Experience and Theory:
A Defense of the Kantian A priori and Kepler's Philosophy of Science in Light of Modern Space-Time Physics

Erfaring og Teori:
Et forsvar av Kants a priori og Keplers vitenskapsfilosofi i lys av moderne rom-tidsfysikk

Philosophiae Doctor (PhD) Thesis

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Summary

This thesis is intended as a contribution to the debate over the relation between everyday thinking and contemporary science. My main aim is to contribute to a more general debate concerning epistemology and scientific reasoning. In order to maintain focus and enter the scientific details, I have limited my discussion here to issues relating to special relativity. Other case studies such as debates over quantum mechanics (see Andersen & Arenhart: 2016) and risk assessment of biotechnology (see Rocca & Andersen: 2017) have therefore been left out.

A common debate in contemporary philosophy of science is whether we should accept scientific theory at face value, or whether we should take direct and everyday experience as our main source to knowledge of the world. The existence of this debate implies that there exist a somewhat hard separation between these two types of knowledge, or at least, that there is a sizeable tension between them.

A primary example of the separation between everyday thinking and scientific theory is the debate over space and time. Since the introduction of special relativity in 1905, physics have adopted notions of space and time that are radically different from the ones we use in our everyday language and experience. What should we do in such a situation? Should we reject our everyday experience at the benefit of scientific data, or should we reject scientific data at the benefit of our everyday experience? Philosophers have argued both positions for the last century and the received view appears to be that we should adopt the scientific theory and think of our own experience as somewhat flawed. The downsides of this solution is that we have become strangers in our own world, and that only a society of experts know the truth. There is however, a third option available, which is to find alternative ways to evaluate the relevant data and thus construct other possible theories. In relation to special relativity, there is already an alternative theory in place. This theory is known as the Lorentz ether theory. At the end of this thesis, I argue that the Lorentzian theory is preferable to special relativity for a series of philosophical reasons. The main reason being that the Lorentzian theory allows a common ground between everyday thinking and scientific theory. We are therefore no longer strangers in our world.

I take Keplerian philosophy of science in combination with contemporary semirealism as my vantage point in part one and argue that a scientific theory par excellence is a theory, which can be coherently stated in a single argument where measuring results as well as measuring apparatus and everyday experience are included. Following Kepler, I argue that such a theory must build from a metaphysical basis.

In part two, I establish a Kantian metaphysical basis for everyday experience as well as theoretical and experimental physics. In so doing, I establish a priori elements of phenomenology, mechanics, dynamics and phoronomy. The existence of a priori elements to any modern physical theory is an unpopular notion in contemporary philosophy of science. Typically, the idea is that non-empirical elements of a particular theory must be conventional in some sense, or they ultimately turn out to be empirical after all. The main reasons for the rejection of a priori elements is that arguably, "non-

Euclidean geometries" and their application in special and general relativity prove that there are no a priori elements. What we used to think were a priori elements have turned out to be wrong. The last two parts of the present thesis are counterarguments to this common opinion.

In part three, I argue there are two distinct versions of "non-Euclidean geometries". The "non-Euclidean" geometries are geometrical models on what turn out to be Euclidean surfaces. They are therefore more expansions of, than alternatives to Euclidean geometry. These geometries are fully in line with the notion of an a priori given structure of space. The non-Euclidean "geometries" however, are attempts at constructing higher dimensional models of the structure of space, and are in direct conflict with the a priori three-dimensional structure of our everyday experience. I argue in part three that these "geometries" are actually arithmetical structures that can only be modelled geometrically as long as we accept a series of philosophically problematic tenets.

In part four, I present the Lorentzian ether theory, special relativity, and the problems they intend to solve. I argue that both theories are able to save the phenomena and thus cover the data, but that the Lorentzian theory is explanatorily superior. Furthermore, I argue that the main differences between the theories can be expressed in a series of postulates posited by Einstein. Among these postulates are the concepts of physically real "practically rigid bodies" and methodological demand that we conflate velocity addition and translation or transformation. In conclusion, I argue that special relativity implies a set of dualisms that are unresolved in both Einstein's original theory and the contemporary versions of it. I conclude that the root of the problem lies in Einstein's adherence to a Humean epistemology that selectively neglects dynamical aspects of physical entities.

If my arguments are granted, we can make a step toward finding an overall framework for everyday thinking and scientific theory. We can then also apply the scientific theory in our everyday thinking and expand our laymen's knowledge of who, what and where we are.

Sammendrag

Formålet med denne avhandlingen er å gi et bidrag til debatten rundt forholdet mellom hverdagslig erfaring og moderne vitenskap. I hovedsak er det mitt mål å bidra i en mer generell debatt rundt epistemologi og vitenskapelig tenking. For å kunne holde et visst fokus og ta for meg detaljene i de relevante teorier, har jeg avgrenset diskusjonen til forhold som direkte angår spesiell relativitetsteori. Jeg har dermed utelatt andre case-studier om debatter som angår kvante-mekanikken (se Andersen & Arenhart: 2016) og risikoanalyser av bioteknologi (se Rocca & Andersen: 2017).

I den moderne vitenskapsfilosofiske diskurs debatteres det hvorvidt vi burde akseptere vitenskapelig teori slik den fremstår, eller om vi heller burde anta vår direkte og hverdagslige erfaring som vår overordnede kunnskapskilde. Implisitt ligger det altså en forestilling om at disse formene for kunnskap er i konflikt, eller i det minste at de ikke med enkelhet lar seg sammenfatte i en helhet.

Skoleeksempelet på skillet mellom hverdagslige tankesett og vitenskapelig teori finner vi i debatten over hva rom og tid er for noe. Siden den spesielle relativitetsteori ble introdusert i 1905 har fysikken operert med begreper om rom og tid som er radikalt forskjellige fra de forestillinger vi opererer med i vår hverdagslige erfaring og omtaler i vårt vanlige språk. Burde vi forkaste vår hverdagserfaring til fordel for vitenskapelige data eller burde vi forkaste disse dataene til fordel for vår hverdagslige erfaring? I de siste hundre år har filosofer argumentert for begge disse løsningene og det som etter hvert har blitt standardsynet er at vi burde akseptere de vitenskapelige data og leve med at vår egen erfaring til en viss grad er feilaktig. Ulempen med en slik løsning er at vi da blir fremmede i vår egen verden og at kun et utvalg av eksperter kjenner den sanne virkelighet. Det finnes dog et tredje alternativ. Det går an å tenke seg andre måter å forstå dataene på og dermed konstruere alternative teorier. Når det gjelder den spesielle relativitetsteori finnes det allerede et etablert alternativ i Lorentz eterteori. Jeg argumenterer avslutningsvis med at denne teorien er å foretrekke fremfor den spesielle relativitetsteori av en rekke filosofiske grunner. Hovedgrunnen ligger i at eterteorien åpner for et felles grunnlag for å forstå hverdagserfaring og vitenskapelig teori. Vi er dermed ikke lenger fremmede i vår egen verden.

I del én tar jeg utgangspunkt i Keplers vitenskapsfilosofi og den moderne semirealismen. Her argumenterer jeg for at en vitenskapelig teori, par excellence, er en teori som kan beskrive sine måleinstrumenter, sine måleresultater og vår hverdagslige erfaring i ett sammenhengende argument. Jeg argumenter også med Kepler for at en slik teori må fundamenteres i en metafysikk.

I del to etablerer jeg en felles, Kantiansk, metafysisk basis for hverdagslig erfaring samt teoretisk og eksperimentell fysikk. I denne sammenheng etablerer jeg også a priori elementer innen fenomenologi, mekanikk, dynamikk og foronomi. Forestillingen om at det finnes a priori elementer i fysisk teori er for tiden en mindre populær forestilling. Standardforestillingen er at elementer som ikke kan avledes fra empirien enten må være konvensjoner i en eller annen form, eller at disse elementene allikevel lar seg avlede fra empiri. Hovedgrunnene til at man antar at det ikke finnes a priori elementer er at man tenker seg at de «ikke-Euklidske geometrier» og deres anvendelse i generell og spesiell relativitetsteori beviser at det ikke finnes slike a priori elementer.

Det viser seg snarere at det vi trodde var a priori elementer er feilaktigheter. De siste to delene av min avhandling består av motargumenter til denne, nå vanlige antakelse.

I del tre argumenterer jeg for at det finnes to former for «ikke-Euklidske geometrier». De «ikke-Euklidske» geometriene er geometriske modeller av det som viser seg å være Euklidske overflater. I så måte er disse geometriene å anse for utbedringer snarere enn alternativer til den Euklidske geometri og fullt ut forenlige med en a priori forestilling om rommets struktur. De ikke-Euklidske «geometriene» derimot er forsøk på konstruksjoner av høyere-dimensjonale modeller av rommet. De er dermed i direkte konflikt med den tredimensjonale struktur vi kjenner fra Euklid og vår hverdagslige erfaring. Jeg argumenterer i del tre for at disse «geometriene» i virkeligheten er aritmetiske strukturer som kun kan modelleres geometrisk dersom vi aksepterer en rekke filosofisk problematiske teser.

I del fire presenterer jeg Lorentz sin eter-teori, den spesielle relativitetsteori og de problemer det var meningen at disse teoriene skulle løse. Jeg argumenterer for at begge teoriene dekker de relevante data og at Lorentz sin eter-teori er å foretrekke fordi den forklarer bedre. I tillegg argumenterer jeg for at den beste måten å forstå forskjellen mellom teoriene på er å ta for oss en rekke postulater som Einstein fremsetter. Blant disse er forestillingen om at det finnes fysisk virkelige «practically rigid objects», eller «fullstendig stive objekter». I tillegg postulerer Einstein at vi metodologisk burde behandle hastighetssammensetting som transformasjon eller translasjon. Jeg konkluderer med at disse og andre postulater leder til en rekke dualismer som hverken er løst i Einsteins opprinnelige teori eller de moderne versjoner av den. Til slutt konkluderer jeg med at hovedproblemet i Einsteins tilnærming er at han antar en Humeansk epistemologi som innebærer at man selektivt unnlater å ta høyde for fysiske objekters dynamiske egenskaper.

Dersom mitt argument aksepteres innebærer det et første steg mot dannelsen av et felles rammeverk for hverdagslig tanke og vitenskapelig teori. Da kan vi også benytte den vitenskapelige teori i vår hverdagslige tanke og utvide vår forståelse av hvem, hva og hvor vi er.

Introduction

Relativity theory has affected both specialist and laymen's perception of reality. It appears that although we perceive our world as set in a Euclidean space with a classical, uniform and one-directed time, the physical reality is at times radically different. There is therefore a gap between reality and our perception of it. In most cases where science is interesting, such a gap exists. Usually, gaps are mended by explanation and the new scientific reality is assimilated into our general perception. For instance, Darwin's theory of evolution presented a shockingly novel history of human origins but the vast majority of criticism faded due to the Darwinian explanations. The criticism against relativity has persisted from the theory's introduction in 1905, and still prevails today. Nevertheless, the theory stands as the pinnacle of modern scientific thought. Most non-specialists argue variations of "I don't get it, but the physicists know how this works" with the added effect that our reality is very strange. Critics have typically been defined as crackpot theorists and scientific illiterates. This was the story more or less from the time of Einstein's death until the start of the 21st Century. Otherwise respected physicists like Janossy and Ives, were largely ignored.

In Einstein's own time, the situation was somewhat different, as Einstein often attempted to explain his theory in both professional journals and the grey media. Notable physicists and mathematicians such as Lorentz and Poincaré maintained throughout that there were reasons to abstain from accepting the relativistic framework. Furthermore, Einstein appeared willing to concede weaknesses of his theoretical framework and sometimes, as in his Nobel lecture, emphasized that the theory rests on an unsatisfactory conceptual foundation. As Einstein died, much of the conceptual debates around relativity theory died with him. More recently, however, a wave of conceptual debates has reappeared. In the philosophical community, the most notable critics have been Brown and Pooley, and Brown's 2005 book on the subject has at least helped to re-legitimize the existence of a debate. Within the physics community, Abreu and Guerra stand out as the more audible objectors. A novelty of the contemporary debate is that the opposition is no longer directed toward the theories of relativity as such, but more toward the idea that the issue is resolved. So for instance, Szabo (2010) along with Abreu & Guerra (see for instance Abreu and Guerra 2005, 2008, 2015) argue in a more conciliatory way that relativity theory and ether theories are mathematically as well as observationally equivalent (provided the discussion is restricted to translatory motion) and thus that the difference between them is mainly philosophical. Furthermore, contemporary physicists such as Levy, along with Mansouri and Sexl are furthering their own versions of ether-theoretical alternatives. It is noteworthy that, although mathematically and empirically successful, these frameworks are all but absent in textbooks in philosophy of physics or physics proper.

The received view on space and time in contemporary philosophy of science remains that not only is the relativistic framework superior to the alternatives, but also that the alternatives need not be taken into account. Still, as all of Einstein's writing is publicly available since 2012 (see http://www.alberteinstein.info/), as well as publications on Einstein's own "heretics" (see for instance Kostro 2000 on Einstein's reintroduction of an ether), arguments against the relativistic framework are likely to persist.

What follows is intended as a contribution to the critical evaluation of special relativity. Special relativity is particularly interesting as it sets the frame for any contemporary debate over space and time. For instance, if you maintain that space and time are intuitions in the Kantian sense and that these intuitions allow you to produce synthetic a priori arguments, there is an expectation that you must at some point admit that such a view is restricted. Typically, the restriction comes in the form of a distinction between "manifest" space and time on the one side and "physical" space and time on the other. The Kantian arguments should then apply to the "manifest" images only. The idea that there exist two images of the world in this sense was popularized in Sellars (1962). However, what restricts me from going the other way? For instance, I could argue that as far as the "physical" image contradicts my direct experience or my metaphysical framework, it only has validity within the "physical" models. The mere suggestion appears to entail an anti-science position. However, is it unreasonable to expect that scientific theories are incorporable into our everyday experience in the way that Darwinian evolution, the principle of relativity, the Galilean law of fall, Mendelian genetics, gravity, and other scientific concepts have been? As representatives of those who find such an expectation unreasonable, Ladyman and Ross (2007: 2), claim: "…there is no reason to imagine that our habitual intuitions and referential responses are well designed for science or for metaphysics" (the "Gorilla Brain Argument").[1] In other words, there are elements of science whose level of sophistication are so high that they are beyond argumentative justification. What such a position displays is the idea that we must choose between reality and our understanding, and thus we get the "manifest" vs. "physical" images. As the argument goes, we should prefer the "physical" image since there is no reason to expect our "manifest" image to ring true. This separation has a long history and a version of it was presented by Osiander (1543), in his introduction to Copernicus' *Revolutions*. In Osiander's version, it is nature itself that is beyond our understanding:

> Maybe the philosopher demands probability instead; but neither of them will grasp anything certain or hand it on, unless it has been divinely revealed to him… And as far as hypotheses go, let no one expect anything in the way of certainty from astronomy, since astronomy can offer us nothing of certain, lest, if anyone take as true that which has been constructed for another use, he go away from this discipline a bigger fool than when he came to it. (Osiander 1543: 4)

We thus have an apparent distinction between the contemporary and the classical version of the Gorilla brain arguments. In the classical debate, Osiander proposes his version of the thought that God works in mysterious ways as a skeptical approach to science. In the contemporary debate, Ladyman and Ross apply what we may call a "Physics works in mysterious ways" approach. The common ground is the insistence that human beings are unable to understand the world in which we find ourselves. In effect, certain elements of certain theories are beyond rational scrutiny.

The most famous opponents of this view in the classical debate were Galileo and Kepler, who both argued that it is the very nature of science to provide explanation. On the question of human abilities and our possible understanding of nature, I side with Kepler and Galileo. This means that unless an element of a theory is explained, or otherwise made understandable, that element is problematic. Whether one assumes all elements of a scientific theory should be rationally defendable or not, in those cases where there are actual explanations, one cannot argue that no

[1] In Andersen and Arenhart (2016), we call this the "Gorilla brain argument".

explanation is possible. In part IV of this thesis, I focus on elements of special relativity that are problematic in the sense that special relativity offers no explanations while other theories do.

One way of distinguishing the contemporary and the classical version of the Gorilla brain argument is to emphasize how the classical version motivates scientific skepticism while the contemporary version motivates scientific realism. There certainly is such a distinction. Nevertheless, both versions appears unreasonably defeatist. As philosophers and scientists, the default position should be that we aim to understand the world of which we are part.

Such understanding can of course be sought after in multiple ways, and the received view appears to be that science is primarily data driven. New data appears, the scientist systematizes and organizes them and produces hypotheses to be tested through the generation of further data. The latter part is then typically illustrated through a Popperian framework. Which hypotheses are generated from the data is largely up to the individual scientist who, by sagacity or serendipity, increases our scientific knowledge. In other words, the received view includes the classical distinction between context of discovery and context of justification, typically associated with Popper and Reichenbach. We go about our lives and encounter something surprising. In order to understand the surprising something, we generate theories and test if they come out as true.

Is this the way knowledge production really works, and is it a fair description of the history of scientific development? In our everyday lives, we tend to appeal to already established models when we encounter surprises. So rather than generating freely imagined and novel hypotheses, we appear to search for solutions in an already established mode of thinking. Still, it is commonly assumed that scientific thinking differs, sometimes radically, from our everyday and practically motivated thought. Hence, we may ask whether the empiricist story can account for scientific discovery although it might arguably be false in our everyday reasoning.

One place to start is Hanson's treatment of Kepler's discovery in *Patterns of Discovery*. Hanson tells a straightforward story that has been incorporated into our thinking about scientific discovery. Kepler came into contact with Tycho Brahe's observational data, cast away his Aristotelian metaphysical convictions and deduced three laws of planetary motion. More recent, and detailed, analyzes of Kepler's work show this story to be false (See for instance Jardine 1979, Wilson 1968, Kleiner 1983, Myrstad 2004, and Tønnessen 2012).

Hanson's analysis is in no way an anomaly, but rather an expression of a generally empiricist reading of the history of science. It is still common to quote Newton on his "Hypotheses non fingo" although Newton's own writing clearly indicates his reliance on the hypotheses of Kepler and Galileo (typically seen as communicated through Jeremiah Horrocks) along with his own.

Multiple lines of interpretation are available in the commentary literature about Galileo's work. The received view being that Galileo opposed Aristotelian metaphysics and deductive methodology and replaced it with empirical evidence and experimental methods. As pointed out by Einstein in his foreword to the Stillman Drake version of Galileo's *Dialogue over the two chief world systems,* this story does not stand up to scrutiny. Einstein emphasizes Galileo's focus on comprehensibility as well as his willingness to draw conclusions from pre-empirical notions. I agree with this emphasis and read Galileo as a representative of a similar attitude to that of Kepler, where one seeks the true system by first establishing a metaphysical layer on which to build. Galileo (1615) explicitly identifies the elements of such a layer as the "primary suppositions" that are to be proven. This yields a stratified philosophy of science that splits into the empirical data, the primary suppositions and the physical hypotheses,[2] whose role it is to explain the empirical data in light of the primary suppositions. Einstein (1936) sets forth a similar stratification.

One indicator of Galileo's approach to scientific knowledge is given in his discussion of the apparent evidence for an immobile earth in Day Two of the *Dialogue*. Here, Galileo's protagonists Salviati (the theoretical man), Sagredo (the practical man), and Simplicio (the peripatetic philosopher) systematically move between everyday experience, theoretical suggestions and different theories of motion. The theories of motion are all a priori in this setting and Galileo's aim is to show how his own theories are superior to those proposed by the peripatetic school of natural philosophy. The dialogue has many interesting elements and clearly shows that Galileo's opposition is not directed toward Aristotle, with whom Salviati and Sagredo often agree. Rather, the aim is to show how Simplicio, due to his adherence to the peripatetic natural philosophy, is in an internal conflict. This conflict is between his own direct experience and the deductions he makes on the one side, and the peripatetic teachings and dogmatic defense of elements of Aristotelian metaphysics on the other. I currently find myself in Simplicio's position. On the one hand, my everyday experience along with my own thinking tells me that I exist within a three-dimensional

[2] Galileo (1615) refers to these as «secondary suppositions».

world with a flat space that evolves in a continuous and one-directed manner. On the other, I am told that this is not true and that this lack of truth is proven by relativity theory. My gut reaction to this situation has been to check whether these latter statements are disputable or not. I find that they are. Galileo's treatment of the Copernican and the Ptolemaic systems in the *Dialogue* have been indispensable throughout this process. For what then, if not simply an introduction of empirical evidence and experiment, is it that Galileo makes use of?

Throughout the *Dialogue* Salviati repeatedly refers to anamnesis or re-collection. In all of these instances the claim is, similar to those of Plato in the *Meno* dialogue, that Simplicio already knows the answers to the relevant physical and mathematical questions. By accepting his recollection as "natural" or rationally given, Simplicio is able to push his scientific understanding further and gradually dissolve his internal conflict. Furthermore, when faced with empirical evidence, Galileo refuses to take these at face value and demands that first one must establish a more general understanding in light of which empirical data can be explained. Faced with multiple instances of "proof" that the earth is immobile, the protagonists finally show, through multiple thought experiments, that the "proofs", and how they should be understood, relies on the more general framework of understanding. I will not go further in depth into Galileo's methodology here, as it suffices to say that in the case of Galileo, as in that of Kepler and Newton, it is by no means clear that his methodology fits the empiricist picture.[3] Rather, it appears Einstein's reading is closer to the truth and that Galileo himself did not treat metaphysical theory, rational speculation and empirical evidence as wholly separate issues, but as elements of a single search for the true astronomical system.

In the contemporary debate, similar distinctions are drawn between structure and content, or structures and entities. Chakravartty's semi-realism is a welcome option in this debate, and the position mirrors the Galilean approach by emphasizing how mathematical structures and everyday objects of experience are both included in our understanding of theoretical content. In effect, the gap between structural realism and entity realism is inflated and the two positions have more in common than typically argued. However, the distinction between entities and structures also have a historical component in that the positivist reading of science has typically focused on the robustness of mathematical elements of theories. This reading is a branch of the empiricist

[3] I will refer to Galileo's approach throughout this thesis.

understanding of the history of science that I oppose here. A strong positivist reading of science suggests that science, fundamentally, is about generating the correct logical structures that in the simplest manner reflect empirical evidence. It thus follows that theoretical content or the nature of entities becomes orthogonal to science as such. If entities or everyday objects have any function in science, it is purely as pragmatic elements that guide us toward a more robust structure. A milder version, that still maintains the distinction, is Hanson's abductive reading that I referred to earlier. My understanding of the scientific project along with its history is that science is primarily about generating understanding. Along with Einstein, I hold that scientific understanding is a development or a sophistication of everyday experience and that comprehension is essentially to draw conclusions from an already accepted system. It has been my intention in this section to point out that to the extent that such an understanding opposes the received view, the received view might be in need of modification. One motivating factor for modifying the received empiricist understanding of science is that the history of scientific development is not a history of empiricism.

Concerning history, Stump (2015) presents a compelling argument against the received view in the history of mathematics. More specifically, Stump argues that the standard story of the development of non-Euclidean geometries is not true. The standard story is that non-Euclidean geometries developed through a series of attempts to solve a crisis in geometry. This story has the pedagogical function of emphasizing the relation between logic and mathematics as well as focusing on the idea that systems develop by replacing metaphysical ideas with strict deduction. However, as Stump points out, one of the major changes in mathematical development is the reconceptualization of geometry as such. It is well known that the parallel postulate can be modified in a logically consistent way, and this knowledge is not new. For instance, Geminus developed such "geometries" two thousand years ago. Furthermore, one needs only to draw a triangle on a ball to realize that a triangle can be produced whose internal angles are not 180°. A radical change in the 19th Century however, was the new belief that these "geometric" structures might represent the structure of space itself. There are still good reasons to object to this belief, but it is one that is essential to the contemporary received view of physical theories.

In combination, the received views of science and geometry construct a compelling narrative in relation to the Kantian doctrine of synthetic a priori judgment. Kant argued that space is necessarily flat and three-dimensional and thus modelled by the Euclidean structure. Along with the idea that space is a form of pure intuition, Kant argued that there are aspects of our reality that can be understood without reference to specific empirical evidence. For instance, that any object of experience is three-dimensional, has extension and is located at some distinct place. Kant also argued that the shape of any object of experience must be a shape within the Euclidean model. The problem for any contemporary Kantian is that special relativity is typically represented in a Minkowski structure that differs from Euclidean space. Furthermore, the general theory of relativity is typically seen to show that space has a Riemann structure. So either there is something wrong in Kant, or there is something wrong in relativity theory.

According to the received view, Kant would not have known about the logically consistent non-Euclidean geometries and one can excuse him on this issue. Furthermore, relativity theory is just another example of how science develops by rejecting metaphysical dogmas and following the data. It seems therefore that there is no hope for a contemporary Kantian. We have also seen that Kantians in the 20th Century largely responded to this narrative by constructing “Neo-Kantian” versions of Kant’s original philosophy. Perhaps the most popular of which is Friedman’s dynamic a priori position.

Stump (2015) makes an illuminating comment concerning Friedman’s position along with that of Reichenbach. These positions are not really positions that apply a priori arguments in any real sense, and they should rather be renamed positions of “constitutive elements”. This grouping of theories has a clear upside. We can separate between the empiricist theories that assume that science progresses and should progress through the discoveries of new data, and the constitutive theories that assume science progresses and must progress by inserting constitutive elements. The latter can be separated into those theorists that believe that constitutive elements are a priori given, such as Kant, those who believe that constitutive elements are free constructs of the imagination, to use Einstein’s phrasing, and those, like Stump and Pap who believe that constitutive elements are ultimately empirical.

My position in the debate

I argue for a classical scientific realism that traces back to Kepler and Galileo. This position has strong similarities to the positions of Chakravartty and the constitutive theorists, but also contains elements that contemporary views either ignore or reject. I endorse Chakravartty's emphasis on the separation between what he calls detection properties and auxiliary properties, but argue that such a distinction, in order to be effective, needs a more substantial metaphysical embedding. I further endorse Chakravartty's insistence that the relation between structures and entities is much more tightly knit than appears in the debate between structural realist and entity realists. In relation to Stump, I endorse the general genus of theories and share his rejection of the positivist history and philosophy of science. In distinction to Stump however, I defend the Kantian option with synthetic a priori elements that have universal validity. In light of modern science, such a defense demands a rejection of what I take to be an erroneous assumption in modern philosophy of science shared by Stump, Reichenbach, Chakravartty and Friedman, i.e., that relativistic space-time successfully replaces the Euclidean structure and thus that Kant's argument for an a priori spatial intuition is in error. I will defend the following central claims:

1) Scientific realism is best understood in terms of classical realism as presented by Kepler and Galileo.

2) Science is a development or sophistication of everyday experience. Everyday experience must be explained in the same framework as our physical experiments.

3) All action is interaction

4) All of science has an underlying metaphysics. Physics contains phenomenology, mechanics, dynamics, phoronomy and geometry. Each with its corresponding set of metaphysical assumptions.

5) There are no competitors to the Euclidean model of space.

6) There is already a scientific theory that explains all the relevant phenomena of special relativity within a Kantian framework and with Euclidean geometry. This is the ether theory of Maxwell/Lorentz.

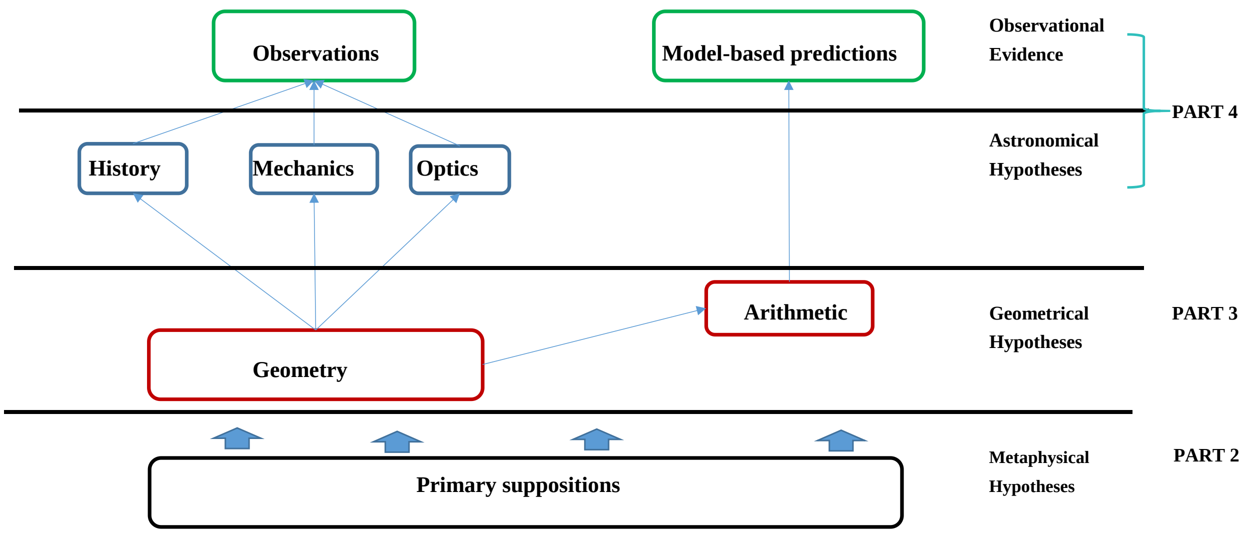
7) Special relativity rests on a series of implausible a priori postulates. In combination, these postulates generate a series of dualisms that are currently unresolved.

8) The Lorentz/Maxwell theory explains the same phenomena as special relativity in a more coherent way, and it explains phenomena that special relativity cannot explain.

9) By applying a Kantian metaphysics and a Lorentzian physics, we can explain the relevant physical phenomena whilst maintaining the world view of our everyday experience.

The thesis is structured into four main parts:

Part 1 establishes the Keplerian philosophy of science and argues that this framework is still a valid option for scientific realists. I focus on establishing the Keplerian theory and its relation to contemporary constitutive theories.[4] During part 1, I construct an illustration of the Keplerian philosophy that will also function as a guide through the rest of the thesis.



In part two, which deals in **metaphysics**, I argue that there are a priori constitutive elements common to everyday experience and scientific theories and that these constitute a metaphysical layer. Through this part, I follow Kant (1786) in distinguishing between four aspects of physics and argue that they all contain a priori constitutive elements. The four aspects are phenomenology, mechanics, dynamics and phoronomy or the theory of motion.

[4] For a more substantial criticism of contemporary naturalism and its relation to contemporary physics, see Andersen and Arenhart (2016).

Part three focuses on the **geometrical** aspects of physical theories and I argue with Frege that there are good reasons to maintain that 1) geometry is based in intuition,[5] and 2) Euclidean geometry has no competitors as models of space.

In **Part four** I discuss the relationship between special relativity and Lorentzian ether theories, and how these **physical theories** account for the **observational data.** I argue that both alternatives account for the observational data but that there are philosophical benefits to the Lorentzian option. I further argue that the thesis shows how the classical Kantian theory is still viable and should be adopted in combination with a Lorentzian physics.

[5] Anschauung

Part I

Science, constitution and metaphysics

Traditionally, there are two main schools of thought in epistemology of science. The empiricist school, which argues that all scientific knowledge is ultimately derived from sense perception, and the rationalists who argue that some scientific knowledge is a priori or independent of sense perception. Since Kant, rationalists typically argue that although science progresses through the accumulation of empirical evidence, such evidence only has meaning or becomes empirical in light of some more fundamental beliefs that are not empirically derived. For instance, although I have never seen the entire surface of an orange in any single moment, since that would require me to have multiple visual perspectives at once, I still have empirical evidence that oranges are spherical. I can only argue that I have this evidence as long as I accept underlying premises: as for instance, that the orange retains its identity when I turn it around, and that the part of the orange, which is outside my visual perspective at any time, remains the same shape as it was when I was looking at it. The underlying premises are arguably the conditions of possibility for any science whatsoever. So the modern rationalist claim is typically that there are some elements in science that are not empirical, and furthermore that these elements are especially important. Stump (2015) gives an overview of such moderate rationalist positions in the 20th century and helps clear up a terminological confusion. Kant's original position was that the most basic elements of scientific theory, as well as in general epistemology, were a priori and apodictic. In other words, the basic structure of experience is rigidly set before any empirical investigation. The modern version of this argument is that although there are conditions of possibility for the empirical investigation of reality, these conditions are not a priori given. Stump (2015) defines the latter theories as constitutive theories without a priori elements. Stump's definition is helpful in clarifying that although more recent writers such as Reichenbach, Pap and Friedman talk of functional, conventional or dynamical a priori elements, these elements are no longer thought of as a priori in the Kantian sense. To avoid confusion therefore, we shall adopt Stumps terminology and call such theories "constitutive theories".

In the following, I shall discuss central elements of constitutive theories as Stump (2015) presents them. I shall further point out that all authors that Stump discusses are philosophers that base their rejection of the Kantian a priori on their realistic reading of "non-Euclidean geometries" and relativity theory. I shall expand on Stump's picture and include the philosophies proposed by Einstein, Kepler and Galileo. I will defend the position of Galileo and Kepler, which I will simply call "classical realism". I will also discuss an element of the classical realist position that mirrors Chakravartty's contemporary distinction between auxiliary properties and detection properties. Finally, I will defend classical realism and argue that there is a need for metaphysical commitment in order to flesh out what are auxiliary and what are detection properties in a given experimental setting. I claim that the best metaphysical position to hold on these issues is my version of the classical Kantian position with synthetic a priori arguments and a Euclidean model of space. A claim for which I will argue in parts II, III and IV.

I 1. Philosophers on constitutive elements in science

Stump (2015) picks out six central theorists within the constitutive tradition: Poincaré, Reichenbach, Cassirer, Pap, Lewis and Friedman. These all believe that there are constitutive elements in science and that these elements are not a priori in the Kantian sense. However, they disagree on the epistemic status and origin of the constitutive elements. For instance, Cassirer argues that there are two types of constitutive elements. Some that are dynamical or changing and others that remain fixed. The fixed element being the assumption of the "unity of nature" (see Stump 2015: 92). The main function of constitutive elements however, is their normative function as guidelines or norms for the formation of, and choice among, theories. This is the *regulative function* of constitutive elements, and one all within the constitutive tradition endorse. Regulatory elements change during scientific revolutions and they are therefore dynamical rather than fixed. However, during what Kuhn called normal science, these regulatory elements dictate what is and what is not acceptable theory. Within normal science, they function - for all intents and purposes - as analytical statements.[6]

[6] This version of an "analytical statement" is a reduced version, equating analytical with unalterable or undebatable. As such, it differs from Kant's "concept containment" distinction between analytical and synthetic statements. Typically, the latter is taken as obsolete after Quine's "Two Dogmas of Empiricism". However, as is

Another central aspect of the constitutive elements that appears generally accepted in the constitutive camp is emphasized by Pap, who calls them "functional a priori". These are generalized sets of empirical facts functioning as constitutive elements in all further investigation. In other words, some of our empirically derived facts show a robust regularity. This regularity we then lift to the status of "law" or "principle" for any further investigation. It would count against any new theory if it contradicts directly or indirectly these principles or laws. Poincaré talks of these as "hardened" empirical facts. Such facts become the core of a theory. An important aspect of Pap's treatment of such elements is that he sees their function as dynamical in the sense that one set of facts can have multiple functions and statuses simultaneously. Stump (2015: 106) endorses Pap's view on functional elements in science. We shall later see that Einstein (1919) discusses a similar issue and adds a twist to it by claiming that such procedures are effective but they do not provide us with an understanding of the phenomena. Friedman's mature theory resembles that of Cassirer on the issue that there are two types of constitutive elements. Friedman's fundamental level consists of what we may call meta-constitutive elements provided by philosophy.

There are clear Kantian elements in the thinking of all of the constitutive theorists. Still, they all reject the Kantian synthetic a priori and they do so on the basis of the same two arguments:

> *The development of consistent non-Euclidean geometries proves that Euclidean geometry is not apodictically true. I.e., since there can clearly be other geometries (they do exist), Kant is wrong in arguing for the uniqueness of the Euclidean model.*

> *Relativity theory shows that space is not Euclidean (it is Minkowskian in the special theory and Riemannian in the General theory), which proves that the basis for Kant's synthetic a priori – his theory of space and time - is wrong. If we had synthetic a priori arguments, this would not be a possibility. Nonetheless, it is a reality of contemporary science.*

shown in Anderson (2004), there are good reasons to maintain Kant's distinction as untouched by Quine's criticism.

Stump claims that the main loss, descending from Kant, is the objectivity that fixed a priori elements would provide. On this issue, I disagree with Stump and I would rather argue that the central function of Kant's synthetic a priori elements is to provide a coherence between our empirical data, our direct experience of the world, and our abstract theories about it. The problem is not so much objectivity, as it is understanding. We will come back to this issue later.

I 2. Scientists on constitutive elements in science

Stump (2015) gives a good overview of the philosophical positions on constitutive elements in science. I wish to expand on this overview by presenting some positions held by the scientists whose theories are most typically discussed. Furthermore, although I will argue against the above arguments, I share with Stump the general idea that science contains constitutive elements. The following is an attempt at strengthening this general position whilst offering a further and more radical possibility.

I 2.1. Einstein's free creations of the human mind

It is widely recognized that Hume was Einstein's main philosophical influence, but there are of course different ways to be a Humean. Einstein started out with a philosophy of science inspired by his teacher, Mach, which only allowed directly and empirically testable elements in a physical theory. There are, however, four central elements of special relativity, which are not empirically justifiable: the measuring apparatus imagined as consisting of "practically rigid bodies", the new rules for velocity addition following from Einstein's new measuring norm, the postulated one-way speed of light, and the strict distinction between inertial and non-inertial motion.[7] If these central elements of special relativity cannot be empirically tested, there is no way in which one ca defend the theory by a Machian epistemology and such an epistemology by the theory. Einstein's solution is to develop his own version of a constitutive theory. A clear statement of this theory is found in his 1936 paper *Physics and Reality,* which I shall discuss throughout this thesis. For now, we shall settle for a very general description.

[7] I will not dwell on these issues here since they are treated in detail and context in part IV. However, it is noteworthy that Einstein (1905) does not include a strict separation between inertial and non-inertial motion. This distinction is only drawn in relation to the well-known clock paradoxes. I discuss these paradoxes in part IV.

Einstein's mature and constitutive theory of science is stratified into multiple layers with empirical data being one. Below this, there are the concepts that give meaning to the empirical data. Importantly, these concepts are not themselves empirically justified in any other sense than that by applying them we can give meaning to the empirical data. There are infinitely many layers as we try to simplify our conceptual apparatus by constructing wider and fewer concepts, and there is, importantly, no bottom layer to this process. Einstein explicitly rejects the Kantian option in both the 1936 paper and elsewhere. His main idea is that what we have called constitutive elements are free creations of the human mind. In other words, there is no rational justification for the constitutive elements in and of themselves. They are justified through their function, which is to give meaning to as much empirical content as possible by utilizing the minimum of constitutive elements. So rather than maintaining his Machian and radical empiricism, Einstein ends up in a position similar to that of Reichenbach and Friedman. However, there is an important point of contention here in relation to the "functional a priori" approach of Pap that is defended in Stump (2015).

In Einstein's set-up, lower level concepts explain the empirical layer. This is a central aspect of Einstein's stratified constitutive theory. The "functional a priori" approach operates only at a single level, i.e. the empirical level. In Einstein's terms therefore, a "functional a priori" principle does not explain a set of empirical phenomena. In his foreword to the Stillman-Drake version of Galileo's *Dialogue,* Einstein gives an informal description of what he means:

> *But to comprehend is to draw conclusions from an already accepted logical system* (Einstein 1953: *xxxix*)

The idea then, is that we can establish constitutive concepts that are free constructs of the human mind and that these make up an underlying set of axioms or a logical system. The role of science is to apply these constitutive concepts to empirical facts and thus explain those facts. So in Einstein's constitutive theory the empirical facts cannot be explained through further empirical facts at the same level. They must be explained through lower level concepts. This, in my understanding, is the central issue at hand in Einstein's much debated 1919 distinction between what he calls principle theories and constructive theories.[8] In summation, we can say that Einstein's mature theory of science is a constitutive theory in line with the theories of Reichenbach and Friedman, but in contrast to Pap and Stump on the issues of explanation. When we discuss Einstein's position in more detail in part IV, we shall see how Einstein's demand for explanation influences his description of relativity theory.

I 2.2. Galileo's primary suppositions

As Einstein, Galileo has been used in support of multiple philosophical traditions. However, a common theme among modern readings is that Galileo's thinking fits in the constitutive tradition. The main reason being that Galileo cannot reach his new conclusions by applying the Aristotelian conceptual apparatus to his celebrated experiments and end up where he ended up. There needs to be a radical shift in understanding at a basic level in order for Galileo to construct the law of fall, the idea of inertial motion and the principle of relativity. There is, however, a debate concerning what ultimately motivated this conceptual shift.

I follow in the tradition of Koyré in reading Galileo's conceptual shift as motivated primarily in everyday experience and metaphysical arguments.[9] The nature of these "metaphysical" arguments is however debatable. Galileo (1615) states that all of astronomy has a threefold structure. At base, there are the primary suppositions, which astronomers adopt as pure philosophers. These are what we have called the constitutive elements. At the other end, there are the empirical facts that often appear to contradict the constitutive elements. In-between, there is astronomical theory whose role it is to remove such apparent contradictions. We thus recognize a continuation from Galileo to Einstein in that they both apply constitutive theories of multiple layers. However, Galileo differs from Einstein on the nature and origins of the constitutive elements. Where Einstein mainly discussed the functionality of constitutive elements as sense-makers, Galilei discussed them in their relation to their truth-value, a view he also attributed to Copernicus. Galileo's threefold set-

[8] I will discuss this distinction in detail in part IV.

[9] Van Dyck (2005) objects to this reading and dismisses Koyré's reading as naive. Van Dyck's position is that the conceptual shift is motivated in experimentation and he uses the example of Galileo's use of pendulums in relation to the law of fall. More recently, Camilleri (2015) rejects the idea that experiments could have motivated the conceptual shift in this way. Camilleri emphasizes the use of thought experiments and analogies to everyday experiences as the source of concept development in Galileo.

up lends itself easily to a syllogistic way of thinking, most clearly illustrated in his treatment of the Peripatetic counter-arguments to the Copernican theory.

Roughly put, the Peripatetics argued that the earth could not possibly be moving due to a set of symmetry arguments. For instance, if the earth rotates eastward and I shoot an arrow eastward, the earth will “catch up” with the arrow and the shot will be short. If, on the other hand, the earth moves eastward and I shoot the arrow with the same strength westward, the earth will “move away” from the arrow and the shot will be long. Empirical evidence shows that there is no difference. The shots are equally long in all directions. Therefore, the earth is not moving.

Galileo’s objection here is to the middle term of the argument, stating the earth would “catch up” or “move away” from the arrow, depending on the direction in which the arrow is shot. The task for Galileo is clear. He must maintain that the earth rotates, which he has established as a primary supposition for the argument and thus as true. He must also maintain there is no connection between the observed distance travelled by the arrow and the direction in which it is shot. However, he can argue that the theory of motion is wrong, which is what he does.

Famously, Galileo argued that when shot, the arrow already has in it the impetus[10] of the motion of the earth. This means that the arrow, when shot with the rotation, is travelling faster and when fired against the rotation it is travelling slower. The rotation of the earth thus plays a double role. It catches up with the eastbound arrow, but due to the same rotation, that arrow is travelling faster. Similarly, the earth moves away from the westbound arrow, but due to the same rotation, that arrow is travelling slower. Since the common causal influence is the rotation of the earth, the speeds cancel out. We do not notice this process because we, the observers, are also rotating with the earth and the arrows.

We thus see how Galileo can set up the argument as a simple syllogism involving all three layers:

P1) The earth rotates (primary supposition/constitutive element)

P2) The rotational motion of the earth is imparted to all elements on the earth (Theory)

[10] Galileo continued to use the term “impetus” after having left the theory of impetus, popular from the 12th century and on, behind. In the dialogue of 1932 he uses the term, but it then covers what we now mean with the term “inertia”, a term and a concept that Kepler had introduced.

C) The rotation of the earth will not influence the distances in which we observe arrows to travel (empirical consequences)

Arguably, the most radical philosophical shift Galileo presents is the idea that the truth is not always apparent in the appearances. Consider a version of the Peripatetic argument here. If you ride a bicycle and throw a rock in the forward direction, the rock will cover more ground than if it were thrown in the backward direction. An observer measuring the distance from the throw to the landing site, will therefore measure an apparent asymmetry. So why should the case not be identical when it is the earth rather than a bicycle that is moving? The solution – which Galileo expresses in terms of shared and relative motions – is that the rotational motion of the earth is hidden from us because we take part in it. Thus, appearances are sometimes misleading. The idea that reality is hidden from us in this way, and must be sought in terms of ideals rather than appearances is a radical shift in philosophy that has its roots in Plato.

Hence, Galileo sets up a view containing three layers of constitutive elements where the primary suppositions are general and true and the astronomic theory is specific. Together they must yield the appearances. The appearances themselves are sometimes misleading and are therefore not a reliable basis for theorizing. Kepler works from the same general set-up.

I 2.3. Kepler's philosophy of science

Kepler describes his philosophy of science in multiple places. In the following, I shall base my reading primarily on the *Apologia pro Tychone contra Ursum* (1600), *Astronomiae Pars Optica* (1604), and his main astronomical work *Astronomia Nova* (1609). (I will refer to these works hereafter as *Apologia*, *Optics* and *Astronomia*, respectively, or with their years of appearance.) Kepler sets up the general scheme of his theory of science as follows:

> Altogether, there are three things in astronomy: geometrical hypotheses; astronomical hypotheses; and the apparent motions of the stars themselves (Kepler 1600:154)

He then makes it clear that the geometrical hypotheses are primary and that the role of the astronomical hypotheses is to ground the geometrical hypotheses causally such that these together can account for the apparent phenomena. Note that Kepler insists on determining the motions of the stars as apparent, rather than as true in this case. The reason is that there might be a difference between the apparent and the true motions. A central role of Kepler's optical theory is to explain how light travels in different media and how this affects our observations in such a way that reality is blurred by observation. This is an aspect of observation, which we often forget, although we are all familiar with the appearance of how a stick "bends" when we put it into water. In the introduction to the *Optics*, he makes the comment:

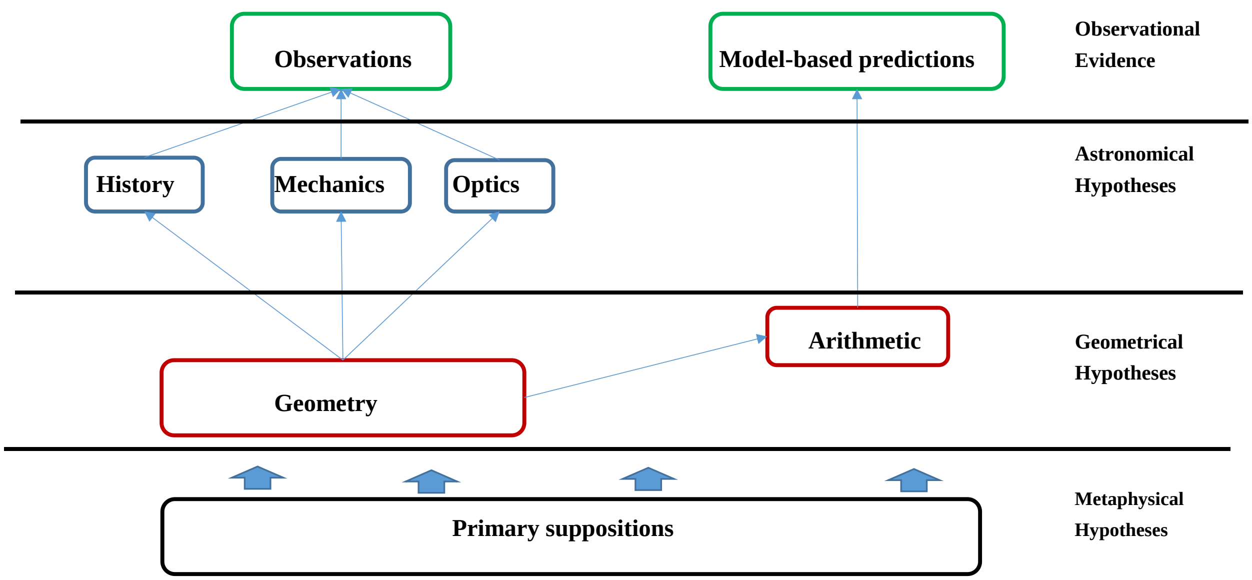
> For because many things, not only about the direct ray, but also about the reflected and refracted ray, were overlooked by Witelo, and many things that should have been explained *a priori* were only brought in extraneously from experience and set up in place of axioms, I thought it a good idea to look a little more deeply into the whole nature of light, and relate it to the things that appear… (Kepler 1604: 16)

We see here how Kepler rejects the "functional a priori" methodology suggested by Pap and Stump. The things that were introduced extraneously from experience *should have been explained a priori*. I emphasize this issue because it could easily be thought that Kepler and Galileo held their positions due to a lack of knowledge or sophistication. I am proposing however, that they considered the options carefully and decided against methods of the kind proposed by Stump and Pap where the constitutive elements of a theory are founded on empirical generalities. Kepler even makes an amusing mockery of the idea that one should draw conclusions directly from appearances.

> For where, walking through the fields, he encounters hedges and things near to his path, he would believe, on the testimony of sight, that distant mountains are really following him. (Kepler, 1600: 155)

As Galileo, Kepler thus argues for a constitutive theory of science not based on appearances, and requiring fundamental levels of concepts, accounting for the appearances. The trick is to explain why the world appears as it does, not to take appearance at face value.

In the *Optics*, Kepler (1604: 13-16) sets up his constitutive theory in a more detailed manner. Here we also see how different fields of inquiry constitute each other in a hierarchy of layers. I will present an illustration of the overall idea. We shall then later use this illustration to show how we can apply Kepler's theory of science in modern physics. The first thing to notice is that Kepler (1604) describes four, rather than three central parts. This is because in the initial 1600 description "Astronomical hypotheses" refers both to physical theory and to metaphysics. In the 1604 description, metaphysical hypotheses treated as a separate layer.



Another important aspect of Kepler's set-up is the direction of the hierarchy, which is in direct opposition to modern naturalism. On Kepler's scheme, the constitution goes *from* metaphysics through geometry and astronomy to observations and predictions. Note how optics follows from metaphysics and geometry, both of which Kepler treated as a priori. It is also important to keep in mind that in Kepler's theory, "hypotheses" are true statements in the same way as they are in Galileo and Copernicus. Kepler (1600) enters a lengthy defense of hypotheses as true statements in order to counter Ursus' claim that hypotheses are intrinsically untrue.

I 2.3.1 Kepler’s hierarchy of constitution

In the *Optics*, Kepler first divides astronomy into two parts represented in the above illustration by the left side, which leads to observations of the apparent phenomena, and the right side, which gives model predictions. Kepler here comments that there is a difference between the two in that one is more philosophical and contemplative, while the other is more practically oriented. Kepler then quotes Plato in claiming that both sides “fly up into the heavens, supported […] by a pair of wings, *geometry and arithmetic*…” The idea here is not that the geometric and the arithmetic parts are fully distinct but rather the opposite. Consider for instance the Rudolphine Tables Kepler “inherits” from Tycho Brahe, and that summarizes Tycho’s observational data. These tables are doubly presented in the above scheme. They represent the arithmetical part as “snapshots” on a geometrical path, but they also fill in the “history” part of the astronomical hypotheses because they constitute observational history. As such, they have a double role that is particularly interesting.

The arithmetic part of the theory should follow from the geometrical hypotheses that are given a priori, i.e. as straight phoronomy or kinematics set in a particular geometry. The geometries used by Kepler are primarily the Euclidean flat geometry and the Apollonian Conics along with the integration and Platonic view of mathematics presented in Proclus. Together these allow Kepler to construct different geometrical shapes and orbits, which would generate predictions concerning positions at particular points in time. The positions should then generate further predictions to compare with the observed phenomena. A set of problems immediately present themselves.

First, one needs an optical as well as a mechanical theory of observation in order to distinguish between the apparent and the real positions. The observational data cannot be taken at face value due to possible refraction and reflection of light on its path from a planet to the observer. What we get therefore are “ideal” positions in the geometrical layer and “distorted” apparent positions in the astronomical layer. Somehow, this situation must be mended or the predictions will not match the observations. Kepler’s solution involved optical theory, mechanical theory, a rewriting of the observational history, along with a series of metaphysical arguments.

Another issue is the well-known issue of under-determination. In the opening part of the *Astronomia*, Kepler shows how, by only slight modifications of the tables, all three “world systems” can yield the same geometrical structure. I.e., Kepler shows that the Copernican, the

Tychonic, and the Ptolemaic system can be treated as different manifestations of the same geometrical structure.[11] In order to solve this issue and ultimately to present what we now know as Kepler's three laws, Kepler had to generate theory both in metaphysics, optics and mechanics. Ultimately, the central resolving argument is an argument from causality and ontological plausibility.[12] We thus see that although the layers convey an integrated whole, there is a hierarchy in which the metaphysics ultimately decides. The emphasis on a sound metaphysical and geometrical ground is hardly surprising considering Kepler's explicit adherence to Plato and Pythagoras whom he describes as his (and Galileo's) "true preceptors" (Kepler: 1597).

I 2.3.2 Kepler on coherence

So far, we have seen that Kepler's philosophy of astronomy is a constitutive philosophy with three layers of constitution (metaphysical, geometrical and astronomical). Together, these three layers must account for and explain the apparent phenomena. There is however an overall condition at play here as well, which Kepler emphasizes. There must be coherence within the constitutive system. Kepler works with two types of coherence that we may call internal and external. The internal coherence of a theory relates to how the different layers of constitution fit together. A simple way of putting it is that the entire constitutive layer must be presentable as a single and coherent argument. In the *Apologia*, Kepler refers to this as the "totality of views"

> We thereby designate a certain totality of the views of some notable practitioner, from which totality he demonstrates the entire basis of the heavenly motions. All the premises, both physical and geometrical, that are adopted in the entire work undertaken by that astronomer, are included in that totality. (Kepler 1600: 139)

At a first glance, the criterion of internal coherence might appear trivial. As soon as one allows some detail, however, the triviality disappears and Kepler's demand comes out as strict. Take for

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[11] An illuminating image of how the three hypotheses or «world systems» relate geometrically in the case of the planet Mars can be found at http://www.keplersdiscovery.com/Hypotheses.html

[12] The astronomical details of Kepler's solution I will not discuss here. For a modern commentary on these issues, see for instance Tønnessen (2012) and Wilson (1968). In this thesis, I am only after the general philosophy of science of Kepler and its possible application to the details of special relativity.

instance the equant point used by previous astronomers as well as Kepler's contemporaries. The equant is a mathematical tool used to generate models of planetary orbits and it is a highly successful one at that. However, Kepler focuses on physical causality in his metaphysics and maintains that all physical phenomena have physical causes. Arguably, the demand for causal explanation is what makes Kepler's astronomy radical enough for him to define it as the "New Astronomy". With a metaphysical demand for physical causes, Kepler cannot accept equant points into his theory, even though they are successful for model generation. Rather, he must ask why it is that equants work at all. For Kepler to be coherent his answer must reveal a physical cause that somehow generates orbits for which equant points successfully generate models. By now, we know that Kepler found his physical cause in the position of the sun and his idea of "gravity" in the sense of a magnetic force inherent in the planets and the sun. However, if Kepler had followed his contemporaries and allowed the equant point to play its standard role, he could still have generated the elliptic orbits but would have no reason to prefer them to the multitude of other possible orbits.

The details of Kepler's work in the *Astronomia* have recently been scrutinized, and among the commentators, we have seen a growing resistance against the standard description of Kepler's work as abductive. Roughly speaking, an abductive inference is a theoretical inference from empirical data. Hanson (1958) famously promoted this, by now, standard description. What I am proposing here however is that Kepler applies a *converse abduction* where we infer our understanding of the apparent phenomena from the constitutive elements of the theory.[13] However, one may still ask whether there is a need for any metaphysics in the general scheme. Arguably, there could be coherent systems that are metaphysically implausible. Why should we not apply such a system if it successfully saves the phenomena?

Kepler's answer to this, which I endorse, relates to his Unitarian view of nature. The Aristotelian or Ptolemaic image of the world was a dualist image wherein the active causes on the earth were radically different from those in the planets. Copernicus weakened this idea by placing the earth among the planets. Kepler, following Copernicus, demanded that whatever physics we end up with, it applies to the earth and the other planets alike. There is only one system!

[13] See for instance Myrstad (2004), Tønnessen (2012), Wilson (1968), and Jardine (1979).

Now, if there is only one system and one believes that the system is regular and explainable, then the constitutive elements of this unifying theory must apply universally. In other words, the metaphysics you use in order to describe how beetles fly must be the same metaphysics that you use in explaining why bricks do not. Furthermore, a phenomenon to be explained is not isolated. Take for instance the historical part of astronomy in which previous observational data is collected. Kepler understood that in order to explain and understand these data, and predict future ones, you must understand how light travels. Once you introduce telescopes, you must also develop an understanding of conics, and the working of lenses. These are all internal matters to some extent. However, if you *also* wish to understand how cells migrate within the human body, you must use microscopes. Would it be acceptable if we found that biologists applied radically different theories to explain the geometry of lenses of microscopes as well as the optics involved?

On this issue, I follow Kepler in his Unitarian view and maintain that such a situation would be unacceptable.[14] In order to avoid problems of this sort, we need a general set of constitutive elements that apply across the fields of inquiry. In this sense, we can talk about a criterion of *external* coherence which demands that knowledge borrowed from other fields comes with the constitutive elements applied in that field. If there is a conflict at the level of constitution, it must be resolved.

Together, the criteria of internal and external coherence motivate some basic metaphysical commitments. Kepler's main metaphysical commitment is that all physical phenomena have a physical cause. Through this commitment to physical causes, we also get a criterion for explanation. A physical phenomenon is explained when the causes that generate the phenomenon are understood. Kepler formulates this idea in the *Apologia*.

> One who predicts as accurately as possible the movements and positions of the stars perform the task of the astronomer well. But one who, in addition to this, also employs true opinions about the form of the universe performs it better and is held worthy of higher praise. The former draws conclusions that are true as far as what is observed is concerned; the latter not only does justice in

[14] Only under a Unitarian view can we speak of science as generating a world-view in which we can find our place as human beings. Arguably, if the world turns out to be ontologically pluralistic and a Unitarian view is impossible, we must revise this position. Indeed, many have argued that modern physics forces a pluralist view. However, in this thesis I will argue that there is no such need, as least not motivated by special relativity.

> his conclusions to what is seen, but also, as was explained above, in drawing conclusions embraces the inmost form of nature (Kepler 1600: 145)

Here we clearly see how Kepler distinguishes between those who draw conclusions concerning *what is seen,* and those who embrace the inmost form of nature and thus draw conclusions concerning what is true. Again, we see the distinction between working with appearances and apparent phenomena on one side, and understanding what generates those appearances on the other. In so doing, Kepler does not reject the results of those working only with appearances as irrelevant or unscientific, but rather as in need of a further reflection. The "functional a priori" approach of Pap and Stump thus becomes a preliminary rather than a final step. What we are seeking is not the regularity of appearances but rather the underlying reality that generates them.

I 3. A priori constitutive elements

I have already stated that Kepler applies causation as an a priori element within his constitutive theory of science. We shall come back to this aspect of Kepler's theory and its relation to the contemporary debate soon. First, however, I wish to elaborate on other a priori elements in both Kepler and Galilei. An interesting element of Kepler's thinking is his focus on the harmonies in the *Harmonices Mundi* (1619). Here, the focus is harmonies in musical and planetary relations as well as relations between the Platonic and Archimedean solids. The main line of thinking in Kepler's approach to these issues is his Unitarianism and his adherence to Pythagoras and Plato. Platonist thinking is also clearly visible in Galileo's repeated reference to *anamnesis* in the *Dialogue*. Here, Galileo shows how an adherence to the peripatetic philosophy generates internal conflicts in the character *Simplicio*. When *Simplicio* gives up elements of the peripatetic doctrine in favor of his own direct experience and intuition, he is told that he *already knew the answer all along.* This "innate knowledge" justifies conclusions drawn from thought experiments. Galileo does not elaborate on the nature of our innate knowledge but maintains that thought experiments have power of demonstration. Thus, in cases where appearance contradict theory, one should of course investigate whether there are problematic issues in the theory. However, if the theory is sound, there is good reason to try to explain the appearance through it (converse abduction). This is for instance how Galileo explained the motion of the earth through the principle of relativity and the notions of shared and relative motion. An interesting issue is then what constitutes a "sound theory". It is not the appearances by themselves. By mere appearance, the earth is immobile. Galileo, as Kepler, refers to principles of simplicity and unity, but also maintains that the ultimate judge is the single person applying his capacities in an honest fashion. Galileo makes the peripatetic *Simplicio* go through such a process throughout the *Dialogue* by asking increasingly abstract questions and comparing the answers to everyday experience. Thus, applying the Socratic Method we know from Plato.

Neither Galileo nor Kepler wrote explicitly metaphysical theories. Rather, they commit to certain claims. Both commit to a Unitarian view and hold that there can be only one physics, thus rejecting the Aristotelian and Peripatetic dualism. Furthermore, they both argue that physical explanation is causal explanation. On the issue of causation, there is a similarity between Kepler and the contemporary view set forth by Chakravartty.

I 3.1. Chakravartty's semi-realism and the detection/ auxiliary distinction

Chakravartty (2007) argues for a new version of scientific realism, which he calls "semi-realism". In order to explicate the position and its relevance for our discussion I will first say something comprehensively about the debate in which Chakravartty takes part.

Chakravartty follows Copernicus, Galileo and Kepler in being a scientific realist. In very rough terms, scientific realism is the idea that science is a good tool for discovering truths about the world. This is a position that I am also endorsing in this thesis. The main idea is that we have reasons to believe that science is not just a perspective among equally valid perspectives but that there is something special about science. However, this special role cannot apply to *all* of science at any given time. Scientific theories are sometimes inaccurate and at other times flat out wrong. A worry for any scientific realist is how to distinguish the true scientific theories from the erroneous ones. Pointing out that a particular theory "works" is not sufficient, as the history of science itself is full of theories that "worked" that nobody believes in anymore.[15] A common strategy of contemporary scientific realists is to look at the history of science and see whether there

[15] This counter-argument is typically referred to as the pessimistic meta-induction, and it is one of the many skeptical arguments that Kepler and Galileo dealt with, and that is still part of the contemporary debate.

are hallmarks pertaining to those theories that stayed relevant and that we still believe in. Chakravartty's position is a conciliatory theory that attempts to reconcile the two main contemporary frameworks of structural scientific realism and entity realism.

Roughly put, structural realism claims that what remains throughout scientific revolutions and thus what survives theory change, is structure. Typically, it is the mathematical structure that remains (at least roughly), and what changes are the entities involved in those structures, according to this theory. The standard example in the literature is how Fresnel's mathematical formulas remain in science, while the ether-field he applied in the explanation of those formulas is no longer part of accepted theory.

Entity realists on the other hand claim that what remain are *interactions among entities*, rather than structure or theory. For instance, if you run double slit experiments on light, you still get interference patterns (typically associated with wave-behavior) even if your contemporary optical theory claims that light is particles or photons.

Chakravartty (2007) makes two central contributions to the debate that we shall bring with us and relate to the Keplerian framework I am setting up here. For one, we can neither understand nor relate to experiments unless we simultaneously consider both structures and entities. Roughly speaking, experimental data in physics tracks the behavior of something through interactions and those interactions, if there is any regularity involved, follow a certain structure. Thus, there can be no *hard* distinction between structures and entities. This aspect of semirealism is central to my overall argument concerning special relativity. As we shall see, the standard reading of special relativity suggests that we only consider the structural aspects of measuring results and the measuring apparatus itself. In consequence, the physical aspects of the measuring apparatus is largely ignored. In part II, we shall see that the relevant physics can be split into four central parts, phenomenology, mechanics, dynamics and phoronomy, and in part IV I argue that the dynamical aspect needs further consideration. As such, the overall argument is in line with semirealism as I understand this position.

Furthermore, some structures end up rejected and some experimental results change in the course of time. We thus need a further criterion that runs deeper than merely singling out one or the other aspect of scientific concepts.

Chakravartty suggests we make a distinction between what he calls “auxiliary properties” and “detection properties”. Chakravartty (1998: 394) defines detection properties as those properties on which detection depends. The idea here is that although a theory of un-observables may contain descriptions of a multitude of properties, we should be minimal in our realism. We should be realist only about those properties that are directly necessary for the relevant detection, and agnostic concerning any other property. In other words, we should hold as true only those elements that constitute conditions of possibilities for the experimental outcome.

Concerning the speed of light for instance, we should then be realist about the mean two-way speed of light being C, but agnostic about the universality of the one-way speed. The former is detectable in experiment, but the latter is not (for an overview see Ahmed et al. (2012)). The one-way speed of light is what Chakravartty (1998: 394) calls an auxiliary property in that it is instrumental in building models of the phenomena and generate further prediction. It is however, not directly connected to the phenomena we observe in the necessary way of detection properties. We should stress, however, that being an auxiliary property is not equivalent to being unreal. Being an auxiliary property is to have a speculative status pending further investigation.

The distinction between detection properties and auxiliary properties is also part of Kepler’s philosophy of science, although applied with a slightly different terminology. A clear instance where Kepler applies this sort of thinking is in his discussion concerning the relative motion of the Earth and the Sun in relation to the Ptolemaic, Copernican and Tychonic theories. The theories disagree on what is moving (earth, sun or both), but as Kepler points out, they agree on the only relevant issue for generating the observational results. This is the relative motion between the earth and the sun. Kepler then brings with him only this element into further investigation.

Although the detection/auxiliary distinction is helpful, it is in need of something more. For instance, in cases discussed in Chakravartty (1998), the detection property is singled out as causal not only in the sense that it is a phenomenon that causes a particular observation (as in the case of the classical theories in astronomy above), but also in the sense that there is something causing the phenomenon itself. I will focus on this issue, as it is one of importance for the discussion of special relativity.

I 3.2. Detection properties and special relativity

Two types of theories can account for the phenomena that gave rise to special relativity. There is of course special relativity itself, and in addition, there is what we call ether theories. Part IV of this thesis is wholly committed to discussing these theories and the relation between them. The two types of theories apply the exact same "arithmetic" hypotheses and make the exact same predictions for the commonly accepted phenomena. However, the physical and metaphysical hypotheses applied in the theories are radically different. The main problem in relation to Chakravartty's distinction is that there seems to be no theory-independent way to single out what the detection properties are. A clear indication of this problem is found for instance in the debates between the ether theorist Ives and his relativistic contemporaries concerning the Ives Stillwell experiments. Ives held that they proved ether theory while his opponents held that the same experiments proved special relativity. One way of approaching the problem is to look for the common detection properties that underlie both theories in the way Kepler did in relation to the motion of the earth and the sun. Once these are established, one can hold that *both theories are functional*. This is the line of argument in Szabo (2010) for instance. Another is to argue in terms of *scientific principles*. One could for instance claim that one theory brings more *unity* than the other does. This is the path taken by Friedman (1983). One could also argue that one theory is *methodologically* superior, such as is argued by Reichenbach (1928). Whether one argues one way or the other, it appears that by themselves, detection properties are insufficient for solving cases of under-determination of this kind. We thus need to make some further commitments.

I 4. What is the nature of a constitutive element?

As we have seen, Kepler, Galileo, Einstein, Copernicus, Pap, Friedman, Stump and Reichenbach all subscribe to a view of science under the constitutive umbrella. Arguably, Chakravartty's causal realism and the insistence on detection properties places him in the same group. Still, there are clear differences between these positions. I focus my interest here primarily on two topics. Firstly, we must ask whether constitutive elements are constitutive of science only or whether they constitute knowledge and experience as such. If constitutive elements only constitute science, we must ask what the relation is between scientific knowledge and everyday knowledge in this case.

On these issues, Einstein holds the most open position and so we shall use this position as our vantage point.

Einstein (1936: 290) states that science is no more than a refinement of everyday thinking. Thus, there is a connection between everyday thinking and science. Einstein points to two common issues in the 1936 paper. First of all, the primary concept in any empirical (whether "everyday" or scientific) investigation is the concept of an object. This concept itself is not derived from empirical investigation, but is a "free creation of the human (or animal) mind" (Einstein 1936: 291). The concept of the bodily object shares this origin with the other constitutive elements or "rules of the game" in Einstein's epistemology. Thus, in Einstein's general (and scientific) epistemology, the constitutive elements are freely chosen. Their only criterion of validity is their ability to help organize the appearances successfully. In the summary of the 1936 paper Einstein presents the issue as:

> Physics constitutes a logical system of thought which is in a state of evolution, whose basis cannot be distilled, as it were, from experience by an inductive method, but can only be arrived at by free invention. The justification (truth content) of the system rests in the verification of the derived propositions by sense experiences, whereby the relations of the latter to the former can only be comprehended intuitively.[16] (Einstein 1936: 322)

Einstein's position then is that there are constitutive elements of scientific theories that *cannot be derived from experience*. Rather they are free constructs whose truth-value is decided entirely on their ability to "save the phenomena" in the best possible way. Einstein's rejection of an empirical base for constitutive elements is on the one side necessitated by relativity theory, where constitutive elements such as "practically rigid bodies" and the one-way speed of light have no empirical justification. However, it brings a problem for Einstein concerning his justification for arguing against an ether theory in the first place. We shall look at Einstein's position in more detail in part IV of this thesis. For now, it suffices to see that Einstein rejects the notion of empirically

[16] The concept of «intuition» Einstein has in mind here appears to be a concept of "hunches" or "apparent obviousness" rather than the Kantian concept.

based constitutive elements while maintaining that there is a continuity and common ground between everyday thinking and scientific thinking. He offers no explanation about the origin of the constitutive elements apart from their being freely chosen and not empirical. This position works both for and against adopting relativity theory whilst leaving us with something of a mystery concerning how to deal with underdetermined cases. For instance, given Einstein's theory of science there are no reasons to prefer the Copernican system to the Ptolemaic alternative. Rather, if we followed Einstein's theory of science further and added a principle that Einstein himself adds (simplicity) we would be compelled to prefer the Ptolemaic system. For, as Galileo (1615) points out, when faced with computational problems, Copernicus himself preferred to use the Ptolemaic system as it is much simpler. Any proponent of Einstein's theory of science would thus have to provide some reason for us to prefer the Copernican model, which is the one we still use.

Pap and Stump argue the opposite of Einstein (1936) and hold that the constitutive elements are empirical. Stump defends this position on a fallibilist argument similar to the skeptic pessimistic meta-induction. The overall argument being that the constitutive elements cannot be a priori because we have seen "a priori" elements come and go throughout the history of science.

Before discussing the empirically based constitutive elements, I shall take a short look at this argument. The fallibilist argument looks entirely empirical and innocent at first glance. We made mistakes before and we will probably continue to do so. However, there is a strong epistemic commitment attached to the fallibilist argument pointed out by Kepler in the *Apologia*. In Kepler's case, his opponent, Ursus, had argued that hypotheses were intrinsically wrong for the very same reason referred to by Stump. Kepler's answer is simple. Do you believe that the earth both moves *and* does not move?

There are ways in which this question can be answered in the affirmative. For instance, one could argue that the earth either moves or does not move, but that we can never know. Thus committing to the classical reading of Kant. Alternatively, one can hold an ontologically pluralist position and claim that there is more than one ontological reality concerning the earth's motion. These are both strong claims that must somehow be defended. However, although he appeals to the fallibilist argument, Stump (2015) does not commit to any of these positions.

Problems involving the fallibilist argument are of course no defense of there being a priori elements in science. All we see here is that when rejecting timeless truths one must either commit to one of the above positions, or appeal to other arguments. Stump (2015) does appeal to other arguments when arguing against the Kantian a priori notions more specifically. His arguments are the same as those of Pap, Friedman, Reichenbach and Cassirer, who argue that non-Euclidean geometry and relativity theory falsify the Kantian a priori elements.

Stump holds that constitutive elements are neither a priori nor "free constructs" and thus differs from both Kant and Einstein. His position, following Pap, is that empirical facts are elevated temporarily to the status of constitutive elements and thus function as the conditions of possibility within a particular framework for some time. In this way, Pap and Stump can avoid the skeptical argument against science by claiming no timeless truths, while maintaining that there are constitutive elements. However, if we look at the non-Euclidean geometries and the theory of relativity, which Pap and Stump argued from above, it seems they are at a loss. Stump (2015: 122) admits that the principle of inertia cannot be derived from empirical investigation, even in principle. The simplest reason being that we make empirical discoveries through interaction, and an inertial system is one that is not being acted upon. Furthermore, inertial motion is defined as "uniform rectilinear motion" which introduces the issue of how we are to empirically define what is "rectilinear". An appeal to empirical evidence here would be viciously circular since such evidence would necessarily presuppose the principle of inertia itself. Stump, as Poincaré before him, admits inertia is problematic, but offers no solution. We know from Galileo's own description in the *Dialogue* that he discovered the principle through thought experiments and not from empirical data. Furthermore, Einstein himself readily admits that there is no empirical foundation for the practically rigid bodies of relativity theory, nor are there empirical justifications for Einstein's claim concerning the one-way speed of light. On the latter issue, Einstein was explicit from the very beginning:

> The latter [a common time for two spatially separated events] can now be determined by establishing *by definition*[17] that the 'time' required for light to travel from A to B is equal to the 'time' it requires to travel from B to A (Einstein 1905: 126)

[17] Original italic.

It thus appears the theories, to which Pap and Stump appeal, contain constitutive elements that do not have empirical origins. Of course, Stump could argue that constitutive elements are sometimes or even *normally* empirical (such as Einstein argues for principle theories). Stump however, holds that constitutive elements are ultimately empirical and must therefore explain issues such as the principle of inertia, practically rigid bodies, one-way speed of light and other such elements that constitute modern physics. There are similar issues in the mathematics applied in modern physics, which we shall discuss in part III.

On the issue of the relation between everyday thinking and experience on one side and scientific thinking on the other, Stump does not offer any theory. It is thus unclear whether these are separate issues or not. Kepler, Kant and Galileo all argue with Einstein that science is a refinement of everyday knowledge. Kant (1781/1787) argues that we constitute our everyday experience through pure intuition and that this very same intuition constitutes the backbone of Euclidean geometry as well as Kant's own laws of mechanics. We shall consider Kant's suggestions in some detail in part II.

Kepler and Galileo repeatedly argue for scientific points through principles that constitute everyday experience. So for instance, when Galileo argues for the principle of inertia, he argues that this principle necessarily constitutes everyday experience and thus it must constitute physical theory. His method of proof is a thought experiment using the idea of frictionless inclined planes. Such frictionless planes, of course, do not exist in the empirical world. Similarly, Kepler intended his argument about the man moving along the hedge referred to above to show how we constitute all of our experience in something non-empirical.

Contemporary theories reject the Kantian notion that constitutive elements are synthetic a priori and give different suggestions concerning their origin. We have seen however, that the contemporary suggestions have issues that appear problematic. Chakravartty (2007) commits to the existence and scientific description of a "mind-independent" reality and thus rejects the Kantian option as being a realist option at all. I will support the Kantian position and argue that the constitutive elements are synthetic a priori. Although not explicitly argued by Kepler and Galileo as such, the Kantian option squares well with both the anamnesis and thought experimental work of Galileo as well as with Kepler's adherence to both the Platonic and the Pythagorean philosophies.

I 5. Summary of part I

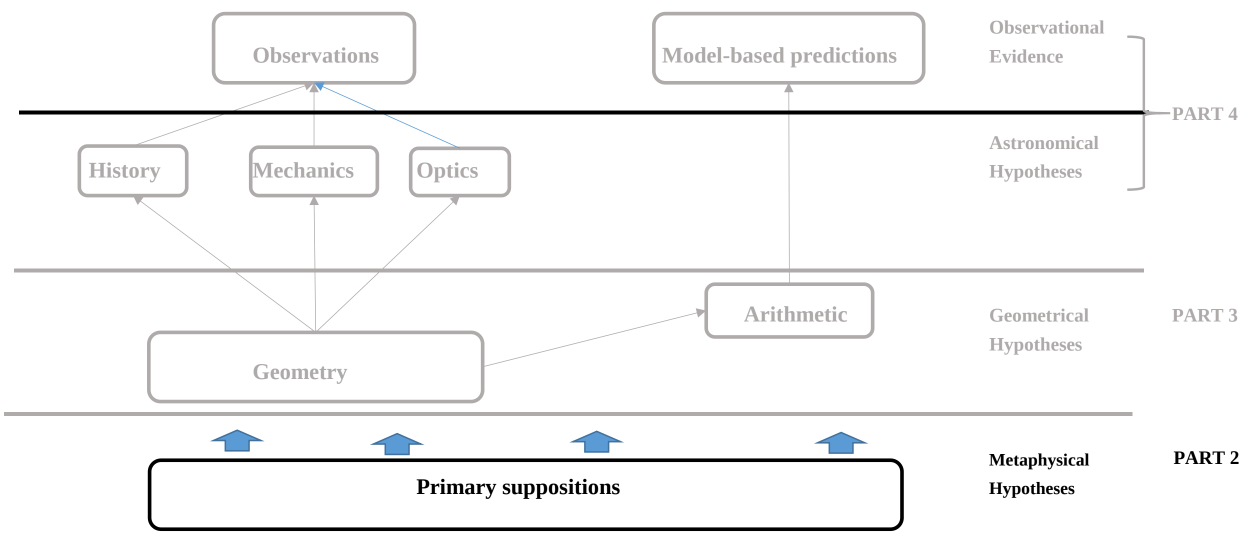
I have argued that there are constitutive elements of scientific theories that provide the conditions of possibility for the expression of empirical facts. We have seen that there is a tradition for thinking in this way that goes back at least to Plato and the Pythagoreans and that we find Copernicus, Kepler, Galileo and Einstein within this tradition. I have positioned myself within the tradition of constitutive theorists and sided with Kepler and Galileo in what I have called classical scientific realism and laid out the overall idea of Kepler's stratified constitutional theory. Through parts II, III, and IV, I shall apply this theory in the modern setting.

In addition to the scientific tradition, there is a philosophical tradition debating the nature and origins of the constitutive elements. This tradition also has its roots in Plato and Pythagoras, but the modern debate traces back to Kant's transcendental philosophy. There is a common notion that Kant was proven wrong by "non-Euclidean geometry" and relativity theory, and thus the philosophical tradition has constructed alternative justifications for constitutive elements. I have indicated some problematic issues for the contemporary theories and claimed that the Kantian option is still preferable. In order to defend this claim I will have to show how Kant's transcendental philosophy applies even in light of contemporary mathematics (part III) and physics (part IV).

Part II

Metaphysics for natural science

In part one, I established myself within the camp of constitutive theorists and argued for a Keplerian version. A central aspect of Kepler's theory is that it brings with it a hierarchy of constitution where the "metaphysical" layer is foundational. This foundational layer is the topic of part two.



In the following, I shall argue that there are a priori elements in the "metaphysical" layer that constitute rules for constructing the upper layers. In so doing I make a standard separation between four aspects of physics (phenomenology, mechanics, dynamics, and phoronomy) and argue for the necessity of a priori elements in all of them. The overall aim is to show how the same a priori elements constitute both our everyday experience and physical theories. Once I have established the a priori elements I move on to part three in which I will discuss the relation between Euclidean and "non-Euclidean" geometries. The overall idea of this structure is that parts one, two and three constitute a basis on which we can debate the observational, physical and philosophical aspects of Lorentzian theories and special relativity, which is the topic of part four.

II 1. What is metaphysics?

In the following, I define metaphysics as "prior to physics". This implies that metaphysics is the study of that which is necessarily part of both everyday experience and physical theories. Being a necessary part here implies that one can know, before doing empirical research, some aspects of our results.

The transcendental philosophy approach to metaphysical investigation is to analyze general aspects of our experience and ask what the conditions of possibility for those are. If we find that there are conditions of possibility that are universal and necessary, these constitute a bottom and prerequisite layer for any empirical investigation whatsoever. Being necessary implies the strong sense of necessary as apodictic, things could not be otherwise and still make sense to us. This sort of necessity is stronger than the necessities implied in most constitutive theories. As we have seen, constitutive theorists argue that there are necessary preconditions (constitutive elements) for scientific research. However, most argue only that there must be some constitutive elements.

Typically, one assumes that there is a choice involved concerning which elements one chooses. This is for example what Poincaré argues when he claims the constitutive elements are conventional. The stronger claim, which is the one I will be defending, is that there is a double necessity. Scientific theories requires constitutive elements, *and* the most basic of these elements are rigidly determinable. Thus, prior to any scientific or everyday experience, we can know what the conditions of possibility are, and there is no choice involved.

Importantly, the necessity involved is not a logical necessity in the sense that the different a priori elements follow logically from each other or that any replacement of a priori elements would constitute a contradiction in and of itself. Rather, the claim is that the a priori elements are necessary and could not be otherwise for *our type of experience in the world*. Thus, it is perfectly conceivable that there could be beings that do not experience temporally for instance. However, this form of experience is foreign to us and we can have no knowledge of it. We are not seeking a view from nowhere.

Einstein (1936: 290) argues that there is a continuation between everyday thinking and science, where the latter is a refinement of the former. He goes on to claim that there is a common and central constitutive element for *all of physics*. This constitutive element is the concept of the bodily object (Ibid: 291). On Einstein's view, we must apply this concept in order to arrive at the possibility of a physics whatsoever. The relation in which Einstein makes this claim is the physicist debate over classical quantum mechanics. Here, Einstein famously argued that quantum mechanics is flawed or limited *because* the theory does not allow us to maintain the concept of an object.[18] We can understand this claim in at least three ways: One possible understanding is that there are rigidly set aspects of the physical world that we experience and thus when we analyze our way to them we are analyzing the world as it really is. That is, the world consists of bodily objects.

Another possible understanding is that we are analyzing the specific way in which humans experience the world. The latter opens up for the idea that we, as experiencing subjects, bring a rigid structure into our experience and thus that our experience exposes but one of many possible ways in which the world could be. In other words, human beings see the world in terms of bodily objects. Einstein argues for a third position in which we cannot claim to know that there "really are" bodily objects or at least, we cannot derive that there are from experience. Neither does he argue that there is a rigidly set structure to the human experience, "there are no final categories in the sense of Kant" (Ibid: 292). His solution is to claim that the concept of the bodily object is a free creation of the human mind that we must suppose are there since without it we cannot successfully account for the phenomena (Ibid: 292-293).[19]

If the concept of an object, or indeed any other candidate for a universally applicable constitutive element of this sort, is a free creation of the human mind, a question arises concerning the nature of our scientific knowledge. At least at a first glance, it appears that a science built on freely chosen concepts is somewhat idealist. Granted, Einstein (1936) emphasizes that the ultimate testing ground for such a concept is whether it is successful in generating the true observations.

[18] I shall not enter into the details of the debate over classical quantum mechanics as this is outside the scope of this thesis. However, it is useful to see how the EPR argument as well as Einstein's (1936) follow up, hinges on there being an object with a continuous identity we can trace through different (experimental) interactions. This concept of object (either as particle or as the medium of wave disturbance) he takes as constitutive of his argument. What is of interest to us is how the concept of object functions as a condition of possibility for any physics, as well as ordinary experience, in Einstein's argument.

[19] Kant famously argued against the idea that the concept of a bodily object is primary in Einstein's sense and his assumption that we can build the notion of a space from the concept of bodily objects and the relations between them. This is because the concept of a bodily object contains the notion of extension. Extension itself presupposes the notion of spatiality or a space in which something is extended. Thus, it appears that space is a condition of possibility for the concept of a bodily object, and not the other way around. One could argue that we could construe this notion of space from relations between non-extended objects (points), but this would lead back to the question of what those relations are. Either, the points must be in the same place and no spatiality arises, or they must be separated and thus already imagined as in a space. It thus appears as if space is necessarily primary to the concept of a bodily object.

Nevertheless, if the concept is freely chosen it is conceivable that there are other concepts that could play the same part. If there are such concepts, we have ultimately chosen our scientific perspective rather than derived it from the available evidence and the worry about idealism remains. In the contemporary debate on scientific realism, the question of the nature of our scientific knowledge is discussed in terms of mind-dependence vs. mind-independence. Einstein's "free creation" theory appears mind-dependent.

II 1.2. Contemporary scientific realism

Chakravartty (2007: Ch. 1) defines scientific realism as consisting of three major commitments (hereafter referred to as the "received view"). The idea is that in order to call yourself a scientific realist, you must commit to the following three claims:

- Metaphysical commitment: *There is a mind-independent world*
- Semantic commitment: *Scientific theories should be read literally*
- Epistemic commitment: *Scientific theories, when read literally, provides knowledge of the mind-independent world*

At first glance, it appears that the received view defines constitutive theories as non-realist positions.[20] This is because they claim that there are constitutive elements of a non-empirical nature that function as conditions of possibility for science. Arguably, these elements must, to some extent be mind-dependent. The alternative route to knowledge is, arguably, mind-independent, as it ultimately relies only on empirical investigation. For instance, Chakravartty argues that we gain knowledge through empirical investigation and that empirical investigation gives access to mind-independent reality.

[20] Provided they are not constitutive theories of the empiricist kind such as the positions of Stump and Pap.

> We believe that our sensory experience is brought about by the very things of which we have experience. Objects exist, and they affect us in such a way that we are confident, by virtue of their affecting us (us perceiving them), that they exist. (Chakravartty 1998: 393)

This is a one-directional causal description of sensation where external objects affect our sense apparatus that signals our brains and that finally leads to a perception.

That knowledge originates in experience is a common claim in all the constitutive theories. The stronger claim Chakravartty makes is that the things of which we have experience bring about this sensory experience. This is also common to anyone who is not wholly idealist. The additional claim from constitutive theorists is that there are aspects of our experience that are due to the perceiving subject, i.e. that sensory experience is not *exclusively* brought about by the things we experience.

We could see this as in breach of the received view on scientific realism as there is a mind-dependent contribution here. However, if we allow a distinction between "mind independent knowledge of mind-independent reality" on one side and "mind-dependent knowledge of mind-independent reality" on the other, it seems the views of Einstein, Kepler, Galileo and Kant can be included in the received view of scientific realism. All we have to accept for this reconciliation is that we gain knowledge of reality in a particular way (through mind-dependent constitutive elements), but that this knowledge is about the mind-independent world. What "mind-independent reality" refers to remains unclear and we shall see that there are some issues with the mind-dependent/independent distinction. First, I shall present Kant's approach to knowledge acquisition in general as containing a priori elements. When Kant's account of general knowledge acquisition is established, I will move on to the more specific a priori prerequisites for physics. Altogether, the aim is to establish what the Kantian position is and why it is preferable to the other constitutive theories.

II 2. Kantian epistemology

The metaphysical framework I wish to defend is Kant's transcendental philosophy as a dynamic approach to metaphysics. We can conceive of Kant's philosophy of science as a continuous restatement of the following question; *how is that possible*? Or; *what are the conditions of* *possibility for x*? What are the conditions of possibility for natural science, mathematics etc. In order to answer these questions, Kant asks the more basic question of how knowledge is possible at all. Such a question requires an analysis of experience.

We start out with everyday experience. This is the reality in which we live and of which we have what Russell (1912: Ch. 5) calls knowledge by acquaintance. In order to understand the phenomena, we can analyze it by abstracting away parts. The first abstraction we can make is to separate the *perceiving subject* from *anything external* to it. We now have two basic components: The perceiving subject on one side and the perceived on the other.

$$\begin{tikzcd} & \textbf{Experience} \arrow[dl] \arrow[dr] & \\ \textbf{Object} & & \textbf{Subject} \end{tikzcd}$$

The perceived we can further analyze by abstracting it into objects that we can analyze in terms of forces that we can analyze in terms of positions and directions of motion. Hence, we must ask what objects there are and how these objects interact (mechanics), the forces that make interactions possible (dynamics), and the motions in and between the objects (phoronomy). The specifics involved we must try to understand through empirical and theoretical investigation. We cannot simply decide how objects interact. Neither can we empirically decide whether this separation is the correct one. Rather, the standard separation into mechanics, dynamics, and kinematics or phoronomy is itself a constitutive element for physics. Furthermore, in a transcendental investigation we must ask whether there are a priori constitutive elements within each part of the separation. If we find such elements, we must accept a contribution from the subject throughout physics. By analyzing pure motion (phoronomy), interactions between forces (dynamics), interactions between objects (mechanics), and finally the interactions between these and the perceiving subject (phenomenology), Kant aims to give a complete and systematic account of how scientific knowledge is possible. A central idea being that we gradually abstract our way from phenomenal experience down to pure motion and keep any a priori elements we find on the way. When the a priori elements are collected, we can investigate the specifics in the world empirically.

II 2.1. Phenomenology or experience in general

The Keplerian philosophy of science that I am proposing dictates that there must be a continuation between our everyday experience and our scientific knowledge. Importantly, as Kepler emphasizes in his example of the man walking along the hedge and thinking distant mountains are moving, the idea is that there are constitutive elements to our everyday experience. These are the ones we must preserve in our science in order to guarantee a continuity. This notion is common to Kepler, Kant and Galileo and it dictates that our science must be able to account for our everyday experience.

Famously, Kant's most basic a priori elements of everyday experience are time and space as pure intuitions (Kant: 1781: A19-50) along with the categories of understanding (Kant: 1781: A68-260). Here I shall exclusively deal with space and time as pure intuitions of sensibility, and ignore the Kantian categories (for a discussion on the relationship between pure intuition and the categories in Kant, see Andersen: 2011, Ch. 3-5). My primary source for discussing what I take to be the Kantian position in the philosophy of science is Kant's *Metaphysical Foundations of Natural Science* (1786). As I am developing my position here, and there are more interpretations of Kant than there are Kantians, I wish to emphasize that where my position departs from the Kantian one I will happily accept it as novel.

As pointed out above, the central aspect of Kant's transcendental philosophy is that in all experience there is a contribution from the subject. This is a *metaphysic's first* approach[21] in which the possibility of formulating an ontology is constrained by possible interactions between subject and object. Some a priori knowledge is neither theory nor culture specific, but rather constitutive of the human condition as such. A supposition of the dynamic view is that there are *no passive material existences*. Material entities, be they forces or objects, only exist in as far as they act. This implies that all properties to some extent are relational, since action is always interaction. This applies also to human beings as perceiving subjects and thus implies that we take an active part in our experience. Thus, *everything acts, always!*

The relation between object and subject within epistemology is a relation between the perceiver and the perceived. Traditionally one has treated this relation roughly in two ways: The object acts

[21] For an argument against the naturalistic approach, see Andersen and Arenhart: 2016

on the passive subject (strict empiricism) or; the subject acts on the passive object (strict idealism). Both approaches imply that the relation is that of agent and patient. The Kantian view is that the object and subject both act and that the resultant phenomenon is a mutual manifestation of the powers of subject and object. Neither subject nor object alone is sufficient for this result.

In order to establish the conditions of possibility for experience in general, we must therefore also investigate what the subject brings to the table. In Kantian terms, we need to investigate systematically the conditions of possibility for the type of experience that human beings have. This is something that each subject must do for him/her-self.

What such an analysis uncovers is that all outer experience (of the material world) is spatiotemporally ordered, and all inner experience (thought) has a temporal order. As we are unable even to imagine an external entity in a non-spatiotemporal way, we have no justifiable grounds to claim that the spatiotemporal order is a property in and of the world itself (as mind-independent). By further analyzing our concepts and experience, we find that spatiotemporality underlies all human construction of objects also in the understanding (See Andersen, 2011 Ch. 3-4). This leaves us with the position that space and time are powers and constraints of the subject that cannot be applied justifiably as properties of external objects, nor as objects in themselves. This view distinguishes itself from the Newtonian view where absolute space is an active existence in and of itself (Newton, 1686: 412-13). In transcendental philosophy, space and time are subject dependent constitutive frames of any physical intuition whatsoever (Kant, 1781/7: A19-50/B33-74). How, then, are we to understand that space and time are not properties of the material world but rather of the experiencing subject?

As subjects, we are part of the world and not external to it. Although it seems an obvious point, we easily get confused on this issue. We wish to understand how things work and we wish to do so in a manner that is objective rather than subjective. It seems therefore that we would do well in excluding the subject altogether. However, it is oxymoronic to claim that we have experiences of which we are not part. Our interactions lay the ground for our knowledge. By analyzing these interactions and abstracting away the subject, we can presumably end up with an image of the world as unfiltered by us. This is problematic for two basic reasons.

Primarily the problem is that we have no references to an interaction free of subjects that is not purely theoretical. The world 'in itself', is not a possible object of investigation. Every time we try to imagine or think determinately about the world in itself, e.g. as in the big bang theory or similar, we import our spatial and temporal intuitions and imaginings into it. When we only think about the world in itself negatively, we end up with thinking nothing specific, only the negation of all the determinations awe apply within our intuitions of space and time. The second problem is the origin of the first. We tend, when imagining the world, to construct temporal and ontological priorities. We think of the world for instance as first empty space, then we add objects and forces, then we add time in order to enable change, and lastly we add the human perceiver. We could think therefore that a theory eliminating the human perceiver is more objective. This is the illusion of a theory of a primary existence of the world independent of us. What one neglects in this illusion is that the entire set-up is imaginary and therefore that it is wholly dependent on the subject's way of formulating theories.

Hence, although the theory of evolution successfully explains the origin of the human species, the theory itself is constrained by the intuition of this species. "The world" must refer to the mutual manifestation of powers of subject and object, as indicated above. Neglecting the subject makes the subject's participation stronger and turns empiricism into idealism where one presents constructs of pure imagination (noumena) as mind-independent reality.

On the other hand, a pure analysis of concepts is equally unhelpful, as this only informs us of how we think. Much criticism of the Kantian position is written under the pretext that Kant's analysis is an analysis of concepts (See for instance Maudlin 2007: 78, and Russell, 1912: 61). However, Kant describes space and time as pure forms of sensible intuition and as in contrast for instance to the categories of the understanding. He explicitly differentiates this from any analysis of concepts (1871: A25/B39A32/B47). As pure forms of sensibility (inner and outer), Kant aims to describe the form of our interactions in the world, not only our thinking about it (See also Friedman, 2012: 254-255).

The basic aspects of our activity as perceiving subjects are thus our temporal and spatial organization of the world. Whatever we do, we do it in time and space. The notion that space and time are primary or constitutive for our everyday experience appears contrary to Einstein's notion that the primary concept of experience is the material object. Einstein's motivation for maintaining the primacy of the material object might be that his description of constitutive elements restricts itself to physics. Although Einstein maintains that science is a refinement of everyday experience, he does not give an account of such experience. Still, the idea proposed by Einstein that the material object is primary and that we can build a concept of a space from it, is highly problematic. Our notion of a material object involves the notion of borders, i.e. the object is present somewhere and absent everywhere else. Furthermore, if the material object is distinct from a mathematical point, it must be extended. Therefore, in order to conceive of such a thing as a material object, one must already presuppose spatiality; position/borders and extension are after all spatial notions. We can therefore establish the prima**c**y of space and time in our everyday experience.

A further problem with Einstein's object-based theory is that it places an undue asymmetry between change and stability. However, stability requires as much explanation as change. The breaking of my coffee cup is no more mysterious than its continued preservation.

In the phenomenological part, where we seek conditions of possibility for experience as such, we have found two a priori elements, time and space. As these constitute experience as such, we cannot contribute them to any specific object of experience but must take them as contributions from the perceiving subject in any (human) interaction whatsoever. Furthermore, we have stipulated that it is not possible to experience any fully inactive object. From this, we can infer that we can conceive of neither space nor time as objects of experience. They are thus not material, as matter is that which we can experience.

II 2.2. Mechanics or the relations bet ween objects

The central term relating to physics in general is "matter" as physics investigates the material world or outer experience. We have seen that conceived as passive and pertaining to the perceiving subject, time and space cannot be material or objects of experience. However, there are multiple ways to conceive of matter. Kant defines matter *mechanically*

> Matter is the movable insofar as it, as such a thing, has moving force (Kant 1786: 75).

Centrally, Kant's definition of mechanics here, i.e. as an investigation into the moving forces of matter follows from the above view on experience as such. Our investigation into matter as an object of external experience presupposes that matter must have the capacity to move us. I.e. unless we allow matter a moving force, understood as the power to move some something, matter cannot influence us and thus cannot be experienced. Material objects therefore are only meaningful determinants as far as they can act and the central action under investigation is the act of moving something else.

Following his definition of matter, as viewed mechanically, Kant states three mechanical laws of motion, two of which for our purposes are similar to Newton's laws.[22] However, Kant's definition of inertia (his second law of mechanics) differs somewhat from Newton's in a way that is of interest in our discussion here.

> *Second Law of Mechanics*. Every change in matter has an external cause.[23] (Kant 1786: 82)

Recall that this is true for a *mechanical* definition of matter, and not proposed as the only definition there is. We shall see in the dynamics that matter has internal forces that can produce change. Nevertheless, in mechanics, the topic of discussion is matter as "the movable", i.e. mechanics is primarily concerned with what we may call objects or entities and the relations between them. If we consider experience as an interaction between the perceived object and the perceiving subject, we see how the principle of inertia makes it impossible to interact with an inertial system. Thus, if we are wholly empiricist in our physics, we should exclude inertial systems from it. Kant's definition thus strengthens Stump's (2015) claim that one cannot empirically discover an inertial system even in principle.

Kant's description of inertia gives a possibility for a further insight in relation to inertial systems, which is that we cannot discover an inertial system *from the outside*. The mechanical investigation of matter is an investigation into how systems, objects or entities influence each other through moving powers or forces. Mechanical inertial systems are intrinsically stable; the law of inertia reflects this stability. Note however, what the law does not state. For instance, it is possible that

[22] For an explication of the differences between Kant's and Newton's mechanics, see Watkins, E., 1997, "The Laws of Motion from Newton to Kant," *Perspectives on Science*, 5: 311–348.
[23] The Newtonian definition then immediately follows in Kant's text.

the system could change due to internal processes. However, in the perspective of mechanics, such internal processes are the interactions among smaller systems that are themselves mechanical. The focus of mechanics is the system as a whole or an entity, and the law of inertia tells us that such a system does not change *mechanically* unless it is subject to an external force.

The mechanical perspective takes as its a priori grounding that there are objects that persist through time and that these have the powers to move each other. We can thus identify five central terms that are constitutive for mechanics, Objects, Time, Space, Power/Force, and Motion. By application of these constitutive terms, we may establish mechanical laws of motion a priori. As we have seen, there is a transcendental grounding of the law of inertia within the Kantian mechanics. Here, matter is that which can be the object of outer experience and a system that nothing is acting upon, also does not act, and thus cannot be an object of experience.

The first law of mechanics states

> In all changes of corporeal nature, the total quantity of matter remains the same - neither increased or diminished (Kant 1786: 80)

If matter disappears, it must go somewhere. However, as we are discussing matter as such, there is nowhere for it to go. The disappearance of matter would therefore require that we go from something to nothing, which breaks with the basic notions of mechanics as the interactions among substances. The appearance of new matter has a similar problem and we must conclude that the quantity of matter is constant.

The third law of mechanics states

> In all communication of motion, action and reaction are always equal to one another (Ibid: 84)

The third law coheres in particular well with the phoronomic principle of relativity we shall establish below. Kant thus establishes his three laws of mechanics a priori and thus, for any investigation into how specific types or systems of matter interact, these three laws are constitutive.

II 2.3. Dynamics, or what powers there are

Kant defines matter *dynamically*

> Matter is the movable insofar as it fills a space. To fill a space is to resist every movable that strives through its motion to penetrate into a certain space. A space that is not filled is an empty space. (Ibid: 33)

Where the mechanical description of matter treats matter as separate and interacting substances, the dynamical description of matter treats matter in terms of acting forces. The first activity in this description is the active filling of a space. In mechanics, the spatial extension of a substance is given. In dynamics however, such an extension requires explanation. What then, is required for matter to fill a space? Kant's answer to this question is that matter must have a repulsive force. An attractive force counteracts this repulsive force. Dynamically then, an object is a persisting equilibrium between repulsive and attractive forces.

In dynamics, objecthood is a predicate of a particularly durable compound of interacting forces. Furthermore, as objects interact they exert forces on each other in such a way that they avoid annihilation. The interactions through which an object remains essentially intact defines it as an object of a particular kind.

This definition is preferable to the idea of an object in itself or an object in 'isolation' as these latter definitions assume that an object would be its identical self across all contexts, and therefore that we can apply a fundamental atomism in the sense that there are basic unchanging entities or physical substances. Such a view implies the ontological dualism between passively existing entities and the forces or powers that make them act. The dynamical view is that *objects are enduring manifestations of continually interacting powers*. In his posthumous publication, Kant argues that there can be no empty space and that the dynamical powers are seated in what he calls an ether. As we shall see in part four, the physics of Maxwell and Lorentz mirrors this idea.

What we need to ask here, is why objects must be thought of in terms of forces and why those forces must be repulsive and attractive. In order to answer this question we must recall that the mechanical description of matter gives an understanding of objects as acting on each other. How do they act on each other? Consider Hume's billiard ball example. Ball A bangs into ball B and ball B moves. Ignoring the Humean skepticism for now, we can view this as a case of causation. Mechanics tells us that the cause of B's motion is the force exerted on it by ball A. That force must be repulsive. Furthermore, the equal forces exerted in the opposite direction must also be repulsive, or ball A would simply have rolled through ball B. We can therefore know that objects act on each other through repulsion. However, an object's repulsive force cannot appear out of nowhere but must - in this case - be a property of the balls prior to the interaction. We can thus know that objects have repulsive forces (or are made up by them). Now, a repulsive force will expand unless it is counteracted. Thus, objects must also have an attractive force that counters the repulsive force.

So far, we have seen that there are five central elements making up our notions of matter as an object of experience. These are Time, Space, Motion, Power, and Object. Furthermore, we have seen that time and space are properties of the perceiving subject, that objects act on each other through their powers and that these powers make up the objects' filling of space. We have also established that there are Mechanical laws grounded in dynamical forces. What we have not looked at, is motion.

As motion will be a central issue in our further discussion, I shall spend some time here on the relevant details. The study of motion, as motion, is phoronomy and constitutes the final or primal part of physics, depending on whether we approach its metaphysics regressively (through analysis of experience and successive abstractions) or progressively (through synthesis of its levels of constitutive principles).

II 2.4. Phoronomy or pure motion

Phoronomy is the study of motion, qua motion, and we have arrived at this study through a series of abstractions. We abstract away the forces that act between entities (mechanics) and the forces inherent to each entity (dynamics). We are then left with moving geometrical figures. By taking a three-dimensional figure such as a cube, we can abstract our way through two-dimensional planes and one-dimensional lines, and finally arrive at dimensionless points. These points do not have the quality of extension, but simply determine a position in a space. A benefit of discussing motion in terms of points is that we do not need to worry about any possible deformation, which would imply dynamical or mechanical influences. Furthermore, these points are prior to any established system of coordinates or rest frames.

II 2.4.1. Absolute and relative space

Kant defines matter *phoronomically*

> Matter is the *movable* in space. That space which is itself movable is called material, or also *relative space*; that in which all motion must finally be thought (and which is therefore itself absolutely immovable) is called pure, or also *absolute space*. (Kant 1786: 15)

Immediately, we see a distinction between (material) relative space, and (immaterial and immovable) absolute space. This distinction is central to Kant's phoronomy and it comes with a clarification that is central to our discussion of physical "space-time" theories in part IV. *Absolute space* in Kantian metaphysics is an overall frame introduced by the subject. Recall that Einstein (1936) proposed that we could build space from the relations between objects. Our counterargument was that for us to derive these spatial relations, the objects must already be positioned spatially.

Consider for instance a single cube. For me to construct this cube in the imagination, I must construct its external borders. However, with the construction of a border I simultaneously construct a distinction between an inside and an outside. The inside of the border is where the cube is, and the outside is where the cube is not. What then, is this outside? The outside here is the space *introduced by me* in order to construct the cube. The inside is a *limitation* of this space. In other words, we do not construct figures from points, but rather from limitations of a space.

Consider further that there is another cube moving in the overall space, we have introduced. The borders and the inside of that moving cube are also moving in the overall space. However, it is perfectly possible to imagine that it is the first cube that is moving and the second that is at rest in the overall space. In order for us to imagine this, we must switch absolute spaces!

A first insight we can get from this is that in all representations of motion, we must introduce an absolute space, where absolute simply means immovable. We can represent the same motions in multiple ways by treating different spaces as immovable. We cannot, however, imagine or represent motions in which all spaces are moving. If we try, we will always find that our representation is set in another immovable and thus absolute space. Galileo's insight in relation to the principle of relativity was that we tend to "forget" that we typically treat ourselves as being at rest and thus representing the absolute frame. Kant's distinction between relative and absolute space is different from that of Newton's Scholium (1687: 408-416), where absolute space is a unique physical entity. Kant's absolute space is simply *that frame which is picked out as immovable for a particular representation of motion*. In addition to there being at least one such frame, there is the limitation that there can be no more than one such frame in a single representation. In summation, for every representation of motion we introduce one, and only one, immovable space. In phoronomy, this space is referred to as absolute. Carrirer (1992) aptly calls Kant's theory of space a "relational theory of absolute space".

II 2.4.2. The phoronomic principle of relativity

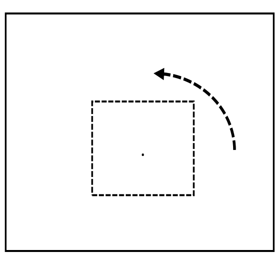
Motion is the change of position in a space and we can distinguish between two general forms. Absolute motion is motion relative to an immovable space; relative motion is motion relative to some other point or set of points in motion (considered as demarcating relative spaces). We can further distinguish between motions that are along the same dimension (rectilinear), motions that alter their direction (curvilinear), motions that are curvilinear and return to their original position (rotation and circular motion) and finally rectilinear motions that return on themselves (oscillations). These are the basic directional determinations of motion and they can of course combine in innumerable ways. A further determination of motions is possible when considering the magnitudes of motion in a single point and over an area. We can then distinguish between motion with varying speeds (acceleration/deceleration) and motion with uniform speed (uniform motion). We shall first consider motion only through the lens of its directional aspect.

A point is at rest if it stays in the same position over time. Being at rest can signify either having no motion, or having multiple motions in opposite directions that cancel each other out. Rest, as the result of multiple opposing motions of a single point, is at the heart of the principle of relativity. In phoronomy, we can apply Kant's definition of this principle.

> Every motion, as an object of possible experience, can be viewed arbitrarily as motion of the [point] in a space at rest, or else as rest of the [point], and, instead, as motion of the space in the opposite direction with the same speed (Kant 1768: 23)

As in physics, the central aspect of the principle of relativity in phoronomy is that there is no detectable or calculable difference between being at rest and moving. The principle of relativity can be applied universally in phoronomy, hence Kant's designation of "every motion".

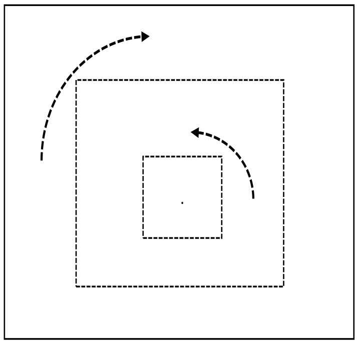
However, it is not immediately apparent that we can meaningfully think of the rotational motion of a single point. Typically, we think of rotation in terms of a one or two-dimensional figure.[24] Any point *on* that figure changes its position in space, but the figure itself remains at rest in the same position. In order, therefore, to illustrate the rotational motion of a point, we can introduce a limitation of space, for instance in terms of a figure, and attach our point to it. If we now imagine this new space as rotating, we get an illustration of the rotating point. However, a rotating space must rotate relative to something, so we introduce a new *absolute* space, to which the rotation is related.



In order to satisfy the principle of relativity, we must be able to construct the same state of affairs but with the point at rest. This we can do through the introduction of a further space encapsulating the former two. By applying to our former absolute space a rotation in the opposite direction, and

[24] Three-dimensional illustrations are of course more realistic, but also introduce expositional complications. For pragmatic reasons I therefore choose two-dimensional planes as illustrations of motion.

imagining the point, the first rotating space, and the second (opposite) rotating space all in a new absolute space, we can combine the motions and end up with our point being at rest.



A less abstract illustration is the motion of a person on a treadmill. The backward motion of the belt cancels out the forward motion of the person running, and the total state of motion relative to the room, is rest. If we wish to illustrate all these motions as relative motions, we simply insert a set of points for every new space and pick out one such set as the “rest system”. In that case, we illustrate the rotation of a single point through the circular motion of a further point around it, and this circular motion as relative to the “rest system” (pick a point on the person, on the belt and in the room).

So far, we have explicitly dealt only with one element of motion, its directionality.[25] However, in order fully to understand the principle of relativity, we must also include magnitudes or quanta of motion. Indeed, there is no possible imagination of any motion whatsoever without a certain speed. What we did in the example above, was simply to leave the determination of a speed to the reader. In this sense, we can talk of the directional aspect of motion as somewhat dubious in that it is a necessary aspect of any motion, but cannot stand on its own. If we think only in terms of direction in a moment, we can only arrive at snapshots of points in particular positions, not motion. This is, however, perfectly familiar to us as it is the exact same relations in which the centre, circumference, radius and area of a circle stand. A similar apparent dubiousness applies to the notion of speed.

[25] We have presupposed implicitly that the clockwise and the counter clockwise motions have the same magnitude.

Numerically, we determine speed as the proportion "distance over time" (S=D/T). However, if we wish to discuss speed as such, we cannot include the direction of motion of the point, as that would give us the velocity rather than the speed. We must therefore think of speed, as the scalar (undirected) quantity of motion, differently from its purely numerical determination. For if we have no direction of motion, we also do not have the possibility of traversing any distance. If we have no distance, but time flows, we simply have rest in the sense that the same point perseveres in the same position over time. In order to understand speed as the undirected magnitude of motion therefore, we must understand it as a magnitude in a non-extended spatial and temporal point, i.e. as an intensive magnitude in a moment or "now".

It is not obvious how we should think of speed as a magnitude that is present in every such moment. For instance, it might appear that we are accepting temporal points as some kind of minimal temporal units; which runs counter to the Kantian notion of space as a continuum.[26] This, however, is not the case. The central and only reason to think of speed as in a non-extended temporal and spatial point is to show that *the magnitude of motion is completely independent of its duration*, i.e. that it is an intensive and not an extensive magnitude. This magnitude does not directly have any corresponding experience.

What we experience, and what we can imagine, are velocities; directed motions. What we have done is to abstract from these actual motions two aspects that are necessary but nonetheless devoid of any possible separate illustration. We simply cannot think of a non-directed speed or a "speed-less" directed motion. The proper modal determination of both speed and directionality of motion therefore, is that of potentials rather than actuals. Speed is the magnitude of potential motion in any direction whatsoever. Directionality is the potential for motion of any magnitude. Any *actual* motion has both direction and speed, and must be determined as a velocity. Thus, it becomes apparent how motion itself is understandable through the constitutive elements speed and direction. There is no logically necessary connection between these elements. Therefore, "motion" is a synthetic construct. Moreover, the components that make up the construct cannot be investigated empirically in separation. They cannot even be imagined as empirically detectible in

[26] Since we consider space as a continuum in all theories relevant to the main discussion of this thesis, I will take the continuum thesis as given.

separation. We thus have a clear instance of a synthetic a priori element, “motion” which grounds our entire investigation of outer experience.

In determining velocities, we always do so in relation to an absolute (rest) system. We must establish such a system (or multitudes of such systems) in order to determine the direction of motion relative to something. In our everyday lives, we may simply use ourselves as the rest system and determine motions as up/down, backward/forward, and left/right. In phoronomy or kinematics, we typically represent these determinations by the axes of a Cartesian coordinate system. The numerical value of a velocity therefore is $V=\frac{\sqrt{dx^2+dy^2+dz^2}}{dt}$

Our definition of a velocity is the displacement of a point in a single or multiple directions with a speed. The speed of rotational motion is somewhat difficult to define. On the one hand, we have the circular motion of a given point around the rotating point. This is an uncomplicated speed to calculate from the length of the circumference of the circle drawn in the circular motion. As the circle grows larger, this motion approximates a rectilinear motion. However, the rotation of the single point is not derivable as a change of position from this circular motion. Indeed, one can pick any circling point and the speed of that point will depend on its distance from the original rotating point. Therefore, we define the quantity of rotation not as a speed but as an amount of rotations during time (rotations/time unit). This we can determine by letting the point be at rest in a larger rotating space.

So far, we have seen that motion is the displacement of a point in a space and that velocity is the combination of the direction and speed of that motion. In the following, we shall see how velocities can combine in such a way that we retain both the initial meaning of our term “motion” and the phoronomic principle of relativity.

Kant (1786) sets forth three examples of combined motions of a single point. These are; the combination of two equal magnitudes of motion of the same point in the same direction, the combination of two equal magnitudes of motion of the same point in opposite directions along a single dimension, and the combination of two magnitudes of motion along different spatial directions. We shall look at them in the context of the phoronomic principle of relativity.

Take a point P moving at a given speed from the position A to the position B

$A \longrightarrow B$

If I wish to combine this motion with an equal motion in the same direction, i.e., doubling the speed of the point, how can I do this? A first approximation is to add space to the situation and illustrate it as

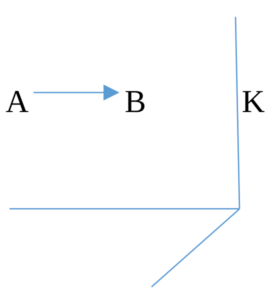
$A \longrightarrow B \longrightarrow C$

By keeping the time variable constant, we increase the speed of the point by adding distance. This, however, is not really a viable solution. Our mission was to double the speed of the point in the same motion, not to show the point as moving at double the speed in a *different* motion. The above solution is of the latter kind, where we have exchanged the initial motion AB with the motion AC (through B). In order to double the magnitude of the motion AB, we must follow the procedure we have used for rotational motions above.

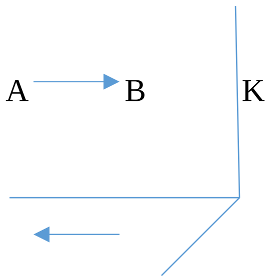
First, we determine the motion AB with a given magnitude in a given amount of time.

$A \longrightarrow B$

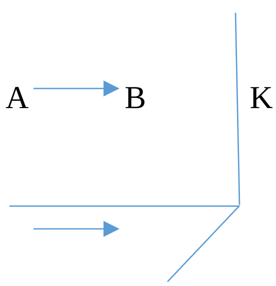
We then imagine this motion as relative to a space K



Finally, we apply an equal, but now opposite motion, to the space K, in relation to the sheet of paper.



Our result is that the same point is moved in the same space (between A and B, now understood to move with K against the space defined by the sheet of paper) at the same time, with double the magnitude of speed. Thus, we have accomplished what we set out to do. The main difference between this (real) solution and the above (attempted) solution, where we simply added distance to the motion, is that we now used and satisfied the principle of relativity. We can illustrate the *same* motion of the *same* point in the *same* space over the *same* amount of time as rest, by simply changing how, in our imagination, we see the direction of motion of the system K. [27]



In this illustration, our point is at rest in the system K, while we imagine the sheet of paper as moving in the opposite direction with the very same magnitude of speed we understood the point

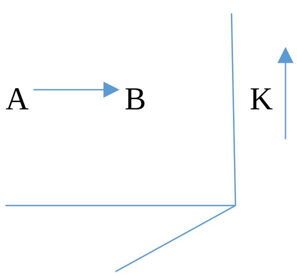
[27] The notion of "measuring motions" is thus clarified in an important sense. Motion, as a synthesis of directionality and speed, can only be imagined measurable if measured through a perspective (absolute space). Thus, in every act of measurement, we pick out an absolute space or "rest system" within which we combine different motions. The phoronomic principle of relativity grounds this understanding by showing how any motion whatsoever can be represented as any other motion whatsoever by a replacement of the absolute space. Thus, any measurement or imagination of a motion comes with a simultaneous determination of an absolute space that determines the perspective of the measurement.

to have relative to the sheet of paper. This is the solution to the second of Kant’s (1786) examples, the combination of two equal magnitudes of motion of the same point in opposite directions.

Kant’s third example, the combination of two magnitudes of motion along different spatial dimensions, is resolved in the same way but now with a vertical motion applied to the space K. First, we establish the motion AB of the point relative to the sheet of paper.

A → B

Then we establish a system K moving vertically with the same magnitude relative to A, B, and therefore also the sheet of paper.



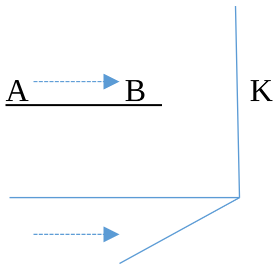
We can thus imagine our point moving diagonally downward in the system K.

The aim of these constructions is to show that our understanding of motion as an intensive directed magnitude allows us not only to represent any moving point as being at rest (thus satisfying the phoronomic principle of relativity), but also that any point in any motion can be represented as that same point in any motion whatsoever.

We have so far implicitly only dealt with uniform motion, but there is no problem involved in including acceleration in our treatment. The only important element that needs treatment in acceleration is what we may call “timing”. Take the point moving from A to B.

A → B

If we assume the point is moving with uniformly accelerated motion and we wish to represent it as being at rest, we apply an equally uniformly accelerated motion to our system K



However, if our point is accelerating unevenly, we must make sure to time the accelerations of K in such a way that the speed of K corresponds to the speed of the point at any single instant. Again, this points to the intensive character of motion.

So far, we have focused on how speed (as an intensive magnitude of motion) can combine with other speeds in order to alter the motion of a specific point at a particular time within a particular space. The last point I wish to make concerning speed as an intensive magnitude is that we must understand speed itself along the same lines. In other words, that we can construct any given speed from an arbitrary set of simultaneous speeds. As an intensive magnitude, we can construe a particular speed through either the successive addition of smaller speeds, the successive subtraction from larger speeds or any arbitrary combination of simultaneous speeds. Similarly, and actually simultaneously, we can construe a particular velocity (as directed speed) through the successive synthesis of smaller velocities. We see this by simply running our previous arguments backward. So, a point in diagonal motion dissolves into two one-directional motions of that same point, a point at rest dissolves into two equal and opposite motions, and any magnitude of motion whatsoever dissolves into an arbitrarily large amount of arbitrarily slow motions in the same direction. Speed, therefore, comes in instantaneous degrees that we can construe from an arbitrarily chosen combination of other instantaneous degrees.

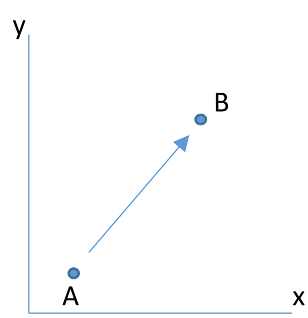
Speed in and of itself is illustrated as a scalar, i.e. as the length of an arrow whose directedness is insignificant. The velocity of a point is the combination of its speed and direction in a vector. For a more comprehensive understanding of the principle of relativity, we shall see how the principle reveals an internal mathematical symmetry between geometry and arithmetic.

Consider a point P moving with velocity v toward the right on the sheet of paper.

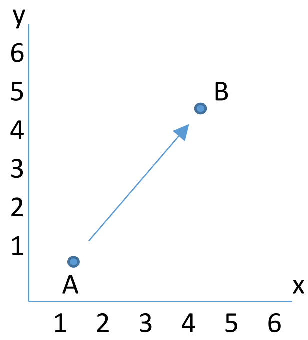
If we wish to illustrate this motion whilst maintaining that the point is at rest, we consider the sheet of paper as moving with an equal magnitude of motion in the opposite direction. As seen from a rest system, R, in which the sheet of paper is moving, the point is now at rest while the sheet of paper is moving.

The motion of the point with respect to the rest system R, we then represent as rest by simple subtraction $v_1 - v_1$. In order to treat our velocities in arithmetic in this way, we must assign each motion an identifiable directionality. We do this by constructing Cartesian coordinate systems. These Cartesian systems must agree on directionality in the sense that for any two coordinate systems K and K', we draw the x-axes along the same dimension. In other words, all systems must share directionality in a single space. Whether this single space is the relational "rest system" we pick out, or the absolute space we insert, it needs to be a single and overall space. Without it we can derive no spatial relations whatsoever since unless they are in the same space, the different systems are simply not spatially related.

Along the axes of a Cartesian coordinate system, we can establish coordinates for positions in a space. We do this by laying down units of length along each axis. The total coordinate (x_n, y_n, z_n) determines the position of the object in that space. We can see a direct relation between phoronomy, geometry, and arithmetic through this procedure. Take the motion of a point P from position A to position B in the following Cartesian coordinate system.



If we introduce unit lengths along the axes, we get numerical values for each position.

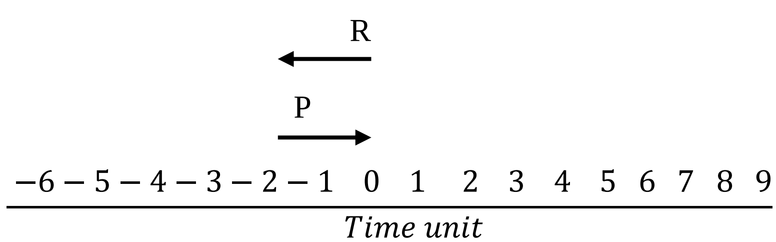


$A=(x_1, y_1)$, $B=(x_2, y_2)$

This again, allows us to calculate the amounts of distances moved in each direction $dx = x_2 - x_1$, $dy = y_2 - y_1$. The total motion of the point we then represent by combining the motions along the axes. In our case, the velocity of the moving point = $\frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt}$, = $\frac{\sqrt{3^2 + 4^2}}{dt}$, = 5/dt.

The basic ideas we are treating here are the ones Galileo applied when he first established the principle of relativity.

We can draw on arithmetic also when adding velocities. Think of the motion of a point in a space as the motion of that same point along the line of natural numbers, where each number represents a speed in a particular direction.



We see that the initial and end positions of the point is the same whether it is moved in the minus direction or the plus direction with equal magnitude.

We can now use simple arithmetic rules of addition in order to combine different motions in such a way that we can apply or exemplify the principle of relativity. Cartesian coordinates allow us to construct alternative "geometries" in the first place, [28] and we see that they also enable us to understand velocity addition in a simple way. It is worth noting, however, that neither the principle of relativity nor the velocity addition, on which it builds, depends on Cartesian coordinate systems. We could do just as well by simply adding vectors (scalars with direction). Nonetheless, Cartesian coordinate systems are helpful in handling these relations.

II. 3. A note on velocities

As we develop the argument in part III and IV, the notion of velocity will be of a central importance. What we must maintain throughout, and have established a priori here, is that the notion of velocity contains the notion of velocity addition. We can illustrate this containment either by reference to scalars and vectors for speed and velocity respectively, or by reference to their numerical determinations, or by reference to our direct experience.

From the latter perspective, we can start from the idea of motion. As we have seen, motion is simply the displacement of something in space. However, for something to move from one place to another, it *must* move with some magnitude, as "motion without magnitude" is simply rest. Thus, we need magnitude of motion in order to distinguish between rest and motion. Note that this consideration is separate from our previous considerations concerning how motion *can be represented* as at rest by inserting *some other motion*. In everyday life, we determine magnitudes of motion by terms like "fast" and "slow". The fastest runner, for instance, is the runner that can get from A to B before anyone else. Cars are faster than humans, and airplanes are faster than cars. The property of "being faster" is also a transitive notion in the sense that if airplanes are faster than cars, and cars are faster than humans, then airplanes are faster than humans. "Being faster" can be thought in two main ways. If, as with running competitions, we determine a particular distance, the one who covers the total distance in the least amount of time, is "being faster" than the competition. If, on the other hand, we determine a particular duration, or amount of time, the one who travels the longest distance during that amount of time, is "being faster" than the competition. If we want to discuss who is "faster" in general, the issue gets more complex as for instance a horse is faster than a human being over a long, but slower over a short, distance. Phoronomically,

[28] This issue is treated extensively in part III.

we ignore issues of fatigue and muscle composition and deal with "being fast" and "being slow" in the abstract. In order to do so, we establish the idea of a speed. This captures "being slow" and "being fast" in a single term where "being slow" equals having a small, and "being fast" equals having a large, magnitude of speed. As any other intensive magnitude, speed is an additive property, meaning that I can achieve any particular magnitude of speed by successively adding smaller speeds or subtracting from larger ones. We have already seen how this is central for establishing the phoronomic principle of relativity.[29] In order to illustrate or imagine these different degrees of speed as *actual*, we need to imagine them as directed i.e., as real motions or displacements in space. We therefore imagine that the speed is in a particular direction. We then get the concept of a velocity. Velocity thus contains within it the intensive character of speed. We can numerically determine velocity as the relation d/t, i.e. as distance or amount of displacement over time. Now we can ask; what is the difference between a fast and a slow moving object?

A fast moving object, as we said, is an object that covers either a given distance in a short amount of time, or a large distance in a given amount of time. Given our idea of velocity we therefore get either $\frac{constant}{small}$ or $\frac{large}{constant}$. In order to operationalize "large", "fast", "small" and "slow" numerically, we apply units for distance and duration. Take for instance a meter (m) for distance and a second (s) for duration. Treating these as our units of distance and duration, we can appeal to standard arithmetic and apply the set of real numbers. Then, $\frac{2m}{1s}$ is faster than $\frac{1m}{1s}$ or $\frac{2m}{2s}$ as the resultant 2m/s is larger than the resultants 1m/s and 1m/s. If we ask how much faster the first object is we see how the concept of a velocity relates to the addition rules of arithmetic. The question simply translates to, how much bigger is 2 than 1. The answer is of course that 2 is 1 bigger than 1, by the basic additive operation n+1. If n=1, then n+1=2. Consider any given velocity, for instance the constant velocity of light, which is 300 000 m/s. How do we determine this velocity mathematically? We start from rest, which is 0m/s and perform the additive operation. So

$$\frac{0+1=1, +1=2, +1=3, +1\ \dots\ 300\,000}{1s}$$

[29] In order to make this notion easier to imagine in everyday terms, consider speed testing a motorcycle on a dynamometer. What is being tested, is the potential displacement ability of the motorcycle while maintaining the actual motorcycle at rest.

We now see how the notion of any velocity whatsoever implies the additive operation in arithmetic and thus that velocity determination contains the classical rules for velocity addition.

In order to clarify, let us consider the difference between an inertial motion and an accelerated one. An inertial motion is a motion where the following is true:

> The average velocity of the motion is identical to the average motion of any segment of the motion. The speeds at any two arbitrarily chosen instants of the motion are identical.

For an accelerated motion, the above statement is untrue. In other words, an accelerated motion is a motion in which speed at different instants differ and the velocity through different segments differ from the average velocity. To bring this back to our everyday lives, we can consider a hundred meter sprint. A sprinter starts out at 0m/s and accelerates for the first 40 meters. Along this distance, the speeds at any instant i_n is lower than the speed at any subsequent instant i_{n+1}, meaning that as i_n increases, the relation Nm/s also increases. We capture this increase by simple addition rules $n+1 > n$. At around 40 meters, a sprinter reaches maximum velocity and moves inertially for the remaining 60 meters. For this segment it is therefore true that Nm/s at i_n = Nm/s at i_{n+1}. This is one way of imagining how an object can achieve a particular velocity. The object starts at rest and accelerates up to the given velocity. In order to understand the end velocity, we must apply standard rules of addition both for speeds and for velocities. Another way to conceive a particular velocity is by treating it as given. I.e., as if the object has the velocity intrinsically and not through acceleration. As we have seen, however, in order to determine the magnitude of that velocity, we work our way up additively in the exact same way. We can therefore see that it is universally true that for any velocity determination whatsoever, we apply standard velocity addition through vector addition and mathematical induction.[30] Thus, the notion of velocity contains the arithmetical operation of addition. If we wish to illustrate this with vectors, we simply add length to the vector as velocity increases.

[30] These must be understood as separate a priori principles. Mathematical induction as a general principle, and vector addition as a principle of phoronomy that incorporates mathematical induction.

II 4. Summary

In this part, I have presented what I take to be central considerations in Kant's metaphysics of natural science that relate to the debate over special relativity. I have followed the Keplerian approach by providing these metaphysical considerations prior to any specific physical theory or data. The main result from this approach is that we can now have metaphysical legs to stand on when we move onto the specifics.

I have chosen to take a regressive approach to Kant's *Metaphysical Foundations of Natural Science* rather than following Kant's original (progressive) set-up. I have done so in order to emphasize that we cannot start from the phoronomy as independent from experience but rather as abstracted from it.[31] Ultimately, human experience is the starting point of this metaphysics. We have seen that we can analyze experience into the contributions made by the perceiving subject and that which it perceives (phenomenology). By further abstraction, we can consider the perceived mechanically as substances that act on each other. The form of action involved is motion, and there are laws of mechanical motion established a priori. Furthermore, we have seen that in order for substances to act on each other they must have forces or powers of motion. By discussing these motions dynamically, i.e., as powers that make up the substances, we have found that there are two types of moving forces at play. The attractive forces, tending toward implosion, counteract the repulsive forces, tending toward expansion. A compound of attractive and repulsive forces that is in stable equilibrium over time constitutes a mechanical substance, or object. We can only understand the moving forces themselves in terms of pure motion, and we have found that ultimately, we must represent motion in an overall and immovable frame called absolute space. However, we have not discussed the characteristics of this space apart from our recognition of it as introduced by the thinking, imagining and perceiving subject. The characteristics of space is the topic of part III, where we shall discuss the relation between different kinds of geometries and their relation to space understood as pure intuition.

As we shall see, special relativity is constructed in an entirely different manner than what I am proposing here. Einstein initially suggests that special relativity is built from empirical experiments

[31] Note that we are abstracting from experience in general here, not from some specific empirical discovery. This distinction is important in order to separate what we are doing here, which is to reveal common constitutive elements of everyday experience and empirical discovery (i.e., constitutive elements of experience as such) from what for instance Stump (2015) suggests, which is to derive relations from empirical specific experiments.

and thus that the abstractions involved are abstractions from specific data. The first problem with this approach, which Einstein calls “practical geometry”, is that the mechanical principles developed here and in particular the principle of inertia, are postulated a priori rather than derived from empirical investigation in special relativity. As we shall see in part IV, the inertia principle is an a priori constitutive element of the theory. Furthermore, we get a confusion between the arithmetical expressions of measuring results and the properties of space (geometry) and motion (phoronomy) when we derive our “geometry” from specific experimental data. The history and debate around this confusion is the topic of the next part of this thesis.

Part III

Space and Geometry

In the first part of this thesis, we pointed to two main objections to the existence of universal a priori elements. The appearance of logically consistent and non-Euclidean "geometries", and space-time as derived from relativity theory. This part is dedicated to answering the first objection. My approach to such an answer is the following claim: *As far as mathematical models are models of space, they cohere with the Kantian theory*. The following is a justification of that claim.

III 1. Foundations of geometry

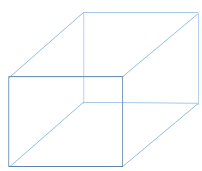
Following our transcendental approach, we analyzed the **phenomena** in terms of the perceiving subject on the one hand and the perceived on the other. This distinction constitutes the basic tenet of the transcendental phenomenology in which the spatial and temporal frames are set by the subject and are thus a priori. Furthermore, we saw that when considering the perceived as objects or substances, we could establish **mechanical** laws of motion a priori, provided the motion is conceived as the motion of matter. Matter being whatever can be an object of outer experience. The laws of motion and the mechanical view in general presupposes that there are forces acting among the objects. Viewed **dynamically**, these forces are what make up the objects. In **dynamics**, we analyzed forces in terms of powers of repulsion and attraction and thus as imploding or expanding motion. In **phoronomy**, we established the notions of speed and direction and saw that these constitute our concept of motion in a synthetic a priori manner. Throughout this gradual abstraction from the phenomena, we have presupposed a particular structure of space in all our two-dimensional illustrations and arguments. We have not built such a structure from more basic definitions but rather applied spatial structure as constitutive of the thought experimental approach to different aspects of physics. In the following, we shall ignore motion and focus on figures in space. Our aim is to establish a common spatial structure that grounds geometry along with the rest of physics. Following our general approach, we start by abstracting away from the phenomena. The main benefit of this procedure, i.e. by abstracting our way down from experience, is that we can know whether we have introduced content that was not there in the initial phenomenon. Weisberg (2007) calls such an abstraction “Galilean idealization” as it is a central aspect of how Galileo sets up his thought experiments as well as his analogies. The main purpose is that while we remove ourselves from the experience through successive phases of abstraction, we make sure we can always find our way back. What then, are the central aspects of the spatial part of our experience?

Geometrically considered, space is the frame for the construction of geometrical figures as constructs of possible objects of outer experience. As a condition of possibility for all human interaction, space itself cannot be an object. Primarily, space allows orientation. By this, I mean that it is through spatial relations that I can know where I am in relation to all other objects, where different parts of me are in relation to other parts as well as my spatial limits as a physical object. Such orientation requires three central characteristics of space.

Space must be horizontal in the sense that I can relate myself to any (real or imagined) object whatsoever. I thus arbitrarily set the “size” of space in any direction. Furthermore, space must be directional in the sense that it allows me to decide where to focus my attention. There are three, and only three, such directions: up/down, left/right, in front/behind. From these we can determine the three-dimensionality of space when constructing our geometry. Directionality also points toward my orientation in space as a set of limitations of the overall horizontal and absolute space. I can limit myself to considering only one or two directions at a single instance. In order to determine the objects I relate to, I also need such a limitation. An object is a limitation of space in the sense that it has borders, which determines the outer limits of the place and space where the object exists. When we wish to set a spatial stage by introducing an absolute space, we determine which direction this space extends into and how far. Thus, I can imagine absolute space as the room in which I sit, or the galaxy in which the earth moves. Depending on my choice of absolute spatial frame, I am either moving or not. Imagined geometrically, i.e. by abstracting away motion, these are simply different limitations of the same overall space. Any limit or border I set to the special extension is wholly dependent on my choice.

These are the most basic aspects of space in relation to our outer experience, and they are thus the framework for any geometry of space. The role of geometry is to model the relations and characteristics of space and the objects within it in the fullest possible way.

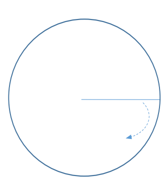
Phenomenologically, every object has a border, which defines where the object exists and where it does not exist. If we ignore the (mechanical) forces between objects, the (dynamical) forces inherent within the objects, and the (phoronomical) motion of the object itself, only a figure remains. **Geometry** is the study of figures and the space in which I represent them. A feature of figures as objects of outer experience is that they are three-dimensional. The three-dimensionality can be seen by applying Galileo's (1632: Day 1, 12-15) argument that there is at least three dimensions and that there are no more than three since adding any further "dimension" is simply a combination of the ones we already have. We can however abstract further. Consider for instance a cube:



By abstracting away the depth of the cube, we get a two-dimensional plane:

By further abstracting away the height of the plane, we get a one-dimensional segment:

By rotating the line segment until it reaches its original position, we can construct a plane circle:



By cutting and combining (elements of) these basic shapes, we can construct the spatial aspects of all geometrical figures. However, as figures with borders, these are all possible limitations of an overall space, which itself is not limited.

Consider the segment above. I can extend this segment arbitrarily far and by maintaining this arbitrariness, I construct a line. In other words, a segment is the limitation of a line. The line itself represents the limitation of a space into only one direction. For instance, I can imagine a line of arbitrary length up/down, left/right or in front/behind. A line is thus a limitation of the directionality of space, while a segment is the limitation of both directionality and horizon. A "straight line" is a line extending in only one direction.

This concludes our regressive argument from the phenomena through mechanics, dynamics, phoronomy and geometry. A central aspect of the approach is that we have always remained within (our immediate relationship to) the phenomena themselves and not introduced any external elements. If we can build geometry and physics on this foundation, we have a unified position from where we can explain our everyday experience as well as our physical theories. If successful, we will then be able to fulfill the coherence demands of classical scientific realism.

III 2. Geometry and physics

In the 19th and throughout the 20th Century, new geometries developed that are not Euclidean. One commonly thinks that this poses a problem for the Kantian position. For, in maintaining that the structure of space is independent of the behavior and relations between entities within space, how do we decide upon the structure of space? It is here that the Kantian position most clearly distinguishes itself. Newtonian mechanics provides an example. In Newtonian Mechanics, space acts on objects (in phenomena of rotation and acceleration), but is itself not acted upon. "…independent in its physical properties, having a physical effect, but not itself influenced by physical conditions" (Einstein, 1921: 59). This puts Newtonian space in an awkward position, as the fundamental entity (space) does not play by the rules of Newtonian Mechanics, more explicitly by the third law.[32] Space then becomes a mysterious entity that, while physically real, breaks the basic laws of physics. There are two distinct paths toward a solution to this problem. Either, space must be acted upon and thereby lose its quality of being absolute, or space is not physically real and does not act on objects at all.

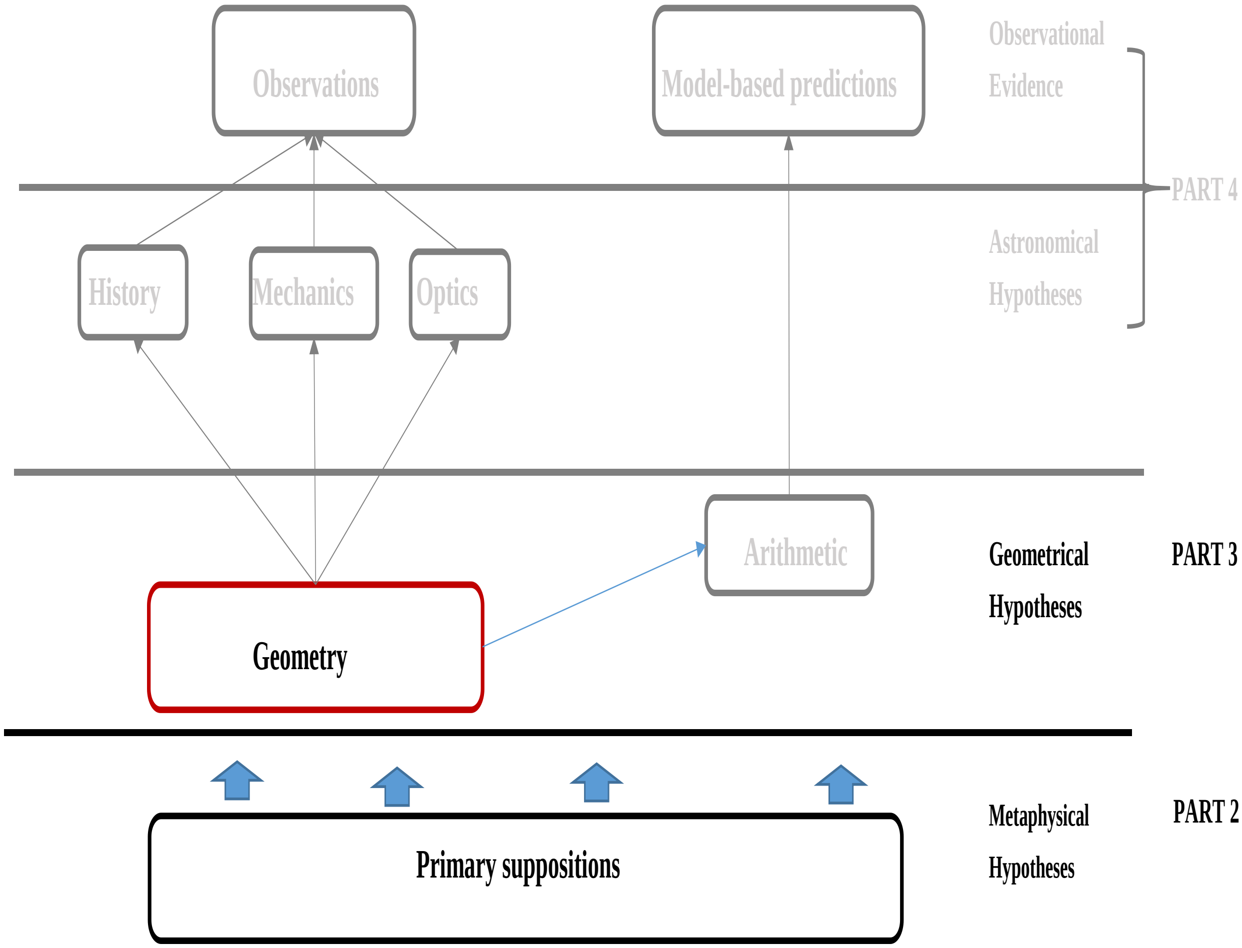
[32] "To every action there is an equal and opposite reaction"

Relativity theory follows the first path.[33] This path leads to a further issue of deciding how to understand space as a physically real existence through its interactions with physical objects. Leibniz and Kant follow the second path which leads us to ask what space can be, and how we can discover its structural qualities if it is not a physically real existence. Leibniz's full-blown relationalist position must ultimately lead to an understanding of the structure of space as derived from the relations between objects in space. Kant argued that one could not do this without first employing a spatial structure in which one had embedded these relations, which again leads to the question of what the structure of space is. If we do not have empirical tools for the discovery of its structure, are we not free to postulate any structure whatsoever?

The transcendental proposal is that we should start from the embodied subject. For instance, space emanates equally and infinitely in every direction with the perceiving subject at its center. We know this directionality visually in our ability to change perspectives by turning our heads or switching position, and we know it non-visually through proprioception (the immediate awareness of the position of our body parts). It is worth noting that the ability to visualize relies on proprioception and our ability to orient ourselves non-visually. In other words, being blind does not deprive people of their abilities of orientation. Following the procedure outlined above, we established that space is directional and horizontal as well as allowing for the positioning and construction of objects as figures through a process of limitation. We shall now look at how Euclidean geometry models these aspects of space.

In other words, we have concluded the regressive abstraction from phenomena to metaphysics, and we now move on to rebuilding the physical framework. The first step in this rebuilding is to establish a geometry. As per the approach we are taking, the geometry we end up with must be coherent with the relevant metaphysical framework as this makes up our set of a priori constitutive elements.

[33] See Friedman (1983: 64) on Einstein's objection to the absoluteness of Newtonian space.



III 3. Euclidean geometry

Euclid's *Elements* starts out with 23 definitions, five postulates and five common notions or axioms. Let us call these the basic rules of Euclidean geometry. In light of our metaphysical position, we intend these as models of our spatial intuition. The basic rules are therefore parasitic on our intuition rather than a freestanding system. Apart from deriving new theorems and finding new geometric relations, we seek to increase the precision by which our definitions, postulates and axioms reflect our spatial intuition. Since we actively apply intuition in any geometric construction, we should expect to find elements of the basic rules that we have not explicated. Since our spatial intuition is itself an abstraction from phenomenal experience, geometry is neither purely mental nor purely empirical. It is rather a reflection over the relations through which we organize our experience. *The Elements* has been widely discussed for millennia, and already the Greek schools of mathematics realized that Euclid's axioms and postulates could not stand on their own. A problem discussed in both Plato and Archimedes is the definition of a straight line. Another, discussed by Geminus in the first century BC (Heath, 1921: 222-234) is the parallel postulate. We have still not resolved these issues in a way that is universally accepted.

These are the postulates and axioms as presented in Euclid:

Postulates:

1. To draw a straight line from any point to any point
2. To produce a finite straight line continuously in a straight line
3. To describe a circle with any center and distance
4. That all right angles are equal to one another
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Common Notions [Axioms]

1. Things which are equal to the same things are also equal to one another
2. If equals be added to equals, the wholes are equal
3. If equals be subtracted from equals, the remainders are equal
4. Things which coincide with one another are equal to one another
5. The whole is greater than the part

(Euclid p. 2)

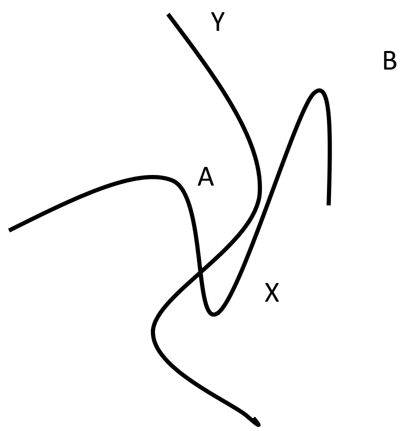
Euclid's choice of priority here is a source of much confusion. By starting from points and gradually building more and more complex structures, we get the impression that we can "build space up" from nothing. As argued in Kant (1768), and discussed above, this requires a previous move in the opposite direction. We start from experience, and, via abstraction, we are left with space as a whole. Through the continuous reduction of this space into fewer and fewer dimensions (from cube to plane to line) we end up in points. The need for this prior approach is apparent in Euclid's definition of a point as that which "has no part" (Ibid: 1). Hence, the point is where the reduction ends. Once we have reached the end of our reductive abstraction, we can rebuild (our knowledge of) space by rigidly applying the Euclidean postulates and common notions. Again, this rebuilding is constrained by the original deconstruction through reductive abstraction.

III 3.1 Directionality and Straight lines

I shall focus primarily on two aspects of Euclid's set-up: the definition of a straight line and the notion of coincidence. Euclid defines a straight line: "A straight line is a line which lies evenly with the points on itself". (Euclid p.1). This definition is notoriously difficult to make sense of and therefore there have been many attempts to construct a better definition. Plato defines a straight line in the following manner: "Straight is that of which the middle is in front of both extremities". This definition relies on the directional and perspectival properties of space. "In front of" is an element of the directional pair "in front of/behind" and being in front of the two extremities implies that we can move from one extremity to the other and check whether the middle has the relational property of being in front of. Plato therefore implicitly introduces directionality into the Euclidean system.

Another and more popular attempt is Archimedes' definition of a straight line as the "shortest distance between two points". The first thing to notice about this definition is that it cannot be built within our procedure of abstraction, since according to that procedure a straight line must be established in order to define distance (length). The question is therefore whether we can construe the Archimedean definition without already presupposing straight lines. The standard procedure is by application of a Cartesian coordinate system and sets of tuples of real numbers. However, aside from the problematic move from geometry to arithmetic implied by this procedure, it seems that any establishment of a Cartesian coordinate system requires a previous establishment of straight lines. We can think of it along the following lines.

Take a Cartesian coordinate system of two dimensions (x, y) with two points (a and b). How do we establish the shortest distance d(ab) in such a way that through this procedure we end up with a straight line?



As long as we have not established our Cartesian system by the use of straight lines, it appears that there is no help in this procedure. As I can find no geometric definition of the shortest distance that does not already imply straight lines, we shall consider it a circular definition, and therefore as inferior to the directional one. Sklar (1974: 14-15) explicitly applies the directional definition in his reformulation of the Euclidean postulates.

1) Two points determine a straight line
2) A straight line may be extended in a straight line in any direction
3) About any point, a circle of specified radius exists
4) All right angles are equal
5) If a straight line falling across two straight lines makes the sum of the interior angles on the same side less than two right angles, then the two straight lines intersect, if sufficiently extended, on that side.

As such, Sklar's reformulation is an improvement on Euclid in that it makes directionality explicit.[34] This leads us to my second point of contention with the Euclidean set-up and its relation to directionality.

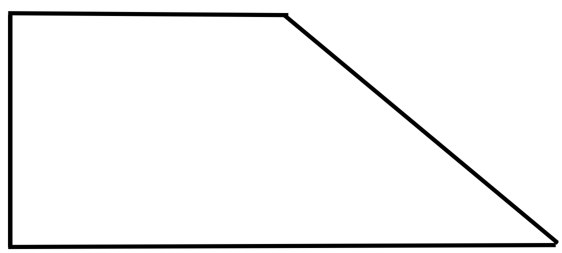
III 3.2 Directionality and coincidence

Euclid's fourth common notion, "Things which coincide with one another are equal to one another" gives a conditional relation between "being equal" and "coinciding". Kant (1768) discussed this relation and found within it a problem that he solved through the relational Kantian

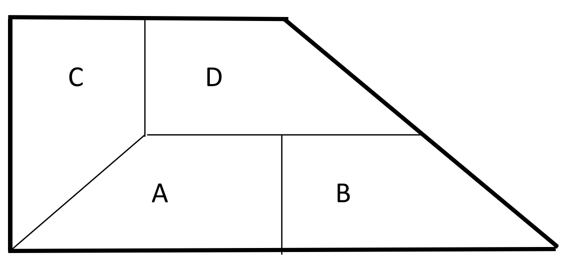
[34] This should motivate Sklar to either define or indicate the origins of directionality. Barring that, Sklar could postulate directionality as primitive. Regrettably, Sklar (1974) does neither. A similar redefinition without discussion is found in Penrose (2004: 28).

view of space. The problem is that of incongruent counterparts. Prior to Kant's text, one assumed that the coincidence/equality relation is a relation of equality. What Kant shows is that there are pairs of objects that we - for all common notions of equality thus far - cannot manage to make coincide. Object A and objects B coincide if we can remove A, and fully replace it with B. In other words, A and B can coincide if A and B can take up the same part of space. Equality of shape is equality in the length of the sides as well as the internal relations, a definition Kant takes from Leibniz.

In order to see the problem, we shall start with the standard two-dimensional example. Take the following object on the surface of a plane.



One can divide this object into four equally shaped figures in the following fashion



The figures A, B, C and D are of equal shape in that all their interior angles as well as the lengths of their sides are equal. However, if we stay on the two-dimensional plane, one cannot make the figure A coincide with any of the other figures (all other figures rotate and/or translate into each other). In order to make it coincide for instance with B, we must flip it over in the third dimension.

We can do this by picking A up by the lower left corner and flipping it along the common line of A and B. We can then ask what it is about A that is different from the other three figures.

We may spell out the difference in terms of directionality. The object A is "left oriented" whilst B, C, D are "right oriented". If we apply this description, we must also accept the origin of directionality. Kant's suggestion is, as explained above, that directionality is subject dependent and immediately known through our embodiment as material beings. In the 1768 paper, Kant discussed the issue further by reference to three-dimensional objects such as a left and a right hand. In the same way as our objects on a plane, hands are of equal shape but we cannot make them coincide. Since hands are three-dimensional, we cannot add and use a supplementary dimension in order to make the hands coincide. As such, a left and a right hand are what we call incongruent counterparts that illustrate the well-known property of chirality. Again, if we reject the subject dependence of our spatial orientation and thereby the origin of directionality, we need an alternative way to explain incongruent counterparts in nature as well as in geometry. To the best of my knowledge, no one has presented such an alternative. My position is along the lines of Kant that the subject dependence of spatial orientation along with directionality is at the base of our geometric reasoning. Geometry, as the modelling of spatial objects and the relations that obtain among them, relies on a reflection on these relations in relation to the subject. As such, geometry is neither a mental nor an empirical study, but a study of the general spatial properties of objects (including our own bodies) as they can be constructed in the intuition. Intuition here is the presentation of the a priori elements of spatial orientation and perception. As a reflection over these general properties, geometry proceeds by abstraction from the phenomena and generalizations over elements that result from such abstractions.

III 3.3 Euclidean geometry as constructions in the intuition

The Euclidean postulates are separable into two groups where 1-3 determine possible constructions. Following Sandmel (2004: 52-55) we see that the first postulate establishes the continuity of space in that any two points are part of a continuum and can be connected by a line drawn from one point in the direction of another. The second postulate states that we can (in our imagination) extend this line indefinitely in the same direction and thereby establish the limitlessness and infinity of space in all directions. The third postulate establishes the possibility of limiting space within an arbitrarily large area by simultaneously drawing lines in all directions, and thereby express the further establishment of the limitlessness and infinity of space.

The latter two postulates are different from the former three in that they establish the results of actions rather than the possibility of these actions. The fourth postulate, which states that all right angles are equal, follows from the directionality relation we established earlier. In the relation between the perceiving subject and any geometric or physical object, there are three pairs of directions: up and down; in front of and behind; to the right and to the left. Any combination of two pairs establishes a plane. So for instance if I draw a line from the left to the right, I can construct a plane from down to up, or from behind to front, with my line as the base. In so doing, I always cut the world into equally large parts. A vertical plane with a horizontal base distinguishes within the world being in front of and behind, a vertical plane with a depth base distinguishes within the world of left and right, and a depth plane with a horizontal base distinguishes within the world between over and under. The angles between these planes are right angles. Another issue is resolved through this procedure, which is the issue of spatial dimensions. We have drawn up three dimensions by reference to directionality: depth, height, and width. There are no more possible dimensions to construct by our procedure of carving up the world. As for our spatial intuition, we know that there are no more than three dimensions and therefore it seems our procedure is a successful establishment of the completeness of a three-dimensional model built from directionality.

The fifth postulate, also known as the parallel axiom, relates straight lines as drawn in different directions. A possible restatement of the definition of parallel lines is that straight lines drawn in the same direction are parallel. The fifth postulate states that such lines never intersect. The model we end up with is thereby a model of our spatial intuition that illustrates how we can build space from basic elements that we have arrived at through an abstraction from the phenomena.

In one central aspect, the abstraction process we have been referring to is similar to what Weisberg (2007) calls Galilean idealization. This is the notion that in every instance where we remove ourselves from experience, we must be able to work our way back into experience. In the case of geometry, this implies that we must not – using the geometric model - lose sight of our ability to orientate ourselves in space. In this lies Kant’s insistence on the priority of Euclidean geometry. Euclidean geometry, once expanded with a directional notion of straightness, is a successful model of our basic spatial form of existence. One often muddles this connection between intuition and geometric construction with the notion of what we can visualize (see for instance Einstein 1921: 240-246). As mentioned, visualization is not central to spatial orientation as is illustrated by proprioception. We do not need any visual data in order to identify which way is up/down/left/right/in front of/behind. All we need is to exist as perceiving material subject-objects. That is, if we admit that geometry is an explication of intuition, intuition is internal to geometry - as its necessary basis - although not explicitly noted as such. Once we reject this role for intuition, we must give non-intuitive reasons for claims of the meaning and truth of geometry. The problem becomes apparent if we consider the use of primitive notions such as directionality. In much the same way as Galileo (1632: 221-223) and Plato (Menon: 80e-86c) present examples rather than definitions in their reference to anamnesis, an intuition-based geometry can refer to examples, and direct the reader in practical construction where there are no available definitions.

If we reject intuition, we must also give up this method. That, however, leaves us with a problem concerning primitive notions. For, if I am to leave my direct experience and implicit knowledge behind, what am I for instance to make of Hilbert's primitive notions and relations? How am I to make sense of a "plane", "point", "line", "between", "congruence" and "containment" if not by appealing to what I already know from my interaction with spatial phenomena in the world? My claim therefore, is that we either implicitly or explicitly always dip into intuition when making geometric constructions. We have seen that the precision of Euclidean geometry in modelling our spatial intuition increases by appealing to directionality. As the overall metaphysical theory I am presenting argues on by a downward abstraction from the phenomena, such an appeal is natural.

III 4. "Non-Euclidean" geometries

As we have seen in section III 3.3., the Euclidean system applies to the form of our outer experience in the sense that we can maintain the horizontal, directional and limitable aspects by applying the Euclidean axioms and adding nothing external. In Euclid, we do so by limiting space to certain figures and primitive terms and performing constructions according to the basic rules.

Euclidean geometry, as presented in Euclid's *Elements*, primarily deals with two-dimensional surfaces that are "flat" in the sense that they can be constructed from straight lines. Other geometries, such as the Apollonian *Conics*, appeal to the same basic space of direct experience and apply it to different figures. Such an application expands our knowledge of geometrical relations and supplements the understanding we get from Euclid.

There is a tradition for calling these geometries "non-Euclidean geometries", which has led to some confusion. The *Conics* deals with cones and conic sections and thus it is "non-Euclidean" in the sense that it applies itself to other geometrical shapes than Euclid did. It does so however, by appealing to the same constitutive spatial characteristics as Euclid (horizontal, directional and limitable). *Conics* limits space into only those shapes that are either conical or can be constructed by cutting sections out of cones by insertion of flat surfaces. As we shall see, classical projective geometry limits itself to our visual perspective, spherical geometry limits itself to the surfaces of spheres and hyperbolic geometry originally limits itself to surfaces construed with the help hyperbolic conic sections. In other words, these geometries are not competitors to Euclidean geometry but rather expansions of geometry as a whole. When presented axiomatically, these geometries must naturally adopt different axioms and postulates and will thus derive different theorems as well.

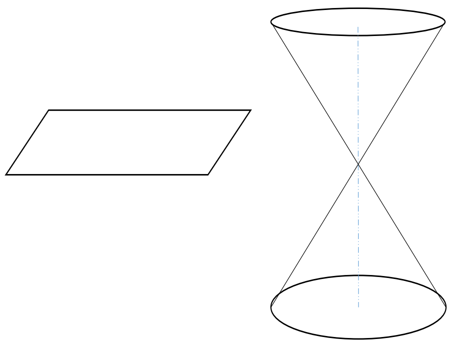
From the Kantian perspective, there is nothing dramatic about such a possibility. After all, it would be strange to believe that Euclid deals with all possible figures there are or that all surfaces of all figures show the same characteristics. On the Kantian theory, "non-Euclidean" geometries simply expose the characteristics of surfaces of different figures, where all these figures are themselves merely limitations of the same overall space. Before contrasting our view with the formalist view defended in Stump (2015), we shall present some examples.

III 4.1. Standard "non-Euclidean" geometries

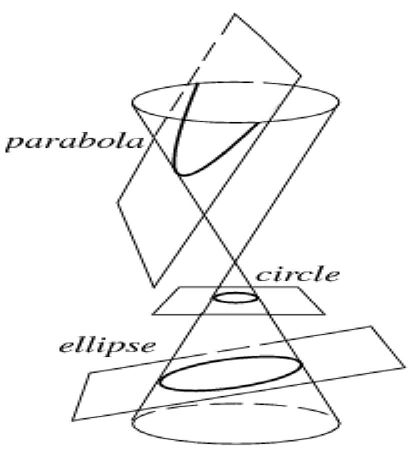
In the following, I present some examples of "non-Euclidean" geometries or geometries on alternative surfaces. I follow the standard procedure of considering only two-dimensional geometries and their model-representation within our familiar form of outer experience. The general idea is to see how we can achieve new geometric knowledge by extending our field of inquiry to the surfaces of conic sections and visual perspectives.

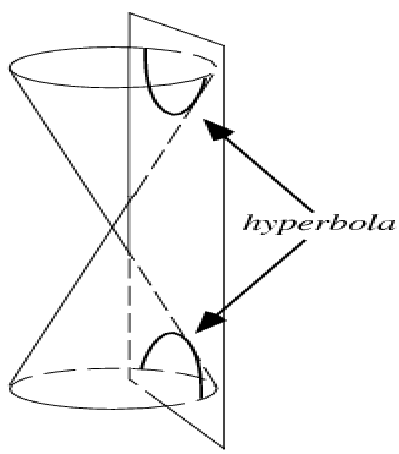
III 4.1.1. Apollonian conics

Conics is the study of a particular type of shapes and it is a part of geometry typically associated with Apollonius of Perga, a geometer in the second century BC. The central aspect of conics for our purposes lies in the notion of conic sections. Consider a two dimensional plane and a three dimensional cone.



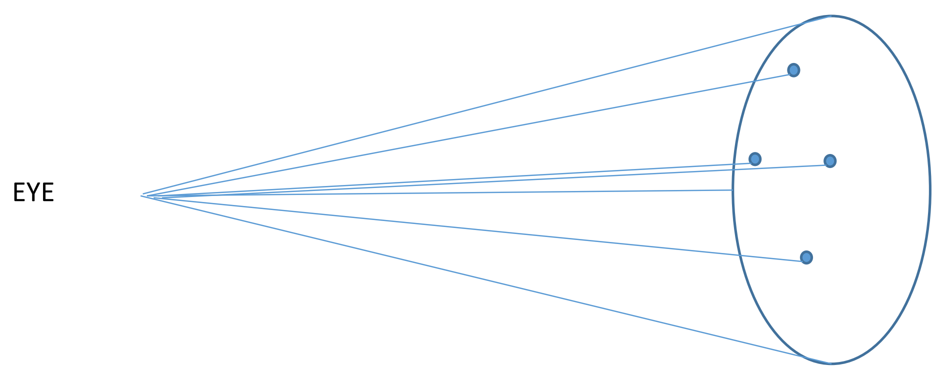
By intersecting the plane with the cone at different angles, we get what we know as the conic sections parabola, hyperbola, ellipse and circle.



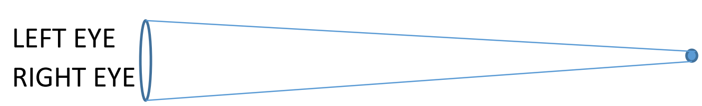


Studying the characteristics of these sections, we find numerous possible applications. For instance, Kepler used his knowledge of the Apollonian conics extensively both in establishing his optics and in exploring the elliptic motion of the planets. The interesting aspect of conics for us is what happens if we try to apply theorems of Euclidean geometry to them. I.e., what happens if I treat the cone itself, or one of the surfaces created with the use of conic sections, as a surface comparable to Euclidean flats and look at their properties? For instance, if I take the cone itself as my surface and draw a triangle on it, I find that it is no longer true that the sum of internal angles are equal to two straight angles. We must note that, as we know them, there are neither straight lines nor rectilinear triangles on a conical surface. Still, if we think of straightness in metric or Archimedean terms - the shortest distance between two points (on the surface in question) – we can construct triangle-like shapes. These triangle-like shapes have different properties from the (rectilinear) triangles we already know. We thus see that the surface of the cone has other properties than the surface of a flat two-dimensional plane.

By continuation from Kepler, we can see that there are interesting relations between conics and our visual perspective. Take for instance four distant stars on the night sky and an earthbound observer. The visual perspective of the observer appears well modelled by a cone.



Furthermore, if we think of the same observer looking at a fixed point in the distance, i.e., establishing a horizon, it seems the cone is a natural model also here.

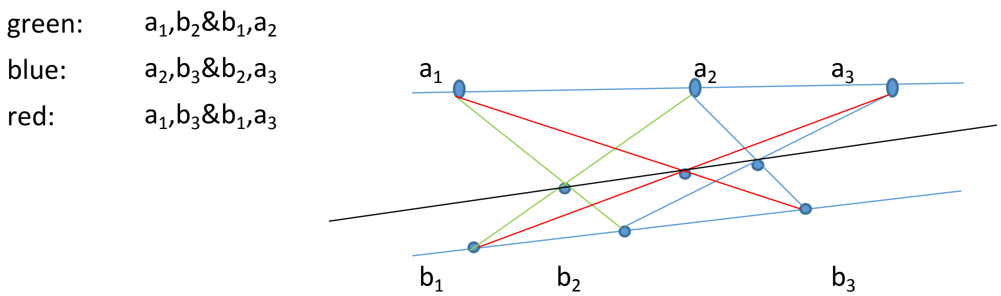


It seems therefore that conics are good initial models of human visual perspective. As we shall see, there are even better geometric models of our visual perception. By now, the best candidate is the projective plane in projective geometry.

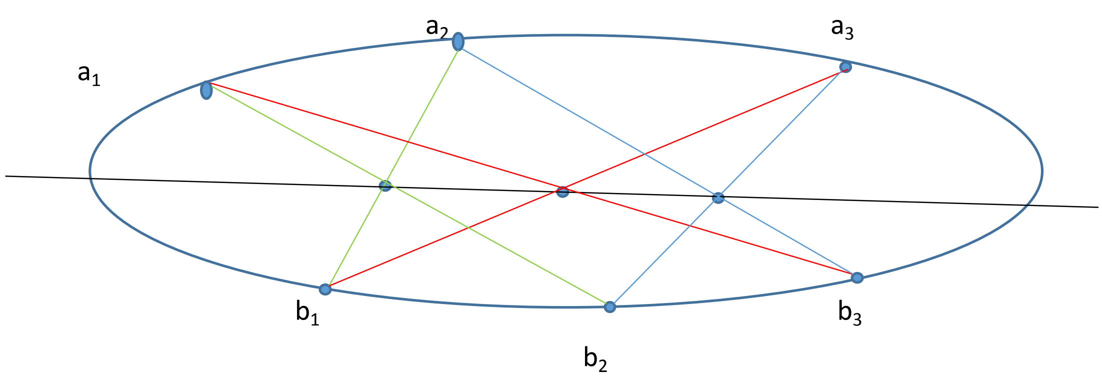
III 4.1.2 Projective geometry

Following Wildberger (2010), I represent classical projective geometry in terms of Pappus' theorem, Pascal's theorem, Desargues' theorem and the notion of a "point at infinity". What we end up with is a geometry that successfully models our visual perspective at any given instant. Projective geometry operates with only two basic elements, "point" and "straight line".

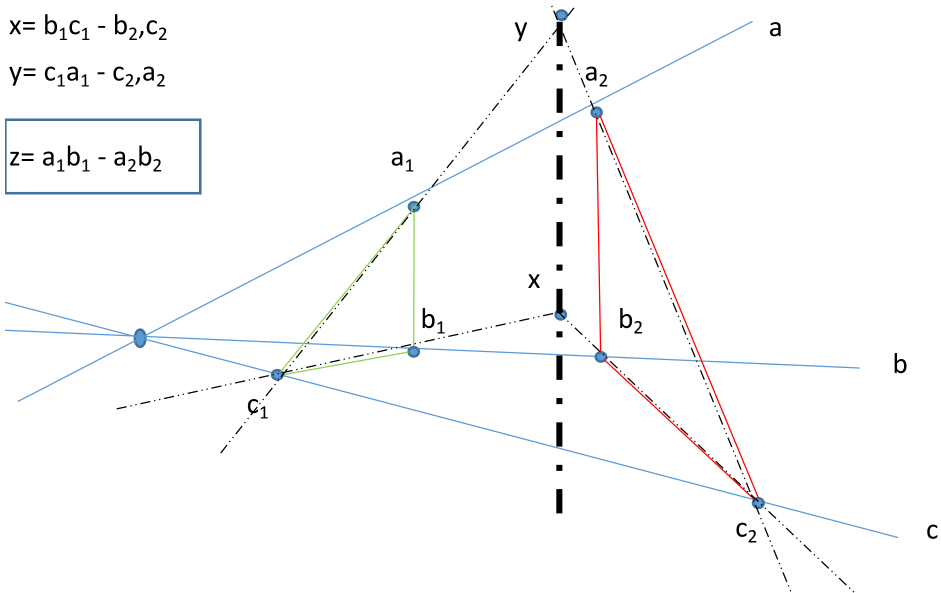
Pappus' theorem states that for any three points, a_1, a_2, a_3 on a line and b_1, b_2, b_3 on another line: If we draw additional lines connecting the points on those lines, the points of intersection produce another line.



Pascal showed that Pappus' theorem holds for all conic sections. Here illustrated by an ellipse.

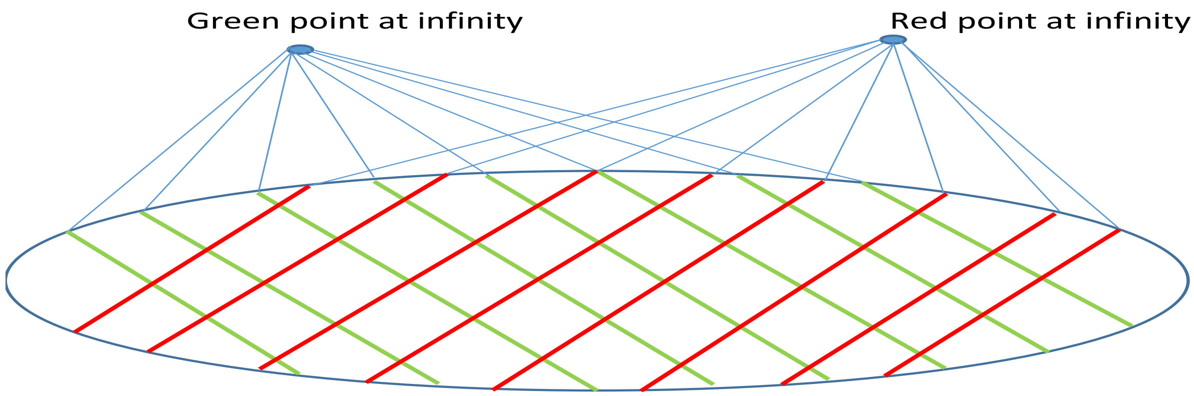


Desargues' theorem states that, given a point with three lines drawn through it, Pappus' theorem holds for any two, perspective triangles.



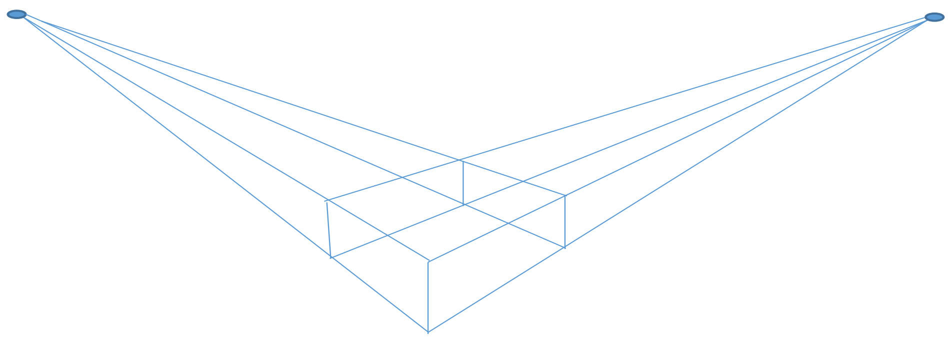
The pertinent question to ask here is what is going on with the point z, which is not in our illustration. We clearly see how the lines c_2b_2 and c_1b_1 meet in x, and the lines c_1a_1 and c_2a_2 meet in y, but the lines a_1b_1 and a_2b_2 are parallel and should not meet anywhere. In order for Desargues' theorem to work therefore, one must deny the Euclidean parallel postulate and devise projective geometry as an alternative system. The way one does this in classical projective geometry is by the introduction of the "point at infinity".

If we think in terms of human perception, we can motivate the "point at infinity" by thinking of perspectives and representations of three-dimensional figures in two-dimensional space. This is what we do when we take a photograph. If I photograph a series of equally shaped and equally sized cubes that stretches from me toward the horizon, my camera will represent them as becoming smaller and smaller the further away they are from me. Even if the cubes align perfectly and have parallel sides, it thus becomes apparent that they approach each other as they approach the horizon. If they approach each other, they must meet at some point. The point in which we imagine them to meet is what classical projective geometry calls the "point at infinity". As long as one grants that the "point at infinity" is an imported concept, projective geometry must replace the Euclidean parallel postulate with the statement "all straight lines meet at exactly one point". According to our initial understanding of parallel lines as "straight lines drawn in the same direction", there are parallel lines in projective geometry. According to the Euclidean insight (and the parallel postulate) that parallel lines do not intersect or meet, there are verbally no parallel lines in projective geometry. However, since we have use for the notion of straight lines pointing in the same directions also in projective geometry, we may think of the Euclidean relation of "being parallel" in terms of all straight lines that only meet at *the same* point at infinity. The green lines below are thus parallel to each other, but not to the red ones.



Importantly, we are willing to accept that the green lines meet at the same point at infinity only if we first apply our constitutively based knowledge of what it means to be parallel. Without this pre-theoretical notion, it is arbitrary which lines meet at which point. Once accepted, we can use our definition of "parallel" in terms acceptable to projective geometry in order to show how lines that are parallel within Euclidean geometry and the form of our outer experience appear in a model of projective geometry.

From the developing system, we thus get a "non-Euclidean" geometry, which seems to better model our perception in that it includes the perspectival nature of our visual orientation. It also gives us an excellent procedure for representing our depth perception on two-dimensional surfaces such as a sheet of paper:

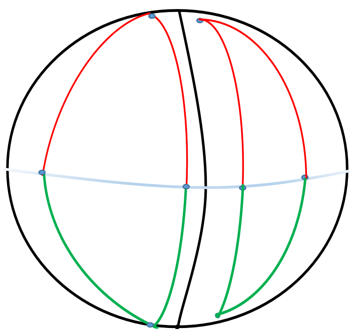


III 4.1.3. Spherical geometry

Spherical geometry studies spatial relations on the surface of spheres. In distinction to surfaces of flat planes, which are the primary subjects of Euclidean geometries, the spherical surfaces have a built in curvature. Due to this curvature, there are no straight lines in the sense of our form of outer experience and Euclidean geometry. Rather, great circles or "geodesics" play the part of "straight lines". There are no parallel geodesics in spherical geometry and thus the Euclidean parallel postulate does not apply here. Again, if we set aside our pre-theoretical knowledge of straight lines and use instead the Archimedean definition of "shortest distance", we can pick our circles[35] that go around the sphere and use them to construct triangle-like shapes. As we see from the illustration below, triangle-like shapes on the surface of a sphere "behave" rather differently than in Euclidean geometry. For instance, the internal angles of a triangle differ with size. The sums of internal angles of the red "triangles" in the illustration below are neither equal to each other nor to two straight angles. Another novel issue is that two triangle-like shapes of equal size and internal angles, but on opposite hemispheres (i.e. the red and the green triangles) sometimes turn out to be incongruous counterparts. Kant (Prolegomena: § 6-13) argues that this proves that geometry is not analytic. If mathematics were analytic, the properties of shapes and figures would follow deductively from the concepts applied. If we analyze the concepts of "straight lines", "triangles", "hemisphere" etc. as we apply them in geometry, we cannot deduce from them the properties of incongruous "triangles". Nevertheless, from the model below we see that in spherical geometry the "triangles" on opposite hemispheres are incongruous. For Kant, this exemplifies how mathematical reasoning involves constructions in pure sensible intuition (what we have referred to as the form of our outer

[35] Typically referred to as «great circles».

experience). If we insisted on a deductive analytic view of mathematics, we could never know that certain pairs of "triangles" have the property of being incongruous on spherical surfaces. We would simply have to give up this knowledge.

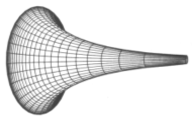


Spherical geometry does not directly display the horizontal characteristics of the form of our outer experience. For instance, every "straight line"[36], if extended, ultimately returns on itself. Furthermore, the sphere's size determines its curvature and therefore, once a definitive curvature is given, a size or external border is simultaneously given. Nevertheless, considered as the study of the surface of spheres – where "sphere" is a figure in space as the form of our outer experience – spherical geometry is highly useful. Apart from our increased geometrical knowledge, spherical geometry has added practical utilities such as for instance determining distances on the surface of the earth, flight routes etc. We can see, then, that although spherical geometry does not model the form of our outer experience directly and ideally, it expands our geometric knowledge.

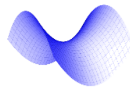
III 4.1.4. Hyperbolic geometry

In hyperbolic geometry, one uses a conic section, the hyperbola, to construct surfaces whose geometric properties naturally differ in the sense that we must interpret differently the primitive terms "line", "parallel" etc. when applied to such surfaces. The most commonly known hyperbolic surfaces are (i) the hyperbolic pseudo-sphere, and (ii) the hyperbolic paraboloid or "saddle surface".

[36] Recall that «straight lines» refers to great circles in spherical geometry.



(i)



(ii)

A common feature of these surfaces, along with the spherical surface, is that there are no straight lines, as we commonly understand them. However, if we recall the metric or Archimedean definition of straight line as “shortest distance”, we can construct “straight lines” between points. If we do so, we will arguably find that the Euclidean parallel postulate does not hold. In order to do so, however, we also need to redefine what we mean by parallel. In order to avoid needless levels of abstraction at this point, we shall first introduce a complication from the formalist view of geometry, and later see how one can think of multiple “lines” as “parallel” within hyperbolic geometry.

III 4.2. Formalist theories and the “crisis” in geometry

The standard story of “non-Euclidean” geometries is that the mere fact that such geometries exist and are consistent constitutes a crisis for geometry itself. After all, commonly defined, geometry is the study of space as such as well as figures within that space. Now, if there are multiple consistent geometries, why should we hold on to the Euclidean one as a model of space as such? I propose an answer to this question below, but for now all we need to know is that there is a common narrative claiming a crisis in geometry. If there were such a crisis, it is a serious blow to human knowledge as we have taken mathematics in general and geometry in particular as the pinnacles of certainty. Apart from logic, mathematics is the only field in which we still accept the notion of “proof”. What then, can we do in order to answer to a crisis in our most fundamental knowledge? The answer given in the standard story is logic. By formalizing all of mathematics within logic, one could reestablish mathematical certainty. As a logical system, there is no reason whatsoever to prefer Euclidean geometry and thus the crisis becomes even more apparent.

Since the early 20th Century, it has been common to think of “non-Euclidean” geometries in this way. As pointed out in Stump (2015: 29-30), and Putnam (1967: 5-6) however, the crisis never happened. Mathematicians working on “non-Euclidean” geometries never considered the existence of multiple consistent geometries as a problem at all. As we saw in relation to spherical geometry, Kant, for instance, saw these developments purely as expansions of geometrical knowledge. Once presented as logical schemes however, the new geometries appear as if they introduce a choice concerning "what the structure of space" really is. Stump makes the following comment

> My claim is that formal logic and the study of the foundations of mathematics affected the writing of history of the non-Euclidean geometries and that the narrative that was established has affected pedagogical writing on geometry to this day (Stump 2015: 31)

Regardless of his understanding of their history, Stump maintains the formalist or logicist conception of geometry as logical systems. Putnam famously argued that no such systems, understood as foundation(s) of mathematics, were ever needed:

> Philosophers and logicians have been so busy trying to provide mathematics with a 'foundation' in the past half-century that only rarely have a few timid voices dared to voice the suggestion that it does not need one. I wish here to urge with some seriousness the view of the timid voices. I don't think mathematics is unclear; I don't think mathematics has a crisis in its foundations; indeed, I do not believe mathematics either has or needs 'foundations'. The much touted problems in the philosophy of mathematics seem to me, without exception, to be problems internal to the thought of various system builders. (Putnam 1967: 5)

On the Kantian view, there is no particular foundation for mathematics. Rather, Kant discusses mathematical knowledge as an exemplar of the possibility of knowledge in general. Other options, such as for instance the analytic method suggested by Cellucci (2013) and Grabiner (1983) are still open. Within the framework of Putnam, Kant and Cellucci's philosophy of mathematics, there never was a crisis of mathematical knowledge. Rather, the perceived crisis can only be perceived as such if one assumes that mathematics is an instance of pure *logical* reasoning. This view, however, is the standard view in contemporary philosophy of mathematics. Penrose (2004) defines mathematical proof as

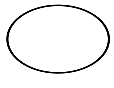
> A proof, in mathematics, is an impeccable argument, using only the methods of pure logical reasoning, which enables one to infer the validity of a given mathematical assertion from the pre-established validity of other mathematical assertions, or from some particular primitive assertions – the axioms – whose validity is taken to be self-evident. Once such a mathematical assertion has been established in this way, it is referred to as a theorem (Penrose 2004: 10)

Provided the "'pre-established' validity of other mathematical assertions" are themselves inferred from the axioms and primitive notions, Penrose here defines the standard formalist view of mathematical reasoning. In order to prove the validity of the "non-Euclidean" systems referred above, one must hence establish new axiomatic systems and derive theorems from them by pure logical deduction. The formalist project has been going on since the late 19th Century and has non-trivial and unsolved problems.

III 4.2.1. Mathematics as pure deductive reasoning

Stump (2015: 25) argues that primitive terms accrue meaning only within an axiomatic system. Thus, the meaning of primitive terms like "line", "straight", "parallel", etc. have no meaning or reference outside such an axiomatic system. Once this claim is accepted, one pretends to establish the logical references of primitive terms deductively, according to the rules of the standard view. If you object for instance to the notion that the following illustration is a representation of a "straight line", it is only because you have accepted the axioms of Euclidean, rather than those of spherical geometry.

"Straight line":



Note that one treats the typical reference for "straight line" of our imagination as invalid. This means that references to proofs such as Kant's demonstration of incongruent counterparts or of the pre-axiomatic theorems of projective geometry (Pappus, Desargues and Pascal's theorems) with the use of intuitive constructions, exemplified through the illustrations presented above, are all treated as invalid on the view presented here. Nevertheless, Stump (2015: 25) argues directly from such references when presenting illustrations of Klein's model of hyperbolic geometry.

> …the assumption that a line forming an acute angle to a perpendicular never intersects the given line does not lead to a contradiction. Of course, taking these models to represent hyperbolic geometry assumes that … (Stump 2015: 25)

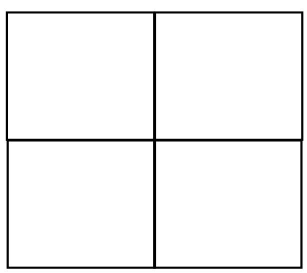
The inconsistency in the argument lies in first denying anything but deduction as mathematical reasoning, and then referring to models we can visualize, as proofs of consistency. Notably, by referring to the models that we create in our own imagination and that we represent on paper, Stump refers directly to the Kantian notion of pure sensible intuition, i.e., to constructions in space as the form of our outer experience. Arguments either directly "proven" from or aided by our imagination in this way are common among formalists, although they explicitly deny such arguments as valid (See for instance Penrose (2004: 34, 35, 36, 38, 39, 40, 41, 45, and 47) for hyperbolic geometry). Such instances indicate that perhaps there is something in geometric reasoning apart from deductions from axioms.

Stump (2015: 49-50) presents the different stances taken by Hilbert and Poincaré on one side and Frege and Russell on the other. The latter two indicated that there is some element of intuitive reasoning involved in metric geometry. Stump quotes Poincaré's somewhat sarcastic treatment of this suggestion (in Stump 2015: 49-50):

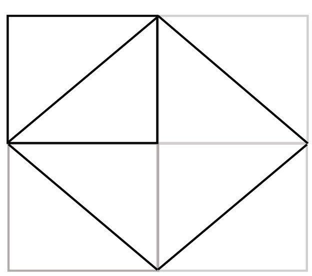
> I find it difficult to respond to those who think they have a direct intuition of the equality of two distances or two durations; we speak very different languages, I can only envy and admire them, without understanding, since I completely lack this intuition (Poincaré 1899: 279)

There are two interesting aspects of Poincaré's claim here. First, people obviously have direct intuition of the equality of two durations. Anyone who ever went dancing, saw an orchestra, or was trained as a musician will recognize that "intuiting the equality of two durations" is nothing more than being able to follow the beat. More importantly, such intuitions are not what Frege and Russell refer to, although they, as Stump points out, provide vague directions. However vaguely indicated by Frege and Russell, we have good reasons to assume that there are extra-logical aspects of mathematical reasoning. One way to think about this issue is in terms of our pre-theoretical ability to understand geometric construction.

Plato (Menon: 80e-86c) provides an example of such construction. Here, Socrates presents a slave boy with a square and asks how it would be possible to double its area. A first and "commonsensical" suggestion would be to prolong the sides of the square. By how much? An initial idea might be to double the sides. This of course gives you a square of four times the area, which is not what we are seeking. However, the knowledge that doubling the sides quadruples the area, gives us a clue on how to proceed since we can now think of the problem as halving our quadrupled square:



The easiest way to halve a square is of course to draw a diagonal. By drawing a diagonal in each of the four squares, we will end up with a square double the size of our initial square.



Which was our task.

This simple construction demands not only that we know how to recognize a square, but, importantly, we must also recognize that straight lines combine in such a way as to construct a square. Furthermore, we must know that the area of the square depends on the length of the lines inscribing it, that "doubling the lines" is not equal to "doubling the area", that although the size changes the shape remains the same, that a diagonal cuts a square in half, that half of the quadruple equals a double etc. In other words, anyone able to understand the demonstration of how to double the area of a square must have geometrical understanding. Without knowledge of the axioms and the proper deductive processes, such understanding should be impossible if mathematical reasoning is purely deductive.

III 4.2.2. Gödel and the completeness of deductive systems

The most common criticism of the formalist axiomatic view of mathematics is motivated in Gödel's incompleteness theorems.[37] In order to understand Gödel's argument we must first make some preliminary commitments. There is mathematical knowledge. Our theory of whole numbers is part of this knowledge. Any acceptable formal axiomatic system must be powerful enough to express the theory of natural numbers. There are mathematical theorems derived independently of axioms (see for instance Pappus, Pascal, and Desargues' theorems). One may then ask how we know that mathematical truths are true.

The logicist axiomatization program is an attempt at answering this question. Their answer is that mathematics can be understood through its rigorous use of logical deductions. Recall Penrose and Stump's claims above. Gödel then applies our theory of whole numbers to show first that

(i) *There are true statements within any mathematical system, which we cannot prove by formal logically valid deductions alone.*

Note that this is a criticism of the description of (all) mathematical proof(s) as logical deduction(s) and not a criticism of mathematical knowledge itself. Indeed, Gödel uses arithmetic in his criticism. However, unless one is willing to give up arithmetic, Gödel's criticism stands. Furthermore, Gödel shows how

(ii) *No consistent system can deductively prove its own consistency.*

[37] I will not treat Gödel's incompleteness theorems in any detail here since all participants in the debate accept them.

If, as the formalist axiomatic view states, mathematics is reducible to logic, one cannot use logic to prove the consistency of mathematics! This is a problematic result for the formalist axiomatic view, which remains unsolved and appears unsolvable.

We thus have the following situation. There are logically reconstructed axiom systems of geometry that, as proven through their successful modeling within the form of our outer experience, i.e., as successful constructions in Euclidean space, are accepted as consistent. Presented as purely logically construed axiom systems, these systems appear as alternatives to the Euclidean one. If one assumes that there is nothing more to geometry than logic, there seems to be a problem of deciding which such axiomatic system of geometric applies to space as such. Furthermore, if one assumes there is nothing more to mathematical reasoning than logic, there are unexplainable mathematical insights that are true (such as the existence of incongruous counterparts) but not derivable. Gödel (i) shows that this situation will not be resolved while insisting that we can reduce mathematics to logic. One could still think, however, that by reducing mathematics to formal axiomatic, one could strengthen our belief in its consistency. That would require that we already know that the underlying logic is itself consistent. Gödel (ii) shows that this we cannot know by the means proposed. Thus, the philosophical theory that mathematics is formal axiomatic, is (i) demonstrably incomplete, (ii) possibly inconsistent. The power of Gödel's arguments is that they show that these deeply problematic features are built into the axiomatic systems, as such. I.e., there is no reason to expect that we will ever be able to solve them!

There is a solution to this perceived crisis though. Give up the formal axiomatic program as a foundation and accept non-deductive reasoning into mathematics. If done, the original, alleged crisis of geometry, as well as the real crisis of the programs of logical foundation for mathematics (i.e. the formalist or logicist reductionism), disappear.

III 4.3. The transcendental view and its preference for Euclidean geometry

Although rejecting the formal axiomatic view of mathematics removes the perceived crisis in geometry, there are many issues left open. For instance, what is the "extra" content in mathematics and where does it come from, and why do not Kantians adopt projective geometry as the more "intuitive"?

Throughout this thesis, we have argued that all action is interaction and that when we perform observational acts, we bring spatiality into the phenomena, as this is how we organize our outer experience. We gain geometric knowledge by performing constructions within this organizational space, i.e., in imagination. When we wish to analyze external shapes and figures, we do so in that same space. Geometry, as the study of the properties of space can therefore legitimately apply model thinking and reference to intuition.[38] In other words, in addition to deductions, we gain geometrical knowledge through analyzes of geometrical objects and their properties within the form of outer experience. Kant (A733/B761) puts it thus:

> Mathematics is capable of axioms because, by constructing the object in intuition, it can connect the object's predicates, a priori and immediately, for instance that three points are always in a plane.[39]

Kant's transcendental geometry is often erroneously presented as a geometry about visualization. Visualization *is* an important aspect of transcendental geometry in the sense that transcendental geometry allows proof by model-construction. However, we must recall that the seat of what we have called "form of outer experience" in transcendental philosophy, is what Kant refers to as "sensible intuition" which is wider than the merely visual. For instance, if I stand at the top of the Eiffel tower and turn my head, I visually get the impression that all of Paris rotates. I immediately reject this idea since my other senses inform me that I am the one moving. However, even if I am not moving I can still find problems in my visual perspective. Consider the following situation.

Four distant humans observe some object.

[38] Provided «intuition» refers to constructions within the form of outer experience and not "common-sense" or "hunches".
[39] My translation

B

A object C

D

They accurately draw their visual data in the following way:

If we take projective geometry as our theory of space, we are forced to accept that *the very same object* has, at least, four different shapes (i.e. the object is directed in four different directions) and two different sizes. The transcendental question would be; how is that possible? The answer is found by thinking about how our visual perspective is formed and in particular how having two eyes provides us with depth perception. We can think of our visual perspective within projective geometry, we can also ask, what would the object look like if I was right next to it, and what is the shape of the object if I account for the depth function of my visual perspective? We then find that the object is a square, which is exactly how Euclidean geometry would model it. The argument for maintaining Euclidean geometry is that Euclidean geometry preserves the characteristics of the form of outer experience and can construct models of the objects whose surfaces the other geometries describe. Following Frege|, who states:

> The wildest visions of delirium, the boldest inventions of legend and poetry, where animals speak and stars stand still, where men are turned into stone and trees turn into men, where drowning men haul themselves up out of swamps by their own topknots – all these remain, so long as they remain intuitable, still subject to the axioms of geometry. Conceptual thought alone can after a fashion shake off this yoke, when it assumes, say, a space of four dimensions or positive curvature. To study such conceptions is not useless by any means; but it is to leave the ground of intuition entirely behind. If we make use of intuition even here, as an aid, it is still the same old intuition of Euclidean space, the only one whose structure we can intuit. (Frege, 1989/1884: §14: p. 20)

In section I 2.3., we presented a Keplerian thought experiment of a man looking at distant mountains while walking next to a hedge. Judging only by his visual perspective, the mountain appears to be following him. Most of us have had similar experiences for instance by looking at the moon through the window of a moving car. The point of Kepler's illustration is to show the difference between the "apparent" and the "true" phenomenon.

Kepler gives many such examples, such as for instance the story, involving his friend who observes a cow through the window of a carriage while there is a spider hanging from the windowpane. The friend calls out, "oh look, a many-legged cow" (Kepler 1600: 240) There are plenty of examples of this sort that all point to how the specific limitations of our visual perspective light might lead us to err. The more familiar one is perhaps the visual perspective of how an oar "breaks" when it is submerged into the water. The common feature of all these examples is that, if we take our visual perspective directly and un-moderated as showing us the true reality, we will be wrong. If you follow your visual experience strictly and try to grab the submerged oar, you will grab nothing but water. Luckily, we rarely act in strict accordance to what our visual perspective may seem to show, but make immediate corrections, such as when I now know that the top of my coffee mug is circular although it appears visually as an oval.

Thus, we cannot *replace* the Euclidean geometry with the projective one. However, by appealing to the same aspects of the form of our outer experience (horizontal, directional, limitable) we can explore spatial elements from the perspective of visual sense data. As a supplement to Euclidean geometry, projective geometry is therefore a welcome expansion of our mathematical knowledge and representational capacities concerning spatial aspects of experience. The most obvious way in which projective geometry is helpful to us is in demonstrating how visual sense data (best modelled in projective geometry) requires further explanation if we wish to understand the underlying phenomenon. Thus, we return to the basic ideas of Galileo in establishing the principle of relativity – where our own motion is hidden from us – and Kepler's distinction between the "true" and "apparent" planetary positions - where the apparent positions fail to account for the perspective of the observer. In all these cases, we increase our knowledge through the transcendental procedure of finding the conditions of possibility for the appearances. We find this possibility in the space of our form of outer experience, and, though it may need supplementation, Euclid's geometry remains the best model of that space. If a better model is constructed, we should simply switch models.

III 4.4. "Practical geometry" from empirical research

Rather than accepting the Kantian option, one could collect the extra-logical content of geometry from empirical research. This approach requires not only that we "reassign meaning" to the primitive geometric terms, but that we remove their meaning altogether. What we are left with is then a purely formal system. As Stump (2015) puts it:

> By the end of the 19th Century, a formal view of mathematics was taking hold in which geometry was viewed as a pure uninterpreted axiomatic system that does not express truths. The primitive terms in geometry are taken to refer to nothing at all; thus, they could be explicitly redefined to refer to any set of objects that maintained the abstract relationships stated in the axioms. (Stump 2015: 20)

What we have then, is just the formal axiomatic view prior to Gödel. In the previous section, we discussed what type of reasoning was involved in mathematics and found that it could not be purely deductive reasoning. Here, we ask what content one should enter into the de-interpreted axioms. The view we shall discuss and which dominated philosophy of physics in the 20th Century, is that we can collect content from empirical research, typically from physics. Since the axiomatic systems themselves are free of content, we now have a more reliable way to check which abstract logical relations hold between which objects in the world, or so the argument goes.

When the axioms of geometry are fully de-interpreted, they are not reasonably described as axioms of geometry, since they can, in principle, just as well be axioms of psychology, football tactics, or anything else. Strictly speaking, they are no longer axioms of mathematics at all, but pure logical systems.[40]

As an example of how this is done, let us look at Euclid's first axiom or "common notion" which originally states the following.

[40] The axioms in this de-interpreted version are only "mathematical" if mathematics reduces to logic, which we have seen that it does not.

Things which are equal to the same thing are also equal to one another

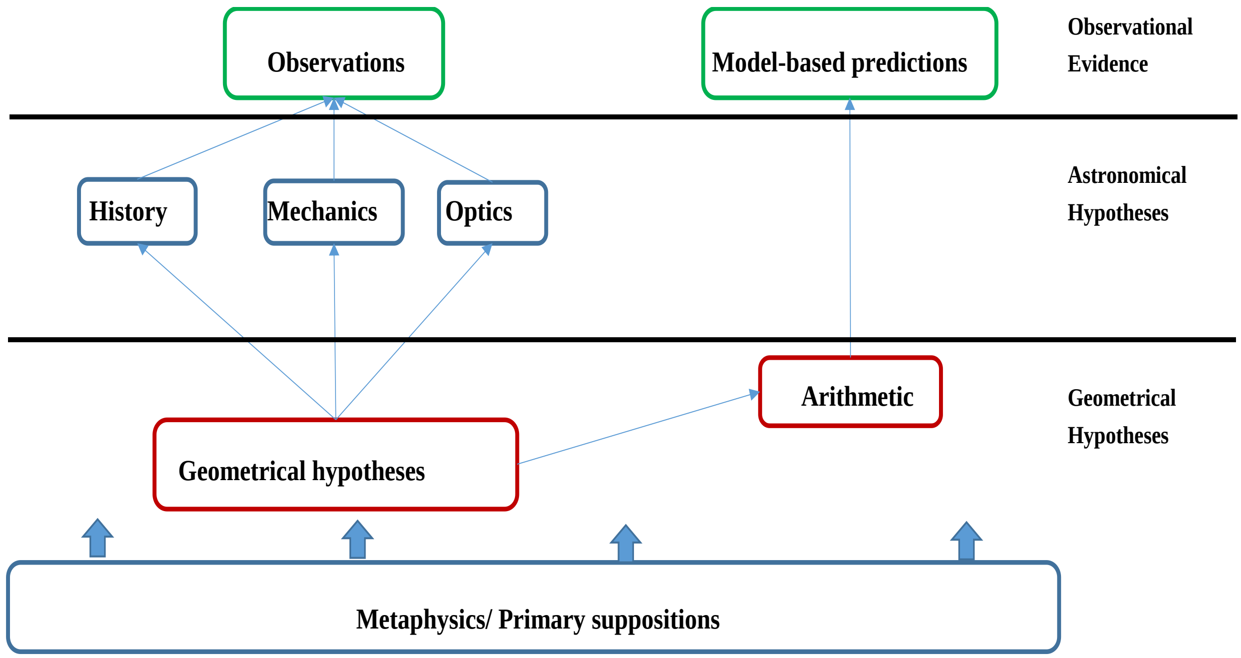
For now, we can write the relation of "being equal to" as the relation E:

$\forall a, b, c: (E(a, b) \wedge E(a, c) \rightarrow E(b, c)$

We see that this common notion simply expresses a transitivity relation. This de-interpreted axiom is about anything that stands in a transitivity relation. As arithmetic we can read the axiom as (3= 6/2 & 3= 4-1) → (6/2= 4-1). As art pricing we can read the axiom as stating that (two Caravaggio = one Munch) & (two Caravaggio = six Bush) → (one Munch = six Bush). Moving away from mathematically related topics altogether (veranda = porch & veranda = patio) → (porch = patio). As pointed out in Sandmel (2004:62), there are major benefits to such a de-interpretation, because it allows us to see structural similarities between different areas of research that would otherwise be very hard to discover. Nevertheless, as stated in its fully de-interpreted form, the axiom is no longer an axiom of geometry, but an expression of a logical relation that we call transitivity. Thus, we cannot use the fully de-interpreted axioms to provide meaning to the primitive terms of geometry, since fully non-geometric content also follows from them. We need to find our content elsewhere.

As mentioned, the standard non-Kantian approach is to find content from empirical research and predominantly from physics.[41] This approach brings with it some complications. First, we do not typically express physics data in geometric terms but rather numerically. We must therefore find a way to guarantee that these numerical expressions are expressions of geometrical relations as well as arithmetical ones. Second, we must know whether the results are due to the measuring process and the apparatus used, or whether it is due to the targeted phenomenon. Third, whether due to measuring apparatus or targeted phenomenon, we need to know what level the mathematical expression represents. Since all levels are expressible in mathematical terms, we must therefore ask if the relevant mathematics express a dynamical, mechanical, kinematical or geometric aspect of the situation. In order to include empirical data from physics therefore, we must move further up our overall model and clarify some new relations.

[41] Note that there are further options. See for instance Cellucci (2013), Hersch (1997), Spelke (2011).



In Kepler's original set-up, the arithmetic expressions of the model-based predictions follow from the metaphysics along with the basic geometric set-up (i.e. conic sections constructed within Euclidean space). However, when considering the arithmetic expressions of the observed phenomena, we must see how the model-arithmetic differs from the observation-based arithmetic in Kepler's set-up. We must separate between arithmetic expressions of the geometry and arithmetic expressions of the measurements. The latter follow not only from geometry and metaphysics, but also from considerations of the behavior of the measuring apparatus, the optics and all aspects of the phenomenon under question.[42] Once these aspects are understood, one can include them in the model and thus make the model-arithmetic match the observational arithmetic.

In the empiricist proposal, the arithmetic involved is simply the arithmetical expressions of the measurements. These expressions one then uses to define the geometry. In order to see the difference more clearly, let us consider Poincaré's celebrated thought experiment:

> Suppose, for example, a world enclosed in a large sphere and subject to the following laws:—The temperature is not uniform; it is greatest at the centre, and gradually decreases as we move towards the circumference of the sphere, where it is absolute zero. The law of this temperature is as follows:—If R be the radius of the sphere, and r the distance of the point considered from the centre,

[42] I.e. the phenomenal, dynamical, mechanical and kinematical/phoronomic aspects.

> the absolute temperature will be proportional to $R^2 - r^2$. Further, I shall suppose that in this world all bodies have the same co-efficient of dilatation, so that the linear dilatation of any body is proportional to its absolute temperature. Finally, I shall assume that a body transported from one point to another of different temperature is instantaneously in thermal equilibrium with its new environment. There is nothing in these hypotheses either contradictory or unimaginable. A moving object will become smaller and smaller as it approaches the circumference of the sphere. Let us observe, in the first place, that although from the point of view of our ordinary geometry this world is finite, to its inhabitants it will appear infinite. As they approach the surface of the sphere they become colder, and at the same time smaller and smaller. The steps they take are therefore also smaller and smaller, so that they can never reach the boundary of the sphere. If to us geometry is only the study of the laws according to which invariable solids move, to these imaginary beings it will be the study of the laws of motion of solids deformed by the differences of temperature alluded to. (Poincaré 1902: 75-76)

What Poincaré is getting at, is the following. Given a set of measurements that satisfy some basic requirements (no erratic behavior of the measuring apparatus etc.); there are at least two ways to see the situation. Either the sphere-dwelling beings could assume some universal but undiscoverable force and maintain a Euclidean geometry + universal force description of their world, or they could develop a "non-Euclidean" geometry directly from their measuring results. In this thesis, I am stating that, provided the sphere-dwelling beings share our form of outer experience, they should maintain their Euclidean model of space and assume a universal force. Note that, as presented in Poincaré's thought experiment, this would be the correct solution. There is a sphere (imagined as a figure in Euclidean space) and there is a universal force present. As long as the sphere-dwellers are able to separate the apparent phenomena (expressed in direct results of measurement) from the true phenomena (understandable only through a construction in the form of outer experience), they will end up in this solution.

The empiricist proposal, which is the topic of our discussion at present, is that the sphere-dwellers would be correct in establishing a "non-Euclidean" geometry directly from measurements. This geometry would lead them to believe they are living in a boundless space.[43] Thus, we see how the empiricist geometry approaches the problem and how it differs from Kepler's, Kant's and our methodology. The overall idea of basing geometry directly on empirical measurements is what

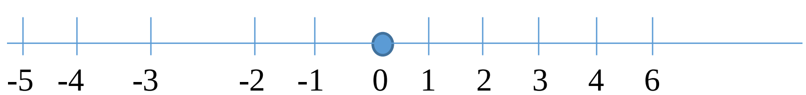
[43] We should note, however, that according to Poincaré's thought experiment, this is the wrong conclusion. Their world is "enclosed in a large sphere".

Einstein (1921) calls "practical geometry".[44] We shall discuss this more in detail below, but first we must treat the issue of how arithmetic relates to geometry prior to measurement.

III 4.4.1. From geometry to arithmetic

The standard way of constructing an arithmetical geometry is to follow Descartes and establish a coordinate system. Recall that we saw earlier (III 3.1.) how establishing a Cartesian co-ordinate system requires a presupposed knowledge of what a "straight line" is. The main point of establishing such a system is to provide a metric, i.e., a system for determining distances by use of units represented by numbers. Once we have established distances in this way, we can use them to establish the relationships between points and figures. First, however, we must include some further presuppositions. The following is a summary of Sandmel (2004: Ch. 5), with some additional comments.

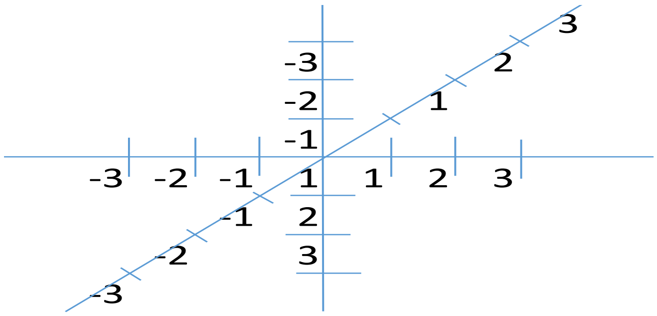
Through a point called the "origin", we can draw a line in any directional pair and call one direction of that pair "positive". We typically choose the directional pair "right/left" and the direction "right" as our positive direction. The opposite direction is then the "negative direction". On this line through the origin, we place increasing numbers in the positive direction and decreasing numbers in the negative direction:



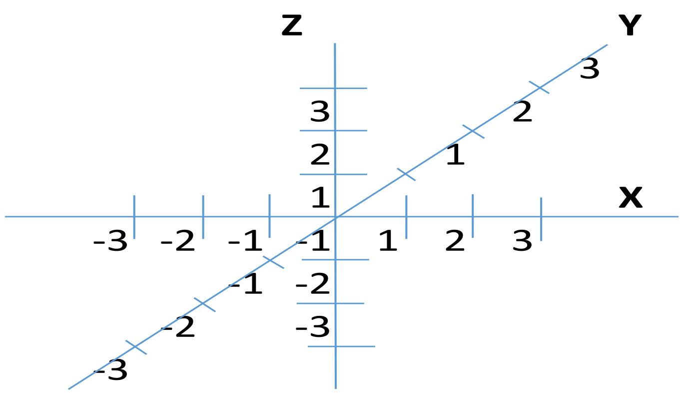
As we can see, there are already problems here, as the negative numbers are not equidistant from each other. We thus need, again, to apply a notion of equidistance known to us prior to the establishment of a co-ordinate system.

Once established with equidistance between the units, we can add more dimensions. In order for new dimensions to be effective, they need to be perpendicular to the dimension we already have.

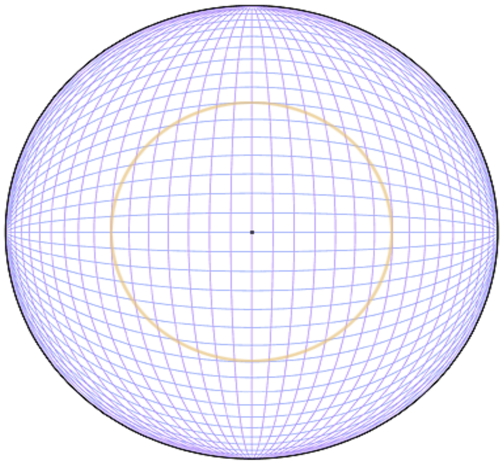
[44] Poincaré's own conclusion is that "[experiment] tells us not what is the truest, but what is the most convenient geometry … Beings educated there would no doubt find it more convenient to create a geometry different from ours, and better adapted to their impressions …" (Ibid: 82). Thus maintaining, with the formalists, that there is no *true*, but only more or less *convenient* geometric descriptions of space.



The Cartesian system is, as we can see, still problematic as the “up/down” pair has the positive direction as “downward” rather than the more common “upward”. As pointed out in Sandmel (2004: 39), such a system is an incongruent counterpart to the standard Cartesian. Thus, we need also to presuppose a particular internal relation among the positive and negative axes before we get an effective system. What we end up with, is a familiar Cartesian coordinate system:

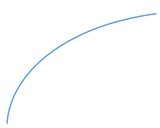


Just as in the phoronomic part established in part II, we can introduce numerical definitions of geometric relations into this system and thus end up with an arithmetized geometry. Once we have established the Cartesian system as properly directed with straight lines and equidistant units, we can imagine the system as determining a grid. By using only two dimensions, we can place our grid on “non-Euclidean” surfaces and thus find arithmetical expressions for geometrical relations on these surfaces.



Furthermore, by taking the numerical determinations of geometrical relations in Euclidean geometry, we can get numerical determinations of how different “figures” and “shapes” appear on different surfaces. Consider for instance a “straight line”. If we apply the metric definition of a “straight line” as the shortest distance between two points, we can see the difference between “straight lines” on a spherical surface and in a flat plane:

A “straight line” drawn between points (x_1, y_1) and (x_2, y_2) will appear as on the flat plane and as

 on the spherical surface. We must recall however, that in order to get to our Cartesian system and thus make sense of the metric definition, we had first to make use of our directional definition of straight lines.

So far we have seen how, starting out with a geometry, we can establish Cartesian coordinate systems intuitively[45] and thus express geometric relations arithmetically. Once the arithmetic expressions and relations are established, we can apply them to other geometric surfaces and thus investigate relations between different kinds of surfaces. Such a procedure maintains the priority of Euclidean space, since we constructed the initial Cartesian coordinate systems in this space. If we wish to doubt this procedure and apply the measuring data directly in order to construct a geometry from them, new problems arise.

[45] I.e. as constructions within the form of outer experience.

III 4.4.2. From arithmetic to geometry

In part II, we saw that there are helpful relations between phoronomy and arithmetic, considering for instance treating motion in terms of vectors in a Cartesian system. Throughout that treatment, as well as in our treatment of arithmetized geometry in the previous section, we started out from spatial relations and found numerical correlates. We cannot directly establish a geometry from arithmetic but must always presuppose a coordinate system or a metric of this kind. What we can do, however, is checking out onto which of our geometric surfaces a particular arithmetic relation (or set of relations) best maps. This one can do according to the de-re-interpretation procedure.

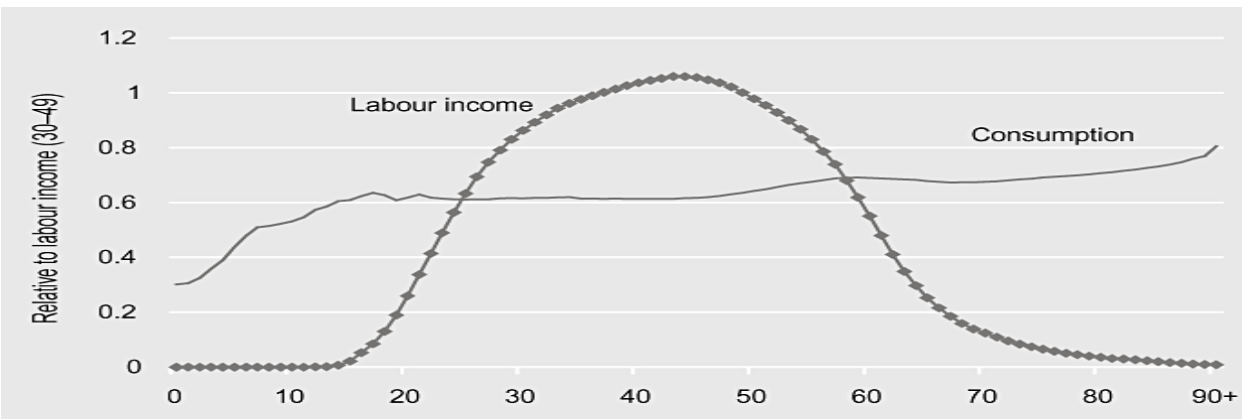
First, we de-interpret our axiom system so that it only contains logical terms and un-interpreted elements. Then we add tuples of numbers, intended to represent shapes and positions in a Cartesian coordinate system. When we have come this far, we can think about our geometries in terms of different curvatures. A flat space has no curvature, a sphere has a constant curvature, and hyperbolas have different curvatures at different positions. As in Poincaré's thought experiment, we can then try to work out which surface best maps our tuples of numbers. If you have a triangle of lines A = (x_1x_2), B = (y_1y_2), C = (y_2x_1), you can measure the sum of the internal angles. If the sum is exactly 180° the surface is Euclidean, if it is more than, or less than 180°, the surface is spherical or hyperbolic.

However, there are also differences between geometry and arithmetic and not all information crosses over. For instance, there is no geometrical correlate to the arithmetical negative numbers as there are no geometric objects of negative size and no possibilities of a negative distance. Phoronomically, a negative number can imply a motion in a particular direction, but no "negative" motion. Thus, there are things we can do with numbers that we cannot do with spatial objects. Therefore, going from arithmetic relations in a Cartesian system to geometry in this way is only appropriate once we know that the arithmetic relations represent geometrical content to begin with.

III 4.4.2.1. Geometric imaging of non-geometric functions

In the previous section, we saw how Pythagoras' theorem can be "generalized" for multiple surfaces through the process of de- and re-interpretation. Importantly, we needed already to have our geometric knowledge at hand for the generalization to be possible. As long as we know which of the lines A, B, C is the hypotenuse, the arithmetic de-interpretation translate back to geometry. If we do not know, the relation does not translate.

This points us to a further issue concerning the relationship between arithmetic, algebra and geometry. Infinitely many relations in the world have algebraic and arithmetical expressions as well as geometrical representability, but have nothing to do with geometry as the study of space. Obvious examples are age/income relations, education/income relations, predator/prey relations and so on. The basic idea here is that we have two variables, x and y, where one is expressible as the function of the other y = $f(x)$. The most famous such relation in physics is perhaps Newton's second law, (F = ma), where the force is expressed as the product of mass and acceleration. We can illustrate these functions geometrically, and thus visualize them. Below is an example of how we can illustrate geometrically income and consumption as functions of age:



Mason et al. (2010)

This way of expressing data is more helpful as an illustration than the original data set or the approximated Gaussian function. Nevertheless, such illustrations do not teach us about geometry apart from the fact that we find illustrations of this kind, within Euclidean plane geometry, to be illuminating. From our Kantian perspective, we can reasonably assume that the visualization is helpful, not only because we can see the actual distribution, but also because we can imagine how it would look if things were slightly different.[46] We can think of this illustration approach as using geometry as a pedagogical tool, and that this tool is effective due to our familiarity with spatial relations prior to any axiomatic knowledge.

[46] For instance, one could generate hypotheses about how debt and wealth accumulation make it possible to maintain consumption at an older age even though income decreases.

The case of age/income/consumption is an obvious example of a geometric illustration of a non-geometric content. Other cases are not that obvious. Consider for instance the Newtonian law of gravity, which relates gravitational force, two masses and the distance between them, $F = G\frac{m_1 m_2}{r^2}$. According to our description of geometry as abstracting from physical forces, gravitation is a dynamical or mechanical matter and not one that affects geometry. On Einstein's practical geometry approach, the answer is not so straight forward, since there it becomes a question more of convenient expression than about categorical separation. If we include the gravitational force into the geometric description and thus "remove" the mechanical aspect, we would be "geometrizing physics" into a field of pure prediction and modelling. In such a physics, actual physical content would ultimately be absent. Einstein famously rejected this approach from the beginning, but his theories are still sometimes supported by such a procedure. Consider for instance Stachel (1983) and his treatment of the light principle in relativity.

III 4.4.2.2. Geometrizing physics

One issue relating to special relativity is the idea that there is an upper speed limit for any direct measurement. Typically, one also considers this as the upper limit for the relative motion between any two objects whatsoever.[47] Stachel (1983: 255-272) aims to treat this issue as a purely kinematical problem in order to avoid problems concerning optics and the physical functioning of the measurement apparatus. His idea is to establish the upper limit without reference to the light principle and only using line segments, inertial frames and inertial (non-accelerated) motion.

> I shall show – sometimes only in sketchy outline- that one may use the concept of a set of ideal entities in rigid inertial motion (RIM) with respect to each other as a sufficient foundation for SRT [special relativity]. (Stachel 1938: 258)

[47] We treat this in detail in part 4.

An important aspect of Stachel's approach is the focus on ideal entities. Stachel's aim however, is not to construct a physical theory of real objects in real relative motion, but a kinematical theory. For such a theory, we need not worry about ideality, as every element is purely mathematical.[48]

Stachel defines a unit length by marking two points on a body and drawing a straight line through them (ibid: 261) and thus secures the ideality of the entire set-up. In other words, we are working purely a priori.

Stachel's choice of calling such line segments "rods" might lead to a confusion concerning whether we are discussing ideal or real objects. In order to avoid such confusion, I shall refer to them as "line segments". Stachel then follows the standard procedure of discussing the relation between Cartesian coordinate systems within classical space and time, in relative (uniform) motion. As we shall see, Lorentz and Einstein tried to explain why we could describe the measured metrical relations between physical entities in relative motion by a particular mathematical equation, which includes the upper speed limit. As Stachel is working purely in the ideal, he need not worry about this. The stated aim of the approach is to establish the equation in pure kinematics so that it can function as a limiting factor for dynamical physical theories (Ibid: 255-260).

Stachel attacks the problem in what is now a familiar manner. First we establish definitions for length and time coordinates in a single system K, then we add a system *k'* in relative (uniform) motion to that system and establish length and time coordinates. Finally, we establish a common set of length and time coordinates for the entire set-up. At this point Stachel has to choose between two distinct alternatives. He can introduce a maximal velocity for the Cartesian systems, or he can assume the possibility of instantaneous (infinite) motion. Stachel opts for the introduction of a maximal velocity (Ibid: 268) and shows how this relates to contractions of the unit length line segments. Stachel concludes:

[48] Stachel has already assumed the concept of rigid inertial motion, so whether he further on considers ideal or real objects, he has already tacitly presupposed everything that follows with the assumption of rigid inertial motion, i.e. original space, time and simultaneity, as well as the universally applicable distinction between uniform rectilinear and accelerated motion.

> This symmetry group fully characterizes the pseudo-metrical structure of Minkowski space (Ibid: 268).

It seems as if Stachel is performing a paralogistic argument here, where the conclusion is postulated rather than generated. I will let Galileo demonstrate another example of the same kind of problem. In Galileo's *Dialogue over the two chief world systems* (1632: 161-163), the character Salviati accuses Aristotle, Ptolemy and the Peripatetics of paralogistic argumentation in the form of *presupposing the middle term*. The protagonists have been discussing the possibility of the earth being in motion as a prelude to the discussion over the assumed supremacy of the Ptolemaic system. The Peripatetic philosopher Simplicio has presented the supporting argument for a motionless earth and is now under attack. I shall quote the entire segment:

> **Salviati:** Aristotle says, then, that a most certain proof of the earth's being motionless is that things projected perpendicularly upward are seen to return by the same line to the same place from which they were thrown, even though the movement is extremely high. This, he argues, could not happen if the earth moved, since in the time during which the projectile is moving upward and then downward it is separated from the earth, and the place from which the projectile began its motion would go a long way toward the east, thanks to the revolving of the earth, and the falling projectile would strike the earth that distance away from the place in question. Thus we can accommodate here the argument of the cannon ball as well as the other argument, used by Aristotle and Ptolemy, of seeing heavy bodies falling from great heights along a straight line perpendicular to the surface of the earth. Now, in order to begin to untie these knots, I ask Simplicio by what means he would prove that freely falling bodies go along straight and perpendicular lines directed toward the center, should anyone refuse to grant this to Aristotle and Ptolemy.
>
> **Simp**. By means of the senses, which assure us that the tower is straight and perpendicular, and which show us that a falling stone goes along grazing it, without deviating a hairsbreadth to one side or the other, and strikes at the foot of the tower exactly under the place from which it was dropped.
>
> **Salv.** But if it happened that the earth rotated, and consequently carried along the tower, and if the falling stone were seen to graze the side of the tower just the same, what would its motion be?
>
> **Simp.** In that case one would have to say 'its motions' for there would be one with which it went from top to bottom and another one needed for following the path of the tower.
>
> **Salv.** The motion would then be compounded of two motions; the one with which it measures the tower, and the other with which it follows it. From this compounding it would follow that the rock

> would no longer describe that simple straight perpendicular line, but a slanting one, and perhaps not straight.
>
> **Simp.** I don't know about its not being straight, but I understand well that it would have to be slanting, and different from the straight perpendicular line it would describe with the earth motionless.
>
> **Salv.** Hence, just from seeing the falling stone graze the tower, you could not say for sure that it described a straight and perpendicular line, unless you first assumed the earth to stand still.
>
> **Simp.** Exactly so; for if the earth were moving, the motion of the stone would be slanting and not perpendicular.
>
> **Salv.** Then here, clear and evident, is the paralogism of Aristotle and Ptolemy, discovered by yourself. They take as known that which is intended to be proved.
>
> **Simp.** In that way? It looks to me like a syllogism in proper form, not a *petition principii*.
>
> **Salv.** In this way: Does he not, in his proof, take the conclusion as unknown?
>
> **Simp**. Unknown, for otherwise it would be superfluous to prove it.
>
> **Salv.** And the middle term; does he not require that to be known?
>
> **Simp**. Of course; otherwise it would be impossible to prove *ignotum per aeque ignotum*.
>
> **Salv.** Our conclusion, which is unknown and is to be proved; is it not the motionlessness of the earth?
>
> **Simp.** That is what it is.
>
> **Salv.** Is not the middle term, which must be known, the straight and perpendicular fall of the stone?
>
> **Simp.** That is the middle term.
>
> **Salv.** But wasn't it concluded a little while ago that we could not have any knowledge of this fall being straight and perpendicular unless it was first known that the earth stood still? Therefore, in your syllogism, the certainty of the middle term is drawn from the uncertainty of the conclusion. Thus you see how, and how badly, it is a paralogism. (Galileo 1632: 161-163)

Galileo's argument is clear, and it fits well, also in the case of Stachel (1983). The stated aim was to show that we could demonstrate the upper speed limit from ideal entities in rigid inertial motion. If we can establish that the theory follows from purely kinematical considerations, we could geometrize the relation and thus think of it as a characteristic of space. However, at the interesting point of the argument, i.e. where we choose to adopt a maximal velocity or not, Stachel offers no argument. He simply chooses to adopt a maximal velocity:

> If K [constant linked to the maximal velocity] is infinite, we get the Galilei transformation laws of Newtonian mechanics, which are thus a degenerate case. If K is finite, set $K=V^2$ giving a one-parameter family of groups of transformations each of which is formally similar to the usual Lorentz transformations, except that V plays the role of the fundamental velocity (Stachel 1983: 268).

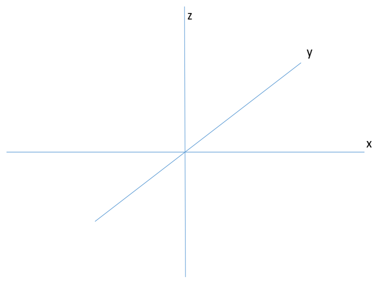
Initially, we set out to decide whether the Lorentz transformations reflect dynamical or kinematical relations. Stachel showed that in a purely kinematical treatment, we could choose between the Galilean and the Lorentz transformations. If we opt for the Lorentz transformations, we can further show the familiar deformations of length and duration. As in the case of the Peripatetics and the ancients, we have therefore simply *chosen* our conclusion and paralogistically presented it as a demonstration.

Attempts, such as Stachel's, to geometrize physics were commonplace in the 20th Century. Even if they had been successful, they would have been problematic since the basic assumption is that anything that has a universal character, i.e., anything that applies generally, is geometric or kinematic, rather than physical (see Janssen 2009 and part IV of this thesis). Read realistically, such approaches imply the metaphysical claim that there are no universal physical relations. This metaphysical claim demands some justification. On our Keplerian and Kantian set-up, geometrizing physics is simply a category mistake where properties and behaviors of physical entities are treated as properties of space. Furthermore, what we typically end up with when following the geometrizing procedure, are geometries that we cannot model.

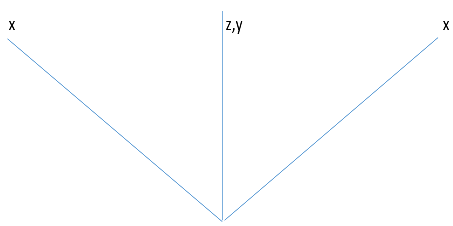
III 4.4.2.3. Higher dimensional geometries

We have already seen how Stump argued for the consistency of "non-Euclidean" geometries by referring to their successful modeling in Euclidean space. So far, we have treated only geometries that one can model in this way. All such models are two-dimensional. However, if one wishes to create an alternative model of space itself, this model has to have more than two dimensions. We then reach a problem, as one have been unable to model any higher-dimensional non-Euclidean "geometries". Sandmel (2004: 167-184) tackles this issue and makes an illuminating point.

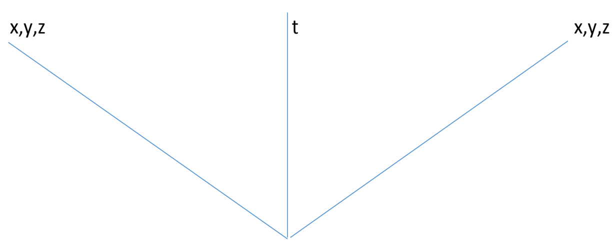
Whenever we are presented with a "model" of a higher-dimensional "geometry" such as Riemann or Minkowski's structures, we are instructed to collapse at least two dimensions into one. The standard rhetoric is that the collapse is there for simplicity. We thus easily forget that the higher-dimensional version cannot be modeled. Take for instance the standard description of Minkowski space-time "geometry". Starting out with a three-dimensional Cartesian coordinate system



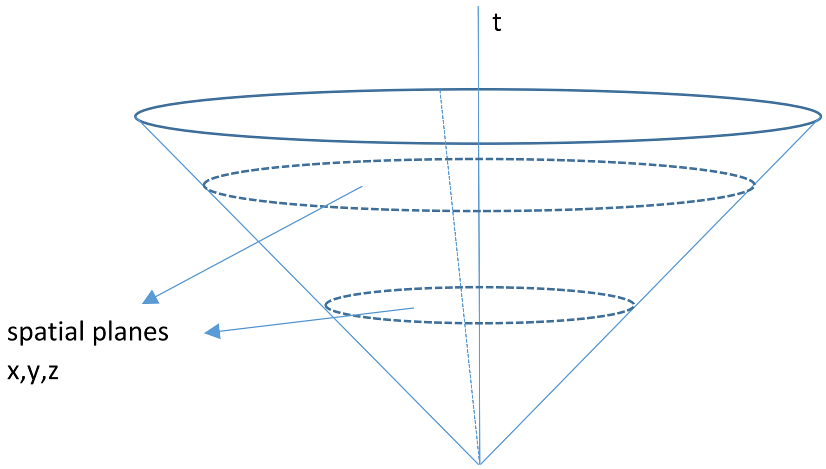
We then introduce the maximum speed limit (as discussed in the previous section). We do this by first removing the "depth" or y-dimension of our illustration and then inserting a spatial boundary. In order to maintain "three-dimensionality", we can consider the z-axis as representing both "depth" and "height". Furthermore, if we want the x-axis to illustrate the speed limit, we may tilt it accordingly.



All we need now is to introduce time, as the "speed limit" makes no sense outside a temporal framework and the "geometry" presented here is supposed to be a space-time "geometry". We do this by moving our z- and y-axes over to the x-axis and inserting "time", t, at the former z, y, axis.



We now have a two-dimensional model of the Minkowski space-time “geometry”. Still, “space” in this model is reduced to a single dimension. We can redeem ourselves slightly by expanding into a two-dimensional space if we make the model three-dimensional and thus get spatial planes



Importantly, we cannot go any further than this, because our form of outer experience, in which we make the initial constructions, does not allow for more than three dimensions. Nevertheless, the dimension-cutting procedure used here – using one dimension as representative of multiple dimensions – informs us somewhat on what the general idea is. For instance, we can see how the edges of the cone represent maximum displacement at any time and that, as time increases, one can move further. As a geometric illustration of the idea that there is a physical speed limit, the above diagram does a decent job. However, it cannot be a model of “space” as such since there is a dimension missing. This is a common feature of all higher-dimensional “geometries” as opposed to their two-dimensional counterparts. We can model two-dimensional surfaces with curvature by representing the curve in a third dimension. For a three-dimensional space with curvature, we would need a fourth dimension. So why do we not just introduce the fourth dimension in our model? Our answer to this is simply that the modelling along with the dimension cutting and our projections are all constrained by our possible imagination within the form of outer experience. This is where we initially model Euclidean figures as well as “non-Euclidean” surfaces. This imagination, along with our form of outer experience, is constrained to three dimensions.

We may now ask whether limited illustrations that cannot model the intended entities without removing a central part of space (at least one spatial dimension), are geometries at all. The fact that we can only make quasi-spatial models indicates that they are not. Einstein (1921) argues that although the “spaces” in such “geometries” cannot be modeled, we can visualize the procedure by which they can be constructed. As we have seen, however, visualizing cannot be the touchstone of a geometry. We can easily visualize the age/income/consumption relation form above, but that in itself is not enough to call it a geometry. It is simply an algebraic function expressed arithmetically and illustrated geometrically. Furthermore, our ability to visualize a procedure would anyway be insufficient. I can visualize a procedure by which I continuously add dimensions to my originally three-dimensional Cartesian system. This process could go on indefinitely without constituting a geometry. What we need above visualization is to maintain our capacity for spatial orientation, which is, as shown in our proprioception, closely connected to our existence as embodied physical objects. This capacity is lost in the higher-dimensional “geometries” where dimension cutting is always necessary. In light of these reflections, we shall take another look at Poincaré’s sphere-dwellers.

III 4.4.2.4. Revisiting Poincaré’s sphere-dwellers

Poincaré’s example is intended as an illustration of the relationship between physics and geometry, the overall idea being that geometry and physics are so closely linked that one does not make sense without the other. Furthermore, there will always be more than one possible option. In the case of the sphere-dwellers, they can either assume a Euclidean geometry + a universal force, or a “non-Euclidean” geometry + absence of force. Poincaré argued for pragmatism in such cases and suggested the sphere-dwellers should opt for the more convenient, “non-Euclidean” option.

Throughout this thesis, I have argued within a transcendental tradition and made use of Galilean idealizations or abstractions. The central methodological constraints are that one should arrive at the field of inquiry by abstracting one’s way down from the phenomena and make sure one does not add anything external on the way. The benefit of this method is that we are sure that we can work our way back to possible experience and thus maintain epistemology of science as a subfield of general epistemology. All of Galileo’s thought experiments work in this way. Poincaré’s thought experiment is of an entirely different nature.

Here we are asked to imagine a sphere (with which we are familiar), three-dimensional beings, and a universal force acting on the length of measuring rods (which we know from standard thermal expansion). The force, however, is such that the further away from the center we get, the colder it gets and the shorter the measuring apparatus gets. Thus, when we approach maximum distance from the center, the measuring apparatus approaches non-extension and the temperature approaches absolute zero. This is, of course, not how thermal expansion works in our world.[49] Contrary to our Galilean method, we are now working within a wholly imagined universe with a different physics. As such, the sphere-world does not inform us about our possible experience. Nevertheless, it provides us with some information. Primarily, as we know that physical forces act on our measuring apparatus, we should take care to factor that action in, when we review our measuring results. This is the "mechanics" part of our Keplerian set-up. Secondly, it is revealing that when we wish to present thought experiments of this sort, or when we wish to construct geometrical illustrations of algebraic functions, we invariably do so in Euclidean space. As soon as Poincaré wishes to guide us out of the Euclidean space, he introduces non-geometrical content, i.e., a physical force. Treating this physical force as geometric content is simply to geometrize the actual physics in a skewed manner, as the resultant "non-Euclidean" (geometry + physics) contradicts the initial Euclidean geometry used to set up the entire system: "Suppose, for example, a world enclosed in a large sphere…" If, on the other hand, we reject the geometrizing and maintain that there is a force acting on the sphere-dwellers and their measuring apparatus, we avoid the contradiction altogether. We are then still within our Euclidean geometry. Importantly, however, the best possible approach for the sphere-dwellers is to maintain their "non-Euclidean" geometry, but not as a theory of space. Rather, the "non-Euclidean" system should be used as a mapping of how temperature influences their measuring apparatus. In this way, they can maintain both a viable geometry and an effective method for comparing measurements.

Stump (2015: 60) introduces Poincaré's sphere-dweller parable as "… part of an argument to prove that a non-Euclidean world is imaginable". This introduction is severely misleading. The "world" in which the sphere-dwellers reside is Euclidean from both perspectives. From the perspective of the sphere-dwellers it is Euclidean and infinite, from the perspective of an outside observer, the

[49] Living beings would not be able to act at absolute zero and although materials expand due to heat increase, they do not shrink indefinitely when it gets colder.

world is Euclidean and limited by an external boundary. There is no reference whatsoever in Poincaré's thought experiment to a non-Euclidean world.[50]

What Poincaré distinguishes between are the differences in methodology applied by us and the sphere-dwellers. Our idea of metric geometry implies what Poincaré calls Euclidean displacement, which means that the measuring apparatus remains exactly the same no matter how or where it moves. I.e., we imagine our measurements as made with rigid objects. As we know, the objects that sphere-dwellers use do not have this quality. Their equipment changes as it moves and thus suffers what Poincaré calls non-Euclidean displacement (Poincaré 1902: 78). In other words, the different "geometries" in Poincaré's example refer to different methodologies, not to different resultant structures or "worlds". When Poincaré tries to give an example of a non-Euclidean "geometry", such as the Riemann structure, he immediately cuts dimensions:

> Riemann's Geometry.—Let us imagine to ourselves a world only peopled with beings of no thickness, and suppose these "infinitely flat" animals are all in one and the same plane, from which they cannot emerge. (Poincaré 1902: 45)

In neither case is Poincaré showing that a "non-Euclidean world" is imaginable. What Poincaré is arguing throughout is that we cannot measure our way to the true geometrical structure. Such measurement always involves postulates concerning the measuring apparatus, and we could always make different postulates. In other words, Poincaré is arguing that if we wish to find the true geometry – which he does not believe to exist – physical measurement will never yield anything more than a conventional result.

III 4.5. Minkowski space-time "geometry"

Poincaré's thought experiment is to some extent analogous to the development of Minkowski's celebrated four-dimensional "geometry". The Minkowski structure is a combination of physical measurement and a formal axiomatic approach to geometry. We shall look at the actual

[50] The misunderstanding is fully understandable as the heading of the section is "The Non-Euclidean World". Nevertheless, the world Poincaré invents, is fully Euclidean.

measurements in part IV, for now we shall look only at the mathematical set-up. As per the standard formalist axiomatic procedure the Minkowski structure is established from arithmetic, i.e., from a set of tuples of real numbers (R). The two-dimensional “non-Euclidean” geometries are representable as two-dimensional “surfaces” in R^3, which indicates two spatial dimensions and a “curvature”. Euclidean geometry is typically referred to as a “flat” R^3 “space”. A three-dimensional “geometry” with “curvature”, such as the full Riemann structure, is a R^4 “space”. The Minkowski “space”, which has a “flat” space of three dimensions plus time, is an R^4 structure. One could equally well represent Newtonian laws of motion – Euclidean space plus absolute time – as another R^4 structure. However, as we are treating geometry as the study of space, the R^4 version of Newtonian laws of motion is not a geometry, but rather an arithmetically expressed kinematic. We treated the Kantian, phoronomic correlate in part II.

As it is impossible for us to construct a full model of a four-dimensional space, the Minkowski “space” is typically represented by a two-dimensional geometric illustration where all spatial directions are reduced to a single dimension, “x”. “x” in the illustration thus represents the real numbers (x, y, z). The temporal dimension is kept separate in the illustration as “t”. Before arriving at the coordinates, however, one must establish a space in which to place the coordinates. As per the standard procedure, the entire set-up rests on Euclidean geometry. The kinematical part of Einstein (1905) starts by stating:

> Consider a coordinate system in which Newton’s mechanical equations are valid. To distinguish this system verbally from those to be introduced later, and to make our presentation more precise, we will call it the ‘rest system’. If a particle is at rest relative to this coordinate system, its position relative to the latter can be determined by means of rigid measuring rods using the methods of Euclidean geometry and expressed in Cartesian coordinates. (Einstein 1905: 125)[51]

Here we see how Einstein sets up an absolute space (the rest system) and applies Euclidean geometry within it. Then we can express the Euclidean geometry arithmetically as tuples of numbers representing positions in a Cartesian coordinate system. We get the first three real numbers of the R^4 structure by this procedure. Einstein goes on to say that if we want to involve

[51] By defining the “rest system” in terms of inertia and Newtonian physics, Einstein implicitly brings along the classical notion of rectilinear motion as well as Galilean rules for velocity addition.

motion, we must express the coordinates as functions of time, thus completing the R^4.[52] So far, we have a standard kinematic or phoronomy built on Euclidean geometry. Einstein maintains this Euclidean, Galilean and Newtonian base when establishing the mathematical correlate of his measuring apparatus.

> Let there be two coordinate systems in the 'rest space', i.e., two systems of mutually perpendicular lines originating from one point. Let the x-axes of the two systems coincide, and their y- and z-axes be respectively parallel. (Einstein 1905: 130)

What we have here are two Cartesian coordinate systems at relative rest. As we have already seen, such systems require determinations of straight angles as well as straight lines, which are ultimately only possible from the directional definition of straightness. "Parallel" here, refers to our classical notion of parallel as "drawn in the same direction".

> Each system shall be supplied with a rigid measuring rod and a number of clocks, and let both measuring rods and clocks be exactly alike. Now, put the origin of one of the two systems, say *k*, in a state of rest with (constant) velocity v in the direction of increasing x of the other system K, which remains at rest; and let this new velocity be imparted to *k'*s coordinate axes, its corresponding measuring rod, and its clocks. (Einstein 1905: 30)

Here, Einstein is making use of our natural notion of time. The motion is to be imparted on the coordinate axes and all elements enclosed by them. He further establishes this on the next page by insisting that, while moving, the axes of the two systems remain parallel. Note that this means that no matter how far you draw out the axes, any motion of the axis is simultaneous with any motion of any other point in the entire system, thus establishing universal simultaneity. Furthermore, Einstein picks out a particular state of motion, which corresponds to his mechanical definition above. The moving system *k* moves at a constant velocity in a single direction (along increasing x). I.e., what Einstein considers, are inertial systems. Einstein is seeking to establish a

[52] Einstein, of course, did not consider the R^4 structure in his mathematical establishment of 1905. Minkowski established this structure three years later. What we are getting at here, is that to the extent that the Minkowski R^4 «space» relies on special relativity, it relies on Euclidean geometry.

mathematical correlate for physical measurement. Mathematical "rods" and "clocks" as indicators of various positions that can be represented arithmetically as four-tuples of real numbers are inserted for this same reason. However, as it is well known that accelerated motions affect physical entities, and Einstein wants to maintain that the entities remain rigid, he must initially consider only systems in inertial motion. The same argument applies to the frames of reference within which "rods" and "clocks" are defined. However, the mechanical definition does not apply within the strictly mathematical realm, and we saw in part II that the phoronomic principle of relativity applies to all systems. Inertial systems, which are defined as not suffering from dynamical or mechanical influence, are initially conceived of as behaving identically in mechanics and in phoronomy or kinematics. Just as in our Kantian phoronomy, any particular such system can be treated as "moving" or as "at rest", but in all cases there must be a "rest system" or absolute space from which we consider the moving system. Recall also that the reason why we can treat any particular system as at rest or as moving is the phoronomic principle of relativity. As we saw in part II, the phoronomic principle of relativity relies on our ability to imagine any motion as "cancelled out" by the motion in the opposite direction. We express this possibility arithmetically as total velocity $w = v_x - v_x = 0$. As we saw in part II, this way of adding and subtracting velocities is a condition of possibility for establishing any magnitude of motion, and it relies on the intensive property of speed along with directionality.

What we have then, is what is typically referred to as a Galilean kinematics: Universal simultaneity, Galilean rules for velocity addition, and Euclidean space. The homogeneity of the space and time considered are simply postulated by Einstein later on when he considers which types of equations one must use in order to relate different measurements made in the rest system and in the moving one.

> First of all, it is clear that these equations must be *linear* because of the properties of homogeneity that we attribute to space and time. (Einstein 1905: 31)

In conclusion, Einstein conceptualizes the measuring apparatus within Galilean kinematics. From this system, one can enter into arithmetical description of relations between coordinates. The *actual* coordinates and their internal relations should be collected from empirical research as in Poincaré's thought experiment. We must therefore find physical objects corresponding to the mathematical notions we have established here. As we saw in Poincaré's thought experiment, we already know that physical interaction affects physical measuring apparatus and that we need physical conventions in order to establish a measuring procedure. We must therefore look closely at how the measuring apparatus is constructed and to which extent it reflects geometric content.

III 5. Summing up part III

Our initial aim was to defend the Kantian notion of geometry as synthetic a priori against the standard objection from contemporary philosophy of science. The standard objection is that "non-Euclidean geometry" proves that mathematics is not a priori. What we have seen, however, is that "non-Euclidean" geometries - studies of the properties of non-flat surfaces – are all constructed within our form of outer experience. Furthermore, in order for the "non-Euclidean" geometries to be treated as axiomatic systems in apparent contradiction to Euclidean geometry, we must "reassign" meaning to the central terms. Invariably, the idea is that we must establish "straight lines" from a metric definition of "shortest distance". This, again, presupposes that we have a system of measurement for distances, which presupposes a Cartesian coordinate system, which is only functional as long as it is constructed with straight lines. In other words, the metric definition is viciously circular unless one accepts "straightness" on the Cartesian coordinate system as intuitively given. Following the intuitive path, however, defeats the overall argument. An easier way out, is to drop the "reassignment" procedure and use a standard terminology. We then end up with statements like "there are no parallel great circles on a sphere", which do not contradict Euclid. Rather, as the spheres in question are figures in Euclidean space, we see how the overall set-up presupposes the validity and truth of flat Euclidean geometry. The resultant structures can therefore not replace Euclidean geometry, although they definitely expand our geometric knowledge. We have also seen that the formalist axiomatic approach is helpful in illustrating how different surfaces and mathematical branches relate to each other. However, we should abandon the formalist axiomatic approach as an approach to mathematical foundationalism, where it arguably constructs more problems than it solves.

As for Non-Euclidean "geometries" –geometric illustrations of arithmetical and algebraic relations – we have seen that these might or might not express geometric content. In some cases, such as when the arithmetical structure expresses socioeconomic relations, the resultant visualizations are simply geometric illustrations of our non-geometrical knowledge. Stump (2015) argued that "non-Euclidean" geometries are proven consistent once we accept the "reassignment" procedure and construct models in Euclidean space. A prerequisite for such modelling is that we only treat two-dimensional surfaces (thus allowing us a third dimension in which to imagine the curvature of that surface). Any model of a higher dimensional structure demands what Sandmel (2004: 167) refers to as "dimension cutting".[53] There is no possible such modelling of the full structure. This is also the case for the Minkowski R^4 structure, which is the standard mathematical representation of special relativity. However, we have also seen that in order to get to this structure, we have based the entire mathematical correlate for the measuring apparatus on classical time with universal simultaneity as well as Euclidean geometry. Thus, the kinematical set-up is the well-known Galilean kinematics. As we go into the thought experiments of special relativity in part IV, we should also be aware that they all follow the following structure:

1) Set up a Cartesian coordinate system K, based in Euclidean geometry with a directional understanding of straight lines.

2) Consider another Cartesian coordinate system *K'* set-up in Euclidean geometry with a directional understanding of straight lines, which is moving relative to K.

What we must remember when looking at such thought experiments, is that the notions of motion and velocity involved, all contain the standard rules for velocity addition that we presented in part II, section 3.0. Any time we determine a velocity, we therefore imply these rules. In other words, as we move into the realm of physics, we must remember to bring with us the basic insights of phoronomy. According to our Keplerian philosophy of science and the coherence criteria established in part I, we should try to maintain the Galilean kinematics throughout. Otherwise, we will end up with two systems rather than one, which breaks with the overall Unitarian ideal. If we introduce a new kinematics or "geometry" for objects in the world, we must separate these objects

[53] "Dimensjonsavkorting»

from the ones that constitute the measuring apparatus, since the latter are defined as Euclidean in special relativity. Einstein was among the few who would admit to this problem:

> It is striking that the theory (except for the four-dimensional space) introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things … (Einstein: 1949: 55)

From the perspective of philosophy of mathematics therefore, we can maintain our transcendental approach and reject the standard position that it has been proven obsolete.

Part IV

Physics

So far, I have argued for a Keplerian approach to scientific realism, which includes the need for a metaphysical vantage point. As such a vantage point, I have presented my version of Kantian metaphysics with emphasis on Kant's transcendental philosophy as explicated in the *Metaphysical Foundations of Natural Science* (Kant: 1786). Following the standard distinction between different areas relating to physics we have abstracted our way down from the phenomena through mechanics, dynamics and phoronomy, and in some detail, geometry. In the geometrical part we found good reasons to maintain the Kantian notion of geometry as a synthetic a priori field of inquiry where the most basic information is revealed through our constructions in representative space, or what we have called the form of outer experience. In building our way up again, we saw that the Minkowski structure, if conceived as a geometric structure, has a dualism built into it. The dualism consists of the fact that the thought experimental "measurements" from which the structure is built, are constructed within Euclidean geometry, Galilean velocity addition, and a classical theory of time.

As indicated in part I, there are two standard objections to transcendental philosophy in contemporary physics. The first is the idea that "non-Euclidean geometry" has proven Kant's transcendental philosophy of mathematics to be obsolete. In part III, we saw that there are good reasons to maintain the Kantian option and replace the, by now standard, formalist axiomatic approach.

The second standard objection to Kantian philosophy in modern physics is that special relativity theory proves that the Kantian theory is wrong. The reason being that since special relativity successfully applies a non-Euclidean space-time "geometry" based in empirical research, Kant's idea of space (and time) as the form of outer experience, and its structure, is obsolete. Part IV, is dedicated to challenging this objection and explaining why we should maintain the Kantian theory.

Dorato (2002), following Gödel (1949), argues that one can defend the Kantian theory by separating between "mental" and "physical" time. This is of course Sellars' (1962) "manifest" and “physical” time dressed up in slightly different terminology. This route leads to a separation between everyday experience and scientific theory that we have argued against throughout this thesis and which breaks with Kepler’s criteria of coherence. My approach to the issue is to argue that we have good reasons to abandon special relativity as a space-time theory altogether and replace it with a Kantian framework. In this way, we get to keep our *mental* or *manifest* time intact throughout physical theory, and thus resolve the dualism implied by special relativity as well as Dorato’s suggestion. In order to defend a Kantian framework for modern “space-time physics”, I will make two general observations.

> *Two general frameworks account for the relevant phenomena. The relativistic framework, which develops from Einstein’s “practical geometry”, and the Lorentz/Maxwell framework, which develops from our Kantian theory.*

We must therefore find out how these frameworks differ and which one is preferable. A step in this direction is to see where the frameworks are identical.

> *As established by Bell (1976), Abreu & Guerra (2005, 2006, 2008, and 2015) and Szabo (2010) there are neither mathematical nor empirical differences between Maxwell/Lorentz theory and special relativity.*

As we shall see, the differences between the theories are differences in theoretical physics as well as metaphysics. Following the Keplerian philosophy of science, we must investigate the coherence and plausibility of each theory in the metaphysical, mathematical and physical realm. I will argue that the Lorentz/Maxwell theory is superior on grounds of coherence.

We shall see that Lorentzian theories are constructed in a fully coherent way. The regressive abstraction process from phenomena left us with a priori elements of phenomenology, mechanics, dynamics, phoronomy and geometry. We now turn around and build the system back up progressively from geometry, through phoronomy, dynamics, mechanics and phenomenology. A central element in relation to coherence is that only the Lorentzian theory allows us to maintain the a priori elements we have established, and thus construct the phenomena coherently in a single argument without additional speculative hypotheses or postulates. We thus maintain the Keplerian philosophy of science (part I), the metaphysical theory (part II) and the geometrical structure (part III) and explain the relevant phenomena (part IV).

Special relativity, on the other hand, introduces a series of postulates of varying ontological plausibility. Among these are the postulated one-way speed of light for multiple systems, practically rigid bodies, a new norm for measurement, a "replacement" of velocity addition with translation, and a hard separation between physically real systems in inertial and slightly non-inertial systems. These postulates lead to a series of dualisms particular to special relativity. What we end up with are two sets of geometry (Euclidean for measuring apparatus and Minkowski for everything else), two sets of kinematics (one where the Galilean rules for velocity addition holds and one where they do not apply), two sets of dynamics (one for the measuring apparatus and one for all other things).

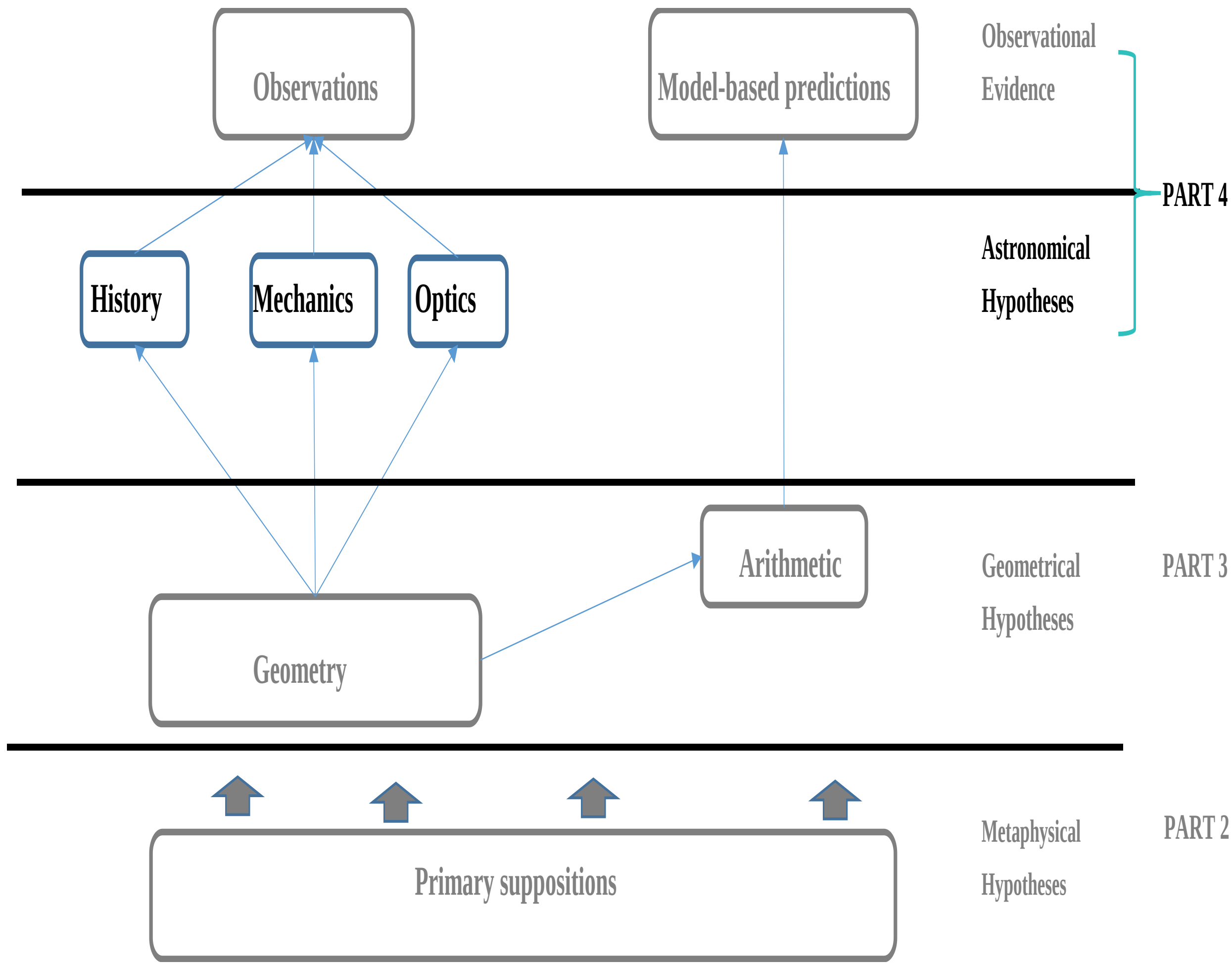
Our main concern will be with the postulate of "physically real rigid bodies". These are what Einstein called "practically rigid bodies" and they lead to the inherent dualism referred to at the end of part III. Furthermore, the theory postulates an ad-hoc norm for measurement that reduces our capacity to account for everyday phenomena and that therefore cannot apply generally. These and other postulates of varying physical as well as philosophical plausibility make special relativity a less coherent theory than the Lorentzian alternative, and I shall therefore argue in support of the latter.

IV 1. The structure of part IV

First I establish the scientific background of Lorentz theory and special relativity and focus on the two main problems that both theories intend to solve (section 2). I then show how Maxwell and Lorentz solve these problems in a coherent manner. We can therefore treat Maxwellian and Lorentzian theory as a single framework (section 3-5). We then move on to Einstein's view of Lorentzian theory and see how the special theory of relativity is constructed through empirical data and a series of postulates (section 6-7) and compare this solution to the Lorentzian theory (section 8). As most contemporary readings of special relativity differ philosophically from Einstein's original proposal, I shall treat the contemporary debate as a separate issue in section 9. Finally, I present some questions that have answers within Lorentzian theory, but are still open is special relativity. These are generally debated issues where there is no clear consensus within the received view (section 10). Before starting however, I make a short note on the notions of clocks, rods and measurement.

IV 1.2. On measurement

Below, we shall establish that the Lorentzian theories are empirically identical to special relativity. We know therefore that the difference is not at the level of observation. Concerning the mathematical structure, we have already seen that both theories perform thought experiments and build their measuring apparatus within Euclidean geometry and the classical theory of time. We shall see below that they also apply the exact same arithmetical structures for relating measurements done in different frames (Lorentz transformations). The difference is therefore of neither an empirical nor an observational nature. This leaves us with the physical aspect of the theories, i.e., what we have established in Kant as mechanics, dynamics, kinematics and phenomenology and the astronomical (here physical) hypotheses of Kepler's set up. This is where we shall find that the theories differ. Moreover, although we shall focus on the physical hypotheses, we must of course account for the other levels as we go along. The focus, however, will be on the measuring apparatus described as "mechanics" in Kepler's philosophy.



Since Aristotle, it has been a fundamental goal of scientific inquiry to arrive at general truths about natural processes and the objects that take part in them. Throughout this period, it has also been a basic requirement that we make measurements in order to validate such generalities. In other words, scientific inquiry involves interaction with the physical world. A measurement, like an experiment, is a particular kind of interaction, which, for its success, requires a set of standards. We need standardized equipment and standard units of measurements so that we can compare measurements made in different contexts as well as measurements made by different people. The benefit of standardized equipment is that we can test our expectation that the same type of measurement made with the same type of equipment yields the same type of result. The benefit of standardized units is that we can quantify our results in a synchronized manner.

Much of modern science requires thought experiments. Thought experiments have two central functions in the setting that we shall discuss. All physically realized experiments and measurements result from a thought experiment, however crude, performed by the scientist. “If I do X to system Y using equipment E, will I get result R?” Another function of a thought experiment is to move the inquiry a step further than what is available from immediate and direct empirical testing. Galileo’s thought experimental method which led to the law of fall is perhaps the hallmark of such a thought experiment. For although the empirically available results indicated that objects fall at different rates, Galilei proved by argument that all that was revealed through these empirical tests was the presence of an interfering factor, air resistance. As is well known, Galilei’s law of fall is empirically valid once we remove air resistance.

Galileo’s example shows a particular feature of the relation between hypotheses, thought experiments and empirically realized experiments. A naive Popperian might argue that if empirically realized experiments yield results that differ from those expected from theory (thought experiment and hypothesis) then the theory is falsified. As we have seen, Galilei argued that the role of the natural philosopher is to explain why the empirical results come out as apparently contradicting theory. In the case of the law of fall, his answer was air resistance. In a similar manner, Kepler rewrote Tycho Brahe’s observational data in order to establish the real positions of the planets as opposed to their apparent positions. In other words, modern science in general and modern physics in particular work under the assumption that our experiments and measurements need further justification in order to fit what we take as the true theory.

For an understanding of 20th and 21st Century physics, the concept of a *rigid body* is central. Roughly defined, a rigid body is a body that maintains its central properties throughout interaction. As we shall discuss physical theories and their importance for the philosophy of space and time, the most important aspects of rigid bodies is that they maintain their shape and size, and that regularly repeating events on the bodies remain regular throughout interactions. It is well known that there exist no physically real and universally rigid bodies in nature, so why do we still need them?

As with any theoretically idealized object, the function of a concept of rigid bodies is their ability to provide us with an ideal for measurement. We know for instance that physical rods suffer thermal expansion, and we can account for any inaccuracy of measurement by considering this expansion. What we are doing then is making our measurement fit the ideal of a measurement made with a rigid rod. Similarly, we know that even our most precise clock, the NIST-F2, is estimated to run slow by about one second per 300 million years. This clock is slow in relation to an ideal or rigid clock. Thus, we see that the function of a rigid body is to provide an ideal from which to evaluate physically real bodies and their abilities to measure spatial and temporal relations.

The most commonly known and used ideal for measurements of spatial relations among objects is a Cartesian coordinate system, which establishes a three dimensional system at rest. Typically, the three axes are equipped with points that are a unit distance apart. One can imagine any stable object placed within such a system and use the system to measure the total size of the object. In this sense, a Cartesian coordinate system functions as an ideal for any physical measuring rod. Similarly, one can imagine the "tics" of a rigid clock as points on a line where there is exactly one time unit of temporal distance between them. We thus establish ideal or rigid clocks and rods in theory.

A central question relating to our previous discussion as well as the remainder of this thesis is the relation between ideal or rigid rods and clocks and their physical approximations. An ideal clock has the following central property:

> The duration between one instance of an event on the clock and the next such event is stable. In other words, the tic-rate of a clock is regular.

An ideal rod has the following property:

> The distance between any two points on the rod remains the same independently of what happens to the rod. I.e., the rod maintains its shape and size whether we move it, kick it, rotate it or otherwise influence it.

No physical object has those ideal properties. A task for physics therefore, is to find the system that best approximates the ideal rods and clocks and to establish under which conditions that system loses its appropriateness. One version of such a clock, typically referred to as a Feynman clock, is a simple system of two reflecting surfaces (mirrors) and a light signal. [54]

Place the two mirrors such that their surfaces oppose each other, and let a light signal bounce back and forth.

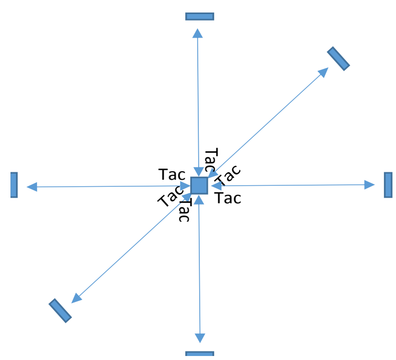
As the signal hits the right hand mirror, we can make a "tic" and as it hits the left hand mirror, it will make a "tac". A time-unit measured by such a clock is one "tic-tac". There are several reasons for why light-signal clocks like these are helpful for the further discussion, but before we look at these we must apply the above, general, assumptions about clocks to our light-signal clock. A well-functioning clock is independent of its orientation, which must also apply to the light-signal clock. This however, informs us that a further assumption of the light-signal clock is that the two-way speed of light (one tic-tac) is the same in all directions, given the standard assumption that distance is a directionally independent quality of space (the distance between the mirrors does not depend on the orientation of the system). Additionally, we assume that the position of the light-signal clock is without significance.

A central issue for the use of clocks is our ability to synchronize multiple clocks. In order to synchronize light-signal clocks, we need to know that the distance between the mirrors of the different clocks are identical and that the two-way speed of light is the same in all directions. The latter assumption follows from standard electrodynamic theory and is a common assumption of all theories discussed in this thesis. That the distances between the mirrors are the same follows from assumptions about the homogeneity of spatial directions presented in part III, and is a common

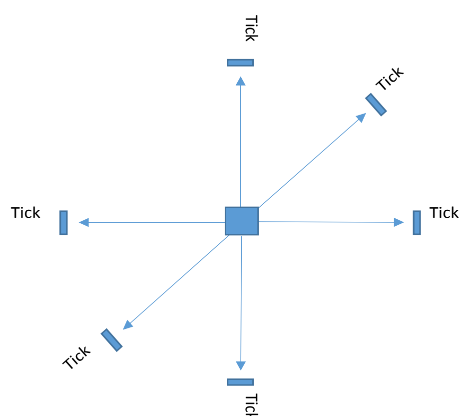
[54] Although very accurate, the NIST-F2 only functions under very specific conditions and is thus not a practical tool for measurement.

assumption of all theories that we consider.[55] Using light-signal clocks thus assumes a series of metaphysical assumptions that relate to the qualities of space and time. It is noteworthy that these assumptions correspond to conceptions of space and time that apply to Euclidean geometry and classical theories of time.

An interesting feature of light-signal clocks is that they rely on the two-way, or roundtrip speed of light rather than directly on the speed of light in a single direction. Einstein (1905: 126) *defines* the one-way speed as half the two-way speed and thus maintains that the speed is identical in all directions. How can such a definition be justified? The two-way speed can easily be tested by the following set up: Take three light-signal clocks and place them perpendicularly to each other.



If the "tacs" are simultaneous, the two-way speeds are identical. At a first glance, the one-way speed can be tested by checking if the "tics" are simultaneous:



[55] Below, we shall see that Lorentz argues that the distance between the mirrors relies on the relative motion between the light-clock and the ether. In other words, there are further theoretical justifications needed once we include the possibility that the measuring apparatus itself is moving.

The problem in this case becomes where to place the observer whose job it is to determine whether the tics are simultaneous. If we put the observer outside the center, the distances to different tics will be unequal. Assuming equal light speed in all directions, this observer will never measure simultaneous clicks, as the different tics will take unequal time to reach his position. On the other hand, if we place the observer at the center, we are again measuring the two-way speed. As measurements go therefore, all we have are measurements of the two-way speed.

In order to say something meaningful about the one-way speed, we must dip back into the metaphysical proposition that space is homogenous and thus that directionality does not alter distances, i.e. the distance Oslo-Copenhagen is the same as the distance Copenhagen-Oslo. This metaphysical assumption, in combination with the physical assumption that light speed is constant, provides us with the necessary tools for discussing the one-way speed of light. As we shall see in section 7.3 below, Einstein maintains that the concept of simultaneity does not have an absolute meaning, implying that different measuring systems will disagree on whether two events are simultaneous or not. This is what we also saw in the above example concerning the simultaneity of the "tics". However, our inability to make measurements agree across systems does not alter the underlying phenomena. By this, I mean that whether or not we can measure it, there is a question of whether the tics are simultaneous or not.[56]

The question of the one-way speed relies on our understanding of the concept of simultaneity and the role of this concept within theories of time. A common feature of simultaneity common to all such theories is that "simultaneous" is distinct from "before" and "after" and that these three notions are the only ordinal notions concerning time. This implies that any event A is either before, or after or simultaneous with another event B. In our case, we cannot simply claim that these notions are frame-dependent in the sense that it all depends on where we measure from, as the question is about that which we measure, not about how we measure it. We cannot measure the one-way speed as of now. Nevertheless, in order for there to be a two-way speed, there must be a one-way speed of the light signals involved. We get no "tacs" if there are no "tics". The "ticing"-events represent the one-way speeds in different directions. Independent of our measurement there

[56] Otherwise, we must contend that there was no gravity prior to Newton's theory.

must be a state of affairs concerning the simultaneity or non-simultaneity of the tics. Let us consider the options.

If the tics are simultaneous, our initial assumptions hold. Distances are not dependent on orientation and the speed of light is constant. If they are not simultaneous, one of the assumptions is false. Either distances depend on orientation and we must construct radically new theories of space and time, or the speed of light differs in different directions and we must start over on the physical theories. Both scenarios are devastating to special relativity as well as Lorentzian theories.

The only plausible option therefore, is that the tics are simultaneous and that the one-way speed of light is constant. What does that tell us? It tells us that the one-way speed of light is constant in at least one inertial system, i.e., there is at least one state of motion that is such that if you are in it, light travels at the same speed in all directions. It does not tell us that this is the case in all systems though. Rather, if we can find a system in which the one-way speed of light is constant in all directions, the one-way speed will be unequal for different directions in all other systems.

We shall see that the principle of relativity prevents any possible knowledge of whether we are "at rest" or moving uniformly. If we apply this principle to the one-way speed of light, we see that we cannot know whether we are moving uniformly or if we are at rest relative to the system in which the one-way speed of light is constant in all directions. Thus, any theory that assumes the metaphysics of homogenous space, the validity of the principle of relativity for all measurement and the constant speed of light, must also assume there exists a system in which the tics are simultaneous and that we cannot know whether we are moving relative to this system.

Regardless of our position on the one-way speed of light, the Feynman-clock gives a good approximation to a rigid clock and it has the further upside that once the clock is established, we can use the same theoretical elements to establish measuring rods. We know that the two-way speed of light is constant. We can therefore calculate the distance between the mirrors by using that constant in the following fashion. Distance = the time needed for one tic-tac/ speed of light. We can use this definition to establish rods of different physical materials and test under which conditions these rods maintain their shape and size. Under all such conditions, they function as good rods. We now have operational definitions of rigid rods and clocks using physical materials.

An important aspect of the way in which we established clocks and rods is the reliance on metaphysical arguments in combination with physical theory. The metaphysical arguments provide us with the theoretical concepts “equal distance” and “equal duration” and thereafter, the physical theory tells us how we can operationalize these concepts in the world. In other words, any measurement relies on metaphysical and physical conceptions that are “brought to life” through experience and operationalization in real physical objects. In order to use clocks coherently therefore, the theoretical concepts we use in establishing rods and clocks must be valid also for the theories we arrive at by using rods and clocks as measuring devices. In other words, for every measurement, the measuring apparatus and its behavior must be taken into account.

A primary proponent of the view that the significance of measurement results relies on our understanding also of the behavior of the measuring apparatus, was the Danish physicist Niels Bohr. Bohr argued that one cannot think of measurement as the interaction between two separable elements, but must rather think of a measuring result as the behavior of a whole system consisting of all the previously separable elements. For instance, Bohr states that

> The essential lesson of the analysis of measurements in quantum theory is thus the emphasis on the necessity, in the account of the phenomena, of taking the whole experimental arrangement into consideration… (Bohr 1938: 101)

Bohr is here emphasizing the need to not only think of the object of investigation as being influenced by interaction, but also the measuring apparatus itself as altered by such interaction. This general idea is in full agreement with the Kantian notion of accounting for contributions of the subject in any human interaction. As we have seen, the Kantian theory also emphasizes that all interaction, whether involving humans or not, must ultimately be conceived of as interactions among the dynamical forces inherent in every physical object. Naturally, Bohr is primarily referring to quantum effects on the measuring apparatus here, but there is also a more general lesson intended; when speaking of measurements we must always also take into account the behavior of the measuring apparatus itself! Bohr goes on to argue that this consideration is a central issue also in the theory of relativity.

> In both cases [quantum mechanics and relativity theory] we are confronted with novel aspects of the observational problem, involving a revision of customary ideas of physical reality, and originating in the recognition of general laws of nature which do not directly affect practical experience… The ultimate reason for the unavoidable renunciation as regards the absolute significance of ordinary attributes of objects, and for the recourse to a relative or complementary mode of description respectively, lies also in both cases in the necessity of confining ourselves, in the account of experience, to comparisons between measurements in the interpretation of which relativity refinements and quantum effects respectively have on principle to be neglected. (Bohr 1938: 105)

Through these short segments, Bohr invigorates a classical debate in epistemology. As I have argued throughout Part II and III, my observation of an object is an interaction between that object and myself. Both bring something into the interaction and both bring something out of it. As with any other physical system, it is possible to isolate me from the object in the imagination. In life, however, no physical object is ever truly isolated but always takes part in some interaction or other. Furthermore, Bohr brings this general epistemological problem into the idea of making physical measurements. For quantum theory, which is Bohr's major concern, the issue to be explained is how quantum systems can possibly appear as wave phenomena in some instances and as particle or corpuscular phenomena in others. For Lorentzian theories and special relativity, we shall see that the central issues involve our understanding of clocks and measuring rods and their relation to duration and distance. In other words, we will be looking into how the metaphysically based theoretical concepts of "duration" and "distance" are operationalized physically by clocks and rods. Operationalization here refers to the way in which we let physically real objects stand in for our theoretically developed concepts. We know that this is not a one to one relation but rather a gradual approximation. As concerns rods and clocks, we know that operationalization of good rods and clocks can be approximately achieved under very specific conditions. Our increasing knowledge of physical objects thus gives us increasingly accurate approximations to concepts through operationalization. This increasing knowledge also informs us about the conditions under which a particular type of interaction occurs and how that interaction affects both the measuring apparatus and that which we wish to measure. Lorentz and Einstein disagree about how physical rods and clocks are affected by interaction and thus under which conditions they successfully operationalize distance and duration. We shall see that this difference leads them to radically different ways of understanding measuring results and ultimately space and time themselves.

Before going into the details concerning the differences between Einstein and Lorentz's theories, we shall take a quick look at the problems they are trying to solve.

IV 2. The scientific background in 19th Century physics

In the early 19th Century, Young and Fresnel's work established the wave theory of light. A central part of this theory is the idea that light is a disturbance in a medium. Young (1803) for instance systematically applied an analogy to sound. As sufferers of Tinnitus know, one can silence sound frequencies by adding sounds of another frequency. Young's interference pattern experiments showed that a similar phenomenon is applicable to light. In these experiments, Young showed that if you shine light at a wall from two sources (established through a double-slit set up) the emerging pattern on the wall is a synthesis of lighter and darker patches represented by a harmonic function. The most plausible explanation being that light waves interfere with each other and some waves accentuate each other while others cancel each other out (See Young 1803, and Buchwald 1989).

Our general understanding of a wave is as a disturbance in a medium. Sound is a disturbance in the air, for instance. However, light reaches us from distant stars, so the medium of light must be a medium that is not dependent on an atmosphere or something similar. The prevailing idea was that the light medium was a universal field, ether. There are multiple ways to understand the ether concept, and in the present discussion, we only need to assume that the ether is a universal and basic field that acts as a medium for the propagation of light. As we shall see, on the theories of Maxwell and Lorentz, ether serves the further purpose of being a basic material field that stand as a base for the electromagnetic field.

Two basic theories for the relation between the ether and the earth were proposed. The *ether drag theory* suggested that the earth in its motion dragged the ether with it. The *ether wind theory* suggested that the earth travelled through the ether. Ether drag theories quickly went out of fashion[57] and the general ether hypothesis was that the earth travels through the ether. However, ether theories struggled to explain two central problems: the negative result of the Michelson

[57] There are, however, proponents of the ether drag theory still. These often base their adherence on Einstein's consideration of the space-time of GR as an ether, and the suggestion that the earth drags space-time along in its motion (See Kostro 2000).

Morley experiments, and the theoretical over-determination concerning moving magnets and conductors. We shall look at these problems in the following.

If ether is the medium of the propagation of light, and the earth is in motion relative to the ether, one should think that this relative motion influences light phenomena on the earth. This was the general idea of the experiments set up by Michelson and Morley in the 1880's. The experiments played on the interferences of light waves established by Young. The general idea was that the ether is stationary, and light travels at a particular velocity relative to it. If the earth moves through the ether, any equipment on the earth moves relative to the ether and therefore moves relative to the motion of the light.

We already know that interacting waves of light create interference patterns (fringes). These patterns will change if we change the temporal and or angular relations between the waves. If we take the ether to be "at rest" and the earth to be moving, we should be able to understand at which angles and how fast the earth moves through the ether by studying how the results of experiments on light change depending on the motion of the earth. This was the working hypotheses of Michelson and Morley when they initiated what are, by now, possibly the most famous experiments in the history of modern physics.

IV 2.1. The experimental problem: Michelson & Morley

The set-up of the experiment of Michelson and Morley is the following.

From a light source, a ray of light is emitted along the line ac. Parts of the light will be reflected in a, bounce off mirror b and continue through mirror a´ and into d. Another part of the light will travel along the path ac, through the mirrors, (a, and a') bounce off the mirror at c, then be reflected at a´ and meet the first ray at d. When the two rays of light meet, they will create an interference pattern (fringes). If the general idea is correct, we should observe changes in the interference patterns (fringe shifts) depending on how we rotate the apparatus.

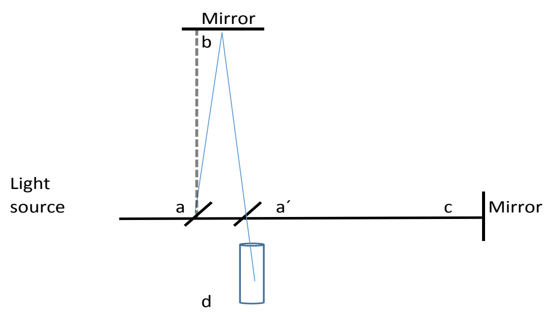


Fig IV 1.

For the purposes of this discussion, we shall stick to the received view, which is that no such shifts were observed, and therefore that it is not possible to detect the motion of the earth through the ether experimentally. As the relative motion between the earth and the ether was central in explaining the aberration of light (Michelson & Morley 1887), the negative result of Michelson and Morley's experiment was a serious problem for physics in the 19th Century[58].

IV 2.2. The theoretical problem: Magnet and conductor

The magnet conductor problem is most famously defined as a problem in Einstein (1905). The relevant issue is that, in the context of Maxwell's theory of electrodynamics, one can imagine the following situation:

There is a magnet and a conductor in the electromagnetic field and there is relative motion between them. Since we have no way of establishing which is "really" moving or "really" at rest,[59] we are free to choose whether it is the magnet or the conductor that moves. Since there is no empirical difference, our choice only concerns the perspective of measurement; there should be no theoretical difference if the magnet or the conductor is moving. However, in his ground breaking "On physical lines of Force", James Clerk Maxwell (1861: part 2: 346-347) presents different explanations for the two cases. I shall quote Einstein (1905)'s presentation of the issue.

[58] With the wisdom of hindsight, Abreu and Guerra (2005) point out that the Michelson Morley interferometer simply is a set of two Feynman clocks and that we could have known in advance what the result of the experiment would be.
[59] This inability is further corroborated by the negative results of the Michelson Morley experiments.

> If the magnet is in motion and the conductor is at rest, an electric field with a definite energy value results in the vicinity of the magnet that produces a current wherever parts of the conductor are located. But if the magnet is at rest while the conductor is moving, no electric field results in the vicinity of the magnet, but rather an electromotive force in the conductor, to which no energy per se corresponds, but which, assuming an equality of the two cases, gives rise to electric currents of the same magnitude and the same course as those produced by the electric forces in the former case (Einstein 1905: 123-124).

As illustrated in Einstein's presentation, there are two distinct theoretical descriptions of what should be the same situation. This means that the theory seems to suggest that the existence or non-existence of certain forces depends on our free choice of perspective. We therefore have the appearance of theoretical over determination. We shall see that there is no theoretical over-determination in this case, but rather a set of different illustrations of the same underlying phenomenon.

The two problems mentioned briefly here (the theoretical and the experimental problem) form the background on which both Lorentzian theories and special relativity was constructed. I shall first present the Lorentzian theories, which follow more directly in the line of classical electromagnetism.

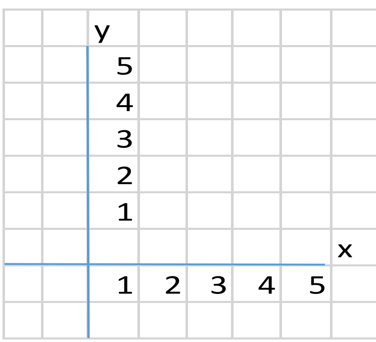
IV 3. Lorentzian theories and the Michelson Morley experiment

In the following, we establish a common theoretical ground between the electromagnetic theories of Maxwell and Lorentz. We shall largely ignore the electron deformation hypothesis of Lorentz[60] and focus on the overall framework of Lorentzian theories. To my knowledge, neither Lorentz nor Maxwell wrote explicitly on their positions in philosophy. However, what I seek to establish here is that the Lorentzian alternative allows for a Kantian reconstruction. We use the Michelson Morley experiment as the starting point for understanding Lorentzian theory.

[60] Lorentz' electron deformation hypothesis was Lorentz' attempt at explaining the macro-level phenomena from micro-constituents. This theory is widely criticized as opposing standard quantum physics (see for instance Janssen 2002: 431).

Relying on the abstraction process described in parts II and III, we have available a Euclidean geometry and a classical theory of time. Furthermore, we can establish Cartesian coordinate systems, which allow us to treat measuring results relating to motion numerically. We also know that, phoronomically; all motion is relative in the sense that any (inertial) motion can be described as any other motion, or as at rest. We can thus construct a metaphysically based theoretical description of the measuring apparatus involved in the Michelson Morley experiment.

In our first approximation, we treat the physical measuring apparatus involved as if it was a truly rigid system described as a Cartesian coordinate system. From the abstraction procedure in parts II and III, we know that this Cartesian system is built from a directional definition of straight lines, and that it determines a Euclidean space. As the relevant motion we shall treat is only along two dimensions, we stick to a two-dimensional Cartesian system with a grid.



The Michelson Morley experiment is an experiment in relative inertial motion, where we imagine the ether as the “rest system” or absolute space, and the interferometer as moving in relation to this system. As the motion is inertial, we could of course treat the ether-system as moving and the interferometer as “at rest”. [61] When the two systems are at rest, they are identical, i.e. they are both geometrically Euclidean and thus homogenous. Phoronomically, this identity is preserved. Therefore, if I construct a Cartesian frame for the “rest system” and another for the “moving system” and introduce a figure, the numerical descriptions of that figure would be identical in the two systems. This is of course just another way of stating that, as long as we do not think of physical objects, but rather mathematical ones in motion, the objects remain rigid. However, the Michelson Morley experiment is a physical experiment involving physically real objects as well

[61] If acceleration or rotation were involved, we would have to account for the additional forces involved.

as light. In order to understand the issue more fully, we must therefore also account for the physical interactions at play.

From classical electrodynamics, we know that the speed of light is constant relative to the ether. From this knowledge, we can establish a common principle of special relativity and Lorentzian ether theory, which one typically refers to as the "light principle". The light principle states that

The speed of light is independent of the motion of its source

Thus, we see how the Michelson Morley experiment makes sense. The assumptions are that there is an ether, understood as the frame or medium that light is moving in, and that at some point during its path, the earth must travel in different directions relative to it. We further assume that duration and distances can be operationalized by light in motion. With these background assumptions, we move an interferometer through the ether by placing it on the surface of the earth. If all our assumptions hold, we should find that the interferometer gives different measurements as the earth moves in different directions. The surprising result of Michelson Morley was that no such difference could be detected, and we must therefore evaluate which of our explicit or implicit assumption(s) were wrong.

In parts II and III, we saw that the measuring system we use, the Cartesian grid, is constructed within Euclidean geometry and classical time. Thus, if we are to replace the general space-time set up, we would need to find a new way of constructing Cartesian systems along with a meaningful notion of motion (see part III) and velocity addition. Lorentz does not follow this route although we shall see later that Einstein to some extent does.

Lorentz solution is to look at the dynamical functioning of the measuring apparatus.[62] Like Kepler, who separated between "apparent" and "true" positions of the stars and Galileo, who separated between "shared" and "relative" motion, Lorentz ends up separating between the mathematical

[62] See for instance Lorentz (1899, 1916), Janssen (1995, 2002), and McCormmach (1970)

description of the measuring apparatus as moving rigid objects and the dynamical physical objects that operationalize them in the experiment.

If we assume that the erroneous assumption is physical rigidity, we see that our description of the measuring situation is somewhat flawed. In Lorentz' full theory [63] the idea is that as any object travels through the ether it changes its shape. Within the ether-theoretical perspective, the line of reasoning is that since the ether communicates forces that affect material objects, these objects are also affected by their motion relative to the ether. In effect, the Lorentzian theory states that there are no dynamically rigid objects, even when they travel inertially.[64]

There are different ways to imagine how physical objects are deformed by their motion through the ether, and Larmor, Fitzgerald and Lorentz were working on slightly different hypotheses. Lorentz' solution was that

> *As any material object moves relative to the ether, it is contracted in the direction of motion*

We know this as the "absolute rod-contraction" hypothesis. Absolute rod contraction implies that two objects that are identical when at rest in the ether will no longer be identical if one of them is moving.

Consider two identical rods, Rod and *Rod'*. As long as they are both at rest in the ether, they remain identical. If we stay with Rod at rest in the ether and move *Rod*' toward the right at a very high velocity, we will measure *Rod'* as being shorter.

At rest:

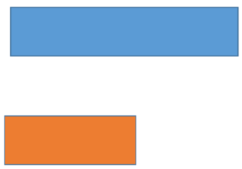
Rod

[63] I.e., what Grûnbaum popularized as the «doubly amended» theory.

[64] Naturally, there are still inertial systems as defined mechanically and phoronomically. Thus, there is still no way to know whether you are at rest or moving inertially.

Rod'

When *Rod'* is moving at high velocity toward the right:



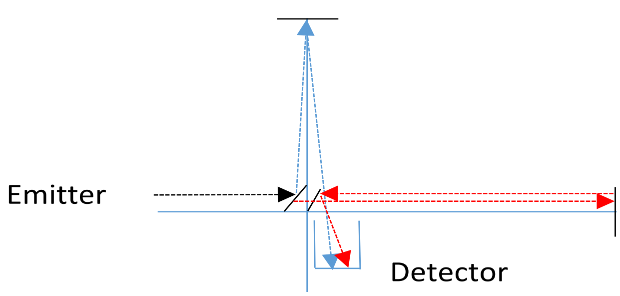
Rod

Rod'

In relation to the Michelson Morley experiment, this means that as the interferometer moves through the ether along with the earth, the earth, the observers and the interferometer contracts slightly in the direction of motion. As it turns out, this explains the null-result of the experiment.

Consider the original set up. There is an ether system E. The (one-way) speed of light is constant in any direction relative to E. For our illustration, let the sheet of paper represent the ether system, E.

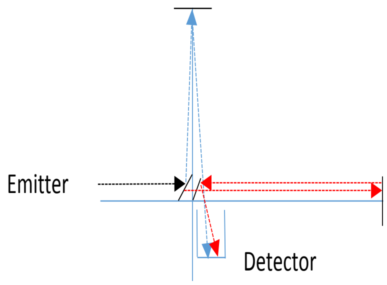
In order to measure the speed of this ray in different directions, we build an interferometer.



Interferometer at rest

As long as the interferometer is at rest in the ether, the rays of light travels equal distances. If the interferometer were rigid, we should detect a difference between the rays when it moves. Recall that the speed of light is constant only in relation to the ether and not in relation to the moving interferometer.

As Lorentz realized however, if the interferometer is a physical object and all physical objects suffer absolute rod contraction in the direction of motion through the ether, then the contraction can cancel out the effect of the motion.



Interferometer moving toward the right

This presupposes that the contraction is of the exact right magnitude and that it increases with velocity. Lorentz finds an expression of this magnitude and relates it to the speed of light relative to the ether in what we now know as the "Lorentz transformations". Before establishing the arithmetical determination of absolute rod contraction in the Lorentz transformations, let us consider Lorentz' procedure here.

By gradual abstraction from the phenomena, we have established a priori elements of mechanics, dynamics, phoronomy and geometry. By maintaining these a priori aspects, we have also built a concept of motion and a notion of velocity addition. We now apply these to the Michelson Morley experiment and find that the experimental results can be explained by reference to a dynamical effect on the measuring apparatus. First, however, we established that the measuring situation is just an instantiation of a wider phenomenon known as absolute rod contraction that applies to all moving objects of a physical nature. The rationale for this generality is that at the base of the argument there is the notion that all physical forces are ultimately seated in the universally present ether. The ether concept is an old and established idea that has been around since Aristotle and was discussed by Descartes, Leibniz, Kant and others. It has been treated as having different specific properties throughout history. For instance, Einstein (1920) describes the "physical qualities of space" in general relativity in terms of an ether, and establishes that; "According to the general theory of relativity space without ether is unthinkable" (Einstein: 1920). More recently, Wilczek (2008: 90) describes the ether in terms of a non-empty metric field in relation to quantum electrodynamics. Kant's notion of the ether, expressed in his posthumous work, is that of the material aspects of space, thus somewhat similar to Einstein's later description.[65]

In other words, rather than introducing an ad-hoc or auxiliary notion, Lorentz refers his explanation to already accepted ideas. From his idea of a dynamical ether, Euclidean space and classical time, Lorentz hypothesizes that there is a central difference between mathematical objects, that maintain their shape no matter what, and physical objects, that suffer a deformation due to their motion in the ether. In giving up rigidity of physical objects therefore, Lorentz is able to maintain the basic framework and explain the observational results. The magnitude of the deformation is expressed in two papers, which together constitute the doubly amended Lorentz theory.

IV 3.1. Classical transformations and Lorentz transformations

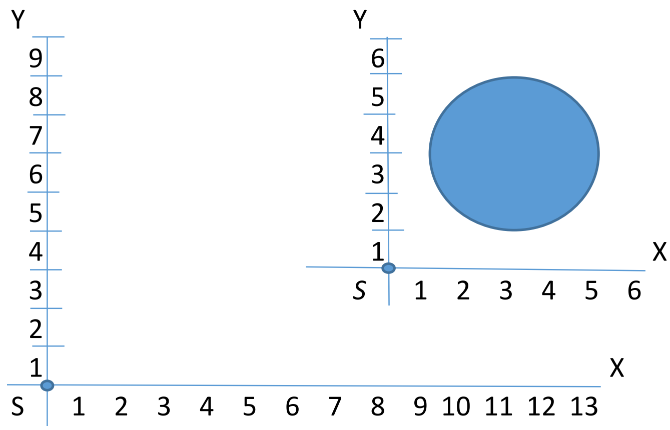
Our theory of concern here is the Lorentz ether theory in its most developed version i.e., the "doubly amended theory". This version of Lorentzian ether theory involves two central elements, rod contraction and clock dilation. Although interrelated, we shall treat these issues separately in order to get a clearer image of what the Lorentzian theory states. Once we have treated the issues in separation, we shall join them together in a more general theory.

IV 3.1.1. Rod contraction or; how motion affects the shape of objects

In order to understand the Lorentz transformations and their function in ether theory, we will first look at the spatial transformations we use for rigid geometrical objects. We shall usually refer to the transformations from rigid geometrical objects as "classical transformations".

[65] A central issue here is the term «entity» and the objectification of the ether as a «thing». From the history of philosophy we know that similar notions, such as the "I" refuse such an objectification. Kant (1796-1803, 21:215-590) discusses matter as consisting of two general aspects. The ether (Caloric) as an unobservable "world system" and "entities" as observable material objects. The ether is in itself unobservable but yet a prerequisite for observable material entities. In a similar manner, we cannot objectify the subjective character of the "I, yet it is a prerequisite for objectification as such. Foundational notions such as ether, "I", space, and time all share this dual role of unobservable prerequisites for observation within transcendental philosophy.

Consider an object as measured from two identical systems, S and *S* at relative rest but at different positions in space:



The object will be described as having the same shape but existing at different positions, depending on which system we choose to treat as the measuring system. Say we only have the measurements of system *S* available, but we want the position of the center of the object expressed as a measurement in system S. We then first determine the position of the origin of the system *S* from the system S as *So* = (8, 3). From this, I know that the positions in S are the positions in *S* plus eight units along the positive x and 3 units along the positive y. Therefore, the center of the object in S= *S*x+8, *S*y+3. The center of the object in *S*= (*x*=3) and (*y*=4). Therefore the position of the center of the object in S= (x=3+8), (y=4+3) = (11, 7).

These are the classical transformations for positions given in one system as expressed in another system as long as the systems are at relative rest. If one system is moving, however, we have to factor that motion in. Consider the following:

We establish a Cartesian coordinate system (K) with three axes (x, y, z) that we call the "rest system". In addition, we establish a Cartesian coordinate system (*K´*) with three axes (x´, y´, z´), moving relative to K. For an object co-moving with *K´*, we can ask, given a distance d(ab) between two arbitrary points on the object as measured in the moving system, what is that distance when measured from the "rest system" K. In other words, we are still transforming the coordinates of one system into those of another. For simplicity, we consider the two points lying along the x-axis and the motion of *K´* being in the direction of positive x in K.

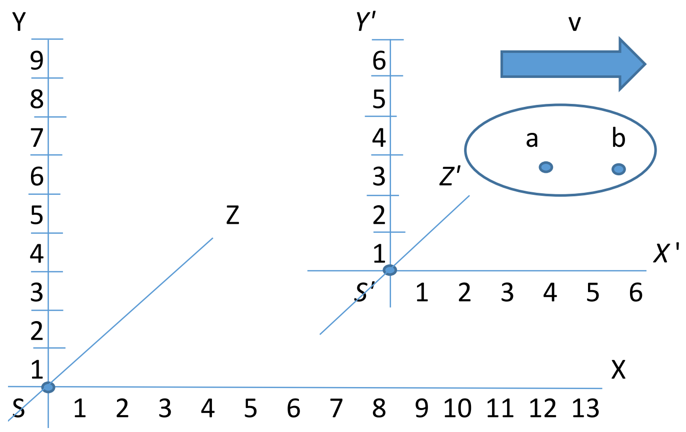


Fig. III 3.3

In the moving system *K´*, the distance d´(ab) is ($x_b´$ - $x_a´$). By a Galilean transformation, the distance d(ab) in the rest system K, is (x_b - x_a), which is identical to d´(ab), provided we are dealing with strictly rigid objects. Thus, when we are working with geometrically based ideal object, spatial measurement from systems in relative motion are identical.

In order to find the position of a and b in K from measurements made in *K*', we must factor in the velocity of *K'* as well as the time of measurement. We then get $x'_a = (x_a - vt)$ and $x'_b = (x_b - vt)$, i.e., the position of a and b in the rest system, K, are the positions in *K'* plus the result of multiplying the duration and velocity of motion of *K'* since the two systems had a shared point of origin and time was set to zero in both systems. These classical transformations hold for rigid objects and measuring frames in relative inertial motion.

Lorentz' task, however, is to find an expression for similar measurements, made with physical measuring instruments in relative inertial motion. As we know, physical objects are not rigid and the classical transformations will thus not hold. Nevertheless, the classical transformations are built from our a priori understanding of velocity, velocity addition, space and time, so they will necessarily be part of the solution. Lorentz' approach is to find a way to express the classical transformations plus the systematic contraction of physical objects. He does this by way of establishing a constant k, by which the original transformations can be multiplied.

For measurements made with a physically real apparatus the general set-up is classical transformations multiplied somewhere with a factor k. Lorentz, Voigt, Fitzgerald and Larmor all found the numerical expression for this constant and the resulting expression is what we know as the Lorentz transformations, which looks like this:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We can see here that Lorentz, Larmor, Voigt and Fitzgerald take the classical transformation (x'= x - vt) and multiply it by the constant $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ which then expresses the inverse of the contraction (in the direction of motion) of the entire measuring system *K*' and everything moving along with it. We see that if v = c, that is if the velocity of the object reaches the velocity of light in a vacuum, the constant yields $\sqrt{1 - \frac{v^2}{c^2}}$ = $\sqrt{1-1}$ = 0 and we must divide by zero which is nonsensical. In other words, the transformations only hold for velocities smaller than that of light in a vacuum. As these transformations express the universal contraction of any object whatsoever due to its motion through the ether, the fact that there is a "speed limit" built into them implies that either, there can be no velocities higher than c, the (one way) velocity of light in the ether, relative to the ether or the transformations do not hold.[66] The contraction and its numerical expression through the Lorentz transformations is enough for Lorentz to explain the null-results of the Michelson Morley experiment and thus to solve what we have called the "experimental problem".

IV 3.1.2. Clock dilation or; how motion affects temporal processes

In order to explain further phenomena, Lorentz applied a similar way of thinking about the functioning of clocks. With our light-clock set-up, we see that such a need quickly presents itself.

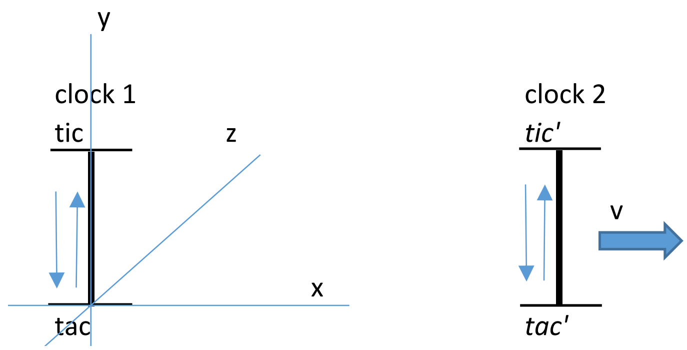
Take two vertically oriented Feynman clocks spatially separated but at relative rest in the ether.

[66] Thus, all the commotion surrounding the claim that one had found neutrons travelling faster than C at CERN in 2012.

Using light signals to synchronize the clocks, send a light signal from the base of clock 1 to the base of clock 2 and back again. As both clocks are stationary in the ether, and light speed is identical in all directions in the ether, we know when the signal reached clock 2 = ½(moment of reception – moment of emission). We can now synchronize the clocks 1 and 2 so that they tic and tac at the same instant. If we let t_0, t_1, t_2 … represent the series of tacs on clock 1, and t'_0, t'_1, t'_2 … represent the tacs on clock 2, we can set the clocks so that ($t_0 = t'_0$), ($t_1 = t'_1$), ($t_2 = t'_2$), …

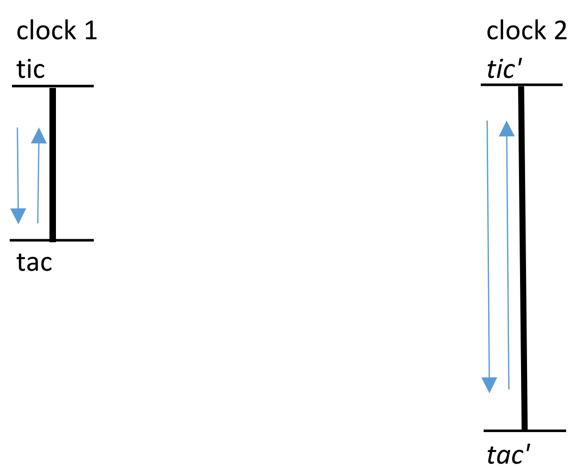
Now place the clocks together and set them such that at $t_0 = t'_0$ they are at the exact same position, but let clock 2 move horizontally, with inertial motion toward the right with velocity v.

This motion introduces two complications. First, we can no longer synchronize the clocks by simply using the distance between them. This distance increases with time, and the amount of increase depends on the velocity of clock 2. We must therefore account for the fact that clock 2 is "running away" from the light signal at velocity v, and it keeps running away after the signal has been reflected. This effect is somehow captured in the formula $t' = t - (vx/c^2)$ where v is the velocity of the clock along the horizontal x-axis, and x is the distance of the clock 2 from the origin along this axis, of an imagined Cartesian coordinate system with clock 1 at the origin, and c is the speed of light.

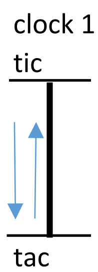


So far, we see how the clocks can be synchronized while in relative motion by sending light signals between them. We have assumed that they run at the same rate throughout since they were made to run at the same rate when they were initially synchronized.

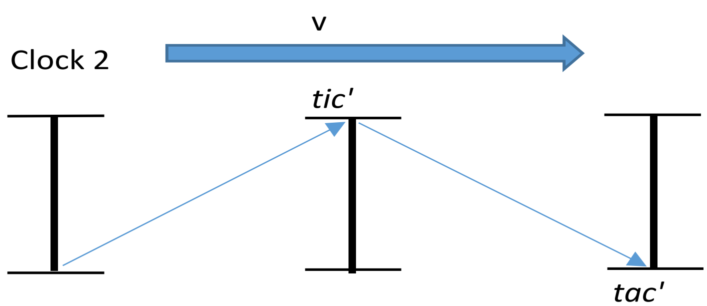
We must however, also consider the following. Recall that the clocks are generating tics and tacs by reflecting light signals. We get time intervals by acknowledging that the speed of light, c, is the amount of distance/time travelled. We also know that this speed is constant. If we increase the distance between the mirrors on clock 2 (and maintain the original distance in clock 1) clock 2 would run slower than clock 1, even if the clocks were stationary.



Hence, the clocks only remain synchronized as long as the distance between the mirrors is kept constant. The reason being that the distance travelled by light has to be the same. Now, we know that rod contraction occurs along the dimension of motion, so the distance between the mirrors remains stable even though we move clock 2 horizontally. However, the distance that light has to travel becomes longer. This is because in addition to the vertical trip, the light now also has to travel horizontally due to the motion of the clock. From the perspective of an observer stationary in the ether along with clock 1, it functions like this:



While clock 2 functions like this:



As the light in clock 2 has to travel further, the duration of each *tic'/tac'* increases and therefore clock 2 is slower than clock 1. We see here that the synchronization procedure depends on both light speed and the velocity of clock 2, just as in our determination of rod contraction above. It is therefore not surprising that the expression for how much the clock slows down is our, by now familiar Lorentz factor

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

In combination, we get the following expression for relating two clocks where one is at rest and the other is moving inertially in the ether, actually the expression for time in the Lorentz-transformations:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

IV 3.1.3. A necessary correction

Although what we have gone through above finally arriving at this expression seems to be the standard description and explanation, it is not (strictly) correct. The following expression of the Lorentz factor is always larger than 1

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We can see this by considering the numerator (1) will be larger than the denominator no matter the value of v. Therefore, whatever factor we multiply by this expression will come out as larger. If the above explanation rang true, and the above expression accounts for the deformation of the clock, we would have to conclude that the moving clock speeds up rather than slows down. This is contrary to experience as well as theory. We must therefore find the clock dilation elsewhere. What we are left with, implicitly containing the clock dilation as well as a compensation for the above factor larger than 1, is the expression

$$\left(t - \frac{vx}{c^2}\right)$$

Placing clock 2's position at the origin of the Cartesian coordinate system *K'*, we can express the value of x of this clock's position as a function of time and velocity, i.e. x = vt.

Thus,

$$\frac{vx}{c^2} = \frac{v(vt)}{c^2} = \left(\frac{v^2}{c^2}\right)t$$

Inserting this into our original expression yields

$$t' = \mathrm{t} - \left(\frac{v^2}{c^2}\right)t$$

Extracting the common factor, t yields

$$t' = \left(1 - \frac{v^2}{c^2}\right)t$$

and we recognize that we are multiplying t with the very same factor whose square root is found in the Lorentz factor. Hence, our full expression for time *t'* of clock 2, when at the origin of *K'* equals:

$$t' = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)t$$

As we know the factor by which t is multiplied is smaller than 1, we know that $t' < t$. Now we can see what we could not see from the Lorentz factor alone, which is that clock 2 slows down.

So far we have only dealt with clock 2 which we have placed at the origin of *K'*. If we consider another clock in *K'* positioned at some distance from the origin, what happens then? Let us consider a clock 3 synchronized with clock 2, and therefore slow relative to clock 1. This clock is synchronized through an addendum (positive or negative) depending on where one has placed it on the x-axis common to K and *K'*.

As an example, we consider clock 3 positioned at distance a, measured in K, from the origin of *K'*, along the positive x-axis and travelling along with clock 2, and thus the system K' as a whole. The x-value of clock 3 is given by the position of clock 2, (vt), and the distance from clock 1 to clock 3, x = (vt) + a at any given time, t. Inserting this into our original expression

$$\left(t - \frac{vx}{c^2}\right)$$

Yields

$$t' = t - t\left(\frac{v^2}{c^2}\right) - \frac{va}{c^2}$$

Or

$$t' = t\left(1 - \frac{v^2}{c^2}\right) - \frac{va}{c^2}$$

Multiplying this by the Lorentz factor

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We get

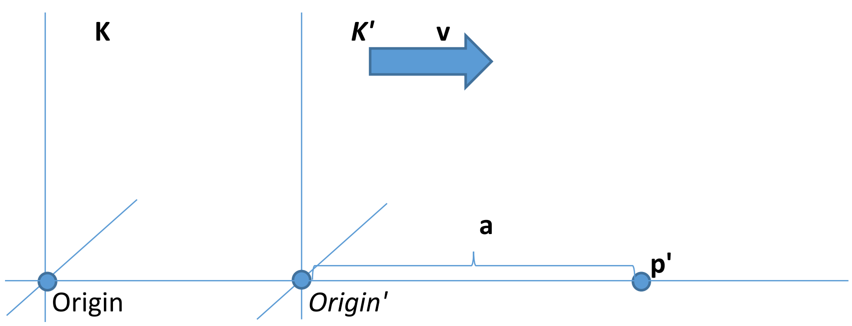
$$t' = \frac{t\left(1 - \frac{v^2}{c^2}\right) - \frac{va}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From which we get

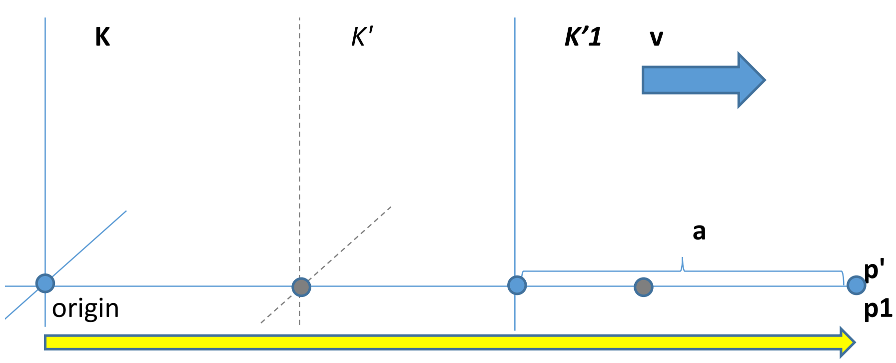
$$t' = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)\mathrm{t} - \left(\frac{\frac{va}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$$

And we see that on the right side of the equation sign, we have first the expression for clock dilation for clock 2 and thus all clocks in *K'*. Following that we have the addendum that follows from the spatial separation between clock 2 and clock 3, and the difference between the clock synchronizations in K and *K'*. Now we can see which elements of the equations follow from which physical assumption or conventional determination. The initial clock dilation for all clocks in *K'* follows from their common motion through the ether and the ignition synchronization of clock 1 and 2 at $t_0=t'_0$ when passing each other. Furthermore, we see as separated, what results from the differences of the synchronization procedures applied in the two systems, e.g. in particular the system *K*'.

We can take a closer look at the dilation by considering the systems K at rest in the ether *K'* travelling with constant velocity v, in the direction of positive x, common to K and *K'*, more directly with respect to their different synchronization procedures. In the travelling system *K'* there is a point *p'* on the x-axis. The point *p'* remains a distance a, from the *origin'* of *K'* throughout.



Emit a light signal from the origin of K in the direction of the point *p'*. As the light travels toward *p', p'* and the entire system *K'* is moving away from the origin of K.



By the time t_1 the light signal reaches *p'*, the *origin* of *K'* as well as *p'* will have moved away from the origin of K.

The distance between origin of K and *origin'* of *K'*:	$O'_1 = vt_1$
The distance between the origin of K and the point p_1:	$p_1 = vt_1 + a$
Another way of expressing the position of the point p_1:	ct_1

In other words,

$$ct_1 = a + vt_1$$

Which is equivalent to

$$ct_1 - vt_1 = a$$

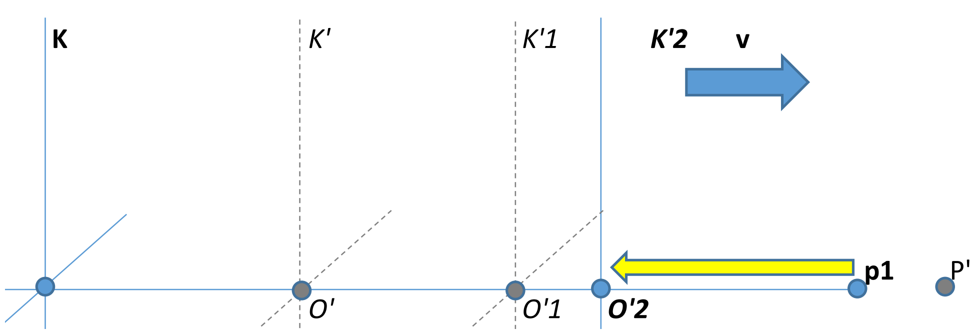
Or

$$(c - v)t_1 = a$$

Dividing by (c - v) on both sides

$$\boldsymbol{t_1 = \frac{a}{(c - v)}}$$

Once the light signal reaches p_1 it is reflected back in the opposite direction. It reaches the *origin'* of the moving system *K'* at time t_2 as measured in K.



The position of the *origin'* at t_2 as measured in K, is $\boldsymbol{O'_2 = vt_2}$

The distance travelled by light between t_1 and t_2 is the distance between p_1 and O'_2

This distance can be expressed either as

$$ct_1 - vt_2$$

Or as

$$a - v(t_2 - t_1)$$

Or alternatively as

$$ct_2 - ct_1$$

Or

$$c(t_2 - t_1)$$

These are all equivalent expressions for the distance light travels from its reflection in p_1 until it meets the travelling *origin'* at t_2.

Therefore

$$c(t_2 - t_1) = a - v(t_2 - t_1)$$

Expressing the distance a as

$$c(t_2 - t_1) + v(t_2 - t_1) = a$$

Extracting the common expression (t_2-t_1)

$$(t_2 - t_1)(c + v) = a$$

Dividing by (c + v) on both sides

$$\boldsymbol{t_2 - t_1 = \frac{a}{(c + v)}}$$

If we then take our expressions for (t_2 - t_1) and t_1,[67] we can add t_1 on both sides

$$(t_2 - t_1) + t_1 = \left(\frac{a}{(c+v)}\right) + \left(\frac{a}{(c-v)}\right)$$

Simplifying the right side by inserting a common denominator (c + v)(c - v), we then have

$$(t_2 - t_1) + t_1 = \left(\frac{a(c-v) + a(c+v)}{(c^2 - v^2)}\right)$$

Which yields

$$(t_2 - t_1) + t_1 = \frac{2ac}{(c^2 - v^2)}$$

Adding up the left side we finally have

$$t_2 = \frac{2ac}{(c^2 - v^2)}$$

We sent a ray of light that passed through *O*' at t'_0 and returned to *O'* at t_2.We also know that according to the synchronization procedure in *K*', the time of reflection t'_1 at p_1 should be at the exact half point between $t_0 = 0$ and t_2. (once we have corrected for clock dilation by the Lorentz factor, $\sqrt{1 - \frac{c^2}{v^2}}$)

Therefore, the clock travelling along with *p'* synchronized in the system *K'* compares to the clock at rest in K and the ether in the following fashion

[67] Marked in **bold**

$$\frac{1}{2}(t_2 - t_0) = \frac{1}{2}t_2 = \frac{ac}{(c^2 - v^2)}$$

However, according to the clock at rest in K, the time when the signal reaches p_1 (which corresponds to *p'* at the time of reflection), is t_1. We must therefore account and correct for the difference between the synchronizations in K and *K'*. We do this by subtracting the difference between t_1 and $\frac{1}{2}(t_2 - t_1)$

$$\Delta t = t_1 - \frac{1}{2}(t_2 - t_1)$$

$$= \frac{a}{(c - v)} - \frac{ac}{(c^2 - v^2)}$$

$$= \frac{(a(c + v) - ac)}{(c^2 - v^2)}$$

$$= \frac{av}{(c^2 - v^2)}$$

As the clocks in *K'* are synchronized with each other, we also know that they are dilated by the same factor, $\sqrt{1 - \frac{c^2}{v^2}}$

We must therefore multiply Δt by this factor in order to get the correct expression to be subtracted from *t'* as a function of t.

$$\Delta t' = \left(\frac{av}{(c^2 - v^2)}\right)\sqrt{1 - \frac{c^2}{v^2}}$$

$$\Delta \boldsymbol{t}' = \frac{\frac{\boldsymbol{av}}{\boldsymbol{c^2}}}{\sqrt{1 - \frac{c^2}{v^2}}}$$

Which is the expression for the correction pertaining to the difference between the synchronizations of the systems K and *K*' in the expression for Lorentz transformation for clocks we have deduced earlier.

We reached this formulation through illustrations in Euclidean space, and crucially, by applying classical or Galilean rules for velocity addition. We have also seen how the notions of straight lines and inertially moving systems are necessary elements of our description.

We also see the crucial role played by the conventionally determined synchronization procedure. We assume a correlation between the initial settings for space and time determination; we assume that clocks and rods are identical in the two systems and that when they are at relative rest they measure the same distance and duration. Finally, we assume that their motion through the ether dynamically influences all rods and clocks in the same way.

IV 3.1.4. Dynamical explanation

The ether theoretical claim is that clocks slow down when travelling through the ether and that, as with contraction, the slowing down increases with the velocity of the clock through the ether. Again, the effect is universally applicable to material objects, and if you travel together with a clock and a rod, you will not see the rod contracting and the clock slow down because you, as a material object, will also contract and your internal processes will slow down. As stated in Bell (1976), an observer travelling in the ether will not observe that her measuring rods are contracted and her clocks are slowing down.

> No, because the retina of her eye will also be contracted, so that just the same cells receive the image of the metre stick as if both stick and observer were at rest. In the same way, she will not notice that her clocks are slow because she will herself be thinking more slowly. Moreover, imagining herself as being at rest, she will not know that light overtakes her, or comes to meet her, with different relative velocities c + v. (Bell 1976: 76)

Once we have both clock-dilation and rod-contraction, we get what Grünbaum called the doubly amended Lorentz theory, which is the framework we consider. Let us recapitulate what we have said so far.

A rigid object is an object such that for any two points on the object, the distance between them is constant. In ordinary language, this translates into “a rigid object maintains its shape”. If we imagine an object as consisting of molecules, we can think of rigidity in terms of the distance between any two molecules in that object. If this distance is constant, the object is rigid. If it changes, the object is non-rigid. In the framework of Lorentzian theories, the Lorentz transformations imply a rejection of this type of rigidity for physical objects travelling through the ether. Importantly, the concept of a rigid object is maintained as a geometrical ideal. The job performed by the Lorentz transformations is to determine the quantity by which a physically real object differs from the geometrical ideal.[68]

The common aspect of all ether descriptions is the notion of a fundamental space-filling material field. A central function of the notion of an ether is the rejection of any truly empty space. In the Lorentzian theory, what the Michelson Morley experiment teaches us is that the ether has properties relating to motion such that any object whatsoever, if moving in the ether, suffers a particular kind of deformation. The theory states that not only do clocks dilate and rods contract as a function of motion through the ether, but so does everything else of a physical nature (supposing it has a mass larger than zero). Lorentz took this to imply that there is no way to detect the relative motion between the earth and the ether. Every time we attempt such a detection, we, as observers, and our equipment, are equally deformed and end up with the same results whether we are actually moving or not. In other words, the ether becomes *unobservable in principle*. Thus, Lorentz ends up in a similar position as that of Kant (1796-1803, 21:215-590).

[68] From the perspective of formal axiomatic geometry as well as Einstein’s practical geometry, our way of thinking here is erroneous. They insist that geometrical expressions only have meaning in relation to an axiomatic system and physical measurements. A measurement is to be taken at face value and if it breaks with the standard axioms (here Euclidean geometry and classical time), one should express it in terms of a new set of axioms. They use the Minkowski structure as their new “space-time geometry” as we shall discuss later. We shall, however, hold on to our description of the issue since we have been able to maintain meaningful explanations of the phenomena by abstracting our way from ordinary experience and rebuilding from the geometry of the form of outer experience. Note that we are also able to maintain our entire phoronomy and thus our concepts of motion, speed and velocity without having to introduce any speculative concept or postulate. Most importantly, we have been able to maintain our original set-up of the measuring apparatus throughout and modified our treatment of such apparatus once we apply physical objects involved with dynamical forces.

On this theory, we plausibly and dynamically explain the Michelson Morley experiment as resulting from a deformation of the measuring apparatus. A problem follows from the assumption that the ether is undetectable. This problem is of a more practical nature. If all physical materials contract as a function of their motion through the ether, and we do not know the state of motion of the ether, from where do we measure? The only solution available from within Lorentz' framework is that we must pragmatically choose a "rest system" and stick to it in the same way as we do when we represent motion phoronomically in absolute space. Once we have made the choice we can transform all measurements made in other systems to coordinates in our rest system. There seems, however, to be no principled way to decide between one rest system and another, as one may consider every such system as equally deformed.

Therefore, we are free to choose any system whatsoever as our rest system, provided it is inertial. As we saw in part II, some external force affects all non-inertially moving systems and in order to account for that force, one must refer to an inertial system ("rest system"/absolute space). Without reference to an inertial system, we will remain unable to construct measuring standards and thus lose our ability to distinguish between systems that are affected by some external force, and systems that are not. Thus losing our ability to determine forces at all. Once we have established inertial systems, we can switch between them, provided we transform all measuring results to the "new" rest system we choose. We see therefore that the relativity principle, which states that we can choose whichever inertial system we wish as our rest system, carries over from its initial statement in phoronomy through the dynamical ether and the mechanical principle of inertia. On all levels, the following statement holds:

> *We must determine an absolute space/"rest system" in order to establish relative inertial motions. The choice of absolute space/"rest system" determines the relative inertial motions and their magnitude, while the choice of absolute space can be made at our convenience.*

By following this procedure and applying the Lorentz transformations, we have an explanation for the null result of the Michelson Morley experiment. However, we also need to solve the theoretical (magnet/conductor) problem in order to establish our theory, since Einstein claimed this problem as the most direct reason for not following Lorentz in his first presentation of the special theory of relativity.

IV 4. Maxwell's ether theoretical response to the magnet conductor problem

In section 2.2., we introduced the theoretical problem of Maxwellian electrodynamics through Einstein's (1905) description. Typically, "Maxwellian electrodynamics" refers only to the mathematical aspects of electrodynamics as presented in Maxwell (1861). Maxwell initially published 20 equations, which Hertz and Heaviside later systematized into four general equations (Fleisch 2008: vii-viii). They are typically referred to either as Maxwell's equations or the Maxwell-Hertz equations. Without entering into the details, what we shall note about these equations is that they distinguish between magnetic fields and electric fields (Gauss' laws) and express relations between these fields (Faraday's law and the Maxwell-Ampere law).

This distinction motivates a view of electromagnetics as an inquiry into two separate types of entities, the magnetic and the electric field. If the appearance of a particular field and particular forces depend on our chosen perspective, we get the "Magnet Conductor Problem". However, Maxwell revised his view in the much less quoted 1865 paper, and we shall discuss this revision in the following. First, let us recall what kind of problem we are facing.

The mathematical structure of Maxwell's theory remained undisputed throughout. Indeed, Einstein (1905) simply inserts the equations into his theory without any further argument. The common assumption therefore, is that the mathematics itself holds. All involved parties also accept the observational data. The problem therefore, is neither prediction, observation, nor mathematics. Rather, it inheres in the realm of theory and explanation (physics/metaphysics).

The problem is that if we take a conductor and a magnet in relative motion, Maxwell (1861: part 2: 346-347) suggests that we get different physical explanations depending on whether we represent the magnet or the conductor as moving. Empirically, i.e., in terms of a resultant and observable current, there should be no difference, as our choice of representation does not change the empirical results. Faraday confirmed this experimentally.

Phoronomically, we have already shown that we can represent any (inertial) motion as rest and that our choice of rest system maintains the relative motion. No phoronomic or kinematic reason demands that the principle of relativity should not apply to this situation. In other words, phoronomically, as well as empirically, it makes no difference whether we choose to represent the magnet, conductor, or both, as moving. There should therefore be no difference in how physical theory describes the two situations. As we have seen however, Maxwell's initial description treats the situations as distinct. The central elements of Maxwell's revision of his theory is captured in the following remark from the 1865 paper:

> I have on a former occasion[69] tried to describe a particular kind of motion and a particular kind of strain, so arranged as to account for the phenomena. In the present paper I avoid any hypothesis of this kind; and in using such terms as electric momentum and electric elasticity in reference to the known phenomena of the induction of currents and the polarization of dielectrics, I wish merely to direct the mind of the reader to mechanical phenomena which will assist him in understanding the electrical ones. All such phrases in the present paper are to be considered as illustrative, not as explanatory.
>
> In speaking of the energy of the field, however, I wish to be understood literally. All energy is the same as mechanical energy, whether it exists in the form of motion or in that of elasticity, or in any other form. The energy in electromagnetic phenomena is mechanical energy. The only question is, Where does it reside? On the old theories it resides in the electrified bodies, conducting circuits, and magnets, in the form of an unknown quality called potential energy, or the power of producing certain effects at a distance. On our theory it resides in the electromagnetic field, in the space surrounding the electrified and magnetic bodies, as well as in those bodies themselves, and is in two different forms, which may be described without hypothesis as magnetic polarization and electric polarization, or, according to a very probable hypothesis, as the motion and the strain of one and the same medium. (Maxwell 1865: 487-488).

The way to resolve the magnet conductor problem therefore, is to realize that the initial distinction that creates the problem is one between two "modes of illustration" and not between two different theoretical *explanations*. This differentiation is of some importance.

[69] Maxwell refers here to his 1861 paper "On physical lines of force" (part II, p.346-347) in which he treats the problem as if the moving magnet and the moving conductor are two different phenomena that result in the same mechanical effect.

Maxwell's modes of illustration are auxiliary or vicarious hypotheses in need of explanation. Thus read, there is a strong similarity to Kepler's use of the equant point and circular orbits as a vicarious hypothesis in the *New Astronomy*, as well as to the auxiliary elements in Chakravartty (2007).

As with Kepler's vicarious hypothesis, Maxwell's modes of illustration must somehow be replaced by some *other* physical hypothesis. In the meantime, the modes of illustration (magnetic and electric field), serve as guiding principles of thought rather than being assumed as elements of reality. If we also adopt the 'very probable thesis' that the forms of mechanical energy represent the motion and the strain of the same medium, we get a full-blown ether theory, where the ether is the medium for material as well as optical motion. The modes of illustration are thus replaced by an ether field. However, as the ether is undetectable in experiment, the modes of illustration maintain their heuristic function. On this reading, the magnet conductor problem relies on a paralogism that follows from reification. Magnetic and electric fields are modes of illustration intended to generate images of phenomena in the electromagnetic field, seated in the ether. If we take them to be actual explanations, we are led to think that there are two separate types of existences: magnetic and electric fields. Furthermore, if we accept that there is no empirical answer to the question "which of the two really moves?" we must arbitrarily choose whether we illustrate the situation through an electric or a magnetic field at some particular instance.

Rather than thinking of magnets, electric fields, currents, magnetic fields, etc. as separate existences, we should understand them all as effects of interactions within an underlying field (ether). Therefore, we should not think of the resulting "electric field" or "electromotive force" as properties of magnets and conductors, but rather as states of the overall system. Whether we initially describe these properties as magnetic or electric is irrelevant, as these are simply modes of illustration, or auxiliary/vicarious concepts. In and of themselves, there are no strictly electric or magnetic fields in the same way that there are no undirected or speed-less motions. However, in order to grasp that there is motion going on, we must pick out some perspective. Our choice of perspective should be irrelevant to the mechanical effects, provided we maintain the same magnitude of motion, as is the case.

As long as the Galilean principle of relativity is accepted, there is no possible distinction between uniform motion and rest at any level of theory (phoronomically, mechanically or dynamically). As such, it is problematic to postulate a physical effect of a magnet or conductor's (uniform inertial) motion. In the Maxwellian framework, this problem is resolved in two ways:

First, the motion of magnet/conductor is not illustrated as motion as such, but as motion relative to the underlying electromagnetic field, ether. It makes theoretical sense to postulate an effect of this relative motion; after all, it follows from the distinction between shared and relative motion central to Galileo's argument.

Second, on the Maxwellian theory, the physical constitution of the magnet and conductor are irrelevant in this setting, the central issue is that there is relative motion between some physical object and the ether.

The Galilean principle of relativity informs us that we cannot know whether it is the magnet or the conductor that is moving relative to the ether. Still, as they are moving relative to each other, at least one of them must be moving relative to the ether. What Einstein refers to as the "magnet conductor problem" concerns the fact that there are two ways of representing this motion. Either as the magnet moving or as the conductor moving. Maxwell (1865: 487-488) argues that both representations are mere illustrations of the fact that there is motion in the electro-magnetic ether. As such, the magnet conductor problem is not really a problem about magnets and conductors, but rather a problem about motion in the ether as such.

In our treatment of the phoronomic principle of relativity, we showed how we could construe any motion as any other type of motion, as long as we are willing to add further moving systems. In this dynamical case, we confront a similar issue. There is a certain amount of motion going on. We can represent it as being the motion of the magnet or of the conductor, or both. Barring any reference to the overall motion of the galaxy or similar, the amount of motion at any instance (speed) and the duration of this motion, must be kept constant. Which direction we determine the motion to be in (i.e., whether the magnet moves toward the conductor or the conductor toward the magnet, or the magnet and conductor toward each other) should be, and is irrelevant.

On Maxwell's revised theory then, "electromotive force" and "electric field" are auxiliary concepts used as modes of illustration. They represent different ways to talk about the potential for mechanical effects. A possible way out is to simply determine electric and electromotive force as electromagnetic energy (i.e., electromagnetic potential for work). However, auxiliary modes of illustration are by no means useless. We need them to guide our understanding in such a way that we can produce some image of the situation. However, as Chakravartty has repeatedly emphasized, we must exclude them from the set of elements of the theory about which we are realists. What we are missing, is a base for this energy, i.e., a place for it to reside. This is the role of the ether. The difference between the electric field and the electromotive force therefore, is no more than a choice of description. The central aspect of the situation is that there is relative motion between magnet and conductor and therefore that there must be motion relative to the underlying field. The resultant current is an effect of this motion through the field and as such, a property of the entire system. Such a description flows naturally from our basic assumption of a dynamical system where forces are expressions of an underlying field, and mechanical properties of interaction are exchanges of such forces.

IV 5. Lorentzian theories

Throughout this thesis, "Lorentzian theories" refers to all modern ether theories that represent the electrodynamic phenomena within a Euclidean space filled with ether. A central aspect of these theories is that they reject the assumption of rigid physical bodies (see also Maxwell 1865: 464). This leads to a clear separation between phoronomy, where points are rigid, and dynamics, where objects are non-rigid or elastic. Another central aspect is the idea of real potentialities as emphasized by Maxwell (ibid). We are already familiar with such potentialities from the phoronomic treatment of speed. We can spell out the Lorentzian theories within our Kantian framework in the following way:

Geometrically: Euclidean geometry models the space of our outer experience and we can construct Cartesian coordinate systems of three, two or one dimension within this space. These coordinate systems require a pre-axiomatic understanding of straight lines and straight angles.

Phoronomically: Euclidean space is combined with classical time in order to construct the possibility of motion. Motion itself is only understandable as a combination of speed and directionality. Speed, as a potential for motion in a particular direction, is maintained as an intensive magnitude, and we can construct any particular speed by combining other speeds. Thus, in phoronomy, any motion is constructible as any other motion through the insertion of multiple relative spaces. This is done through Galilean velocity addition, which can also be illustrated as a universally applicable vector addition. The motion of all these spaces is representable only as long as we establish an absolute space or "rest system".

Dynamically: There are no physically real rigid objects, and all physical objects contract as an effect of their motion through the ether. Thus, all physical objects are ultimately elastic. Material objects exist within a universal field (ether). We can *illustrate* the inner workings of this field as separate magnetic and electric fields. However, the real existence of these fields is as integrated in a single totality. The totality itself we cannot directly observe, but must infer from experience. As such, the results of the Michelson Morley experiment indirectly proves the existence of the ether by showing how the ether affects physical objects such as an interferometer.

Mechanically: We treat objects *as if* they were separate existences affecting each other through the medium. However, the mechanical effects all result from interactions within the ether in the sense that the ether mediate all interaction, apart from direct contact.

Phenomenologically: The perspectives we take in a particular measurement do not influence the measuring result. I.e., whether I choose to measure from a system in which a magnet or a conductor, is "at rest", I get the same data. Motion through the ether is undetectable as any deformation applies equally to the measuring apparatus (including human beings) as it does to the target objects.

In relation to the Keplerian philosophy of science, we have determined that the central issues plaguing physics in the late 19th and early 20th Century were of a theoretical kind. Einstein, Föppl and others had relied on the Maxwell-Hertz equations and the theoretical explication of Maxwell (1861). Here, we have emphasized how Maxwell in his (1865) corrected the initially dualist image in his (1861). From this, we see that Maxwell was able to provide an ether theoretic description of the phenomena that avoided the dualism by distinguishing between the auxiliary "entities" of his initial theory and the underlying dynamical explanation. Lorentz later developed the ether theoretical perspective and was able to explain the Michelson Morley experiment. In his explanation, Lorentz followed Galileo and Kepler in emphasizing the importance of accounting for the measuring apparatus. In so doing, Lorentz leaves all other elements of the theory unaltered and fully accounts for the observational data. What we end up with is a dynamical ether theory set in Euclidean space with classical time. This theory successfully accounts for all relevant phenomena. Furthermore, the Minkowski structure is fully valid in Lorentzian ether theory as the mathematical relations established by Lorentz make up the foundation of that structure. Importantly however, the Minkowski structure is not a geometry here. Rather, the Minkowski structure is a well-behaved mapping of measurements from different measuring systems made by non-rigid physical objects. As such, it is a convenient tool for relating measuring results. In order to give a geometrical description of the phenomena, we must apply the Lorentz transformations so that we can model the entire set-up within a single space. This space remains Euclidean and thus we avoid any geometrical dualism where there is one "space" for setting up the measuring apparatus and another "space" for moving physical objects. By avoiding this geometrical dualism, we also avoid the physical dualism inherent in special relativity, i.e., the dualism between measuring rods and clocks, and all other things. The Lorentzian theory is therefore a fully coherent system by Kepler's criteria, and we can conclude that whatever the special theory of relativity does, it cannot disprove the Lorentzian ether theory as a possible physical framework. With it, we see that the Kantian theory remains valid as a metaphysics for natural science.

So far, we have argued against the formal axiomatic view of mathematics and seen that, as long as we reject their main tenet – all mathematical reasoning must be deductive reasoning from a system of uninterpreted axioms expressed in pure logic – we can coherently explain the relevant phenomena. Einstein, who started from a Machian empiricist position and gradually moved toward the formal axiomatic theory, argued that there is a better way to explain the same phenomena. Einstein's solution, special relativity, has long been the standard framework for understanding the Michelson Morley experiment as well as the problems inherent in Maxwell's original conception of electrodynamics. In the following, I present Einstein's solution and argue that it is inferior to the Lorentzian option. Before so doing, let us consider Einstein's initial motivation for the 1905 paper.

IV 5.1. Einstein's view of ether theory

Einstein (1905b) argues that light exists and is propagated as particles (photons) rather than as rays or waves. If the photon idea could be fully applicable, there is no real need for an ether-based understanding of the propagation of light.[70] Furthermore, we have already seen that the way

[70] By 1909 Einstein states that "…it is my opinion that the next phase of the growth of theoretical physics will bring us a theory of light which will reveal itself as a kind of mixture of wave- and emission theory" (Einstein 1909: 817).

Einstein sets up the "magnet conductor problem" in Maxwell's theory indicates that he did not know about Maxwell's proposed (1865) solution. That Einstein did not consider Maxwell (1865) becomes even more plausible when we see below that the solution he proposes is strikingly similar to that of Maxwell.[71] Additionally, Lorentz initially presented his theory in terms of an electron deformation hypothesis that was quickly replaced by quantum mechanics. With these uncertainties in mind, Einstein approached the issues with caution concerning explanatory mechanisms and physical speculation. As the special theory of relativity introduces its own speculative concepts, Einstein maintained throughout that it is a temporary solution, rather than a full explanatory system. The 1905b paper, which introduced the light-quantum or photon, is titled, *On a heuristic point of view concerning the production and transformation of light*, and in 1907 he describes special relativity in these terms:

> The principle of relativity, or, more exactly, the principle of relativity together with the principle of the constancy of velocity of light, is not to be conceived as a 'closed system,' in fact, not as a system at all, but merely a heuristic principle which, when considered by itself, contains only statements about rigid bodies, clocks, and light signals. (Einstein 1907: 206)

A similar point is emphasized in Einstein 1919, where he establishes both special and general relativity as theories of principle that do not explain. As a former student of Mach, Einstein initially approached the problems in what he took to be a strictly empiricist way, without accounting for the fact that he had introduced elements such as Euclidean geometry and the principle of inertia, a priori. As we shall see, there are also less plausible elements of special relativity that are introduced a priori, such as physically rigid bodies and a measuring norm which leads to the relativistic "rules for velocity addition". We should note however, that whenever Einstein realized he had introduced metaphysically implausible elements, he justified them as temporary solutions due to a lack of knowledge. So for instance, in the 1921 paper *Geometry and experience*, Einstein makes the following comment concerning rigid bodies:

[71] We also know that Einstein's textbook, written by Föppl, omits Maxwell's 1965 solution.

> The idea of the measuring-rod and the idea of the clock coordinated with it in the theory of relativity do not find their exact correspondence in the real world. It is also clear that the solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which must not play any independent part in theoretical physics. But it is my conviction that in the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. (Einstein 1921: 236-237)

As relativity theory became the orthodoxy of modern space and time physics, these cautionary reflections faded, and as we shall see below, the standard modern philosophical view is to embrace rather than resolve any apparent conflict within the theory. We should keep in mind, however, that, as Bell (1976) puts it:

> The facts of physics do not oblige us to accept one philosophy rather than the other
>
> (Bell 1976: 77)

IV 6. Special relativity and Einstein's philosophy of science

For almost every position in philosophy of science, there is at least one instance of someone arguing that Einstein held that position. A thorough and detailed investigation into Einstein's personal philosophical position has developed from this confusion, and the contemporary consensus is that Einstein's main philosophical influence was David Hume. This consensus is well documented in Einstein's own writings as well as in commentary literature by his companions (see for instance (Norton 2005), (Howard 2005) and (Stachel (2002)). In the following, I adopt the view of Slavov (2016) that Hume's influence on Einstein was double. Einstein adopts the Humean notion that space and time are empirical concepts as well as the general Humean epistemology of impressions laying the ultimate grounds for ideas and concepts. As we shall see, Einstein's adherence to Humean epistemology is important both for understanding the way special relativity was constructed and the way the theory was later debated by Einstein and his peers.

IV 6.1. Einstein and Hume on concept generation

Special relativity, commonly understood, is a theory of space and time that generates a new kinematics for physics.

The Kantian as well as the Newtonian theories of space and time, treat the structure of space and time as a priori given. On a Humean epistemology this is unacceptable as every concept should be generated either from formal laws (in which case the concept is purely formal and says nothing of the world), or the concept must be confirmed by sense impressions. Einstein (1916) takes a clear stand on this issue and discusses simultaneity, space, and time in the following manner.

On simultaneity

> The concept does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled in an actual case. We thus require a definition of simultaneity such that this definition supplies us with the method by means of which, in the present case, he can decide by experiment whether the two lightning strokes occurred simultaneously. As long as this requirement is not satisfied, I allow myself to be deceived as a physicist (…), when I imagine that I am able to attach a meaning to the statement of simultaneity. (Einstein 1916: 16)

On space

> In the first place we entirely shun the vague word 'space', of which, we must honestly acknowledge, we cannot form the slightest conception, and we replace it by 'motion relative to a practically rigid body of reference' (Ibid: 8)

On time

> In order to have a complete description of the motion, we must specify how the body alters its position *with time*; i.e., for every point on the trajectory it must be stated at what time the body is situated there. These data must be supplemented by such a definition of time that, in virtue of this definition, these time-values can be regarded essentially as magnitudes (results of measurements) capable of observation. (Ibid: 8)

Here we see Einstein's connection to Hume as regards both general epistemology and the idea that space and time are empirical concepts. The general line of argument is that neither space, time, nor simultaneity is a priori given. Rather, he claimed that we must reduce these concepts through operational definitions so that what they signify are detectible through experiment. Time, space, and simultaneity are therefore not primary concepts, but rather derived concepts that all ultimately rely on the concept of 'motion relative to a practically rigid body of reference'. Einstein further confirmed these ideas and established in his Einstein (1934):

> Now as regards the concept of space: this seems to presuppose the concept of the solid body. (Einstein 1934: 278).

The contrast to our Kantian set-up is thereby clear. Our approach was to treat geometry and phoronomy purely a priori and thus prior to any empirical investigation. Einstein's approach is to postulate the concept of a "practically rigid body" and use this concept to determine space, time, and simultaneity. His objection to the Kantian notion of a priori concepts is explicit; "There are no final categories in the sense of Kant" (Einstein1936: 292).

In our set-up, we established speed as an intensive magnitude, as well as the Maxwellian electromagnetic potentials. We can test neither empirically. As such, they are problematic notions according to Einstein's general epistemology. This means that Einstein ahould not only introduce new definitions of space, time, and simultaneity, but but also remove the notion of potential forces. A result of this Humean-Einsteinian programme of re-defining is that the terms "space", "time", "simultaneity" etc. take on new meanings, not only in the sense that we learn new things about space, but also that we mean something radically different by that term.

IV 6.1.1. Lorentzian theories and special relativity, notation and terminology

A problem of terminology follows from the radical difference between the general philosophical perspectives of Lorentzian theories and special relativity. On a Lorentzian theory, "time", "space", and "simultaneity" refer to a priori given notions. In special relativity, the same terms refer to results of measurement. Lorentz referred to the results of measurements with postulated physical clocks as "local time" that needs modification through the Lorentz transformations. Einstein's solution was to take the measuring results at face value:

> One had only to realize that an auxiliary quantity introduced by H. A. Lorentz and named by him 'local time' could be defined as 'time' in general (Einstein 1907b: 253).

For the last 10-15 years, we have seen an increased attention to this issue among those who argue that the Lorentzian theory is a valid alternative to special relativity. Three notable authors in this setting are Szabo along with Abreu and Guerra, who in a series of papers[72] argue for the empirical and mathematical equivalence of Lorentzian theories with special relativity. Theirs are conciliatory projects aiming to show that special relativity and Lorentzian theories are simply the same theory. We find the current culmination of the work of Abreu and Guerra in their 2015 paper *Speakable and unspeakable in Special Relativity: Time Readings and Clock Rhythms*[73]

> … both viewpoints are merely different perspectives of one and the same theory. (Abreu & Guerra 2015: 183)

A general line of argument in Abreu and Guerra is that we are misinterpreting the theories as radically different because the language of the theories is blurred.[74] Such blurriness can lead to paralogistic arguments, and according to Abreu and Guerra, this is what is going on in the standard debate over the two types of theories. The kind of paralogism involved here is visible in a simple example

> Bishops only move diagonally
> Francesco Barberini was bishop of Sabina
> Francesco Barberini only moved diagonally
>
> Deinterpreted: F = Bishop, G = only moves diagonally, a = Francesco Barberini

---

[72] (Abreu & Guerra 2005,2008 and 2015), and (Szabo 2010)

[73] Abreu and Guerra recognize that this is a restatement of Bell (1976)

[74] We have already seen, in part III, how the formalist approach to geometry systematically blurs the meaning of familiar terms such as "straight line", "straight angle" and so on. Szabo, Abreu and Guerra recognize that something similar is going on in relation to the terms "space" and "time" in modern physics.

Yields Modus Ponens

$\forall(x)\ F_x \rightarrow B_x$
$\underline{F_a \qquad}$
B_a

In order to avoid confusion, Szabo (2010) introduces a notation that we shall apply in the following. This notation takes into account that in all cases where we wish to denote a spatial or temporal coordinate of some event or process, we must operationalize such coordinates through the application of some measuring apparatus. As we shall see, synchronization procedures for clocks, application of the Lorentz transformations, assumptions about the measuring apparatus, etc., differ in the two theories. We therefore introduce the following notation

Time coordinates as defined in Lorentzian theory = $\widehat{time}$

Time coordinates as defined in special relativity = $\widetilde{time}$

Space coordinates as defined in Lorentzian theory = $s\widehat{pac}e$

Space coordinates as defined in special relativity = $s\widetilde{pac}e$

Concepts directly related to space and time, and that differ in the theories are noted according to the same scheme. So $simu\widetilde{ltan}eity$ refers to simultaneity as defined in special relativity.

IV. 7. Special Relativity On the electrodynamics of moving bodies

The groundwork for both the special and the general theory of relativity is laid in Einstein (1905): Here we find the famous loss of absolute simultaneity and the relativity of time and space that follows from it. The paper divides into two main parts with a set of sub-sections. In the kinematical part, Einstein establishes a new kinematics, and in the electromagnetic part, he inserts the Maxwell-Hertz equations into the new structure. As Einstein makes the most radical changes in the kinematical part, I shall focus on this. The aim of this discussion is to show the importance of Einstein's Humean insistence on operational definitions along with the postulation of rigid or absolute measuring apparatus.

IV 7.1. On rigid bodies

Einstein (1905) starts out by defining the magnet conductor problem. His worry is of the same kind that motivated Lorentz, Larmor, Fitzgerald and Poincaré to develop the Lorentz transformations. Einstein's solution is to alter the kinematics and the methodology of measurement in such a way that the difference between the magnet and conductor perspectives disappears.

The approach is to start from a concept of rigid bodies and establish a new kinematics from relations between such bodies. Although we have seen that neither Maxwell nor Lorentz accepted physically rigid bodies, Einstein insists, "*Like all electrodynamics*, the theory to be developed here is based on the kinematics of a rigid body…" (Einstein 1905: 124 my italics). In order to see the difference between Einstein and Lorentz, I will discuss rigidity in some further detail here.

IV 7.1.1. The postulate of physically rigid bodies

In geometry, a rigid object is one that does not change its shape. If we understand a physical object as rigid, it becomes an absolute object in the sense of Friedman's (1983: 64-70) distinction between absolute and dynamical. A rigid physical object therefore, is one that is not affected by other goings on in world.[75]

> That there are bodily objects to which we have to ascribe, within a certain sphere of perception, no alteration of state, but only alterations of position, is a fact of fundamental importance for the formation of the concept of space (in a certain degree even for the justification of the notion of the bodily object itself). Let us call such an object "practically rigid". (Einstein 1936: 296)

As we have seen, Lorentz and Maxwell argued that there are no physical bodies rigid in this sense. On the ether theory, as in direct perception of rapidly moving objects, physical objects are deformed when displaced. Brown (2005) emphasizes Einstein's neglect of this deformation and seeks a reason for it. As it stands, Einstein's insistence that there exist physical bodies that are rigid in this sense is a postulate particular to special relativity, and we shall treat it as such.

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[75] Newton's absolute space has this feature. It is a physical entity, by virtue of having causal powers, but it is a rigid physical entity in that nothing else can influence it.

First postulate of special relativity:

There exist bodies that are physically real and geometrically rigid

We have already discussed the plausibility of this postulate and seen that it ultimately leads Einstein to accept that the theory inserts an ontological dualism into physics, between the measuring apparatus (thought as rigid) and all other things into physics.[76] In relation to Stump (2015), we should also note that this postulate cannot even in principle, be derived from empirical investigation. It is an a priori postulate that functions as a constitutive element of special relativity. As Einstein notes in the above quoted segment, it is a fact of fundamental importance for Einstein's conception of space and bodily objects.

What we see here, is an instance of Hume's separability principle.[77] An object that, by definition, does not alter its state, is separated from the dynamical interactions of the world and thus a rigid or absolute object. If we think of *all* physical objects as practically rigid bodies, we also face the problem of immediate propagation of forces.

Consider a material rod of 2 meters. Kick the rod from one end. If the rod is mechanically and dynamically rigid, the opposite end must immediately move (i.e. the rod is completely un-elastic). However, an immediate motion implies that the force you imparted on the rod at your end propagates instantaneously to the other end. Another way of putting this is that the force propagates at infinite speed. Now, as we have mentioned, the Lorentz transformations break down at speeds higher than the speed of light. If you have any force propagation at higher speeds, the theory does not work. In working with an electrodynamics that obeys the Lorentzian rules therefore, the propagation of force is maximally that of light which again means there is time between the kicking at one end and the motion at the other. During this time, the rod is contracted, and therefore not rigid. This example works with an impulse and accelerated motion, but according to the Lorentz

[76] See summary of part III

[77] We can see a continuity in Einstein's writing on separability across inquiries. It is generally accepted (see for instance Fine: 2016) that the separability principle is central in Einstein's resistance to quantum mechanics.

transformations, the same applies to objects in inertial motion.[78] The way Lorentz understood this was by maintaining a clear distinction between space, time, and physical objects.

Einstein's solution is radically different. The Humean epistemology leaves him with no possibility of working along the lines of Lorentz. If space and time are operationally defined and empirically determined, they cannot play the role awarded them in the Lorentzian set up. Rather, what Einstein suggests, is that once we have established a material object as a clock, it *always* measures $\widetilde{time}$ correctly. Similarly, a measuring rod measures the correct $\widetilde{length}$. In other words, once we have picked out a particular object as operationalizing a "rest system", we have postulated it as geometrically rigid! Hence the ontological dualism.

We thereby lose our motivation for maintaining a privileged measuring system since any system is equally valid. In relation to $\widetilde{space}$ and $\widetilde{time}$ therefore, material measuring rods and clocks, are rigid. If the Lorentz transformations are correct as applied to systems in relative motion, we must find a way around the problem. We shall see how Einstein finds a way. For now, all we need to know is that as rigidity is concerned,

> *practically rigid rods and clocks give the correct $\widetilde{space}$ and $\widetilde{time}$ measurements. If my measurement differs from yours, we are in different $\widetilde{spaces}$ and $\widetilde{times}$ according to special relativity.*

IV 7.2. Clock synchronization and the postulate of one-way light speed

The kinematic part of Einstein (1905) starts with a section on *the definition of simultaneity*. We have already seen that Einstein requires simultaneity to be experimentally testable in order for it to be justifiably applied in any physical theory. He instructs us to consider a mechanically inertial coordinate system, i.e., a system in which the laws of Newtonian mechanical equations are valid. This is a way of establishing that the system is not accelerated or under influence from external forces. He calls this system the 'rest system'. The position of any object within the system is

[78] Recall that the Lorentzian solution to Michelson Morley is that the apparatus contracts due to non-accelerated motion.

determined by "…means of rigid measuring rods using the methods of Euclidean geometry and expressed in Cartesian coordinates" (Ibid: 125). Thus, Einstein not only maintains that there are physically rigid bodies, but also that these are established in Euclidean space. This is the familiar procedure used to measure distances in our everyday lives:

Take a folding rule, starting from your chosen point of origin, lay it down repeatedly until you reach the object whose position you wish to determine. However, if we wish to determine the *motion* of an object, we need to include coordinates for time and the motion as functions of these. Einstein then insists: "all our judgments of time are always judgments of simultaneous events" (ibid: 125). His example is that of an event coinciding with a clock reading.[79] The aspect of time we are after, in other words, is a time coordinate given a temporal scale represented by a clock. In order to determine a time coordinate, we must find an equivalent system to that which we have for spatial coordinates (a Cartesian system erected in Euclidean space). At first approximation, a single Feynman clock will do.

The time coordinate shown on the clock at the same time as a physical event, is the time coordinate of that event. In order to account for events removed by a large distance from the clock, we need more clocks. We can imagine clocks at any position in our Cartesian system of coordinates. The problem is how to guarantee that they are synchronized. Einstein suggests the procedure of using light signals with the assumption that light always travels at the same speed in every direction (isotropy of light).

Consider two events A and B removed from each other by a large distance. By the above procedure we know the time coordinate of A by reading it off the clock at A's position. We know the time coordinate of B by reading it off the clock at B's position. However,

> So far we have only defined an "A-time" and a "B-time," but not a common 'time' for A and B. The latter can now be determined by establishing *by definition* that the 'time' required for light to travel from A to B is equal to the 'time' it requires to travel from B to A. (Einstein 1905: 126)

[79] Note that this definition of simultaneity is simply a physical operationalization of the a priori pre-theoretical notion of simultaneity as two events coincide. Again, this is a central and constitutive element of special relativity that, contrary to Stump and Pap's "functional a priori" does not have an empirical origin.

We have thus established a second postulate of special relativity:

The one-way speed of light is constant in every direction

As with the first postulate of special relativity, this is inserted a priori or 'by definition' rather than from empirical investigation. As we saw in section IV 1.1., the one-way speed of light is notoriously difficult to measure and its determination is an active field of modern physics.[80]

Using this postulated feature of light, Einstein defines synchronous clocks as those clocks for which it is true that $t_B - t_A = t'_A - t_B$ where t_A and t'_A are the times on the clock A at the sending and receiving of the synchronising light signal, and t_B is the synchronized time on the clock B. We further add a transitivity relation such that if the clock A is synchronous with B, then B is synchronous with A, and if C is synchronous with B, then C is synchronous with A. We can thereby synchronize an arbitrary amount of clocks within our Cartesian system.

We now have a full determination of time coordinates for measurements in this system given that the clocks do not move relative to the system. The time coordinate of an event is the reading obtained simultaneously from the clock at the event, and hence defined to be simultaneous with the clock used for synchronizing the whole system. Finally, we assume that the velocity of light (in empty space[81]) is a constant. How does this work?

Consider two events taking place at points, A and B, in a Cartesian coordinate system. If I wish to know the distance between them, I can measure it by laying down a measuring rod. Now I can set up a clock at point B and let it reflect light signals from and back to point A. When the signal reaches me (at A) from the point B, I know its time of arrival at B by definition. This follows from the postulate that the one-way speed of light is identical in all directions, and the further stipulation

[80] Again, we must emphasize that, as a constitutive element of special relativity that one has not and possibly cannot derive from empirical investigation, the postulate of constant one-way speed breaks with Stump and Pap's theory of constitutive elements.

[81] On the Lorentzian theory, the constancy of the speed of light is in relation to the ether. Einstein switches here to determining this constancy in relation to empty space. I am inclined to follow Wilczek (2008) in assuming that Einstein is drawing here on the possibility of describing light as propagated as particles (photons) rather than waves. This hypothesis is central to Einstein (1905b).

that this speed is a universal constant. I can establish the actual value of the speed of light by the following formula

$$Speed\ of\ light = \frac{The\ distance\ from\ A\ to\ B\ and\ back\ again}{The\ time\ used\ by\ a\ signal\ sent\ from\ A\ to\ B\ until\ it\ returns\ to\ A} \quad \text{Or } V = \frac{2(dAB)}{t'_A - t_A}$$

We now have a total system for determining the time and space coordinates of any event at any point in the system. It is vital that we remember that the clocks and rods in the system are all at rest in relation to each other and the Cartesian coordinate system we imagined at the outset. If something is moving, it is not a clock or a measuring rod *in this system*. Einstein concludes the section by stating

> It is essential that we have defined time by means of clocks at rest in the rest system; because the time just defined is related to the system at rest, we call it the 'time of the rest system' (Einstein 1905: 127).

Here we see that Einstein's use of the term "time" refers to time coordinates as measured with clocks distributed and synchronized in the system at rest. He then moves on to the next section, *On the relativity of lengths and times,* where he finally concludes that there is no universal simultaneity. Note that as long as we consider only one system (the rest system) we have universal simultaneity. All clocks run at the same rate and give the same coordinates while they are all at relative rest. So far, therefore, we have a classical system of spatial and temporal coordinates, operationalized physically by distribution of clocks synchronized with light signals. As concerns any result of measurement within this system, Galilean, Newtonian, Lorentzian theories and special relativity agree. All clocks are synchronized and all unit-defining measuring rods are of equal length. Thus,

For all measurements in a single system, Lorentzian theories and special relativity give the exact same result

IV 7.3. The loss of universal simultaneity

Before considering any further thought experiment, Einstein introduces two basic principles, the light principle and the principle of relativity. In so doing, he is entering the core issue discussed at the time. The principle of relativity was typically understood as a principle of mechanics,[82] while the light principle is a principle of electrodynamics. If the principles can be made to function together, the prospects of a full reduction of mechanics to electrodynamics are greatly enhanced. Einstein states the principle in the following fashion.

The principle of relativity

> If two coordinate systems are in uniform parallel translational motion relative to each other, the laws according to which the states of a physical system change do not depend on which of the two systems these changes are related to (Einstein 1905: 128)

This is a restatement of the Galilean principle of relativity and the main intention of the relativity principles is to establish that as long as you are within such a system, you cannot tell whether you are moving or not.

For instance, when we sit in an airplane mid-air, everything behaves the same way as it would were we on the ground. If there is turbulence or the plane is landing or taking off, we immediately realize that we are moving. The relativity principle states that as long as everything behaves the same way, there is no way to know whether you are "really moving". In Einstein's terminology, the idea of "really moving" is nonsensical as it insinuates an underlying space in relation to which you are moving (for instance Newtonian absolute space). He therefore expresses the same notion in relation to physical laws. [83]

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[82] The difference between the mechanical and the phoronomic principle of relativity is that while the latter applies to all motion whatsoever, the mechanical principle only applies to motion with unchanging velocity.

[83] Einstein sets up an antinomy in this case where the only two options are «a physically detectible absolute space» and the lack of an absolute space. As the principle of relativity shows that we cannot measure any difference between uniform motion and rest, we can reject a detectable absolute space in relation to these motions and thus conclude that absolute space is a false idea. However, as argued throughout part II, there are other versions of absolute space that are not reliant on physical detectability. Rather, space, in the Kantian version, is seen as a

The light principle

> Every light ray moves in the ‘rest’ coordinate system with a fixed velocity V, independently of whether this ray of light is emitted by a body at rest or in motion. Hence,
>
> $$velocity = \frac{light\ path}{time\ interval}$$
>
> Where ‘time interval’ should be understood in the sense of the definition given in section 1 (ibid: 128). [84]

As presented here, the light principle establishes that the motion of the emitting body of light is irrelevant for the velocity of its propagation. As such, the formula for determining the velocity of light, as established in the previous section, still holds. What Einstein wishes to show in the following is that the apparent conflict between the two principles can be resolved. The assumed conflict lies in the following consideration.

Imagine a “rest system” with a rod in it. Measure the length of the rod by laying down a measuring rod next to it (as you would do with a normal meter-stick). Then impart a translational uniform parallel motion to the rod (move the rod uniformly in one direction). If we wish to know the length of the rod, while it is moving, there are two ways to go about it.

1) We can move with the rod and lay down our measuring rod next to it again. The difference being that the rod, the measuring rod and we are all now moving. Importantly, we are not moving relative to each other, but only relative to the “rest system”.

On this procedure, we would measure the length of the rod to be exactly what it was the first time we measured it. This is independent of whether we apply the perspective of relativity or Lorentzian theory. Since we are not moving relative to each other, we share a state of motion. If there is any

prerequisite for any detection whatsoever. Kant and Einstein agree that there is no Newtonian space, but disagree on what other alternatives there are. The Kantian alternative is, however, not available to Einstein as Einstein takes the concept of bodily objects as the primary concept from which the concept of a space must be derived. By formulating the principle of relativity in relation to laws, Einstein maintains that the principle deals only in relations among bodily objects. We thus see that a central element of disagreement is the philosophical question of epistemic (and ontological) primacy.

[84] Note that, as expressed here, the light principle claims only that the speed of light is independent of the motion of the emitting source. It does not claim an independence from the motion of observers. Furthermore, Einstein emphasizes that light speed is constant in the “rest system”. In Lorentzian theory, this is represented by whichever system we establish as at rest in the ether. So far, therefore, the theories agree concerning the light principle.

deformation therefore, it applies equally to all of us and the same result follows as in the initial measurement. This satisfies the principle of relativity, but we have not included the light principle. There is, however, another way to go about measuring.

2) Instead of laying down a measuring rod next to our rod, we can measure its length using light signals. First, we synchronize clocks according to the synchronization procedure established above, which guarantees that light is measured to travel at the same speed in every direction within a given system. We establish such clocks for the "rest system". We can use the same light signals to establish distances and spatial coordinates throughout the "rest system". We are now able to measure the length of the moving rod *from the "rest system"*, and just as in the Lorentzian theories, we find that the rod appears shorter

When measured by a measuring rod at relative rest, the Lorentzian theory and special relativity agree that the length of the rod is l. This is independent of whether the rod and the measuring rod are "moving" together, or if they are "at rest" together. As long as there is no relative motion between the rod and the measuring rod, the length stays the same.

As indicated by Einstein, this result no longer holds when we follow the second procedure and measure the moving rod from the "rest system". In both Lorentzian theories and special relativity, the result of such measurement gives the length of the moving rod as

$$L_{moving} = L_{\text{at rest}} \sqrt{1 - \frac{v^2}{c^2}}$$

Which indicates that the higher the velocity of the rod, the shorter it appears from the system at rest. We can therefore establish that the results of length measurements depend on the measuring procedure and the frame from which one measures. In special relativity, this is referred to as the relativity of $\widetilde{length}$ which is identical with the rod contraction of Lorentzian theories. If, however, we ignore the Lorentzian terminology for now, and simply refer to the result of length measurements between relatively moving systems as $\widetilde{length}$, we can establish another equivalence

For every measurement of $\widetilde{length}$, Lorentzian theories and special relativity give the same result

Einstein insists at this point that,

> current kinematics tacitly assumes that the lengths determined by the above two operations are exactly equal to each other, or, in other words, that at the time t a moving rigid body is totally replaceable, in geometric respects, by the *same* body when it is *at rest* in a particular position (Einstein 1905: 129).

The truth of this statement depends on what is intended by the term "kinematics". If "kinematics" refers to the motion of physical objects (i.e. $\widetilde{kinematics}$), we must include physical considerations and Einstein's claim would be false. There is no such tacit assumption in the Lorentzian theories. On the contrary, Lorentzian theories predict a contraction of the moving physical body. The only difference being that on a Lorentzian theory we would call such an investigation "dynamics" rather than "kinematics".

If, on the other hand, "kinematics" refers to strictly mathematical objects (i.e. phoronomy or $\widehat{kinematics}$), Einstein would be right that there is such an assumption.

We have established that the length of a material rod is frame-dependent in the sense that if we measure the rod's length from the "rest system" we get a different result than if we measure it while moving along with the rod. This is what is meant by the "relativity of length" in the special theory of relativity.

Einstein then uses the relativity of length to make a further and more fundamental point. If we place a clock at each end of the moving rod and synchronize them in the "rest system" according to the above procedure, we get a novel result. In order to motivate this result, let us consider the criterion for synchronization again.

Two clocks, A and B, are synchronized if the following relation holds for light signals $t_B - t_A = t'_A - t_B$, where, again, t_A and t'_A are the measurements on the clock A of the emitting and arrival of the signal sent from A and reflected at B, and t_B is the time of the arrival and reflection at B measured on the clock B. In other words, we have organized the measurements on the clocks so that we measure light as moving at the same speed in all directions. Hence, we have synchronized the clocks of our thought experiment in the “rest system”, meaning that the above relation holds for objects at rest in the “rest system”. The relation does thus not hold in the system of the moving rod. This means that if you are moving along with the rod, you will measure different speeds of light from clock A to clock B and from clock B to clock A. For an observer moving along with the rod therefore, the clocks are not synchronized, while for an observer at rest in the “rest system”, they are. As “being synchronized” refers to “ticking simultaneously”, Einstein concludes that

> Thus we see that we cannot ascribe *absolute* meaning to the concepts of simultaneity; instead, two events that are simultaneous when observed in some particular coordinate system can no longer be considered simultaneous when observed from the system that is moving relative to that system. (Einstein 1905: 130)

Again, we must emphasize that, as long as we follow the same procedure, we get the exact same result in the Lorentzian theories. Since the speed of light is assumed isotropic only in the priority system of such theories, it follows that speeds of light in opposite directions are not understood as identical in other systems. However, on the Lorentzian theories, what Einstein calls the “relativity of simultaneity” is called a loss of synchronization between clocks in relative motion. If, again, we temporarily dispel the Lorentzian terminology and simply apply the term $s\imath m\widetilde{ulta}ne\imath ty$ to all results of measurements following Einstein’s procedure, the two theories fully agree on the result. More generally,

For all measurements of $s\imath m\widetilde{ulta}ne\imath ty$, the Lorentzian theories are equivalent to special relativity

The difference between our Kantian approach and the Humean approach of Einstein is thus further established. Einstein insists that the concept of simultaneity has meaning only as far as it can be confirmed in experiment. Experiments tell us that clocks in relative motion are not synchronous and that events measured as simultaneous in one system are measured as not simultaneous in another. From this, he concludes that one cannot attach an absolute meaning to the concept of simultaneity. We can thus see how Einstein instantiates the Humean notion of concept building and acceptance. A notion he restates in later writings such as for instance “Concepts can only acquire content when they are connected, however indirectly, with sensible experience” (Einstein 1934: 176).

On a Kantian view, the connection with sensible experience is established transcendentally through analysis of the general content of experience. As such, the concept of simultaneity underlies the entire discussion. For although Einstein rejects the absolute meaning of simultaneity, he must presume simultaneity in each system as well as in the establishment of any rest system whatsoever. Additionally, as we have seen, his notion of simultaneity is fully determined a priori and thus prior to the operationalization. The only element of simultaneity that has no absolute meaning is its determination in a particular set of direct measurements. In Einstein (1905) that is treated as the only meaning there is.

IV 7.4. On the relativity of time

There is another way in which one can synchronize the clocks on the rod in our previous thought experiment. Take a clock at each end of the moving rod and synchronize them so that the synchronization relation holds for the rod when it is moving. Our results will then be the opposite of what we found in the previous section. I.e., the observer moving with the rod, will see the clocks as synchronized while anyone measuring from the “rest system” will judge them as not synchronized.

We thus get two different sets of “time coordinates” for a particular event, depending on our choice of measurement. The time coordinate of the event as measured from the rest system, and the time coordinate of the event as measured from clocks synchronized within the moving system. The relation between them is given by the Lorentz transformations

$$time\ from\ the\ moving\ system = \frac{time\ from\ the\ "rest"\ system - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

The Lorentzian theories and special relativity agree on this result.[85] The difference being that special relativity treats $\widetilde{time}$ coordinates, i.e., coordinates that result from measurements in the moving system, as time coordinates, while the Lorentzian theories treat these coordinates as indicators of the physical dilation of clocks due to their motion in the ether. We can thereby establish another equivalence

> *For any measurement of* $\widetilde{time}$*, the Lorentzian theories and special relativity give the exact same result.*

We have thus established the empirical equivalence of Lorentzian theories and special relativity as forewarned in the introduction to this chapter and previously established in Abreu and Guerra (2015) and Szabo (2010). Furthermore, we have seen that time and space in special relativity refer to $\widetilde{time}$ and $\widetilde{space}$ coordinates. In our ether theoretical exposition, we saw that these terms should refer to the dynamical deformations of physical objects. Einstein's postulate of physically real rigid objects prevents him from considering this option.

What we are missing then, is a relativistic treatment of velocity and velocity addition. In the Lorentzian theories, this was established a priori through a phoronomic treatment. As we are dealing in $\widetilde{time}$ and $\widetilde{space}$ however, such a treatment is not obviously available.

IV 7.5. $\widetilde{Velocity\ addition}$ = Translation

Einstein (1905:140-142) introduces what he calls a new «addition theorem for velocities». In so doing, he is introducing a confusion in terminology. The argument as he sets it up is that we consider two Cartesian coordinate systems K and *K'* and a point moving relative to both. He then asks what the velocity of that point is, relative to *K'* once we know the velocity of *K'* and the velocity of the point, as measured from K. Naturally, he finds that this must be expressed by using

[85] See section 3 above.

the Lorentz transformations. We have already seen above how these transformations work. We must, however ask a pertinent question: what is velocity addition?

Velocity is the rate of displacement in one or more directions. Velocity addition is the adding and or subtracting of multiples of such displacements. So total velocity equals velocity$_1$ plus velocity$_2$, or w = ($v_1 + v_2$). Now let us bring back Einstein's set-up. We have two systems K and *K'* in relative motion. Additionally, we have a point p moving relative to both. What Einstein is asking us to do is, given that we know the velocity of p relative to K; find the velocity relative to *K'*. This is exactly the same type of task we had earlier when comparing measuring results in different systems. The action that we are to perform is to translate our results from measurements in one system (in this case K) into measurements of another system (in this case *K'*), which is a translation or transformation. In other words, there is no velocity addition involved in the operation we are told to perform. Einstein (1905: 142) refers to the operation as a "parallel transformation" which further indicates that what we are actually doing here is to take a set of velocities measured in one coordinate system and map them onto some other coordinate system.

What we have then, is simply a confused terminology. What Einstein refers to as velocity addition, is not velocity addition. It is the translation of a set of velocities from one system to another (in the form of a parallel transformation). Recall however, that when we set up the Lorentz transformations to begin with, we used velocity addition in order, not to translate from one system to another, but to find the position of a ray of light that is first travelling in one direction and then reflected back in another direction within one single system. The rules we used were the Galilean w = ($v_1 + v_2$). We should note therefore that as far as velocity addition is concerned, special relativity relies on the Galilean rules. When considering $veloci\widetilde{ty\ ad}dition$ we are simply referring to translation. Therefore

> *The translation rules and the rules for velocity addition are identical in Lorentzian theory and special relativity*

As a simple misnomer, we should not be overly worried about this issue. However, we shall see that Einstein builds on this misnomer in his suggested solution to the magnet conductor problem. For now, let us consider a more familiar issue.

Clap your hands together and then make the following evaluation:

Call the right hand Right and the left hand Left. Consider a point exactly between your eyes as the origin of a measuring system Body. From the perspective of Body, the total velocity is Left + Right as dictated by the Galilean rules for velocity addition. If we wish to contrast this to the relativistic conception, how would we go about it? The apparent answer is that we should apply the Lorentz transformations. In other words, we should measure the speed of Left from the perspective of Right or vice versa. This, however, does not give you the added velocities. It gives you the same situation as seen or understood from another perspective. What we wish to know is what the velocities are from the perspective of Body. To this, the special theory of relativity offers no suggestion as it reformulates all instances of explicit velocity addition as translation operations.[86] Thus, $\widetilde{velocity\ addition}$ is unable to capture the direct experience I have of clapping my hands together. This breaks with our Keplerian demand for continuity as well as Einstein's own description of scientific theory as a sophistication of everyday experience. We should therefore recall that, as long as we stay within one system (for instance Body), we must use Galilean rules for velocity addition.

Einstein's addition theorem for velocities is an application of the Lorentz transformations in cases where we wish to add velocity measurements made in different coordinate frames in relative motion. The central idea in these velocity addition rules is that each system carries with it a different set of $\widetilde{lengths}$ and $\widetilde{times}$.

Take two Cartesian coordinate systems K and *K'* in relative motion. Take an object in motion relative to K and *K'*. Measure the velocity v of the object in the system *K*'. measure the velocity v' of the system *K'* relative to K. We now wish to know, if I measure the velocity of the object in K, what will I get?

[86] The implicit velocity additions that make up the Lorentz transformations remain Galilean throughout.

For simplicity, we imagine the systems organized in such a way that both the system *K'* and the object, as measured from the systems K and *K'*, are moving along the x-axis in the direction of positive x. By traditional thinking we would therefore assume that the velocity w of the object as seen from K, is the velocity of the moving system plus the velocity of the object as measured from that system w = (v + v'). However, we know that the measurements in K and *K'* are made with $\widetilde{length}$ and $\widetilde{times}$. In order to account for this difference, we therefore apply the Lorentz transformations a final time, and get $w = \frac{v+v'}{1+\frac{vv'}{c^2}}$. Hence, we account for the fact that the systems contain clocks that do not run synchronously with each other and simultaneously guarantee that there are no velocities higher than that of light. The last point is important in order to maintain a meaningful application of the Lorentz transformations. However, what we have done does not correspond to the title of the section. We have transformed velocities from one system to another, but we have not added velocities measured in a single system.

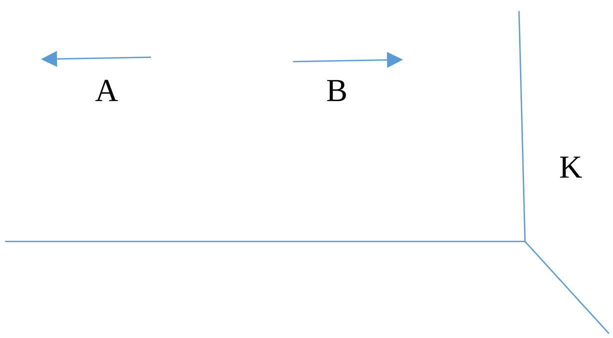
Consider a Cartesian coordinate system K. In it, two objects move in opposite directions at 90% of the speed of light. What is the relative velocity between the two objects in K? The answer according to standard velocity addition is 1.8 x (velocity of light). Physics textbooks[87] typically assume that the velocity between two objects in a single system has to be lower than the speed of light, but there simply is no reason for this in Einstein's theory. Once the velocity of light is established as isotropic within the "rest system", the relative velocity between light and a moving object is ≠ C (the velocity of light). It was from this knowledge we concluded that on Einstein's theory we lose the absolute meaning of simultaneity.

Einstein (1905) introduces the Lorentz transformations as rules for translation and in so doing, saves both the principle of relativity and the light principle. This $\widetilde{kinematics}$ is an instance of the practical geometry approach where physical principles (in this case the light principle) are included in the space-time geometry. A second noteworthy aspect of the new $\widetilde{kinematics}$ is that velocity addition is simply ignored. Einstein applies the language of velocity addition and composition of velocities in "The addition theorem for velocities," (Einstein 1905: section 5) but nonetheless

[87] See for instance (Rohlf 1994: 100)

avoids adding velocities. In order to substantiate this claim, let us define velocity addition and composition of velocities.

Velocity addition requires that we can treat two or more velocities in the same system, say objects A and B travelling at the same speed v in opposite directions in the system K



The observational perspective in this situation is the system K, or more specifically, any point at rest relative to the system K. Now, if I am to add the velocities of A and B, the traditional way of doing this is by simple addition of the velocity of A in the -x direction and the velocity of B in the +x direction. Since the speeds are equal, i.e. v, the relative velocity between A and B, as calculated from K, is 2v.

Einstein's procedure requires that we use the Lorentz transformations as rules for velocity addition, rather than the standard Galilean ones used above. We should also recall that when we established contraction and dilation in section 3 above, we used Galilean velocity addition. There can therefore be no replacement of these. What we have is therefore another dualism, in this case between velocity addition, used in establishing measuring coordinate systems, and $veloci\widetilde{ty\ ad}dition$ coordinating measurements made in different coordinate systems.

The way the Lorentz transformations are used in the theory is as translation rules between *different* coordinate systems. What we are seeking here are rules for adding velocities in *the same system*. Otherwise, we are not adding velocities, but rather performing translations between different systems. The way Einstein sets up the issue is also a clear indication that what he is doing, is simply repeating the translation rules.

In the system k moving with velocity v along the x-axis of the system K, let a point move in accordance with the equations

$$\xi = \omega_\xi \tau$$

$$\eta = \omega_\eta \tau$$

$$\zeta = 0$$

where ω_ξ and ω_η denote constants. We seek the motion of the point relative to the system K. (Einstein 1905: 140)

In order to find the motion of the point relative to the system K, Einstein translates the motion into the system K. There is of course nothing wrong with this; it is just not velocity addition. It is translation. Einstein thus argues paralogistically by conflating translation with addition.

As is typical for a paralogistic argument, the error is not obvious at first sight. In the classical system, the rules for translation and the rules for velocity addition are the same rules (simple addition). This is also the case in Einstein's kinematics, only now the rules are different. What we must ask however, is why we introduced the Lorentz transformations in the first place.

The Lorentz transformations were introduced in order to account for a series of measurements regarding motion relative to different frames. In the ether experiments, it was initially assumed that we could detect the relative motion between the earth and the ether system. The Lorentz transformations were introduced as part of an explanation of why no such motion was detected. Einstein's entire Kinematical part of the 1905 paper is dealing with measurements. However, velocity addition is not a pure case of measurement. It also represents a particular mental process.[88]

[88] It is important to recall that the term "system" here refers to multiple theoretical levels reflecting different aspects of reality. At the base of the argument, there is the idea of different phoronomical systems (spaces) in relative motion. Measuring systems however, are dynamical systems of interacting forces that we imagine attached to the phoronomical systems. We can bring our knowledge from phoronomic systems into our establishment of the (dynamical) measuring systems. We cannot, however, go the other way and derive phoronomical systems from the dynamical ones. The simple reason for this is that there might be dynamical forces within the system that do not reflect the underlying phoronomy. For instance, there might be thermodynamic forces that expand the (dynamical) measuring system in such a way that it stops reflecting the properties of the phoronomic system. In order to compare measurements it is essential that we can know that the measuring apparatus measures in the same way on all relevant occasions. Velocity addition requires that we use the same measuring apparatus to measure different phenomena within a single phoronomic system (i.e. from a single state of motion) and relate them to each other. Having accomplished this much, one can investigate into how these relations would appear from another phoronomic system. This latter process is translation. In special relativity, the velocity addition is simply ignored, and one is supposed to jump directly into translation between different systems by applying the Lorentz transformations.

In velocity addition, we measure the velocity of A and the velocity of B from the perspective of K. We then add these velocities (considered as vectors) together. We, the observers, do not move anywhere. In translation, we measure the velocity of B in K and ask what the velocity of B is in the system where A is at rest. What we do therefore is to move (mentally) from the system K to the system where A is at rest. Although represented by the same functions in classical physics, the mental operations are different. We can see this more clearly in the case of Lorentzian mechanics.

Here, the rules for velocity addition are the same in kinematics as they are in dynamics. The translation rules, however, are not. In Lorentzian dynamics, motion through the ether deforms every physical system. The dynamical rules of translation are therefore the Lorentz transformations. The rules for velocity addition stay the same. Indeed, they stay the same also in special relativity as long as we do not conflate velocity addition with translation. Einstein ignores this difference, as there is no purely mathematical treatment of the phenomena of velocity addition, and indeed of conceiving of velocities and speed being (intensive quanta) as such, to be gathered from his procedure. In the following, we shall therefore refer to Einstein's rules as rules for $\widetilde{velocity\ addition}$.

IV 7.6. Summing up

Until now, we have seen that Einstein insists on operational definitions of basic terms such as space, time, and simultaneity. He also insists on there being no privileged rest system. Furthermore, he establishes a procedure for the synchronization of material clocks such that clocks synchronized in this way always give the correct $\widetilde{time}$ coordinates. Since we know from empirical results that otherwise identical clocks in relative motion do not agree, Einstein concludes that each system has its own $\widetilde{time}$ and its own $\widetilde{space}$ and will pick out a unique set of events as $\widetilde{simultaneous}$. This opens up a new set of possibilities in the manner Einstein derives the Lorentz transformations from the basic principles of electrodynamics and mechanics. The cost of

To put it concisely, if you wish to know the relative velocity between two trains travelling in opposite directions as observed by you when you are standing on the pavement, special relativity offers no answer whatsoever. Rather, the theory implies that you must get onto one of the trains in order to make that measurement, i.e. you must ignore velocity addition.

Einstein's solution is that we must allegedly give up the idea of universal time and space given in a single structure. Read realistically, this implies that there really is no space and time as we typically imagine them. Read instrumentally, it implies that the best way to model the phenomena is within a framework lacking explicit references to universal space and time. Einstein (1907: 206-207) promotes the latter view.

Read instrumentally, it implies that the best way to model the phenomena is within a framework lacking universal space and time. Einstein (1907: 206-207) promotes the latter view.

> The principle of relativity, or, more exactly, the principle of relativity together with the principle of the constancy of velocity of light, is not to be conceived as a "closed system," in fact, not as a system at all, but merely as a heuristic principle which, when considered by itself, contains only statements about rigid bodies, clocks, and light signals.

If we take Einstein at his word, we could ask; what exactly is the relation between Lorentzian theories and Einstein's theory of relativity? More specifically, we could ask; what is the nature of space, time, simultaneity, rigid rods, and rigid clocks in the two theories? In order to answer these questions, we shall look closer at the findings of Szabo (2010).

IV 8. Space-time measurement and operational definitions

The special theory of relativity and any Lorentzian alternative makes use of operational definitions of central terms. The utility of operational definitions is to connect our theories to the empirical world in cases where such a connection is not obviously given. In most cases, there are elements of a scientific theory of such a nature that direct connection is unavailable. Consider for instance planetary motion. The motion of a planet through the heavens might initially appear a straightforward case. You look at the sky, recognize the planet, plot the position and repeat the procedure. Over time, you see the repetition of positions. Now you have a pattern and can plot an orbit by simply connecting the dots. The case however, is not that simple. First, you have to account for the fact that what you observe is the light from a planet hitting your eyes. In order to find the position of the planet therefore, you need to know the path that light took in order to reach you. You also have to understand how different media affect this path so that you know whether light travelled in a straight line or if it was refracted or reflected somewhere. However, unless you can experiment outside the earth's atmosphere, you do not know the density of "empty space" in relation to the density of the earth's atmosphere. What you can do then, is to assume that "empty space" is not at all dense, and that it is equivalent to an earthbound vacuum. Through this assumption, you can operationalize the medium of "empty space", and send light signals through vacuum and see how it refracts when it leaves your vacuum chamber. You note the position that you observed, and consider the refraction in order to find the real position of the planet. Thermometers and barometers for temperature and pressure are more obviously examples of operational definitions. Not all cases are this straightforward.

Consider for instance the notion of "genetic stability" in contemporary biology. As a part of the ongoing debate concerning the safety of genetically modified organisms, one question is whether a modification interferes with the genetic stability of a plant. However, there is no obvious way in which to understand the meaning of the term "genetic stability". Typically, a molecular biologist will look for alterations in the DNA sequence, ecologists will look for adaptive ability, and plant breeders will look for phenotypical stability. They will therefore apply different operational definitions when they go looking for genetic stability. As long as this is clear and we are aware of the differences, the situation poses no problem. However, consider the following statement: "Plants x, y, and z are genetically stable". There is no straightforward way to know the meaning of this statement without checking which theory and operational definition is implied. If we do not find out which meaning is intended, we can easily become victims to paralogistic fallacies.[89]

In developing his new theory of motion, Galileo (1632) pointed out a similar case concerning the meaning of "straight lines" in the peripatetic defense for the Ptolemaic system. A more current example is Lawrence Krauss' use of the term "nothing" as referring to a quantum field without particles. In our case, the problematic terms are "space", "time", "simultaneity" and "velocity".

Throughout his career, Einstein insisted that these terms simply have no meaning unless we can test them in experiment. What we need therefore are operational definitions through which we can test. However, all these terms have different operational definitions in the Lorentzian and

[89] For a more detailed discussion, see Rocca & Andersen (2017)

Einsteinian theories, and consequently, when using the terms “space”, “time”, “simultaneity” and “velocity”, Lorentz and Einstein are simply talking about different things.

Lorentz did not intend to define space and time operationally in any meaningful way. He is presupposing the classical notions of space and time, and within this framework, he tags the coordinates of events. If you find a material clock running at a slower rate than your synchronized clock at rest, that clock is deformed. The route of investigation is to figure out why or by what magnitude deformation occurs. This is a purely dynamical question. $\widetilde{Time}$ and $\widetilde{space}$ coordinates tell you something interesting about clocks and rods as physical objects. In Einstein’s special relativity, we have an entirely different way of thinking about those same physical effects, since the same coordinates represent space-time relations between postulated practically rigid objects in motion.

IV 8.1. Operational definitions of space and time in special relativity

Einstein (1905 and 1916) insists that a concept has no meaning unless it is testable, at least in principle, in experiment. He traces this attitude back to his reading of the philosophical works of Mach and Hume (Einstein 1949: 51). For the present discussion, this reference is of particular interest as the Humean empiricist attitude was a partial motivation for the Kantian philosophy I am defending. On a Machian reading of Hume it seems reasonable to reject any non-empirical content for a physical theory.[90] As noted, Einstein also rejects the notion of a space that can affect but not be affected by physical objects, such as the absolute space of Newton. Within this school of thinking it seems the only way out is either to reject the notions of space and time altogether or find ways empirically to test them. The first option eliminates the possibility of physics. Einstein follows the second option. What are we really operationalizing, then?

Szabo (2010) discusses the different space and time tags of the Lorentzian and the Einsteinian theory. He takes these to be different versions of “space” and “time”. This is perfectly reasonable on Szabo’s logico-empirical construction of the theories. However, the idea that different operational definitions directly indicate “space” and “time” (i.e. the logico-empiricist approach) is

[90] Such an attitude is also implied by Stump and Pap’s view on constitutive elements.

in itself problematic. For it seems that any operational definition whatsoever presupposes space and time in a more original and basic sense.

Whether we are operationalizing: stability, pressure, temperature or any other phenomenon, we either explicitly or implicitly presuppose our ordinary concepts of space, time and motion, for making sense of the operational set-up. Kant's argument, that I am defending here, is that we not only presuppose space and time, but that we presuppose that they have a particular structure. This can explain why Einstein applies classical notions in his thought experimental set-up. In order to see what is going on in Einstein (1905), we shall anyway follow Szabo (2010)'s set-up, but with the following amendment:

In neither Lorentz' nor Einstein's theory are we dealing directly with space and time.

We are dealing with different ways to conduct measurements of phenomena that we already conceive of as given in the form of outer experience. The relevant measurements concern how to determine the spatial and temporal coordinates of any event, which we as such already conceive of as given in space and time. The suggestion that these coordinates represent space and time as such is an additional (and problematic) philosophical assumption.

If we lay aside this philosophical assumption, and think only of different coordinates, we see that there is no difference between the theories. We know that since the relevant phenomena are velocity dependent, if we measure an event from two systems at relative rest, they will yield the same result. Szabo (2010) suggests the following set-up:

There is a meter stick at rest in the International Bureau of Weights and Measures BIPM in Paris. Let this be our measuring rod. Let us also place a clock next to this meter stick. We then introduce two Cartesian systems K and *k'*, both at rest relative to the clock and rod. If an event occurs, how do we measure its coordinates? Both theories agree on the following procedure[91]:

[91] In the following we will use the notation introduced in the previous segment $\widehat{classical/Lorentzian}$ and $\widetilde{relativistic}$

(D1) Time tag in K according to special relativity

> Take a synchronized copy of the standard clock at rest in the BIPM, and slowly move it to the locus of the event A. The time tag $\tilde{t}^{K}$ (A) is the reading of the transferred clock when the reading occurs.

(D2) Space tag in K according to special relativity

> The space tag $\tilde{x}^{K}$(A) of event A is the distance from the origin of K of the locus of A along the x-axis measured by superimposing the standard measuring rod, being always at rest relative to K. (Szabo 2010: 5)

We achieve the time and space tags of the Lorentzian theory in the exact same manner (D3): $\hat{t}$(A) = $\tilde{t}$(A) and (D4): $\hat{s}$(A) = $\tilde{s}$(A). As we know, the difference between the theories comes out when the systems K and *k'* are in relative motion. The space and time tags in special relativity for the system *k'* in motion relative to the BIPM are defined as

(D5) Time tag in *k'* according to special relativity

> Take a synchronized copy of the standard clock at rest in the BIPM, gently accelerate it from K to *k'* and set it to show 0 when the origins of K and *k'* coincide. Then slowly (relative to *k'*) move it to the locus of event A. The time tag $\tilde{t}^{k'}$ (A) is the reading of the transferred clock when A occurs.

(D6) Space tag in *k'* according to special relativity

> The space tag $\tilde{x}^{k'}$(A) of event A is the distance from the origin of *k'* of the locus of A along the *x*-axis measured by superimposing the standard measuring-rod being always at rest relative to *k'*, in just the same way as if it were at rest. Szabo (2010: 6)

Here we find the difference between the two theories. Einstein has postulated that the measuring apparatus is practically rigid, meaning it measures correctly when synchronized within any system. Lorentz argues that for any system set in motion, *the entire system and all elements co-moving with it* suffers a physical deformation. I.e. the clock and rod of the moving system *k'* slows down and contracts by factors given by the Lorentz transformations. In order to get the $\widehat{time}$ and $\widehat{space}$ tags, therefore, we first follow the relativistic procedure, and then correct for the deformation.

(D7) Time tag of an event in *k'* according to [Lorentz theory]

Take a synchronized copy of the standard clock at rest in the BIPM, gently accelerate it from K to *k'* and set it to show 0 when the origins of K and *k'* coincide. Then slowly (relative to *k'*) move it to the locus of event A. Let *T* be the reading of the transferred clock when the reading occurs. The time tag $\hat{t}^{k'}$(A) is

$$\hat{t}^{k'}(A) = \frac{T + X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

(D8) Space tag of an event in *k'* according to [Lorentz theory]

Let X be the 'distance' from the origin of *k'* of the locus of A along the *x*-axis measured by superimposing the standard measuring-rod, being always at rest relative to *k'*, in just the same way as if all were at rest. The space tag $\hat{x}^{k'}$(A) is

$$\hat{x}^{k'}(A) = X\sqrt{1-\frac{v^2}{c^2}}$$ (Szabo 2010: 6)

As definitions 5 and 6 disagree with definitions 7 and 8, we see that the two theories apply different tags. When we say, therefore, that special relativity gives us an entirely new theory of space and time we should remember that this difference consists in using *other ways to determine* spatial and temporal coordinates. We should also note that the resultant $\widehat{\mathrm{time}}$ and $\widehat{\mathrm{space}}$ transforms into the resultant $\widetilde{time}$ and $\widetilde{space}$ by an application of the Lorentz transformations and vice versa. For any clock and any rod measuring any event therefore, Lorentz and Einstein agree on what their value will be. We summarize this in the following statement:

Whenever we apply Lorentz theory and Einstein theory to the exact same events in the world, they give the exact same results

Any disagreement concerns what one takes a certain event to represent. The postulated practical rigidity of rods and clocks in Einstein's theory prevents him from accepting the Lorentzian deformation of rods and clocks. Similarly, Lorentz' theory that physical clocks and rods slow down and contract prevents him from accepting Einstein's "practically rigid rods and clocks". We can therefore add the following statement:

Whenever Lorentz theory and Einstein theory appear to disagree, they are referring to different events

For the further concepts of velocity and simultaneity, their definitions follow from the space and time tags: $\widehat{velocity} = \frac{\widehat{space}}{\widehat{time}}$ and $\widehat{simultaneous}$ events are events that have the same $\widehat{time}$ coordinate. $\widetilde{velocity} = \frac{\widetilde{space}}{\widetilde{time}}$ and $\widetilde{simultaneous}$ events have the same $\widetilde{time}$ coordinate. For any measurement, the two theories still agree on what the $\widehat{velocity}$ and $\widetilde{velocity}$, of an event is, as well as whether any set of events are $\widehat{simultaneous}$ or $\widetilde{simultaneous}$.

However, the latter concepts differ from our classical notions of velocity and simultaneity as there are multiple possible systems from which to measure. As we are to treat all of these on an equal footing, we get "frame-dependent" concepts. This leads us to the velocity addition rule introduced in section 5 of the 1905 paper. As we pointed out, both theories suggest that as long as we are adding velocities in a single frame, the Galilean rules for velocity addition hold. However, Einstein insists on adding velocities in different systems without accounting for the fact that we have synchronized these systems in different ways.

To summarize;

Lorentz theories and special relativity agree on all empirical data

What then, are we to make of the commonly assumed radical distinction between the two theories? Following Abreu and Guerra (2008: 33), we shall conclude that assertions such as "time dilation", "length contraction", "relative simultaneity" and so on are indeed philosophical statements that come in addition to any empirical determination. On the operationalist view, initially proposed by Einstein, we can now make sense of Einstein (1916: 29)

> … the electrodynamics of Maxwell-Lorentz, on which the original theory was based, in no way opposes the theory of relativity.

As far as empirical results and mathematical structure are concerned, Einstein is of course correct. The theories predict the same empirical results from the same mathematical structure. But this covers only two of the Keplerian levels of scientific theory. As soon as we enter into the theoretical realm, the theories are radically different. Einstein's practically rigid objects have no existence in a Lorentzian theory and the ether is abolished from special relativity. A point often ignored in debates over special relativity and Lorentzian theories is that neither practically rigid objects, nor the ether, are testable in any direct way. They can only be given a status as detectable once the overall framework is established. Once we adopt a Lorentzian theory, the ether is detectable through the deformation of objects in motion. Once special relativity is established the practically rigid objects are detectable, indirectly, through the space-time coordinates. However, neither concept has any theory-independent measure. As such, a practically rigid object as well as the ether, functions as an unexplained explainer in the form of a postulate. Its justification thus relies wholly on its explanatory value. We shall therefore have to look at the theoretical differences in more detail.

IV 8.2. Classical space and time in special relativity

We know that Einstein concludes that there is no universal or absolute meaning to the notion of simultaneity and that our notions of space and time should depend on our ability to determine simultaneous events. He correctly states that contemporary kinematics tacitly assume classical notions of space and time. However, what Einstein refers to as kinematics here is a different thing altogether from what he proposes as the result of his investigation. The contemporary kinematics that tacitly assumes classical notions of space and time, is a pure kinematic, i.e. a kinematic that deals only in motion as such, prior to any empirical investigation. What Einstein wishes to develop is a physical kinematics based on physical knowledge. This much is clear from his use of the light principle. However, what Einstein seems incompletely aware of is that we gain this physical knowledge through an application of a pure kinematics. In other words, Einstein is operating with two notions of kinematics – one explicitly and one tacitly.

In building his arguments and set-up on thought experiments, he uses classical pure kinematics. Let us call this $\widehat{k\imath nemat\imath cs}$. What he ends up with, is an empirically based $\widetilde{k\imath nemat\imath cs}$. Consider the following set-up from Einstein (1905):

> Let there be two coordinate systems in the 'rest' space, i.e., two systems of three mutually perpendicular lines originating from one point. Let the X-axes of the two systems coincide, and their Y- and Z-axes be respectively parallel. Each system shall be supplied with a rigid measuring rod and a number of clocks, and let both measuring rods and all the clocks of the two systems be exactly alike (Einstein 1905: 130).

What we have here are two Cartesian coordinate systems at relative rest with clocks and rods distributed within them. From part III of this thesis, we know that the establishment of such systems require notions of orthogonal and straight lines. We also see that Einstein does not define what he means by "parallel" here, but for the successful application of the Lorentz transformations, "parallel" must refer to the classical notion. There is also nothing dubious about Einstein's use of these systems as long as everything is at relative rest. In situations where there is no motion, the space and time of Einstein's special relativity is straightforwardly Euclidean. When everything is at rest, there is no velocity at play. However, Einstein maintains the tacitly assumed classical notions of time and space also when he introduces motion. We shall not concern ourselves with the result of measurements here, as what we are looking for are the presupposed notions of space and time in the set-up.

> Now, put the origin of one of the two systems, say *k*, in a state of motion with (constant) velocity v in the direction of increasing x of the other system K, which remains at rest; and let this new velocity be imparted to *k's* coordinate axes, its corresponding measuring rod, and its clocks. (ibid).

What Einstein asks us, as readers, to do here, is to move all the elements of k (in thought), *simultaneously*. The determination of this simultaneity is common to both systems, as it must be. For if the internal relation between the x, y, and z-axes is not maintained throughout the motion, the systems cannot be related through the Lorentz transformations. If the moving system *k* keeps on moving, the motion of the three axes (x, y, z) keeps on being simultaneous and constant from both perspectives. We can therefore maintain a notion of simultaneity at an arbitrarily long distance as well as at arbitrarily high velocities. These qualities reflect rigid systems in motion through classical time and space, i.e. $k\imath n\widehat{emat}\imath cs$. Note that we are not discussing the measuring results gathered from the rods and clocks that are moving simultaneously with the coordinate axes, but only their simultaneous motion. Therefore, the geometrical system itself is phoronomically rigid in this case. If the moving system is to maintain its function as a measuring system in the Einsteinian thought experiment, it must display this feature. In order for such a system to have *physical* validity, we must further assume that these phoronomical relations hold for physical systems of measuring apparatus. Any non-simultaneous motion that alters the internal relations would produce radically different results.

Why does the simultaneous motion of the axes, rods and clocks determine classical notions of space and time? Let us look at the spatiotemporal features of the system in motion. It has three orthogonal lines that remain orthogonal. There is no change of internal angles. Furthermore, we describe these lines as parallel to the lines of the system at rest. In other words, Einstein tells us to accept the Euclidean parallel postulate throughout the thought experiment. In part II, we saw that what partially determines the non-Euclidean geometries and "geometries," is their denial or replacement of Euclid's parallel postulate. When we enter Minkowski's space-time "geometry", for instance, we find that a different parallel postulate holds. However, we build this Minkowski space-time "geometry" from the *results of measurements* and it is thus of a different category than the $\mathrm{k\imath n}\widehat{\mathrm{emat}}\mathrm{\imath cs}$ applied here.

What we are inquiring into, is the production of results in the thought experimental set-up. In addition to retaining the parallel postulate, we also retain the separation between space and time. We have a coordinate system consisting of three spatial axes (x, y, z) and a measuring rod for the determination of spatial coordinates, and a clock for the determination of time-coordinates. Ignoring the clock and rod, we shall in the following only focus on the three spatial axes (x, y, z).

The axes are three mutually perpendicular lines (ibid). The lines have to be straight in order to give meaning to the terms "perpendicular" and "parallel". We can choose to move the system *k* along any single or any combination of these lines. Einstein moves *k* in the direction of positive x. There is no boundary set for this motion and we can move *k* arbitrarily far along x. We thus satisfy the first two postulates of Euclidean geometry as determined in Sklar (1974: 14-15).[92] The mutual perpendicularity of the three lines satisfy the fourth postulate.[93] Einstein's insistence that the moving lines *y* and *z* remain parallel to the lines y and z of the system K at rest only makes sense as long as "being parallel" picks out a distinct direction. In other words, we presuppose with the fifth Euclidean postulate that being parallel is a distinct property singing out one line through each point outside a straight line.[94] All that remains in order to satisfy all Euclidean postulates, is the third postulate which states; "about a point a circle of a specified radius exists" (Sklar 1974: 14). This postulate illustrates the inward and outward infinity of space, which we have already established through the arbitrariness of the relative distance and direction of motion between the systems *k* and K in any direction, and which Einstein further implicitly confirms through the conception of the spherical expansion of light behind the idea of the synchronization procedure.

In addition to the correspondence with the Euclidean postulates, Einstein (1905: 131) announces homogeneity as a property he accepts as an attribute of space. In conclusion, Einstein's thought experiments tacitly assume the classical notions of space and time and the very same classical $k\imath n\widehat{emat}\imath cs$ he criticizes. Hence,

> *The Lorentzian and Einsteinian theories are both constructed within the perspective of classical notions of space and time*

This is in perfect agreement with Kant and Frege's notions of intuition based geometry. Einstein instructs us to derive a set of relations from empirical data, but once we ask how to arrive at these data in the first place, we are back to our initial presuppositions. If we have to presuppose classical notions of space and time in order to arrive at a particular structure, how can we reject these notions arguing that they conflict with the (mathematical) features of that structure? This is a breach of Kepler's coherence criteria established in part I.

[92] 1: Two points determine a straight line, 2: A straight line may be extended in a straight line in either direction

[93] 4: All right angles are equal

[94] I.e. given a line l and a perpendicular line *l*. Through a point p on *l*, there is a single and distinct line that can be drawn such that it is parallel to l.

However, if we simply apply the notions “kinematics”, “space” and “time”, we might fail to notice that we are working with entirely different things. The $\widetilde{space},\ \widetilde{t\imath me}$, and $\widetilde{k\imath nemat\imath cs}$ of special relativity are results of measurements based in an operationally defined apparatus. In order to get to them, we set up thought experiments that use the classical notions of $\widehat{space},\ \widehat{t\imath me}$ and $\widehat{k\imath nemat\imath cs}$. Rejecting the latter arguing from the former is tantamount to tearing down the building blocks of the theory. So far we have not seen any successful attempt at arriving at $\widetilde{space}$, $\widetilde{t\imath me}$, and $\widetilde{k\imath nemat\imath cs}$ without presupposing $\widehat{space}$, $\widehat{t\imath me}$ and $\widehat{k\imath nemat\imath cs}$.

As it stands, special relativity and the Lorentzian alternative both tacitly assume classical notions of space and time. The difference between the theories is a difference between accepting these assumed $\widehat{not\imath ons}$, and an acceptance of the same $\widehat{not\imath ons}$ in addition to the empirically based $\widetilde{not\imath ons}$. Such an analysis is not available within Einstein’s Humean epistemology. For although Einstein accepts that some scientific notions are “free constructs of the imagination”, he consistently rejects the Kantian idea that some basic concepts are given a priori. The Humean *tabula rasa* strictly opposes the possibility of anything stronger than a convention concerning elements that are not directly testable. On the relation between Kant and Hume on these issues, Einstein writes:

> I am reading Kant’s Prolegomena here, among other things, and am beginning to comprehend the enormous suggestive power that emanated from the fellow and still does. Once you concede to him merely the existence of synthetic a priori judgments, you are trapped. I have to water down the “a priori” to “conventional,” so as not to have to contradict him, but even then the details do not fit. Anyway it is very nice to read, even if it is not as good as his predecessor Hume’s work. Hume also had a far sounder instinct. (Einstein 1918b: 7)

What I am arguing, is that the watering down to “convention” is misplaced when we are dealing with Einstein’s use of Euclidean geometry in his construction of the very theory that supposedly *proves* that Euclidean geometry is not a priori given.

IV 8.3. Classical space and time in the Lorentzian theory

Roughly stated, Lorentz claims the following: Consider two identically constituted objects, O and O' that function as measuring apparatus for the determination of space and time coordinates. If O and O' are at relative rest, they determine the same spatial and temporal coordinates. If they are in relative (constant translational)[95] motion, the moving object (considered as moving relative to the ether) will contract and physical processes within it will slow down. It will therefore show different time and space coordinates from those determined by the object O. There is no way to determine in any ontological manner whether O or O' is "really moving". What we have to do is therefore to choose pragmatically one of the objects as "at rest" and the other as moving. Which one we choose is irrelevant for our current purposes. However, once we have chosen a "rest" system, we must stick to our choice when introducing further objects. In other words, we pragmatically choose a perspective from which to view the world. If we measure from an object in relative motion to our perspective, we apply the Lorentz transformations so that we get the spatial and temporal coordinates we would have gotten had we measured from our chosen "rest" perspective. If we apply this method, we never get into any issue with the nature of space and time.[96] Such a pragmatic procedure might initially appear unwanted, but we should recall that it is perfectly in line with our general approach to the world as well as the procedures of general physical thinking. We know that there are systems from which we can measure the world as if the earth is standing still. Nevertheless, we ignore such perspectives and stick to a single model in which the earth is moving. We do this for two reasons. It allows us to maintain a single world-view, and it allows us to maintain laws of nature. However, in order to justify the statement "the earth moves", we need to assume a privileged "rest" system that we cannot derive directly from our empirical observations.

Pragmatically to choose a system as privileged is therefore less of a new invention and more of an application of what we usually take to be sound physical thinking. Lorentz never explicitly states it, but the notions of space and time he applies are classical space and time (Euclidean three-

[95] A central point of agreement between Lorentz and Einstein is the fact that they take inertial motion as given, and thus that the motions relevant for the frames of the measuring apparatus are limited to translational inertial motions. Later re-formulations by Stachel (1983), Frisch (2011), Lange (2013) and others, attempt to circumvent this limitation by defining the class of relevant motions indirectly (See section 7.2 for a treatment of this approach).

[96] Abreu and Guerra (2006) define this set-up as a set of "Galilean clocks"

dimensional space and a one-dimensional time). In the Lorentzian theory therefore, we assume the same notions of space and time in the thought experimental set up and in our conclusions. What changes from classical mechanics as well as electrodynamics is that *the shape of physical objects and the ongoing processes within them* somehow depend on their velocity. As velocity affects all physical objects in the same way there is no experiment by which an observer within a system in motion can detect any difference, and the mechanical principle of relativity holds. Furthermore, the observer's shape and internal processes change in exactly the same way as any other object.

Knowledge concerning whether space is absolute in the Newtonian sense, relational in the Leibnitzean sense, or intuitive in the Kantian sense, is not a prerequisite for the Lorentzian theory. All the theory does, is apply a particular and familiar spatiotemporal structure. According to the phoronomic principle of relativity, any system in any state of motion can thus be treated as "at rest" relative to absolute space. The mechanical principle of relativity however, limits the amount of physical systems that can be treated in such a way to the inertially moving ones. Once, we have picked out a system as the "rest system," it represents rest relative to absolute space, or the ether system. The Lorentzian interpretation of the Michelson Morley experiment is that any object moving relative to the ether system dilates and contracts in relation to objects "at rest". As this dilation and contraction is predictable, moving rods and clocks maintain their function as indicators of distance and duration. If you are a Leibnitzean about space and time, you could argue that "space" and "time" are concepts built from the relations between objects in the rest system and objects moving relative to them. If you are a Newtonian, you could argue that the rest system is but one of many possible representations of the physically real absolute space. If you are Kantian, you could argue that the absolute space is a necessary prerequisite for the whole exercise and that the ether is best understood as its physical properties. This means that the Lorentzian theory is primarily a theory of dynamics. If physical deformations are included in a kinematical representation of the theory (for instance in the way special relativity was read at the time of its publication), we are no longer working in a pure kinematics derived from principles, but rather in a kinematical model of the dynamics of moving bodies. The philosophical discussion concerning relationalism or substantivalism of space is thus an independent worry, not directly linked to Lorentz' theory or to special relativity.

IV 8.4. Operationalized space and time and paralogistic arguments

Considering the operational procedures of Einstein (1905), there is a dissonance in the use of several central terms. Einstein does not distinguish between the classical space and time of the thought experimental set-up and the $\widetilde{space}$ and $\widetilde{time}$ that follows from his use of the light principle and the principle of relativity along with a specific synchronization procedure and the assumption of practically rigid rods and clocks. He simply denotes them as “space” and “time”. We have seen that this leads to a confusion concerning what the theory actually tells us. Now that we have a clearer view on the relationship between these terms, we can ask whether the standard realist reading of special relativity as a space-time theory is justified. More specifically, we can ask whether we deem it a reasonable procedure to apply relativistic concepts of space and time when we already know that we have constructed these by applying the classical notions.

The relation is exactly as described by Frege concerning non-Euclidean geometries. To the extent that there are intuitable models of non-Euclidean geometries, they are models drawn on familiar objects in Euclidean space. To the extent that there are relativistic notions of $\widetilde{space}$ and $\widetilde{time}$, we have to construe them within the framework of classical space and time. If there were options to this procedure, we should at least have seen attempts at reconstructing Einstein’s thought experiments without classical space and time by now. We may also ask whether the notion of operationalizing space and time reductively makes sense to begin with.

Einstein’s procedure was to synchronize clocks and then postulate that these clocks continue to give correct time coordinates independently of their motion. This leads to problematic dualisms. The dualism between the *practically* rigid measuring apparatus and everything else was apparent to Einstein (1949: 55-57), and a reasonable ground for him to maintain that the theory is not finished. The second dualism is between the space and time of the set-up and the resultant $\widetilde{space}$ and $\widetilde{time}$ of the theory. Einstein appears to have been unaware of this internal dualism in his theory. However, this dualism points to the problem of measuring something that you have already postulated from the outset. Typically, such a procedure just leads to circular argumentation and question begging. These points would be good criticisms of Lorentz if he had intended his theory as a theory of time and space. However, in Einstein’s theory, the result is not circular definitions as much as it is contradictory ones.

There is, of course, a way out of this by postulating a further dualism between “manifest space and time” and “physical space and time”, where the notions of manifest space and time are our classical notions and the notions of physical space and time refer to “how things really are”. However, we know that the possible empirical data counts identically well for Lorentz’ and Einstein’s theories and therefore “how things really are” must be postulated.[97] There are no solutions to this problem in the literature, and it seems that any operational definitions of space and time will suffer from similar problems simply because any measurement whatsoever presupposes the notions of space and time.[98] I therefore conclude with Szabo (2010) that special relativity, by itself, tells us *nothing new* about space and time. It simply picks out a set of measurement results and calls them “space” and “time”.

IV 8.5. Lorentzian theories, special relativity, and Minkowski space-time “geometry”

Minkowski (1908) presented a new “geometrical” structure that radically simplifies the coordination of $\widetilde{space}$ and $\widetilde{time}$ coordinates. Standardly, this structure is treated as the new structure of space-time in contemporary literature. Treating Minkowski’s structure as a “space-time geometry” only makes sense as long as you are willing to treat $\widetilde{space}$ and $\widetilde{time}$ as the true measurements of space and time. From a Lorentzian perspective, the Minkowski structure does not model space and time, but it is a very helpful tool for coordinating the dynamics of the measuring apparatus. Minkowski’s structure is therefore equally valid for both theories.

The Minkowski structure is ultimately a systematization of four-tuples of numbers. There are two steps necessary to treat it as a “space-time geometry”. The first step is to provide geometrical meaning to the four-tuples (as they stand they are un-interpreted symbols x, y, z, t). There is nothing about these symbols and the relations between them that indicates that they represent space and time. We then need to attach meaning to the terms, and in so doing, we introduce a series of philosophical commitments. Furthermore, we must introduce relations between the supposed

[97] As we shall see below, this identity only applies as long as we limit ourselves to the phenomena treated in special relativity (uniform translational motion). The Lorentzian theories can also account for rotational and accelerated phenomena but these are excluded from the ontology of special relativity as they lead to paradoxes in that theory.
[98] This point is also emphasized in Abreu and Guerra (2005, 2006 and 2008).

spatial and temporal coordinates (such as that there are no more spatial directions; the directions are orthogonal to each other etc.). In other words, we must dress the structure up in a full-blown space-time theory.

The second step we must take is to treat the measuring results that Minkowski structures, as "space" and "time" measurements. Again, this is a free philosophical choice and does not follow from the data. A Lorentzian will treat the same results as measurements relevant to determination of the dynamics of moving bodies. We can now make our final point of equivalence.

> *The Minkowski structure works equally well for Lorentzian theories as it does for special relativity. The theories agree on $\widetilde{space}$ and $\widetilde{time}$ measuring results and the Minkowski structure simplifies the organization of such results. The only remaining disagreement concerns what the structure represents.*

We therefore conclude that as concerns any prediction of measurement or any mathematical modelling, special relativity and Lorentzian theories are identical. However, in distinction to Abreu, Guerra and Szabo, I suggest that this does not make the overall theories identical. They differ in the theoretical realm.

IV 8.6. How are Lorentzian theories different from special relativity?

Abreu & Guerra (2015), as well as Szabo (2010) claim that there is actually only one theory. Their approach to the term "theory" here is limited to what can be determined from experiment (Szabo 2010, and Abreu & Guerra 2015: 185). On a positivist reading of theory, they are correct. The theories are empirically and mathematically identical as long as we limit ourselves to discussing issues that are directly related to experiment and that exclusively deal with inertial motions. If, however, we apply a wider notion of the term, the theories are radically different in that they present distinct world-views, motivate different amounts and types of further study, and apply different basic intuitions about what constitutes a good theory.

The empirical and mathematical content dictates the following: As translation laws, we must apply the Lorentz transformations rather than the Galilean transformations. Empirically this means that material clocks synchronized in a common system and then put into relative motion do not remain synchronized. Material rods placed next to each other in a single system, and found to be of identical length, are no longer of identical length when they are in relative motion. The theoretical question is why. Since these new insights apply to all physical systems, independently of their material constitution, it is natural to search for the answer in the most general terms possible. A major difference between Maxwell and Lorentz on the one side, and Einstein on the other, is that Maxwell and Lorentz find their answers in the dynamical theory while Einstein finds his in $k\imath n\widetilde{ema}t\imath cs$.

We can spell out the philosophical difference between the two approaches within a classical distinction in philosophy, i.e. in terms of rationalism and empiricism. Maxwell and Lorentz takes a rationalist approach to the extent that they take space, time, and simultaneity as a priori given. Their only option is therefore to alter the most basic physics, which, according to their electro-dynamic project, is dynamics.

Einstein, on the other hand, accepts no a priori definitions of this kind. Most notably in (Einstein 1916 and 1921), he emphasizes the need to operationalize every concept that is to play a role in physical theory. He sides with Hume in thinking that among these concepts we find space and time.[99] Space and time in relativity are empirical concepts subject to measurement like any other such concept, hence the new kinematics in Einstein (1905). As we have seen however, Einstein is unable to establish space and time in this fully empirical way. The measuring apparatus presupposes Euclidean space and classical time, and the physically rigid bodies along with the constant one-way speed of light are postulated a priori. In the next segment, we shall see that there

[99] Hume (1739-40: Book I, Part I) sets out to reduce space and time to sense impressions. In Book I, part II he retreats from this position and determines space and time as manners of ordering impressions (See also Slavov 2016). Einstein's task therefore is to either solve Hume's philosophical problems concerning time and space in light of empiricism or to successfully operationalize the ordering of impressions itself. Arguably, Hume's existential crisis in the *Conclusions of this Book* points toward an insight that the project cannot be completed. "The intense view of these manifold contradictions and imperfections in human reason has so wrought upon me, and heated my brain, that I am ready to reject all belief and reasoning, and can look upon no opinion even as more probable or likely than another. Where am I, or what?" Hume (1739-40: 268-269).

is a final postulate of special relativity, which we have only hinted at so far. This postulate concerns the measuring norm Einstein introduces in order to solve the magnet conductor problem.

IV 8.7. Einstein's solution to the magnet conductor problem

In order to understand Einstein's approach to the magnet conductor problem and its relation to Maxwell's solution, we shall start from Einstein's (1905) opening statement

> It is well known that Maxwell's electrodynamics – as usually understood at present – when applied to moving bodies, leads to asymmetries that do not seem to be inherent in the phenomena (Einstein 1905: 123).

The first noteworthy element of this statement is the 'as usually understood at present'. Hon and Goldstein (2005) gives a thorough overview of what Einstein probably refers to as the usual understanding, and as parts of their treatment are relevant for our problem here, I shall briefly run through the issues.

Maxwell (1865: 459-460) distinguishes his approach from that of Weber and Neumann who have assumed action at a distance between particles in the relevant objects.[100] Maxwell's novel suggestion is to explain the mechanical (observable) phenomena as resulting from "… actions that go on in the surrounding medium as well as in the excited bodies…" (Ibid: 460). It is within this framework that Maxwell determines the electromotive force as a mode of illustration. Furthermore, Maxwell writes

> … energy in two different forms may exist in the medium, the one form being the actual energy of motion of its parts, and the other being the potential energy stored up in the connexions, in virtue of their elasticity (Ibid: 464)

Neither Einstein nor, according to Hon and Goldstein (2005), Hertz, Heaviside, nor Föppl develop this aspect of Maxwell's theory. Rather, one takes 'Maxwell's theory' as equivalent to Maxwell's

[100] Maxwell refers to theories such as these as "mathematical theories".

equations. Hertz and Heaviside developed Maxwell's equations in such a manner that the equations are fully symmetric. By the time Einstein enters the field, the 'usual understanding' of Maxwell's theory is simply that it consists of the Maxwell-Hertz equations and the reflections provided in Maxwell (1861).

Given that the equations are symmetric and the empirical data (the appearance of a current) are univocal, there is the appearance of theoretical over-determination, since we apparently can describe and explain the same phenomenon in two different ways depending on the framework we choose. Einstein's problem, as he sees it, is therefore to dissolve the 'asymmetry' between the univocal phenomena and mathematics on the one hand, and the two sets of physical theoretical explanations, on the other. From Einstein's treatment of the issue, it is apparent that he took the 'usual understanding' of the physical theory at the time to be that electromotive force, electric field and magnetic field were all physically real and distinct existences.[101]

IV 8.7.1. Einstein's approach to solving the problem

Einstein first alters the $\widetilde{\mathrm{kinematical}}$ framework by introducing the Lorentz transformations as kinematical principles and applying notions of $\widetilde{space}$ and $\widetilde{time}$. Once the new $\widetilde{kinematics}$ is established, he inserts the Maxwell-Hertz equations (i.e. the mathematical formulation of Maxwell's electrodynamics) without alteration. Then Einstein returns to the magnet conductor problem.

> By way of interpreting these equations, we note the following remarks. Imagine a pointlike electric charge, whose magnitude measured in the rest system is 'unit,' i.e., which, when at rest in the rest system exerts a force of 1 dyne on an equal charge at a distance of 1cm. According to the principle of relativity this electric charge is also of 'unit' magnitude if measured in the moving system. (Einstein 1905: 145)

[101] Understood in this way, Maxwell's theory does provide two radically different stories for the same empirical phenomenon. If we take our understanding from the later Maxwellian theory, Maxwell agrees with Einstein that the asymmetries are not inherent in the phenomena. Rather, they are inherent in the dynamical base for such phenomena, i.e., in the ether. It is apparent however, that Einstein reads electromotive force, magnetic field etc. as explanatory rather than, as Maxwell (1865) presented them, as illustrations.

He then continues to set up the old mode of description of this situation, i.e., the situation, as it is "usually understood" including the electromotive force. Finally, he suggests a new mode of description.

> If a unit point electric charge moves in an electromagnetic field, the force acting on it equals the electric force at the location of the unit charge that is *obtained by transforming the field to a coordinate system at rest relative to the unit charge.* (Ibid: 146, my italics).

The procedure suggested by Einstein is therefore to ignore all systems other than the system in which the point unit charge is at rest. Note that this is described as the procedure leading to the determination of the force acting on the point charge, and it is thus presented as the true description. As we saw in section 7.5., we are thereby compelled to ignore rest systems in which both the magnet and the conductor are moving. What Einstein presents therefore, is a new norm for measurement, which plays the role as the final postulate of special relativity

> *Whenever two physical objects are in relative motion, a good measurement must treat one of the objects as 'at rest' in the rest system.*

If we apply this measuring norm, there is never a problematic electromotive force.

There is a similarity to Maxwell's solution here in that Maxwell defines the electromotive force as an "illustration", while Einstein defines it as an "auxiliary concept" (Einstein 1905: 146). The difference is that Maxwell imagines the situation from a standpoint of the ether where it does not matter whether it is the magnet or the conductor that is moving.[102] All that matters is that there is motion relative to the ether, and as long as there is relative motion between the magnet and the conductor, there must be motion relative to the ether. Einstein's solution on the other hand is to focus on measurement.

[102] Indeed, in Maxwell's set-up we can always describe the situation as if they are both moving.

> We can see that in the theory developed here, the electromotive force only plays the role of an auxiliary concept, which owes its introduction to the circumstance that the electric and magnetic forces do not have an existence independent of the state of motion of the coordinate system. (Einstein 1905: 146)

At first sight, this seems a somewhat odd claim. How can it be that elements of reality (magnetic and electric forces) have existence or not depending on our choice of coordinate system? On Maxwell's point of view, we consider electric and magnetic forces as not being truly independent phenomena. They are treated as different but integrated aspects of a unified electro-magnetic field. Since our motion in this field determines how we experience and measure forces, we get different ways of generating the results depending on our own state of motion. The coordinate system we choose reflects our state of motion with respect to the underlying basis of these phenomena. This again means that electric and magnetic forces are treated as frame-dependent existences, and that we must reflect this in the theory we develop about the underlying reality.

According to Einstein, however, the true description of the magnet conductor situation is to translate all measurements into the system where the point charge is at rest,[103] hence the asymmetry between observation, mathematics and theory disappears. There is now only one description allowed also in the theory.

We thus have two theoretical suggestions for solving the magnet conductor problem. The primary difference being that Maxwell is analyzing two phenomena (magnet moving and conductor moving) and must ask why it is that these different phenomena result in the same current. His answer is that it is the fact that there is motion in the overall system that creates the current. What moves is, for this particular question, irrelevant.

Einstein on the other hand sees the situation as being two differing descriptions of the same phenomenon. His solution is then to adopt a *norm of measurement* stating that one should always measure from the system in which the unit charge is at rest, in effect ignoring the alternative descriptions. As such, Einstein's solution to the magnet conductor is more of a pragmatic

[103] Einstein applies an identical approach to problems in optics. "All problems in the optics of moving bodies can be solved by the method employed here. The essential point is that the electric and magnetic fields of light that is influenced by a moving body are transformed to a coordinate system that is at rest relative to that body. By this means, all problems in the optics of moving bodies are reduced to a series of problems in the optics of bodies at rest." (Einstein 1905: 152)

convention than an attempt at a resolution of the real issue. This specific norm of measurement is also highly problematic when applied to other cases. For instance, Galileo in imagination places the reader outside the moving earth in order to illustrate the similarity between measurements on the surface of the earth and measurements on boats. From here, Galileo shows that the earthbound measurements yield the same results whether or not the earth is moving relative to that perspective (or the sun). Hence, shifting perspectives in this way was instrumental to discover and demonstrate the basic principles of classical physics, indeed, the original principle pf relativity and the concomitant principle of inertia, still presupposed by Einstein. Choosing an approach similar to Einstein's pragmatic one, at the time of Galilei, would have ended the scientific revolution before it happened.

Einstein (1936) argues that, since the most basic elements of a theory are not derived from experience but rather free constructs of the human (or animal) mind, scientific theories must ultimately rely on conventions of this kind. In effect, there exists no solution to this kind of problem apart from pragmatism. We are however, not provided with a rationale for this norm apart from its pragmatic value in relation to the magnet conductor problem.

IV 8.8. Summing up special relativity

As initially conceived, special relativity was to be a cautious description of the empirically known facts at the time and we have seen that Einstein remained cautious throughout. What he took to be the proper Humean position gradually replaced the philosophical heritage from Mach. As such, Einstein gave up on the idea that all physical theory builds inductively from empirical knowledge.

> However, a merely casual look at factual development already teaches us that big advances in scientific knowledge originated this way [inductively from empirical facts] only to a small degree. For if a researcher would approach things without preconceived notions, how would he be able to pick the facts from the tremendous richness of the most complicated experiences that are simple enough to reveal their connections through laws? Galileo would never have found the law of free-fall without the preconceived opinion that the situations as we find them are complicated by the effects of air resistance, and therefore, that one has to focus on cases where this effect has only negligible influence. The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction. (Einstein 1919b: 1)

In order to evaluate physical theory therefore, we must also evaluate the philosophical elements that are presupposed prior to any measurement. We have seen that Einstein makes a series of postulates that do not appear in the Lorentzian theory. The least plausible of which are the existence of physically real but rigid bodies (see section 7.1.), and the measurement postulate (section 7.5. and 8.7.). Only when these postulates are accepted, do we end up in special relativity. Even if we accept these postulates, we do not have a replacement of classical time and Euclidean geometry. These are prerequisites for the measuring apparatus that Einstein uses throughout. At best, we can generate a series of dualisms where we have two geometric structures, two notions of simultaneity and time along with two sets of kinematics. Or, as we saw Einstein state in our summation of part III, a dualism between the measuring apparatus and all other things.

IV 9. The contemporary philosophical debate over special relativity

Although Einstein remained cautious when discussing the philosophical implications of his theory, the standard reading of special relativity in contemporary philosophy is a realist reading. In other words, the received view is that special relativity provides us with new concepts of space and time. Nevertheless, there are criticisms and thus there is a continuing philosophical debate over these issues. The contemporary debate focuses around three main issues.

(i) Brown and Pooley, among others, have argued that there must be a dynamical explanation of the measuring apparatus. Such an explanation is not present in the contemporary version of special relativity. Dorato (2007), Van Camp (2011), Lange (2013), Janssen (2002), Stachel (1983) and others, in their approaches to this alleged problem, contrast with this view. The latter group defends special relativity as a kinematical theory where the dynamics is explained by the kinematics. There is, according to the latter group, no need for a separate dynamical explanation.

(ii) Einstein (1919) presented a distinction between principle and constructive theories, where only the latter type of theory yields explanations. In the same paper, Einstein defines relativity theory as a principle theory. The debate concerning the principle/constructive distinction focuses on whether a constructive alternative is possible, as well as whether it is true that only constructive theories provide explanations.

(iii) The standard view is that modern physics disproves Kant's transcendental argument for Euclidean geometry. However, as there is no straightforward alternative theories of space and time, Einstein, Friedman, Reichenbach and others have established what Stump (2015) calls "adjective a priori" approaches. The general idea being that we must adopt a foundational set of concepts in order to investigate the world, but that contrary to Kant's idea, these concepts are free constructs of the imagination. Once established, these concepts play the part as a priori given. As we have seen, Szabo, along with Abreu and Guerra challenge this interpretation of the empirical and mathematical data relevant to the theory of relativity, and thereby reopen the debate. I will argue that the Kantian option is not only available, but also preferable to the "adjective a priori" approaches.

IV 9.1. Dynamical and kinematical approaches to central elements

We have seen that a central difference between special relativity and the Lorentzian theory concerns their positions on the measuring apparatus. Einstein views the measuring apparatus as practically rigid and Lorentz views it as dynamically elastic. Ives (1951), Brown (2005), Szabo (2010), and others point out that it appears as if Einstein chose simply to ignore what they take to be an empirical fact; material clocks slow down and rods contract due to their motion.[104] There is merit to this complaint, but there is also a defense of Einstein's choice with roots in a more general philosophical problem.

Consider a simple mathematical object like a square. A natural way to test whether this object is rigid is to pick out two arbitrary points (a and b) on the object and measure their distance d(ab). If this distance remains the same while the object is moved or acted upon, the object is rigid. All strictly mathematical objects are rigid in this sense. However, as argued by Einstein (1921), Minkowski (1908) and more recently in Disalle (2006), the Minkowski structure is not an instance of pure mathematics. It is a model of a series of measurements on (imagined) physical objects. In order for a physical object to be strictly rigid, any force distribution within that object has to be instantaneous. As the speed of light is a limit in the Minkowski structure (as it is in special relativity

[104] As pointed out in Part I section 3.1., ignoring the dynamics of the entities involved breaks with the general outlook of semirealism. As the physical contraction is ignored, we are left only with the structural aspects of the situation.

and the Lorentzian theory), rigidity in the above sense cannot be a property of any physical object. Physical objects must therefore be dynamically elastic. Still, in approaching dynamics in this way, we have made multiple steps. First, we have attempted to find a property of an object by analyzing a relation between two of its constituents (points a, and b). Second, we have assumed that these constituents are themselves rigid. If the relation d(ab) is to be a proper indicator of rigidity, a and b cannot change in size and shape during the measurements. If we imagine a physical rod and replace the points (a, b) with arbitrarily short segments s_1 and s_2, we have a possible physical measurement. Still, we have to treat these segments *as if* they are rigid even if we wish to determine the dynamical nature of the rod. In a next step, we can ask whether these segments really are rigid, and in order to do so we must establish smaller segments that we treat *as if* they were rigid, and so on. What I wish to show is that any element can be treated as rigid or non-rigid depending on which question we wish to answer.

From the Lorentzian dynamical perspective, we can treat clocks and rods *as if* they were rigid and simply ask what their measured coordinates will be. We can then take these coordinates as indicators of physical contraction and dilation in relation to our privileged measuring system. Furthermore, we can ask whether this system in its entirety is rigid. In order to do so, we must postulate a larger system, such as the ether, and treat this system *as if* it were rigid. This allows us to evaluate the rigidity of the privileged system. We can then further ask whether the ether system itself really is rigid, which demands the postulation of an underlying rigid (absolute) space.

These two examples are intended to show that whether we are reductionist or holists, we must always switch between which elements we take as foundational/stable/absolute for a given scientific inquiry. Furthermore, we see that there is always another step in the inquiry. Dorato (2014) correctly identifies this kind of interplay as a central element of scientific revolutions, but he neglects that in any such interplay, all rigid (or in Dorato's case, structural) elements must be open for a further dynamical treatment. As soon as we pick out an element as *ontologically* rigid rather than an element that can be treated *as if* it is rigid for a particular investigation, we block future investigation. A rigid non-dynamical object is one that cannot meaningfully be separated into further interacting parts. Ontologically therefore, a rigid object forces a dualism by picking out a particular set of objects that follow different rules. In special relativity, this particular set of objects are the postulated practically rigid rods and clocks.

Einstein is therefore entirely correct in pointing out that there is an ontological dualism in special relativity and that this dualism concerns the measuring apparatus on one side and everything else on the other (Einstein 1949: 55). The problem is not so much that there are these rigid elements in the theory. Such elements could be treated dynamically later on (as Einstein suggests in 1921 and 1949). The problem is rather that the vast majority of contemporary literature in philosophy of physics argues against a dynamical treatment. This is done through advocating a realist reading of the theory as if it is already finished. In other words, the majority argument is that there is no need for development of special relativity. This has led to a series of neo-Kantian positions, which Stump (2015) calls constitutive theories. These fall into an overall debate on Einstein's (1919) distinction between principle and constructive theories. Before entering that debate, we shall enter the debate concerning whether or not we can live with a purely kinematical theory.

IV 9.2. Kinematics vs dynamics

The problematic nature of the presumably practically rigid measuring apparatus of special relativity has led to an alternative tradition for understanding the Lorentz transformations. Rather than reading Minkowski's structure in the way it was introduced (as a model over the physical phenomena common to the Lorentzian theory and special relativity), the proponents of this tradition argue that the Lorentz transformations can be derived without reference to the light principle.

It is believed that the upshot of such an approach is that we can find the Lorentz transformations purely from an application of the principle of relativity and thereby avoid operationalization and synchronization procedures altogether. It is noteworthy that this implies a divergence from Einstein's initial insistence that nothing can play a part in our theory unless it is (in principle) testable in experiment. There of course is no sin involved in departing from Einstein's theories in physics or philosophy of science, but it should be clear that special relativity in this new form, is radically different from Einstein's theory.

Einstein built his theory from an empiricist standpoint with the intention to define every active element in the theory operationally.[105] The new version is an idealist theory where we replace the

[105] As we have seen, the operational definitions themselves are naturally dependent on a priori postulates.

dynamical light principle and the mechanical measuring rods with idealized mathematical objects. I shall take Stachel, Lange, and Janssen as examples of contemporary proponents of this departure. Janssen (2009) and Lange (2013) both argue from the premise that Stachel (1983) has already established the possibility of a purely kinematical interpretation of special relativity. In part III section 4.4.2.2., we saw that Stachel's attempt boils down to a simple postulate of there being an upper speed limit. If we pretend that we did not know that this idea comes from electrodynamics, we at least get the apparent choice between a Euclidean and a Minkowskian kinematic. In deciding between the Minkowskian and the Euclidean structure, Stachel (1983) offers no solution. We should also recall that the Minkowskian kinematic itself follows from $\widetilde{space}$ and $\widetilde{time}$ coordinates generated by Euclidean measuring systems in Euclidean space and classical time. All motion in that structure is initially conceived through the Galilean rules for velocity addition. However, there are other suggestions for determining the kinematics as Lorentzian in order to avoid the ontological dualism between measuring apparatus and all other things. One such approach is given in Janssen (2002/2009).

IV 9.2.2. Janssen's redefinition of kinematics

Rather than directly entering the philosophical debate on classical vs. relativistic space and time, Janssen (2002) argues from those problems in electrodynamics that gave rise to both the Lorentzian theory and special relativity. He finds that special relativity is superior as it can apply a common cause argument in explaining a range of problems in electrodynamics. However, Janssen argues that this common cause is the Minkowski structure:

> The argument basically turns on the observation that the Lorentz invariance of all physical laws, on which Einstein and Lorentz agree, points to a Minkowskian rather than a Newtonian space-time structure (Janssen 2002: 438).

We have seen that this is debatable. That the Minkowskian structure maps $\widetilde{space}$ and $\widetilde{time}$ is established. What Janssen needs to show is that these are the correct operationalizations of space and time. In absence of such an argument, all we have is the Minkowskian structure, which maps both theories equally well, although with differing interpretations.

Janssen (2002: 431) observes that the Lorentzian theory suffers in light of quantum physics. While it is correct that quantum theory superseded the Lorentzian electron theory, arguing for special relativity as a space-time theory on these grounds, is a category mistake. Neither special relativity nor the Lorentzian theory have explanations of phenomena at the quantum level.[106] The established relation between special relativity and quantum physics is the establishment of Lorentz invariant quantum physics by Dirac in "The Principles of Quantum Mechanics" (Dirac: 1930). However, Lorentz invariance is common to the theories we are investigating here and cannot serve in favor of one or the other.

In invoking an argument from quantum physics, Janssen appears to conflate two elements of the Lorentzian theory. The first element is the electron theory on which Lorentz was working. The second element is the deformation theory when generalized to all material objects and all physical phenomena. It is correct that the deformation theory was an attempt to save the electron theory, but it does not depend on it. Neither does the underlying ether theory that Lorentz was working with in order to understand both electrons and deformations. The relevant aspects of the Lorentzian theory in our discussion are the deformations of all material entities due to motion, their determination by the Lorentz transformations and their explanation in the ether theory. These are untouched by quantum physics.[107]

The main argument in Janssen (2002) is that special relativity offers a common cause argument where Lorentz does not. Again, we are dealing with a conflation of different elements of the theories. One could refer to a common cause in the Lorentzian alternative simply through the deformation hypothesis; "The reason why phenomenon x is Lorentz invariant is because all

[106] See Giovanelli (2014) and section 9.3.4.1., below, for the discussion between Weyl, Pauli, and Einstein concerning both special and general relativity and the relation to quantum physics.
[107]What appears to be a real threat from quantum physics, are the recently discovered loophole-free Bell inequality violations in Hensen et al. (2015) These affect not only the Lorentzian theory but the entire mathematical framework. Roughly speaking, Hensen et al. have found that two spatially separated physical entities can influence each other instantaneously, which means that influences between them are communicated at infinitely high velocities, which contradicts the upper speed limit of the Lorentz transformations. The discoveries are "loophole-free" if there are no other explanations for the results.

material objects are deformed by velocity".[108] This cannot be a final answer. We would need to know why all objects deform due to their velocity. However, temporarily taking deformation as a brute fact is no more mysterious than taking $\widetilde{space}$, $\widetilde{t\imath me}$, and practically rigid objects as brute facts.

A similar objection was suggested in Brown (2005: section 8.2). Brown asks how the structure of space-time can causally guide the behavior of physical objects. I.e. how can it be that space-time is the cause of a particular behavior? Janssen responded to this objection by somewhat changing his position in 2009. Here Janssen argues that

> Special relativity is preferable to those parts of Lorentz's original ether theory it replaced because it shows that various phenomena that were given a dynamical explanation in the former theory are actually kinematical (Janssen 2009: 26)

What was a common *cause* argument in 2002 has now become what Janssen (2009: 27) calls a Common Origin Inference.[109] The common origin Janssen refers to is of course the Minkowski $\widetilde{space}$-$\widetilde{t\imath me}$ $k\imath\widetilde{nemat}\imath cs$. However, Janssen's argument for why the Lorentz invariance of physical laws determines them as kinematical rather than dynamical effects is somewhat surprising. It seems as if Janssen is suggesting that we redefine what we mean by kinematics. The suggestion is that we establish two ways in which a phenomenon is kinematical.

> A phenomenon is kinematical in the broad sense if it is *independent of the specifics of the dynamics*. It is kinematical in the narrow sense if it *is an example of standard spatio-temporal behavior* (Janssen 2009: 28).

[108] Here, Janssen is mainly arguing against the position of Brown (2005), who according to Janssen postulates *specific* causal or constructive arguments for the issues at hand. Although in itself interesting, I will not go deeply into Brown's account. However, the Lorentz/Maxwell approach I am arguing for is an approach to the overall perspective. The deformation is thence a general effect concerning all matter.

[109] Janssen rejects any causal influence from space-time, as this would force acceptance of an absolute space-time acting on objects.

Janssen then argues along a series of complex examples that the special relativistic approach is superior because if we assume it, a range of otherwise dynamical phenomena becomes kinematical. My objection to Janssen's approach is that his new definition of a kinematical phenomenon effectively blurs the distinction between dynamics and kinematics.[110] For instance, Oxford dictionary of physics (1985) defines dynamics as "the branch of mechanics concerned with the motion of bodies under the action of forces", and kinematics as "the branch of mechanics concerned with the motions of objects without being concerned with the forces that cause the motions". Consider two rectangular objects leaning on each other in the manner of two playing cards in a house of cards. On the dictionary definition, we must refer to dynamics if we wish to understand why the objects remain erect; i.e., we must refer to forces inherent in the objects and external forces acting on them if we wish to understand their behavior. We do not need to discuss any *specifics* in the dynamics, as any two rectangular objects would display the same behavior. Our house of cards is a kinematical phenomenon in Janssen's broad sense.

As neither object is moving, and therefore are inertial, we also have a kinematical phenomenon in the narrow sense. The problem is that being an instance of standard spatio-temporal behaviour is needlessly vague, while being independent of the specifics in the dynamics is too wide. Take for instance Newton's third law of motion:

> *If one body exerts a force on another, there is an equal and opposite force, called a reaction, exerted on the first body by the second*.

This law is clearly independent of the specifics of the dynamics, but to call it a kinematical law simply blurs the meaning of kinematics into a branch of physics that deals with general behaviour. If we finalize a Standard Model and show that all of physics reduces to it, it seems we have kinematics all the way down. I opt for maintaining the standard distinction between dynamics and kinematics and disregard Janssen's attempt at a re-definition. We now move on to the latest attempt

[110] Insisting that special relativity shows that the relevant phenomena are kinematical in nature is thus a paralogism in the form of an equivocation as we have seen with the notions of space, time and velocity. I shall use the established notation in the following. I.e. $K\imath n\widehat{emat}\imath cs$ for a standard definition and $k\imath n\widehat{emat}\imath cs$ for Janssen's new version.

at establishing the Lorentz transformations as kinematical of which I am aware. This is found in Lange (2013).

IV 9.2.3. Lange's meta-law description

Lange (2013) focuses the debate on a series of objections from Brown (2005) and sets out to argue against Brown's position. As Brown's position differs from mine – Brown argues for a "dynamical special relativity" – I shall focus on Lange's positive account. The main idea put forth by Lange is that there is a way to distinguish dynamical from kinematical versions of special relativity if one views the Lorentz transformations as meta-laws. The difference between a meta-law and a first order law is spelled out in terms of places on a hierarchy of generality. Roughly, a meta-law constrains a lower level law, but a lower level law does not constrain a meta-law.

Lange's argument is similar to that of Janssen (2009) in focusing on the specific laws demanded to account for phenomena in Maxwellian (classical) electrodynamics. However, Lange makes the same paralogistic argument as Stachel (1983) in claiming to derive the Lorentz transformations from the principle of relativity only. As in Stachel, there is the assumption of an upper speed limit for interaction and propagation. Again, the choice of such an upper limit is at best conventional, and in light of our knowledge that, if we do not assume it, we get the classical notions of space and time, it seems arbitrary in this context. Once the convention is adopted as a space-time kinematic, Lange makes an interesting point concerning meta-laws.

If all measurements give the same invariances, it seems strange to look for solutions in the specific laws at play for the particular phenomenon. We should therefore expect to find an explanation only if we look into the general aspects of nature. The special relativistic position is that the generality is found in the space-time structure itself. The Lorentzian suggestion is that we should look at the basics of dynamics. For Lorentz and Maxwell, this means looking at interactions within the ether. Neither of these positions refer to specific force-laws. As such, the role of the Lorentz transformations in these theories are on equal footing concerning Lange's hierarchy of laws. As a mechanism for choosing between Lorentzian dynamics and relativistic kinematics therefore, Lange (2013) is ill fitted.

IV 9.2.4. General issues in the Kinematical approach

We have glanced at the arguments in Lange, Stachel and Janssen and found that they do not provide sufficient reason to choose special relativity over the Lorentzian alternative. However, I think the entire rephrasing of the debate over the Lorentz transformations into a debate over special relativity with or without the light postulate (kinematics vs. dynamics) is an unhappy development.

Let us assume that there really was a way to derive the Lorentz transformations as exclusive solutions to transformations of Cartesian coordinates in relative motion. How would this inform us about the state of natural phenomena? As they are set up in Lange (2013), Janssen (2002 and 2009) and Stachel (1983), all we have are tuples of real numbers (x, y, z, t). We are then given the Lorentz transformations as solutions for relating these real numbers to another set of real numbers (*x', y', z', t'*). We do this by performing algebraic operations on them. However, how do these real numbers connect to space and time in general, and to "real physical space and time" in particular? In the actual arguments of Lange and Stachel, it is clear that the connection is established through the classical notions of space and time:

We set up Cartesian coordinate systems in Euclidean space with a universal time. This gives us space and time for each system. We then relate the systems through Galilean transformations. Thereafter, we postulate an upper limit for speed along with relativistic rules for *velocity addition*.[111] The total system is intended to represent a new kinematics. This kinematic is set up in Euclidean space with universal time and then altered into a pseudo-Euclidean 'space' with system-relative time. It is also severely limited as it deals exclusively with relations between rigid systems in inertial motion. Accelerated motion, rotational motion and motions along curved lines demand another kinematical set up. The Galilean kinematics that the new Minkowskian structure is supposed to replace deals with all these types of motion. It therefore seems odd to talk of a replacement of kinematics. Furthermore, the Galilean kinematics is set up in and maintains Euclidean space and universal time. Since Ignatowski started the kinematization project in 1909, we still have no suggested plausible explanations or solutions to these severe limitations and discrepancies. All we have are spurious re-discoveries of the following fact:

[111] Recall that these are actually translation laws.

> *If you assume the principle of relativity, homogenous time, Euclidean space for every single system, isotropic space for the overall system, inertial motion, and practically rigid objects, you only need to postulate an upper velocity limit in order to get the Lorentz transformations.*

When this is achieved, there is still work to be done in showing how this idealist system connects to physical reality. It seems the only way to construct such a connection is to introduce operational definitions, which would bring us back where we started.

The kinematization procedure has little to do with the worries of either Einstein or Lorentz. Stachel (1995) and Lange (2013) seek such a connection by quoting the same passage from Einstein (1935).

> The special theory of relativity grew out of the Maxwell electromagnetic equations. So it came about that even in the derivation of the mechanical concepts and their relations the consideration of those of the electromagnetic field has played an essential role. The question as to the independence of those relations is a natural one because the Lorentz transformation, the real basis for the theory of the special relativity theory, in itself has nothing to do with the Maxwell theory … (Einstein 1935: 223)

Stachel and Lange both take this quote as establishing Einstein's real insight into the matter, which is that the theory turns out to be a theory of space-time that is independent of dynamics. This story, however, does not stand up to scrutiny. Einstein's goal from the start was to maintain the operationalist attitude and to found all elements of the theory on physically testable phenomena rather than idealized speculation. The independence from Maxwell's theory is an independence from the specific laws of Maxwellian electrodynamics. As we have seen, the special theory of relativity relies on the postulation of practically rigid rods and clocks to derive the necessary operational definitions of space and time tags. Einstein quickly realized that these cannot play an independent part in the theory, but should rather be described from reflections within the theory. However, as the state of physics at the time, there was not - as is still the case - a viable theory for the micro-constitution of macro objects.

Janssen (2002) addresses this problem and points out that the new quantum physics was a big threat to the Lorentzian molecular theory, the only viable theory that combined the Lorentz transformations with Maxwell. The fact that the Lorentz transformations are independent of Maxwell's specific theory just shows that whether the molecular laws of matter are Maxwell's or some other laws does not matter. We still have the Lorentz transformations and we still have no explanation as for the specifics of how matter is constituted. Special relativity must therefore rely on something else. Stachel and Lange's suggestion that the entire dynamics is irrelevant appears mysterious in light of their own quote from Einstein, which continues:

> … and because we do not know the extent to which the energy concepts of the Maxwell theory can be maintained in the face of the data of molecular physics. In the following considerations, except for the Lorentz transformations, we will depend only on the assumption of the conservation principles for impulse and energy. (Ibid)

It should be clear therefore that Einstein's view is that the theory remains the way it is due to a lack of knowledge.[112] I emphasize this difference between the kinematical approach and Einstein's approach in order to clarify that even if the kinematical approach fails, this does not hurt Einstein's position or the theory of special relativity in general. The kinematical approach is a theoretically independent suggestion. It appears therefore that, apart from the paralogism inherent in the kinematical approach, there is a further parallel to the classical debate.

One of Salviati's main objections to the Peripatetics in Galileo (1632) is that the authors claim authority by reference to Aristotle without staying true to the Aristotelian approach. There seems to be a similar thing going on here. Lange and Stachel claim the authority of Einstein as a quasi-justification for the kinematical hypothesis by reference to Einstein. However, as also pointed out by Giovanelli (2014), Einstein repeatedly rejected any notion of "geometrizing" the laws of physics in this way.

> It is common knowledge that it is impossible to base a theory of the transformation laws of space and time on the principle of relativity alone. As we know, this is connected with the relativity of

[112] For a fuller description of Einstein's position on kinematical approaches, see Giovanelli (2014).

> the concepts of 'simultaneity' and 'shape of moving bodies'. To fill this gap, I introduced the principle of constancy of the velocity of light, which I borrowed from H.A. Lorentz' theory of the stationary luminuferous ether, and which, like the principle of relativity, contains a physical assumption that seemed to be justified only by the relevant experiments (Fizeau, Rowland, etc.). (Einstein 1912: 131)

My evaluation of the kinematical approach in general is that it brings with it a conflation of categories. The main idea behind it is that we get a new kinematics by a priori means. Therefore, we should replace the old and outdated Galilean kinematics. As we have seen, however, every construction of the Minkowski structure presupposes the Galilean kinematics. There can therefore be no replacement, only additions. Concerning the Lorentz transformations and the Minkowski structure, it seems Einstein, Lorentz and Minkowski already presented the more plausible views. Physical facts have led to a structure that maps both the Lorentzian and the relativistic theory. Our role therefore, is to understand those physical facts. Not to treat them as if they were mathematical. The kinematical/dynamical debate is intertwined with the debate concerning principle and constructive theories that I shall now turn to.

IV 9.3. Principle and constructive theories[113]

Much of the contemporary debate concerning special relativity relates to Einstein's 1919 article *What is the theory of relativity*. In it, Einstein distinguishes between what he calls constructive theories and principle theories. What has caught the attention of most commentators is Einstein's claim that both theories of relativity (the special and the general theory) are principle theories, and that such theories do not provide explanations (Einstein 1919: 228). Einstein likely borrowed the distinction from Lorentz (1900).

In the following, I provide a context for Einstein's distinction and show how it relates to his general view on scientific knowledge. I will also show how the principle/constructive distinction connects well to the history of philosophy of science. Finally, I shall discuss complementary and opposing views in the contemporary debate.

[113] This section is an elaboration on a previous manuscript "What is a space-time theory", in collaboration with my supervisor Johan Arnt Myrstad.

IV 9.3.1. What is the theory of relativity?

Einstein (1919) first describes what he takes to be a constructive theory.

> We can distinguish various kinds of theories in physics. Most of them are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out. Thus the kinetic theory of gases seeks to reduce mechanical, thermal, and diffusional processes to movements of molecules – i.e., to build them up out of the hypothesis of molecular motion. When we say that we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the process in question (Einstein 1919: 228)

The constructive theories are then contrasted with the principle theories.

> Along with this most important class of theories there exists a second, which I will call 'principle-theories'. These employ the analytic, not the synthetic method. The elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. Thus the science of thermodynamics seeks by analytical means to deduce necessary conditions, which separate events have to satisfy, from the universally accepted fact that perpetual motion is impossible. (Ibid)

Finally, he compares the two types of theories and places relativity in the latter camp.

> The advantages of the constructive theory are completeness, adaptability, and clearness, those of the principle theory are logical perfection and the security of the foundations. The theory of relativity belongs to the latter class. (Ibid)

What are we to make of this distinction, and why should we think of relativity, or in our case, special relativity, as a principle theory? My suggestion is that we take our cue from Einstein (1936).

IV 9.3.2. The multiple layers of a scientific theory

Einstein (1936) lays out a schematic description of scientific theories from the viewpoint of general epistemology. In it, the first and primary idea is that of the bodily object. This is not identical in any non-arbitrary way to a mind-independent existence, but rather related to sense impressions. Einstein opts out of the philosophical discussion concerning the relation between these and goes on to discuss the concept of the bodily object in terms of sense experiences.

> Considered logically, this concept is not identical with the totality of sense impressions referred to; but is a free creation of the human (or animal) mind. On the other hand, this concept owes its meaning and its justification to the sense impressions which we associate with it. (Einstein 1936: 291)

These concepts, i.e., concepts that relate to sense impressions although they cannot, logically, be derived from them, are what Einstein refers to as primary concepts (Ibid: 293). The totality of concepts that relate directly to sense experience and relations among them, constitute the first layer of a scientific theory.

In order to increase unity, simplicity and generality of our theories, a second layer is added. This includes concepts that have no direct connection with sense experiences. However, the second layer is justified by the ideal that the first layer of the theory can be derived from them. Take for instance the material point in Newtonian mechanics. Although there is no direct experience of such a thing, it is justifiable. I have experience of pressure created by heat when I make coffee. This pressure can be explained by reference to the motions of material points (water molecules) within the water-chamber of my coffee maker. As the motions become increasingly fast, the water molecules push their way up through the coffee. The general utility of the concept of a material point, thus justifies it as a lower level concept, even in the absence of direct experience.

An important aspect of lower level concepts in Einstein's stratified philosophy of science is that they are chosen freely. As long as the lower level concepts are useful in divulging the goings on between the primary concepts of the first layer, they are justifiable. The ultimate test of their utility is, in other words, whether they are instrumental in saving the phenomena. Einstein gives the analogy of a game:

> All that is necessary is to fix a set of rules, since without such rules the acquisition of knowledge in the desired sense would be impossible. One may compare these rules with the rules of a game in which, while the rules themselves are arbitrary, it is their rigidity alone which makes the game possible. However, the fixation will never be final. It will have validity only for a special field of application (i.e., there are no final categories in the sense of Kant). (Ibid 292)

There can be many such layers of freely chosen concepts, and the ultimate goal for science is to construct the highest degree of precision at the first layer through the minimal degree of complexity at the lowest level.

We can draw a historical line here from Galilei (1615) who described the basic outset of a theory as primary suppositions. Hume (1739-40) insisted that all elements must be either derived from impressions or laws of reason and principles of association. Kant (1781/87) argued that the most basic aspect of any theory whatsoever is pure intuition in space and time. Einstein (1934 and 1936) argues that the basic elements of theories are necessarily conventional.[114] He argues that the primary idea in our epistemology is the bodily object and that this idea is a free construct of the human mind.

However, a bodily object is an object that takes up space (extension), has a position and resists penetration (i.e. the object has dynamical forces). As argued by Kant and throughout this thesis, there is no possible conception of a bodily object that is not intrinsically framed in spatial relations. There is thus a problem concerning the *primacy* of the bodily object in Einstein's epistemology. Given that Einstein is following Hume's epistemological intuitions, it makes sense for him to maintain that the bodily object has primacy. Indeed, if we are to measure our way to reality, we must have something to measure with and this something must be a bodily object. As special relativity proposes measurement procedures for the measurement of time and space, this becomes a pressing issue where the primacy or non-primacy of the bodily object must undergo more scrutiny. For, if we take bodily objects to be epistemically primary, as well as having the ontological qualities needed, we end up in special relativity. If we reject this primacy and assume space and time to be primary, we get Lorentzian theories. Defenders of the relativistic conception

[114] Thus siding with Reichenbach in the debate over constitutive elements.

must therefore explain how and why the bodily object is primary. The standard way of doing this is by reference to convention.

We *can* take the primacy of the bodily object as a brute fact and develop theories from there. We have seen, however, that even theories that start from this assumption make use of the classical concepts of space and time in order to define the bodily objects involved and their properties. In the case of special relativity, the problem is particularly pressing since Einstein not only claims primacy of the bodily object, but also postulates practically rigid objects as a separate category. If the bodily object has primacy over space and time, it seems one should be able to construct a theory of space and time from concepts of bodily objects and the relations between them. However, it is exactly at this point that Einstein replaces the bodily object with the unexplained practically rigid bodies. These practically rigid bodies function as a separate category of reality which Einstein himself argues are different from everything else. It therefore seems reasonable to ask whether it is the practically rigid body or the bodily object that has primacy in relation to space and time.

As we have pointed out, a practically rigid body has properties of rigidity that are distinctly phoronomic, i.e., the rigidity of a practically rigid object is not a property of a bodily object. In special relativity however, the structure of space and time is constructed from practically rigid bodies and not from bodily objects. It thus appears that although Einstein is arguing for primacy of the bodily object in his more philosophical writing, the theory he proposes takes a different route. One could argue that the practically rigid bodies are conventional and parts of the "basic rules of the game". However, the practically rigid bodies only have one single function, which is to measure time and space; apart from that role they are wholly absent from the framework. If the goals of conventionally chosen basic concepts are unification and simplification, as argued in Einstein (1936), it seems that practically rigid bodies are ill fitted.

Furthermore, given that we are unable to construct meaningful alternatives to Euclidean space in our imagination, why should we assume that our spatial concepts are *freely* chosen? I can freely choose whether I view a specific phenomenon from this or that position, but it appears impossible for me to choose a perspective from which space has for instance seven dimensions. It seems therefore that space and time do not belong on the list of freely chosen concepts, but rather function as a basis on which such concepts can be freely constructed. Einstein (1934) discusses the relation between spatial qualities and bodily objects as an intimate connection in our sensible experience, but treats the concept of space as separate from that of spatial relations. His list of primacy is "solid bodies; intervals; space" (Einstein 1934: 278). This implies that space is built outward from a point or closed interval. We have argued in the exact opposite direction and seen that even the concept of a "solid body" requires notions of shape, which is inherently spatial.

IV 9.3.3. Single vs. multiple layer theories; the principle constructive distinction

There has been much confusion surrounding the principle constructive distinction given by Einstein (1919). Rather than wade through the details of the multiple suggestions in the literature (See for instance; Janssen 2002, Brown 2005, and Lange 2013)[115], I shall present my view. In order to maintain this view, we must assume that Einstein held a coherent position in philosophy of science throughout his career. The findings of Giovanelli (2014) favors such an assumption, as does Einstein's repeated claim that his theory was never finished.[116]

Einstein and Lorentz both use thermodynamics and the kinetic-molecular theory of gases as their examples of principle and constructive theories (respectively). If we view these in light of the 1936 stratification, we see why they are of distinct kinds. The laws of thermodynamics are generalizations over observable empirical facts, for instance that one cannot make a perpetual motion machine. However, they do not explain why this is so. Such an explanation entails dipping down into the lower level concepts in the way we did with the coffee maker. As such, thermodynamics is a single layer theory, i.e., a theory of principle.

The kinetic-molecular theory of gases on the other hand, expresses the same laws in terms of the motions of molecules (material points). This theory therefore allows you freely to move among multiple layers of concepts all the way from the most abstract concepts of Newton's laws of motion, material points and forces, to the description of a particular such as a coffee maker, a power plant, or a car. The kinetic-molecular theory is a theory that allows you to derive any element of it from the basic rules of the lower level conceptual framework of Newtonian mechanics. It is therefore a constructive theory.

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[115] All of these are concerned with either the kinematic/dynamic distinction, or the debate concerning the microstructure of macro-size objects. As we shall see, my view opposes both approaches.

[116] See for instance Einstein (1907: 236), (1921: 236), (1923: 483) and (1949: 55-57)

As pointed out in Einstein (1919), the strength of a principle theory is its certainty. As a generalization over empirical facts, it is highly unlikely to change. This is also the case with thermodynamics. If the mechanical world-view is fully replaced by a view in which there are no particles, the thermodynamic theory will still stand strong while its explanations through the kinetic-molecular theory will have to be revised or replaced.

At this point, I wish to emphasize that the higher/lower levels of concepts, do not reflect a macro/micro level set of descriptions. For instance, in ecology the lower level concepts that explain, are related to higher levels of complexity and are therefore far removed from the microstructures it can explain.

Before analyzing special relativity in terms of the principle/constructive distinction understood here, I wish to highlight an upshot to this understanding. Understood in terms of underlying concepts vs. generalizations over empirical findings, the constructive/principle distinction provides us with a connection to the history of science. Copernicus (1543: 5) argued the need for a unified approach to astronomy, and accused the Ptolemaics - who would regularly invent disconnected hypotheses - of creating a “monster rather than a man” out of the limbs of astronomical data. Calling for such a unified approach is calling for the development of a constructive theory of astronomy.

Galileo (1615), as we have seen, introduced a hierarchy and argued that any scientific inquiry starts from basic, lower level premises that might appear contrary to evidence. The role for a philosopher of science is showing how the higher-level evidence can be made to comply. We can see this for instance in Galileo’s discovery of the law of fall and the relativity principle. The Peripatetics thought they had evidence for the immovable earth hypothesis, but this evidence turned out to rely less on the empirical facts and more on the basic assumptions. A new constructive theory led to a new description of that same evidence.

Kepler’s (1609) insistence on finding a causal base for the motion of planets led him to the correct laws. Newton (1687) applied these laws and gave a full constructive theory. In so doing, he claimed to have made no hypotheses. On this point, *and this point only*, Einstein accuses Newton of making an error. The error is believing that theory comes inductively from experience (Einstein 1936: 301). Hence, we can connect the great scientists in light of Einstein and Lorentz’s distinction. Osiander, Ursus, and Ramus on the other hand, all argued that the limit of astronomy is generalization over observations, or principle theories (see Jardine (1979) and (1984: 211-224). As Osiander puts it

> For it is sufficiently clear that this art [astronomy] is absolutely and profoundly ignorant of the causes of the apparently irregular movements. And if it constructs and thinks up causes – and it has certainly thought up a good many – nevertheless it does not think them up in order to persuade anyone of their truth but only in order that they may provide a correct basis for calculation (Osiander 1643: 3-4).

This quote by Osiander is usually seen as the dogmatic rant of a priest protecting the authority of scripture against his contemporary scientists, but it is a position still firmly footed in the philosophy of science. The work of Galileo, Tycho and Kepler, was intended to establish a constructive alternative, which culminated in the Newtonian theory. We shall now depart from this historical interlude and ask whether special relativity is a principle theory or a constructive one.

I side with Einstein and claim that special relativity is a principle theory. It is a theory that does not offer an explanation although it offers a generalized picture of phenomena. In order to be a constructive theory, special relativity would have to account for the measuring apparatus used in the theory (rods and clocks imagined as practically rigid), in light of meaningful lower level concepts. As we shall see, no such explanation is available.

IV 9.3.4. The problems concerning rods and clocks

In dealing with rods and clocks in special relativity, we shall deal with two problems both related to the concept of rigidity. The first problem concerns explanations of rigidity in terms of Born's principle of the physical identity of units of measure. This issue is dealt with in Giovanelli (2014), and I shall simply explicate the central points of his findings. The second issue concerns the form of existence of rods and clocks and the ambiguity related to their description by Einstein as "practically-rigid" bodies. As the former also informs the latter, we shall start with Giovanelli's description of what we may call the epistemic problem.

IV 9.3.4.1. Einstein, Weyl, and Pauli; the epistemological problem

Giovanelli (2014) focuses on a problem concerning rods and clocks in (special and general) relativity theory. The first issue comes into the discussion in a somewhat roundabout way. It is a well-established fact that if two material rods and two material clocks measure the same distance (length unit) and have the same tic-rate (time unit) when they are at relative rest on one occasion, they will measure the same distance and tic rate *whenever* they are at relative rest. This relation holds regardless of what happens to the rods and clocks in the meantime. In other words, if you and I have a set of synchronized rods and clocks when we are together, your clock and meter stick might well behave differently as you travel the world. However, if you come back to me, our sticks and clocks will measure the same distance and tic at the same rate.[117] Born referred to these relations as the 'principle of the physical identity of the units of measure' (Born, 1922: 211 in Giovanelli 2014: 21).

Although implicitly assumed in special relativity, the principle has no obvious justification. Weyl suggested that the principle could only be justified through the application of a field theory from which the principle would emerge. As a field theory is necessarily dynamical, and one would need a dynamical theory to explain stability, Weyl's option makes sense. However, in order for such a theory to do the job required, it must develop from within the framework of general relativity. I.e., the general theory of relativity would have to yield a theory of matter that could explain the assumption of rigidity in this specific sense. General relativity has not yielded such a theory and neither did Weyl's own field theoretic approach. Still, one should expect that relativity theory explains central features of its measuring apparatus, or that these features are explained from some external theory.

Pauli, on the other hand, argued that Weyl's approach was flawed because a field theory would include continua that cannot be applied within a realm where they cannot even in principle be observed. The problem being that a field theory of matter would make claims about distances below the Planck scale. Such distances are empirically meaningless according to quantum physics.

Einstein is therefore squeezed between two demands. On the one hand, Weyl is demanding an explanation of rods and clocks as physical bodies, on the other; Pauli is demanding that rods and

[117] Note that it is a separate discussion whether our clocks have ticked the same amount of times.

clocks yield empirically meaningful observable results. Giovanelli gives a detailed description of Einstein's many formulations of his middle of the road solution and the relations in which they are given. In this simplified rendition, I shall refer to the standard quotation from Einstein (1921). The set-up for this argument is Einstein's proposal for a "practical geometry" which consists of the de- and re-interpretation of geometrical content discussed previously. The axioms of geometry are taken as free of any content and thus as providing only a set of relations. Content is introduced from physics. This, in Einstein's opinion, just as in that of Minkowski, makes "practical geometry" an empirically testable structure. Einstein justifies his approach through the following statement.

> The idea of the measuring-rod and the idea of the clock coordinated with it in the theory of relativity do not find their exact correspondence in the real world. It is also clear that the solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which must not play any independent part in theoretical physics. But it is my conviction that in the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. (Einstein 1921: 237)

What Einstein is claiming, in other words, is that there is, at the present stage of development of theoretical physics, no lower level set of concepts from which one could derive a theory of material clocks and rods. In other words, there is no possibility of establishing a constructive theory (for instance along the lines of Weyl). In such a situation, it must be permissible to bring in concepts that are not founded in a lower conceptual base. The provisional insertion of the ad-hoc "practically rigid" bodies is thus a temporary "solution" only. This part of the statement squarely establishes Einstein's position as describing the relativity theories as principle theories. Furthermore, the clocks and rods that are inserted provisionally into the theory do not find their exact correspondence in nature.[118] The latter issue is solved by referring to Born's principle of the physical identity of the units of measure as a principle intrinsic both to Euclidean and Riemannian geometry. Hence, the solution is to reject the demand for a full explanation due to the absence of

[118] As such, they are auxiliary rather than detection properties in the language of Chakravartty (2007).

a meaningful theory of matter, and argue that rigidity in this specific sense follows from the relations given in axiomatic geometry.

To summarize, there are no *explanations* given because there is no reliable lower level conceptual apparatus, but there is sufficient empirical evidence for a generalization. Hence, relativity theory is a principle theory.

Typically, we aim to use principle theories as a starting point for developing constructive theories. Einstein's insistence that rigid bodies are temporary also points in this direction. However, no further empirical investigation can lead us to reject the notion of rigid bodies. They are postulated as rigid and thus as not having dynamical properties. There is, in other words, nothing to investigate. By holding on to rigid bodies *as if* they were determined empirically, we are stuck. The only way out of the temporary solution, is to give up the notion of physically real rigid bodies and investigate the dynamical properties of the actual measuring apparatus. This however, implies giving up the entire framework. As a principle theory, therefore, special relativity is somewhat flawed. The constitutive elements are not empirically discovered, general characteristics of natural processes, but rather theoretical postulates kept outside empirical investigation.

A similar problem is inherent to Stachel's (1983) solution. Stachel aims to produce a purely kinematical version of the theory by strictly a priori methods. Even if the solution were unambiguous, we do not find a solution through this strategy. Be removing the empirical content, we are left merely with another abstract structure. We do not know how or whether this structure applies to the world. If we introduce empirical content, we re-introduce all the problems we were trying to solve. Thus, in different ways, Einstein and Stachel's approaches deny the possibility of an empirical investigation into the issue. Einstein by postulating the empirical content (rigid measuring apparatus), Stachel by removing the empirical content altogether.

IV 9.3.4.2 A Lorentzian explanation of Born's principle

If we reject Einstein's practically rigid bodies and the physical geometry, we can explain Born's principle to some extent. Provided clocks and rods are regular and thus elastic physical bodies, we must ask under which conditions they are rigid and under which conditions they are not. As established, on the Lorentzian theory rods and clocks are rigid, in the sense that they measure the "correct" length and time-units, only when they are at rest in the ether. When moving relative to the ether, good rods and clocks vary according to the Lorentz transformations. Thus, the fact that previously synchronized clocks tic at the same rate whenever they share their state of motion relative to the ether follows directly from how the Lorentzian theory explains the Lorentz transformations as formulas for clock dilation. The same argument, naturally, goes for rods.

On the issue of Born's principle, Einstein appears to be caught up in a series of conflations. First, in the 1921 quotation above, he writes. "It is also clear that the solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which must not play any independent part in theoretical physics", which appears intended to answer to Pauli's objection from quantum physics. From quantum physics, one could argue that there should be a natural motion from quantum to macro-phenomena. However, this objection could be raised against all theories of macro entities that do not have a quantum description. As such, Einstein is correct in pointing out that no such theory exists and that we need to withhold our judgement. The problem in special relativity however, is that the measuring apparatus is imagined as practically rigid bodies. Not only do these lack a description concerning their micro-constituents, they are external to the entire theoretical set-up. This is a distinct problem from that of having a full-blown *atomic* theory.

Dirac (1930) shows how quantum physics can be made to comply with the Lorentz transformations. In other words, supposing we had a full-blown theory of atomic physics, we would still have to explain why the Lorentz transformations apply. The answer from Lorentzian theories is that objects deform as a function of their motion through the ether. Einstein on the other hand appears to conflate basic *theoretical elements* with *atomic theory* at this point, thus presenting an apparent justification for his reference to practically rigid bodies. What is missing is the clarification that atomic theory as well as rods and clocks must be explained from the same basic theoretical elements. This is what one attempts in the Lorentzian theory. Until such an explanation is provided in special relativity, the theory remains at best, a principle theory.

IV 9.4. constitutive theories and contemporary philosophy of science

As we have seen, the stratified schematic description of scientific theories presented in Einstein (1936) is recognizable throughout the history of science. However, three elements of Einstein's description raise questions. One is Einstein's insistence on the primacy of the concept of bodily object. A central problem here is the establishment of physical clocks as part of generating operational definitions of time. A clock is any regularly repeating event or set of events. However, how can we establish that the relevant events are repeating *regularly?* I.e. how do we establish regularity? It seems that the only way this can be done is through a postulate, and indeed Einstein (1905: 131) tacitly assumes the "homogeneity that we attribute to space and time". If the concept of the bodily object is primary with respect to the concepts of space and time, it is problematic in a circular manner to bring in a tacit assumption about space and time in order to distinguish a particular set of objects as clocks. However, this is how clocks are typically established. It therefore appears as if the primacy of the concept of the bodily object can only be established under certain assumptions (in this case homogeneity) about space and time. If homogeneity is not to be accepted a priori, as it is in the Kantian theory, it requires an empirically established argument that Einstein does not provide. This is a problem also for Pap and Stump who argue that constitutive elements are ultimately empirical. As we have seen, the "functional a priori" theory proposed by Pap and Stump, is simply what we have determined here as a principle theory. However, the empirically constituted elements are not enough to generate special relativity, as that theory also involves a series of a priori postulates. Stump and Pap cannot apply this solution if they follow through on their theory and thus it is hard to see the how the "functional a priori" perspective helps us understand special relativity.

Einstein insisted that lower level concepts such as practically rigid bodies are freely chosen, provided they yield an empirically sound theory, which corresponds to Friedman and Reichenbach's idea that scientific theory involves coordinative principles or conventional a priori elements. Such a connection is further motivated by Einstein's reactions to Kant's *Prolegomena*, already cited above

> I am reading Kant's Prolegomena here, among other things, and I am beginning to comprehend the enormous suggestive power that emanated from the fellow and still does. Once you concede to him merely the existence of synthetic a priori judgments, you are trapped. I have had to water down the 'a priori' to 'conventional', so as to not have to contradict him, but even then the details do not fit. (Einstein ca 1918b)

The central aspects can be spelled out in terms of Einstein's rules of the game. In the same way as a game needs rules, a complete scientific theory needs basic concepts through which empirical discoveries can be shown to make sense and be explained. Such concepts cannot be derived from experience in the sense of naïve naturalism, but experience must be accounted for.

Reichenbach (1920) and Friedman (1983) discuss their positions in light of the rejection of Kant's original conception of a priori knowledge as something given. This rejection is motivated by the success of relativity theory. However, as shown above (section 8) Reichenbach and Friedman have overstated the difference between Lorentzian theory and special relativity in this question. Einstein, on the other hand, arrives at this position through his Humean insistence on the primacy of the concept of the bodily object. His suggested procedure is that we imagine two solid bodies touching each other at some point or being distant from one another. The relations between these objects give spatial relations from which arises the concept of a space. Strictly speaking, this does not work. Our conception of a bodily object that can either touch at some point or be distant from another object, involves the central spatial aspect of extension. A bodily object, in contrast to a point, is necessarily extended. As we are talking of an extended object rather than extension as such, there is a boundary to this extension, which presupposes a larger space, which would include not only the two objects and the perceiving subject, but also the space outside that boundary. It is simply impossible to build a conception of space from spatially limited elements in this way.

What can be, and is, done in both of Einstein's theories, is to simply ignore the initial spatiality of the concept of an object, and use that object to construct alternative geometries. We have seen this in the construction of special relativity and the concepts of practically rigid bodies, simultaneity, time, and space. Euclidean spaces expressed through Cartesian coordinate systems are used to build up the framework of possible measurements. On that basis, measurements are connected through a new kinematical system. However, we never get rid of the initial Euclidean structure. I suggest that there is a good reason for this, which is that there is no alternative way in which to imagine the basic elements of any geometry whatsoever. Every non-Euclidean system is imaginable only as the structure of a figure in Euclidean space.

The point I am trying to make here is that there is a similar ambiguity in Einstein's suggestion that the bodily object is primary. The bodily object is extended and therefore given spatially. If it were not, and we were dealing with points, we would have to introduce extension in order meaningfully to say that the two points are separated. Their separation consists in their occupation of different parts of the same space. The space-less object and the purely non-Euclidean model do not exist as possible (spatial or temporal) representations in imagination and they are not empirically experienced objects. Einstein's suggestion therefore, that the bodily object is primary, is a problematic feature of his general epistemology.

The idea that we cannot derive lower level concepts directly from experience is an appealing feature of Einstein's epistemology, which removes him from the school of radical naturalists.[119] However, the fact that one cannot directly derive basic concepts from experience does not imply that there is no relation. Kant's idea that the formal aspects of experience (space and time) are aspects of experience although they do not have their origin in experience, is still a viable option. Note that here we are talking of experience in general, not experience of individual and separable objects in particular. A non-Kantian might prefer to start with something other than time and space in this connection. Consider for instance the Platonic ideas in the mature Platonic theory and their role in thinking. These are relations such as "larger than", "similar to", "containment", "contradiction" etc. Relations such as these are distinct from concepts such as chair, table, quark, etc. The way I understand Plato, is that he separates these relations from concepts in general because the general concepts are definable only using such relations. They therefore serve a transcendental organizational function in thinking as such.

Kant's idea is that space and time serve the same function in our interactions in the world. Therefore, no single experience will yield a spatial theory, but spatiality can be unveiled from considering interactions in general. Proponents of non-Kantian constitutive epistemologies cannot simply scoff at this possibility as something that has been empirically disproven. We have already

[119] Einstein (1944) explicitly warns against a radically naturalist position.

seen that there is an empirical equivalence between relativity theory and Lorentzian theories, and that relativity theory is less than fully theoretically developed. Hence, If one wants to reject the Kantian option, this must be done on philosophical rather than physical grounds.

Therefore, I maintain that the final aspect of Einstein’s epistemology, the idea of freely chosen lower level concepts, is in need of moderation. We do not freely choose space and time as basic and constitutive elements. These constitute the form of outer experience within which we not only have actual experience but also generate thought experiments and imagined manipulations of familiar objects. In light of this perspective, we can explain why no models of non-Euclidean geometry are imaginable to us without the aid of Euclidean space, and why no construction of measuring apparatus is cast in terms of the resultant alternative structures. If there is no special role for the Euclidean model, this remains a mystery.

IV 10. Lorentzian theories and special relativity, a comparison

So far, we have discussed special relativity and Lorentzian theories separately, and seen how problems are resolved within the two frameworks. I have defended the Lorentzian way of thinking and discussed the standard contemporary defenses for special relativity. However, we have ignored some issues that we can now discuss in a more complete manner.

IV 10.1. Coherence and ontological dualism

In part I, we saw that Kepler and Galileo demand that everyday experience and scientific theory must be representable within a single coherent argument. This is not possible in special relativity. Special relativity introduces “practically rigid bodies” and thus an ontological dualism. This dualism necessitates two sets of explanatory schemes. Euclidean geometry, classical time, and Galilean kinematics for the practically rigid measuring apparatus, and; Minkowski “space-time geometry”, and $k\widetilde{\imath nemat}\imath cs$ for that which we measure. This all follows from reading Einstein’s notion of practically rigid objects as objects that are rigid in practice.

If we give up this idea, and rather think of these objects as approximately rigid, we get the Lorentzian theory where the Lorentz transformations give the magnitude of deformation. In other words, the Lorentz transformations inform us about the difference between physical objects and their rigid ideals. Thus, the Lorentzian theory follows the constructive route where space, time, velocity etc. are constitutive elements, and dilation and contraction are plausible dynamical explanations. Special relativity follows the principle route and ends up in a series of dualisms. If we include the dualisms in a single argument, we end up in contradictions. Inserting a "manifest" and a "physical" space, just replaces the former dualisms with another dualism.

Giovanelli (2014) cleaned up aspects of the contemporary debate on the kinematics/dynamics distinction relating to the question of ontological dualism, in an impressive way, but does not offer the only interpretation of Einstein's much debated confession:

> It is striking that the theory introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking, measuring rods and clocks should emerge as solution of the basic equations (objects consisting of moving atomic configurations), not, as it were, as self-sufficient entities. The procedure justifies itself, however, because it was clear from the beginning that the postulates of the theory are not strong enough to deduce from them equations for physical events sufficiently free from arbitrariness in order to base upon such a foundation a theory of rods and clocks. If one did not wish to forego a physical interpretation of the coordinates in general (something that, in itself would be possible), it was better to permit such inconsistency – with the obligation, however, of eliminating it at a later stage of the theory. But one must not legitimize the sin just described so as to imagine that distances are physical entities of a special type, intrinsically different from other physical variables ('reducing physics to geometry', etc.). (Einstein 1949: 56-57)

My reading is that the passage refers to an ontological as well as an epistemological problem. The epistemological reading suggests that, if a material theory were available, the use of measuring rods and clocks to operationally define space and time determinations reductively would be acceptable. The ontological reading suggests that even if we had such a theory, measuring rods and clocks would be different from all other things. This ontological problem is not resolved through a theory of matter. A dynamical theory that explains the constitution of physical objects, as well as how they are spatially deformed and their internal processes are altered by motion, would however explain Born's principle of the physical identity of units of measure. In being dynamical, such a theory would have to treat all physical objects in a reciprocal interplay of forces, which would resolve the dualism. Still, such a theory requires that we treat space and time as a priori given and hence treat the measuring apparatus as non-rigid. In other words, it would be a Lorentzian theory.

There is a parallel here to the debate during the scientific revolution. The Peripatetics, in their defense of what they took to be the Aristotelian theory, were willing to separate between different forms of motion (natural and unnatural) and to introduce two sets of physical laws. Planetary motion was then distinct from motions on the surface of the earth. In order to resolve the dualism, Copernicus, Galilei, and Kepler applied the philosophy of science I am proposing here. A central move in the classical debate was to give up the idea of two sets of rules as well as the idea of the primacy of circular motion. If, in contemporary physics, we give up the idea of physically real rigid bodies, the dualisms resolve themselves and we regain a unitarian world-view. One way of getting rid of physically real rigid objects, is to adopt the Lorentzian theory. As of today, this is not the standard view either in philosophy of physics nor in physics proper. In effect, we maintain a series of conceptual issues. We shall run through some of these, but first we shall briefly mention an option that we have not considered.

IV 10.1.1. A structuralist reading of the data

During the scientific revolution, the mathematician Ramus famously stated that the only proper view of astronomy is the structuralist view. All we can ever achieve is the proper structure and since we can interpret the structure in different ways, we shall never know anything above structure. When Kepler (1609) published his *New Astronomy*, he spent the entire first page reminding (the by then deceased) Ramus that the book contained a new theory which was superior to the others and thus was more than just a description of structure.

If one wishes to remain structuralist, however, there is a possibility for this in light of the data we have discussed. The structuralist solution is to geometrize the whole of physics and not just the rods and clocks. This would mean depriving physics of any meaningful understanding whatsoever, concerning forces, objects, space, time, fields, etc. As such, we remove the final difference between the Lorentzian theories and special relativity. What we end up with is a purely structural theory. However, as soon as we try to generalize this structuralist view, we find problems. For instance, how to explain quantum physical entanglement of spatially separated objects when there are no objects.[120] We also get a problem concerning the distinction between mathematical structure and physical structure. The structuralists have not yet offered viable solutions to any of these problems.

IV 10.2. The world-system

Relativistic $\widetilde{kinematics}$ does not include any new rules for velocity addition. As we have seen, Einstein's treatment of velocity addition in the final part of the kinematics of the 1905 paper is translation or transformation rather than velocity addition. The paralogistic reference to translation or transformation as velocity addition follows from the practical geometry approach where the addition rules should follow from measurements. We have seen that all that follows from measurement are the translation or transformation rules. The velocity addition rule remains the same.

The motivation for replacing addition with translation or transformation is to ensure that we cannot get superluminal velocities by composition of velocities. However, if we replace all velocity addition by translation or transformation, we lose a vital part of physical and metaphysical theory, which is to represent the physical system as a whole. Such a representation necessitates an overall space in which to relate the appearances. In the Newtonian system, this space is a physical entity. In the Kantian theory, it is an absolute space inserted by us. In the Maxwell/Lorentz, dynamically space is filled with ether and one can choose whether to apply a Kantian or a Newtonian attitude towards it. Since there is no overall space in the special relativistic picture, there is no overall world-view to represent. However, if one accepts the initial understanding of space given in Einstein 1905 prior to any physical synchronization procedure, the "rest frame" we choose plays exactly the same role as a privileged frame in Lorentzian theory and absolute space in Kantian metaphysics. We know that different reference frames give different measurements. We also know that in order to measure anything, we must measure it in some frame or other. Special relativity tells us that every frame is equally valid. However, since every frame is unique, we always make a choice when we decide which frame to use. This choice does not alter the phenomena, but it alters our theoretical description. The same goes for Lorentzian privileged frames and Kantian

[120] See for instance Ladyman (2005).

absolute space where we set a single frame for *all* phenomena, including the measuring apparatus. Our choice of frame is arbitrary in either case. The standard objection to Lorentzian theories, that there is no detectible privileged frame, is therefore equally applicable to any measurement whatsoever in special relativity. In both cases, the choice determines our numerical measuring results, in neither do the phenomena change.

The special relativistic solution to the arbitrariness of an overall frame is to reject the notion that there is such a frame. The cost of such a rejection is that, even in principle, we cannot describe multiple phenomena within a single system. This, again, leads us into the problematic issue of how different systems are spatially related. For, if system A and system B are in any form of spatial relation, they must exist in the same space. If they exist in the same space, they should be describable as such. However, through the relativistic rules for $Veloc\imath\widetilde{ty\ ad}d\imath t\imath on$, we are instructed to translate motions into multiple perspectives and at no point do we end up with an overall world-system.

IV 10.3. Where is the speed?

Speed can never appear empirically as a distinct magnitude. Empirically, speed is always combined with direction. This forces us to think of speed as an intensive magnitude in phoronomy and therefore as a potential magnitude in dynamics. The inclusion of such potentials is wholly in line with Maxwell's idea of electrodynamics (Maxwell 1865: 464). Speed as a potential combines with direction in order to give velocity. Additionally, we can talk of the speed of a point at an instant, and, given that we know the direction of motion in previous instants, we can meaningfully talk of instantaneous velocity.

The practical geometry approach of starting from measurement rejects such an idea, and is thereby limited to discussing average velocities. The traditional infinitesimal approach and its more modern reflection in standard analysis do not solve this problem. Either an infinitesimally small distance is extended, in which case it provides us with an average, or it is not extended, in which case it allows no measurement. The case is similar concerning the point at the limit in standard analysis (see Sandmel 2004). Thus, in light of contemporary mathematics, the methodology of practical geometry simply cannot lead to a notion of speed at an instant. This might seem a less than detrimental effect. After all, instantaneous speed is not a measurable phenomenon. However, the point becomes more apparently problematic once we try to address cases of acceleration.

Acceleration is the motion of a point from one state to another through all intermediate states without ever remaining in any such state. Therefore, we must think of particular speeds at particular instances. On the other hand, we must think of these instances as limitations of a homogenous and continuous space. An object uniformly accelerating from rest to any given velocity (i.e. an object with a constant rate of change of velocity) can always be determined as having a particular speed at a particular instant, but never as maintaining that speed over any amount of time. In other words, there is a difference between being in a permanent state of motion (which applies to any moving object at any instant) and persevering in that state or motion (which only applies to objects in uniform motion). On our treatment of acceleration therefore, the notion of an instantaneous speed is vital. By practical geometry, it seems we can never attain a meaningful concept of such a notion. We therefore also lack the ability to separate between uniform and accelerated rectilinear motions, and thus inertial motions become meaningless. This is a problem for special relativity, which deals exclusively in inertial motions. We do of course have the necessary mathematical tools to calculate velocities as well as accelerations. What we are missing in special relativity is the conceptual apparatus justifying these tools. A Kantian approach to Lorentzian theories provides us with such an apparatus.

IV 10.3. Losing internal mathematical relations

I have argued that the relativistic rules for $\widetilde{velocity\ addition}$ are not so much new rules for velocity addition as they are the replacement of velocity addition by translation or transformation. However, if we were to accept the new rules as rules for velocity addition, we must take into account the effect it has at the mathematical level of the theory. Einstein (1905) writes that by replacing the standard addition rules by Lorentzian ones, "… the vector addition for velocities holds only to a first approximation" (Einstein 1905: 140). However, the connection between the standard velocity addition rules and the Lorentz transformations is somewhat muddled. Through the Kantian approach to velocity addition, we saw that velocity addition rules follow from standard addition rules in arithmetic. The velocity addition rules therefore apply directly, and without any conceptual alteration. In the special relativistic system, it is not clear how one can uphold such a relation. The Lorentz transformations involve a constant that represents a highest possible velocity. In physics, this velocity represents the velocity of light in a vacuum. In Einstein's practical geometry, it represents the highest possible velocity. However, we initially borrowed the velocity addition rules from an application of arithmetic in phoronomy. The question therefore becomes what such a constant or upper limit represents in arithmetic proper. It is clear that the constant must represent the notion of a highest number, which, in arithmetic, is an absurdity. [121]

We can interpret the fact that the velocity addition rules holds to an approximation according to two distinct meanings. Either, the velocity addition rules accidentally hold to a first approximation, which is no more than a mathematical oddity, or they hold to a first approximation only, because they are in need of modification. My understanding is that Einstein intends the latter meaning. However, if read in this way, the moderation needed is that we include the notion of a highest number into arithmetic. As this cannot be done universally, we need to alter arithmetic only in those cases where we apply it to relativistic $\widetilde{velocity\ addition}$. In effect therefore, we are removing the mathematics of special relativity from mathematics in general. Not only in the traditional sense of applying certain formulas in specific areas of inquiry, but also in the more radical sense of altering the foundational operations. In any other part of arithmetic, it is true that for any given number, n, there exists a higher number n+1.

IV 10.4. The clock paradox

At the end of section four, *Physical meaning of the equations…* of the 1905 paper, Einstein introduces a thought experiment which has led to much debate in the last century, and that still generates literature; [122] the clock paradox.

> If at the points A and B of *K* there are clocks at rest that, considered from the rest system, are running synchronously, and if the clock A is transported to B along the connecting line with velocity v, then upon arrival of this clock at B the two clocks will no longer be running synchronously; instead, the clock that has been transported from A to B will lag $\frac{1}{2}tv^2/V^2$ sec (up to

[121] Mathematical induction, as is involved in the statement that one can always add another larger number, was the single synthetic a priori element based in intuition that even Poincaré accepted.

[122] See for instance Matvejev et al (2016)

quantities of the fourth and higher orders) behind the clock that has been at B from the outset, where *t* is the time needed by the clock to travel from A to B.

> We see at once that this result holds even when the clock moves from A to B along any arbitrary polygonal line, and even when the points A and B coincide. If we assume that the result proved for a polygonal line holds also for a curved line, then we arrive at the following result: If there are two synchronously running clocks at A, and one of them is moved along a closed curve with constant velocity until it has returned to A, which takes, say, *t* sec, then, on its arrival at A, this clock will lag $\frac{1}{2}t(v/V)^2$ sec behind the clock that has not been moved. (Einstein 1905: 139)

As described here, there is no paradox involved but rather an oddity. The oddity is most apparent if we replace Einstein's clocks with a pair of twins. The oddity then spells out in terms of one twin being older than the other. The *paradox* arises only if we try to include the principle of relativity and therefore describe the situation from both the perspective of the twin that remains at B and the perspective of the twin that travels from B to A and then returns to B. If the principle of relativity applies, we end up in a paradox. For, according to the twin that remains at B (hereafter the B-twin), the "moving" twin (hereafter the AB-twin) is younger. According to the AB-twin, the B-twin moves and therefore the B-twin is younger. The result being that each twin is younger than the other.

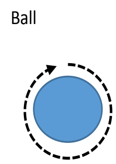
IV 10.5.1. Clock paradox and rotational motion

Although the $t\imath me\ d\imath lat\imath on$ effect applies to accelerated, as well as rotational motion, the mechanical principle of relativity does not (see Einstein 1918 and Feynman 1963: 77-79). One hence claims there is a privileged reference frame in the situation, since only the B-twin can give proper measurement. What we should expect to see, from the perspective of special relativity, therefore, is that once the twins are reunited, all biological processes are now occurring at the same rate, but the AB-twin has suffered less biological deterioration. In terms of clocks, the clocks now tic at the same rate but a simultaneous reading of both clocks shows that the AB-clock has ticked fewer times. Hence, the principle of relativity only applies as long as both systems are moving inertially.[123] This implies that special relativity is not applicable to rotational phenomena since

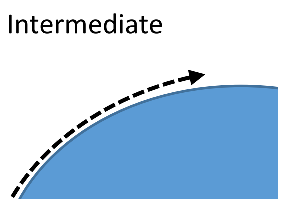
[123] The behavior of clocks (and rods) that give rise to the clock paradox in its different versions could be compared to and treated together with Born's principle of the equivalence of physical units of measure, as well as the epistemic and ontological dualities of special relativity. All three issues appear to refer to a dynamical relation

such phenomena require a privileged frame (that is, unless one moves to general relativity in which there is simultaneity at a distance).[124] Our concern here is not to attempt an argument from paradox, but rather to ask whether it is reasonable to assume a theory that makes a radical separation between rotational and rectilinear motion. I shall follow the argumentative line of Selleri (1998).

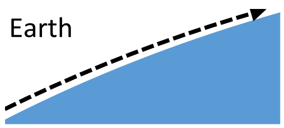
Imagine an association football, which roughly has a circumference of 70cm. Further, imagine measuring out a single trip around the ball. This seems radically different from a rectilinear trip on the pitch. Now imagine increasing the ball size gradually, say until it reaches the size of the earth, whilst maintaining the length of the trip. What we see is that the circular motion involved around the ball, gradually approaches a straight line.



The trip around a regulation size ball



The same distance around an intermediate size ball (segment)



The same distance around the earth (segment)

The larger our ball gets, the more our trip looks like a rectilinear motion. Given that we perform experiments in the real world, it seems overly drastic to assume a radical separation between

between bodily objects and some underlying frame of reference. However, as special relativity denies any such frame outside convention, there appears to be no possible solution.

[124] See Norton (2000)

rectilinear and curvilinear motions of this kind. Indeed, we can make our ball so large that it would be impossible to detect any difference between a curvilinear motion along its circumference and a straight line.

Phoronomically, it makes sense to separate the two types of motion. However, in Einstein's practical geometry there is no clear line between the mathematical and the physical realm. It is therefore hard to see why we should make a radical separation in theory between phenomena that are inseparable in practical physics. Indeed, the separation between curvilinear and rectilinear phenomena in special relativity is strikingly similar to the magnet conductor problem in Maxwell's first approximation. An open question for proponents of special relativity is therefore how the separation between curvilinear and rectilinear motions can be made within the practical geometry approach. The problem does not occur within Lorentzian theories as such theories account for rotational phenomena, i.e. there is no separation to argue for here.

IV 10.5.2. The clock paradox and acceleration

Another way to set up the clock paradox is to imagine two clocks A and B at rest and synchronized in a system K. We can then move the B-clock in uniform rectilinear motion, say, in the direction of positive x in K, for some time t, and then turn it around so that it will re-join the clock A. Again, the paradox arises if we assume that both the A-clock and the B-clock are functional reference systems.

Einstein (1918) resolves the paradox by insisting that only the clock A is in inertial motion throughout the process. Therefore, only the clock A is a possible reference system according to special relativity. Feynman (1963: 77-79) makes exactly the same argument and emphasizes that the clock B (or the moving person) will suffer physical influences in the process of turning around (acceleration) and therefore is not an inertial system. The accelerated version of the clock paradox is therefore not within the realm of special relativity. In order to solve it, one must move to the general theory. [125]

[125] One problem with this solution is that it ignores Born's principle of the physical identity of units of measure, which applies in every other part of special relativity. As we have seen, Born's principle states that a measuring device will measure correctly once it returns to a state of uniform motion

In the contemporary philosophy literature, Einstein and Feynman's solutions are found wanting. Perhaps most famously in Maudlin's evaluation "Everything in this 'explanation' is wrong" (Maudlin 2012: 81). Maudlin insists that the proper approach to the paradox is within a Minkowski space-time. The way to think is therefore in the following terms.

There are three distinct events involved.

Event 1, clocks A and B are synchronized (i.e. showing the same amount of ticks and ticking at the same rate) and are separated as B moves in the direction of positive x.

Event 2, the clock B turns around and moves back toward A.

Event 3, the two clocks are back at the same place and their tick rates are identical, but the clock B shows a smaller amount of ticks.

In order to see how this solution works, we must remember that in the Minkowski space-time, every event has a specific space-time coordinate. This coordinate is absolute, meaning that the space-time "distance" between two events (= space + time) is independent of how far you travelled in space. The more space you traverse, the less time you spend. From this, we can immediately see a solution to the first part of the paradox; how the situation looks if we maintain that A is at rest. A, being at rest, must have spent the maximal amount of time since, in the duration between the events 1 and 3, A has not traversed any space whatsoever. B must have spent some of his "space-time units" on his motion through space and must therefore have spent less than the maximal amount of time. Thus, B shows a smaller amount of ticks at event 3 (when the clocks are re-joined).

Maudlin's objection to Einstein and Feynman's solution is that acceleration is orthogonal to the solution of the paradox. Indeed, he shows that within the Minkowski framework one could bump the clock at A, back and forth for a short time. Even if the acceleration of A during the "bump" is more violent, we get the same result. B still traverses more space and therefore displays fever ticks

independently of what happened to it in the meantime. We can therefore make the acceleration period arbitrarily small and expect correct measurements once the system is moving uniformly. If we do so, what happens during the acceleration period becomes reliant on how far separated the two measuring systems are, which is an absurdity.

at event 3. This leaves us with the question of how we are to understand the situation from the perspective of the moving clock B. To this, there are multiple suggestions:

a) In Einstein and Feynman's solution (acceleration causes the difference of age), both clocks represent inertial systems at all other stages than event 2, where the clock B changes direction.

b) Norton[126] argues that B sees the clock A as running slow throughout the trip, but that the change of motion involved in turning around changes the clock A (in B's perspective) to such an amount that the event 3 difference is accounted for.

c) Maudlin (2012: 83) eventually argues that "The twins phenomenon is explained without having to attribute any 'motion' or 'speed' or 'rest' to anyone: It is a simple matter of space-time".

d) Bohm (1965) argues from the relativistic Doppler shift, Debs and Redhead (1995) argue for a geometric parallelogram solution based in Reichenbach and Grünbaum, whilst others again argue that they find the solution by realizing that the clock B switches between multiple frames of reference.[127]

Note that if we set the paradox up with three or more clocks, all the above suggestions fail.[128] Consider the following:

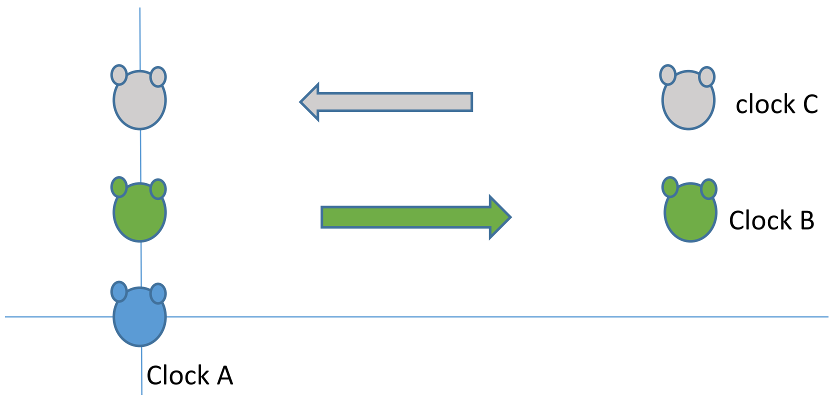
Clock A and Clock B are synchronized at $t_0 = t'_0$ when clock B moves past clock A (at rest in the ether); clock B then meets clock C and the latter is synchronized with the first, at $t'_2 = t''_2$ when they are passing each other. As clock C moves in the opposite direction of clock B, it will meet with clock A. When clocks A and C meet, clock C is showing fewer ticks passed than clock A, and there is neither acceleration nor rotation involved. In the system that we have chosen as at rest

[126] http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/spacetime_tachyon/)

[127] http://www.fnal.gov/pub/today/archive/archive_2014/today14-05-02_NutshellReadMore.html

[128] This set-up is taken from Myrstad & Sandmel (1987)

in the ether, the clocks ticks faster than any clocks that move in it, and a clock moving with velocity v in the ether will slow according to the Lorentz factor. All observations of the moving clocks from a distance and all clock setting according to the different synchronization procedures relevant for the different inertial systems in motion in the ether, follow with definite arithmetical determinations from this assumption. Furthermore, the Lorentz transformations imply that for any choice of "rest" system in an inertial frame moving rectilinearly and uniformly with respect to the ether, all the phenomena and measurements will "mimic" what happens in the ether system.



Note also that, again, we see a difference between Einstein's physics based approach - basing the suggested solution in physical differences between systems in different types of motion – and purely mathematical solutions, such as Maudlin's direct reference to the abstract Minkowskian structure. I emphasize that these are radically different approaches to physics, but that they share a common strategy in denying rather than resolving the paradox. If we accept Lorentz' absolute clock dilation and deny the idea that all measurements in all (inertial) systems are equally valid, the paradox is resolved.

I will not enter into the debate as to how the twin or clock paradox is properly resolved within special relativity. Rather I wish to point out the parallel between the multiple theoretical explanations given to the clock paradox, and the multiple theoretical explanations of the magnet conductor problem that bothered Einstein. Furthermore, it is worth pointing out that the twin/clock paradox keeps generating papers and there is still no agreement in sight. It seems rather that the multiple explanations solution is generally accepted. As a case in point, Matvejev et al. (2016) argue for two physically separate and contradictory solutions to a version of the twin paradox they call the trolley paradox. It seems therefore that rather than resolving theoretical ambiguity in the way Einstein, Maxwell and Kepler aimed to do, parts of modern theoretical physics and philosophy of physics has embraced such ambiguity.

Whether the acceptance of ambiguities is a strength or a weakness of modern physics, it is clear that one cannot reasonably argue against classical electrodynamics on the grounds of theoretical ambiguity or asymmetry on the one hand, only to embrace the same kind of ambiguity within special relativity on the other. My position in this debate is that ambiguities of this kind are problematic and that the ambiguity concerning magnets and conductors in Maxwell's electro-dynamic theory was resolved in Maxwell (1865). The multiple solutions to the clock paradox in special relativity is a weakness of the theory in its current form.

IV 10.5.3. Ether, Light speed, and types of motion

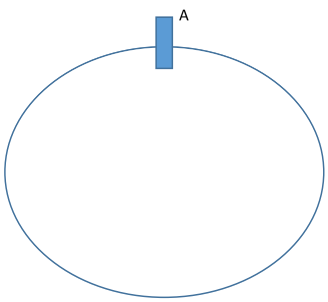
The standard approach in special relativity remains to relegate issues of rotational motion, such as those examples that led to the clock paradox, to the general theory. Rotational, and indeed accelerated, motions are thus excluded from the standard ontology. There is a long precedence for this kind of separation of motions stemming from the Aristotelian physics and cosmology.

On the Aristotelian model, rectilinear motions (up and down) were considered natural motions whose driving forces were inherent to earthly objects. Rotational motions were considered perfect motions of the ethereal stars and the planetary disc, while rectilinear motions sideways (along with earthbound rotations) were considered unnatural motions demanding an external mover. Galileo's splitting of a parabolic motion into two rectilinear ones, was in direct opposition to the general metaphysics at his time. A core element of Galileo's insight is that all motions should be treated within the same basic framework. His most celebrated argument concerning this was the argument for the relativity of motion and the importance of perspective. The Peripatetics had presented the tower argument as a falsification of a moving earth theory. Galileo, using the principle of relativity, showed how the resultant phenomena following from dropping a stone off a tower were independent of the motion of the earth. The important aspect for us in this setting is the relation between rectilinear motion and rotational motion.

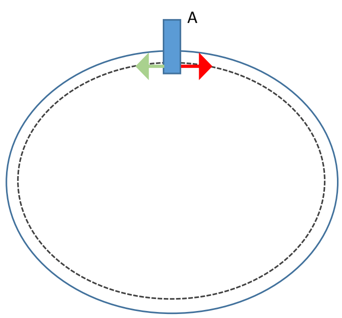
Selleri (1998) argues that there is no good reason for why one should draw a strict separation between these two types of motion. Galileo's use of the principle of relativity in relation to the rotational motion of the earth is a vindication of this view. Indeed, if Galileo had accepted the then standard view that such motions are inherently different, he would have no justification for applying the principle. There is a resemblance between the tower argument presented by the Peripatetics and modern experiments on rotational phenomena. Sagnac (1913) presented results of experiments on rotational phenomena as a vindication of ether theory. I will present a simplified version of his argument. In order to establish the relevant concepts, we shall first introduce the issue of compensation.

Consider a person bicycling at a constant velocity while passing a game of tennis. The tennis players stand still and strike the ball with exactly the same strength and angle. You, however, are bicycling along the tennis court in such a way that you are bicycling away from one player and toward the other. If you measure the speeds of the ball with reference to yourself therefore, your results will indicate that one player strikes the ball harder than the other does. However, the average speed of the ball in a motion *back and forth* will be the same whether you are bicycling or standing still. This follows from the fact that the speed you subtract from one player is the exact same speed you add to the other. This is compensation and as we shall see, compensation is a key feature in understanding why the principle of relativity in special relativity theory relies on measurements of the two-way motion of light.[129] Now to Sagnac's experiment on rotational phenomena. Take a cylinder with a light cannon A on it that also functions as a signal detector

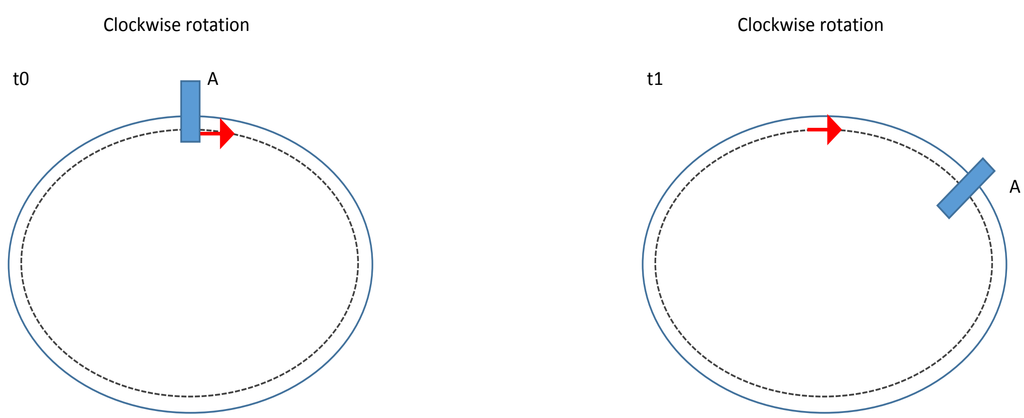
[129] Within special relativity theory, the principle of relativity applies to inertial systems only. Due to Einstein's clock synchronization procedure the one-way speed of light as measured by an Einstein synchronized clock will always be C. For accelerating or rotating clocks, compensation does not apply and therefore neither does Einstein's assumption of the one-way speed being half the two-way speed. Special relativity therefore postulates only the global equivalence of measurements made in measuring systems attached to inertially moving bodies. A general principle of relativity appears in the general theory.



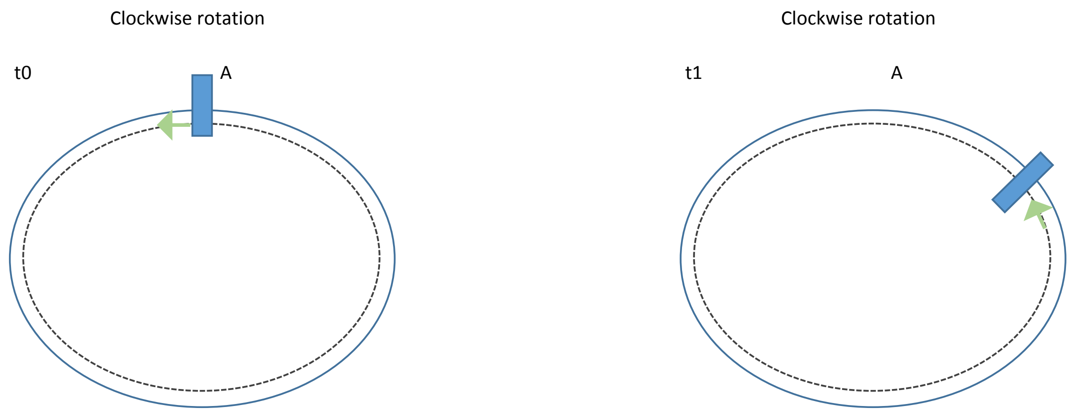
Now send two signals S and S' in opposite directions around the cylinder.



We can now ask the following question: Can we, from measurements of these two signals, determine whether the cylinder is rotating? The intuitive answer is yes. Because, if the cylinder is rotating, then whichever signal travels with the rotation would travel a longer distance as the detector is moving away from it.



The signal travelling against the rotation would travel a shorter distance as the detector travels towards it.



Now, if we were throwing rocks, the intuitive answer would be wrong, as the two rocks would be travelling with different speeds. It is essential to Galileo's argument that the speed of the travelling object varies with the speed of the earth. However the light principle as defined by Einstein, states:

> Every light ray moves in the 'rest' coordinate system with a fixed velocity V, independent of whether this ray of light is emitted by a body at rest or in motion (Einstein 1905: 128).

If we use light rays, therefore, Galileo's argument does not apply and the intuitive answer holds. We can detect whether the cylinder is rotating or not by checking whether the light rays take the same amount of time to reach the cylinder. This implies different things for special relativity and Lorentzian theories.

As the speed of light in any direction is constant only in the ether system, a Lorentzian theory concludes that the cylinder is rotating relative to the ether. For special relativity, the case is more problematic as the question becomes "what constitutes the 'rest' system?" On the one hand, special relativity claims that there is no privileged system and thus that the speed of light is identical in all directions in all systems.

Following Einstein's measurement norm, we should measure the speed of light from the perspective of the cylinder. From its own perspective of course, the cylinder is at rest. This implies that the two signals travel the same distance at the same speed whether the cylinder is rotating or not. We should therefore expect that the two signals reach the detector simultaneously whether the cylinder is rotating or not. They do not.

Furthermore, the difference in signal approach is fully predictable from knowledge of the size of the cylinder and the speed of the rotation. Special relativity has no account of this phenomenon, and rotational phenomena are subsequently defined out by the ontology of the theory. Within the Lorentz type theories, one only needs to establish an arbitrarily chosen "rest" system to represent the ether, and calculate from that perspective. How are we to think of this issue?

First, we can replace our ring-laser inspired image with the one initially used by Sagnac.

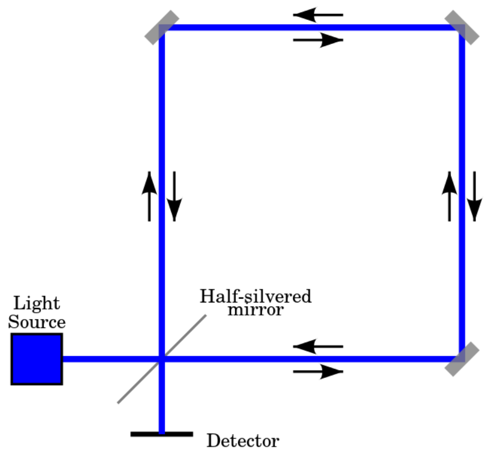
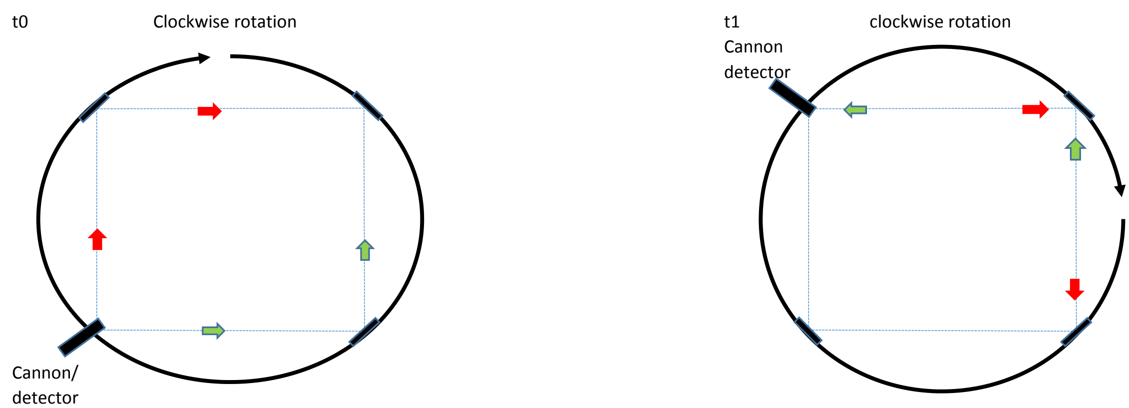


Image taken from wikiwand.com

The benefit of this illustration is that it visually shows that the problem has nothing to do with the rotational motion of the light signal. Any of the traditional arguments used in the twin paradox setting (i.e. something happens to the moving object) is therefore excluded. The only rotational motion relevant is that of the apparatus. According to the light principle, the speed of light should be independent of the motion of the light source, which it is. Within the ether theories, this independence follows from the fact that the speed of light is constant relative to the ether. As such, one should expect that when we have a detector that we know is moving relative to the ether (for instance by imparting rotational motion to it), this should affect the measured speed of light, which it does. The light principle in special relativity however, assumes that the speed of light is constant in relation to the light source and the detector (assuming these are not in relative motion) independent of their motion relative to the ether. In cases where we can know that the whole apparatus is moving (as in Sagnac's experiments), we see that this assumption does not hold.

Furthermore, the Sagnac set-up shows that we can illustrate the motion of the light signal as a set of one-way motions. This helps us establish that the one-way speeds involved are indeed identical. The reason for this is that if the speeds were different, the measuring results would not be predictable purely from our knowledge of the length of the path and the speed of rotation of the apparatus. Rather, we would have to know exactly where the detector is at the times of detection as well. If the apparatus did a half turn before detection, the light would have taken the “up” path twice and the “down” path only once.



Thus, there would be no possible compensation. The only thing that can explain the predictability of the different detection times is that the light principle holds for the system in which the apparatus is rotating, but not in the system of the apparatus itself. On a Lorentzian theory, this implies that the rotating apparatus is not at rest in the ether. We therefore see that the Lorentzian theory cannot only account for the goings on in rotational phenomena, but also gives an explanation for why these phenomena occur.

We saw that from the perspective of special relativity, rotational and accelerated phenomena must be treated differently from phenomena of systems in inertial motion. However, we do not know why this is so. The typical answer is that gravitational and centrifugal forces come into play in rotation and acceleration. From Einstein’s practical geometry approach, it thus follows that these forces must be accounted for within the kinematics, which is what is done in general relativity. What we have seen above is that the Lorentzian theory can explain the rotational phenomena without introducing any extra class of forces. Furthermore, in order to see why the principle of relativity only applies to systems in rectilinear uniform motion, we can imagine an accelerated rectilinear motion of the apparatus.

Moving the apparatus in an accelerated motion toward the right, means that as the green arrow moves "right", the apparatus moves away from it at velocity v. As the green arrow moves "up" there is no relevant motion of the apparatus. As the green arrow moves "left", the apparatus moves toward the green arrow at a velocity >v. In other words, the total distance travelled "left"/"right" is shorter than half the total distance (which would be the case if the apparatus was at rest). Again, results of such experiments are predictable only under the assumption that the one-way speed of light is constant in all directions *in the system in which the apparatus is accelerated*. Thus, as long as we follow the standard measurement procedure of Lorentzian theories, we get the correct results. If we move the apparatus with a uniform rectilinear motion toward the right, the distances "right"/"left" add up to half the total distance and we get the same result as if the apparatus was at rest.

We see therefore that the Lorentzian theories can explain rotational, rectilinear, uniform and accelerated motions within a single explanatory frame. This frame states that the one-way speed of light is constant in the ether system. This is an ontological statement in the theory. Epistemologically however, there is no way to distinguish the ether system from any other system moving with a rectilinear uniform motion relative to it. The reason for this is that there is no possible way to measure directly the *one-way* speed of light between two points, without making assumptions about the synchronization of the measuring apparatus. All our direct measurements, made without such preconditions, are of the two-way speed. As pointed out, even in a system where the "left" and "right" speeds differ, they will differ in such a manner that the two-way speed comes out the same.

Einstein's solution to this issue was simply to postulate that the *one-way* speed is constant in all systems with non-accelerated non-rotational motion. This is an ontological claim for which we are offered no rationale. Rather, we are invited to revisit the Aristotelian notion of ontologically different classes of motions, based solely on kinematical criteria, with no backing up by causal characteristics. This was the exact issue Galileo treated and altered in his partial establishment of classical physics.

IV 11. Physics and epistemology

In part IV, we have seen that there exist two theoretical frameworks that successfully describe a set of physical phenomena. For the phenomena that are described by both, the theories are mathematically and empirically equivalent. However, at the ontological level the theories differ. The most central difference being that Lorentzian theories assume that light travels in a medium, called the ether, and that the assumption of the constant one-way speed of light in all directions is valid in relation to the ether only. Special relativity assumes that the one-way speed is constant in all directions in all inertially moving systems. Epistemically, the theories agree that no inertially moving system can be singled out as being "at rest" in relation either to the ether or a possible physically absolute space. Nevertheless, the theories differ in what they claim about the world.

For a century, physicists and philosophers of science have almost uniformly held that special relativity is the better option among the two. Still, there exists a counter tradition from Sagnac, Lorentz, Janossy, Ives, Levy and others, which maintains that the ether theories are preferable. Recent debates between these traditions are not univocally in favor of special relativity. The paper series of Abreu and Guerra clarify, with Szabo, the mathematical and empirical equivalence of the theories. Furthermore, concerning presumably problematic issues of conceptualization, such as the twin paradox, there is no general agreement within the special relativity camp. Issues relating to the ontological dualism between rods, clocks and everything else, the mathematical dualism between Euclidean and non-Euclidean geometry, the loss of mathematical unity stemming from the relativistic rules for velocity addition, the ontological difference between different types of motion and so on, do not have generally agreed upon solutions. While the internal debate within philosophy of physics and physics itself is interesting, there is a more general epistemological debate underlying it. This debate relates to the classical problem: How far does pure empiricism take us?

As we have seen, Einstein started out from a position of pure empiricism generally attributed to his teacher, Mach. The most basic idea of such an approach is that we should ultimately be able to reduce every concept to an operational definition. We see this approach particularly clearly in Einstein (1916) where he insists that concepts of space, time, and simultaneity simply have no meaning in physics unless we can measure them. Einstein gave up on the puritan version of this approach as it became increasingly apparent that all the central concepts of special relativity, i.e.

time dilation, spatial contraction, the loss of universal simultaneity and so on, are wholly reliant on a set of postulates. These postulates are using the notions of practically rigid measuring rods and clocks, as well as the constant one-way speed of light. None of the phenomena that confirm these necessary conditions experimentally can be measured without postulating the theory they ground. As such, special relativity is a point in case against the more puritan empiricism that Mach advocated. Thus, Einstein explicitly warns against a Machian understanding of Hume's empiricist project:

> … by his clear critique, Hume did not only advance philosophy in a decisive way but also – though through no fault of his – created a danger for philosophy in that, following his critique, a fateful 'fear of metaphysics' arose which has come to be a malady of contemporary empiristic philosophizing: this malady is the counterpart to that earlier philosophizing in the clouds, which thought it could neglect and dispense with what was given by the senses. (Einstein 1944: 24)

The idea that the 'fear of metaphysics' is a malady is a natural development from Einstein's earlier claims that his theories are principle theories in (Einstein: 1919) and that theories come in strata with basic rules of the game that are free constructs of the imagination (Einstein: 1936). Why this development?

Poincaré had argued that there is no empirical way to answer a question concerning the correct structure or proper geometry of space. Any empirical investigation is a combination of a theory of objects as well as a theory of structure.[130]

This thesis includes an argument along Poincaré's lines. At the level of measuring results, one can freely choose a Galilean space-time structure with elastic objects (Maxwell/Lorentz) or a relativistic structure with practically rigid objects. Einstein (1921), discussed this issue directly and held that there is no procedure by which you can measure your way directly to the spatiotemporal structure or the practical (non-)rigidity of the measuring apparatus. One must apply metaphysical arguments as well as empirical ones. The problem then becomes which metaphysical position to hold.

[130] A recent argument defending of Poincaré in opposition to Einstein can be found in Valente (2017)

In order to decide which version of (geometry + physics) theory to adopt, we must take certain metaphysical claims for granted. Concerning both object theory and space-time theory, Einstein maintained a Humean metaphysics. The first challenge to such a metaphysics is that the most basic elements of the theory – practically rigid rods and clocks, and rest systems with isotropic light speed – are theoretically constructed, and cannot be detected empirically. Hence, Einstein's (1936) version of a constitutive theory of science where the a priori elements are free constructs or conventions. The neo-Kantian approaches of Friedman and Reichenbach follow the same philosophical impulse. Arguably, we can see elements of the general line of thinking already in Hume's description of time and space as "manners of having impressions" (Slavov 2015: 257).

Another Humean element of Einstein's approach is the common insistence that physical objects are epistemically prior to space and time (Einstein 1934 and 1936). Hume himself argued for this on the basis that we do not have singular impressions of space and time. However, we do have singular impressions of physical entities:

> Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that, at last you lose sight of it; it is plain, that the moment before it vanished the image or impression was indivisible. It is not for want of rays of light striking on our eyes, that the minute parts of distant bodies convey not any sensible impression; but because they are removed beyond that distance, at which their impressions were reduced to a minimum, and were incapable of any farther diminution. A microscope or telescope, which renders them visible, produces not any new rays of light, but only spreads those, which always flowed from them; and by that means both gives parts to impressions, which to the naked eye appear simple and uncompounded, and advances to a minimum, what was formerly imperceptible. (Hume 1739-40: Book 1, part 2, Section 1: 27-28)

A problem with Hume's argument is that the entire set up for this thought experiment relies on there being a dynamical relation between the observing subject and the observed object (ink spot). This relation is set in space and time. Thus, Hume's argument for singular impressions of physical objects relies on their spatiotemporal relation to an experiencing subject. Nevertheless, for Einstein as well as for Hume, the singularity thesis translates into an analysis of physical phenomena in terms of separable events and instances. The coordination of such events and instances are, as we have seen, central to Einstein's (1905) argument.

Pragmatically, in order to do physics we need operational definitions as well as notions of events and instances. Only through such definitions can we establish a clear connection between the world and our theories. If reified however, operational definitions, events and instances become problematic. In other words, what defines our possible measurements does not directly give us a meaningful ontology. Einstein's realization of this point led him to moderate his initially Machian position. This leaves us with the following question: Are conventionally established concepts the best we can come up with?

The Kantian answer to this question is "no". On the Kantian epistemology, we start from the basic tenet that every physical object interacts in the world in a dynamical manner. The spatial and temporal way in which human beings interact is thus an intrinsic property to being human.

Taking the Kantian theory as our epistemological base, we find that the most basic elements of a physical theory are also the most basic aspects of human interactions. If we maintain these as synthetic a priori, we find that space and time must have a Euclidean structure. For a Kantian, the main question is whether such a spatiotemporal structure is possible in light of modern physics. In this part, I have argued that, through the theories of Maxwell and Lorentz, the Kantian option is not only possible. It is preferable. By maintaining the Kantian epistemology within physics therefore, we can adhere to Einstein's own view on the relationship between everyday experience and abstract scientific models: "The whole of science is nothing more than a refinement of everyday thinking" (Einstein 1936: 290).

The main point being that there should be a possible unifying approach to different aspects of the world applying different methodologies in different fields, but still coherent within an overall epistemological theory. We have seen that such Unitarianism is possible within the Kantian framework. We do not see how the same form of Unitarianism is possible on Einstein's own approach to special relativity. The basic element of Einstein's approach is that concepts are without content when we are unable to connect them to direct experience (Einstein 1934: 276-277). So although we must start with pre-scientific notions of for instance space, these concepts only acquire content, and thus meaning, when operationally defined. However, there are no unambiguous ways to operationalize basic elements of our experience such as speed, motion, space and time. The operational definitions used are rather justified as functionally successful conventions. This move toward convention is an abstraction that leads away from everyday thinking and experience. In our everyday thinking and experience, there is a Euclidean structure to space and we have universal simultaneity. As long as there is no proposed way in which we can alter our everyday experience through the theory of relativity, the four-dimensional space-time structure with absolute rods and clocks remains an abstract idea separated from our everyday lives.

When Galileo introduced the principle of relativity and his new theory of motion, he was able to incorporate everyday experience. Thus, rather than separating theory from experience, the new theory provided new insight into everyday experience. With special relativity, we have yet to find a way to perform this feat. For instance, the ontological difference between rotational and rectilinear motion has no everyday correlate. Rather, our everyday experience of rectilinear motion completely ignores the fact that we are placed on a rotating spheroid planet. As such, there is a clear way in which the relativistic approach is inferior to the Kantian approach combined with Lorentz/Maxwell type theories.

Concluding remarks

In this thesis, I have emphasized a common aspect of Galileo, Kant, Einstein and Kepler's philosophies of science, which is their insistence that science is a development from, or refinement of everyday thinking and experience. This common aspect implies that if we are to understand our world, we must also understand our part in it. Galileo, Kant, and Kepler further insist that a part of everyday and scientific experience is that it contains more than mere sense impression.

Einstein initially thought he could build a theory on empirical data alone. When he found that neither he nor any of the great physical thinkers had been able to succeed in this fashion, he gave up his radical empiricist position. Nevertheless, Einstein remained a Humean thinker until the end, and held with Hume that if we find ideas that are neither empirically derived nor laws of reason, these ideas are dubious. His final position therefore, is that the secure foundation of science is empirically derived, all else are free inventions of the human mind. There are different ways to understand Hume however, and different lessons to draw from different readings. For instance, in the introduction to the *Treatise*, Hume comments

> And as the science of man is the only solid foundation for the other sciences, so the only solid foundation we can give to this science itself must be laid on experience and observation (Hume: 1738-40: xvii)

How are we to understand this claim? If we read Hume's emphasis on experience as a reference to experience in general, Hume makes the same general claim as Galileo, Kepler and Kant that knowledge must relate to and ultimately relies on general aspects of everyday experience. A more plausible reading of Hume, which Einstein endorsed, is that "experience" refers to particular instances and events. For Hume, a solid empirical foundation is only available as long as we can derive our knowledge from separate events and instances. Thus, when Hume cannot find particular instances of causal necessity and productive power, he rejects our knowledge of them. There are, of course, other argumentative routes available. In relation to causation, we can reject necessity (as Anjum and Mumford have systematically argued over the last decade). Another possibility is to look at Hume's basic set-up, which is the idea that our experience is a mosaic of separable events and instances, joined together by association rules. There are at least two ways to challenge this idea. A modest proposal would be to ask whether the separate instances and events come to us as in some way already related, which implies that the relation is part of the initial experience and thus as real as any separable aspect of it. This way of thinking is intrinsic to the semirealist position in relation to scientific knowledge. Entities are available to us through their (structural) relations, and our knowledge of those relations rely on there being identifiable entities. It is also possible for the semirealist to adopt this view concerning everyday knowledge and thus adopt a more general view of experience than the strictly Humean notions of distinct events and instances.

A more radical proposal, which is the proposal I understand Kant to make, is to deny the primacy of events and instances altogether. We see this particularly in relation to space. Hume, like Leibniz, initially thought we could build space from relations among separated objects in the way Einstein argued in the 1936 paper. Kant on the other hand, finds that we must understand the objects themselves as limitations of an overall space. Thus, Kant starts out with a totality already given. We then separate the totality into objects, events and instances. On the Kantian theory, if we find aspects of experience, which we cannot treat in separation, these represent a different class or category of experience. On Hume's theory, either they do not exist or we cannot know of their existence.

We have seen that Hume treated space and time as "manners of having experiences", but why? Although the project is still ongoing, there has been no successful theory of how one can build space and time progressively from relations among objects. Neither do we have separable experiences of space and time. However, without space and time, we do not have any science or indeed any content to experience at all. Therefore, the Humean theory dictates that we must keep space and time, but cannot justify their existence. As such, they are described as "manners of having impressions" without any further justification. Einstein, hence, attempted building space more or less directly from the concept of the solid object, and the structure of space from relations among such objects. However, he implicitly postulates an entire theory of space and time a priori in order to conceive of a measuring apparatus.

We see a similar issue in relation to geometry. Kant's view on geometry as a synthetic a priori science, builds from the idea that, as embodied physical beings, space is familiar to us. It is a central aspect of the totality of experience and the interactions into which we are constantly involved. For Kant, geometry is the study of abstract features of that same experience and interaction by excluding physical content. Again, we start from a totality as given and work our way down to the specifics. The formalist axiomatic approach goes in the opposite direction and claims that we must start from axioms and work deductively from there. Naturally, we cannot deduce the axioms themselves and like Einstein's theory of physics, we are instructed to take the axioms as free inventions. What we ultimately end up with is then an alleged geometry and a physics that is empirically valid, but chosen freely by a community of mathematicians and physicists, i.e., an idealist position.

My claim is that we fare better by rejecting this type of approach and accept that we generate different types of knowledge in different ways. I agree with Stump and the constitutive theorists that all scientific theory involves constitutive elements, some of which are empirical. However, not *all* constitutive elements are empirical, such as for instance the law of inertia and the mechanical laws that follow from it. Following Hume - Friedman, Reichenbach, Einstein, Stump and others have no category for non-empirical constitutive elements other than free inventions or conventions in some version or other. It seems unreasonably coincidental that the law of inertia, and the mechanical principle of relativity which, along with the laws of motion remain valid through the centuries although they allegedly are mere inventions. If we approach the issue from a Kantian perspective, the law of inertia follows from the rules for vector addition and the notion of speed as an intensive magnitude in phoronomy. However, phoronomy is a purely a priori science. The constitutive elements of mechanics therefore, are a priori established. Still, we must accept that some constitutive elements are conventional in physics, and thus could be otherwise. For instance, there are other ways to synchronize clocks than by using light signals. The Kantian version of constitutive theories I am arguing for is a layered theory of a priori, conventional, and empirical constitution. A priori constitutive elements limit the range of plausible conventional constitutive elements, which again limit the range of plausible empirical constitutive elements. We can see this in Lorentz' theory, where the a priori constitutive elements are given by Euclidean geometry, Kantian phoronomy and the principle of inertia. The conventional constitutive element is the synchronization procedure by light signals, and the empirical constitution of all further physical theories is that clocks in relative motion lose their synchronicity.

We must beware, however, of dogmatism at any level. If there is a meaningful investigation into issues that challenge either layer of constitution, it must be welcome. Thus, if there are successful proofs that Kant's a priori elements are erroneous, they must be revised or replaced. I have argued that no such proof has been presented. Rather, what we have is a suggestion that the a priori elements are all conventional or in some sense freely chosen. On this suggestion, neither the principle of inertia nor our ability to understand geometrical proofs prior to formal education can be explained. Moreover, any investigation into the issues is blocked. The formalist axiomatic approach to geometry blocks the possibility of pre-axiomatic geometrical knowledge, Hume's empiricism blocks investigation into parts of our knowledge that cannot be investigated in terms of events and instances, and Einstein's postulate of practically rigid bodies (when upheld as real entities) blocks investigation into their dynamical constitution. In all cases, methodology, not empirical fact blocks further investigation.

Since the scientific revolution, physics has adopted an increasingly abstract approach to knowledge production, where talk of fall and light speed in a, strictly speaking, non-existent vacuum or "empty space" are commonplace. Furthermore, physics adopts thought experiments that entail causal necessitation, rigid bodies, closed systems, separable events, perfectly straight lines, inertial motions and so on. Imagined as real entities, these are all suspect. They do however play a crucial role in our investigation into the world. For instance, in order to understand what a dynamically elastic object is, we adopt the notion of rigidity and negate it. Thus, the concept of rigidity and the notion of rigid objects play a crucial part in our understanding of dynamics. Necessity, separable events, inertial systems, and vacuum play a similar role. We apply these notions as ideals against which we may test elements of our reality, and as such, they are crucial parts of knowledge production in physics. We apply the very same ideals in everyday life. When we tell our life stories, we talk in terms of separable events, we use counterfactual arguments based in necessities when we talk politics, and we see perfectly straight lines when we look at buildings. In other words, in all aspects of our lives, we apply ideals that do not strictly apply anywhere else than in us. And we do so with great success. However, if we reify these ideals and demand that the world complies, we get lost. This is, ultimately, what happens in the Humean epistemology, where separate events and instances are taken as not only real, but also primary.

My suggestion is that we leave the Humean empiricism, the axiomatic formalist geometry, and the realist reading of special relativity behind. We can then start asking how these types of knowledge are possible. As suggested by Hume, this must start from an investigation into the human subject whose experience we are investigating and the interactions in which we humans take part. The common aspect of all such interactions is that all actors participate actively. Subjects interact by exchanging powers with objects, objects interact by exchanging powers with each other, objects are possible due to the interactions of their internal powers, and the powers themselves reside in the dynamically elastic ether. In other words, by opening up every interaction to investigation, we find that there are indeed powers all the way down!

References

Abreu, R. & Guerra V. (2005) The conceptualization of time and the constancy of the speed of light, *European Journal of Physics*, 26: 117-123

Abreu, R. & Guerra V. (2006) On the consistency between the assumption of a special system of reference and special relativity, *Foundations of Physics*, 36: 1826-1845

Abreu, R. & Guerra V. (2008) The principle of relativity and the indeterminacy of special relativity, *European Journal of Physics*, 29: 33-52

Abreu, R. & Guerra V. (2015) Speakable and unspeakable in special relativity: time readings and clock rhythms, *Electronic Journal of Theoretical Physics*, 12: 183-204

Ahmed, F. D et al. *(2012) A review of one-way and two-way experiments to test the isotropy of the speed of light, Indian Journal of Physics, 86: 835-848*

Andersen, F. (2011) *Time and Space in Special Relativity: A Critique of the Realist Interpretation*, Germany, Lambert Academic Publishing

Andersen, F. & Becker Arenhart, J. R. (2016) Metaphysics within science: against radical naturalism, *Metaphilosophy*, 47: 159-180

Anderson, R. L. (2004) It adds up after all: Kant's philosophy of arithmetic in light of the traditional logic, *Philosophy and Phenomenological Research*, LXIX, 3: 501-40

Bell, J. S. (1976) How to teach special relativity, in *Speakable and Unspeakable in Quantum Mechanics*, Cambridge, Cambridge University Press, 1987, 3rd edition 2010: 67-81

Bohm, D. (1965) *The Special Theory of Relativity*, Taylor and Francis e-Library, 2009

Bohr, N. (1938) The causality problem in atomic physics, in Faye, J. and Folse, H.J. (Eds) *The Philosophical Writings of Niels Bohr Vol. IV Causality and Complementarity*, Conneticut, Ox Bow Press, 1998: 83-92

Born, M. (1922) *Einstein's Theory of Relativity*, in Brose, H.L (Ed., Trans.) New York, EP Dutton and Company Publishers

Brown, H. R. (2005) *Physical Relativity Space-time Structure from a Dynamical Perspective*, Oxford, Oxford University Press

Buchwald, J. Z. (1989) *The Rise of the Wave Theory of Light: Optical Theory and Experiment in The Early Nineteenth Century*, Chicago, The University of Chicago Press, 1998

Camilleri, K. (2015) Knowing what would happen: the epistemic strategies in Galileo's thought experiments, *Studies in History and Philosophy of Science Part A*, 54: 102-112

Carrirer, M. (1992) Kant's relational theory of absolute space, *Kant-Studien*, 83: 399-416.

Cellucci, C. (2013) Philosophy of mathematics: making a fresh start, *Studies in History and Philosophy of Science*, 44: 32-42

Chakravartty, A. (1998) Semirealism, *Studies in History and Philosophy of Science*, 29: 391-408

Chakravartty, A. (2007) *A Metaphysics for Scientific Realism: Knowing the Unobservable*, New York, Cambridge University Press

Copernicus, N. (1543) *De Revolutionibus Orbitum Coelestium,* in Wallis, C. G. (Trans.) *On the Revolutions of Heavenly Spheres*, New York, Prometheus Books, 1995

Daintith, J. et al. (1985) *Oxford Dictionary of Physics*, 6th edition, Oxford, Oxford University Press, 2009

Debs, T. A. and Redhead, M. L. G. (1996) The twin 'paradox' and the conventionality of simultaneity, *American Journal of Physics,* 64: 360-373

Dirac, P. A. M. (1930) *The Principles of Quantum Mechanics,* Great Britain, 4th edition (1958) Oxford University Press, 2010

Disalle, R. (2006) *Understanding Spacetime*. *The Philosophical Development of Physics from Newton to Einstein*, New York, Cambridge University Press

Dorato, M. (2002) Kant, Gödel and relativity, in Gardenfors, P., Kijaria-Placek, K. and Wolenski, J. (eds) *Proceedings of the Invited Papers for the 11th International Congress of the Logical Methodological Philosophy of Science*, Dordrecht, Springer Library: 329-346

Dorato, M. (2007) Relativity theory between structural and dynamical explanations, *International Studies in the Philosophy of Science*, 21: 95-102

Dorato, M. (2014) Dynamical versus structural explanations in scientific revolutions, *Synthese*: 1-21

Einstein, A. (1905) On the electrodynamics of moving bodies, in Stachel, J (Ed.) *Einstein's Miraculous Year*, UK, Princeton University Press, 2005: 123-164

Einstein, A. (1905b) On a heuristic point of view concerning the production and transformation of light, in Stachel, J (Ed.) *Einstein's Miraculous Year*, UK, Princeton University Press, 2005: 177-198

Einstein, A. (1907) Comments on the notes of Mr. Paul Ehrenfest: 'The translatory motion of deformable electrons and the area law'. In Beck, A. (Trans.) *The collected papers of Albert Einstein, Vol. 2*. *The Swiss years: Writings, 1900-1909*, Princeton: Princeton University Press, 1989: 236-238

Einstein, A. (1907b) On the relativity principle and the conclusions drawn from it, in Beck, A (Trans.) *The Collected Papers of Albert Einstein Vol. 2 The Swiss years: Writings, 1900-1909,* New Jersey, Princeton University Press, 1989: 252-311

Einstein, A. (1909) On the development of our views concerning the nature and constitution of radiation, in Beck, A (Trans.) *The Collected Papers of Albert Einstein Vol. 2 The Swiss years: Writings, 1900-1909,* New Jersey, Princeton University Press, 1989: 379-395

Einstein, A. (1912) Relativity and gravitation reply to a comment by M. Abraham, in Beck, A (Trans.) *The Collected papers of Albert Einstein Volume 2: The Swiss Years:* Writings, *1912-1914*, New Jersey, Princeton University Press, 1996: 130-134

Einstein, A. (1916) *Relativity the special and the general theory,* Milton Keynes, BN Publishing, 2009

Einstein, A. (1918) Dialog über Einwände gegen die relativitätstheorie, *Die Naturwissenschaften* 48: 697-702
Einstein, A. (1918b) Letter to M. Born [undated, untitled] in Born, I. (Trans.) *The Born-Einstein letters Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955*, Great Britain, The Macmillan Press LTD, 1971

Einstein, A. (1919) What is the theory of relativity*, London Times*, November 28, 1919, in *Ideas and Opinions*, USA, Crown Publishers 1954, fifth printing 1960: 227-232

Einstein, A. (1919b) Induction and deduction in physics, *Berliner Tageblatt*, 25 December 1919, in *The collected papers of Albert Einstein Vol 7: The Berlin Years: Writings, 1918-1921* (English translation supplement) UK , Princeton University Press, 2002: 108-110

Einstein, A. (1920) Ether and the theory of relativity, in *The collected work of Albert Einstein Vol 7: The Berlin Years: Writings, 1918-1921* (English translation supplement) UK, Princeton University Press, 2002: 160-183

Einstein, A. (1921) Geometry and experience, in *Ideas and Opinions,* USA, Crown Publishers 1954, fifth printing 1960: 232-246

Einstein, A. (1923) Nobel Lecture: Fundamental ideas and problems of the theory of relativity, Nobelprize.org. Nobel Media AB 2014. Web. 19 Nov 2014
http://www.nobelprize.org/nobel_prizes/physics/laureates/1921/einstein-lecture.html

Einstein, A. (1934) The problem of space, ether, and the field in physics, In *Ideas and Opinions*, USA, Crown Publishers 1954, fifth printing 1960: 276-285

Einstein, A. (1935) Elementary derivation of the equivalence of mass and energy, *Bulletin of American Mathematical Society*, 41: 223-230

Einstein, A. (1936) Physics and reality, In *Ideas and Opinions*, USA, Crown Publishers 1954, 5th printing 1960: 290-323

Einstein, A. (1944) Remarks on Bertrand Russell's theory of knowledge, in *Ideas and Opinions*, USA, Crown Publishers 1954, fifth printing 1960: 18-25

Einstein, A. (1949) *Autobiographical Notes*, in Schlipp, P.A (Tans. Ed.) *A. Einstein Autobiographical notes*, La Salle, Illinois, Open Court, 1996

Einstein, A. (1953) Foreword, in Galilei, G. (1632) *Dialogue concerning the two chief world systems*, In Gould (Ed.) & Drake (Trans.) New York, Modern Library 2001

Euclid (ca. 300 BC) *στοχεία*, in Heath, T.L. (Trans.) and Densmore, D. (Ed.) *Euclid's elements*, USA, Green Lion Press, 2013

Feynman, R. (1963) *Six not-so-easy pieces*, London, Penguin Books 1999

Fine, A. (2016) The Einstein-Podolsky-Rosen argument in quantum theory, *The Stanford Encyclopedia of Philosophy* (Fall 2016 Edition) Edward N. Zalta (Ed.) URL = http://plato.stanford.edu/archives/fall2016/entries/qt-epr/

Fleisch, D. (2008) *A students guide to Maxwell's equations*, New York, Cambridge University Press

Frege, G. (1892) *Über Sinn und Bedeutung*, in Geach, P. and Black, M. (Eds) *Translations from the philosophical writings of Gottlob Frege*, Oxford ,Blackwell Publishers 1993: 56-79

Friedman, M. (1983) *Foundations of Space-Time Theories Relativistic Physics and Philosophy of Science*, New Jersey, Princeton University Press

Friedman, M. (2012) Kant on geometry and spatial intuition, *Synthese*, 186: 231-255

Frisch, M. (2011) Principle or constructive relativity, *Studies in History and Philosophy of Modern Physics* 42: 176-183

Galilei, G. (1615) Considerations on the Copernican opinion In M. A. Finocchiaro (Ed.) *The Galileo Affair: A Documentary History*, USA, University of California Press: 70-86

Galileo, G. (1632) *Dialogo Dei Massimi Sistemi*, in Drake, S. (Trans.) *Dialogue concerning the two chief world systems, Ptolemaic and Copernican*, New York, The Modern Library, 2001

Giovanelli, M. (2014) ‘But one must not legalize the mentioned sin’: Phenomenological vs. dynamical treatments of rods and clocks in Einstein′s thought, *Studies in History and Philosophy of Modern Physics,* 48: 20-44

Gödel, K. (1949) A remark about the relationship between relativity theory and idealistic philosophy, in P.A.Schilpp (Ed.) *Albert Einstein Philosopher-Scientist*, LaSalle, Illinois, Open Court: 557-562

Grabiner, J. V. (1983) The changing concept of change: The derivative from Fermat to Weierstrass, *Mathematics magazine*, 56: 195-206

Hanson, N. R. (1958) *Patterns of Discovery*, London, Cambridge University Press

Heath, T. L. (1921) *A History of Greek Mathematics, Vol II From Aristarchus to Diophantus*, Oxford, Oxford University Press

Hensen, B. et al. (2015) Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, *Nature,* 526: 682-686

Hersch, R. (1997) *What is Mathematics, Really?* Oxford, Oxford University Press

Hon, G. & Goldstein, B. R. (2005) How Einstein made asymmetry disappear: symmetry and relativity in 1905, *Archive for the History of Exact Sciences*, 59: 437-544

Howard, D. (2005) Albert Einstein as a philosopher of science, *Physics Today*, 58: 34-40

Hume, D. (1739-40) *A Treatise of Human Nature: Being An Attempt To Introduce The Experimental Method of Reasoning Into Moral Subjects*, in Selby-Bigge, L.A. & Nidditch, P.H. (Eds) Oxford, Oxford University Press, 2nd edition, 1978

Ives, H. E. (1951) Revisions of the Lorentz transformations, *Proceedings of the American Philosophical Society*, 95: 125-131

Janssen, M. (1995): *A Comparison Between Lorentz's Ether Theory and Special Relativity in Light of the Experiments of Trouton and Noble,* PhD Thesis, University of Pittsburgh.

Janssen, M. (2002) Reconsidering a scientific revolution: The case of Einstein versus Lorentz, *Physical perspectives* 4: 421-446

Janssen, M. (2009) Drawing the line between kinematics and dynamics in special relativity, *Studies in History and Philosophy of Modern Physics* 40: 26-52

Jardine, N. (1979) The forging of modern realism: Clavius and Kepler against the sceptics, *Studies in the History and Philosophy of Science*, 10: 141-173

Jardine, N. (1984) *The Birth of History and Philosophy of Science. Kepler's A Defense of Tycho Against Ursus With Essays on Its Provenance and Significance*. Cambridge, Cambridge University Press

Kant, I. (1781/1787) *Kritik Der Reinen Vernunft*, in Mathisen, Serck-Hansen and Skar (Trans.) *Kritikk av den rene fornuft*, Oslo, Pax forlag A/S, 2005

Kant, I. (1783) *Prolegomena Zu Einer Jeden Künftigen Metaphysick, Die Als Wisseschaft Wird Auftreten Kõnnen*, in Von Kohl, L (Trans.) *Prolegomena*, Denmark, Det lille FORLAG, 1994

Kant, I. (1786) *Metaphysische Anfangsgrunde der Naturwissenschaft*, in Friedman, M. (Ed. Trans.) *Metaphysical Foundations of Natural Science,*UK, Cambridge University Press, 2004

Kant, I. (1796-1803) *Opus Postumum* in E. Förster (Ed. Trans.) and M. Rosen (Trans) Cambridge UK, Cambridge University Press, 1993

Kepler, J. (1597) Letter from Kepler to Galileo, 13 October 1597, in Caspar, M. (Ed.) *Johannes Kepler Gesammelte Werke,* CH Beck'sche Verlagsbuchhandlung, Munich, 1937, xiii, 76: 13-16

Kepler, J. (1600) *Apologia Tychonis contra Ursum,* in Jardine, N. *The Birth of History and Philosophy of Science. Kepler's Defence For Tycho Against Ursus With Essays On It's Provenance and Significance*, New York, Cambridge University Press, 1984

Kepler, J. (1604) *Paralipomena to Witelo, & Optical Part of Astronomy* in Donahue W.H. (Trans.) New Mexico, Green Lion Press, 2000

Kepler, J. (1609) *New Astronomy Based Upon Causes or Celestial Physics Treated By Means of Commentaries on The Motion of The Star Mars From The Observations of Tycho Brahe, Gent*, in Donahue, W. H. (Trans.) *Johannes Kepler New Astronomy*, UK, Cambridge University Press, 1992

Kepler, J. (1619) *Harmonices Mundi*, in Aiton, E. J., Duncan, A. M., & Field, J. V. (Trans.) *The Harmony of The World,* American Philosophical Society Vol. 209, 1997

Kleiner, S. A. (1983) A new look at Kepler and abductive argument, *Studies in the History and Philosophy of Science* 14: 279-314

Kostro, L. (2000) *Einstein and the Ether*, Canada, Apeiron

Ladyman, J. & Ross, D. (2007) *Every Thing Must Go: Metaphysics Naturalized*, Oxford, Oxford University Press

Ladyman, J. (2005) Mathematical structuralism and the identity of indiscernibles, *Analysis* 65.3: 218-21

Lange, M. (2013) How to explain the Lorentz transformations, in Mumford, S. and Tugby, M. (Eds) *Metaphysics and Science,* Oxford, Oxford University Press: 73-101

Lorentz, H. A. (1899) Electromagnetic phenomena in systems moving with any velocity less than that of light, *Proceedings of the Section of Sciences, Amsterdam,* 1904: 809-831

Lorentz, H. A. (1900) Elektromagnetische theorien physikalischer erscheinungen, in *Collected papers,* Netherlands, Springer,1935 Vol. 8: 333-352

Lorentz, H. A. (1916) *The Theory of Electrons and Its Applications To The Phenomena of Light and Radiant Heat*, New York, G E Stechert & Co

Mason, A., Lee, R., & Lee, S. H (2010) Population dynamics: Social Security, Markets, and Families, *International Social Security Review*, 63: 145-75

Matvejev, V. N., Matvejev, O. V. & Grøn, Ø. (2016) A relativistic trolley paradox, *American Journal of Physics*, 84: 419-425

Maudlin, T. (2012) *Philosophy of Physics Space and Time,* New Jersey, Princeton University Press

Maudlin, T. (2007) *The Metaphysics Within Physics,* New York, Oxford University Press

Maxwell, J. C. (1861) On physical lines of force, *Philosophical Magazine*

Maxwell, J. C. (1865) A dynamical theory of the electromagnetic field, *Philosophical Transactions of the Royal Society*, 155: 459-512

McCormmach, R. (1970) H.A. Lorentz and the electromagnetic view of nature, *Isis*, 4: 459-497

Michelson, A. & Morley, E. W. (1887) On the relative motion of the earth and of the luminiferous ether, *Sidereal Messenger*, 6: 306-310

Minkowski, H. (1908) Space and time, in Levertoff, F. and Petkov, V. (Trans. Eds) *Space and Time Minkowski's Papers on Relativity,* Canada, Minkowski Institute Press, 2012: 37-52

Myrstad, J. A. & Sandmel, T. (1987) *Einstein, Kant og Fysikkens Metafysiske Basis,* Filosofiske Undersøkelser nr.6, Oslo, Institutt for filosofi, Universitetet i Oslo

Myrstad, J. A. (2004) The use of converse abduction in Kepler, *Foundations of Science*, 9: 321-338

Newton, I. (1686) *Principia*, in Cohen, B, and Whitman, A. (Trans.) *The Principia: Mathematical Principles of Natural Philosophy*, Los Angeles, University of California Press, 1999

Norton, J. D. (2000) What can we learn about the ontology of space and time from the theory of relativity?, *PhilSci Archive*, http://philsci-archive.pitt.edu/archive/00000138/

Norton, J. D. (2005) How Hume and Mach helped Einstein find special relativity, in Dickson, & Domski, M. (Eds) *Discourse on a New Method: Reinvigorating the Marriage of History and Philosophy of Science*. Chicago and La Salle, IL: Open Court, 2010: 359-386

Osiander, A. (1543) To the reader concerning the hypotheses of this work, in Wallis, C. G. (Trans.) *On the Revolutions of Heavenly Spheres*, New York, Prometheus Books, 1995: 3-4

Pal, P. (2003) Nothing but relativity, *European Journal of Physics*, 24: 315-319

Penrose, R. (2004) *The Road To Reality A Complete Guide To The Laws of The Universe*, New York, Vintage books, 2007

Plato (387-367 BC) *Meno*, in Løkke, H. (Trans.) *Platons Samlede Verker bind III*, Norway, Vidarforlagets Kulturbiblitek, 2006: 271-321

Poincaré, H. (1899) Des Fondements de la géométrie: à propos d'un livre de M. Russell, *Revue de Métaphysique et de Moreale* 7: 251-279, In Stump, D. *Conceptual Change and the Philosophy of Science: Alternative Interpretations of the A priori*, New York, Routledge 2015: 49-50

Poincaré, H. (1902) *Science and Hypothesis*, in Greenstreet, W. J. (Trans.) *Science and Hypothesis*, New York, Walter Scott Publishing, 1905, digital edition, Project Gutenberg 2011 http://www.gutenberg.org/ebooks/37157

Putnam, H. (1967) Mathematics without foundations, *The Journal of Philosophy*, 1: 5-22

Reichenbach, H. (1920) *Relativitãtstheorie und Erkenntnis A Priori*, in Reichenbach, M. (Trans.) *The Theory of Relativity and A Priori Knowledge*, Berkley, University of California Press, 1965

Reichenbach, H. (1928) *Philosophie der Raum-Zeit-Lehre* in Reichenbach, M. and Freund, J. (Trans.) *The philosophy of space and time*, New York, Dover Publications Inc., 1958

Rocca, E. & Andersen, F. (2017) How biological background assumptions influence scientific risk evaluation of stacked genetically modified plants. an analysis of research hypotheses and argumentations, *Life Sciences, Society and Policy*, (forthcoming)

Rohlf, J. W. (1994) *Modern Physics From α to z^0*, USA, Wiley and Sons Inc.

Russell, B. (1912) *The Problems of Philosophy*, New York, Barnes & Noble Inc., 2004

Sagnac, G. (1913) L'éther lumineux démontré par l'effet du vent relatif d'éther dans un interféromètre en rotation uniforme," C. R. Acad. Sci. 157, 708-710, in Rizzi, G. and Ruggiero, M (Trans., Eds) *Relativity in Rotating Frames*, Dordrecht, Kluwer Academic Publishers, 2004, Chap. 2: 5-7

Sandmel, T. (2004) *Matematikkens Indre Arkitektonikk og Dynamikk, Konstruksjonstyper I Noen Utvalgte Matematiske Disipliner*, Oslo, Unipub Forlag

Sellars, W. (1962) Philosophy and the scientific image of man, in Colodny, R. (Ed.) *Frontiers of Science and Philosophy*, Pittsburg, University of Pittsburgh Press, 1962: 35-78

Selleri, F. (1998). On a Physical and Mathematical Discontinuity in Relativity Theory. in Selleri, F. (Ed.) *Open Questions in Relativistic Physics*, Montreal, Apeiron

Sklar, L. (1974) *Space Time and Spacetime*, London, University of California Press, 1977

Slavov, M. (2016) Empiricism and relationism intertwined: Hume and Einstein's special theory of relativity, *Theoria* 31: 247-263

Spelke E. S. (2011) Natural number and natural geometry. In Brannon, E. & Dehaene, S. *Space, Time and Number in the Brain Searching for the Foundations of Mathematical Thought Attention & Performance* XXIV. Oxford University Press: 287-317

Stachel, J. (1983) Special relativity from measuring rods, in Cohen, R. S. and Laudan, L. (Eds) *Physics, Philosophy and Psychoanalysis*: 255-72

Stachel, J. (2002) What song the sirens sang: How did Einstein discover special relativity? In *Einstein from B to Z.* Einstein studies, Vol. 9, Boston, Birkhäuser: 157-170

Stump, D. J. (2015) *Conceptual Change and The Philosophy of Science: Alternative Interpretations of the A priori*, New York, Routledge

Szabo, L. (2010) Lorentzian theories vs. Einsteinian special relativity - a logico-empiricist reconstruction, in Maté, A., Rédei, M. & Stadler, F. (Eds) *Vienna Circle and Hungary - Veröffentlichungen des Instituts Wiener Kreis*, Dordrecht, Springer, 2011: 191-227

Tønnessen, S. (2012) *The Birth of Mathematical Physics: Kepler's Proof of The Planets' Elliptical Orbits From Causal Hypotheses*, Tromsø, University of Tromsø, PhD thesis: http://munin.uit.no/handle/10037/4484

Valente, M. B. (2017) The conventionality of simultaneity in Einstein's practical chrono-geometry, *Theoria, Revista de Teoría, Historia y Fundamentos de la Ciencia*, 32: 177-190

Van Camp, W. (2011) On kinematic vs. dynamic approaches to special relativity, in *Philosophy of Science Assoc. 22nd Biennial Mtg (Montréal, QC)*

Van Dyck, M. (2005) The paradox of conceptual novelty and Galileo's use of experiments, *Philosophy of Science*, 72: 864-875

Weisberg, M. (2007) Three kinds of idealization, *Journal of Philosophy*, 104: 639-659

Wilczek, F. (2008) *The Lightness of Being Mass, Ether and The Unification of Forces*, New York, Basic Books

Wildberger, N. (2010) Universal hyperbolic geometry II: A pictorial overview, https://arxiv.org/abs/1012.0880v1

Wilson, C. (1968) Kepler's derivation of the elliptical path, *Isis* 59: 5-25

Young, T. (1803) The Bakerian lecture. Experiments and calculations relative to physical optics, in *Philosophical Transactions of the Royal Society of London*, 1804, 94: 1-16

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