Environmental Policy and the Direction of Technical Change*

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Abstract

Should governments direct R&D from "dirty" into "clean" technologies? How important is this compared to carbon pricing? We inquire into this, introducing two novelties compared to recent literature. We introduce decreasing returns to R&D, and allow future carbon taxes to influence current R&D decisions. Our results suggest that governments should prioritize clean R&D. Dealing with major environmental problems requires R&D to shift to clean technology. However, with most researchers working with clean technology, both productivity spillovers and future risks of being replaced increase. Consequently, the wedge between private and social value of an innovation is largest for clean technologies.

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1 Introduction

Reducing the share of fossil fuels in the energy mix is a major challenge for climate change policy.[1] Research and development (R&D) drives down costs and improves technologies, and hence, facilitates the diffusion of new, clean technologies. On the other hand, this mechanism is also present for dirty technologies. For instance, recent improvements in "fracking" technology have made it profitable to extract oil from under-ground shale layers, putting downward pressure on the oil price, and thus reducing the relative attractiveness of electric vehicles.

Economists normally argue that putting a tax on carbon emissions is the single most important instrument for tackling climate change. Moreover, although most economist agree that research and development of new carbon free technologies should be subsidized, few advocate prioritizing public R&D funds for clean technologies. This view has, however, recently been challenged in the most recent literature linking climate and R&D policy, see for instance Acemoglu, Aghion, Bursztyn, and Hémous (2012) and Dechezleprêtre, Martin, and Mohnen (2013).

Our paper builds on this recent literature, and group technologies into either *clean* or *dirty*.[2] We then pose the following research questions: (I) Under what circumstances should governments actively direct research effort away from dirty technologies into clean technologies? and II) To what extent can a clean research subsidy replace a carbon tax?

Although it is hard to find data on total global R&D spending on dirty and clean technologies, several sources indicate that the former greatly outperforms the latter.[3] Since

[1] In order to keep global warming below the 2^0C target, a third of oil reserves, a half of gas reserves, and more than 80 percent of coal reserves must stay in the ground (McGlade og Ekins, 2015), while IEA (2011) predicts a 50% growth in total energy demand in the next 25 years. Hence, the production of clean energy must increase dramatically.

[2] Dirty technologies can be defined with point of departure in the fossil fuel value chain, that is, discovering and extracting fossil fuel resources, and improving end-use technologies utilizing fossil fuels such as road transport, coal and gas power plants. Clean technologies can mainly be defined as renewable energy from for example solar and wind, and transport based on electricity and hydrogen.

[3] Aghion, P., Dechezlepretre, Hemous, Martin, and Van Reenen (2016) find that the number of new patents is higher within dirty transportation technologies. The EU Industrial R&D scoreboard (2014) lists companies with respect to their R&D spending. Typical clean technology companies such as Vestas (windmills), First

public spending on R&D tends to follow private spending (e.g., due to tax rebates in proportion to R&D spending), turning this around may require drastic intervention. Hence, more knowledge about potential mechanisms leading to under-provision of clean R&D seems essential. Another policy relevant question is to what extent a clean research subsidy can replace carbon pricing. Implementing a global price on carbon has proven very difficult, and concerted global action on supporting clean R&D may thus be an alternative.

A central analysis of the competition between clean and dirty technologies is the contribution by Acemoglu, Aghion, Bursztyn, and Hemous (2012) (henceforth AABH). They analyze optimal R&D subsidies and carbon taxes in a model with clean and dirty technologies in which the latter technology starts off as more advanced. They argue that a targeted subsidy to clean R&D should be used to shift all R&D effort from dirty R&D to clean R&D, either immediately or within a few years.

In our opinion, it is not clear whether AABH's results are robust to other modelling choices for the innovation sector. First, AABH assume that a scientist only enjoys the current period monopoly profits, which implies that future climate policies are unable to redirect research today.[4] Second, in AABH's model there is constant returns to R&D within a period. As a result they obtain a corner solution for the allocation of the R&D effort: Either all scientists do dirty R&D, or they all do clean R&D. Third, in their numerical simulations they only consider high elasticities of substitution between clean and dirty inputs.

Our point of departure is the AABH model with clean and dirty inputs to final goods production. However, we model the innovation sector differently. First, we let scientists retain profits on an innovation until it is replaced by an innovation of better quality. Second, we introduce duplication effects by having decreasing returns to the number of scientist inno-

Solar (solar panels) and Tesla (electric vehicles) are far behind both oil companies and traditional car producers on the list. Finally, use of fossil fuels are also subsidized more. The International Energy Agency (IEA, 2014) has estimated consumer subsidies to fossil fuels at US$548 billion in 2013, while subsidies to renewable energy amounted to US$121 billion.

[4] In their numerical simulations, each period lasts five years. In the literature on economic growth, innovators typically enjoy monopoly profits for an extended period of time, see Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1991) and Acemoglu (2002; 2009).

vating in a technology.[5] Third, we run simulations with both high and moderate elasticities of substitution between clean and dirty inputs.

At first glance all these changes should make targeted R&D support less crucial, given that optimal carbon taxes are implemented. Innovators will expect future environmental policies to be more stringent, and thus redirect their research in accordance with their expectations. Furthermore, decreasing returns to R&D within a period, and lower elasticities of substitution between clean and dirty inputs, make it profitable to do R&D in both sectors independent of the level of accumulated productivity. Surprisingly, both our theoretical results and our numerical simulations suggest that governments should nonetheless support clean R&D more than dirty R&D. However, to our knowledge, the causal mechanisms leading to this result have not been discussed in the literature before.

Dealing with the environmental problem effectively requires R&D effort to shift from dirty technologies to clean technologies. When most scientists start to work with clean technology, the clean productivity growth rate will be higher. However, scientists do not take into account positive knowledge spillovers on future innovators and the difference in growth rates implies that knowledge spillovers are greater in clean R&D. Hence, clean R&D should receive more subsidies.[6]

André and Smulders (2014) also find that with directed technological change the market directs too little R&D investments to growing sectors. In their paper there are either energy-saving R&D or labor-saving R&D. Due to future resource scarcity, energy-saving R&D will become relatively more valuable, and this makes the knowledge spillovers in this sector relatively more valuable today, which should be reflected in the allocation of R&D resources.

The difference in the social value of the knowledge spillover is not the only motivation to reallocate researchers to clean R&D. When a majority of scientists start to work with clean technology, the future risk of someone coming up with a better innovation increases,

[5] Hémous (2013) also allows for decreasing returns to scale in innovation effort.

[6] Heggedal (2015) finds a similar relationship between growth rates and R&D subsides in a Romer (1990)-type growth model without environmental considerations.

and so does also the probability of losing the income from an innovation. This accelerating replacement effect also implies that the incentives for clean R&D compared to the incentives for dirty R&D are too small from a social welfare perspective.

In contrast AABH write about their result that [*citation*]*...the subsidy deals with future environmental externalities by directing innovation towards the clean sector, whereas the carbon tax deals more directly with the current environmental externality by reducing production of the dirty input.* In our model, the carbon tax can deal with future environmental externalities by redirecting current research, however, as explained, there are two other mechanisms leading to underprovision of clean R&D.

It is well known that when there is more than one market failure, it is socially optimal to have a set of policy instruments, each targeting one of the market failures, e.g. a tax on carbon emissions and a subsidy for R&D. The subsidy is then used to increase the total amount of R&D.[7] However, in our model the total amount of R&D is given, and thus a subsidy to either clean or dirty R&D directly indicates that this field of research should be prioritized.

We also simulate the model numerically in order to investigate to what extent the two instruments can replace each other in a second best world. If the elasticity of substitution between clean and dirty inputs is relatively high, we find that a subsidy to clean R&D welfare dominates an emission tax as a stand-alone policy. That is, welfare with only an R&D subsidy is higher than welfare with only a carbon tax. On the other hand, if the elasticity of substitution between clean and dirty inputs is more moderate, the conclusions are less clear.

Independent of the elasticity of substitution between the two inputs, R&D subsidies are essential for correcting the two market failures in R&D: Knowledge spillovers in clean R&D have higher social value, and the accelerating replacement effect makes the incentives for private clean R&D too small. When R&D resources are channeled to clean R&D, the clean

[7] See eg. Goulder & Schneider (1999); Rosendahl (2004); Gillingham, Newell & Pizer, (2008); Fischer & Newell (2008); Popp et al (2010).

input will eventually replace the dirty input almost entirely even without a carbon tax if the two inputs are close substitutes. The simulations suggest that both these two market failures are important arguments for subsidizing clean R&D and thus directing technical change towards the clean sector.

Emission taxes, on the other hand, help directing innovation towards the clean inputs, and if dirty and clean inputs are not that close substitutes, help to keep emissions down. Thus, this paper not only explains why there should be targeted subsidies to clean R&D, but also clarifies the role of the emission tax in a framework of directed technological change.

1.1 Related literature

As mentioned we let successful innovators keep a patent until it is replaced by a better patent. Several contributions in the environmental economics literature have shown that the length of the monopoly period has implications for environmental policy as well as innovation policy, see e.g. Gerlagh, Kverndokk and Rosendahl (2014) and Greaker and Pade (2009). However, none of these contributions include both clean and dirty technologies.

Decreasing returns to R&D within a period is standard in the economic growth literature, see e.g. Jones and Williams (2000). Jones and Williams (2000) argue for a "stepping-on-toes" effect. This effect will decrease the return to an additional unit of R&D effort since the chance of coming up with the same idea as your fellow researchers increases the more researchers there are at each point in time.

The literature on directed technological change and the environment is steadily increasing in size (see Heutel and Fischer (2013) for an overview on macroeconomics and the environment). Several papers modify and simulate the AABH model, though in different directions and analyzing other problems than in the present paper: Hourcade, Pottier, and Espagne (2011) discuss parameter choices related to the climate part of the model; Mattauch, Creutzig, and Edenhofer (2015) add learning-by-doing effects to the framework ; Durmaz and Schroyen (2014) extend the model by adding abatement technology (carbon capture and storage); David Hémous (2013) and van den Bijgaart (2015) extend the model to include more than one country and analyze unilateral environmental policies in a global context. Importantly, none of these papers explore profits in the innovations that are retained until replaced by a better quality, so that future emission policies affect innovation decisions today.

A key assumption in our model is that innovation is path (state) dependent. A new innovation builds on past quality and increases the productivity of future innovations. Such path dependency is found by Aghion, P., Dechezlepretre, Hemous, Martin, and Van Reenen (2016). They analyze clean and dirty technologies in the automotive industry, and find that there is path dependence in innovation following from spillovers and the firms' histories. Moreover, that productivity spillovers is a rationale for subsiding clean innovation has empirical support. Further, in a recent paper Dechezleprêtre, Martin, and Mohnen (2013) find that spillovers are larger in clean than dirty technologies. The driving force behind the result seems to be that clean technologies are newer technologies than dirty, and that a new technology field has larger spillovers than an old technology field.

On the theory side, Acemoglu, Akcigit, Hanley, and Kerr (2016) develop another model of endogenous growth with clean and dirty R&D where they model the R&D sector differently from us. In their model clean and dirty machines within a product line are perfect substitutes, and hence, in order to have a market, a new clean machine must in most cases outcompete the dirty machine within the same product line. This only happens rarely, and thus, innovators may not get any profits from clean R&D at all even if they improve the clean machine. As us, they also find that carbon taxes may be expensive to relay on alone, and that targeted subsidies to clean R&D are a crucial part of climate policy.

The paper is organized as follows. Section 2 presents the model and the decentralized market allocation, while Section 3 shows the socially optimal R&D allocation and discusses efficient innovation and emission policies. Section 4 provides an extension with patent infringement problems. The model is simulated numerically and the results for optimal policies are given in Section 5, while Section 6 provides a conclusion.

2 The model

The model is an infinite-horizon discrete-time economy with households, a final goods sector, a clean and a dirty intermediate input sector, a machine sector that delivers machines of different qualities to the intermediate input sectors, and finally, an innovation sector that may improve these qualities. The major difference between AABH's model and our model is the innovation sector. We therefore emphasize the innovation sector in the presentation of the model, and cover the rest of the model more briefly.

2.1 Final goods

The final good is used for the production of machines and for consumption C_t, and it is produced by combining dirty and clean intermediates. The production function for this good is given by:

$$Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where Y_{ct} and Y_{dt} is the input of clean and dirty inputs, respectively, and ε is the elasticity of substitution. It is hard to know *a priori* what the elasticity ought to be, but it seems reasonable that the two inputs cannot substitute each other perfectly e.g. solar- and wind energy are intermittent and may require dirty back-up power.

2.2 Production of intermediates with a carbon tax

The production of dirty and clean intermediates uses labor and machines. Machines are given in different varieties i which are specific for either clean or dirty intermediate production. The production function for clean and dirty intermediates in sector $j \in \{c, d\}$ is given by:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di \tag{2}$$

where $\alpha \in (0, 1)$, L_{jt} is labor use in sector j, A_{ijt} is the quality (productivity) of machine type i in sector j at time t, x_{jit} is the input of machine type i in sector j at time t, and the number of machine types is 1. Every time a new innovation is made in one of the sectors, one particular machine type i is replaced by a better machine of the same type. The innovation is drastic, implying the older version of the machine type no longer can be sold with positive profits.

The intermediate firm's problem is:

$$\max_{L_{jt},x_{jit}} \left\{(p_{jt}-\tau_{jt})L_{jt}^{1-\alpha}\int_0^1 A_{jit}^{1-\alpha}x_{jit}^{\alpha}di - w_t L_{jt} - \int_0^1 p_{jit}x_{jit}di\right\},$$

where p_{jt} is the price of the intermediate input of type j, τ_{jt} is the carbon tax ($\tau_{ct}=0$) and p_{jit} is the price of machine type i in sector $j \in \{c,d\}$. The demand for machine type i is found from the first order condition for the optimal use of machine i:

$$x_{jit} = \left(\frac{(p_{jt}-\tau_{jt})\alpha}{p_{jit}}\right)^{\frac{1}{1-\alpha}} L_{jt}A_{jit}, \tag{3}$$

Equation (3) is the demand function for clean and dirty machines. We note that demand depends positively on their productivity A_{jit} and the amount of labor L_{jt} entering either the clean or the dirty sector.

The demand for labor in sector j is given from the first order condition for the optimal use of labor in each sector:

$$(1-\alpha)(p_{jt}-\tau_{jt})L_{jt}^{-\alpha}\int_0^1 A_{jit}^{1-\alpha}x_{jit}^{\alpha}di - w_t = 0. \tag{4}$$

By rearranging (4) we have:

$$L_{jt} = \frac{(1-\alpha)(p_{jt}-\tau_{jt})Y_{jt}}{w_t}. \tag{5}$$

Both (4) and (5) will be used later when we compare the decentralized market allocation with the socially optimal allocations of researchers.

2.3 Production of machines

A producer of a machine type ji is a monopolist and solves:

$$\max_{p_{jit}}[(p_{jit} - \psi(1-s))x_{jit}], \tag{6}$$

where demand x_{jit} is given by (3) above, ψ is the unit cost of a machine (measured in units of the final good), and s is a subsidy to correct for the static monopoly distortion. Costs are normalized to $\psi = \alpha^2$, and the efficient subsidy rate that gives price equal to marginal cost is $s = 1 - \alpha$, which we assume is implemented. Then, solving (6) gives the profit maximizing price on machines $p_{jit} = \alpha^2$. Inserting back into (6), and using (3), we obtain for the per period profit π_{jit} of a machine producer:

$$\pi_{jit} = \bar{\alpha}(p_{jt} - \tau_{jt})^{\frac{1}{1-\alpha}} L_{jt} A_{jit},$$

where $\bar{\alpha} = (1-\alpha)\,\alpha^{\frac{1-2\alpha}{1-\alpha}}$. Note that profits are only derived from holding a patent with the highest quality in each machine type.[8]

2.4 Innovation and allocation of scientists

In each period, a scientist engages in either clean or dirty innovations, and gains profits if she innovates. When a new innovation is made in machine type i, A_{jit} bumps up to $(1+\gamma)A_{jit}$, where $(1+\gamma)$ is the quality step rate. A scientist can choose sector, but not target a specific machine type; instead a scientist is randomly allocated to a machine type in the specific sector. Thus, the scientist makes her decision based on the average machine quality in sector A_{jt} which is given by:

[8] We assume that the quality difference between a new and old machine is sufficiently large, so that firms can charge the unconstrained monopoly price of the new machine. Further, the quality difference is large enough to avoid infringement problems related to patent breadth. This latter assumption is relaxed in Section 4.

$$A_{jt} \equiv \int_0^1 A_{jit} di. \tag{7}$$

A scientist engaged in innovation in sector j then expects a quality $(1+\gamma)A_{jt}$ upon successful innovation.

The mass of scientists in one sector is given by ℓ_{jt}, and we normalize the number of scientists such that $\ell_{ct} + \ell_{dt} = 1$. We assume that scientist earn profits on an innovation until their machine is replaced by a new machine of better quality. At each point in time there is a probability that someone successfully invents a better quality which we denote by z_{jt}.

Further, we assume that there may be duplication by other scientists, i.e. more than one scientist may have the same successful innovation in a given period. We let the duplication effect be represented by decreasing returns to labor input on aggregate sector innovation given by the function ${\ell_{jt}}^{\varpi}$ where $\varpi \in (0,1)$. The probability of a successful innovation in sector j is then given by $\eta_j {\ell_{jt}}^{\varpi}$ where η_j is a parameter.

The expected discounted profits Π_{jt} of a single scientist entering sector j at time t is then given by:

$$\Pi_{jt} = \eta_j (\ell_{jt})^{(\varpi-1)} \bar{\alpha}(1+\gamma) A_{jt-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1 - z_{j,t+v}}{1 + r_{t+v}} \right) \left((p_{j,t+k} - \tau_{j,t+k})^{\frac{1}{1-\alpha}} L_{j,t+k} \right), \tag{8}$$

where r_t is the scientist's discount rate and $(\ell_{jt})^{(\varpi-1)}$ is the average productivity of a scientist in sector j. Since the average productivity of a scientist is declining in the number of scientists, we do not get a corner solution for the allocation of researchers as in AABH.

Furthermore, equation (8) includes the multiplicative term $\Pi_v(1 - z_{j,t+v})$ which denotes the probability of an innovation in technology j surviving from period t until period v. The probability of being replaced z_{jt} is given by $\eta {l_{jt}}^{\omega}$, that is, the probability that an innovation occurs divided by the number of machine lines, which is normalized to unity. Thus, the multiplicative term will be declining in the amount of researchers working with technology

j.[9]

Equation (8) also includes the discounted stream of future profits from an innovation $\Sigma_k (p_{j,t+k} - \tau_{j,t+k})^{\frac{1}{1-\alpha}} L_{j,t+k}$, which among other things, depends on future tax rates. In contrast, AABH only allow the scientists to retain profits in the same period as the innovation occurs. After that period the ownership of the technology is returned to the machine producers without compensation.

Introducing long-lived patents may have significant implications for policy. Let's say that the current per period profits are greater in the dirty sector and that the carbon tax rate rises over a number of future periods. The tax increases the value of clean machines relative to dirty machines over time. Scientists do not take into account the effect of future taxes if patents last for one period and they engage in dirty innovations. On the other hand, if patents are long-lived, scientists take into account that the value of clean machines improves over time. A switch to clean innovation may then be induced today without the need for innovation subsidies.

The decentralized allocation of scientist is given by that in equilibrium the expected profits must be the same for both sectors:

$$\Pi_{ct} = \Pi_{dt} \tag{9}$$

$$\Leftrightarrow$$

$$\frac{\ell_{ct}}{1-\ell_{ct}} = \left(\frac{\eta_c A_{ct-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1-z_{c,t+v}}{1+r_{t+v}}\right) \left((p_{c,t+k})^{\frac{1}{1-\alpha}} L_{c,t+k} \right)}{\eta_d A_{dt-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1-z_{d,t+v}}{1+r_{t+v}}\right) \left((p_{d,t+k} - \tau_{d,t+k})^{\frac{1}{1-\alpha}} L_{d,t+k} \right)} \right)^{\frac{1}{1-\varpi}},$$

where $1 - \ell_{ct} = \ell_{dt}$. We will discuss equation (9) and how it relates to optimal policies in Section 3.2.

Note that in every period, scientist only base their choice of sector on the average past

[9] In the benchmark model we assume that the quality difference between a new and old machine is large enough to avoid infringement problems related to patent breadth. In Section 4 we discuss implications of such infringement problems.

quality of machine types. Given the allocation of scientist, the average quality of the machine types develops according to:

$$A_{jt} = (1 + \gamma\eta_j(\ell_{jt})^{\varpi})A_{jt-1} \tag{10}$$

This is also different from AABH as the total productivity of the scientist depends on the number of scientists through the term $(\ell_{jt})^{\varpi}$.

2.5 Consumers and the environment

There is a continuum of households with measure 1 that all have preferences:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t),$$

where ρ is the discount rate of the households, C_t is consumption, and S_t is the environmental quality. The instantaneous utility function $u(C_t, S_t)$ has positive first-order derivatives.

There is no storage technology in the economy so all final goods are consumed or used as (converted) inputs in the production process in each period. The households hold equal shares of all the assets in the economy (labor income and R&D firms' (scientists') profits). Then the discount rate for the R&D firms follows from the households' valuation of getting income in a future period. Hence, the firms' discount factor β_t for a payoff in period t seen from period zero is:

$$\frac{1}{(1+\rho)^t} \frac{\partial u(C_t, S_t)}{\partial C} = \beta_t, \tag{11}$$

where $\beta_t \equiv \prod_{v=0}^{t} \left(\frac{1}{1+r_v}\right)$ and r_t is the interest rate following from a standard Euler equation (see Appendix A.1).

The law of motion for the quality of the environment is:

$$S_{t+1} - S_t = -\xi Y_{dt} + \delta(\bar{S} - S_t), \tag{12}$$

where ξ denotes the rate of degradation stemming from emissions from the dirty input Y_{dt}, δ is the rate of environmental regeneration, and $\bar{S}$ denotes the maximum environmental quality. Note that S_t only takes values in the range $(0, \bar{S})$. The law of motion given by (12) is different than the one specified in AABH. This is to facilitate the common assumption in integrated assessment models that the rate of CO_2-depreciation in the atmosphere is increasing in the stock of CO_2 (e.g., Hwang et al., 2013, who use a simplified version of the DICE model). See Appendix A.7 for more discussion of this issue as well as other details about the numerical model.

3 Socially optimal policies

In this section we first calculate the first order conditions of the planner's problem. Subsequently, we compare the socially optimal allocation of scientist to clean and dirty R&D with the decentralized market allocation of scientists and then discuss optimal policies.

3.1 Socially optimal allocation

The planner's problem reads:

$$
\begin{aligned}
\max_{L_{jt},\ell_{jt}} \quad & \sum_{t=0}^{\infty} \tfrac{1}{(1+\rho)^t} u(C_t, S_t) \\
s.t \quad & C_t = Y_t - \psi\left(\int_0^1 x_{cit}di + \int_0^1 x_{dit}di\right) \\
& Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} d \\
& A_{jt} = (1 + \gamma\eta_j(\ell_{jt})^{\varpi}) A_{jt-1} \\
& S_t = -\xi Y_{dt-1} + (1-\delta) S_{t-1} + \delta\bar{S} \\
& L_{ct} + L_{dt} \leq 1 \\
& \ell_{ct} + \ell_{dt} \leq 1,
\end{aligned}
\tag{13}
$$

given $A_{c0} < A_{d0}$ and S_0, where $\psi\left(\int_0^1 x_{cit}di + \int_0^1 x_{dit}di\right)$ is the total expenditure of final goods in the production of intermediate goods.[10]

[10] The solution to the planner's problem exists and is unique as the objective function is continuous and strictly concave with a convex constraint set.

The full set of first order conditions following from the planner problem is given in Appendix A.2. In this section we discuss aspects of the planner solution that are directly relevant for optimal policy.

First, from the first order condition with respect to consumption:

$$\frac{1}{(1+\rho)^t}\frac{\partial u(C_t, S_t)}{\partial C} = \lambda_t, \tag{14}$$

we see that the shadow value of the final good λ_t is given by the discounted marginal value of consumption in period t, i.e. the social discount factor. Notice the close connection between λ_t and the market discount factor β_t from equation (11). We are later going to utilize that $\lambda_t = \beta_t$ if the market solution is efficient.

The shadow value of environmental quality ω_t is given by

$$\omega_t = \sum_{v \geq t} (1-\delta)^{v-t} \frac{1}{(1+\rho)^v}\frac{\partial u(C_v, S_v)}{\partial S} I_{S_v < \bar{S}},$$

where δ is the rate of environmental regeneration. Note that $I_{S_t<\bar{S}} = 1$ if $S_t < \bar{S}$, and $I_{S_t<\bar{S}} = 0$ otherwise, since $S_t = -\xi Y_{dt-1} + (1-\delta)S_{t-1} + \delta\bar{S}$ only in the interval $(0, \bar{S})$.

Next we have, for the optimal production and use of the two intermediates, Y_{ct} and Y_{dt}:

$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{ct}^{\frac{-1}{\varepsilon}} - \frac{\lambda_{ct}}{\lambda_t} = 0 \tag{15}$$
$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} Y_{dt}^{\frac{-1}{\varepsilon}} - \frac{\lambda_{dt}}{\lambda_t} - \frac{\xi\omega_{t+1}}{\lambda_t} = 0.$$

The first term in both expressions in (15) is the marginal increase in final goods production from an additional unit of the input. The next term in both expressions e.g. λ_{jt}/λ_t, is the shadow value of the inputs measured in consumption units. In a *laissez faire* market economy these are equivalent to the prices of the inputs.

The last term in the equation for the dirty input is the marginal value of the external effect of this input (measured in consumption units). That is, ξ is the factor that links the use of dirty inputs to the deterioration of environmental quality, and ω_{t+1} is the shadow value of environmental quality. In a *laissez faire* market economy the environmental deterioration caused by dirty input usage is likely not taken into account, however, a Pigovian tax equal to $\xi\omega_{t+1}/\lambda_t$ would internalize this effect.[11]

Lastly, the socially optimal allocation of scientists can be written:

$$\frac{\ell_{ct}}{1-\ell_{ct}} = \left(\frac{\frac{\eta_c A_{ct-1}}{A_{ct}} \sum_{k=0}^{\infty} \lambda_{c,t+k} Y_{c,t+k}}{\frac{\eta_d A_{dt-1}}{A_{dt}} \sum_{k=0}^{\infty} \lambda_{d,t+k} Y_{d,t+k}} \right)^{\frac{1}{1-\varpi}}, \tag{16}$$

where λ_{ct} and λ_{dt} are the shadow values of the clean and dirty intermediate goods, respectively. The term A_{jt-1}/A_{jt} can be substituted by using (10) and we obtain:

$$\frac{\ell_{ct}}{1-\ell_{ct}} = \left(\frac{\eta_c[1+\gamma\eta_c(\ell_{ct})^{\varpi}]^{-1} \sum_{k=0}^{\infty} \lambda_{c,t+k} Y_{c,t+k}}{\eta_d[1+\gamma\eta_d(1-\ell_{ct})^{\varpi}]^{-1} \sum_{k=0}^{\infty} \lambda_{d,t+k} Y_{d,t+k}} \right)^{\frac{1}{1-\varpi}}. \tag{17}$$

When $\eta_c = \eta_d$, we have the following lemma on the relationship between the allocation of scientists and the social value of clean and dirty inputs.

Lemma 1 *Along the socially optimal growth path, the social planner allocates more scientists to the innovation sector in which the net present value of the total future use of intermediate inputs is greater.*

Proof. See Appendix A.4. ■

3.2 The decentralized versus the social allocation of scientists

Now we will compare the decentralized market and the social allocations of scientists to innovation. Denote the social allocation to clean innovation ℓ_{ct}^S and the decentralized market allocation to clean innovation ℓ_{ct}^M.[12] In the following we assume the probabilities of a

[11] See Appendix A.3 for a derivation of the Pigovian tax rate.

[12] Note that there need not exist one unique decentralized market allocation of researchers. As van der Meijden and Smulders (2017) shows, current and future allocation of researchers could depend on expecta-

successful innovation to be equal across industries, i.e. $\eta_c = \eta_d = \eta$.

We need to get the expression for the decentralized allocation of researchers (9) on a form that is comparable to the socially optimal allocation of researchers (16). First, by inserting for x_{jit} from (3) into (4), and using both that $p_{jit} = \alpha^2$ and the expression for average machine quality (7), we get the following expression for the wage rate:

$$w_t = (1-\alpha)\alpha^{\frac{-\alpha}{1-\alpha}}(p_{jt} - \tau_{jt})^{\frac{1}{1-\alpha}} A_{jt}.$$

Inserting this wage rate into the demand for labor (5), we can rewrite the demand for labor as:

$$L_{jt} = \alpha^{\frac{\alpha}{1-\alpha}}(p_{jt} - \tau_{jt})^{\frac{-\alpha}{1-\alpha}} \frac{Y_{jt}}{A_{jt}}. \tag{18}$$

Then, finally, by inserting (18) into the decentralized allocation of researchers (9) we obtain:

$$\frac{\ell_{ct}^M}{1-\ell_{ct}^M} = \left(\frac{\frac{A_{ct-1}}{A_{ct}} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1-\eta {l_{c,t+v}^M}^{\omega}}{1+r_{t+v}} \right) p_{c,t+k} Y_{c,t+k} \frac{A_{ct}}{A_{c,t+k}}}{\frac{A_{dt-1}}{A_{dt}} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1-\eta {l_{d,t+v}^M}^{\omega}}{1+r_{t+v}} \right) (p_{d,t+k} - \tau_{d,t+k}) Y_{d,t+k} \frac{A_{dt}}{A_{d,t+k}}} \right)^{\frac{1}{1-\varpi}}. \tag{19}$$

There are three major differences in (19) from the social optimal allocation (16):

1. First, the shadow prices $\lambda_{j,t+k}$ are substituted by the discounted market prices $\prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) (p_{j,t+k} - \tau_{j,t+k})$. The negative external effect of using dirty intermediates may lead to a difference between these terms as already indicated above.

2. Second, the replacement probability $\prod_{v=1}^{k} \left(1 - \eta {l_{j,t+v}^M}^{\omega}\right)$ is not a part of (16). This term will reduce expected future profits. Furthermore, as can be seen directly from the expression, the reduction in profits is larger the more scientists $l_{j,t+v}^M$ there are working in a sector.

tions. That is, if all researchers believe that clean will replace dirty, it could be individually profitable to go into clean R&D already today without any public support.

3. Third, the term $\frac{A_{jt}}{A_{j,t+k}}$ inside the summation term in (19) is not present in (16). This term will get exceedingly smaller, the higher the growth in A_{jt}.[13]

Apart from the three points above, there is also a difference between the decentralized and the social allocation of scientists due to differences in current and future state variables A_{ct} and A_{dt}. Thus, to compare the allocations given by (16) and (19) we need comparable paths of the state variables. To this end, let there exist an optimal policy programme in which the planner commits to implementing the first best allocation in each period. In particular, the programme consists of three elements:

First, the optimal subsidy $s = 1 - \alpha$ for the use of machines is implemented (as assumed from before). Second, the planner sets the Pigovian-tax on the use of dirty input. This tax internalizes the environmental externality perfectly, and thus, together with the subsidy s, this must imply that $\lambda_{d,t+k} = \prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}}\right)(p_{d,t+k} - \tau_{d,t+k})$ for all periods.

Third, a subsidy to either clean or dirty innovation is implemented in each period so that the first best allocation of scientist is achieved. Under this policy programme, all market failures are corrected for and the social allocation is achieved in the decentralized market equilibrium, i.e. the left hand side of (16) and (19) are the same for all periods.

Then, we pose the following question: Given the optimal policy programme, what innovation sector must be subsidized in a given period in order to implement that period's efficient allocation of scientists? We attribute the difference between the social and decentralized allocation ratio, along the policy programme path, to two effects:

- **The replacement effect** listed as number two above. In the decentralized market allocation ratio the future replacement rates matter. The replacement rate is not taken into account in the social allocation ratio, and the replacement effect is a market failure. Thus, innovation in the sector with the larger replacement rate, *ceteris paribus*, is lower than optimal for the decentralized allocation.

[13] In the fraction $\frac{A_{jt}}{A_{j,t+k}}$, the numerator stays constant, while the denominator grows over time as long as researchers are allocated to sector j.

- **The productivity spillover effect** listed as number three above. Research in an input sector today benefits all future research in the sector through the standing-on-shoulder effects, i.e. every subsequent innovation involves a larger absolute step in product quality. This will increase the future use of the input, however, researchers today do not take this into account, and the private value of an innovation falls short of the social value. Thus, innovation in the sector with the largest growth rate in the knowledge stock is lower than optimal for the decentralized allocation.

Note that using equation (10) we can write the productivity growth rate in a sector as

$$\frac{A_{jt}}{A_{jt-1}} = (1 + \gamma\eta_j(\ell_{jt})^{\varpi}). \tag{20}$$

Thus, the sector with more scientists has the higher growth rate, as well as the largest replacement rate. We then have the following proposition:

Proposition 2 *Along the optimal policy programme path, if the current and future productivity growth rates are larger in one sector, then innovation should be subsidized in that sector.*

Proof. See Appendix A.5. ■

Innovation in the sector with largest growth should be subsidized since market failures due to both the replacement effect and the spillover effect are largest there. As hinted to at the end of Section 3.1, by putting some more restrictions on the problem, we can say more about which kind of R&D that should be subsidized. In their Proposition 6, AABH states that all innovation should switch to the clean input in finite time, that the optimal R&D subsidy which achieves this is temporary, and that the emission tax also is temporary if $\varepsilon > 1/(1-\alpha)$. This result does not carry completely over to our model because in our model there is no corner solution for the R&D sector. Clearly, if initially $A_d > A_c$ and $\ell_{dt} > \ell_{ct}$ in the market solution, clean R&D would at some time need to be subsidized since in the long run only growth in Y_{ct} can be allowed when $\lim_{S_t \to 0} u(C_t, S_t) = -\infty$. Moreover, the subsidy might have to be permanent to avoid too high growth in A_{dt} and consequently in Y_{dt}. The reason is that even with $\lim_{S_t \to 0} u(C_t, S_t) = -\infty$, we can have some production of dirty inputs at all times. As long as there is a tiny production of dirty inputs, some use of R&D effort in the sector will be profitable since the marginal productivity of R&D tends to infinity as the R&D effort tends to zero in our model.

As shown by AABH (see their online Appendix B, p. 4), the emission tax can be used to limit the use of Y_{dt}, but the emission tax may be temporary. The reason is that along the optimal path A_{dt} will stagnate, and A_{ct} will grow perpetually. Then, as long as $\varepsilon > 1/(1-\alpha)$[14], Y_{dt} will go towards zero, and the level of environmental quality will reach its maximum value in finite time after which there is no need for an emission tax. In our model, for $\varepsilon > 1/(1-\alpha)$ and A_{dt} constant, Y_{dt} will also go towards zero. However, A_{dt} will not be constant in the optimal solution of our model, and we cannot say whether the emission tax can be completely removed. We will return to these topics when analyzing the numerical results in Section 5.

It may be helpful to characterize the innovation subsidies in terms of net present values of the clean and the dirty inputs instead of by the growth rates of the technologies. The relationship between the value of the inputs and optimal subsidies to clean innovation is stated in the following corollary:

Corollary 3 *Along an optimal policy programme path, clean innovation should be subsidized if the net present value of the total future use of the clean input is higher than the net present value of the total future use of the dirty input.*

Proof. See Appendix A.6. ■

This result highlights the role the value of the environment and emission taxes play for optimal subsidies to innovation. In the event that emissions have a large impact on

[14] With $\alpha = 1/3$, this amount to $\varepsilon > 1.5$. If, on the other hand, $\varepsilon < 1.5$, Y_{dt} will grow for a fixed A_{dt}, and one will have to use an emission tax to shut off this growth.

environmental quality and this quality again is important for utility, the value of clean inputs will be large relative to the value of dirty inputs and it will be optimal to direct innovation more towards clean technologies. In this case the optimal growth rate of clean technology is higher than for dirty so the market failures are larger for clean innovation in a decentralized market. In contrast, if emission impacts are small and not so important for utility, it is optimal to direct innovation more towards dirty technologies to build on their productivity advantage. In this case the optimal growth rate is higher for dirty technologies and subsidies to innovations on dirty innovations are needed to implement the efficient allocation. Analysis of the value of the inputs, the relative growth rates of the technologies, and policies are done by numerical simulations in Section 5.

4 Patent infringement

As in the benchmark model, the quality difference is sufficiently large so that no one would buy the old machine if a new machine is available at the monopoly price. However, we now assume that the scope of patents is so broad that patent right holders of past innovations can block the commercialization of new innovations. Patent life is infinite, so the current producer of a machine type needs to hold the patent rights to past innovations in that machine type, or have licence agreements with holders of such patent rights. We assume that when a scientist makes an innovation, she buys the patent rights from the incumbent market leader at a price that exactly compensates for the loss of future profits. Thus, the current producer holds all patent rights in the relevant machine type, which again is sold to future innovators.

The expected discounted profits $\tilde{\Pi}_{jt}$ of a single scientist entering sector j at time t is then given by:

$$\tilde{\Pi}_{jt} = \eta_j (\ell_{jt})^{(\varpi-1)} \left[\bar{\alpha}(1+\gamma) A_{jt-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) \left((p_{j,t+k} - \tau_{j,t+k})^{\frac{1}{1-\alpha}} L_{j,t+k} \right) - P_{jt} \right],$$

where P_{jt} is the expected price a successful scientist needs to pay the incumbent for the patent rights. This price is the net present value of profits from producing machines with the average quality in sector j in the last period, i.e. A_{jt-1}, and the price can be written:

$$P_{jt} = \bar{\alpha} A_{jt-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1}{1 + r_{t+v}} \right) \left((p_{j,t+k} - \tau_{j,t+k})^{\frac{1}{1-\alpha}} L_{j,t+k} \right).$$

Thus expected profits of entering sector j can be written:

$$\tilde{\Pi}_{jt} = \eta_j (\ell_{jt})^{(\varpi-1)} \bar{\alpha} \gamma A_{jt-1} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1}{1 + r_{t+v}} \right) \left((p_{j,t+k} - \tau_{j,t+k})^{\frac{1}{1-\alpha}} L_{j,t+k} \right). \tag{21}$$

There are two differences between equation (21) and equation (8) from the benchmark model. First, the replacement rate z_{jt} does not enter into (21), as a scientist always gets the full net present value of selling its machine in the market. Thus the replacement effect is not present, and, compared to the benchmark model, the gains to innovate are increased in the sector with the higher productivity growth rate. Second, an entrant needs to pay out the incumbent, and this lowers the gains to innovate in both sectors.

Similarly to (19), we can write the decentralized allocation of researchers:

$$\frac{\ell_{ct}^{\tilde{M}}}{1 - \ell_{ct}^{\tilde{M}}} = \left(\frac{\frac{A_{ct-1}}{A_{ct}} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) p_{c,t+k} Y_{c,t+k} \frac{A_{ct}}{A_{c,t+k}}}{\frac{A_{dt-1}}{A_{dt}} \sum_{k=0}^{\infty} \prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) (p_{d,t+k} - \tau_{d,t+k}) Y_{d,t+k} \frac{A_{dt}}{A_{d,t+k}}} \right)^{\frac{1}{1-\varpi}}, \tag{22}$$

where $\ell_{ct}^{\tilde{M}}$ denotes the decentralized market allocation to clean innovation. Then, as in Section 3.2, we do a comparison of the decentralized allocation and the social optimal allocation along the optimal policy programme path. There are two major differences in (22) from the social optimal allocation (16):

1. First, the shadow prices $\lambda_{j,t+k}$ are substituted by the discounted market prices $\prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) (p_{j,t+k} - \tau_{j,t+k})$. The negative external effect of using dirty intermediates may lead to a difference between these terms as already discussed.

2. Second, the term $\frac{A_{jt}}{A_{j,t+k}}$ inside the summation term in (22) is not present in (16). This

term will get exceedingly smaller, the higher the growth in A_{jt}.

Point 2. refers to the *productivity spillover effect*, and it enters the expression exactly in the same way as in the market solution of the benchmark model, see (19). Then, as in the benchmark model, innovation in the sector with largest growth should be subsidized since the spillover effect is largest there.

However, comparing (22) with (19), we note the replacement effect is not present in (22). Since the replacement effect pulls in the same direction as the spillover, and is not present, the overall market failure is likely smaller with patent infringement than in the benchmark model without infringement. Consequently, we conjecture that less R&D support is needed to implement the social optimal allocation. This is confirmed in the numerical analysis, see Subsection 5.3.

5 Numerical analysis

In this section we present numerical analysis that builds on the analytical model above. The utility function and other details are specified in Appendix A.7. This includes the links between emissions (Y_{dt}), concentration in the atmosphere, temperature increase, and environmental quality (S_t). We assume a quasi-linear utility function with separable preferences between consumption and environmental quality. Furthermore, the utility function is linear in consumption, which implies that interest rates are constant over time. This reduces the complexity of the simulations and allows us to focus on how future carbon taxes influence innovation decisions today.[15]

When calibrating the model, we mostly follow AABH. In our benchmark case we assume a substitution elasticity of $\varepsilon = 3$. AABH also simulate $\varepsilon = 10$, which we find rather high. Instead we also examine the effects of a lower elasticity of substitution case with $\varepsilon = 1.5$.[16] Following AABH, we set machine share $\alpha = 1/3$, probability of a successful

[15] Since our model is numerically more complex to solve than the AABH model, we implement these simplifications. We also use a different specification of environmental utility than AABH, as the one in AABH implies relatively low damages for temperature levels close to the "disaster" level of 6 degrees.

[16] Most CGE models apply substitution elasticities around 1 or below when it comes to substitution of

innovation $\eta = 0.02$ (per annum) for both sectors, the quality step $\gamma = 1$, and the discount rate $\rho = 0.015$ (AABH also consider $\rho = 0.001$). The initial productivities A_{d0} and A_{c0} are calibrated so that clean inputs constitute 20% of total inputs, which is in line with the current share of non-fossil energy in worldwide energy use.

The value of ϖ is set to $\varpi = 0.7$. This implies that the initial share of scientists in the clean sector is 18% in our BaU scenario (with $\varepsilon = 3$). This is somewhat below the current share of clean energy R&D in global energy R&D, which is around 25-30%; however, the current R&D investments may reflect that investors expect a future policy development that lies between a BaU scenario and an optimal climate policy scenario.[17]

We simulate the model over 70 five-year periods, i.e., 350 years, but only displays the first 250 years (like in AABH). At the end of the time horizon, the temperature is falling in the policy scenarios as there is almost no use of dirty energy anymore (this is different with $\varepsilon = 1.5$, see below). Hence, extending the time horizon has negligible effects on the variables in the policy scenarios.[18]

5.1 Results: Benchmark case $\varepsilon = 3$

In the Business-as-Usual (BaU) scenario, most scientists move to the dirty sector, so that after 50 years only one percent remains in the clean sector. Production mostly consists of dirty inputs, and the temperature increase passes the assumed threshold level of six degrees after 110 years.

The optimal policy consists of a tax on dirty inputs and a subsidy to either clean or dirty innovation (note that the subsidy can only affect the distribution of scientists between sectors, as the total number of scientists is fixed). Figures 1A and 1B show the optimal

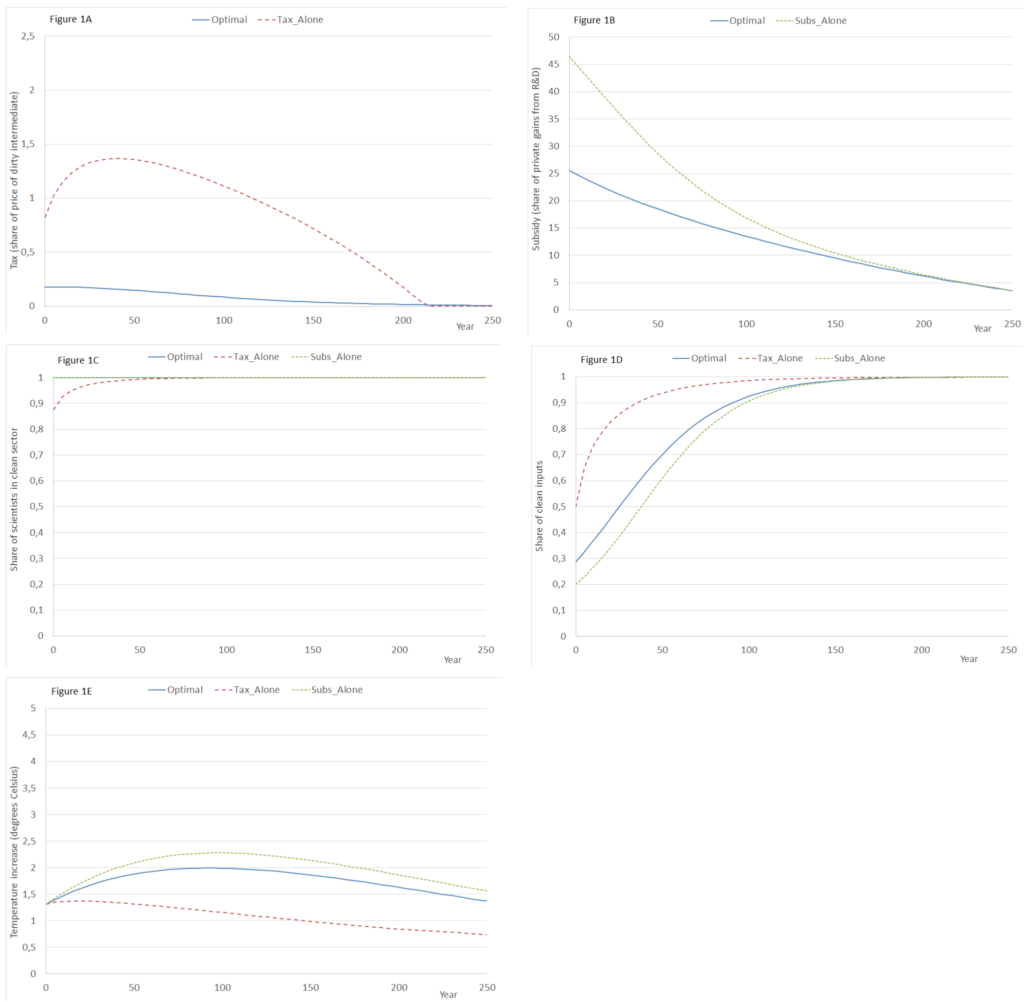
different energy goods at the sectoral level. For instance, Böhringer et al. (2014) apply elasticities in the range of 0.25 - 1. These elasticities may be interpreted as relevant for the intermediate term, whereas we are more interested in long-term elasticities.

[17] In the case with $\varepsilon = 1.5$ we recalibrate the value of A_{c0} but not ϖ. As a consequence, the initial share of clean input is not changed, but the initial share of scientists in the BaU scenario increases to 37% (since ϖ is held fixed). If we recalibrate ϖ to get the same initial share of scientists as with $\varepsilon = 3$, ϖ becomes approximately one, and the comparison of different elasticities would be difficult to interpret.

[18] The optimal subsidy level, though, depends quite a lot on the time horizon.

combination of tax and subsidies. The figures also show the optimal tax in the case without any subsidy, and the optimal subsidy in the case without any tax. The subsidy is expressed as a share of the expected discounted profits (excluding the subsidy) for scientists in the clean sector (Π_{ct}), whereas the tax is expressed as a share of the price of dirty intermediates (p_{dt}).

Figure 1: First- and second-best environmental policies ($\varepsilon = 3$)



First, we notice from Figure 1A that the tax starts at a fairly moderate level, and then gradually declines over time in the first-best solution, which reflects a combination of a low discount rate and that environmental quality starts to improve again after 100 years, when the temperature increase peaks at two degrees Celsius (cf. Figure 1E).

Second, Figure 1B shows that it is optimal to subsidize clean research quite heavily. Initially, the optimal subsidy is in fact 25 times higher than the private returns from clean research in this scenario. The subsidy gradually declines over time relative to the private returns from clean research (as shown in the figure), but increases over time when measured per unit research effort. The main finding in this figure, i.e., that clean research should be subsidized in order to direct technical change towards the clean sector, supports the findings in AABH.[19] However, note that there is a distinct difference in the scientist's incentives in the two models. AABH assume that the scientist can only benefit from its innovation in the first 5-year period, whereas we assume long-lived patents where scientists *do* take into account future changes in the value of clean innovations due to climate policies. Rather, the reason for the subsidy in our model is the following (cf. Section 3): As practically all scientists move to the clean sector immediately (see Figure 1C), the risk of replacement is biggest in this sector. Moreover, as most scientists are in the clean sector, the productivity growth rate is highest in this sector and, thus, the spillover effects related to standing on shoulder are also highest. In the sensitivity analysis in Section 5.3, we examine the effects on the optimal policy of removing the replacement effect (cf. Section 4 above). Then we are able to see how important the two externalities are for the optimal subsidy level.

If taxes for some reason are not used, the second-best subsidy increases notably, especially in the beginning (see Figure 1B). Without a future tax on dirty inputs, innovators are less incentivized to do clean R&D, and hence need a higher subsidy to enter the clean research sector. From Figure 1D we notice that the share of clean inputs is lower in this scenario than in the first-best case, as there is no tax to stimulate the use of such inputs. However, the productivity growth of clean inputs is slightly higher than in the first-best scenario, and gradually it becomes profitable to switch from dirty to clean inputs. Nevertheless, the temperature increase is somewhat higher in the second-best scenario with only subsidy

[19] The corresponding subsidy path in AABH is initially zero, before it jumps suddenly after 50 years and then declines towards zero again after 100 years. This pattern is driven by the fact that AABH assume constant returns to inputs from scientists, leading to corner solutions in the innovation sector (either only dirty or only clean innovation within a period).

compared to the first-best case, cf Figure 1E.

If instead subsidies are not used, the second-best tax increases dramatically (see Figure 1A). The explanation is that particularly high taxes are needed to move scientists to the clean R&D sector. Nevertheless, the share of scientists in the clean sector is below the corresponding share in the optimal scenario, cf. Figure 1C. Note, however, that the tax scenario is likely to be time inconsistent, as the future tax rates are imposed mainly to stimulate early innovation into clean inputs. Hence, when future periods arrive, the regulator would like to reduce the tax level (see for example Golombek et al, 2010). We notice that the share of clean inputs is much higher than the optimal share, and that the temperature increase is significantly smaller than in the first-best case.

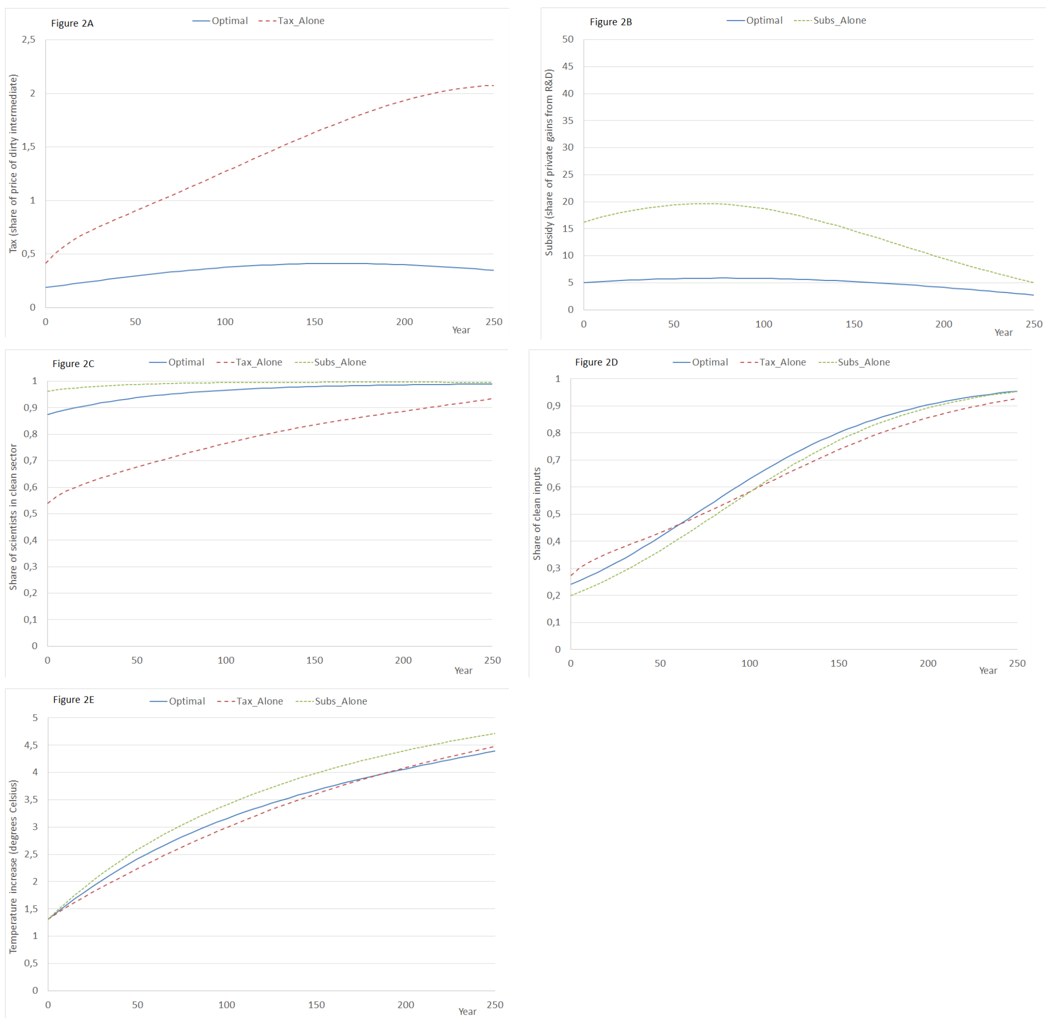
Since we assume that utility is linear in consumption, our utility function implicitly assigns a monetary value to different levels of environmental quality. We can then compare the utility of the three policy scenarios directly measured in consumption equivalents. First, we find that the number of consumption equivalents is reduced by merely 0.4% in the subsidy-alone scenario compared to the optimal policy scenario, whereas the number of consumption equivalents is reduced by 5.4% in the tax-alone scenario. Second, as the latter scenario may be time inconsistent as well, our results suggest that the subsidy to clean R&D is even more important than the tax on dirty inputs in this case. This is due to the relatively high substitution elasticity between clean and dirty inputs, which implies that once clean technologies become sufficiently developed, they can take over most of the market without depending on a tax on dirty inputs.

5.2 Results: Lower elasticity of substitution case $\varepsilon = 1.5$

With $\varepsilon = 1.5$, clean and dirty technologies are less substitutable than in the benchmark case with $\varepsilon = 3$. The optimal tax on dirty inputs now increases over time (see Figure 2A). The reason is that even if clean inputs eventually become cheaper than dirty inputs, consumers will prefer to use a combination of inputs. Hence, a higher tax level is needed to keep emissions down.[20]

As seen in Figure 2B, the optimal subsidy level is much lower than in the case with $\varepsilon = 3$. This is partly because the higher tax makes innovation into clean R&D more profitable. Hence, a lower subsidy is needed to direct innovation into the clean sector.[21]

Figure 2 : First- and second-best environmental policies ($\varepsilon = 1.5$)



An additional reason for the lower optimal subsidy level in the case with $\varepsilon = 1.5$ is that it

[20] In the figure the optimal tax, relative to the price on dirty inputs, starts declining after 150-200 years. However, the absolute level of the tax increases throughout our time horizon.

From Figure 2E we see that the temperature is increasing steadily, and would probably pass the disaster level of 6 degrees after another 100 years if the tax continued to decline after the end of our simulated time horizon.

[21] If we rather increase the substitution elasticity to e.g. $\varepsilon = 5$ we get the opposite result for both the tax and the subsidy.

is more valuable from a welfare perspective to use a combination of inputs instead of relying mostly on either clean or dirty inputs when the substitutability is lower. This is clearly seen when comparing Figures 1D and 2D, which show the share of clean inputs in aggregate production. Thus, a social planner would prefer that both clean and dirty inputs become cheaper to use. Hence, the share of scientists in dirty innovation is much higher than with $\varepsilon = 3$, see Figures 1C and 2C.

A consequence of using more dirty inputs is higher emissions and increased temperature. Figure 2E shows that the temperature rises steadily throughout the time horizon. [22]

If taxes are not used, the second-best subsidy increases significantly compared to the first-best policy. When the goods are less substitutable, the subsidy has to stimulate clean R&D quite heavily so that clean inputs become so cheap that consumers eventually buy only small amounts of dirty inputs. Although the subsidy relative to the private returns from research peaks after some decades, the subsidy per unit research effort increases throughout the time horizon.

If subsidies are not used, the second-best tax increases significantly, but less than with $\varepsilon = 3$ (in relative terms) as the cost difference between clean and dirty inputs is less important when the substitution elasticity is lower.

The loss in consumption equivalents of second-best policies are now 3% if only subsidies are used, and 10% if only taxes are used. Thus, even in this case subsidies seem to be more important. However, the costs of relying only on subsidies are more expensive than with $\varepsilon = 3$, as the market is less interested in switching very much towards the clean technology even if its price become cheaper than the dirty technology.

[22] As pointed to in Section 3.2, there is a notable difference in the optimal solution depending on whether ε is above or below 1.5 (given $\alpha = 1/3$). In AABH's model, the use of dirty inputs will eventually drop towards zero by itself if $\varepsilon > 1.5$, i.e., the optimal emission tax is temporary. With $\varepsilon < 1.5$, there will always be an incentive to increase the use of dirty inputs. In our model, things are less clear-cut, but also in our model $\varepsilon = 1.5$ can be seen as a borderline. As the temperature does not peak in this case, the optimal solution depends to some degree on the time horizon of the simulations.

5.3 Results: Sensitivity analysis

The numerical model uses a number of uncertain parameters, which to a large degree are taken from AABH. Above we have considered the importance of the substitution elasticity. Here we want to look into some of the other important parameters. We also examine the patent infringement issue analyzed in Section 4. Last but not least, we consider the effects of potential spillover effects between clean and dirty technologies. We focus on the optimal solutions, and show the optimal taxes and subsidies in the various cases in Figures 3A and 3B. The analysis in this section assumes $\varepsilon = 3$, and the benchmark case shown in the figures refer to the results in Section 5.1. Table 1 displays the sensitivity cases we consider.

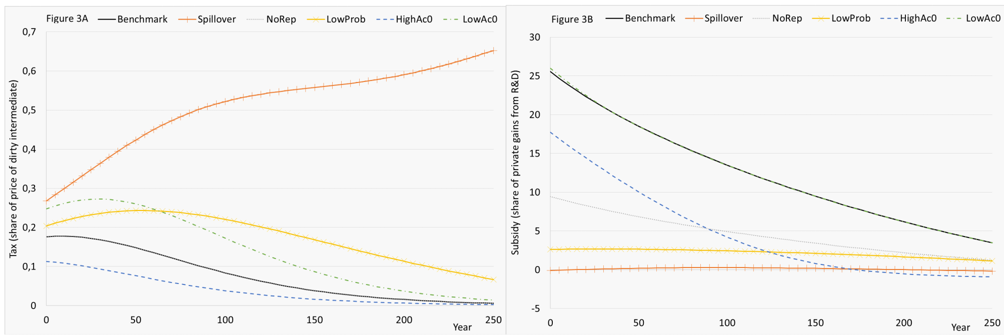
Table 1. Sensitivity analysis

LowAc0	The initial technology level in the clean sector (A_{c0}) is halved
HighAc0	The initial technology level in the clean sector (A_{c0}) is doubled
LowProb	The probability of successful innovation (η_j) halved (both sectors)
NoRep	No replacement effect for innovators
Spillover	Limited spillover effects between clean and dirty technologies

The initial technology level for the clean sector (relative to the dirty sector) is calibrated but still uncertain. Thus, we first consider the effects of either halving or doubling this initial level of clean technology. In both these cases, the optimal subsidy is substantial. With a higher initial technology level, the subsidy is somewhat reduced as less public support is needed to switch from dirty to clean research (private returns from clean research are higher). On the other hand, with a lower initial technology level, the optimal subsidy is only marginally increased. Furthermore, Figure 3A shows that the optimal tax decreases (increases) if the initial clean technology level is increased (decreased), as the relative price of clean inputs compared to dirty inputs decreases (increases) and makes the switch to clean inputs more (less) profitable (before the tax is imposed).

The probability of successful innovations is uncertain, and here we examine the effects of halving this probability (from 0.02 per year). The implication of this is to reduce the optimal subsidy quite a lot, the reason simply being that innovations are less effective than with the benchmark assumption. Instead the regulator will have to rely more on the emission tax, which is substantially increased in this case.

Figure 3 : First-best environmental policies in sensitivity analysis[23]



As discussed in Section 4, it can be questioned to what degree existing innovators lose out when new innovations arrive. If there is no replacement effect for the innovators, the expected profit of an innovation changes, and hence the allocation of researchers also change, see equations (21) and (22). Here we consider the effects on the optimal policy if there is no such replacement effect. As seen in Figure 3B, the optimal subsidy is then more than halved. Thus, the replacement effect is an important argument for subsidizing clean research according to our simulations. However, the optimal subsidy is still substantial without the replacement effect, implying that also the spillover effect to future research is an important argument for the clean research subsidy. As the replacement effect is only affecting the distribution of profit, and has no real effect on e.g. research productivity, the optimal scenario becomes identical to the benchmark case in all other respects than the subsidy level. Thus, the emission tax is the same as before (see Figure 3A).

Finally, we consider the implications of spillover effects between clean and dirty technologies. We model this by changing equation (10) to:

[23] Note that the curves Benchmark and NoRep overlap in Figure 3A, while the curves Benchmark and LowAc0 overlap in Figure 3B.

$$A_{jt} = A_{jt-1} + (1-\vartheta)\gamma\eta(\ell_{jt})^{\varpi} A_{jt-1} + \vartheta\gamma\eta(\ell_{(-j)t})^{\varpi} A_{(-j)t-1}, \tag{23}$$

where $(-j)$ denotes the other sector and ϑ denotes the spillover rate between clean and dirty. We consider the case where $\vartheta = 1/4$, which means that internal spillovers (within a technology) are still more important than external spillovers. In this case, the initial contributions from the clean and dirty sectors to the growth in the clean technology (i.e., the second and third terms in (23)) are almost identical in the optimal solution. As shown in Figure 3B, the optimal subsidy in this case is close to zero throughout the time horizon. The reason is that directing research into one particular sector is less important when there are spillovers across the sectors. Moreover, as we assume decreasing returns to scale in each of the two research sectors, this tends to favour a balanced share of researchers in the clean and dirty sectors. However, another implication of these spillovers is that the dirty technology level also grows substantially over time, and hence the incentives to use dirty inputs are strong throughout our time horizon. The optimal emission tax is therefore much higher in this scenario than in the benchmark case, especially at later periods. Nevertheless, the temperature does not peak in this scenario. Furthermore, whereas the tax-alone scenario is only marginally more costly than the optimal scenario, the subsidy-alone scenario is not able to avoid the environmental disaster as it is impossible to avoid too much use of dirty inputs when the dirty technology is deemed to improve significantly no matter how few researchers are allocated to the dirty sector. This scenario highlights that the optimal combination of emission taxes and subsidies to clean technology research depends crucially on to what degree there are learning spillovers between the two types of technology.

6 Conclusion

We have studied to what extent governments should actively direct research effort away from dirty technologies into clean technologies. The novelty in our analysis is how we model the innovation sector: We allow innovation profits to survive longer than one period and introduce decreasing returns to R&D at any point in time. In addition, we look at two ways of modelling patents: with and without patent infringement.

At first glance long lived patents and decreasing returns to R&D should make targeted R&D support less crucial. That is, innovations that not only give instantaneous profits implies that future environmental policies can redirect research today, and decreasing returns force R&D to take place in both sectors independent of the level of accumulated productivity. Surprisingly, we find that governments should nonetheless support clean R&D and not dirty R&D. Dealing with a major environmental problem effectively requires R&D effort to shift to clean technologies. However, when most researchers work with clean technology, both productivity spillovers and the future risk of being replaced increase. Consequently, the wedge between the private and the social value of an innovation is larger for clean technologies than for dirty technologies along the transition path. This also holds with patent infringement even though the innovator in this case experiences no economic loss if being replaced by another innovator with a newer patent.

We have also analyzed to what degree a clean research subsidy can replace a carbon tax. We then find that an R&D subsidy-alone policy outperforms a carbon tax-alone policy. At least that is the case with a relatively high elasticity of substitution between clean and dirty inputs ($\varepsilon = 3$). This suggests that the subsidy to clean R&D is even more important than the tax on dirty inputs if the two inputs substitute quite well. If the elasticity of substitution between clean and dirty input is more moderate, the case is less clear, however, a clean research subsidy can still replace fully an emission tax.

Given that implementing a sufficiently high global price on carbon has proven very difficult, concerted global action on support to clean R&D may thus be worth aiming for in international negotiations. It is however a topic in itself how such R&D cooperation should be organized. It is not trivial to subsidize R&D, that is, some international body working for many governments must pick and reward projects on a grand scale. This aspect of R&D policy is clearly downplayed in our analysis.

There are several more aspects of our model that could be discussed and that will likely affect the desirability of R&D subsidies for the clean sector. First, there is a fixed number of scientists in the R&D sector. This assumption is not so important for the qualitative results, as the purpose of our paper is to analyze policies related to the relative allocation of scientists between two classes of technology (although it simplifies solving the model). However, if there were more technology classes in the economy – for instance a general technology in addition to a clean and a dirty energy technology – the no free entry assumption might be less innocent, as subsidies to clean technologies then would also crowd out innovation in the general technology. This is something we plan to study in a future project. Second, there are no spillovers between the two classes of technologies in our theoretical analysis and most numerical simulations. As the sensitivity analysis showed, assuming some spillovers across the two sectors may significantly diminish the necessity of directing R&D to clean technologies today, as it may be better to develop the more productive technology before making the switch to clean. This is also a venue for future research.

References

Acemoglu, D. 2002. Directed Technical Change. *Review of Economic Studies*, 69, 781-809.

Acemoglu, D. 2009. *Introduction to Modern Economic Growth.* Princeton University Press.

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hémous. 2012. The Environment and Directed Technical Change. *American Economic Review* 102, 131-166.

Acemoglu, D., Akcigit, U., Hanley, D., & Kerr, W. 2016. Transition to clean technology, *Journal of Political Economy*, 124(1), 52-104.

Aghion, P., Dechezleprêtre, A., Hémous, D., Martin, R., & Van Reenen, J. 2016. Carbon Taxes, Path Dependency and Directed Technical Change: Evidence from the Auto Industry. *Journal of Political Economy* 124(1), 1-51.

Aghion, P. and P. Howitt. 1992. A model of economic growth through creative destruction, *Econometrica* 60, 323-351.

André F. J. and S. Smulders. 2014. Fueling growth when oil peaks: Directed technological change and the limits to efficiency. *European Economic Review* 69, 18-39.

Böhringer, C., K.E. Rosendahl and J. Schneider. 2014. Unilateral Climate Policy: Can OPEC Resolve the Leakage Problem? *The Energy Journal* 35 (4), 79-100.

Dechezleprêtre, Antoine, Ralf Martin, and Myra Mohnen 2013. Knowledge spillovers from clean and dirty technologies: A patent citation analysis. Grantham Research Institute on Climate Change and the Environment Working Paper 135.

Durmaz, T. and F. Schroyen 2014. Evaluating Carbon Capture and Storage in a Climate Model with Induced Technical Change. unpublished wp

EU Industrial R&D scoreboard 2014. see http://iri.jrc.ec.europa.eu/scoreboard14.html

Fischer, C. and R.G. Newell. 2008. Environmental and Technology Policies for Climate Mitigation. *Journal of Environmental Economics and Management* 55, 142-162.

Gerlagh, R., S. Kverndokk and K.E. Rosendahl 2014. The optimal time path of clean energy R&D policy when patents have finite lifetime, *Journal of Environmental Economics and Management* 67, 2-19.

Gillingham, K., R.G. Newell, and W.A.Pizer. 2008. Modeling Endogenous technological Change for Climate Policy Analysis, *Energy Economics* 30, 2734-2753.

Golombek R., M. Greaker and M. Hoel. 2010. Carbon Taxes and Innovation without Commitment, *The B.E. Journal of Economic Analysis and Policy* 10-1 (Topics), Article 32.

Goulder, L.H. and S.H. Schneider. 1999. Induced technological change and the attractiveness of CO2 abatement policies. *Resource and Energy Economics* 21, 211–253.

Greaker M. and L. Pade. 2009. Optimal CO2 abatement and technological change: Should emission taxes start high in order to spur R&D? *Climatic Change* 96.

Grossman, G.M. and E. Helpman. 1991. Innovation and Growth in the Global Economy. Cambridge: MIT Press.

Heggedal, T. R. 2015. Knowledge spillovers and R&D subsidies to new, emerging technologies, *Economics of Innovation and New Technology*, 24(7), 710-733.

Hémous, D. 2013. Environmental policy and directed technical change in a global economy: the dynamic impact of unilateral environmental policies, *CEPR Discussion Paper* No.DP9733

Heutel, G., & Fischer, C. 2013. Environmental macroeconomics: Environmental policy, business cycles, and directed technical change. *Annu. Rev. Resour. Econ.* 5(1), 197-210.

Hourcade, J.C, A. Pottier, and E. Espagne. 2011. The Environment and Directed Technical Change: Comment. *FEEM Working paper* 95/2011.

Hwang, I.C., F. Reynès and R.S.J. Tol. 2013. Climate Policy Under Fat-Tailed Risk: An Application of Dice, *Environmental and Resource Economics* 56, 415-436.

IEA 2014 . World energy outlook 2014.

IPCC, 2013. Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Chapter 6 [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A.

Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

Jones, C.I. and Williams, J.C., 2000. Too Much of a Good Thing? The Economics of Investment in R&D, *Journal of Economic Growth* 5, 65–85.

Mattauch, L., F. Creutzig, and O. Edenhofer 2015. Avoiding carbon lock-in: policy options for advancing structural change. *Economic Modelling*, 50, 49-63.

McGlade, C., & Ekins, P. 2015. The geographical distribution of fossil fuels unused when limiting global warming to 2 C. *Nature*, 517(7533), 187-190.

Popp, D., R.G. Newell and A.B. Jaffe. 2010. Chapter 21 – Energy, the Environment, and Technological Change. In *Handbook of the Economics of Innovation* (eds B.H. Hall & N. Rosenberg).

Romer, Paul M. 1990. Endogenous Technological Change, *Journal of Political Economy* 98(5), S71-S102.

Rosendahl, K.E. 2004. Cost-effective environmental policy: implications of induced technological change. *Journal of Environmental Economics and Management* 48, 1099–1121.

van den Bijgaart, I. 2015. The Unilateral Implementation of a Sustainable Growth Path with Directed Technical Change. *FEEM Working Paper* No. 11.2015 .

van der Meijden G. and S. Smulders. 2017. Carbon Lock-In: The Role of Expectations. *International Economic Review*, forthcoming.

Appendix

A.1 The Euler equation

Let a_t be a (representative) household's asset value. The household's problem is then

$$\begin{aligned} & \max_{C} \sum_{t=0}^{\infty} \tfrac{1}{(1+\rho)^t} u(C_t, S_t), \\ s.t \qquad & a_{t+1} = (1+r_t)a_t + w_t + \pi_{dt} + \pi_{ct} + \tau_{dt} Y_{dt} - C_t, \end{aligned} \tag{24}$$

where the only firm profits are from selling the machines since profits will be zero in the final goods and intermediate sectors. Note that labor income is w_t as wages will be the same in the two intermediate sectors, and $L_{dt} + L_{ct} = 1$. Further, note that since there is no saving asset, all resources are spent in every period, i.e. the interest rate will be such that $C_t = w_t + \pi_{dt} + \pi_{ct} + \tau_{dt} Y_{dt}$. It follows that the Euler equation can be written

$$\frac{\partial u(C_t, S_t)}{\partial C} = \frac{(1 + r_{t+1})}{1+\rho} \frac{\partial u(C_{t+1}, S_{t+1})}{\partial C}.$$

A.2 Solving the Planner problem

The Lagrangian from the problem given by 13 is:

$$\begin{aligned}
\mathbf{L} = & \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u\left((Y_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right), S_t \right) \\
& - \sum_{t=0}^{\infty} \lambda_t (Y_t - \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}) \\
& - \sum_{t=0}^{\infty} \lambda_{ct} (Y_{ct} - L_{ct}^{1-\alpha} \textstyle\int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di) \\
& - \sum_{t=0}^{\infty} \lambda_{dt} (Y_{dt} - L_{dt}^{1-\alpha} \textstyle\int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di) \\
& - \sum_{t=0}^{\infty} \mu_{ct} (A_{ct} - (1 + \gamma \eta_c (\ell_{ct})^{\varpi}) A_{ct-1}) \\
& - \sum_{t=0}^{\infty} \mu_{dt} (A_{dt} - (1 + \gamma \eta_d (1 - \ell_{ct})^{\varpi}) A_{dt-1}) \\
& - \sum_{t=0}^{\infty} \omega_t (S_t + \xi Y_{dt-1} - (1-\delta) S_{t-1} - \delta \bar{S}),
\end{aligned}$$

where λ_t is the shadow value of final goods, λ_{jt} is the shadow value of intermediate goods in $j \in \{c, d\}$, μ_{jt} is the shadow value of the average machine quality (the technology stock) in $j \in \{c, d\}$, and ω_t is the shadow value of the environmental quality. Note that we have substituted in $C_t = Y_t - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right)$ and that we have set $L_{ct} + L_{dt} = 1$ and $\ell_{ct} + \ell_{dt} = 1$ for simplicity.

The FOC wrt C_t is

$$\frac{1}{(1+\rho)^t}\frac{\partial u(C_t,S_t)}{\partial C}-\lambda_t=0. \tag{25}$$

The FOC wrt S_t is

$$\frac{1}{(1+\rho)^t}\frac{\partial u(C_t,S_t)}{\partial S}-\omega_t+\omega_{t+1}(1-\delta)I_{S_t<\bar{S}}=0, \tag{26}$$

where $I_{S_t<\bar{S}}=1$ if $S_t<\bar{S}$, and $I_{S_t<\bar{S}}=0$ otherwise, since $S_t=-\xi Y_{dt-1}+(1-\delta)S_{t-1}+\delta\bar{S}$ only in the interval $(0,\bar{S})$. Equation (26) can be solved recursively to get

$$\omega_t=\textstyle\sum_{v\geq t}(1-\delta)^{v-t}\frac{1}{(1+\rho)^v}\frac{\partial u(C_v,S_v)}{\partial S}I_{S_t,\ldots,S_v<\bar{S}}.$$

The FOCs wrt Y_{ct} and Y_{dt}

$$\begin{aligned}\lambda_t\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}Y_{ct}^{\frac{-1}{\varepsilon}}-\lambda_{ct}=0\\ \lambda_t\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}Y_{dt}^{\frac{-1}{\varepsilon}}-\lambda_{dt}-\xi\omega_{t+1}=0.\end{aligned} \tag{27}$$

These are discussed in the main text. Further, the FOC wrt machines x_{jit} is

$$-\frac{1}{(1+\rho)^t}\frac{\partial u(C_t,S_t)}{\partial C}\psi+\lambda_{jt}L_{jt}^{1-\alpha}A_{jit}^{1-\alpha}\alpha x_{jit}^{\alpha-1}=0,$$

which using equation (25) and that the cost of a machine is given by $\psi=\alpha^2$, can be written

$$x_{jit}=\left(\frac{1}{\alpha}\frac{\lambda_{jt}}{\lambda_t}\right)^{\frac{1}{1-\alpha}}A_{jit}L_{jt},$$

where we have used the cost $\psi=\alpha^2$. The market solution yields the following use of machines:

$$x_{jit}=\left(\frac{(p_{jt}-\tau_{jt})}{(1-s)}\right)^{\frac{1}{1-\alpha}}A_{jit}L_{jt}.$$

With a Pigovian tax equal to $\xi\omega_{t+1}/\lambda_t$, we have $p_{jt} - \tau_{jt} = \lambda_{jt}/\lambda_t$ (see below). Thus, with a subsidy $s = 1 - \alpha$, we obtain the optimal production of machines.

The relevant FOC for the allocation of scientist is

$$\mu_{ct}\varpi\gamma\eta_c\ell_{ct}^{\varpi-1}A_{ct-1} - \mu_{dt}\varpi\gamma\eta_d(1-\ell_{ct})^{\varpi-1}A_{dt-1} = 0 \tag{28}$$

$$\Leftrightarrow$$

$$\frac{\ell_{ct}}{1-\ell_{ct}} = \left(\frac{\mu_{ct}\eta_c A_{ct-1}}{\mu_{dt}\eta_d A_{dt-1}}\right)^{\frac{1}{1-\varpi}}.$$

In order to get (16) we need to substitute for μ_{jt}. First, we use the FOC wrt the average quality A_{jt} which is given by

$$\lambda_{jt}L_{jt}^{1-\alpha}(1-\alpha)\int_0^1 A_{jit}^{-\alpha}x_{jit}^{\alpha}di - \mu_{jt} + \mu_{jt+1}(1+\gamma\eta_j\ell_{jt+1}^{\varpi}) = 0 \tag{29}$$

Next we use $A_{jt} \equiv \int_0^1 A_{jit}di$ and the definition of Y_{jt} to rewrite equation (29)

$$\lambda_{jt}(1-\alpha)\frac{Y_{jt}}{A_{jt}} - \mu_{jt} + \mu_{jt+1}(1+\gamma\eta_j\ell_{jt+1}^{\varpi}) = 0. \tag{30}$$

Then we use equation (10) to rewrite equation (30):

$$\mu_{jt} = \lambda_{jt}(1-\alpha)\frac{Y_{jt}}{A_{jt}} + \mu_{jt+1}\frac{A_{t+1}}{A_t}. \tag{31}$$

Notice that equation (31) can be written as a sum of the form $\mu_{jt} = \lambda_{jt}(1-\alpha)\frac{Y_{jt}}{A_{jt}} + \lambda_{jt}(1-\alpha)\frac{Y_{jt+1}}{A_{jt+2}}\frac{A_{jt+1}}{A_{jt}} + \lambda_{jt}(1-\alpha)\frac{Y_{jt+2}}{A_{jt+2}}\frac{A_{jt+1}}{A_{jt}}\frac{A_{jt+2}}{A_{jt+1}} + ...$ We use this to obtain

$$\mu_{jt} = (1-\alpha)\frac{1}{A_t}\sum_{v\geq t}\lambda_{jv}Y_{jv}. \tag{32}$$

Last, combining equations (28) and (32) gives the following expression for the optimal allocation of scientist

$$\frac{\ell_{ct}}{1-\ell_{ct}} = \left(\frac{\frac{\eta_c A_{ct-1}}{A_{ct}} \sum_{v \geq t} \lambda_{cv} Y_{cv}}{\frac{\eta_d A_{dt-1}}{A_{dt}} \sum_{v \geq t} \lambda_{dv} Y_{dv}} \right)^{\frac{1}{1-\varpi}}. \tag{33}$$

which we use in Subsection 3.1.

A.3 The Pigovian tax rate

The Pigovian tax rate is the tax rate that ensures that we get the optimal use of the clean and dirty intermediates. In the decentralized market solution the uses of the two intermediates by the final goods sector are given by:

$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} Y_{dt}^{\frac{-1}{\varepsilon}} - p_{dt} = 0$$
$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} Y_{ct}^{\frac{-1}{\varepsilon}} - p_{ct} = 0$$

In order to obtain the optimal use of the two inputs, we see from (27) that we must have:

$$p_{ct} = \frac{\lambda_{ct}}{\lambda_t} \tag{34}$$
$$p_{dt} = \frac{\lambda_{dt}}{\lambda_t} + \frac{\omega_{t+1}\xi}{\lambda_t}$$

In a *laissez fair* market equilibrium prices on the intermediates will adjust such that $p_{jt} = \lambda_{jt}/\lambda_t$. Hence, the Pigovian tax rate τ_{dt}^* must be equal to $\xi\omega_{t+1}/\lambda_t$. Along an optimal growth path in which the Pigovian taxes and the subsidy to machines are both implemented, we claim in the text that:

$$\lambda_{d,t+k} = \prod_{v=1}^{k} \left(\frac{1}{1+r_{t+v}} \right) (p_{d,t+k} - \tau_{d,t+k}) \tag{35}$$

We have that $\lambda_t = \prod_{v=0}^{t} \left(\frac{1}{1+r_v} \right)$, and we see that (34) and (35) are equivalent.

A.4 Proof of Lemma 1

We have from (15) that $\lambda_{ct} = \lambda_t \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} Y_{ct}^{\frac{-1}{\varepsilon}}$ and $\lambda_{dt} = \lambda_t \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} Y_{dt}^{\frac{-1}{\varepsilon}} - \xi\omega_{t+1}$. Thus, λ_j is the marginal social value of input j, and $\sum_k \lambda_{j,t+k} Y_{j,t+k}$ is the net present value of the total use of intermediate input j from period t. If $\eta_c = \eta_d$, then (17) implies $\ell_{ct} > 1/2$ if $\sum_k \lambda_{c,t+k} Y_{c,t+k} > \sum_k \lambda_{d,t+k} Y_{d,t+k}$.

A.5 Proof of Proposition 2

The summation in (19) has two terms not appearing in the summation in (16): $\prod_{v=1}^{k} \left(1 - \eta {l_{j,t+v}^{M}}^{\omega}\right)$ and $\sum_k A_{jt}/A_{j,t+k}$. If $l_{c,t+v}^M > l_{d,t+v}^M$ along the optimal policy programme path, A_c will grow faster than A_d. Thus, with $l_{c,t+v}^M > l_{d,t+v}^M$, we have both $\prod_{v=1}^{k} \left(1 - \eta {l_{c,t+v}^{M}}^{\omega}\right) < \prod_{v=1}^{k} \left(1 - \eta {l_{d,t+v}^{M}}^{\omega}\right)$ and $\sum_k A_{ct}/A_{c,t+k} < \sum_k A_{dt}/A_{d,t+k}$. Compared to the social allocation of scientists, the fraction $l_{c,t}^M / l_{d,t}^M$ in any period would then be too small without subsidies.

A.6 Proof of Corollary 3

First, Lemma 1 states that $\ell_{ct}^S > \ell_{dt}^S$ iff $\sum_{k=0} \lambda_{c,t+k} Y_{c,t+k} > \sum_{k=0} \lambda_{d,t+k} Y_{d,t+k}$. Next, using (20), it follows that $\frac{A_{ct}}{A_{ct-1}} > \frac{A_{dt}}{A_{dt-1}}$ iff $\ell_{ct} > \ell_{dt}$. Then, combining this with Proposition 2, the result follows.

A.7 Specification of the numerical model

In this appendix we present how the utility function and the environmental quality function are specified in the numerical model. The rest of the model is specified before.

The instantaneous utility function is given by

$$u(C_t, S_t) = C_t + \phi(S_t), \tag{36}$$

where $\phi(S)$ is the valuation of the environmental quality. The linearity of utility with respect to consumption implies that the interest rate is exogenous and constant over time, cf. Section 2.5.

The general function $\phi(S)$ can in the context of climate change be expressed as $\phi(\Delta)$, where Δ denotes the temperature increase relative to the pre-industrial level ($\phi'(\Delta) < 0$). In order to specify the function $\phi(\Delta)$, we first follow AABH and assume the following relationship between temperature increase and CO_2-concentration in the atmosphere measured in parts per million (ppm) (C_{CO2}):

$$\Delta = 3\log_2(C_{CO2}/280), \tag{37}$$

where 280 ppm is the pre-industrial level of CO_2-concentration. Further, $C_{CO2,disaster}$ denotes the concentration level associated with the disaster temperature increase, which AABH sets to $\Delta_{disaster} = 6$ degrees.

We assume a constant depreciation rate (δ) of the CO_2-concentration in the atmosphere (above the pre-industrial level):

$$C_{CO2,t+1} - C_{CO2,t} = \xi Y_{dt} - \delta\left(C_{CO2,t} - 280\right),$$

where we assume $\delta = 0.005$ (per year).[24] The parameter ξ is calibrated so that annual concentration level increases by 2 ppm initially in the BaU-scenario.[25]

As explained in the main text, the environmental damage costs in AABH are quite low as long as the temperature is not too close to the disaster level of 6 degrees. For this reason, and the fact that we use a separable utility function, our specification of the $\phi(\Delta)$ function is different from theirs:

$$\phi(\Delta) = -\frac{\lambda\left(2^{\Delta/3} - 1\right)^2}{\Delta_{distaster} - \Delta}, \tag{38}$$

[24] The carbon cycle in the atmosphere is much more complex. On the one hand, over the first few decades after the emissions takes place, the decay of CO_2 is more rapid than 0.5% per year, see IPCC (2013, pp. 472-3 and 544-5). On the other hand, a non-trivial part of the CO_2 remains in the atmosphere for several millennia. Thus, our assumption can be seen as a simplification and compromise between the medium- and long-term effects.

[25] http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_growth

where $\lambda>0$ is a parameter to be calibrated. The specification in (38) implies $\phi(0) = 0$, $\phi'(\Delta)<0$, and that the numerator is quadratic in the concentration level of CO_2 (above the pre-industrial level). However, as the temperature increase approaches the assumed disaster level of 6 degrees, $\phi(\Delta)$ declines towards minus infinity.

Finally, the parameter λ is calibrated so that the temperature increase peaks at 2 degrees in the optimal solution when $\epsilon = 3$.[26] The motivation for this choice is the fact that the 2 degrees target has been established by the world leaders since the UNFCCC meeting in Cancun in 2010. An alternative calibration strategy could be to use an estimate of the social cost of carbon – however, these estimates vary quite substantially.

[26] Note that in the case with $\epsilon = 1.5$, the optimal temperature path does not peak at 2 degrees, cf. Figure 2E, as we do not want to change the ϕ-function when we change the value of ϵ.