_t = Q_t + X_t. \tag{10}$$

Demand function

Following the example of Dvir and Rogoff (2010), I assume an iso-elastic demand function of oil such that the oil price cycle is a function of oil demand and income,

$$P_t = \left(\frac{Q_t}{Y_t} \right)^{-\gamma}, \tag{11}$$

[10] Alhajji and Huettner (2000) and Golombek et al. (2018) find that OPEC or OPEC core exerts market power.

[11] Deaton and Laroque (1992, 1996) assume a non-zero depreciation rate on the storage.

where $\gamma > 1$. According to equation (11) the price P_t is a Constant Relative Risk Aversion (CRRA) function that provides the effective demand of oil at rate γ. Accordingly, $\frac{1}{\gamma}$ is the price elasticity of demand. In this case, I implicitly assume a unit income elasticity of demand where the percent change in demand is equal to the percent change in income ($\partial \ln Q / \partial \ln Y = 1$). This assumption is in line with empirical results from several recent studies. For example, Gately and Huntington (2002) have found that income elasticities are around 1 for non-OECD countries; more recently, Golombek et al. (2018) have used a structural model to estimate an income elasticity of demand of 1.11.

Inserting equation (10) into (11), the inverse demand function can be written in terms of availability and income:

$$P_t = \left(\frac{A_t - X_t}{Y_t} \right)^{-\gamma}. \tag{12}$$

Income and production shocks

The production of oil is sensitive to the events in the producing area, which would suggest oil production amount fluctuates over time. Similar to the work of Deaton and Laroque (1992, 1996)[12], I assume that the production cycle Z_t fluctuates around a constant level $\bar{Z}$, such that

$$Z_t = \bar{Z} \exp\left(e_t^z\right), \tag{13}$$

where e_t^z is a stochastic production shock that follows a normal distribution $e_t^z \sim N\left(0, \sigma_z^2\right)$. $\bar{Z}$ denotes a constant parameter. The logarithm of production is distributed at mean $\ln \bar{Z}$ and the standard deviation at σ_z, $\ln Z \sim N\left(\ln \bar{Z}, \sigma_y^2\right)$. The production shock reflects the disturbances from the supply side. It can be recognized as the wars, political events or any unobserved failures of production in the crude oil producing countries. This simplified assumption of exogenous production is standard for short run. For instance, Kilian (2009) also assumed a vertical short-run supply of oil, where supply of oil adjusts infrequently to changes in demand.

As a revised version of the work of Deaton and Laroque (1992; 1996), I also consider that the income cycle follows a stochastic process, such that

$$Y_t = \bar{Y} \exp\left(e_t^y\right), \tag{14}$$

where e_t^y is a normally distributed shock at mean zero and with a standard deviation at σ_y, $e_t^y \sim N\left(0, \sigma_y^2\right)$. $\bar{Y}$ is a constant of the income level. Subsequently, the logarithm of income is distributed at mean $\ln \bar{Y}$ and the standard deviation at σ_y, $\ln Y \sim N\left(\ln \bar{Y}, \sigma_y^2\right)$. A

[12] Deaton and Laroque (1992; 1996) derive the implications for different commodity prices, mostly agricultural crops but also copper, i.e., a non-renewable mineral.

positive income shock can be recognized as a boom in the world economy; on the contrary, a negative income disturbance can be seen as a global recession.

Speculation equilibrium

I solve for the rational expectation model of maximizing expected profits under competitive economy. The arbitrage conditions imply that when storage is positive, the current price equals the expected price in period $t+1$, apart from marginal storage cost. Otherwise, the storage becomes zero when the current price is higher than the marginal gain from storage. That is to say,

$$P_t = \beta E_t \left[P_{t+1}\right] - C, X_t > 0 \tag{15}$$

$$P_t > \beta E_t \left[P_{t+1}\right] - C,\ X_t = 0 \tag{16}$$

where C denotes the cost of storage. β denotes the discount factor.

Thus, the current price depends not only on the current quantity to income ratio as shown in equation (11), but also on the future demand through choosing the storage level X_t according to the future expectation, as in equation (15) and (16).

4.2 The role of income and productions shocks in the rational expectations equilibrium: the mechanism

In order to detect the role of income and production shock on the price in the commodity storage model, in this section, I discuss the results from two simplified storage models: 1) a storage model with a stochastic production process that assumes a constant income $Y_t = \bar{Y}$,[13] 2) a storage model with a stochastic income process that assumes constant production $Z_t = \bar{Z}$.

The rational expectations equilibrium

The solutions from the two commodity storage models specify a rational expectations equilibrium of storage as as function of state variables. In the first storage model with stochastic production, availability A_t is the state variable. Recall that production Z_t is an element of A_t, the optimal storage rule is denoted as $X\left(A_t\right)$. In the second model with stochastic income, both availability A_t and income Y_t are state variables. The optimal storage rule is written as $X\left(A_t, Y_t\right)$. I solve the two nonlinear rational expectations commodity storage model numerically. Following Miranda and Fackler (2002), I solve the dynamic models by using a spline collocation method for function approximation. The collocation approach in details is summarized in Appendix B.

[13]This model is similar to the case assuming a strictly convex price function in Deaton and Laroque (1992).

Simulated moments

To discuss the effect of shocks on equilibrium prices, I simulate prices by employing the optimal storage rule from the two models. The simulation is performed for 20,000 quarters in order to obtain stationary moments. In each period, a production shock or an income shock is drawn from a normal distribution with a mean value of 0 and a standard deviation of 0.1. I impose the same production shock and income shock in absolute values, $\epsilon_t^Y = -\epsilon_t^Z$, for each period.[14] Using simulated shocks and transition functions, I compute state variables in the next period. Subsequently, the storage level can be interpolated by using the optimal storage rule of $X(A_t)$ and $X(A_t, Y_t)$ in each case. After this, I compute the equilibrium price by using the demand function in equation (12).[15] I perform 1,000 repetitions of the simulation in each model.[16] The first 100-period simulations are deleted in order to eliminate the effects of initial values.

Table 3 shows the moments of simulations in the case of production shock and income shock, respectively. The values in Table 3 are the mean value of the simulated series' moments over 1,000 repetitions.

From Table 3, I find that the storage model with a stochastic production process has a stronger and more intensive smoothing effect than the model with a stochastic income process.[17] To be specific, the simulated price in the production-shock model has lower standard deviation with a value of 0.23 (versus 0.25 in the income-shock model) and a higher persistence with a value of 0.53 (versus 0.51 in the income-shock model), although the production-shock model generates a higher percentage of stock-out with a value of 3.5 percent (versus 1.2 percent in the income-shock model).

Impulse responses

The reason for the stronger smoothing effect in the production-shock model is mainly due to the higher inventory level. As represented in Table 3, I find that the mean level of storage has a value of 0.22 in the production-shock model and 0.19 in the income-shock model.

To support this point, I illustrate the impulse responses of availability, storage, and prices of the two models in Figure 4 using the same initial values in period zero.[18] I impose one standard deviation of positive production shock and negative income shock,

[14]For comparison, I impose reversed income shocks and production shocks in order to have similar (positive or negative) effects on the price in both models.

[15]Other parameters used for simulation are set as $\gamma = 5$, $\beta = 0.97$ and $C = 0$

[16]The standard error of simulated moments are very robust to changes of larger numbers of repetition in the simulation.

[17]In Table 3, the 95 percent confidence intervals of simulate moments in two models do not overlapped. Thus, the differences are significant.

[18]The initial values of state variables for the simulation are $A_0 = 1.1$ and $Y_0 = 1$.

Table 3: Simulated price moments in the storage model with production shock and income shock

Models	Production (Z) shock	Income (Y) shock
σ_z	0.1	0
σ_y	0	0.1
	(1)	(2)
% of stock outs	3.511	1.186
Moments of P		
mean(P)	0.997	1.039
std(P)	0.226	0.252
1st-order a.c.(P)	0.529	0.513
Moments of A		
mean(A)	1.222	1.192
std(A)	0.176	0.125
1st-order a.c.(A)	0.816	
Moments of X		
mean(X)	0.217	0.192
std(X)	0.144	0.125

Notes: Column (1) shows the moments of price and demand by using the simulated data from the storage model with stochastic production shock. Column (2) shows the moments of simulated data from the storage model with stochastic income shock. The simulations are performed through using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from the initial values. The measures shown in the table are the average over 1,000 repetitions.

respectively, in period one, with $\sigma_Y = \sigma_Z = 0.1$. The shocks are simulated for 20,000 repetitions in period one. The simulation in the subsequent 13 periods assume zero shock on production and income in both cases; thus, $\epsilon_t^Z = \epsilon_t^Y = 0$ for $t > 1$. The series in Figure 4 shows the average over 20,000 repetitions of the simulation across 14 periods.

As represented in the panel (a) of Figure 4, there is an immediate response of availability to the positive production shock in period one in the production-shock model. It simultaneously determines a high storage level in period 1 through the policy function (as shown in the panel (b)). On the other hand, in the income-shock model, the income shock does not have a direct effect on the availability. The availability remains at the same level in period 1.[19] It results in a relatively lower level of storage than in the production-shock case in the shock period in panel (b), although the storage increases due to the negative income shock and thus lower expected price. In the production-shock model with a higher storage level, panel (c) shows a limited drop of price in the shock period and a fast recovery toward a steady state after period 2. Thus, the storage level is more sensitive to the production shock than to the income shock, which introduces a stronger smoothing effect in the production-shock model. This is mainly because income shock is a multiplicative shock on availability, whereas supply shock is an additive shock.

To summarize, the storage model is more sensitive to the production shock than to the income shock. The production shock introduces a more intensive smoothing effect on the equilibrium price, with a low variance and high persistence. In the next section, I will go back to my extended storage model with both production and income shocks, and fit the model to the data. I will also discuss the relative importance of income shock and production shock.

5 Matching the extended storage model and the data

In this section, I apply the extended storage model with both a stochastic production and income process to the cyclical components of oil prices, world oil consumption, and global GDP. I employ a moment-matching method for estimation. To discuss the relative fitness to the data, I also compare the estimates and simulated moments from the extended storage model and the model with production shocks. Later, using estimated parameters from the extended storage model, I explain the dynamic behavior of oil prices. Later, I discuss the relative importance of income and production shock through counterfactual analysis.

[19] This is consistent with the moments of simulated availability shown in Table 3. In the income-shock model, the availability has a lower mean with a value of 1.19 and with a standard deviation of 0.125, whereas, in the production-shock model, the mean is 1.22 and the standard deviation is 0.18.

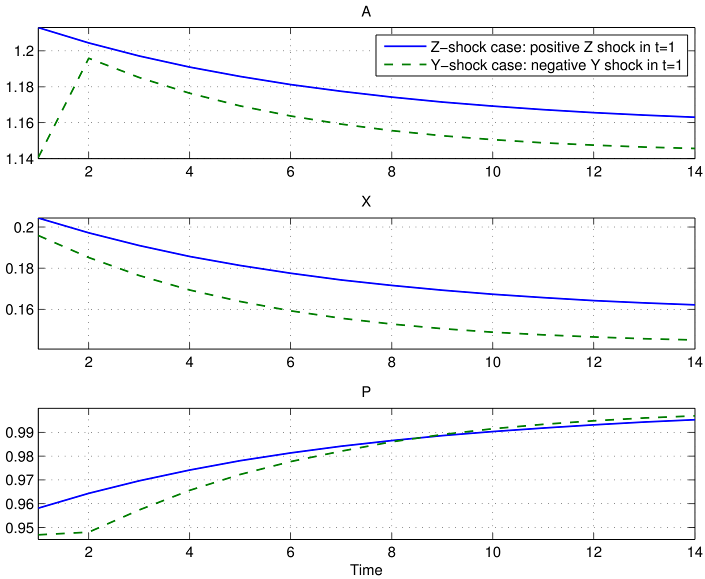

Figure 4: Impulse responses in storage model: The Income-shock and production-shock case

Notes: The initial values of state variables for the simulation are set as $A_0 = 1.1$ and $Y_0 = 1$. One-standard deviation production shock and income shock are imposed in period 1 with $\sigma_Y = \sigma_Z = 0.1$, and let $\epsilon_1^Z = -\epsilon_1^Y$. The shocks are simulated for 20,000 repetitions in period 1, and then the series are simulated for the next 13 periods. The series shown in the figure are the average values over 20,000 repetitions.

Finally, I compare the impulse response of oil price to the income shock in the extended storage model and the VAR analysis.

5.1 Estimation

Calibrated parameters

By assuming an exogenous process on income and production in equation (13) and (14), some parameters can be calibrated independently from the storage model. Through the construction of BN decomposition, the logarithm values of oil cycles fluctuate around 0. It is reasonable then to have $\bar{Y} = 1$ and $\bar{Z} = 1$. Moreover, knowing the log-normal distribution of income with $\ln Y \sim N\left(\ln \bar{Y}, \sigma_Y^2\right)$, I calibrate the standard deviation of income shock at $\sigma_Y = 0.02$ through using an observed GDP cycle.[20]

SMM

Subsequently, I employ a SMM estimation method introduced by Lee and Ingram (1991), which estimates structural coefficients – the demand elasticity parameter γ, production volatility σ_z, and discount rate β. The SMM estimation is implemented by first solving the extended storage model (with a stochastic production and income process) through using a collocation method for a set of parameters, and then simulating series of price $\tilde{P}_t$, oil demand $\tilde{Q}_t$, and income $\tilde{Y}_t$ by using the policy function of storage.[21] The optimal coefficients are determined when they minimize the weighted sum squares of the difference between empirical and simulated data moments, such that the following equation can be formulated:

$$\hat{\theta} = \arg\min_{\theta} M(\theta)' W M(\theta). \tag{17}$$

$\theta \equiv [\gamma, \beta, \sigma_Z]$ is the structural coefficient vector of interest, which is a $l \times 1$ $(l = 3)$ vector. $M(\theta)$ is a $k \times 1$ moment condition which is the difference between empirical and simulated data moments, such that

$$M(\theta) = \frac{1}{T}\sum_{t=1}^{T} m(P_t, Q_t, Y_t) - \frac{1}{N}\sum_{t=1}^{N} m\left(\tilde{P}_t(\theta), \tilde{Q}_t(\theta), \tilde{Y}_t(\theta)\right), \tag{18}$$

where T denotes the number of observation; N denotes the sample size of simulated data, and $N = T \times H$ where $H \geq 1$.[22] I employ $H = 50$ in the estimation.[23] $m(P_t, Q_t, Y_t)$ and

[20] Since the production of oil is unobserved, the standard deviation of production shock σ_z can not be calibrated and is estimated using the moment-matching method.

[21] I assign $\bar{Z} = 1$ and calibrated parameters of $\bar{Y} = 1$ and $\sigma_Y = 0.02$ using world GDP data.

[22] Following Michaelides and Ng (2000), the data are simulated for $T \times H \times 1.1$ periods, and the first 10 percent-period simulated data is trimmed.

[23] Michaelides and Ng (2000) found a good sample performance when the simulated sample is approxi-

$m\left(\tilde{P}_t(\theta), \tilde{Q}_t(\theta), \tilde{Y}_t(\theta)\right)$ are moments computed by using observed data and simulated data.[24] In equation (17), W denotes the optimal weighting matrix evaluated as the inverse of the covariance matrix of empirical data moment $m(P_t, Q_t, Y_t)$; accordingly, the following equation applies:

$$W = \frac{1}{T}\left[m(P_t, Q_t, Y_t)' m(P_t, Q_t, Y_t)\right]^{-1}. \tag{19}$$

In the SMM estimation, the moment function includes the mean, variance, auto-covariance of price and quantity demand, and income-quantity covariance, such that the following equation applies:

$$m(\theta) = \begin{bmatrix} \left(P_t - \bar{P}\right)^2, \left(Q_t - \bar{Q}\right)^2, \left(Y_t - \bar{Y}\right)\left(Q_t - \bar{Q}\right), \\ \left(P_t - \bar{P}\right)\left(P_{t-1} - \bar{P}\right) \\ \left(Q_t - \bar{Q}\right)\left(Q_{t-1} - \bar{Q}\right) \end{bmatrix}. \tag{20}$$

Furthermore, the asymptotic distribution of θ is given by

$$\sqrt{T}\left(\hat{\theta} - \theta_0\right) \to^d N(0, \Omega),$$

where Ω denotes the $k \times k$ covariance matrix, such that

$$\Omega \equiv \left(1 + \frac{1}{H}\right)\left[\frac{\partial M(\theta)}{\partial \theta} W \frac{\partial M(\theta)}{\partial \theta'}\right]^{-1}.$$

Estimation results

Following equation (17)–(20), I obtain the SMM estimates as shown in Table 4. The first column in Table 4 shows the estimated coefficient, and the second column presents the standard error of the estimates.

The coefficient γ, which measures the relative changes of price with respect to the change in the Q to Y ratio, is estimated at 5.22, which is statistically significant with a 95 percent confidence interval. This implies a price elasticity of oil demand of $-\frac{1}{\gamma} = -\frac{1}{5.22} = -0.19$.[25] This estimate is close to the results of other scholars who have studied the oil price elasticity of demand. Dahl (1993), Gately and Huntington (2002), Cooper (2003), and Dées et al. (2007), among others, estimated single-equation models of oil demand, and

mately 10 times as large as the actual data. I also find that the estimates are robust to different numbers of H, when $H \geq 10$.

[24] In the SMM estimation, I assign the same random values for the production shock and income shock in each iteration. This is in order to satisfy the property of "stochastic equicontinuity" for simulation estimators as shown in McFadden and Ruud (1994).

[25] The standard error for the price elasticity of demand is 0.014, which is computed through the delta method.

Table 4: SMM estimates of parameters for the extended storage model

	Extended storage model		Z-shock storage model	
	coeff.	s.e.	coeff.	s.e.
γ	5.215	(1.343)	3.156	(0.197)
β	0.989	(0.006)	0.975	(0.196)
σ_Z	0.055	(0.016)	0.006	(0.021)
J-statistics	1.064		12.605	
p-value	58.8%		0.6%	
Degree of freedom	2		3	

Notes: The heteroskedasticity and autocorrelation consistent (HAC) standard errors are shown in parentheses.

obtained demand elasticities for crude oil between -0.2 and -0.6. Alhajji and Huettner (2000) estimated a structural model of crude oil market and arrived at a price elasticity of demand of -0.25.

The quarterly discount factor is estimated to be 0.99 (with a standard error of 0.01). It implies a quarterly interest rate at 0.01, and a annual interest rate at 0.04.[26] This estimated interest rate is close to the results in Deaton and Laroque(1992; 1996), in which the annual interest rate, for example, is estimated to be 0.05 for copper. Furthermore, I obtain a statistically significant production volatility of 0.06, which is five times larger than the volatility of income shock.

In addition, an overidentifying test is implemented to test the model specification. The Sargan-Hansen's J-statistic is computed as the fraction of the optimal value of the SMM objective function. The J-statistic follows a Chi-square distribution at $k-l$ degrees of freedom, as shown in the following equation:

$$T\left(1+\frac{1}{H}\right)\left[M\left(\hat{\theta}\right)' WM\left(\hat{\theta}\right)\right] \xrightarrow{d} \chi^2\left(k-l\right).$$

In the case of this analysis, I have $k=5$ and $l=3$, and thus $k-l=2$. Table 4 shows a J-statistic of 1.064. This suggests that we can not reject the null hypothesis that model moments match data moments, $H_0 : E\left[M\left(\theta\right)\right]=0$, at a significance level of 5 percent. Thus I conclude that the model is correctly specified and the parameters are estimated consistently.

As a comparison, I also estimate the storage model with only production shock as in

[26]The discount factor is function of interest rate r, such that $\beta=\frac{1}{1+r}$.

Deaton and Laroque (1992) using SMM.[27] The estimates are shown in the last column in Table 4. I find that γ is estimated significantly at 3.16 with standard error 0.20. I employ a Z-test to investigate the equality of estimate γ from the extended storage model and the production-shock model.[28] Using the estimates of γ from these two models, I compute a Z-statistic at 1.52 which implies a p-value at 12.9 percent. Thus I can not reject the null hypothesis of equal γ estimated from the two models at 10 percent significance level. Next, the estimated discount rate β in the production-shock model is 0.98, which is very close to the result from the extended storage model. However, in the production-shock model, the production shock volatility is estimated statistically insignificant (0.006 with standard error 0.02). Meanwhile, the Sargan-Hansen's J test is computed 12.61 with p-value at 0.6 percent, which suggests a rejection of the null hypothesis of model moments matching data moments at 5 percent significance level. The last two results reveal that the commodity storage model with only production shock may be misspecified. The empirical results also show that the extended commodity storage model represents a major improvement over the original Deaton and Laroque (1992)'s model for the crude oil market.

5.2 Fitness of the storage model

Using the estimated parameters from the extended storage model in Table 4, I discuss in this section the applicability of the extended storage model to the data by comparing the moments of empirical data and the simulated data from the model. I also compare the moment of simulated data from the extended storage model and the storage model with production shock only to discuss which model has better fitness to the data. The results are presented in Table 5.

The simulations are performed by using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from initial values. Column 1 in Table 5 shows the data moments of oil price cycles and oil consumption cycles. Column 2 shows the moments of simulated series from the extended storage model that allow for positive storage. Column 3 shows the moments of simulated series that impose zero storage in all periods. Column 2 and 3 show the mean value of moments over 1,000 repetitions.

In Table 5, column 1 and 2 reveal a good fit between the model and the data. The columns show that the storage model generates a price with a mean value of 1.00 and with

[27] The moment function includes the mean, variance, and auto-covariance of price and quantity demand.

[28] The Z-statistic is compute as $Z = \frac{\gamma_1 - \gamma_2}{\sqrt{(SE\gamma_1)^2 + (SE\gamma_2)^2}}$, where γ_1 and γ_2 denote the estimates from the extended storage model and from the production-shock model respectively, and $SE\gamma$ denotes the standard error of γ.

Table 5: Moments of oil price and simulated prices in the extended storage model and the storage model with production shock only

	Data	Extended storage model		Z-shock storage model
		$X \geq 0$	$X = 0$	$X \geq 0$
	(1)	(2)	(3)	(4)
% of stock outs		3.191	100	0.229
mean(P)	1.005	1.000	1.043	1.000
std(P)	0.113	0.124	0.308	0.011
1st-order a.c.	0.591	0.536	-0.001	0.514
skewness	3.547	3.422	0.911	0.364
kurtosis	21.191	23.046	4.506	3.630
mean(Q)	1.001	1.002	1.002	1.000
std(Q)	0.024	0.021	0.055	0.003
1st-order a.c.	0.402	0.515	-0.001	0.515
skewness	0.097	-1.511	0.165	-0.311
kurtosis	3.437	7.969	3.049	3.557
corr(Q,Y)	0.301	0.367	3.049	–

Notes: Column 1 presents data moments by using cyclical components of the oil price and the world oil consumption. Column 2 shows the moments of price and demand by using the simulated data from the extended storage model. Column 3 shows the moments of simulated data assuming storage is zero in all periods. Column 4 shows the moments of price and demand by using the simulated data from the storage model with production shock only. The simulations are performed using estimated parameters for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects from initial values.

a standard deviation of 0.12, which is close to the data moments with a mean value of 1.01 and a standard deviation of 0.11. Furthermore, the extended storage model is able to capture the persistence in the oil price. The persistence is 0.54 for the simulated price and 0.59 for the price cycle data.

Although the skewness and kurtosis are not included in the moments condition in equation (20) for the SMM estimation, the extended storage model reveals a good capability to reproduce the higher moments as in the data. As Table 5 shows, the skewness is 3.42 for the simulated price and 3.55 for the data. Similarly, the kurtosis is 23.05 for the simulated price, and 21.19 for the data. The close match to the data skewness and kurtosis also confirms the good fit of the model to the data.

Column 4 in Table 5 also shows the simulated moments of the storage model with single production shock using estimates from Table 4. Comparing to column 2, I find that the production-shock model is not able to match many moments in the data, especially the standard deviation, skewness and kurtosis of the oil price and consumption. This also reveals that the production-shock model does not fit well to the data, whereas the extended storage model with both production and income shocks has better fit.

Furthermore, a comparison between column 2 and 3 in Table 5 reveals the effect of speculative behavior, wherein column 3 presents the moments of simulated prices that impose zero storage for all simulation periods. I use the same parameters for simulation in column 2 and 3.

The extended storage model behaves similar to that of Deaton and Laroque (1996), such that the storage dampens shocks by means of lowering the price volatility and introducing persistence. As Table 5 illustrates, the volatility of simulated prices is 0.12 with possible storage and 0.31 without storage. The persistence of the simulated price is 0.54 with possible storage and close to zero when storage is impossible.

These results reveal that the storage has a smoothing effect that mitigates shocks. Intuitively, when the current price is high, caused by a high (low) GDP (production), it is most likely that the future GDP (production) shock is low (high) due to the zero shock persistence. Since the income (and production) process is mean reverting, speculators expect a lower future price, and thus have incentives to reduce storage in the current period. This leads to a decrease of oil availability in the market in the subsequent period. It then implies a high future price following the high price in the current period. Therefore, speculative behavior smoothens the price cycles.

In Figure 5, I also show a simulated price of oil from the storage model as an instructive illustration, (see Deaton and Laroque (1992)). The series is simulated by using estimated parameters. The simulation is implemented for 300 periods. The first 100-period simu-

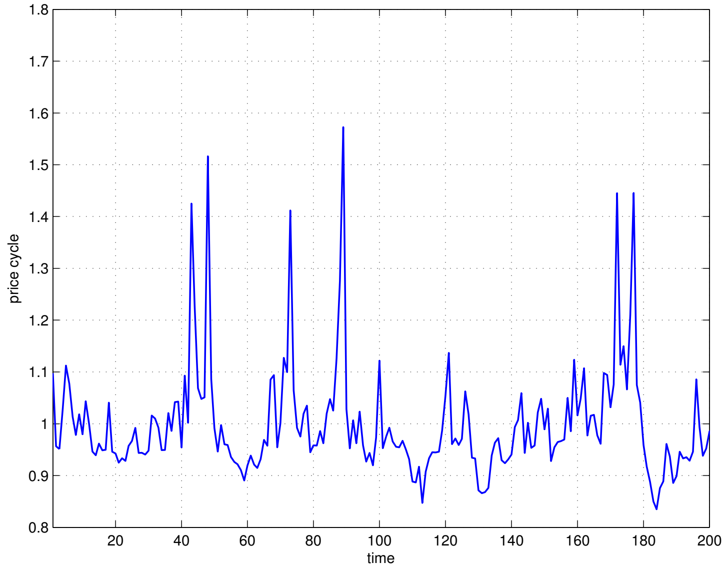

Figure 5: Simulated price of oil from the extended storage model

Notes: The price of oil cycle is simulated for 300 periods. The first 100-period simulations are deleted in order to eliminate the effects from initial values. In the simulation, the global GDP data is used as the exogenous process of income.

lations are deleted in order to eliminate the effects from initial values.[29] Figure 5 shows marked resemblances in the features of the simulated price to the price cycle data that are shown in Figure 1. From Figure 5, one can observe that the model generates occasionally large upward spikes (see plot in Figure 1). Moreover, the model is able to produce the low-variance phase more often when the oil price cycle is low.

5.3 The importance of income and production shocks

Using estimated parameters of the extended storage model, I perform a counterfactual analysis to discuss the relative importance of the income and production shocks as a means of explaining the variance and autocovariance of the oil price cycle for the period 1986–2009. I also employ the impulse responses so as to illustrate the response of the oil price cycle to income shock, and so as to compare it with the evidence from the data in the VAR analysis in Section 3.

Counterfactual analysis

[29]While simulating the price of oil, I use world GDP cycle data as the income process.

Table 6: Moments of simulated prices with zero production and income shock

	$\epsilon_z, \epsilon_y \neq 0$ (1)	$\epsilon_z = 0, \epsilon_y \neq 0$ (2)	$\epsilon_z \neq 0, \epsilon_y = 0$ (3)
mean(P)	1.000	1.000	0.999
std(P)	0.124	0.055	0.118
var(P)	0.015	0.003	0.013
corr(P_t,P_{t-1})	0.536	0.495	0.539
cov(P_t,P_{t-1})	0.008	0.001	0.007

Notes: This table shows the moments of simulated prices in different scenarios. Column 1 presents the moments of simulated price in the extended storage model with both income and production shocks. Column 2 refers to the case that assumes zero production shock in the simulation. Column 3 refers to the case that assumes zero income shock in the simulation. For the simulation, in column 2 and 3 I use the same policy function of storage as in column 1. The simulations are performed through using estimated parameters in Table 4 for 20,000 periods and 1,000 repetitions. The first 100 periods of simulations are deleted in order to eliminate the effects of initial values.

Table 6 presents a counterfactual analysis in three different cases of price simulation. The first column presents the moments of simulated price cycles in the benchmark model with a stochastic production and income process. Estimated parameters in Table 4 are used for simulation. The values are consistent with those in column 2 in Table 5.

I derive two counterfactual exercises. First, I impose zero production shock over the simulation period ($\epsilon_Z = 0$), and I use identical estimated parameters, as in the benchmark model. Moreover, I employ the policy function of storage derived from the benchmark model for interpolation and simulation. The moments of the simulated price with zero production shock is shown in column 2 in Table 6. Second, I let the income remain constant at its mean value over time (thus $\epsilon_Y = 0$), and similarly I use the same parameters and the policy function as in the benchmark model. The moments of the simulated price with zero income shock are shown in column 3 in Table 6.

As column 2 illustrates, setting a zero production shock strongly impacts the volatility and persistence of the oil price cycle. With a similar mean of 1.00, the variance of the simulated price is 0.003 with zero production shocks–much lower than the 0.015 in the benchmark case. These results indicate that the production shocks contribute 80 percent of the price variance in the benchmark case. At the same time, the autocovariance of the simulated price with a zero production shock is 0.001, versus 0.008 in the benchmark case. Thus the production shock accounts for 87.5 percent of the autocovariance of price in the benchmark case.

However, the income shock moderately impacts the volatility and persistence of the price cycles. The variance and autocovariance of the simulated price with zero GDP shock are 0.013 and 0.007, which, respectively, account for 13 percent and 12 percent of the corresponding moments in the benchmark case.[30]

The higher importance of production shock to the oil price cycle is mainly because of the large estimated production volatility (see Table 5). Due to the unobservable crude oil production in the data, the SMM estimation method searches for the proper production shock volatility (and other system parameters) in order to match model-implied moments to the data moments. On account of the GDP volatility of 0.01 through the calibration, the estimated production volatility is obtained when the simulated price have the closest persistence (together with other moments) to that of the data (0.59). As I explained in Section 4.2, the production shock motivates storage and increases the smoothing effect. Subsequently, the SMM estimates a production-shock volatility of 0.06—five times as large as the volatility of GDP shock; in other words, the large estimated volatility of production shock reveals the importance of the production shock to the oil price cycles.

5.4 Comparison with the VAR model

In order to observe whether the extended storage model is in line with the empirical evidence in the VAR analysis, I compute impulse responses of the oil price cycle to income shocks (in the extended storage model), and I compare them with the data of oil cycles from the VAR model in Section 3. Figure 6 depicts the impulse responses of income shocks on the cyclical components of world GDP and oil price in the storage model and VAR model. The impulse responses in the VAR model are identical to those in Figure 3. I impose the same positive income shock in period 0 in both the storage model and the VAR model, and I assume the shock will return a value of 0 in the subsequent 14 periods. The production shock is set to be 0 for all the periods. Panel (a) illustrates identical response of initial world GDP at 0.67 percent in period 0, and panel (b) plots the impulse responses of price.

In the storage model, as in equation (14), the income process is assumed to have 0 persistence. Thus, the impulse response on world GDP, as shown in panel (a), returns to 0 percent right after the period with positive income shock. Due to the autoregressive feature of the VAR model, the impulse response of income shock on GDP process gradually fades out after 12 periods.

By imposing the same income shocks in period 0 in both the storage model and VAR model, I find that the oil price cycle has limited impulse response to world GDP shock in the storage model. Panel (b) shows that facing the same positive demand shocks and zero

[30]The sum of the contribution to the overall price variance and autocovariance in the two counterfactual cases does not equal 100 percent. This is mainly because the simulations of price vary in each case of the 1,000 repetitions, and I compute the fraction using a mean value of the 1,000 repetitions of simulations.

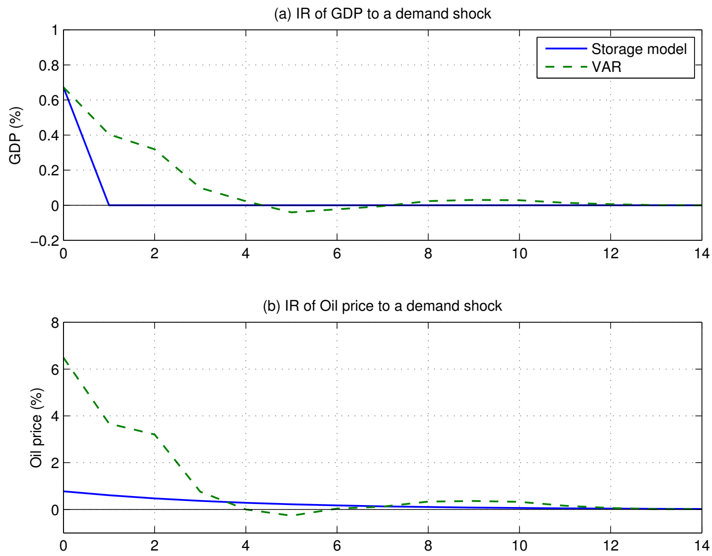

Figure 6: Impulse responses of GDP and oil price cycle with respect to income shock in the storage model and VAR model

Notes: Panel (a) plots the impulse responses of GDP with respect to the same positive income shocks in period 0 in the storage model and VAR model. Panel (b) shows the impulse responses of price cycles with respect to positive demand shocks (as shown in the panel (a)) in the storage model and VAR model. The income shocks are simulated for 5,000 repetitions in period 0. The shocks returns to 0 in the subsequent 14 periods. The production shock is assumed to be 0 for all the periods. Figures plot the median value for the 5,000 repetitions of the simulations.

supply shock, the oil price cycle increase only 1 percent in the storage model, compared to 6.30 percent in the VAR model.

In summary, the revised commodity storage model, extended with both stochastic production and income processes, is capable of capturing the large volatility and persistence in the cyclical component of the oil price. The autocorrelation of the oil price cycle can be fully attributed to the speculation effect. However, the model generates a moderate response towards income shocks.

6 Conclusions

In this paper I employ a multivariate method to extract the cyclical component of the oil price. I find a large and positive effect of global GDP shock on the oil price cycles in a

VAR model. I apply an extended commodity storage model with both income shock and production shock to the cyclical component of oil prices, after all long-term trends have been removed. This model is estimated using SMM for the period 1986–2009.

I obtain encouraging results in several respects. First, I find significant and meaningful coefficients that are estimated from the commodity storage model. Second, through employing estimated parameters, the model is capable of replicating some stylized features of oil cycles—in particular, the volatility and persistence of the oil price cycle. Comparing to the model with only production shock, the extended storage model has better fitness to the data. Furthermore, in the extended storage model, income shocks explain 13 percent of the variance in oil prices, whereas production shocks explain 87 percent. This is due to the large volatility of production and the intensive smoothing effect on the equilibrium price that is triggered by the production shock. A comparison of impulse responses between the extended storage model and the VAR model shows that the GDP shock generates more moderate effect of the oil price cycles in the extended storage model than in the VAR analysis.

The limitations of this paper are mainly related to the simplified assumption of a stationary stochastic income process. The world GDP as a measure of world income is concluded as a non-stationary series. For further extension of my model, it would be advisable to assume an income process with both transitory and permanent components in an empirical storage model. I can then assess the income effect on the price of crude oil and compare with the results in the current paper. Through such an estimation, I will be able to evaluate the fraction of transitory and permanent components in the income process.

Furthermore, this paper does not have a detailed setup for a speculator, such as the OPEC core, with market power on controlling the oil price through production. Therefore, possible research can be extended to an endogenous production process in the commodity storage model. From the estimation of such a model, I may be able to show the importance of having the endogenised production process. The estimates will also give an indication of the market power of the speculator.

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Appendix

A The Implementation of the Beverage-Nelson decomposition: a state-space approach

This appendix summarizes the state-space approach for the BN decomposition with detected number of cointegrating relationship as discussed in Cochrane (1994) and Morley (2002).

I decompose the cyclical components using estimates from the VEC model as equation (6) with one cointegration relationship. Due to the existence of one cointegration relationship among variables, it is possible to identify speed of adjustment coefficients, denoted as γ, and cointegration vector, denoted as α, using maximum likelihood, such that

$$\hat{\Pi} = \hat{\gamma}\hat{\alpha},$$

where $\hat{\Pi}$ denotes the estimates of Π in the VEC model in equation (6). $\hat{\gamma}$ is a 3×1 coefficient matrix. $\hat{\alpha}$ is a 1×3 structure coefficient matrix for the long-run stationary relationship

$$\hat{\alpha}\mathbf{X}_t = \hat{\alpha}_y \ln Y_t + \hat{\alpha}_q \ln Q_t + \hat{\alpha}_p \ln P_t \sim I(0),$$

where $\hat{\alpha} = \begin{bmatrix} \hat{\alpha}_y & \hat{\alpha}_q & \hat{\alpha}_p \end{bmatrix}$.

Cochrane (1994) suggests a stylized method by transforming the VEC model into an AR(1) format when computing the trend and cyclical components. Following Cochrane (1994) I transform the VEC in (6) into an AR(1) format such that

$$\begin{bmatrix} \triangle\mathbf{X}_t \\ \hat{\alpha}\mathbf{X}_t \end{bmatrix} - \hat{\mu} = \hat{\mathbf{B}}\left(\begin{bmatrix} \triangle\mathbf{X}_{t-1} \\ \hat{\alpha}\mathbf{X}_{t-1} \end{bmatrix} - \hat{\mu}\right) + \begin{bmatrix} u_t \\ \hat{\alpha}u_t \end{bmatrix}, \tag{21}$$

where

$$\hat{\mu} = \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \begin{bmatrix} \hat{\beta} \\ \hat{\alpha}\hat{\beta} \end{bmatrix}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} \hat{A}_1 & \hat{\gamma} \\ \hat{\alpha}\hat{A}_1 & \hat{\Pi} + 1 \end{bmatrix}.$$

$\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, $\hat{A}_1$ and $\hat{\Pi}$ are the estimates of coefficient in the VEC model (6).

Beveridge and Nelson (1981)define the trend component of $\mathbf{X}_t$ as the expectation of the h-step ahead forecast where $h \to \infty$. Following Cochrane (1994), the trend component is computed as

$$\mathbf{X}_t^T = \mathbf{X}_t + \left[\begin{array}{cc} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 1} \end{array}\right] \hat{\mathbf{B}} \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \left(\left[\begin{array}{c} \triangle \mathbf{X}_t \\ \hat{\alpha} \mathbf{X}_t \end{array}\right] - \hat{\mu} \right),$$

and the cyclical component is computed as

$$\begin{aligned} \mathbf{X}_t^C &= \mathbf{X}_t - \mathbf{X}_t^T \\ &= -\left[\begin{array}{cc} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 1} \end{array}\right] \hat{\mathbf{B}} \left(\mathbf{I} - \hat{\mathbf{B}}\right)^{-1} \left(\left[\begin{array}{c} \triangle \mathbf{X}_t \\ \hat{\alpha} \mathbf{X}_t \end{array}\right] - \hat{\mu} \right). \end{aligned}$$

B Solving the Nonlinear Rational Expectations Commodity Market Model

This appendix summaries the collocation method of solving a simplified rational expectations commodity market model with one state variable and one control variable following Miranda and Fackler (2002). The extended model in this paper is solved in the same logic.

The simplified model

Let us consider a simple commodity storage model with a stochastic production process, where at beginning of period t, the availability of the commodity is A_t. Meanwhile, suppose that an amount Q_t is sold to consumers at a market clearing price $P_t = P\left(\frac{Q_t}{\bar{Y}}\right)$. At each time the producer can either produce or store the product, therefore I have that in each period the availability equals the sum of storage and consumption, $A_t = X_t + Q_t$. Speculators observe the availability and make decisions on the storage amount X_t following an arbitrage equilibrium condition, as shown in equation (15) and (16), derived from maximization of expected profit. Then I can write the complementarity problem as follow:

$$\begin{aligned} f_t &= \beta E_t \left[P\left(\frac{A_{t+1} - X_{t+1}}{\bar{Y}}\right)\right] - P\left(\frac{A_t - X_t}{\bar{Y}}\right) - C \qquad (22) \\ X_t &\geq 0, f_t \leq 0, \\ X_t &> 0 \Longrightarrow f_t = 0 \\ X_t &= 0 \Longrightarrow f_t < 0. \end{aligned}$$

Finally, the storage in the next period X_{t+1} depends on the current states A_t and Y_t, the control variable X_t and the exogenous production shocks e_{t+1} which is realized after time t. Then the transition function of the state variable can be written as follow

$$A_{t+1} = g\left(A_t, X_t, e_{t+1}\right) = X_t + \bar{Z}\exp\left(e_{t+1}\right)$$

In this problem, the state space is $A \subseteq R^{d_a}$, and the response space is $X \subseteq R^{d_x}$. The production shock e is normally distributed with mean 0 and variance σ^2.

Collocation method

To solve this rational expectation model with non-smooth policy function, I first specify the state variable with N number of nodes, such that A_i for $i = 1, 2, ..., N$. After that, I approximate the equilibrium price function $P\left(\right)$ in (22) as follow

$$P\left(A, X\left(A\right)\right) = \sum_{j=1}^{N} c_j \phi_j\left(A\right), \tag{23}$$

where the equilibrium price function is a linear combination of known basis function ϕ_j with coefficients c_j for $j = 1, 2, ..., N$.[31]

Then I can rewrite the original complementary problem into to the form

$$\begin{aligned} f\left(A_i\right) &= \beta \sum_{k=1}^{K} \sum_{j=1}^{N} w_k \left[c_j \phi_j \left(X\left(A_i\right) + \bar{Z}\exp\left(e'\right)\right)\right] - P\left(A_i - X\left(A_i\right)\right) - C \qquad (24) \\ X\left(A_i\right) &\geq 0, f\left(A_i\right) \leq 0, \\ X\left(A_i\right) &> 0 \Longrightarrow f\left(A_i\right) = 0 \end{aligned}$$

for each $i = 1, 2, ..., N$. In equation (24), the random production shock e in the transition function is substituted with discrete approximation of e_k and probabilities w_k for $k = 1, 2, ..., K$. This method transfers the model into N nonlinear equations and N unknown coefficients c_j for $j = 1, 2, ..., N$.

I use a two-layer-iteration-loop method to solve equation (24). First, I give an initial guess of coefficient c_j for $j = 1, ...N$. In the inner loop, with given initial guess of coefficients, I find the optimal solution of the control variable X at each state nodes A_i in equation (24).[32] Second, using optimal solution of the control variable from the inner loop, I am able to compute the updated coefficient c_j using equation (23) in the outer loop. After that, the newly updated coefficient enters the inner loop to compute optimal control variable at each state variable again. The iteration carries on until the coefficients convergence.

[31] I employ cubic spline method for the basis function.

[32] Following Miranda and Fackler (2002), I solve the complementarity problem using min-max root-finding method.

PAPER 4

Optimal Asset Allocation for Commodity Sovereign Wealth Funds*

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Abstract

This paper solves a dynamic asset allocation problem for a commodity sovereign wealth fund under incomplete markets. We calibrate the model using data from three countries: Norway, UAE and Chile. In our benchmark calibration for Norway, we find that the fund's manager should initially invest all her wealth to stock and reduce this fraction gradually over time. We find that the solution is particularly sensitive to the assumption about the volatility of commodity prices. The solution for Chile implies that for relatively high risk aversion coefficients the manager should start at a small fraction of her wealth to increase later over the life cycle of the fund.

JEL Classification: *E21, G11.*

Keywords: *Dynamic asset allocation, portfolio management, sovereign wealth fund, income risk.*

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1 Introduction

Sovereign wealth funds (SWFs hereafter) are institutional investors that engage in long-run policies with the objective of ensure gradual transfers of wealth across generations in order to maximize long-run expected returns. Most SWFs' source of income comes from the commodity sales revenues and/or the accumulation of foreign exchange reserves. Although these investment funds have existed for decades, there has been a significantly increase of SWFs since 2000. As of 2013 there are 55 different commodity based SWFs in the world administering US$4 trillion in assets, corresponding to US$1,211 billion more than the estimated size of hedge funds worldwide, and to 3 percent of the global investment industry (SWF Institute, 2014 and Research, 2015). As prominent actors in the global asset market, the SWFs are expected to enhance the performance of the funds through optimal management of portfolio investments.

Given their importance in the global asset markets, this paper investigates, from a normative perspective, the optimal portfolio choice of a commodity SWF with a long-term investment horizon. To do so, we set up and solve an otherwise standard dynamic asset allocation problem following the contributions of Bodie et al. (1992) and Campbell and Viceira (2002). In particular, we focus on the following problem faced by the fund's manager: given his preferences for risk, how much of the fund's wealth should be invested in a risk-free asset and how much should be invested in a risky asset (stocks) over a long investment horizon. At this point we abstract from any political and fiscal considerations, but we will include them in future research. What distinguishes a commodity SWFs from other institutional investors is the source of their income and the correlated income risk with the asset market. In general, the revenues from commodities are highly volatile, decreasing over time as the resource is depleted and partially correlated with the stock market. To gain analytical tractability and insight on the mechanisms behind the asset allocation choices made by the fund's manager we first solve the model under the assumption that all the income risk is perfectly spanned by the stock market and hence markets are said to be complete. Following Munk and Sørensen (2010), we show that under complete markets the main determinants of the optimal investment strategy over time are the dynamic behavior of the commodity-wealth to financial-wealth ratio, the fund's manager level of risk aversion and the income volatility.

We later abandon the assumption of complete markets and solve the model using global approximation methods. We study the optimal asset allocation, and solve for both unconstrained (where agents can borrow/lend assets) and short-sales constrained problems by calibrating the model in such a way that it matches salient features of the Norwegian

SWF. We use the calibrated model to draw quantitative results. The predictions of the unconstrained problem indicate that the Norwegian Petroleum Fund should leverage its investment in the risky asset in order to allocate more than its total financial wealth in stocks during its first year of operation and then gradually decrease its position to a long-run fraction of 60 percent after 30 years. When we impose the constraint on short-sales, the model predicts that the fund should keep all its financial wealth invested in the risky asset for the first five years, and then start decreasing the investment share on stock gradually to a long-run share of 60 percent in about 40 years. Both implications are explained by an initially large oil-wealth to financial-wealth ratio that allows the fund's manager to take large risk positions when the natural resource is far from being depleted. To take into account the effects of high and low volatilities in the price of oil we perform a sensitivity experiment. We find that changes in the volatility of the oil price can substantially affect the size of the hedging demand that lead to differences of up to 30 percent in the investment share of stocks when moving from a low to a high variability level of prices.

We conduct a similar analysis for the commodity SWFs of Chile and UAE. Given that UAE has larger levels of oil reserves and similar current financial wealth, the optimal constrained allocation suggests the fund's manager to invest a larger share of the risky asset throughout the investment horizon. For the case of Chile, we find that for reasonable values of the risk aversion and given the observed cross-correlation between the price of copper and the stock returns there are scenarios for which the SWF should invest only a small fraction of its financial wealth in the risky asset at the beginning of the investment horizon and then increase it substantially as the resource is depleted.

Our setup builds on the dynamic asset allocation framework of Merton (1971), Campbell and Viceira (2002), Stoikov and Zariphopoulou (2005) and Chacko and Viceira (2005) extended to include the stream of risky labor income that the investor receives over time. Svensson and Werner (1993) and Henderson (2005) have derived closed form solutions for such a problem by assuming that investors have preferences that can be represented by a negative exponential utility function and the presence of imperfect correlation between income growth and stock returns. More recently, Munk and Sørensen (2010) restore to numerical methods to provide a solution to an asset allocation problem with stochastic interest rate and labor income under incomplete markets, and find that the shocks to labor income and interest rate have considerable effects on the optimal investment decisions.

Our work also relates to a number of recent contributions that study the asset allocation problem faced by SWFs. Dyck and Morse (2011) use panel data to examine the objectives driving SWF investments. Scherer (2011) formulates a portfolio allocation problem where the fund's manager must choose how much to invest in risky assets under a mean-variance framework where his payoff is determined not just by the funds wealth but the entire government's economic wealth. He finds that the optimal investment on risky asset is a function of financial-wealth to oil-wealth ratio, assuming constant government budget relative to financial wealth. van den Bremer et al. (2016) study the investment implications for a SWF in a model for oil exporter countries by combing theories of portfolio allocation and optimal extraction rate. Using an approximated solution of the model under the assumption of incomplete markets, they find that the policy maker should consider the below-ground wealth in his optimal investment strategy for the SWF. Although providing a natural framework to perform different qualitative experiments, their model is not able to match the observed data.

The remainder of the paper proceeds as follows. In Section 2 we provide some stylized facts about the three SWFs studied in this paper. Section 3 describes the asset allocation problem for a commodity SWF and provides a closed form solution for the optimal portfolio choice under complete markets. In Section 4 we relax the assumption of complete markets and solve the dynamic asset allocation problem using the numerical method proposed in Munk and Sørensen (2010). The model is calibrated to mimic some of the characteristics of the Norwegian SWF. We also perform a sensitivity analysis to different levels of oil price volatility. Section 4 also compares the optimal investment strategies of the SWFs in Norway, UAE and Chile. Finally, Section 5 concludes.

2 Commodity prices and SWFs in different countries

In this section, we describe the data used in this study and characterize some statistical moments among commodities and stock prices. Then, we describe financial and technological conditions for the SWFs of Norway, UAE and Chile.

Data. We use annual data on commodities and stock prices for the period 1900-2013. The price of oil corresponds to the West Texas Intermediate (WTI) reference price, while for copper we use the high grade copper price. For the stock market we use the S&P 500 composite price index. The commodity and stock prices are collected from Global Financial Data (2014). Nominal prices are deflated using the Consumer Price Index reported by the U.S. Bureau of Labor Statistics (2014). The base year is 2010 when computing real values. Commodity production for Norway and the UAE are taken from the U.S. Energy Information Administration (2014), while the reserves of crude oil are obtained from BP (2014). For the Chilean SWF we use cooper production data from the Chilean Copper Commission (2014), while the data on copper reserves is collected from the U.S. Geological

Table 1: Empirical moments: commodity and stock prices

Moments	1900-2013	1900-1941	1942-1973	1974-2013
Stock price				
$\mathbb{E}\left(\Delta \log S_t\right)$	0.018	-0.002	0.041	0.020
$\mathrm{std}\left(\Delta \log S_t\right)$	0.195	0.229	0.162	0.184
Oil prices				
$\mathbb{E}\left(\Delta \log P_t^{oil}\right)$	0.005	-0.009	-0.020	0.040
$\mathrm{std}\left(\Delta \log P_t^{oil}\right)$	0.247	0.240	0.042	0.336
Copper price				
$\mathbb{E}\left(\Delta \log P_t^{copper}\right)$	-0.004	-0.027	0.010	0.008
$\mathrm{std}\left(\Delta \log P_t^{copper}\right)$	0.240	0.235	0.143	0.303
Cross Correlations				
$\mathrm{corr}\left(\Delta \log S_t, \Delta \log P_t^{oil}\right)$	0.039	0.121	0.133	-0.034
$\mathrm{corr}\left(\Delta \log S_t, \Delta \log P_t^{copper}\right)$	0.321	0.605	0.075	0.137

Notes: The table reports empirical moments for commodity and stock prices using annual data. The stock price corresponds to the S&P 500 composite price index, oil price is measured by the WTI reference price and the copper price is collected from the high grade copper price. Commodity and stock prices are from the Global Financial Data (2014). Nominal prices are deflated by the U.S. CPI deflator from U.S. Bureau of Labor Statistics (2014).

Survey (2014). The data on financial wealth for Norway, the UAE and Chile are obtained from Norges Bank Investment Management (2013), SWF Institute (2014) and Chilean Copper Commission (2014), respectively. Finally, the GDP data is taken from World Bank (2014).

Table 1 summarizes some empirical moments for the real price of oil, copper and stocks for four different subsamples. The calculations reveal some interesting facts for commodity prices regarding their price volatility and correlation with the stock market over long periods of time. In particular, we find that the volatility of variations in the oil price increases from 4.2 percent per year in the post war period (1942-1973) to 33.6 percent per annum during 1974-2013. Similarly, the correlation between variations in the price of oil and stock returns goes from a positive value of 13.3 percent per year to a mildly negative value of -3.3 percent per year in the same subsamples. In what follows, we will calibrate our benchmark model using the moments for the whole sample period (1900–2013) and later on perform a series of sensitivity analysis using the volatility of commodity prices found in the different subsamples.

Three commodity based SWFs. In this paper we study three commodity funds: Norway, the UAE and Chile. Figure 1 provides some information regarding their current and expected levels of wealth. Panel (a) plots the real value of their asset holdings in 2013. Norway is one of the world largest established sovereign fund with income source corre-

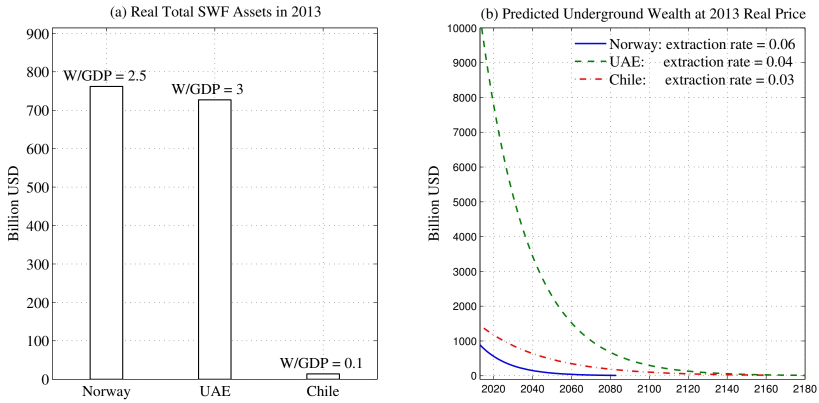

Figure 1: Current financial and underground wealth for different countries.

Notes: Panel (a) plots the real asset holding in 2013 for the SWF of Norway, the UAE and Chile. The financial wealth GDP ratio in 2013 for each country appears at the top of each bar. Panel (b) predicts the underground commodity wealth for Norway, the UAE and Chile since 2013. The wealth paths are computed assuming constant real prices of commodities as of 2013, zero exploration and constant extraction rate at the 2013 level for the three countries. WTI oil price and real high grade copper price are collected from Global Financial Data (2014). Nominal prices are deflated using the U.S. CPI deflator from U.S. Bureau of Labor Statistics (2014). The production of oil in Norway and the UAE in 2013 are from U.S. Energy Information Administration (2014). Chilean copper production in 2013 is from Chilean Copper Commission (2014). The crude oil reserve values are obtained from BP (2014). The Chilean reserve of copper is from U.S. Geological Survey (2014). The financial wealth in Norway is from Norges Bank Investment Management (2013). The financial wealth data for the UAE is from SWF Institute (2014). The financial wealth of Chile is from Ministry of Finance Gobierno de Chile (2013). The GDP data of three countries is from World Bank (2014).

sponding to oil revenues. For the year 2013, the Norwegian Government Pension Fund Global (GPFG) held US$770 billion in assets, nearly two and half times the Norwegian real GDP. In practice, the Norway's Ministry of Finance owns the fund and determines a investment strategy in 2009–2013 with a constant of 60 percent equity, 40 percent bonds[1]. According to the Ministry of Finance Norway (2015), the Norwegian government commits to a budgetary rule that is 4 percent of the fund asset each year. Furthermore, the UAE SWF is another world largest sovereign fund based on oil revenues. The Abu Dhabi Investment Authority (ADIA), the UAE SWF, held US$720 billion USD in assets, around three times the UAE's real GDP in 2013. On the other hand, The Economic and Social Stabilization Fund is the SWF in Chile which is a relative young fund based on the copper revenue. Their 2013 asset holdings amounted to US$14 billion, a 10 percent of Chile's real GDP.

[1]Now it is 60 percent equity, 35 percent bonds and 5 percent real estate (Norges Bank Investment Management 2013, 2015)

Panel (b) in Figure 1 depicts the value of underground reserves for the three commodity SWFs by assuming zero further exploration after 2013[2], and a constant production rate of the remaining reserves at each point in time. In particular, we assume that the future extraction-reserve rate remains constant at its 2013 level which was 6 percent for Norway, 4 percent for UAE and 3 percent for Chile. Therefore, using the effective levels of reserves in 2013 we predict the trajectories of underground reserves in Norway, the UAE and Chile. In order to make the numbers comparable we value the predicted reserves of oil and copper at 2013 real prices and assuming no price changes thereon[3]. Moreover, we also compute the time of resource depletion[4] as the terminal date with positive underground wealth for each country[5]. Given their large initial reserves of oil, our calculation indicate that the 2013 underground wealth for the UAE amounts to US$10 thousand billion, and the resource will be completely depleted in 170 years. Norway, starting with US$900 billion in underground oil wealth will reach resource depletion in around 70 years. Finally, given the low rate of extraction, Chile will deplete their reserves of copper in 150 years. We use these values when computing optimal portfolio allocations in Section 4.2.

3 An asset allocation model for a SWF manager

In this section, we describe the problem faced by a commodity SWF manager. At this point we abstain from any fiscal and political considerations and instead we focus on the optimal asset allocation made by the fund's manager in order to ensure a stable stream of transfers to the government when it is known *a priori* that the revenues of the fund are decreasing over time due to the depletion of the resource. To do so, we first assume that markets are complete and derive a closed form solution for the optimal allocations that will give us some intuition about the mechanism and driving forces behind the model. We later study the allocation implications under the assumption of market incompleteness. Our framework build on the early contributions of Merton (1971) and assumes that time in the economy evolves continuously and that all the uncertainty faced by the investor can be represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with associated filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$.

[2]The assumption of zero exploration may underestimate their real financial wealth. Future analysis should include a time-varying production rate with possible resource exploration.

[3]The 2013 reserves of crude oil in Norway and the UAE were 8.7 billion barrels and 101 billion barrels, respectively. The 2013 reserves of copper in Chile were 190 million metric tons. The real price of oil in 2013 was US$102 per barrel while the real price of copper was US$7651 USD per metric tons.

[4]With constant extraction rates, production quantities of oil and copper decline infinitely. In the calibration, we compute the time of depletion corresponding to date when the terminal production revenues are less or equal to US$100 million. The terminal production revenue is computed as the product of terminal extraction and real commodity price in 2013.

[5]The underground wealth is assumed to be zero after the depletion time.

3.1 Description of the model

Asset returns. Let us assume that the SWF manager has a costless access to two tradable assets. A risk free bond with instantaneous return r, and a risky asset (stocks) with instantaneous return $r + \psi$, where ψ denotes the expected excess return of stocks over the risk free bond. The price of the stock S_t is assumed to evolve according to:

$$\frac{dS_t}{S_t} = (r + \psi)\, dt + \sigma_S dz_{St}, \tag{1}$$

where σ_S is the volatility of the stock price and z_{St} is a standard Brownian motion.

Commodity revenues. Assuming zero exploration (and discoveries) of new reserves, the availability of the natural resource q_t decreases at a constant extraction-reserve rate $\alpha_q > 0$ for $\hat{T}$ periods:

$$\frac{dq_t}{q_t} = -\alpha_q dt. \tag{2}$$

The price of the commodity is assumed to follow a geometric Brownian motion with a drift α_p and volatility σ_y:

$$\frac{dp_t}{p_t} = \alpha_p dt + \sigma_y \left(\rho_{yS} dz_{St} + \sqrt{1 - \rho_{yS}^2} dz_{yt} \right) \tag{3}$$

where z_{yt} is a standard Brownian motion and $\rho_{yS} \in [-1, 1]$ denotes the correlation between variations in the commodity price and the stock returns. For simplicity, we assume that z_{yt} and z_{St} are independent. Therefore, and assuming zero cost of production, the disposable income for the SWF is given by $y_t = p_t q_t$. Using Itô's Lemma, the dynamics of the fund's revenues follow:

$$\frac{dy_t}{y_t} \quad = \quad \alpha dt + \sigma_y \left(\rho_{yS} dz_{St} + \sqrt{1 - \rho_{yS}^2} dz_{yt} \right) \quad \forall t \leq \hat{T}, \tag{4}$$

where $\alpha = (\alpha_p - \alpha_q)$ represents the expected income growth. In what follows, we will assume that the fund's income is equal to zero after the full depletion of the exhaustible resource, that is, $y_t = 0$ for all $\hat{T} \leq t \leq T$ where $T > \hat{T}$ corresponds to the investment horizon of the fund's manager. Given our set of assumptions, ρ_{yS} also denotes the instantaneous correlation between the fund's income growth and the stock returns. When $\rho_{yS} = 1$, the income risk is fully spanned by the tradable stock. We refer to this scenario as the complete market case.

The SWF manager's decision problem. Following Campbell and Viceira (2002) and Munk and Sørensen (2010), we assume that the manager of the SWF faces the classical consumption-investment problem. Starting from instant $t$and given the current level of wealth, W_t, and prevailing prices in the economy, the manager chooses the paths of consumption and portfolio allocation, $\{c_u, \theta_{Su}\}_{u=t}^{T}$, that provide him the maximum level of life-time utility, $J(W, y, t)$. The latter is also known as the value function. Formally, he solves:

$$J(W, y, t) = \max_{\{c_u, \theta_{Su}\}_{u=t}^{T}} \mathbb{E}_t \left[\int_t^T e^{-\delta t} U(c_u)\, du + \epsilon e^{-\delta(T-t)} U(W_T) \right] \tag{5}$$

subject to the budget constraint:

$$dW_t = (rW_t + \psi\theta_{St} + y_t - c_t)\, dt + \sigma_S \theta_{St} dz_{St} \tag{6}$$

where $\delta > 0$ is the discount rate, $U(\cdot)$ is an increasing and concave utility function and $J(W, y, T) = \epsilon U(W_T)$ is a given terminal condition with parameter ϵ for terminal level of wealth W_T. The portfolio allocation θ_{St} denotes the amount, in nominal terms, invested at time t in the risky asset. Similarly, the consumption choice should be understood as transfers that the fund's manager makes to the central government which will eventually become part of the government's budget constraint. Therefore, in what follows we will refer to c_t as consumption or government transfers at time t interchangeably.

Finally, we assume that the fund's manager preferences are described by a standard, time separable, power instantaneous utility function over consumption:

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \tag{7}$$

where $\gamma > 0$ is the coefficient or relative risk aversion and $1/\gamma$ the intertemporal elasticity of substitution of consumption.

3.2 Optimal investment strategies

In order to compute the optimal investment strategy for the SWF, we will use the dynamic programming approach originally proposed in Merton (1971) for the case of continuous-time models. At every point in time t, the fund's manager gathers information on his financial wealth W_t and commodity revenue y_t to make his optimal choices. With these values for the state variables of the problem, the dynamic programming equation, also known as the Hamilton-Jacobi-Bellman (HJB) equation, is given by:

$$\delta J(W_t, y_t, t) = \max_{c_t, \theta_{St}} U(c_t) + J_t(W_t, y_t, t) + J_W(W_t, y_t, t)[rW_t + \psi\theta_{St} + y_t - c_t]$$
$$+ \frac{1}{2} J_{WW}(W_t, y_t, t)\sigma_S^2 \theta_{St}^2 + \alpha y J_y(W_t, y_t, t)$$
$$+ \frac{1}{2} J_{yy}(W_t, y_t, t) y_t^2 \sigma_y^2 + J_{Wy}(W_t, y_t, t) y_t \theta_{St}\sigma_S\sigma_y\rho_{yS} \quad (8)$$

with terminal condition

$$J(W_T, y_T, T) = \epsilon U(W_T) = \frac{\epsilon W_T^{1-\gamma}}{1-\gamma}, \quad (9)$$

where J_i denotes the first order partial derivative the value function with respect to the state variable i, and J_{ij} the second order derivative with respect to the state variables i and j.

Let $\pi_{St} = \frac{\theta_{St}}{W_t}$ denote the fraction of wealth invested in the risky asset. Given the manager preferences, the first order conditions for an interior solution are given by:

$$c_t = J_W^{-\frac{1}{\gamma}} \quad (10)$$
$$\pi_{St} = \underbrace{\frac{1}{-\frac{W_t J_{WW}}{J_W}} \frac{\psi}{\sigma_S^2}}_{\text{Myopic-demand}} + \underbrace{\frac{y_t J_{Wy}}{J_W} \frac{1}{-\frac{W_t J_{WW}}{J_W}} \frac{\sigma_y \rho_{yS}}{\sigma_S}}_{\text{Hedging-demand}}. \quad (11)$$

Equation (10) defines the optimal level of transfers from the fund to the government such that at the maximum an extra unit of consumption should be as valuable, to the decision maker, as an extra unit of wealth that could be used to finance future transfers.

Equation (11) determines the optimal portfolio allocation to the risky asset as the combination of two components. The first term is usually referred to as the *myopic portfolio rule* and corresponds to the investment strategy that an investor with a short investment horizon will follow. It corresponds to the standard investment recommendation from the mean-variance analysis of Markowitz (1952) that suggest that the optimal fraction of wealth invested in the risky asset should be proportional to the asset's risk premium over the risk free asset, ψ, and inversely proportional to the asset's volatility, σ_S, and the manager's risk aversion, the latter measured by the curvature of the value function, $-\frac{WJ_{WW}}{J_W}$.

The second term is the intertemporal hedging component against commodity income risk and represents the extra demand required by an investor with a long horizon investment. In particular, the hedging demand will be determined by the volatility of income, σ_y, and its correlation with the stock returns, ρ_{yS}. The term $\frac{\sigma_y \rho_{yS}}{\sigma_S}$ can be seen as the regression coefficient of commodity revenue innovations on stock return innovations. It is also a function of the investor's risk aversion and his aversion to income revenue risk, measured by $\frac{yJ_{Wy}}{J_W}$. Importantly, this component suggests that the manager should increase his holding of the risky asset for increased levels of aversion to income risk and whenever it returns covary negatively with the commodity based income growth.

3.3 The case of complete markets

To obtain some intuition about the determinants of the demand for risky assets we first assume that markets are complete. Under complete markets, the fund's income is spanned by the risky asset and hence it is possible to derive a closed form solution following Munk and Sørensen (2010). To do so, we first compute the certainty equivalent value of the stream of future oil income, in other words, the commodity wealth. Therefore we introduce $\mathbb{Q}$ to be the risk-neutral probability measure and let $O(y_t, t)$ denote the time t net present value of all future commodity revenues until depletion time $\hat{T}$:

$$O\left(y_t, t\right) = E^{\mathbb{Q}}_{y,t}\left[\int_t^{\hat{T}} e^{-r(s-t)} y_s ds\right],$$

i.e. the value that the SWF would obtain if it was possible to sell all the future revenues from oil in the financial markets. The next proposition provides an expression to compute the commodity wealth:

Proposition 3.1 (Commodity wealth and income multiplier)**.** *Under complete markets, i.e. for $\rho_{ys} = \pm 1$, the commodity wealth of the SWF at time t is given by:*

$$O\left(y_t, t\right) = y\mathbf{1}_{\{t \leq \hat{T}\}} M\left(t\right) \tag{12}$$

where $M\left(t\right)$ is called the commodity income multiplier and it takes the form:

$$M\left(t\right) = \frac{1}{r - \alpha + \sigma_y \rho_{yS} \frac{\psi}{\sigma_S}} \left(1 - e^{-\left(r - \alpha + \sigma_y \rho_{yS} \frac{\psi}{\sigma_S}\right)\left(\hat{T} - t\right)}\right) \tag{13}$$

for $\left(r - \alpha + \sigma_y \rho_{yS} \frac{\psi}{\sigma_S}\right) \neq 0$ and all $t \leq \hat{T}$. The indicator function $\mathbf{1}_{\{t \leq \hat{T}\}}$ reflects the fact after the resource has been depleted the commodity income is zero, i.e. $O\left(y_t, t\right) = 0$ for all $t = \hat{T}, ..., T$.

Proof. See Appendix A. □

To gain some intuition about the role played by the commodity income multiplier, Figure

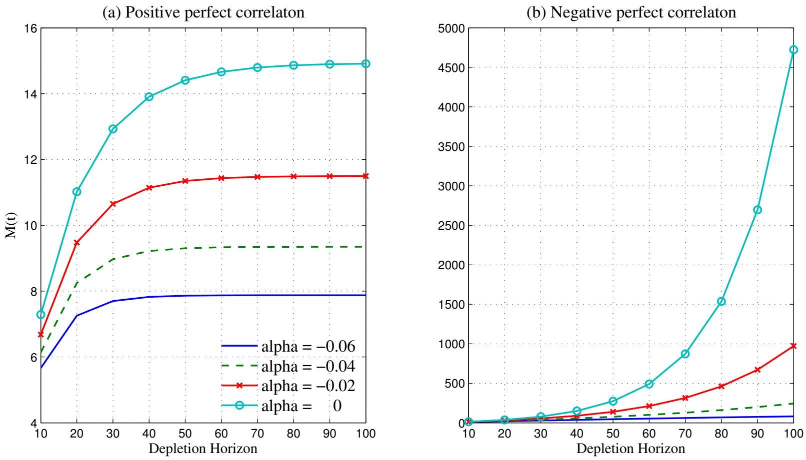

Figure 2: Commodity income multiplier $M(t)$

Notes: Panels (a) and (b) plot the commodity income multiplier $M(t)$ for different depletion time horizons $\left(\hat{T}-t\right)$ and expected income return α under complete markets. Panel (a) reports the multiplier behavior when income growth and stock returns are perfectly correlated in the same direction, $\rho_{yS}=1$, and Panel (b) when the direction is opposite, $\rho_{yS}=-1$. The remaining parameters are set to $r=0.006$, $\psi=0.05$, $\sigma_s=0.2$.

2 plots how the commodity wealth as a fraction of current commodity income, $M(t)$, as a function of the depletion horizon, $\left(\hat{T}-t\right)$, for different expected income growth rates, α, and both perfect positive and negative income-stock correlation. The remaining parameters are set to reasonable values.

Panel (a) in Figure 2 plots the commodity income multiplier when income growth and stock returns are perfectly and positively correlated. Panel (b) plots the multiplier with perfect negative commodity income and stock correlation. Our calculations indicate that the income multiplier is increasing in the expected income growth rate α and the depletion horizon $\hat{T}-t$. The commodity income multiplier is high when resource depletion is far away: concave for a positive correlation, and convex for a negative correlation. For example, if income and stocks exhibit a perfect positive correlation, then a fund's manager that expects the natural resource to be depleted in 70 years from now will value his current commodity wealth to be around 11 times the actual level of revenues. However, in the case of perfect negative correlation, the fund's manager could trade his stream of revenue in the market in order to perfectly insure against negative fluctuations in the stock market. This leads to a higher valuation of his current commodity wealth (more than 1000 times his current income) when depletion of the resource is 70 years ahead.

To solve the closed-form optimal investment strategy we still need to know the form of the value function $J(W_t, y_t, t)$ that solves the partial differential equation described by the HJB Equation (8). We use a *guess-and-verify* method to find the exact form of the value function. Following Munk and Sørensen (2010) we conjecture that the value function takes the form:

$$J(W_t, y_t, t) = \frac{1}{1-\gamma} g(t)^{\gamma} (W_t + O(y_t, t))^{1-\gamma} \tag{14}$$

where $g(t)$ is generally unknown. To solve for the function $g(t)$ we insert our guess, its derivatives, and the first order conditions (10) and (11) into the HJB Equation (8) and derive find that the functions: (A full derivation can be found in Appendix B):

$$g(t) = \frac{1}{A}\left[1 - e^{-A(T-t)}\right] + \epsilon^{\frac{1}{\gamma}} e^{-A(T-t)}$$

$$A = \frac{1}{\gamma}\left[\delta - r(1-\gamma) - \frac{1-\gamma}{2\gamma^2}\left(\frac{\psi}{\sigma_S}\right)^2\right]$$

satisfy the dynamic programming equation for all possible values of W_t and $O(y_t, t)$ and $t = 1, \ldots, T$. Hence, the guess is correct and can be used to find exact expressions for the optimal level of transfers and the optimal asset allocation. In particular, Equation (10) indicates that optimal consumption is proportional to total wealth:

$$c(W_t, y_t, t) = \frac{1}{g(t)}(W_t + O(y_t, t)). \tag{15}$$

Alternatively, the consumption to financial wealth ratio is given by:

$$\frac{c_t}{W_t} = \frac{1}{g(t)}\left(1 + \frac{O_t}{W_t}\right) = \frac{1}{g(t)}\left(1 + \frac{y_t}{W_t}\mathbf{1}_{\{t \le \hat{T}\}} M(t)\right) \tag{16}$$

suggesting that, provided that $g(t) > 0$, consumption is an increasing function of the ratio of commodity wealth to financial wealth which changes over the life cycle of the fund. In fact, after resource depletion, i.e. for $\hat{T} \le t \le T$, the commodity wealth to financial wealth ratio is zero and hence consumption only depends proportionally of financial wealth, with proportionality factor given by the inverse of $g(t)$.

On the other hand, the optimal investment share on the risky asset is given by:

$$\pi_S(W_t, y_t, t) = \frac{1}{\gamma}\frac{\psi}{\sigma_S^2} + \frac{O(y_t, t)}{W_t}\frac{1}{\sigma_S}\left(\frac{1}{\gamma}\frac{\psi}{\sigma_S} - \sigma_y \rho_{yS}\right). \tag{17}$$

The first term, $\frac{1}{\gamma}\frac{\psi}{\sigma_S^2}$, is the myopic investment rule. Clearly, investors will increase their holdings of risky asset if they are less risk averse. The second term, $\frac{O_t}{W_t}\frac{1}{\gamma}\frac{\psi}{\sigma_{S^2}}$, adjusts the optimal allocation due to the deterministic effects of having the commodity income, while the last term defines the hedging demand due to income risks and their (perfect) correlation to fluctuations in stock returns. If $\rho_{yS} = 1$, the hedging demand is negative: the manager should take less risk, as the income substitutes stock holdings. On the contrary, if $\rho_{yS} = -1$ income composites stock holdings and the hedging demand is positive. Therefore the manager should increase his position in the risky asset.

To better understand the effects of oil revenues on the optimal investment decision, let us consider the case of a deterministic flow of oil income, i.e. $\sigma_y = 0$. In this case, the investor's allocation is determined by the time path of the commodity wealth to financial wealth ratio:

$$\pi_S(W_t, y_t, t) = \frac{1}{\gamma}\frac{\psi}{\sigma_S^2}\left(1 + \frac{O\left(y_t, t\right)}{W_t}\right).$$

Hence, for a decreasing trajectory, the investor will take more risk initially and will eventually approach the myopic rule as the resource is depleted and the fraction $\frac{O_t}{W_t}$ goes to zero. In general, SWF with large initial $\frac{O_t}{W_t}$ ratio will take more risk. Now, let us suppose that the commodity income is risky. Now whether the investment strategy approaches the myopic rule from above or below will depend on the sign of the term in brackets in Equation (17). If $\left(\frac{1}{\gamma}\frac{\psi}{\sigma_S} - \sigma_y\rho_{yS}\right) < 0$ then the share of the risky asset decreasing over time. In the opposite case, the share is initially lower but increasing over time.

A numerical example. To illustrate some features of the solution under complete markets we perform some numerical simulations. Figure 3 plots the manager's yearly optimal allocation in risky assets over the SWF's investment horizon for two income volatility scenarios and fixing the remaining parameters at economically reasonable values. In particular, we assume an expected oil income growth $\alpha = -5\%$, a risk-free return of $r = 0.6\%$, a stock volatility of $\sigma_s = 20\%$, a relative risk aversion of $\gamma = 2$, and a discount rate of $\delta = 3\%$. We further assume that the reserves of the natural are depleted in 100 years ($\hat{T} = 100$), and that the initial ratio of financial-wealth to commodity-income is $\frac{W_0}{y_0} = 13$. We simulate 10000 samples of income, y_t, and stock prices, S_t, each of 100 observations, and compute the optimal investment share using Equations (12) and (17).

The results in Figure 3 correspond to the average value over the 10000 simulations for each point in time. The first scenario is represented by the solid line and it corresponds to the case where there is no uncertainty in the commodity income, i.e. when $\sigma_y = 0$. It is possible to show by rearranging Equation (17), that in this case the optimal investment in stocks relative to total wealth is constant and equal to $\frac{1}{\gamma}\frac{\psi}{\sigma_S^2} = 60\%$. In terms of financial

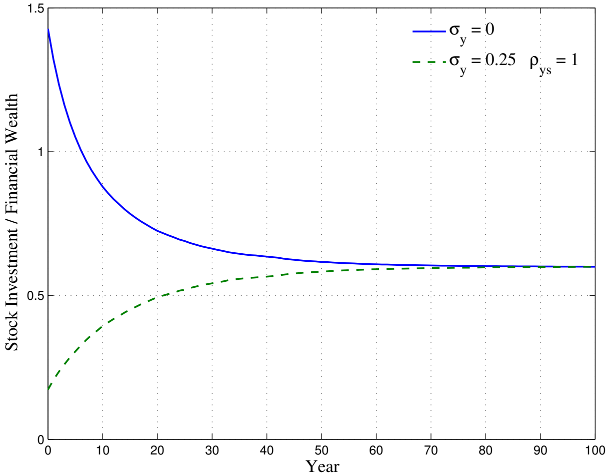

Figure 3: Optimal asset allocation under complete market: deterministic and stochastic income

Notes: The figure plots the optimal fraction of financial wealth invested in the stock under complete market, π_{St}. We report the mean over 10000 simulations. Each of the simulated sample paths for optimal investment share is computed using Equation (17). The solid line represents the expected investment share on stock with $\sigma_y = 0$. The dash line represents the expected investment share on stock with $\sigma_y = 0.25$ and $\rho_{ys} = 1$. The remaining parameters are set to be $\alpha = -0.05$, $r = 0.006$, $\psi = 0.05$, $\sigma_s = 0.2$, $\gamma = 2$, $\delta = 0.03$, $\hat{T} = 100$ and $\frac{W_0}{y_0} = 13$.

wealth and given our initial wealth to income ratio, the average optimal investment on stocks goes from 143 percent at the beginning of the investment horizon and decreases to the myopic rule of 60 percent at depletion time $\hat{T} = 100$.

The second scenario, represented by the dash line, considers the case of stochastic oil income with $\sigma_y = 0.25$ and $\rho_{yS} = 1$. In this case, the investor receives an additional source of risk from oil revenues that needs to be weighted when making investment decisions. Since oil income growth and stock returns are assumed to be perfect and positive correlated, the fund's manager hedges against oil income fluctuations by reducing his demand for stocks. Our simulation results suggest that the optimal investment share should increase gradually and monotonically over time from 17 percent to the 60 percent long run share.

4 Quantitative predictions

This section explores the quantitative predictions of the model introduced in Section 3. We begin our analysis by calibrating the parameters of the model to reflect relevant and salient features of the Norwegian SWF. In particular, we discuss the implications for the optimal investment strategies under incomplete markets and short-sale restrictions. Finally, we compare our results with two other SWFs, the UAE and the Chilean.

4.1 Model calibration

Table 3 summarizes the parameter values for the case of the Norwegian SWF. Time is measured in years and parameters should be interpreted accordingly. The sample period used in the calibration is 1900-2013. The expected growth of oil prices, α_p, is set to be 1 percent as suggested by the mean growth rate of oil price. As discussed previously, we assume a constant extraction rate equal to its 2013 level of 6 percent, which implies an expected growth rate of oil income, α, of -0.05 (0.01-0.06). The volatility of income growth, σ_y, is computed as the standard deviation of oil price growth and the covariance parameter ρ_{yS} as the correlation between oil price growth and stock returns. The expected equity returns, $\psi + r$, is set to 0.055 and their volatility parameter, σ_S, to 0.2. The parameter values for the stock returns are consistent with the magnitude of the equity premium reported in Beeler and Campbell (2012) for similar sample period.

The fund's manager preferences are calibrated as follows. We set his risk aversion parameter, γ, to be 2, the discount factor, δ, equal to 0.03 and the risk free rate of 0.6 percent. Together with the values for the stock returns, our calibration of preferences ensure a myopic investment rule of about 60 percent. Finally, we solve the dynamic asset

Table 2: Benchmark calibration for the Norwegian SWF

Description	Parameter	Value
Drift for oil price	α_p	0.010
Extraction rate	α_q	0.060
Drift for oil income	α	-0.050
Risk free rate	r	0.006
Excess return	ψ	0.049
Income volatility	σ_y	0.250
Stock price volatility	σ_s	0.200
Correlation between stock and oil prices	ρ_{yS}	0.040
Risk aversion coefficient	γ	2.046
Discount rate	δ	0.030
Time to depletion	$\hat{T}$	70
Initial financial wealth income ratio	W_{2013}/y_{2013}	13
Initial financial wealth GDP ratio	W_{2013}/GDP_{2013}	2.500
GDP growth rate	$\log(GDP_t/GDP_{t-1})$	0.015

Notes: Calibrated parameters use Norwegian annual data for the period 1900–2013. Nominal prices are deflated by the U.S. CPI deflator from U.S. Bureau of Labor Statistics (2014). α_p is calibrated as the mean growth rate of real WTI oil price. WTI oil price is collected from Global Financial Data (2014). α_q is computed as the extraction rate at 2013 level in Norway. The production of oil is obtained from U.S. Energy Information Administration (2014). The crude oil reserve is obtained from BP (2014). α is computed following $\alpha = \alpha_p - \alpha_q$. The parameter values of r and ψ are from Campbell and Viceira (2002). σ_y and σ_s are the standard deviation of real oil price growth and real stock price growth. U.S. S&P 500 composite price index, the measure of stock price, is obtained from Global Financial Data (2014). ρ_{yS} is the correlation between the stock price growth and oil price growth. γ is computed by targeting the optimal $\pi_s = \frac{1}{\gamma}\frac{\psi}{\sigma_S^2} = 60\%$ at period $\hat{T}$. The value of δ follows Munk and Sørensen (2010). $\hat{T}$ is computed using Norwegian reserve of crude oil by assuming a constant extraction rate α_q. The W_{2013}/y_{2013} and W_{2013}/GDP_{2013} are computed using Norwegian financial wealth, oil income and GDP in 2013. The financial wealth is obtained from Norges Bank Investment Management (2013). The oil income is computed as the product of real oil price times extraction quantity, assuming zero production cost. The Norwegian GDP is collected from World Bank (2014). $\log(GDP_t/GDP_{t-1})$, a measure of expected GDP growth rate in Norway, follows the average growth of real mainland GDP in Norway for the period 2008–2015 reported by IMF (2014).

allocation problem assuming an investment horizon larger than the time to depletion. The latter is set to be $\hat{T} = 70$ for Norway.

4.2 Incomplete markets

As Table 3 reports, the correlation coefficient between income growth and stock returns suggests that the risks associated with the former are not fully spanned by the latter. Therefore, markets are said to be incomplete since the fund's manager can not fully insure/hedge against income fluctuations in the financial markets. Under incomplete markets, the model introduced in Section 3 does not have a closed form solution and therefore we need to restore to numerical methods to compute an approximated solution. More specifically, we solve the maximized HJB equation which results after replacing the first order conditions (Equations 10 and 11) and the terminal condition in Equation (9) back into the dynamic programming Equation 8.

To simplify the numerical procedure we exploit the homogeneity of the value function, $J(W, y, t)$, as shown in Munk and Sørensen (2010), and reduce the dimensionality of the problem by rewriting the model in terms of two state variables as opposed to three. In particular, we redefine the value function in terms of only wealth-income ratio and time. Then, for a given terminal condition, we combine a backward iterative procedure in the time dimension starting at $\hat{T}$ with an implicit finite difference approximations in the wealth-income ratio dimension to solve the system of equations formed by the HJB equation at a set of grid points for the two state variables. Following Munk and Sørensen (2010), the first-order partial derivatives of the value function with respect to the state variables are discretized using an upwind differencing scheme, while the second-order partial derivatives are approximated using a second-order central difference method. The model is solved with and without short-sales constraints. Similar methods to solve continuous-time dynamic programming have been employed in Brennan et al. (1997), Candler (1998), Munk and Sørensen (2010) and Achdou et al. (2015). The details on the algorithm can be found in Appendix 5.

Unconstrained solution. We first illustrate the quantitative implications for optimal asset allocation for the case where the manager of a commodity SWF does not face any leverage constraints. Figure 4 plots the mean and 95 percentile interval across 10000 simulations of the model's solution with each of the simulations covering a period of 70 years, i.e. until complete depletion of the resource. Panel (a) displays the optimal investment strategy in equity. It suggests that the manager should initially invest 20 percent above of his financial wealth in stocks in the first year. This result is explained by the relative large amount of underground commodity wealth available at the beginning of the investment horizon that allows the manager to borrow money against it and invest it in the risky asset which provides a higher return over the risk-free alternative. However, as the reserves of the commodity are depleted, the manager should decrease his holdings of the risky asset gradually until he reaches the myopic investment rule about 30 years. Panel (b) depicts the hedging demand component on the investment strategy given by Equation (11). Initially, this term represents around 6 percent of the initial share, then approaches zero monotonically as oil revenues gradually decrease.

The results in Panels (a) and (b) suggest that since the fund's commodity revenues are uncertain and the manager has a long term investment horizon, the optimal strategy should be 6 percent lower than what recommended by the standard myopic investment rule for short term investors. The smaller share in equities insures the fund against negative

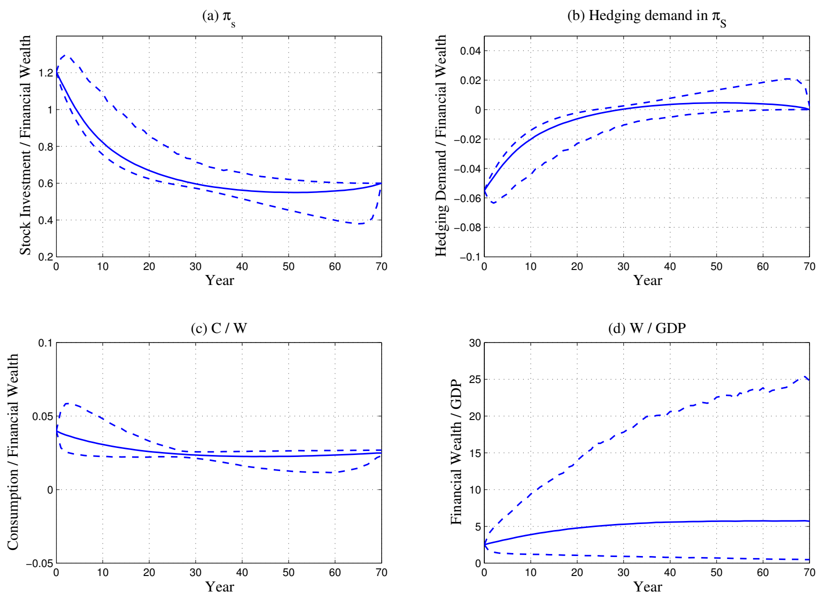

Figure 4: Optimal asset allocation under incomplete markets for Norway

Notes: Panels (a)–(d) plot the optimal investment strategy in equity, the hedging demand, the consumption rule as a fraction of financial wealth and the evolution of financial wealth to GDP ratio under incomplete markets for Norwegian economy following the calibration in Table (2). Solid lines represent the average value over 10000 simulated paths. Each of the paths contains 70 sample points. The dashed lines represent the 5 and 95 percentile from the empirical sampling distribution of simulated series.

future fluctuations in the fund's income. This strategy is implemented by the manager after reducing the amount of short-selling operations used to leverage his otherwise high exposition to risky assets.

Panel (c) indicates that the consumption to financial wealth ratio is relatively constant at 4 percent over the sample period, which is close to the current consumption/transfer policy followed by the Norwegian government. Finally, Panel (d) plots the financial wealth to GDP ratio by assuming that the non-oil GDP of Norway will grow at a constant rate of 1.5 percent per year during the next 70 years. The GDP growth ratio is computed as the average growth of real mainland GDP in Norway for the period 2008–2015 reported by IMF (2014). Therefore, the initial financial wealth-GDP ratio is set to its 2013 value of 2.5 and according to our model it will increase to 5.8 by the time the natural resource is fully depleted.

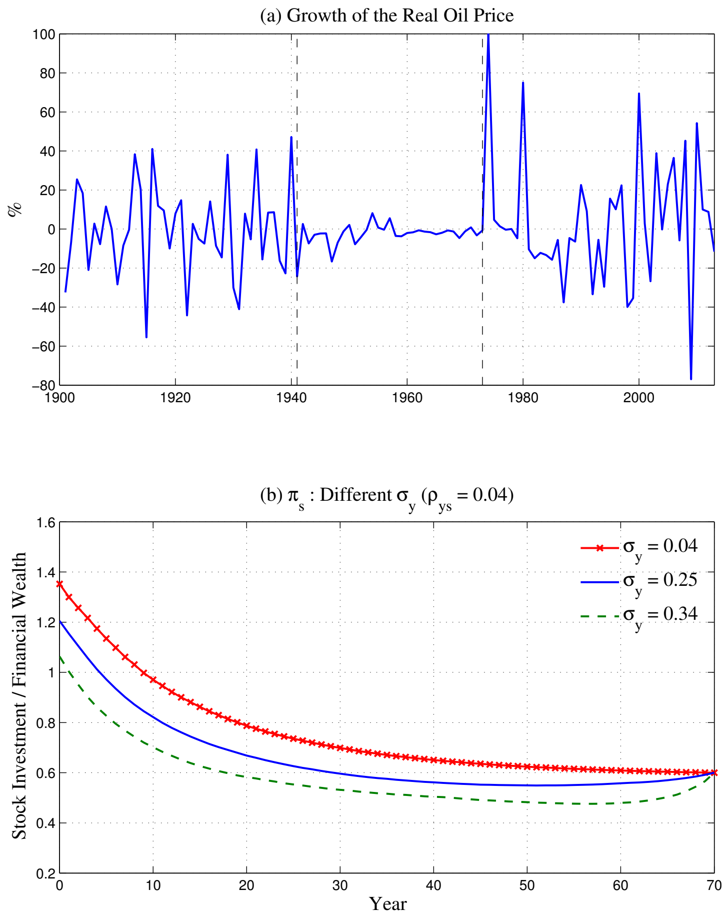

Figure 5: Optimal allocation for different stock-income correlations and oil volatilities

Notes: Panel (a) illustrates the annual growth of real oil price for the period 1900–2013. The vertical dash lines indicate the sub-periods for real oil price. The volatility of oil price growth is 0.04 for the period 1942–1973, 0.34 for the period 1974–2013 and 0.25 for the period. Panel (b) illustrates the sensitively of optimal investment on stock to the oil income volatility σ_y. The dash-dot curve is for $\sigma_y = 0.04$. The solid curve is for $\sigma_y = 0.25$. The dash curve is for $\sigma_y = 0.31$. For other parameters, the benchmark values from Table 2 are used. Panel (b) plots the mean value of 10,000 simulated paths.

The effect of oil price volatility. Panel (a) in Figure 5 depicts the annual variation of the oil price from 1900 to 2013. The plot clearly indicates that the volatility of the oil price changes over time in long-run samples. In particular we notice two different regimes of volatility: a high variability regime during the periods 1900-1940 and 1973-2013 and a low volatility episode from 1942 to 1973. To understand the role played by the volatility of oil price growth we assess how the optimal investment strategy changes for different volatility scenarios. Recall that in our benchmark calibration we have simply taken the average during the entire sample period 1900-2014. Therefore, in panel (b) we plot the optimal investment share in risky assets for three different volatilities of the oil price while keeping the remaining parameters as in the benchmark calibration, in particular a correlation coefficient between oil price changes and stock returns of 4 percent: (i) $\sigma_y = 0.01$ representing the low volatility period in 1942-1973, (ii) $\sigma_y = 0.34$ for the high volatility period of 1974-2014, and (iii) $\sigma_y = 0.25$ which corresponds to the variability in the whole sample.

The results suggest that for low expected commodity income volatility, the fund's manager should take a higher position in risky assets relative to the case where the expected oil price volatility is high. This is due to the low demand for hedging resulting from reduced income risks. The opposite holds when the expected volatility of oil price is high. In this case, SWF managers should allocate less of the financial wealth in the stock market. In general, we find that the allocation decision is substantially affected by the volatility regime. From the perspective of time $t = 0$, the difference in equity allocations can be as high as 30 percent of the fund's financial wealth when moving from a low to high volatility state[6]. So far our experiment only considers the case of either low or high oil price volatility. Future research should allow for a time varying volatility of oil price changes such that the fund's manager includes his expectations about the future development of the volatility in his optimal asset allocation. At this point, we conjecture than the optimal allocation in Equation (11) will be extended to include a new hedge-demand component that will allow the manager to insure his portfolio returns against volatility risks.

Effect of constraints. As discussed previously, the optimal investment strategy dictates that the investor should hold an initial position in stocks above his initial financial wealth. In practice, however, many commodity SWF are liquidity constrained in that it may not be feasible to borrow money to achieve such a portfolio. To understand the effects of such type of constraints we compute the optimal investment when the SWF's ability to short-sale risk free assets to increase his position in equity is restricted according to $0 \leq \pi_{St} \leq 1$. Figure

[6] The share invested in the risky asset is 1.35 for the low volatility scenario and 0.94 for the high volatility case.

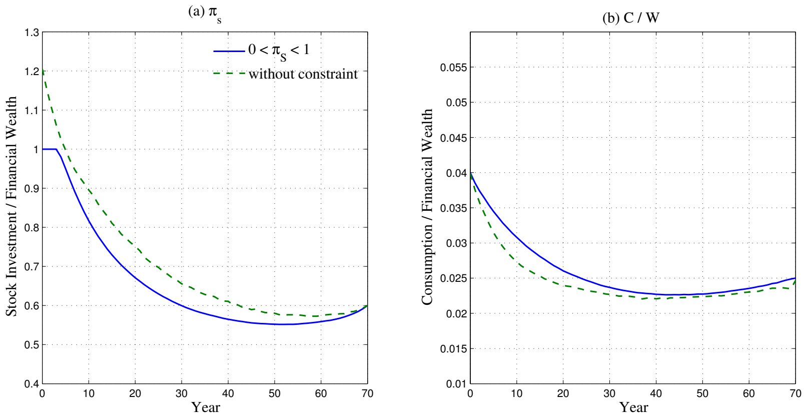

Figure 6: Optimal asset allocation with liquidity constraint

Notes: Panel (a) and (b) illustrates the sensitivity of the optimal investment on stocks and consumption to financial wealth ratio to liquidity constraints of the type $0 \leq \pi_{St} \leq 1$. The solid lines represent the average value over 10000 simulated paths of the optimal policy functions with liquidity constrains. Each of the paths contains 70 sample points. The dashed lines denote the case where the policy functions are unconstrained. The parameters used in the solution and simulation of the model are those in Table 2.

6 plots the results. In general, we find a moderate effect of short-sale constraints in the optimal allocation of financial wealth in equity for the case of Norway. Indeed, as expected since Norway has an initial high wealth-income ratio the constraint is likely to be binding with low probability. The only effect is a moderate adjustment initially in the share of the risky asset. Also the consumption-wealth ratio has mild initial adjustment to reach higher consumption-wealth levels at the end of the period.

Loss due to suboptimal investments. Our optimal investment policy adjusts continuously the holdings in equities such that they reflect the economic conditions given by time variation in the state variables $(W_t, O(y_t, t), t)$. In reality, however, commodity SWFs are sometimes bounded and regulated to keep their investment policies within some ranges set through sovereign mandates. To shed some light into the potential losses of following a constant investment rule versus the optimal time-varying rule, we compute the real financial wealth gains of behaving optimally as predicted by our model. Panel (a) in Figure 7 plots the optimal and constant rule. The latter is fixed to the myopic strategy of 60 percent while the former replicates the investment rule in Figure 6. Panel (b) reports the gross gains in financial wealth from following the optimal rule versus the constant rule. It is clear from the results, that real financial wealth is almost doubled close to the depletion time of the natural resource when following the time-varying asset allocation strategy, and is never

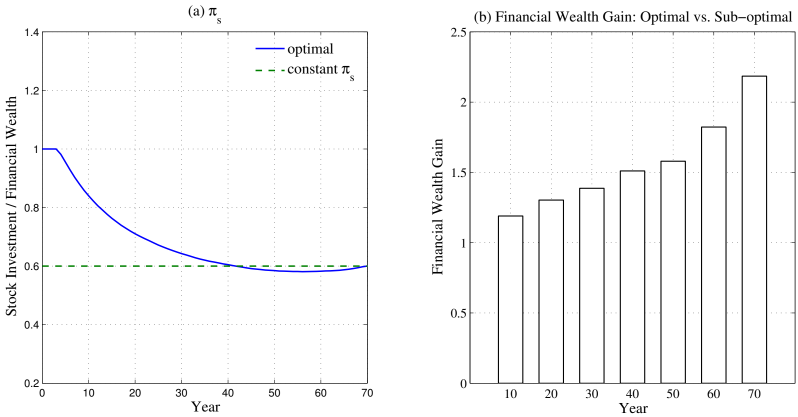

Figure 7: Comparison between optimal and constant investment share

Notes: Panels (a) and (b) compare the investment share and wealth-GDP ratio between the case with constant investment share on stock and the optimal investment strategies with liquidity constraints $0 \leq \pi_s \leq 1$. The solid curve is for the case with optimal investment strategies, as in Equation (11), and consumption rule, in Equation (10). The dash line is for the suboptimal solution assuming constant investment share on risky asset, $\pi_s = 60\%$. The simulation employs parameters as shown in Table 2. The graph plots the mean value of 10,000 simulated paths.

below the wealth that can be achieved by following the myopic rule[7].

4.3 Comparison among countries

This section compares the optimal investment strategies among different SWFs. In particular, we contrast the dynamic behavior of the funds in Norway, UAE and Chile. Table 3 reports the calibration used for each the investment funds. The first column replicates the values used in the benchmark calibration in Table 2. For the SWF in the UAE (Abu Dhabi Investment Authority) we use the same commodity price and stock return parameters as for Norway, but different technical parameters associated with the extraction rate, initial wealth and income values. As shown in the second column, we fix the extraction rate of crude oil in the UAE at 4 percent per year which implies a negative drift for oil income of 3 percent per annum. The financial wealth is set at 7 times that of oil income in 2013. With these assumptions we estimate that complete depletion of the resource will be achieved in 170 years. In the third column we summarize the parameters used for the SWF in Chile (The Social and Economic Stabilization Fund in Chile) in which case the main funding source is that from copper exports. The average growth of the copper price is -0.4 percent over the period 1900-2013 as reported in Table 1. The annual production/extraction rate

[7]To facilitate the comparison we use the optimal rule $c(W_t, y_t, t)$ in both cases. Future research should adjust the optimal consumption rule to the constant investement rule.

is set to be 3 percent. Therefore the expected copper income growth is -3.4 percent. Due to the relatively low extraction rate, the depletion time of copper is estimated to be 150 years. The correlation between the copper price inflation and the stock returns is 0.32 for the period 1900-2013. As a young sovereign wealth fund (created in 2007) the cumulated financial wealth is low and as of 2013 it only accounts for 38 percent of the copper income.

Comparing the UAE and Norway. Figure 8 compares optimal investment strategy for fund managers of Norway and the UAE under short-sale constraints. Panel (a) plots the average investment share on equities over 10000 simulations of the model's solution, each of them with 70 sample points. The results suggest that, given the current calibration, the UAE holds a higher position in the risky asset over time. This can be explained by two interconnected factors. First, the time to depletion is further out in time which implies their underground oil wealth is higher from the perspective of today. Second, the initial financial wealth to income ratio is lower than the one reported for Norway leading to a higher demand for equities as can be seen from Equation 11. Furthermore, the hedging demand plotted in panel (b) combined with a higher expected growth rate in oil revenues over time indicate that the SWF in the UAE can invest a higher fraction of its financial wealth into stocks relative to Norway without compromising the fund's ability to smooth its transfers to the government. Ceteris paribus, this leads us to conclude that the optimal share of financial wealth invested in risky assets at every point in time is an increasing function of time to depletion, income to financial-wealth ratio and expected growth in commodity revenues.

The case of Chile. We would like to finish this section with some comments regarding the Chilean SWF where copper is the source of income. While the price of copper exhibits a similar level of volatility as the price of oil for all the sub-periods considered in this paper, its correlation with the stock returns is higher than the one reported for the price of oil. In particular, for the period 1900-2013, the sample correlation between stock returns and price changes in copper is about eight times larger than that for the oil price. Meanwhile, Chile may have higher risk aversion. Given the parameter values for Chile, Figure 9 plots the optimal investment strategy for different values of the coefficient of relative risk aversion. Interestingly, we find that for a coefficient of relative risk aversion $\gamma = 3$, the optimal allocation policy in the risky asset is reverted in the sense that now the manager should start investing all his financial wealth in risk free assets for the first 30 years (even short-sale equities if possible to leverage this strategy) and then increase gradually his position in equities to reach a steady state fraction of about 40 percent in the long run.

Table 3: Calibrated parameters for different countries (period 1900-2013)

Parameter	Norway	UAE	Chile
	Oil	Oil	Copper
α_p	0.010	0.010	-0.004
α_q	0.060	0.040	0.030
α	-0.050	-0.030	-0.034
r	0.006	0.006	0.006
ψ	0.050	0.050	0.050
σ_y	0.250	0.250	0.250
σ_s	0.200	0.200	0.200
ρ_{yS}	0.040	0.040	0.321
γ	2.046	2.000	2.000
δ	0.030	0.030	0.030
$\hat{T}$	70	170	150
W_{2013}/y_{2013}	13.00	7.00	0.38
W_{2013}/GDP_{2013}	2.5	3.0	0.1
$\log\left(GDP_t/GDP_{t-1}\right)$	0.015	0.015	0.015

Notes: Calibrated parameters use annual data for the period 1900–2013. Nominal prices are deflated by the U.S. CPI deflator from U.S. Bureau of Labor Statistics (2014). α_p is calibrated as the mean growth rate of real WTI oil price and real high grade copper price. Commodity prices are collected from Global Financial Data (2014). α_q is computed as the extraction rate at 2013 level in Norway, UAE and Chile. The production of oil is obtained from U.S. Energy Information Administration (2014). The copper production in Chile is from Chilean Copper Commission (2014). The crude oil reserve is obtained from BP (2014). The Chilean reserve of copper is collected from U.S. Geological Survey (2014). α is computed following $\alpha = \alpha_p - \alpha_q$. The parameter values of r and ψ are from Campbell and Viceira (2002). σ_y and σ_s are the standard deviation of real commodity price growth and real stock price growth. U.S. S&P 500 composite price index, the measure of stock price, is obtained from Global Financial Data (2014). ρ_{yS} is the correlation between the stock price growth and commodity price growth. γ is computed by targeting the optimal $\pi_s = \frac{1}{\gamma}\frac{\psi}{\sigma_S^2} = 60\%$ at period $\hat{T}$. The value of δ follows Munk and Sørensen (2010). $\hat{T}$ is computed using reserve of crude oil and copper by assuming a constant extraction rate α_q for each country. The W_{2013}/y_{2013} and W_{2013}/GDP_{2013} are computed using financial wealth, oil income and GDP in 2013. The financial wealth in Norway, UAE and Chile are obtained from Norges Bank Investment Management (2013), SWF Institute (2014) and Ministry of Finance Gobierno de Chile (2013) respectively. The commodity income is computed as the product of real commodity price times extraction quantity, assuming zero production cost. The GDP of three countries are collected from World Development Indicators (2013). $log\left(GDP_t/GDP_{t-1}\right)$, a measure of expected GDP growth rate in Norway, follows the average growth of real mainland GDP in Norway for the period 2008–2015 reported by IMF (2014).

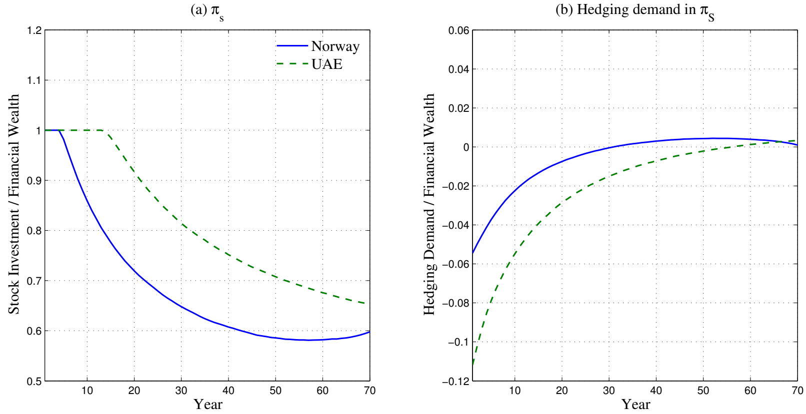

Figure 8: Optimal allocation under incomplete market for Norway and UAE:

Notes: Panels (a) and (b) plot optimal investment strategies and financial wealth GDP ratio for Norway, UAE and Chile. The solid line is for Norway, the solid-cross line is for UAE, and the dash line is for Chile. The simulations employ parameters as shown in Table 3 for each country. The liquidity constraints $0 \leq \pi_s \leq 1$ are imposed when solving the model numerically. The graph plots the mean value of 10,000 simulated paths.

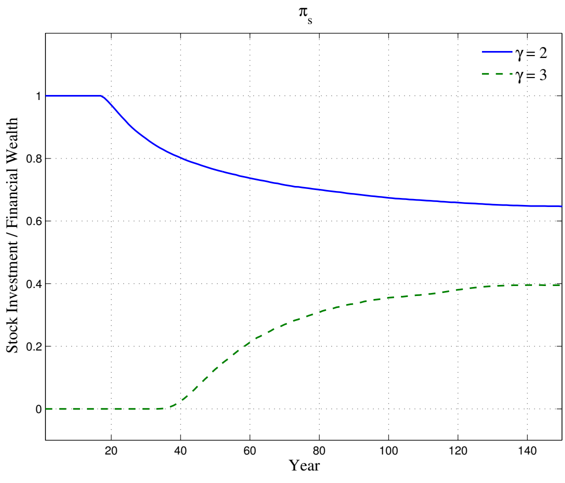

Figure 9: Optimal allocation with different relative risk aversion under incomplete market for Chile:

Notes: The figure plots optimal investment strategies on risky asset for Chile using different relative risk aversion parameter γ. The solid line is for $\gamma = 2$, the dash line is for $\gamma = 3$. The simulations employ other parameters as shown in Table 3 for Chile. The liquidity constraints $0 \leq \pi_s \leq 1$ are imposed when solving the model numerically. The graph plots the mean value of 10,000 simulated paths.

5 Conclusions

Sovereign wealth funds have become an important player in the global asset markets during the last 15 years. They represent 3 percent of the global investment industry and their asset holdings far exceed those of hedge funds. Therefore, this paper investigates the optimal portfolio allocation of a commodity SWF with a long-term investment horizon by setting up and solving an otherwise standard dynamic asset allocation problem under incomplete markets along the lines of Bodie et al., 1992 and Campbell and Viceira (2002). To bring the model closer to the economic environment faced by a SWF, we calibrate the fund's income process to replicate the volatility and correlation of commodity price changes and stock returns.

To gain analytical tractability and useful insights about the dynamics and mechanisms behind the model, we first solve the model under the assumption of complete market following Munk and Sørensen (2010). We find that when income risk is fully spanned by the stock market, the manager's optimal asset allocation depends mainly on the dynamics of the oil wealth to financial wealth ratio, where the former is defined as the net present value of all future revenues coming from commodity sales. We also find that the decision to increase or decrease the demand for risky assets depends crucially on the level of relative risk aversion, the volatility of commodity income and its correlations with the stock returns.

With this information at hand, we then proceed to solve the model under the assumption of incomplete markets in an attempt to bring the model closer to reality. Two types of experiments are carried out. We first compute the optimal behavior for unconstrained investment strategies and then we impose liquidity constraints of the short-sales type. The model is calibrated to resemble the conditions for the Norwegian SWF and simulated to quantitatively assess different investment policies.

For the unconstrained case we find that the SWF in Norway should initially allocate more than 100 percent of their financial wealth into risky assets and then gradually decrease its holdings of stocks to reach a long run share of 60 percent. This position should be achieved after 30 years. When the fund's manager is short-sale constrained, the optimal solution suggests to keep all the financial wealth invested in stocks for the first five years and then start decreasing gradually its holdings to the long run share of 60 percent.

In long-run samples, the growth in the price of oil exhibits periods of low and high volatilities. Therefore, we conduct a sensitivity analysis to assess the effect of different levels of commodity income volatility. We find that different volatility regimes can substantially affect the size of the hedging demand with differences of up to 30 percent in the share invested in stocks when going from a low to a high volatility episode.

We finally perform a similar analysis for two other commodity SWFs in the UAE and Chile. We find that given the higher underground oil wealth reserves than Norway, the UAE should follow an investment strategy in equities that far exceeds the optimal rule in Norway. This experiment leads us to conclude that, ceteris paribus, the optimal share of financial wealth invested in risky assets at every point in time is an increasing function of time to depletion, income to financial-wealth ratio and expected growth in commodity revenues. For the case of Chile, we find that for reasonable values of the relative risk aversion coefficient, it may be optimal for the fund's manager to start investing all his wealth in risk-free assets and then gradually decrease his position to create a portfolio that also includes risky assets.

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Appendix

A. Commodity wealth under complete markets

In this appendix we solve for the reduced form solution of oil wealth under complete market with $\rho_{yS} = \pm 1$.

When the income is spanned with stock price, the commodity income can be valued as the dividend stream from the traded stock

$$\frac{dy_t}{y_t} = \alpha dt + \sigma_y \rho_{yS} dz_{St}. \tag{18}$$

The risk-neutral income process can be written as:

$$\frac{dy_t}{y_t} = \left(\alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S}\right) dt + \sigma_y \rho_{ys} y dz_{St}^{\mathbb{Q}}. \tag{19}$$

where $z_{St}^{\mathbb{Q}}$ is the Wiener process under risk neural probability $\mathbb{Q}$, where

$$z_{St}^{\mathbb{Q}} = \frac{\psi}{\sigma_S} t + z_{st}.$$

According to the Feynman-Kac Theorem, the value of a future commodity income at time t takes the form:

$$O(y_t, t) = E_t^{\mathbb{Q}} \left[\int_t^{\hat{T}} e^{-r(s-t)} y_s ds \right] \tag{20}$$

given the boundary condition $O\left(y, \hat{T}\right) = 0$. To be noticed that the expectation is under the risk-neutral probably $\mathbb{Q}$.

According to (19), we may solve for y_s as:

$$\begin{aligned} y_s &= y_t \exp\left\{ \left(\alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S} - \frac{1}{2}\sigma_y^2\right)(s-t) + \int_t^s \sigma_y \rho_{yS} dz_{Su}^{\mathbb{Q}} \right\} \\ &= y_t \exp\left\{ \left(\alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S} - \frac{1}{2}\sigma_y^2\right)(s-t) + \sigma_y \rho_{yS} \left(z_{Ss}^{\mathbb{Q}} - z_{St}^{\mathbb{Q}}\right) \right\}, \end{aligned}$$

and then

$$e^{-r(s-t)} y_s = y_t \exp\left\{ \left(-r + \alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S} - \frac{1}{2}\sigma_y^2\right)(s-t) + \sigma_y \rho_{yS} \left(z_{Ss}^{\mathbb{Q}} - z_{St}^{\mathbb{Q}}\right) \right\}. \tag{21}$$

Known that $z_{Ss}^{\mathbb{Q}} - z_{St}^{\mathbb{Q}}$ is normally distributed, we may apply the standard rule for expectations of exponential of normal random variables on Equation (21). That is

$$\begin{aligned} E_t^{\mathbb{Q}}\left[e^{-r(s-t)}y_s ds\right] &= y_t E_t^{\mathbb{Q}} \exp\left\{\left(-r+\alpha-\sigma_y\rho_{yS}\frac{\psi}{\sigma_S}-\frac{1}{2}\sigma_y^2\right)(s-t)+\sigma_y\rho_{yS}\left(z_{Ss}^{\mathbb{Q}}-z_{St}^{\mathbb{Q}}\right)\right\} \\ &= y_t e^{F(t,s)-r+\alpha-\sigma_y\rho_{yS}\frac{\psi}{\sigma_S}-\frac{1}{2}\sigma_y^2} \end{aligned} \tag{22}$$

where $F(t,s)$ is some function can be computed easily. Integrating (22) over s we obtain equation (12).

B. Closed form solution under complete market

This appendix solves for the closed form solution under complete market with $\rho_{yS} = \pm 1$.

The social planner faces the problem of maximizing the expected present value of utility such that

$$J(W_t, y_t, t) = \max_{\{c_u, \theta_{Su}\}_{u=t}^T} \mathbb{E}_t\left[\int_t^T e^{-\delta t}U(c_u)\,du + \epsilon e^{-\delta(T-t)}U(W_T)\right].$$

We assume CRRA utility function: $U(c_t) = \frac{1}{1-\gamma}c_t^{1-\gamma}$.

We assume the investor has access to two tradable assets. The bond is the risk-free asset whose price evolves following:

$$\frac{dB_t}{B_t} = rdt. \tag{23}$$

The stock is the risky asset, whose price evolves as the process:

$$\frac{dS_t}{S_t} = (r+\psi)\,dt + \sigma_S dz_{St}. \tag{24}$$

Under complete market, the income rate is spanned by the risky asset. The income evolves following:

$$dy_t = y_t\left[\alpha dt + \sigma_y\rho_{yS}dz_{St}\right]$$

where $\rho_{yS} = \pm 1$.

The dynamic of wealth evolves following:

$$dW_t = \theta_S\frac{dS_t}{S_t} + (y_t - c_t)\,dt + (W_t - \theta_{St})\,rdt. \tag{25}$$

Inserting (24) into (25), we obtain that:

$$\begin{aligned} dW_t &= \left[(r+\psi)\,\theta_{St} + r\,(W_t - \theta_{St}) + y_t - c_t\right]dt + \sigma_S\theta_S dz_{St} \\ &= \left[rW_t + \psi\theta_{St} + y_t - c_t\right]dt + \sigma_S\theta_S dz_{St}. \end{aligned}$$

The terminal condition of the value function $J(W, y, t)$ is given such that:

$$J(W, y, T) = \epsilon U(W_T) = \frac{\epsilon W_T^{1-\gamma}}{1-\gamma} \tag{26}$$

where W_T is the terminal level of wealth.

Then the HJB function under complete market is given by:

$$\begin{aligned} \delta J(W_t, y_t, t) &= \max_{c_t, \theta_{St}} U(c_t) + J_t(W_t, y_t, t) + J_W[rW_t + \psi\theta_{St} + y_t - c_t] \\ &+\frac{1}{2}J_{WW}(W_t, y_t, t)\,\sigma_S^2\theta_{St}^2 \\ &+\alpha y J_y(W_t, y_t, t) + \frac{1}{2}J_{yy}(W_t, y_t, t)\,y^2\sigma_y^2 \\ &+J_{Wy}(W_t, y_t, t)\,y_t\theta_{St}\sigma_S\sigma_y\rho_{yS} \end{aligned} \tag{27}$$

where J_i denotes the first order partial derivative the value function with respect to the state variable i, and J_{ij} the second order derivative with respect to the state variables i and j.

The first order conditions gives solution for θ_S and c as follow:

$$\theta_{St} = -\frac{J_W}{J_{WW}}\frac{\psi}{\sigma_S^2} - \frac{y_t J_{Wy}}{J_{WW}}\frac{\sigma_y\rho_{yS}}{\sigma_S} \tag{28}$$

$$c_t = J_W^{-\frac{1}{\gamma}}. \tag{29}$$

While looking for analytical solution of value function $J(W_t, y_t, t)$, we first conjecture the value function takes the form:

$$J(W_t, y_t, t) = \frac{1}{1-\gamma}g(t)^\gamma (W_t + O(y_t, t))^{1-\gamma} \tag{30}$$

with unknown function form of $g(t)$. $O(y_t, t)$ is given in the complete market case which follows

$$\begin{aligned} O(y_t, t) &= \begin{cases} \frac{y_t}{r-\alpha+\sigma_y\rho_{yS}\frac{\psi}{\sigma_S}}\left(1 - e^{-\left(r-\alpha+\xi\frac{\psi}{\sigma_S}\right)(\hat{T}-t)}\right) & \text{if } r-\alpha+\sigma_y\rho_{yS}\frac{\psi}{\sigma_S} \neq 0 \\ y_t\left(\hat{T}-t\right) & \text{if } r-\alpha+\sigma_y\rho_{yS}\frac{\psi}{\sigma_S} = 0 \end{cases} \\ &= y_t M(t). \end{aligned}$$

According to (30), we may get the derivatives of value function

$$\begin{aligned}
J_t &= \frac{\gamma}{1-\gamma} g^{\gamma-1} (W+O)^{1-\gamma} g_t + g^{\gamma} (W+O)^{-\gamma} O_t \qquad (31) \\
J_W &= g^{\gamma} (W+O)^{-\gamma} \\
J_y &= g^{\gamma} (W+O)^{-\gamma} O_y \\
J_{WW} &= -\gamma g^{\gamma} (W+O)^{-\gamma-1} \\
J_{yy} &= g^{\gamma} \left[(W+O)^{-\gamma} O_{yy} - \gamma (W+O)^{-\gamma-1} (O_y)^2 \right] \\
J_{Wy} &= -\gamma g^{\gamma} (W+O)^{-\gamma-1} O_y .
\end{aligned}$$

We drop the time subscript t and functional notations for simplicity, for example $O = O(y_t, t)$ and $g = g(t)$. Moreover, J_t denotes the first order partial derivative the value function with respect to time t. Inserting into the first order conditions, we get

$$\begin{aligned}
\theta_S &= \frac{(W+O)}{\gamma} \frac{\psi}{\sigma_S^2} + y O_y \frac{\sigma_y \sigma_S \rho_{yS}}{\sigma_S^2} \\
&= \frac{1}{\sigma_S^2} \left\{ \frac{(W+O)}{\gamma} \psi + y O_y \sigma_y \sigma_S \rho_{yS} \right\} \qquad (32)
\end{aligned}$$

and

$$c = \frac{W+O}{g(t)}. \qquad (33)$$

Inserting (31), (32) and (33) into (27), and knowing that $O_y = M$, we get

$$\begin{aligned}
\delta \frac{1}{1-\gamma} g^{\gamma} (W+O)^{1-\gamma} &= \frac{\gamma}{1-\gamma} \left(g^{\gamma-1} (W+O)^{1-\gamma} \right) + \frac{\gamma}{1-\gamma} g^{\gamma-1} (W+O)^{1-\gamma} g_t + g^{\gamma} (W+O)^{-\gamma} O_t \\
&\quad + g^{\gamma} (W+O)^{-\gamma} \left[rW + \frac{1}{\sigma_S^2} \left[\frac{(W+O)}{\gamma} \psi^2 + yM\sigma_y (\sigma_S \rho_{yS}) \psi \right] - g^{-1} (W+O) + y \right] \\
&\quad - \frac{1}{2} \gamma g^{\gamma} (W+O)^{-\gamma-1} \left[\frac{1}{\sigma_S^2} \left\{ \frac{(W+O)}{\gamma} \psi + yM\sigma_y \sigma_S \rho_{yS} \right\}^2 \right] \\
&\quad + \alpha y g^{\gamma} (W+O)^{-\gamma} O_y + \frac{1}{2} g^{\gamma} \left[(W+O)^{-\gamma} O_{yy} - \gamma (W+O)^{-\gamma-1} (O_y)^2 \right] y^2 \sigma_y^2 \rho_{yS}^2 \\
&\quad - \gamma g^{\gamma} (W+O)^{-\gamma-1} O \theta_S \sigma_y \sigma_S \rho_{yS} .
\end{aligned}$$

Rewriting the term $rW = r(W+O) - rO$ and factorizing terms with $g^{\gamma} (W+O)^{1-\gamma}$ and $g^{\gamma} (W+O)^{-\gamma}$, we obtain

$$\begin{aligned}
0 &= g^{\gamma} (W+O)^{1-\gamma} \left\{ -\delta \frac{1}{1-\gamma} + \frac{\gamma}{1-\gamma} g^{-1} + \frac{\gamma}{1-\gamma} g^{-1} g_t + r + \frac{1}{2} \frac{1}{\gamma} \frac{\psi^2}{\sigma_S^2} \right\} \\
&\quad + g^{\gamma} (W+O)^{-\gamma} \left\{ O_t - rO + y + \left(\alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S} \right) y O_y + \frac{1}{2} O_{yy} y^2 \sigma_y^2 \rho_{yS}^2 \right\} . \qquad (34)
\end{aligned}$$

Applying Feynman-Kac theorem, with given the solution of O under complete market in Equation

(20) and risk-neutral process of y in (19), we obtain the PDE in $O(y,t)$ as follow

$$O_t + \left[\alpha - \sigma_y \rho_{yS} \frac{\psi}{\sigma_S}\right] y O_y + \frac{1}{2}\sigma_y^2 \rho_{yS}^2 y^2 O_{yy} - rO + y = 0. \tag{35}$$

Therefore the second term in the (34) is zero. The (34) turns to

$$g_t = \frac{1}{\gamma}\left[\delta - r(1-\gamma) - \frac{1-\gamma}{2\gamma}\frac{\psi^2}{\sigma_S^2}\right] g - 1$$

Then we can solve for the function $g(t)$ give as:

$$g(t) = \frac{1}{A}\left[1 + (BA-1)\, e^{-A(T-t)}\right]$$

where

$$A = \frac{1}{\gamma}\left[\delta - r(1-\gamma) - \frac{1-\gamma}{2\gamma^2}\|\lambda\|^2\right]$$

and B is a unknown constant. According to the terminal condition (26), we get

$$B = g(T) = \epsilon^{\frac{1}{\gamma}}.$$

Therefore

$$g(t) = \frac{1}{A}\left[1 + \left(\epsilon^{\frac{1}{\gamma}} A - 1\right) e^{-A(T-t)}\right] \tag{36}$$

or

$$g(t) = \frac{1}{A}\left[1 - e^{-A(T-t)}\right] + \epsilon^{\frac{1}{\gamma}} e^{-A(T-t)}.$$

Inserting into (30), we obtain the solution of value function.

$$J(W,y,t) = \frac{1}{1-\gamma} g(t)^{\gamma} (W + O(y,t))^{1-\gamma}$$

where

$$g(t) = \frac{1}{A}\left[1 - e^{-A(T-t)}\right] + \epsilon^{\frac{1}{\gamma}} e^{-A(T-t)}$$
$$A = \frac{1}{\gamma}\left[\delta - r(1-\gamma) - \frac{1-\gamma}{2\gamma^2}\lambda_S^2\right].$$

Thus the closed-form solutions for consumption and investment are

$$c = \frac{W + O(y,t)}{g(t)} \tag{37}$$

$$\theta_S = \frac{1}{\gamma}(W + O(y,t))\frac{\psi}{\sigma_S^2} - O\frac{\sigma_y \rho_{yS}}{\sigma_S} \tag{38}$$

for $t \in [0,T]$. Dividing both (37) and (38) by W, we arrive at (16) and (17).

C. Numerical solution method

This appendix explains the algorithm of the numerical solution for the asset allocation model under incomplete market in Section 4.2.

C.1 The original problem

We are looking for solution of $J(W, y, t)$ that solves the Bellman Equation (8) which is a non-linear partial difference equation shown as

$$\begin{aligned}\delta J &= \frac{c^{1-\gamma}}{1-\gamma} + J_t + J_W\left[rW + \theta_S\sigma_S\lambda_S - c + y\right] \\ &\quad + \frac{1}{2}J_{WW}\theta_S^2\sigma_S^2 \\ &\quad + J_y\alpha y + \frac{1}{2}J_{yy}y^2\sigma_y^2 \\ &\quad + J_{Wy}y\theta_S\sigma_S\sigma_y\rho_{yS}\end{aligned} \tag{39}$$

where J_i denotes the first order partial derivative the value function with respect to the state variable i, and J_{ij} the second order derivative with respect to the state variables i and j.

According to the first order conditions, θ_s takes the form that

$$\theta_S = -\frac{J_W}{J_{WW}}\frac{\lambda_S}{\sigma_S} - \frac{yJ_{Wy}}{J_{WW}}\frac{\sigma_y\rho_{yS}}{\sigma_S}$$

and

$$c = J_W^{-\frac{1}{\gamma}}.$$

The terminal condition at time T is known as

$$J(W, y, T) = \epsilon U(W) = \frac{\epsilon W^{1-\gamma}}{1-\gamma}.$$

We know that after depletion time $\hat{T}$ the oil income is zero. There exists an analytical solution for the asset allocation model without endowment, that is

$$J(W, y, t) = \frac{1}{1-\gamma}g(t)^\gamma W^{1-\gamma}, \text{ for } \hat{T} \leq t \leq T$$

where

$$\begin{aligned} g(t) &= \frac{1}{A}\left[1 - e^{-A(T-t)}\right] + \epsilon^{\frac{1}{\gamma}}e^{-A(T-t)} \\ A &= \frac{1}{\gamma}\left[\delta - r(1-\gamma) - \frac{1-\gamma}{2\gamma}\frac{\psi^2}{\sigma_S^2}\right].\end{aligned}$$

Thus at depletion time $\hat{T}$, we have

$$J\left(W, y, \hat{T}\right) = \frac{1}{1-\gamma} g\left(\hat{T}\right)^{\gamma} W^{1-\gamma} \tag{40}$$

where

$$g\left(\hat{T}\right) = \frac{1}{A}\left[1 - e^{-A\left(T-\hat{T}\right)}\right] + \epsilon e^{-A\left(T-\hat{T}\right)}. \tag{41}$$

In this equation δ,γ, r, ρ_{yS}, α, σ_s, σ_y, ψ, ϵ and $\hat{T}$ are parameters.

C.2 The transformed problem

To drop one state variable we use some properties of the function J. Knowing that the function J is homogeneous of degree $1-\gamma$ in W and y (See Munk and Sørensen (2010)), i.e.

$$J\left(k\left(t\right)W, k\left(t\right)y, t\right) = k\left(t\right)^{1-\gamma} J\left(W, y, t\right) \Longrightarrow J\left(W, y, t\right) = k\left(t\right)^{\gamma-1} J\left(k\left(t\right)W, k\left(t\right)y, t\right).$$

Using it with $k\left(t\right) = e^{-\beta t}/y$, we obtain

$$J\left(W, y, t\right) = y^{1-\gamma} e^{-\beta(\gamma-1)t} J\left(x, e^{-\beta t}, t\right) \equiv y^{1-\gamma} F\left(x, t\right) \tag{42}$$

where we define $x = e^{-\beta t}W/y$.

Employing the relationship in (42) and $x = e^{-\beta t}W/y$, we obtained the relationship as follow:

$$\begin{aligned} \frac{\partial x}{\partial y} &= -e^{-\beta t}\frac{W}{y^2} = -\frac{x}{y} \\ \frac{\partial x}{\partial t} &= -\beta x \end{aligned}$$

$$\frac{\partial x}{\partial W} = \frac{e^{-\beta t}}{y}$$

$$
\begin{aligned}
J_t &= y^{1-\gamma}\left(F_t - \beta F_x x\right) \\
J_W &= y^{1-\gamma} F_x \frac{\partial x}{\partial W} = e^{-\beta t} y^{-\gamma} F_x \\
J_{WW} &= e^{-\beta t} y^{-\gamma} F_{xx} \frac{\partial x}{\partial W} \\
&= e^{-2\beta t} y^{-\gamma-1} F_{xx} \\
J_y &= (1-\gamma) y^{-\gamma} F + y^{1-\gamma} F_x \frac{\partial x}{\partial y} = (1-\gamma) y^{-\gamma} F - y^{1-\gamma} F_x \frac{x}{y} \\
&= y^{-\gamma}\left[(1-\gamma) F - F_x x\right] \\
J_{yy} &= y^{-\gamma-1}\left[-\gamma(1-\gamma) F + 2\gamma F_x x + F_{xx} x^2\right] \\
J_{Wy} &= e^{-\beta t}\left(-\gamma y^{-\gamma-1} F_x + y^{-\gamma} F_{xx} \frac{\partial x}{\partial y}\right) \\
&= e^{-\beta t}\left(-\gamma y^{-\gamma-1} F_x - y^{-\gamma-1} F_{xx} x\right) \\
&= e^{-\beta t} y^{-\gamma-1}\left(-\gamma F_x - F_{xx} x\right).
\end{aligned}
$$

Inserting into (39) and dividing both side of the equation by $y^{1-\gamma}$, we get

$$
\begin{aligned}
\delta F \;=\; & \frac{\left(\frac{c}{y}\right)^{1-\gamma}}{1-\gamma} + F_t - \beta F_x x + \left[r \frac{W}{y} e^{-\beta t} + \pi_S \frac{W}{y} e^{-\beta t} \sigma_S \lambda_S - \frac{c}{y} e^{-\beta t} + e^{-\beta t}\right] F_x \\
& + \frac{1}{2} \pi_S^2 \sigma_S^2 \underbrace{\left(\frac{W}{y} e^{-\beta t}\right)^2}_{x^2} F_{xx} \\
& + \alpha\left[(1-\gamma) F - F_x x\right] \\
& + \frac{1}{2} \sigma_y^2\left[-\gamma(1-\gamma) F + 2\gamma F_x x + F_{xx} x^2\right] \\
& + \sigma_y \pi_S \sigma_S \rho_{yP} e^{-\beta t} \frac{W}{y}\left(-\gamma F_x - F_{xx} x\right).
\end{aligned}
\tag{43}
$$

Define $\hat{c} \equiv \frac{c}{y}$ and $\pi_s \equiv \frac{\theta_S}{W}$. Factoring terms with F in the Equation (43), then we are aiming to solve for $F(x,t)$ which solves the following PDE.

$$
\begin{aligned}
\hat{\delta} F \;=\; & \frac{\hat{c}^{1-\gamma}}{1-\gamma} + F_t \\
& + \left\{(1-\hat{c}) e^{-\beta t} + x\left[r - \alpha + \pi_S \sigma_S\left(\frac{\psi}{\sigma_S} - \sigma_y \rho_{yP} \gamma\right) + \sigma_y^2 \gamma - \beta\right]\right\} F_x \\
& + \frac{1}{2}\left[\pi_S^2 \sigma_S^2 + \sigma_y^2 - 2\sigma_y \sigma_S \rho_{yS} \pi_S\right] x^2 F_{xx}
\end{aligned}
\tag{44}
$$

where

$$
\hat{c} = F_x^{-\frac{1}{\gamma}} e^{\frac{\beta t}{\gamma}}
\tag{45}
$$

and

$$\pi_S = -\frac{F_x}{xF_{xx}}\frac{\frac{\psi}{\sigma_S}-\gamma\sigma_y\rho_{yS}}{\sigma_S}+\frac{\sigma_y\rho_{yS}}{\sigma_S}. \tag{46}$$

In Equation (44), β, γ, r, ρ, σ and α are parameters, and $\hat{\delta}$ is

$$\hat{\delta} = \delta + \alpha(\gamma-1) - \frac{1}{2}\sigma_y^2\gamma(\gamma-1).$$

Thus we transform the original two-state PDE (39) into a problem solving for $F = F(x,t)$ with one state variable as shown in (44).

When solving Equation (44) numerically, we also impose a constraint that

$$0 \leq \pi_{St} \leq 1. \tag{47}$$

C.3 Solution algorithm

Define the state space. Before solving the PDE Equation (44) using finite difference methods, we set up an equally spaced lattice in (x,t) defined by the grid points

$$\{(x_i,t_n) \mid i=0,1,...,I;\ n=0,1,...,N\}$$

where $x_i = x_0 + i\triangle x$ and $t_n = n\triangle t$. We define initial values $x_0 = 0.1$, $x_I = 60$, and $N = 24 \times \hat{T}$ with $\hat{T} = 70$.

Terminal condition at $\hat{T}$. Given the condition at depletion time $\hat{T}$ in Equation (40), we may transform it into the condition for $F(x_i,t_N)$ at each value of x_i at time t_N ($t_N = \hat{T}$), which is

$$F_{i,N} = \frac{1}{1-\gamma}e^{-\beta(\gamma-1)\hat{T}}g\left(\hat{T}\right)^{\gamma}x_i^{1-\gamma} \tag{48}$$

where the function $g(\hat{T})$ follows Equation (41).

Terminal value of $\hat{c}_{i,N}$ is computed following (45). The terminal investment share is computed using (46)

$$\pi_S(x_i,t_N) = -\frac{F_x(x_i,t_N)}{xF_{xx}(x_i,t_N)}\frac{1}{\sigma_S}\left(\frac{\psi}{\sigma_S}-\gamma\sigma_y\rho_{yS}\right)+\frac{\sigma_y\rho_{yS}}{\sigma_S}.$$

While computing for control variables at time t_N, we use central difference method to approximate first and second derivatives of function F

$$\begin{aligned} F_x(x_i, t_N) &= \frac{F_{i+1,N} - F_{i-1,N}}{2\triangle x} \\ F_{xx}(x_i, t_N) &= \frac{F_{i+1,N} - 2F_{i,N} + F_{i-1,N}}{(\triangle x)^2}. \end{aligned}$$

Meanwhile, we employ forward difference and backward difference methods to approximate F_x and F_{xx} at the lowest and highest value of space variable x, $i = 1$ and I, respectively, such as

$$F_x(x_0, t_N) = \frac{F_{1,N} - F_{0,N}}{\triangle x} \tag{49}$$

$$F_x(x_I, t_N) = \frac{F_{I,N} - F_{I-1,N}}{\triangle x} \tag{50}$$

$$F_{xx}(x_0, t_N) = \frac{F_{1,N} - 2F_{0,N} + 0}{(\triangle x)^2} \tag{51}$$

$$F_{xx}(x_I, t_N) = \frac{0 - 2F_{I,N} + F_{I-1,N}}{(\triangle x)^2} \tag{52}$$

at time t_N.

Iterations. There are two steps to solve the Equation (44). First, we use the implicit finite different method to solves (44) and iterate $F(x_i, t_n)$ backward from time $\hat{T}$ to 0 with given terminal condition (48) using a guess on the optimal controls $\pi_s(x_i, t_n)$ and $\hat{c}(x_i, t_n)$. Second, we employ the solution of $F(x_i, t_n)$ at time t_n from the first step and compute a new guess of the optimal controls which can again generate another solution of $F(x_i, t_n)$. The procedure carries on until the value of $F(x_i, t_n)$ gets convergence.

Backward iteration over time with a guess of control variables. With given terminal solution (48), we are going to iterate backward starting from $\hat{T}$. At any time t_n, we know the approximated value function $F_{i,n+1}$, and optimal control variables $\hat{c}_{i,n+1}$ and $\pi_{i,n+1}$ at time t_{n+1}. We start with a guess that the value of optimal control at time t_n is $\hat{c}_{i,n} = \hat{c}_{i,n+1}$ and $\pi_{i,n} = \pi_{i,n+1}$.

Defining that

$$\begin{aligned} a_{i,n} &= (1 - \hat{c}_{i,n}) e^{-\beta t} + x_{i,n} \left[r - \alpha + \sigma_y^2 \gamma - \beta + + (\psi - \gamma \rho_{yS} \sigma_y \sigma_S) \pi_{S;i,n} \right] \\ B_{i,n} &= \frac{1}{2} \left[\sigma_S^2 \pi_{S;i,n}^2 + \sigma_y^2 \right] x_{i,n}^2, \end{aligned}$$

we may write (44) as

$$\hat{\delta} F_{i,n} = \frac{\hat{c}_{i,n}^{1-\gamma}}{1-\gamma} + F_t(x_i, t_n) + a_{i,n} F_x(x_i, t_n) + B_{i,n} F_{xx}(x_i, t_n). \tag{53}$$

We rewrite Equation (53) using up-wind approximations of derivatives, and get

$$\hat{\delta}F_{i,n} = \frac{\hat{c}_{i,n}^{1-\gamma}}{1-\gamma} + D_t^+ F_{i,n} + D_x^+ F_{i,n} a^+ - D_x^- F_{i,n} a^- + D_x^2 F_{i,n} B \tag{54}$$

where $z^+ = \max\{z, 0\}$ and $z^- = \min\{z, 0\}$. $D_t^+ F$, $D_x^+ F$, $D_x^- F$ and $D_x^2 F$ represents

$$D_t^+ F_{i,n} = \frac{F_{i,n+1} - F_{i.n}}{\triangle t} \tag{55}$$

$$D_x^2 F_{i,n} = \frac{F_{i+1,n} - 2F_{i,n} + F_{i-1,n}}{(\triangle x)^2} \tag{56}$$

$$D_x^+ F_{i,n} = \frac{F_{i+1,n} - F_{i,n}}{\triangle x} \tag{57}$$

$$D_x^- F_{i,n} = \frac{F_{i,n} - F_{i-1,n}}{\triangle x}. \tag{58}$$

Inserting up-wind approximations of derivatives (55)–(58) into (54), we obtain

$$\begin{aligned} \hat{\delta}F_{i,n} = {} & \frac{\hat{c}_{i,n}^{1-\gamma}}{1-\gamma} + \frac{F_{i,n+1} - F_{i,n}}{\triangle t} \\ & + \frac{F_{i+1,n} - F_{i,n}}{\triangle x} a^+ - \frac{F_{i,n} - F_{i-1,n}}{\triangle x} a^- \\ & + \frac{F_{i+1,n} - 2F_{i,n} + F_{i-1,n}}{(\triangle x)^2} B. \end{aligned} \tag{59}$$

Moving variables with $n+1$ subscripts and $\hat{c}_n$ to the LHS and those with n subscripts to the RHS in Equation (59). I also factorize terms on the RHS. We got

$$\begin{aligned} \underbrace{-\frac{\hat{c}_{i,n}^{1-\gamma}}{1-\gamma} - \frac{F_{i,n+1}}{\triangle t}}_{d_{i,n}} = {} & F_{i,n} \underbrace{\left\{ \begin{array}{c} -\hat{\delta} - \frac{1}{\triangle t} - \frac{1}{\triangle r}\left(a_{i,n}^+ + a_{i,n}^-\right) - \frac{1}{\triangle x}\left(a_{i,n}^+ + a_{i,n}^-\right) \\ -\frac{2}{(\triangle x)^2} B_{i,n} \end{array} \right\}}_{E_{i,n}} \\ & + F_{i+1,n} \underbrace{\left\{ \frac{1}{\triangle x} a_{i,n}^+ + \frac{1}{(\triangle x)^2} B_{i,n} \right\}}_{H_{i,n}} \\ & + F_{i-1,n} \underbrace{\left\{ \frac{1}{\triangle x} a_{i,n}^- + \frac{1}{(\triangle x)^2} B_{i,n} \right\}}_{I_{i,n}} \end{aligned}$$

or

$$d_{i,n} = F_{i-1,n} I_{i,n} + F_{i,n} E_{i,n} + F_{i+1,n} H_{i,n}. \tag{60}$$

Writing Equation (60) into vectors and matrixes. At one specific time t_n, we have

$$\mathbf{d_n} = \mathbf{X_n F_n}$$

where $\mathbf{X_n}$ is a matrix with dimension $(I+1)\times(I+1)$. $\mathbf{F_n}$ is a vector with dimension of $(I+1)\times 1$. In the matrixes below, parameters are drop for the time subscript n, for example $E_i = E_{i,n}$.

$$\mathbf{X_n} = \begin{bmatrix} E_0 & H_0 & 0 & \cdots & 0 \\ I_1 & E_1 & H_1 & 0 & \vdots \\ 0 & I_2 & E_2 & H_2 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & I_I & E_I \end{bmatrix}$$

and

$$\mathbf{F_n} = \begin{bmatrix} F_0 \\ F_1 \\ \vdots \\ F_I \end{bmatrix}_{(I+1)\times 1}$$

At time t_n, $\mathbf{d_n}$ and $\mathbf{X_n}$ are known. Thus we can solve for unknown $\mathbf{F_n}$ as

$$\mathbf{F_n} = inv\left(\mathbf{X_n}\right)\mathbf{d_n}.$$

Iteration for value function at time t_n. Given the solution of $\mathbf{F_n}$ from the first step, we can compute approximated derivatives using central difference method. For $i = 2, ..., I-1$, we compute the derivative of F as

$$\begin{aligned} D_x F_{i,n} &= \frac{F_{i+1,n} - F_{i-1,n}}{2\triangle x} \\ D_x^2 F_{i,n} &= \frac{F_{i+1,n} - 2F_{i,n} + F_{i-1,n}}{(\triangle x)^2}. \end{aligned}$$

We employ forward difference and backward difference methods to approximate F_x and F_{xx} at the boundary of space variable x_i for $i = 1$ and I. The method is the same as shown in Equations (49)–(52).

After that we can compute a new guess on the optimal controls $\hat{c}_{i,j,n}$ and $\pi_{i,j,n}$ according to (45) and (46).

The constraints (47) are employed when deriving control variables. The investment share on stocks is binding if $\pi_S > 1$ or $\pi_S < 0$.

Given the new guess on the optimal controls, we solve again the system of equations and obtain a new guess on the value function at time t_n. We continue the iterations until the largest relative change in the value function over all i and j is below some small threshold (use tolerance at 0.1%).

Then we move to t_{n-1} until time 0. The relative change is computed as

$$RelChange = \max\left\{\max\left[\frac{\left|F^k - F^{k-1}\right|}{\max\left(10^{-5}, \left|F^{k-1}\right|\right)}\right]\right\}$$

where k is the index of iteration, $k = 1, ..., maxIter$.

Using this method, we obtain the optimal solution of $F\left(x_i, t_n\right)$, $\pi_S\left(x_i, t_n\right)$ and $\hat{c}\left(x_i, t_n\right)$ for all x_i and t_n.

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Lin Ma was born in Beijing, China. She holds a BA Degree in Economics and International Trade from China Agricultural University, China (2008), and an MSc Degree in Environmental and Development Economics from the University of Oslo, Norway (2011).

The purpose of this thesis is to study the role of oil in the macroeconomy. The thesis consists of an introduction and four independent papers. The first paper studies factors drive the oil prices in the long run using a dominant firm-competitive fringe model. The result reveals that the global GDP is the main driving force of long-run oil prices. The evidence is provided for OPEC exercising market power.

The second paper concerns the effect of permanent income shocks on the oil price using a time series model with cointegrating features. The findings reveal that the price of oil can be predicted by using a stable long-run relationship and, furthermore, that the GDP shock has a permanent effect on the oil price and that the shocks to the oil price are almost transitory.

The third paper studies elements explain the formation of short-run oil prices in an extended storage model. The findings reveal that GDP shock generates moderate effects on the oil price in the short run, and the production shock plays an important role in the variance of the oil price.

The final paper solves a dynamic asset allocation problem for a commodity sovereign wealth fund (SWF) under incomplete markets. The model is calibrated for the Norwegian SWF that accumulates fluctuated oil income. The solution of the optimal investment strategy suggests that the Norwegian financial wealth should be invested in stocks at a high level initially and then decreased over time.

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