THE OPTIMAL TIME PATH OF CLEAN ENERGY R&D POLICY WHEN PATENTS HAVE FINITE LIFETIME

ABSTRACT

We study the optimal time path for clean energy innovation policy. In a model with emission reduction through clean energy deployment, and with R&D increasing the overall productivity of clean energy, we describe optimal R&D policies jointly with emission pricing policies. We find that, while emission prices can be set at the Pigouvian level independent of innovation policy, the optimal level of R&D subsidies and patent lifetime change with the stages of the climate problem. In the early stages of clean energy development, innovators find it more difficult to capture the social value of their innovations. Thus, for a given finite patent lifetime optimal clean energy R&D subsidies are initially high, but then fall over time. Alternatively, if research subsidies are kept constant, the optimal patent lifetime should initially be long and fall over time.

1. INTRODUCTION

Worldwide emissions of greenhouse gases are growing, and it is recognized that technology improvements are an important element for achieving the deep emission cuts that are suggested in the climate negotiations (see, e.g., surveys in Carraro et al., 2003, and Jaffe et al., 2005). For instance, they are essential for the success of the European Union's Roadmap for moving to a low-carbon economy, which suggests that the EU by 2050 should cut its emissions to 80% below 1990 levels.[1] The question we address in this paper is whether, in general, setting the emission prices right is sufficient to trigger the required technological developments, or whether there is need for extra policies directed specifically at the enhancement of abatement technologies, e.g. the development of clean energy. Furthermore, if the answer to the latter question is affirmative, what characterizes the profile of such policies?

Our first main result follows from establishing a benchmark. If innovation markets function perfectly, e.g., through complete patents with infinite lifetime, then the stage of technological development plays no role in optimal emissions pricing. The emissions price can be set at the Pigouvian level, where the marginal costs to the emitter equals the present value of the future stream of marginal damages associated with the emissions.[2] Technology response to environmental policy does not change this fact. In other words, climate policy can be set independently of climate innovation policy.

Various studies on climate R&D, or more broadly environmental R&D, implicitly assume such perfect markets for innovation (cf Goulder and Mathai, 2000). It is believed, though, that the market for innovations is imperfect, and it is important to extend the analysis of economic policy to imperfect economies (Stern, 2010). Nordhaus (2002), Popp (2004, 2006), and Gerlagh and Lise (2005), for example, in their numerical analyses of R&D and climate policy, assume that the social value of innovations exceeds the private value of innovations by a constant factor 4. Under these circumstances, the apparent question becomes whether environmental policy needs to complement the Pigouvian tax with innovation policy directed at environmental technology.

The case for a dedicated climate technology policy is often contested by economists who point out that it is not implied as such by an imperfect market for innovations. If the gap between social and private returns on innovation is identical over different economic sectors, then a generic innovation policy can correct the innovation market failure for all sectors jointly. Only recently have there been studies pointing to reasons why clean energy R&D should be treated differently (Popp and Newell, 2009; Acemoglu et al., 2012). But the arguments brought forward do not include the main focus of this paper, which is that patents typically

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[1] http://ec.europa.eu/clima/policies/roadmap/index_en.htm

[2] We limit the interpretation of the Pigouvian tax to include only environmental damages. This is a choice for convenience, common in environmental economics. In this paper we specify a cumulative absorption capacity for the atmosphere and define the Pigouvian tax as the marginal social costs of meeting the target.

expire after a certain period and this creates a temporal structure that links the state of the climate to the attractiveness of clean energy R&D for private entrepreneurs. Krysiak (2011) makes such a connection, but does not address the time pattern of optimal R&D policy as in this paper. The mechanism that we recover in our model is that private and social returns on clean energy R&D follow their own, quite different, dynamic patterns. The gap between social and private returns on innovation then changes over the life-cycle of the climate problem, and optimal clean energy R&D policy varies along. Hart (2008) studies how this affects the optimal time path of CO_2 taxes, whereas Goeschl and Perino (2007) study R&D sequences when human kind is confronted with repeating cycles of various environmental problems. Our paper can be considered a more detailed study of one such cycle, such as climate change. In this context, when we refer to a cyclical pattern, we refer to the increase and decline of a pollutant over the life-cycle of an environmental problem as typical for an Environmental Kuznets Curve; we do not imply a repetition of cycles.

Our second and most interesting main finding is that the optimal clean energy R&D policy has a cyclical pattern counter to the pricing policy (e.g. carbon pricing): Assuming finite and constant patent lifetime, the optimal R&D subsidy should initially be high when carbon prices are low, and then gradually decline over time while carbon prices increase; optimal research subsidies might even become negative when carbon prices reach a maximum. After sufficient knowledge has been produced so that carbon emissions fall close to zero, at moderate carbon prices, the innovation subsidy should increase again and converge to a constant rate (not necessarily positive). In a similar way, if R&D subsidies are kept constant, the imperfections in the clean energy market can be corrected by the patent lifetime. It will have a similar pattern as the R&D subsidy when patent lifetime is constant, i.e., decrease monotonically when carbon prices increase and increase again when carbon emissions drop to zero.

If we focus on innovation subsidies, the intuition for this pattern is that innovations will be biased towards technologies that pay back within the patent's lifetime, so that there is insufficient support through markets to develop and improve abatement technologies when the climate problem is emerging and (e.g. carbon) prices are still low. Yet at the point in time when the carbon price is close to its maximum, the market offers innovators a large incentive for emission-reducing research. Innovations will peak without the need for research subsidies. Such a pattern has been seen for SO_2 emissions. SO_2 is an interesting pollutant to evaluate as its emissions peaked a few decades ago in most industrialized countries. While there was no supporting research policy, patents spiked for SO_2 abatement technologies when more stringent regulatory standards came into effect (Dekker et al. 2012, Fig 2). In case that 'clean energy' research tends to crowd out other research, the incentive for clean energy innovations might as well be 'too much', e.g. when carbon prices are at a temporarily high level. In the long run when the environmental life-cycle has ended, there is no reason anymore to treat clean energy research differently from other research. That is, in the long run the optimal subsidy may rise again because the proportion of social returns captured by the innovator is declining.

Thus, the level of the clean energy subsidies must vary over time, targeted to the early phases of the technology development. The mechanism laid out here resembles the learning-by-doing models; in this paper we present conditions on patent-lifetime when the same mechanisms play a role in a learning-by-research model. The model we present bridges part of the gap between the learning-by-doing and learning-by-research strands of literature. The time-dependence of optimal policies has generally been overlooked in earlier R&D models. Nordhaus (2002), Popp (2004) and Fischer and Newell (2008) combine and compare carbon prices and research subsidies for clean innovation, but they only consider constant research subsidies. Our analysis shows how their results would change if they had explicitly included the expiration of patents in their numerical models.

The basis of our analytical framework we borrow from the early literature on endogenous growth and environmental policy. Much of the early work in this field studied balanced growth paths (cf. Bovenberg and Smulders, 1995), or transition dynamics where the environment moves from a dirty to a clean steady state (cf. Bovenberg and Smulders, 1996). However, apart from the questions analyzed, there are two major differences in our analytical model compared to this strand of literature.

First, we do not consider a closed economy but for convenience apply a partial analysis. This choice is based on the observation that the climate problem is mostly associated specifically with the energy sector. For climate change, the single most important question concerns the costs, speed, and policies required to guide the transition of the energy supply sector towards carbon neutral energy sources. Working with a closed economy model will complicate the analysis unnecessarily.[3] Yet the partial model may create a bias in results as it does not trace the effects of sector-specific policies on other sectors. Stimulating research in the abatement sector that we describe may crowd out research in other sectors outside the model, causing welfare losses not accounted for. We control for this problem by adding a crowding out parameter. A more comprehensive assessment is provided in Section 5.

Second, while most of the endogenous growth literature referred to above studies a one-directional move from a dirty to a clean state, the transition we consider is more cyclical in nature, starting from a clean state. This is based on empirical evidence: In the context of climate change (and most other environmental problems), the life-cycle of the environmental problem starts with low emission levels and a clean environment, moving to high emissions and a large pollutant stock. To prevent an ecological collapse, at some point in time, the economy must move back to a state with low emissions. Emissions thus follow a hump-shaped curve (cf. Stokey 1998, Smulders et al, 2011; Hart, 2008). At the initial stage, the Pigouvian tax will rise sharply, but after the first stage, the growth rate of the Pigouvian tax will gradually fall (Hoel and Kverndokk, 1996). The growth in the use of abatement technologies will follow a similar pattern.

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[3] Hourcade et al. (2011) elaborately argue that a closed-economy representation of an environmental problem that is essentially partial in nature, easily leads to misguided conclusions as the model employs unrealistic assumptions to be closed and tractable.

Kverndokk and Rosendahl (2007), Gerlagh et al. (2009), Heggedal and Jacobsen (2011), and Acemoglu et al., (2012) find that this abatement cycle[4] generates a high optimal subsidy rate for abatement when the abatement technology is first adopted, but the subsidy falls significantly over time as the abatement technology matures. Kverndokk and Rosendahl derive these conclusions from a numerical model with learning by doing (LbD), while Heggedal and Jacobsen employ a computable general equilibrium model. Gerlagh et al. (ibid) and Acemoglu et al. (ibid) both combine a formal analysis with numerical simulations. The analysis here complements the formal analysis in the last two papers, which we now discuss in more detail.

Both previous papers assume discrete time, a patent lifetime of one period, and a positive externality from existing knowledge to innovation ('standing on shoulders'). This set of assumptions enables the authors to characterize the equilibrium sequentially: the innovation payoffs only depend on the current state of the economy, so that innovation decisions, even when taken a period ahead, are only part of the equilibrium analysis of the current period.[5] Despite these assumptions that enable a sequential equilibrium, both Gerlagh et al. and Acemoglu et al. do not succeed to fully characterize the dynamics through formal propositions. These papers rely on quantitative simulations to present the pattern of clean innovation subsidies. Gerlagh et al. present propositions that are conditional on the pattern of abatement expenditures in equilibrium, and these conditions are shown to hold in a numerical equilibrium (their Fig 3). Acemoglu et al. present propositions that state whether subsidies and taxes are temporary or permanent, but not whether they have one peak, or multiple, whether they start increasing and then decrease, or are monotonic. Acemoglu et al. also rely on numerical simulations in Section V.B to show emerging patterns.

The assumptions by Gerlagh et al and Acemoglu et al. are convenient, but not innocent. Specifically the one-period validity of patents reduces the empirical relevance (Greaker and Heggedal 2012). Gerlagh et al. (2009) also assume that technologies are available for production for only one period; the patent life-time fully covers the use of the technology in production, and the mechanism that drives their result comes from the spillover from the stock of knowledge to new innovations. Acemoglu et al. assume that knowledge does not depreciate so that it is not clear whether their results come from the knowledge-innovation externality, or from the one-period patent life-time, or from a combination of both.

In our paper we focus on the role of patents' lifetimes, and we aim for a full formal characterization of equilibrium. We relax the one-period patent life-time assumption, and consider continuous time and an arbitrary patent length. We even consider patent lifetime as an adjustable policy parameter. The main contribution of the current paper is to examine analytically within a conventional R&D model the

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[4] Acemoglu et al (2012) do not use the term abatement, but close reading of their application reveals that their clean production is interpreted as non-CO_2 emitting energy, which can be interpreted as abatement of emissions.

[5] For Gerlagh et al. (2009), the mechanism is analyzed in detail in the working paper version (FEEM Nota di Lavoro 35.2007) For Acemoglu et al (2012), this can be seen from their equation (17).

dependence of the time profile of optimal clean energy R&D policy on different assumptions about patent lifetime. We are not aware of any studies using a formal, conventional R&D model, taking into account patent lifetime and also considering the long-run dynamics towards a balanced growth path; thus this is the core distinction between our R&D model and earlier R&D models in the environmental economics literature. The more comprehensive treatment of patent lifetimes complicates the analysis considerably. To keep the analysis tractable, we allow for other simplifying assumptions. We abstract from spill-overs between the stock of technology and innovation, we assume that technologies remain in use forever even when patents expire, and we abstract from energy savings focusing on clean energy development as a perfect substitute for carbon-emitting energy. Even if we do several simplifying assumptions, we think that we still are able to capture some main features of optimal innovation policies. This will be supported with evidence for the patterns of innovation for a more mature environmental problem (SO_2).   As   we focus on the time path of clean energy policies, we also connect to the literature on the time path of abatement. Various applied studies on climate change policy have concluded that there is a need for up-front investment in abatement technologies to stimulate innovation (van der Zwaan et al., 2002; Kverndokk and Rosendahl, 2007). Others have argued that this finding is an artefact of the typical models in use where innovation occurs through learning by doing mechanisms. It has been suggested that models that describe innovation through R&D would not support early abatement (Goulder and Mathai, 2000; Nordhaus, 2002). As in this strand of literature, we analyze optimal time paths, but we focus on time paths of *abatement policies* rather than on time paths of *abatement levels*.

Central to our analysis is the expiration of patents, and the third strand of literature we contribute to considers the optimal lifetime of patents. Patent policy has obvious welfare implications (see Nordhaus (1969) for an early study). In general, an increase in the patent length is growth enhancing by raising the rate of return on R&D (Judd, 1985). On the other hand, patents create a static inefficiency as patents allow monopolistic supply by the patent holder (David and Sinclair-Desgagné, 2005; Requate, 2005; Perino, 2010). Longer patents thereby reduce output, and thus consumption, by increasing the portion of the monopolistic sector. Hence, patents have two opposite welfare effects. Chou and Shy (1993) show that in an overlapping generations economy, long-duration patents crowd out new R&D investment and this plays a key role in obtaining the result that a one-period lifetime is preferred to an infinite lifetime. Iwaisako and Futagami (2003) find an optimal finite patent lifetime to trade-off the two opposite effects. This is followed up in Futagami and Iwaisako (2007) where a finite patent length maximizes social welfare in a growth model that does not exhibit scale effects. These studies focus on balanced growth paths. We extend this literature by also considering optimal patent length along a transition path.

This paper is organised in the following way. In Section 2 we develop the basic model describing the evolution of knowledge through R&D, abatement output, emissions and the stock pollutant. Technological change is driven by the Romer (1987, 1990) type of endogenous growth. We analyze the social optimum, differentiating between short-run and long-run dynamics, by establishing a unique

balanced growth path, and show how the optimal path of R&D would develop over time to reach this path. We are then interested in how the social optimum can be implemented in a market and describe in Section 3 the market equilibrium for abatement goods, abatement equipment and innovation. Then, in Section 4, we analyse optimal climate and innovation policies in the first-best setting. Methodologically, the approach is similar to Hartman and Kwon (2005) and Bramoullé and Olson (2005). In Section 5 we discuss general vs. partial equilibrium effects, whereas in Section 6 we summarise results and conclude.

2. OPTIMAL ABATEMENT AND RESEARCH

We consider an economy with a stock pollutant such as greenhouse gases (GHGs). In the economics literature on climate change, two alternative perspectives have mainly been used with respect to absorption capacity or depreciation of CO_2 emissions. The first perspective, which has been the standard approach in much of this literature, assumes that emissions depreciate through the carbon cycle, and that damages are more or less proportional to income and to CO_2 concentrations. These assumptions imply that optimal emission prices approximately increase with income (see e.g. Golosov et al., 2011; Gerlagh and Liski, 2012; van der Ploeg and Withagen, 2012).

The second, more pessimistic approach builds on more recent conclusions by natural scientists, emphasizing that the CO_2 absorption capacity of the oceans is limited. That is, a non-negligible part of anthropogenic CO_2 emissions, between 15 and 20%, remains in the atmosphere for thousands of years – the other part is taken up by oceans – before long-term geochemical processes convert the CO_2 into other carbon substances. The ultimate implication of this understanding is that if we are concerned with the risk of Greenland melting and other large-scale long-term climate changes, it is not so important whether emissions occur in 2020 or 2050. In economic terms, the absorption capacity should be treated more as an exhaustible stock than as a pollution stock with depreciation (Kharecha and Hansen 2008, Allen et al. 2009, Zickfeld et al. 2009).

In our model we will take the second perspective, which we consider more realistic, and put a ceiling on cumulative emissions. However, as shown in Appendix 3, all our results carry over if we rather take the first perspective and assume that the emissions price increases exogenously with income.

The abatement production model has a similar structure as the model in Iwaisako and Futagami (2003), except that we assume decreasing instead of constant returns to scale for each technology (see below). The model is based on Romer's endogenous growth model, with horizontal innovation of the 'love of variety' concept (Romer, 1987, 1990; Barro and Sala-i-Martin, 1995; Dixit and Stiglitz, 1977; Gancia and Zilibotti, 2005). The model explicitly describes patents as in Futagami and Iwaisako (2007), but extends their model as it has an infinite horizon with continuous time t. Further, the model shares similarities with the one in Gerlagh et al. (2009), but here we have continuous time, variable patent lifetime, and blueprints remain available for use after the patents expire. These are important differences that make it possible to study the optimal patent lifetime. There is one

representative abatement sector, which could either be interpreted as abatement of emissions (e.g., carbon capture and storage), or as an alternative, emission-free, resource sector (e.g., renewables). There are H_t different abatement technologies at each point of time t, which, e.g., could be different wind mill designs (onshore/offshore), solar panels, hydro power technologies, carbon capture technologies etc. An R&D sector develops new technologies. Technological progress takes the form of expansion in the number of different abatement technologies, i.e., increased variety of abatement equipment.

The social planner aims at minimising the present value of social abatement costs, discounted at a constant rate ρ, subject to an upper bound on cumulative emissions. We can think of this upper bound as the assumed cumulative absorption capacity. Current emissions exhaust the absorption capacity, so that in economic terms, the absorption capacity acts as an exhaustible resource.

Let E_t be emissions and let S_t be the remainder of the cumulative absorption capacity. Initial absorption capacity is given by S_0, the capacity constraint by $S_t \geq 0$, and the dynamics are as follows:[6]

$$\dot{S}_t = -E_t. \tag{1}$$

This gives a cyclical pattern of the climate problem. We start from a clean state, then emissions are positive, but they approach zero when S_t approaches zero.

The overall economy grows exogenously, and we assume that benchmark emissions Y_t increase at a fixed rate g, while emissions can be reduced by abatement effort A_t:[7]

$$E_t = Y_t - A_t \geq 0. \tag{2}$$

Typically one can think of three main mechanisms for GHG emissions reductions: (i) through energy savings within a sector, (ii) through energy carrier substitution in the energy sector (including also the use of carbon capture), and (iii) through a shift

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[6] By 2010, cumulative emissions of CO_2 have reached about 525 GtC. Annual CO_2 emissions related to fossil fuel use and deforestation are currently around 8 GtC/yr. The numbers exclude other GHGs, which also provide a substantial contribution to global warming. The papers cited in the main text above implicate that, in order to maintain a high probability that global mean temperatures will not increase by more than 2 degrees Celsius (compared to 1900), we should keep cumulative CO_2 emissions below ca. 1000 GtC.

[7] Y can be interpreted as energy demand, which is then treated as price-inelastic throughout the analysis. The relation between emissions and benchmark emissions is specified as a linear function for convenience of notation (a common assumption, cf. e.g. Goulder and Mathai, 2000). The restriction $E_t \geq 0$ is imposed to simplify the formal analysis. In reality negative emissions may be feasible by combining bio energy and carbon capture. Given that the costs of such measures are high, our qualitative results will likely carry over to this case, too, as negative emissions would have been followed by positive emissions given our restriction on S_t, which cannot be optimal if negative emissions are more costly to achieve vis-a-vis reducing positive emissions.

between energy-intensive and energy-extensive sectors. In our model we only consider (ii), i.e., substitution from emission-intensive to emission-free energy. For the long run, we think this is the most important mechanism. For the feasibility of economic growth combined with zero emissions, the dynamics of clean energy will likely be more important than energy savings and product substitution.[8]

Production of abatement requires the input x_i of abatement equipment, where subscript $i \in [0, H_t]$ refers to variety i, and H_t is the number of equipment varieties. H_t can also be interpreted as the state of knowledge. Building on the horizontal innovation literature (see also Goeschl and Perino, 2007, Greaker and Pade, 2009, and Gerlagh et al., 2009), abatement is produced according to:[9]

$$A_t = \int_0^{H_t} x_{t,i}^\beta \, \mathrm{d}i, \tag{3}$$

where $0 < \beta < 1$, i.e., each type of abatement technology has decreasing productivity when expanded. The different varieties of abatement equipment are neither direct substitutes nor direct complements to other specific equipment. That is, the marginal product of each abatement equipment is independent of the quantity of any particular other type of equipment. Examples of this are different abatement equipments to produce alternative energy (such as wind power, hydro power and solar power). Each variety (technology) has its own ideal site specifics, but the potential of each variety is limited so that new varieties have to be developed to increase the total amount of alternative energy that can be produced at certain marginal costs. For instance, wind power is most valuable in areas with strong wind, and offshore wind power technologies expand the potential for wind power. Further, hydro power offers potential in areas with large waterfalls, and solar power in areas with high solar radiation inflow.[10] For our analysis we assume that decreasing returns to scale for varieties are not too strong, that is, $\beta > \frac{1}{2}$. As we will see in the next section, this condition also follows by assuming that the mark up on prices under monopolistic competition, where each innovator owns his own variety, is less than 100% (which seems reasonable). Due to symmetry, we find that aggregate production becomes:

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[8] As a comparison, in the DICE model (cf. e.g. Nordhaus and Boyer, 2000), the three mechanisms are implicitly lumped together, whereas for Acemoglu et al., (2012) it is ambiguous whether they consider the second or third mechanism, but it is clear that they do not consider energy savings. As they model an economy-wide shift between dirty and clean sectors, this suggests that they consider the third mechanism, but when looking more carefully at their calibration (second line of p155), it becomes clear that their interpretation goes along the lines of the second mechanism.

[9] We disregard any time lags between the instalment of abatement equipment (investment) and the use of equipment (payoff). We also disregard time lags in the innovation process. These time lags are of course important in a short- to medium-run analysis, but of less importance in our long-term context (cf. also the horizontal innovation literature).

[10] Similar arguments can be made about carbon capture, where different technologies exist and can be used to capture CO_2 from different sources (e.g., production of coal power, gas power, steel, cement etc.). Post-combustion technologies can often be used on several sources, whereas pre-combustion technologies are more process-specific.

$$A_t = H_t x_t^{\beta} \tag{4}$$

Individual innovator j develops an amount $dH_{t,j}$ of new varieties proportional to his individual effort $dR_{t,j}$; $R_t = \int dR_{t,j} dj$ denotes aggregate research efforts by all innovators at time t. We assume that research partly duplicates new varieties found by other researchers, with increasing 'standing on toes' when total research efforts rise, so that the following production function for new knowledge applies:

$$dH_{t,j} = dR_{t,j} \, R_t^{\psi-1}, \tag{5}$$

where $0<\psi<1$ measures the rate of return on R&D at the aggregate level. Thus, equation (5) implies a negative externality from research. The externality is more severe the lower is the value of ψ. On the other hand, there is a positive spillover of research unless the innovator is able to reap all future profits from production of the new variety. Thus, as we will see below, patent rules are of major importance.[11]

Aggregation of (5) gives R_t^{ψ} as the aggregate number of new innovations, or the flow of new varieties that adds to the pool of knowledge, H_t:

$$\dot{H}_t = R_t^{\psi} \, . \tag{6}$$

Comparison of (5) and (6) shows that whereas a single researcher exhibits constant returns to scale, the sector as a whole bears diminishing returns to scale. This could be motivated by congestion externalities originating from different researchers' efforts on the same product. This externality has been pointed to by e.g. Stokey (1995), Jones and Williams (2000) and Greaker and Pade (2009). The empirical evidence of this effect is somewhat unclear, however.[12]

As we study a partial model, there is the possibility that additional research in the abatement sector goes at the expense of (i.e., crowds out) research in sectors outside the model. Popp and Newell (2009) estimate that new clean energy R&D indeed partly crowds out other R&D. Even if the other R&D has lower social value, any crowding out will dampen the social value of extra clean energy R&D. Let $\kappa-1$

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[11] There are other imperfections of research that could be introduced. For instance, this model does not specify a dynamic spillover effect based on earlier research, such as "standing on shoulders", "fishing out" or "learning by doing". In particular, the "standing on shoulders" mechanism, which means that dH increases in H, is commonly assumed, see, e.g., Romer (1990), Goulder and Mathai (2000) and Gerlagh et al. (2009). Inclusion of such spillovers would likely strengthen the main results, i.e., that innovation should be stimulated strongest initially.

[12] The 'standing on toes' assumption implies decreasing returns to scale *within a period*. This assumption is consistent with a smooth research path over time. Assume instead constant returns to scale, i.e., $\psi=1$. Then the conclusion from the optimisation problem below would be that we should delay all abatement until the pollution problem is so severe that the safe pollution threshold is reached. At this point of time, research spikes, and abatement costs and pollution levels drop close to zero.

denote the crowding out factor. Then, the social abatement costs are the sum of the costs of abatement equipment $H_t x_t$ and the social costs of research κR_t, where all unit costs are equal to one (note that all varieties are equally productive). Thus, we have negative externalities of research both within the abatement innovation sector (ψ), and in other research sectors ($\kappa-1$).

Social Optimum

The social planner minimizes the net present value of all future costs consisting of both abatement equipment expenditures and research costs:

$$V(H_0, S_0, Y_0) = \min \quad \int_0^\infty e^{-\rho t}[H_t x_t + \kappa R_t]\mathrm{d}t, \qquad (7)$$

subject to the restriction on the carbon absorption capacity $S_t \geq 0$, stock accumulation dynamics (1) and (6), and production equations (2) and (4), with x_t, and R_t as the control variables. We notice that for $H_0 = S_0 = 0$, there exists no solution because emissions cannot be decreased to zero without a prior knowledge stock. However, as long as either knowledge is strictly positive, $H_0 > 0$, or the cumulative emission allowance is positive, $S_0 > 0$, a solution exists.

The current value Hamiltonian, $\mathcal{H}_t$ for the cost minimization problem (7) reads

$$\mathcal{H}_t = H_t x_t + \kappa R_t - \theta_t \dot{S}_t - \eta_t \dot{H}_t - \varepsilon_t E_t - \lambda_t S_t, \qquad (8)$$

where ε_t and λ_t are the dual variables for the non-negativity constraints for E_t and S_t, respectively. We have changed sign for θ_t and η_t such that they are positive and can be interpreted as the shadow prices for the absorption capacity and knowledge, respectively. The first-order conditions read (where we omit the time subscripts):

$$0 = \mathcal{H}_x = H - \beta(\theta-\varepsilon)Hx^{\beta-1} \qquad (9)$$

$$0 = \mathcal{H}_R = \kappa - \psi\eta R^{\psi-1} \qquad (10)$$

$$\dot{\theta} = \rho\theta + \mathcal{H}_S = \rho\theta - \lambda \qquad (11)$$

$$\dot{\eta} = \rho\eta + \mathcal{H}_H = \rho\eta - (\beta^{-1}-1)x \qquad (12)$$

$$\lambda S = 0; \quad \varepsilon E = 0 \qquad (13)$$

In Appendix 1 we rewrite the first-order conditions in intensive form to help with the interpretations. The first two first-order conditions state that the abatement effort (x) is more than linearly proportional to the shadow price of emissions (θ), and that the innovation effort (R) is more than linearly proportional to the knowledge shadow price (η). The third first-order condition states that the emission shadow price (θ) is constant in present value as long as $s > 0$, while the fourth first-order condition states that the shadow price of knowledge (η) equals the present value of its future use for abatement. The last equation presents the typical complementarity conditions for λ and ε.

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Long-term dynamics

As explained in Appendix 1, we use the following normalization: $h_t = H_t / Y_t^{\gamma\psi}$, $s_t = S_t / Y_t$, $\chi_t = x_t / Y_t^{\gamma(1-\psi)}$, and we define $\gamma = \dfrac{1}{\psi + \beta(1-\psi)} > 1$. We also explain that the dynamics of the social optimum are fully captured by the intensive-form pollution stock s_t and the intensive-form knowledge stock h_t. Then we prove the following proposition:

PROPOSITION 1. *A unique balanced growth path exists with $s_t=0$, $h_t=h^*$, $\chi_t=\chi^*$, so that*

> $\dot{H}_t/H_t = \gamma\psi g$, $\dot{x}_t/x_t = \gamma(1-\psi)g$. *Off the balanced growth path, if $s_T=0$ and $h_T<h^*$, then for all $t>T$:*
>
> $h_t<h^*$, $\dot{H}_t/H_t > \gamma\psi g$ *and $\dot{H}_t/H_t$ is decreasing (< and increasing if $h_T>h^*$),*
>
> $x_t>x^*$, $\dot{x}_t/x_t < \gamma(1-\psi)g$ *and $\dot{x}_t/x_t$ is increasing (> and decreasing if $h_T>h^*$).*

The proposition states that if we start with a low knowledge stock, then the rate of growth will be high initially, but will fall. This is an intuitive result. The use of abatement equipment will start at a high level, but its rate of growth will start at a low level, and increase over time. Loosely, we can say that the number of clean energy types increases fast in the early phase, and less so at a later phase. The number of equipment per type shows a complementary path.

Short-term dynamics

We now turn to the short-term dynamics in state space (h_t, s_t). The main idea of the short-term analysis is to show that when the initial knowledge stock is small, say $h_0=0$,[13] then throughout time the knowledge stock will remain small (in a precise way defined below), and when the absorption capacity of the carbon stock is exhausted, $s_T=0$, the balanced growth path is approached from below. These properties will then enable us to sufficiently characterize the short- plus long-run dynamics so as to establish all required properties regarding the private and social value of knowledge.

This is summarized in two propositions that are proved in Appendix 1:

PROPOSITION 2. *For any $s_0>0$, there is a unique h_0, with $\partial h_0/\partial s_0>0$, such that the optimal paths for initial conditions (s_0,h_0) enter balanced growth in finite time.*

PROPOSITION 3. *For initial conditions $s_0>0$, $h_0=0$, when the optimal path enters the long term dynamics at $t=T$, we have $s_T=0$ and $h_T<h^*$.*

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[13] Note from (34) in Appendix 1 that h_t may increase even if $h_0=0$.

We have now established that if we start without initial specific abatement knowledge, the knowledge stock will still be below the balanced growth level when we enter the long-run dynamics. The last proposition describes mathematically the idea that, to set a ceiling to future climate change, at some future date we will have to move towards emission-free energy sources. The current stock of knowledge on emission-free energy sources is so low that we will approach the balanced growth from below. The result is intuitive and it will be essential to establish how the value of knowledge develops over time.

3. MARKET EQUILIBRIUM

We now take a look at how we can implement the first-best allocation through research subsidies, or changing the lengths of patents. Thus, we first explore the precise structure of innovation.

The producers of the abatement equipment own patents and, therefore, receive monopoly profits. However, they have to buy the innovations from the R&D sector, where innovators are competitive and use research effort as an input.[14] We assume that patents have a certain lifetime L_t, and that the equipment can be produced free of charge by anyone after expiration of the patent. Notice that we allow for the patent lifetime to change over time, and to be used as a policy instrument. Free entry is assumed in all markets, including the market for innovation. Thus, in this model there are four imperfections related to innovations: Too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovation as innovators do not include that part of the social value of their innovations that is realized after the expiration of the patent, negative spillovers of total research effort on new innovations due to 'stepping on toes', and crowding out of innovations in other sectors. The level of innovations supported by the market can therefore exceed or fall short of the social optimal level. As innovation is taking place in private firms, the role of the government is to create incentives to achieve the social optimal levels of innovation.

We disregard the fact that patents only to a certain extent prevent the imitation of new innovations (cf. Mansfield et al., 1981). Also, we abstract from creative destruction, which may turn existing innovations obsolete (see e.g. Aghion and Howitt, 1998). Both phenomena imply that very long patent lifetimes may be legally feasible, but practically irrelevant. We return to these issues below where we discuss optimal research policies.

We distinguish between two different types of equipment; those with patents expired ($y_{t,i}$), and those with running patents ($z_{t,i}$). The number of varieties with expired patents is denoted M_t, and the number of varieties with running patents is denoted N_t. Adding up both gives the total knowledge stock

$$H_t = N_t + M_t. \tag{14}$$

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[14] Alternatively we could assume that the innovators are producing the abatement equipment, so that they own the patents and receive the monopoly rent. This would not change the arguments or conclusions of the analysis.

Due to symmetry, all varieties have the same unit production costs. The varieties with expired patents are produced competitively, and thus sold at unit price. Because of symmetry between the varieties, in equilibrium the same quantity will be employed of each equipment with expired patent, i.e., $y_{t,i}=y_t$. The varieties with running patents are produced by the patent holder, and sold at a mark up price $w_{t,i}$. Again, because of symmetry, we have $w_{t,i}=w_t$ and $z_{t,i}=z_t$ for equipment with running patents.[15] The abatement production identity then becomes:

$$A_t = M_t y_t^\beta + N_t z_t^\beta. \tag{15}$$

The flow of new varieties R_t^ψ adds to the pool of patented knowledge, N_t, but after a period L_t these varieties leave the pool of patented knowledge and enter the pool of patent-free knowledge M_t:

$$\dot{M}_t = R_{t-Lt}^\psi \tag{16}$$

$$\dot{N}_t = R_t^\psi - R_{t-Lt}^\psi \tag{17}$$

We now describe the market equilibrium, given a set of policy instruments. In the next section we search for the first-best policy.

Abatement goods

The public agent implements an emission tax τ_t, or more generally a climate policy that induces a cost of emission in the market. From (2) we see that this translates into a market price for abatement A_t, as E_t and A_t are perfect substitutes. Equipment with running patents is subsidized at rate ω_t to correct for market power.[16] The abatement producer maximises the value of production minus the input costs:

$$\text{Max } \tau_t A_t - M_t y_t - N_t(1-\omega_t)w_t z_t, \tag{18}$$

subject to (15), where y_t and z_t are the control variables.

The first order conditions of this maximisation problem determine the abatement producer's demand for patent-free and patented varieties, respectively:

$$y_t = (\beta\tau_t)^{1/(1-\beta)}, \tag{19}$$

$$z_t = (\beta\tau_t/(1-\omega_t)w_t)^{1/(1-\beta)}, \tag{20}$$

The first order condition for patent-free varieties y_t in (19) is similar to the corresponding condition under the social optimum given by (9), with the exception that the social price of abatement, θ_t, is replaced by the market price of abatement,

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[15] In the following we will therefore omit the subscript *i*.

[16] Other policy instruments such as licensing and contracts could also be used to correct for market power due to the patent system, see, e.g., Maurer and Scotchmer (2006) and Scotchmer (1991).

τ_t (recall that $\varepsilon_t=0$). In other words, the shadow cost of emissions is replaced by the (Pigouvian) emission tax. For patent-holding varieties z_t, the market equilibrium (20) can be matched to the social optimum if we set a subsidy $\omega_t=1-1/w_t$ jointly with implementing the Pigouvian tax, i.e., $\tau_t=\theta_t$.

Monopolistic supply of abatement equipment

Acting as monopolists, the producers of patented abatement equipment maximise profits at each point in time, π_t, taking into account the falling demand curves for abatement equipment (again we omit subscript *i*):

$$\text{Max } \pi_t = z_t(w_t-1), \tag{21}$$

subject to (20). We notice that 'profits' refer to the rent value of the patent and not to a surplus. Free entry ensures the zero-profit condition: net revenues from selling the equipment minus production costs equal the rent that the monopolist pays to the patent holder.

The first order condition from maximising (21) with respect to w_t determines the price of the abatement equipment:

$$w_t = w = 1/\beta. \tag{22}$$

From (20) and (22) we find the market equilibrium level of z_t:

$$z_t = (\beta^2\tau_t/(1-\omega_t))^{1/(1-\beta)}. \tag{23}$$

Using (21) we find the rent value of a patent:

$$\pi_t = (\beta^{-1}-1)z_t. \tag{24}$$

The value of a patent can now easily be calculated as the present value of the future patent rents, over the patent lifetime L_t:

$$V_t = \int_0^{L_t} e^{-\rho t}\, \pi_{t+u}\, du = (\beta^{-1}-1)\int_0^{L_t} e^{-\rho t}\, z_{t+u}\, du. \tag{25}$$

Notice that the value of a patent increases with the patent lifetime, the deployment subsidy and the emission tax, as the demand for equipment increases with both the subsidy and the tax (cf. (23)). Thus, all these policy instruments affect the incentives for research.

Markets for innovation

The innovators maximise profit with respect to research effort, where the price of the innovation equals V_t, i.e., the present value of the patent over its lifetime. The government subsidizes research expenditures at a rate σ_t. Thus, the innovators' maximization problem is:

$$\text{Max } V_t \mathrm{d}H_{t,j} - (1-\sigma_t)\mathrm{d}R_{t,j}, \tag{26}$$

subject to (5) .

The first order conditions give that the unit cost of research, which is set equal to one, is equal to the value of the patent, V_t, multiplied by the productivity of $\mathrm{d}R_{t,j}$, $R_t^{\psi-1}$. Due to the zero-profit condition, in equilibrium the value of all patents is equal to the value of all research effort:

$$V_t\, R_t^{\psi} = (1-\sigma_t)R_t. \tag{27}$$

The eight equations (15), (16), (17), (19), (23), (24), (25) and (27) define a market equilibrium through the variables A_t, M_t, N_t, y_t, z_t, π_t, V_t, R_t, for a given carbon tax policy τ_t, subsidies ω_t and σ_t, and patent lifetime L_t. It is straightforward to see that given a path for the policy instruments, the equilibrium exists and is unique; this is a prerequisite for the public agent to steer the economy towards the efficient allocation. Equations (19) and (23) determine the equipment inputs y_t and z_t, respectively. Substitution of (23) in (24) provides π_t, and subsequent substitution in (25) gives an unambiguous value for a new patent at time t, V_t, as dependent on future taxes and deployment subsidies. Subsequently, (27) determines the research effort dependent on the current research subsidy, and (16) and (17) determine the state of knowledge for all t. Finally, (15) determines the abatement level.

4. FIRST-BEST R&D POLICY

Note that innovations depend on the tax and subsidy policies for the coming L_t periods. When patent lifetime L_t goes to infinity, innovators take into account benefits over the full future horizon. On the other hand, when patent lifetime is finite, then innovators are short or medium-sighted, and thus there is a positive externality from innovations. This feature is the core distinction between our R&D model and earlier R&D models in the environmental economics literature.

We now compare the social optimal research effort (10) with the market equilibrium research effort (27). We rewrite the latter as (using (25)):

$$R_t^{1-\psi} = (1-\sigma_t)^{-1}(\beta^{-1}-1) \int_0^{L_t} e^{-\rho u}\, z_{t+u}\, du \tag{28}$$

A comparison with (10), using (12) and $x_t = z_t$, quickly reveals the optimal research subsidy level:

$$\sigma_t = 1 - (\kappa/\psi) \int_0^{L_t} e^{-\rho u}\, z_{t+u}\, du \, / \int_0^{\infty} e^{-\rho u}\, z_{t+u}\, du. \tag{29}$$

Note that the subsidy rate can be negative if negative externalities from abatement research, i.e., stepping on toes ($\psi<1$) and crowding out research in other sectors ($\kappa>1$), dominate the positive externalities that appear after the patent has expired (i.e., the second ratio which is less than one).

Comparing the social optimum in equation (9) with the market equilibrium in (19) and (20), and using the market price defined by (22), we find the optimal policy instruments to be $\tau_t=\theta_t$ and $\omega_t = 1-\beta$ when emissions are positive. When emissions are zero, the tax is set exactly such that abatement equals benchmark emissions, while the optimal subsidy remains the same.

We are now able to define the first best policy to obtain the social optimum. Through a Pigouvian tax on emissions, $\tau_t=\theta_t$, a subsidy on patented abatement equipment equal to $\omega_t = 1-\beta$, and a patent lifetime L_t combined with an R&D subsidy/tax σ_t that satisfies (29), the first-best outcome can be implemented. The reasoning is clear. There are three groups of imperfections in the model; i) emissions, ii) imperfect competition in the market for patented abatement equipment, and iii) positive and negative externalities of research effort. Remember that the last group of imperfections comprises three externalities, one positive and two negative (crowding out effects). Therefore, we would need three (combinations of) policy instruments to implement the social optimum: a tax on emissions, a subsidy to production of patented abatement equipment, and a combination of research subsidy/tax and patent lifetime. Policy makers can choose to either fix the patent lifetime and adjust the research subsidy, or to fix the research subsidy and adjust the patent lifetime.

In order to shed light on the optimal combination of patent lifetime and research subsidy given by (29), we will consider three specific cases. As noted in the introduction, we are particularly interested in the dynamics of the instruments. First, the following proposition considers the implications of having patents that remain valid infinitely.

PROPOSITION 4. *For patents with infinite lifetime, $L_t \to \infty$, the efficient R&D subsidy/tax that implements the first-best outcome is constant for all t: $\sigma_t = 1 - \kappa/\psi$.*

The proof follows straightforwardly from (29) and looks simple, but its meaning is more subtle. If innovation markets are complete, i.e., infinite lifetime of patents, innovation policy can be separated from climate policy. That is, the stage of the climate problem has no effect on the R&D subsidy. As mentioned in the introduction, this result resembles the typical assumption in integrated assessment models with R&D (Nordhaus 2002, Popp 2004, 2006, Gerlagh and Lise 2005). The level of the subsidy now depends on the stepping on toes effects in the abatement sector (ψ), and the costs or benefits of pulling research effort from other sectors (κ). With infinite patents, the private sector captures the entire social value of knowledge.[17] However, as innovators increasingly develop the same knowledge as other innovators when their expenditures increase, research has a negative externality ($\psi<1$) and a tax is appropriate. On the other hand, if other sectors have

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[17] In reality, infinite patents may not be sufficient for the innovator to capture the full social value of knowledge. As mentioned in footnote 11, we do not model the "standing on shoulder" mechanism. Inclusion of this mechanism would likely imply that even with infinite patents, we would get similar result as in Proposition 5, i.e., that the R&D subsidy initially should decline over time.

similar negative research externality characteristics, we should expect that $\kappa<1$, too, reducing the optimal tax level. The proposition suggests that, in the case of infinite patents, abatement research should face the same tax or subsidy as other research activities, given that the different research activities have similar characteristics. Indeed, this also seems intuitive when abatement is not a different type of activity when compared to other sectors.

As noted in the introduction, the abatement sector differs from other sectors through its cyclical behaviour as studied through the short-term analysis of the previous sections. In the case of finite patents, that is, when innovation markets are incomplete, the cyclical behaviour is cause for a non-constant subsidy level. This case is highly relevant, as real-world patent lifetime is not infinite.[18] Moreover, as explained in the previous section, imitation of patented innovations implies that the effective patent lifetime may be finite even if the legal patent lifetime were set to infinity. Hence, considering finite patent lifetimes seems more relevant than infinite patents. The following proposition states that if patents have *constant* but finite lifetime, we must dynamically adjust the research subsidy to implement the first best.[19]

PROPOSITION 5. *Consider the case that patents have constant finite lifetime, $L_t=L<\infty$, and the initial knowledge stock is zero, $h_0=0$. Then there is a t^* with $T–L<t^*<T$ such that the research subsidy that implements the first-best decreases monotonically for $0\leq t\leq t^*$, and increases afterwards (for $t\geq t^*$).*

This proposition is consistent with the first proposition of Gerlagh et al. (2009). However, whereas the result in Gerlagh et al. (ibid) is derived by invoking assumptions on the path of the abatement sector, Proposition 5 follows from the propositions above by deriving the path dynamics. The full proof is provided in Appendix 2, but the conceptual mechanisms are readily understood, using Figure 1 below.

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[18] For instance, patent lengths in the US and the EU are 20 years.

[19] Creative destruction (cf. the previous section) may imply that the social value of an innovation goes to zero before the (finite) patent expires. If this was the case in general, and there were no imitation or "standing on shoulder" either (cf. footnote 17), we would be back to Proposition 4 with constant research subsidy over time.
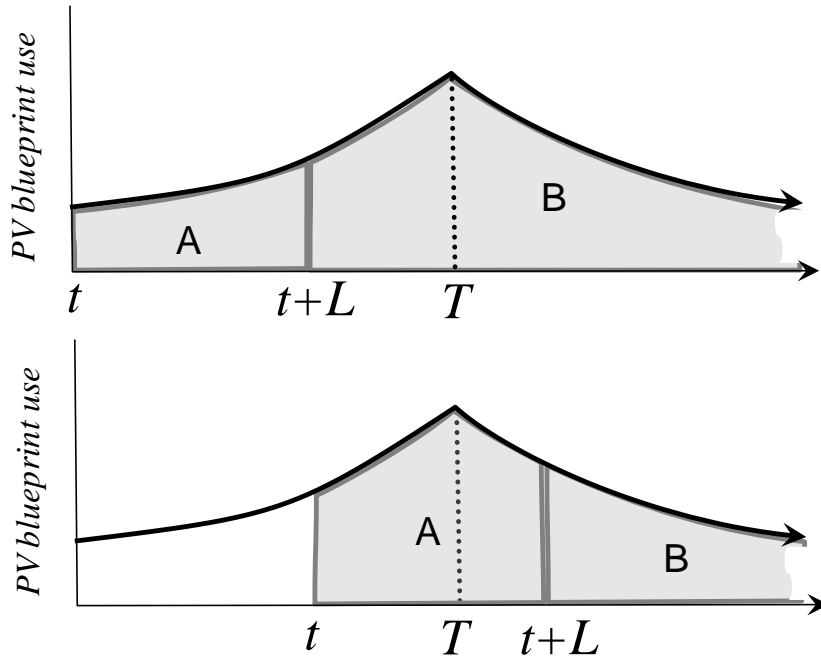
FIGURE 1. *Dynamics of Private versus Social value of blueprints*

The figure shows schematically the rent value of a blueprint for abatement technology, i.e., π_t in (21), evaluated at time zero (discounted). In the early stages, the price of emissions and the use of blueprints are low, so that the rent value is low. As the emission price grows rapidly, faster than the interest rate, the present value rent goes up from $t=0$ to $t=T$. After the first phase of rapid growth, from time T onwards, the growth of abatement drops to the growth of benchmark emissions Y_t. The intensity in the use of knowledge grows slower and the present value decreases. In the figure, at time t, the private value of a new patent is equal to the aggregate rent value over the next L periods, that is, area A. The social value is equal to the private value plus the rent value after expiration, $A+B$. The increase and decline shown in the figure resemble empirical data for SO_2 abatement technologies: Dekker et al. (2012, Fig. 2) show the number of SO_2-reducing (mother) patents spiked around 1985, a few years before the 1990's when more stringent SO_2 standards were implemented and emission reductions in signatory countries were at their highest.[20] SO_2 is an interesting example to evaluate as its emissions and (mother) innovations peaked a few decades ago in most industrialized countries.[21]

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[20] We consider 5-year moving averages of changes in emissions in signatory countries, using data from http://sedac.ciesin.columbia.edu/data/set/haso2-anthro-sulfur-dioxide-emissions-1850-2005-v2-86

[21] SO_2 is considered a flow pollutant when local air quality is concerned, though acid-rain causes longer-lasting damages. The stock pollutant model in the main text is not directly applicable, but the model of Appendix 3 captures a flow pollutant equally well, assuming that the social costs of SO_2 emissions are increasing in income, which we find to be a reasonable assumption. A natural interpretation of T in this case is then when emission reductions peak. Hence, given patent lengths of

It is immediately clear from the top diagram that in the early phase, the private value A is small compared to the social value $A+B$. With finite patent lifetime, the private benefits of innovation will typically be low compared to the social benefits. Consequently, the optimal subsidy should be relatively high.

As time passes, and we move from the top to the bottom diagram, the share of private value A in total social value $A+B$ increases. That is, the main benefits of the technology come at later stages, when the price of emissions has risen. Innovations developed during this stage yield a high rent value to the innovators, during the lifetime of the patent, and thus the need for research subsidies diminishes.[22] A straightforward interpretation of our results is that initially climate policy should focus on knowledge development, while employment of abatement technology becomes relatively more important at a later stage of the policy cycle.

From the figure we can also see that the higher the lifetime of the patent L is, the larger is the share of private value A of the total social value $A+B$. Thus, this will lower the optimal subsidy both at present and in the future. Note, however, that the time path of the subsidy still follows from Proposition 5 as long as $L<\infty$.

To understand why optimal research subsidies go up again after t^*, we need a more subtle argument. Innovations rapidly increase the knowledge stock during the first phase, but at time T, the level of knowledge has still not reached the balanced growth level. This means that the growth rate of knowledge is still high and decreasing, and consistently the intensity of knowledge use, which is the rent value of blueprints, is rapidly decreasing. But if the rent value is rapidly decreasing, that means that the current rents, which make up the private value, are high compared to future rents, which make up the social value. That is, at t^* the ratio between the private and social value of knowledge is *above* its balanced growth level. In the case where we consider the balanced growth state as the reference in which no environment-specific research policy is warranted (because environmental research externalities are no longer fundamentally different from general research externalities), the implication is that at the peak of carbon prices, optimal clean-energy research subsidies could be negative. Over time, as the knowledge stock reaches its balanced growth path, the private versus social value of knowledge goes down and converges to a constant ratio.[23] Based on this, the private value falls more rapidly before convergence than the social value, and, therefore, the subsidy goes up and converges.

From this last argument, it also becomes clear that the last part of the proposition is reversed if the initial knowledge stock h_0 is sufficiently large so that knowledge at $t=T$ exceeds the balanced growth level, $h_T>h^*$, In that special case,

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20 years (cf. footnote 18), the peak in innovations for SO_2 around ten years ahead of the peak in emission reductions seems to be in accordance with our model results.

[22] Qualitatively, the argument does not rely on the indefinite future use of technologies. In Appendix 4 we present the figures for technologies for which the social value diminishes to zero after some time. Gerlagh et al. (2009) find the same result numerically for the case when technologies do not remain in use forever.

[23] Appendix 2 shows (Figure 5) and discusses the profile of the growth rate.

the research subsidy that implements the first-best decreases monotonically for all time t.

Rather than varying the research subsidy over time, we could instead adjust the patent lifetime. Though there are various practical problems to dynamically adjust the patent lifetime, the question of the optimal patent lifetime is considered a relevant question in the literature (cf Futagami and Iwaisako, 2007). For completeness we thus translate the above result to the dynamic lifetime context:

PROPOSITION 6. *Consider the case with constant research subsidies, $\sigma_t=\sigma$, and a varying patent lifetime L_t. Then there is a t^* with $T-L_t<t^*<T$ such that the first-best patent lifetime decreases monotonically for $0 \leq t \leq t^*$, and increases afterwards (for $t \geq t^*$).*

Proof: Similar as for Proposition 5, see Appendix 2, but with L_t instead of ratio V_t/η_t. ∎

We notice that granting longer patent lifetime is not without social costs. As they grant longer monopoly power, they distort future production, or alternatively, require future public funds to correct for market power in the market for abatement equipments. On the other hand, the need for public funding of R&D is reduced accordingly.

Together, Propositions 4-6 show that policy makers have some flexibility in their choice of research policy. They can either choose an infinite patent lifetime combined with a fixed subsidy/tax on research (Proposition 4), or, if they want to avoid infinite patent lifetime, they can pick a constant research subsidy *or* patent lifetime, and adjust the other instrument in line with the stage of the climate problem (Proposition 5 and Proposition 6). That is, with incomplete innovation markets, there is a clear link between the first-best innovation policy and the stage of an environmental problem.

5. GENERAL VS PARTIAL EQUILIBRIUM

A more precise interpretation takes into consideration the partial equilibrium context of our analysis. In general equilibrium, there is competition for inputs to research and development, which means that an increase in abatement-related research can crowd out other research and dampen overall growth. Also, we do not explicitly model the distortionary effects of taxes needed to pay for research subsidies. Such issues are in the domain of general equilibrium models (Bovenberg and Smulders 1995). Integrated models are useful to answer general questions for balanced growth paths, e.g. whether clean R&D should receive special treatment compared to R&D, generally. On the other hand, Golosov et al. (2011) and Gerlagh and Liski (2012) find that general equilibrium analyses make almost no difference to a partial analysis. Moreover, to consider explicitly the dynamic and cyclical nature of the climate is practically impossible in a general equilibrium setting; a partial analysis that studies the transitional dynamics over the life-cycle of the climate problem is more appropriate.

Anyway, the qualitative outcome of our analysis is intuitive and can be superimposed on results from general equilibrium models. Proposition 4 informs us that, if patents have infinite lifetime in all sectors, then the cyclical nature of the climate problem has no traction on optimal policy. Proposition 5 informs us that, if patents have constant finite lifetime, then clean innovation policy should be dynamically adjusted vis-a-vis a typical balanced growth general equilibrium analysis. When clean energy needs a quick start to address the emerging climate change problem, the private value of patents is relatively low compared to the social value of the increase in knowledge, and more public support for innovation is warranted. At a later stage, when the clean technology has matured, it needs less support. For dirty fossil fuel technologies (e.g. tar sands), an inverse pattern holds. When the future use of a particular cluster of dirty technologies will drop as part of a policy to address an environmental problem, then the private value of patents might still be high due to the expected use of the technology in the next couple of decades, but the social value of knowledge is relatively small, because of the expected reduction in the use of the technology in the longer term. The analysis thus supports a more favourable fiscal treatment of research for renewable energy and a less favourable fiscal treatment for research in fossil fuel exploration, but only for the coming decades.

Another caveat of the analysis is its limited attention to practical constraints on innovation policies. The first-best policy is impractical to implement, especially in the early stage of development when the optimal subsidy rate (patent lifetime) is very high (long). For instance, in reality the public agent cannot provide near 100 per cent subsidy to research firms, without strict control of the research effort carried out.[24] Infinite patents also cannot be implemented in practice. Public R&D can be employed as a proxy optimal policy at the early stages of development, where private R&D firms take over at a later stage.

6. CONCLUSION

In this paper we have studied the links between (the time path of) innovation policies and abatement policies under different assumptions of innovation policies such as the possibilities to use patent lifetime as a policy instrument. The latter follows from a core distinction between our model and earlier R&D models in the environmental economics literature, namely that the lifetime of patents is finite. Our analysis is based on an R&D model supplemented with emission-abatement-pollution dynamics, and four imperfections related to innovations; too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovations due to finite patent lifetime, negative spillovers through stepping on toes effects within the abatement technology sector, and crowding out effects in other sectors. Innovation policy instruments include deployment subsidies to patented equipment, research subsidies (or taxes), and the lifetime of patents. Our main result demonstrates that the positive spillovers of innovations due to finite patent lifetime are particularly strong at the early phase of

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[24] In the EU there is an upper limit to the legitimate rate of R&D subsidy.

the climate problem, and to account for this, optimal innovation policy needs to adjust dynamically.

Our results share the tone of earlier papers on the timing of abatement *efforts* in the sense that we find a focus on technological development in the early phase, while the *use* of abatement technologies mainly occurs at later stages when the technology is more mature. But in terms of *policies*, our findings sketch a different picture, in a subtle way. The efficient carbon tax should equal the Pigouvian tax so that, in this sense, climate policy is independent of innovation dynamics. Innovation policy, however, changes with the nature of the climate problem. If the patent lifetime is finite, the optimal subsidy starts at a high level, giving an incentive to accelerate R&D investments, and then falls over time as the climate problem becomes more immediate. In a similar way, if the research subsidy is constant, the optimal lifetime of a patent should be very high initially and then fall. This result on innovation policy signals an important difference with previous energy-emissions-environment models with innovation, where implicitly infinite patents are assumed.

The intuition behind our results is that, at the early stages when the climate problem emerges, the private incentives for innovation are modest only, while the social benefits are large. Over time, private incentives increase more relative to the social value. The reason for the modest private incentives is that, typically, the price of emissions and the value of total abatement activity is low, initially. As a result, the private value of owning technological knowledge is modest, and the incentive to innovate is low. Yet, the social value of knowledge also includes gains further in time, beyond the patent expiration date. When time passes and the future benefits of knowledge enter the patent's lifetime horizon, they increase the private incentive and the need for fiscal compensation decreases.

Thus, climate change calls for public intervention, not only through emission or resource use taxes, but also through subsidies or other measures that stimulate clean energy innovation. Possible measures include public R&D, targeted subsidies on private clean R&D, or longer lifetime patents.

APPENDIX 1: ANALYZING THE SOCIAL OPTIMUM

For the analysis of the social optimum, it is convenient to use the intensive form of the stock variables, i.e., knowledge H_t and cumulative absorption S_t per benchmark emissions Y_t. The equation system does not have constant returns to scale, however, so the intensive form uses a non-linear normalization for knowledge H_t. The lemma below specifies the normalization that defines the analysis in intensive form. We omit time subscripts for convenience.

LEMMA 1. *Let* $h_t = H_t / Y_t^{\gamma\psi}$ *and* $s_t = S_t / Y_t$ *with* $\gamma = \dfrac{1}{\psi + \beta(1-\psi)} > 1$ *Then there exists a function* $v(h,s)$, $v_h < 0$, $v_s < 0$, *such that present value costs satisfy*

$$V(H,S;Y) = Y^{\gamma} v(HY^{-\gamma\psi}, SY^{-1}) = Y^{\gamma} v(h,s) \qquad (30)$$

Proof. We make two notes about notation. First, even though the equation holds for any value of H, S and Y, it is convenient for the proof to consider the values as initial conditions, i.e., with the $t=0$ subscript. Second, whereas the intensive form for the absorption capacity has to be defined by S/Y, for the knowledge intensive form we could equally use $H^{1/\gamma\psi}/Y$, or $H/Y^{\gamma\psi}$. It turns out more convenient to use the latter.

Consider two initial conditions (H_0,S_0,Y_0) and (H_0^b,S_0^b,Y_0^b) with $S_0^b/S_0=Y_0^b/Y_0=\lambda$, and $H_0^b/H_0=\lambda^{\psi\gamma}$, where λ is the scale ratio between the two initial conditions. We must now show that costs satisfy $V^b/V=\lambda^{\gamma}$. To do this, we show that for any feasible path (x_t,R_t) for the initial conditions (H_0,S_0,Y_0) with associated present value costs V, we can construct a solution (x_t^b,R_t^b) for (H_0^b,S_0^b,Y_0^b) with present value costs $V^b/V=\lambda^{\gamma}$. The construction guarantees that $V^b/V \le \lambda^{\gamma}$. Using the inverse construction provides the weak inequality the other way around.

We construct the path (x_t^b,R_t^b) that maintains the ratios $H_t^b/H_t=\lambda^{\psi\gamma}$ and $S_t^b/S_t=\lambda$ for all t. For this purpose, take $R_t^b=\lambda^{\gamma}R_t$, which ensures that $H_t^b/H_t=\lambda^{\psi\gamma}$ throughout. Furthermore, we take $x_t^b=\lambda^{(1-\psi)\gamma}x_t$, so that the abatement ratio is proportional to the gross emissions ratio λ, $A_t^b = H_t^b(x_t^b)^{\beta} = \lambda^{\psi\gamma+\beta(1-\psi)\gamma}H_t(x_t)^{\beta} = \lambda A_t$, while costs increase by factor λ^{γ}: $H_t^b x_t^b = \lambda^{\psi\gamma+(1-\psi)\gamma}H_t x_t = \lambda^{\gamma} H_t x_t$. Q.E.D.

The normalization that is implied by the lemma is as follows: $h_t = H_t / Y_t^{\gamma\psi}$, $s_t = S_t / Y_t$, $\chi_t = x_t / Y_t^{\gamma(1-\psi)}$, $r_t = R_t / Y_t^{\gamma}$, $p_t = v_h(.) = Y_t^{\gamma(\psi-1)}V_H(.)$ and $q_t = v_s(.) = Y_t^{1-\gamma}V_S(.)$, *with* $\gamma = \dfrac{1}{\psi + \beta(1-\psi)} > 1$. Notice that the normalization implies $h_t\chi_t^{\beta}=1$ if $E_t=0$ and $h_t\chi_t^{\beta}<1$ if $E_t>0$, and that social abatement costs become

$$H_t x_t + \kappa R_t = Y_t^{\gamma}(h_t \chi_t + \kappa r_t) . \qquad (31)$$

On a balanced growth path, normalized variables remain constant and the social abatement costs increase at rate γg. To ensure finite net present costs, we require that the discount rate is at least equally large:

$$\rho > \gamma g. \tag{32}$$

Lemma 1 informs us that we can conveniently analyze the dynamics using normalized variables, h_t for H_t, s_t for S_t, χ_t for x_t, e_t for E_t, r_t for R_t, $p_t=v_h(.)$ for the shadow price of knowledge, and $q_t=v_s(.)$ for the shadow price of the emission absorption scarcity.

Bellman's principle tells us that the relation (30) holds for all t, and that two optimal paths will not cross in (h_t,s_t) space. Thus, the lemma shows that the dynamics of the social optimum are fully captured through the two state variables h_t and s_t, and their dual variables p_t and q_t. The dynamics for the pollution stock s_t and the knowledge stock h_t, from (1) and (6), are in intensive form rewritten as follows:

$$\dot{s}_t = -gs_t - (1 - h\chi^\beta) \tag{33}$$

$$\dot{h}_t = r_t^\psi - \frac{\psi}{\psi + \beta(1-\psi)} gh_t \tag{34}$$

The difference between the extensive and intensive form is that both the pollution absorption capacity s_t as well as the knowledge stock h_t have a tendency to decrease, relative to the overall size of the economy, all other things equal. Also, notice that since $\dot{S}_t \leq 0$, and $\dot{Y}_t > 0$, we must have $\dot{s}_t < 0$ iff $s_t > 0$, and $\dot{s}_t = 0$ iff $s_t = 0$.

First order conditions in intensive form

We can rewrite the first-order conditions for the social optimum from the main text in intensive form, with $p_t = Y_t^{\gamma(\psi-1)}\eta_t$ and $q_t = Y_t^{1-\gamma}\theta_t$, to derive

$$\chi_t = (\beta q_t)^{1/1-\beta} - \varepsilon \tag{35}$$
$$r_t = (\psi p_t/\kappa)^{1/1-\psi} \tag{36}$$
$$\dot{q} = [\rho - (\gamma-1)g]q - \lambda \tag{37}$$
$$\dot{p} = [\rho - \gamma(1-\psi)g]p - (\beta^{-1}-1)\chi \tag{38}$$
$$\lambda s=0; \ \varepsilon e=0 \tag{39}$$

Long-term dynamics

We first establish properties for the long run, when $s_t=0$, and thus also $e_t=0$. Define the time T as the earliest time at which $s_T=0$. In the long run, the absorption capacity is fully exhausted and we only need to analyze the dynamics for the knowledge stock h_t, and its co-state variable p_t. Since emissions are zero, we have $Y_t = H_t x_t^\beta$, which we can rewrite as $h_t\chi_t^\beta=1$. By substitution of (36) in (34), and of $h_t\chi_t^\beta=1$ in (38), we find the two-equation dynamics for the state-co-states h_t and p_t:

$$\dot{h}_t = (\psi p_t / \kappa)^{\psi/(1-\psi)} - \gamma\psi g h_t \tag{40}$$

$$\dot{p}_t = [\rho - \gamma(1-\psi)g]p_t - (\beta^{-1} - 1)h_t^{-1/\beta} \tag{41}$$

The state-co-state dynamics produce the phase diagram depicted in Figure 2. As explained above, we require that $\rho > \gamma g$ in order to ensure finite net present costs, Thus, the locus for $\dot{p}_t = 0$ lies in the positive quadrant and slopes downwards.
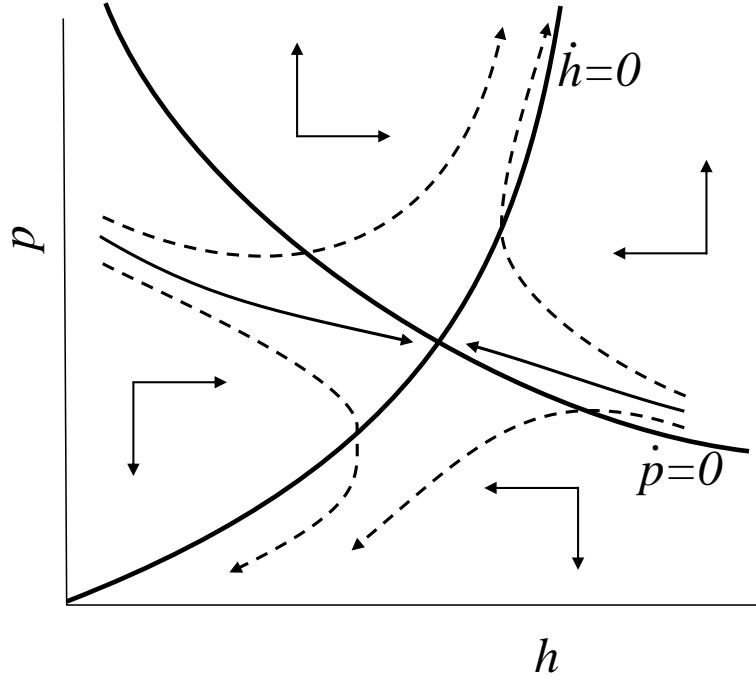

FIGURE 2. *Phase diagram for the long-run optimal path*

It is immediately clear from the phase diagram that a unique balanced growth path exists where the normalized variables $h_t = h^*$, $\chi_t = \chi^*$, $p_t = p^*$ and $q_t = q^*$ are constant. Furthermore, the balanced growth path has saddle-point stability, and the unique saddle path to the balanced growth path has h increasing (decreasing) and p decreasing (increasing). Thus, when the initial knowledge stock is below the balanced growth level, $h_t < h^*$, the balanced growth path is approached from the upper-left and the price of knowledge p_t decreases. Along this path, the growth rate of h_t will decrease as it is increasing in p_t and decreasing in h_t (cf. (40)). From $h_t \chi_t^\beta = 1$, it then follows that χ_t (and thus also x_t) will have an increasing growth rate. We summarize this in the Proposition 1 in the main text, where we use the fact that

$\dot{H}_t/H_t = \gamma\psi g + \dot{h}_t/h_t$ and $\dot{x}_t/x_t = \gamma(1-\psi)g + \dot{\chi}_t/\chi_t$.[25]

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[25] The lower and upper bounds for the growth rates of H_t and x_t follows from the definitions of h_t and χ_t.

Short-term dynamics

To analyze the short term, we run the dynamics of (33)-(39) backwards in time. That is, we take some pair (h_T, p_T) on the stable manifold of Figure 2, and let $\lambda_T = \varepsilon_T = 0$. Then we consider what happens if $\lambda_t = \varepsilon_t = 0$ for all $t \leq T$. The emissions shadow price q_t increases exponentially at rate $\rho - (\gamma - 1)g > 0$ up to $t = T$ (cf. (37)), and so χ_t increases at rate $[\rho - (\gamma - 1)g]/(1 - \beta) > 0$ (cf. (35)). Thus, if the path enters balanced growth at $t = T$, so that (40) and (41) are zero for $t = T$, it follows that the right-hand-side of (38) is positive for $t < T$. That is, p_t increases for $t < T$. It then follows from (36) that r_t is also increasing for $t < T$. Hence, the right-hand-side of (34) is negative, so that h_t decreases for $t < T$. The path is depicted as line *B* in Figure 3. If we include the dynamics for s_t in (33), we can construct a corresponding path $\{(s_t, h_t)\}$ that goes backwards in time from $t = T$ to $t = 0$. From Bellman's principle it is then obvious that any element on this path can be taken as initial condition. This path is depicted as line *B* in Figure 4. Proposition 2 describes the features of this line.
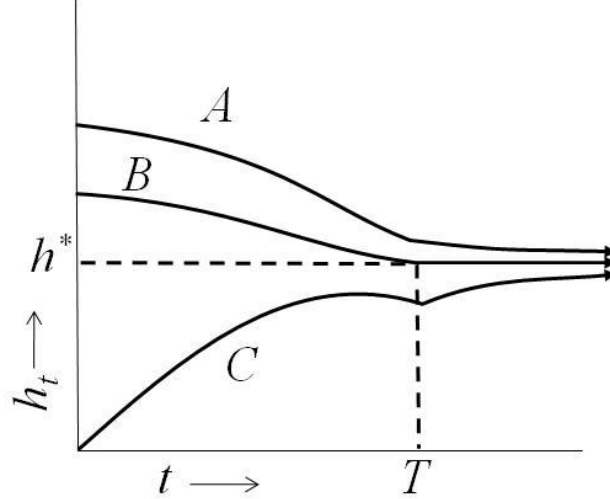

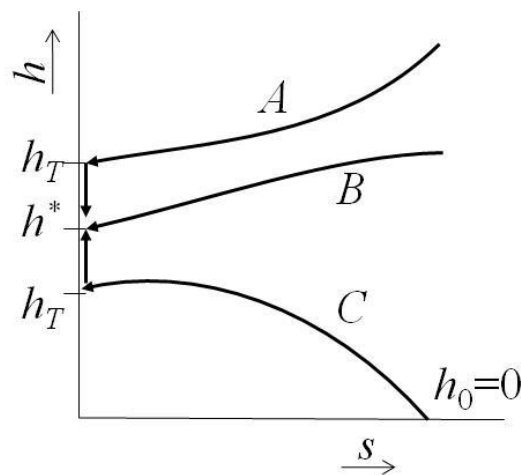FIGURE 3. *Dynamics of knowledge*

FIGURE 4. *Dynamics in state-space*

Bellman's principle also informs us that in state space as shown in Figure 4, optimal paths cannot cross. Therefore, as all paths move to the left (cf. (33)), any initial

condition (s_0, h_0) with h_0 below (above) line B will reach $s_T=0$ in finite time with $h_T<h^*$ $(h_T>h^*)$, cf. line C (A) in Figure 4. This proves Proposition 3.

APPENDIX 2: PROOF OF PROPOSITION 5

PROPOSITION 5. *Consider the case that patents have constant finite lifetime, $L_t=L<\infty$, and the initial knowledge stock is zero, $h_0=0$. Then there is a t^* with $T-L<t^*<T$ such that the research subsidy that implements the first-best decreases monotonically for $0\leq t\leq t^*$, and increases afterwards (for $t\geq t^*$).*

From (29), we can see that $\dot{\sigma}_t>0$ iff V_t/η_t decreases. To study optimal research subsidies, we thus must understand the dynamics of the value of innovations, both the social perspective (η_t) as from the private perspective (V_t). The innovation value depends on the development of the use of equipment, x_t, over time. From (9) and (10) we have that in the short-run ($\varepsilon_t=\lambda_t=0$), i.e., for $t<T$,

$$\dot{x}_t/x_t=\rho/(1-\beta). \tag{42}$$

Combining Proposition 1 and Proposition 3, we know that if $h_0=0$, then at $t=T$, $h_T<h^*$ and the growth rate for x_t will sharply drop and then slowly increase towards a level below the initial growth rate (since $\rho/(1-\beta)>(1-\psi)\gamma g$). The growth in the use of abatement equipment path looks as in Figure 5, where the dotted line denotes the balanced growth level $(1-\psi)\gamma g$.
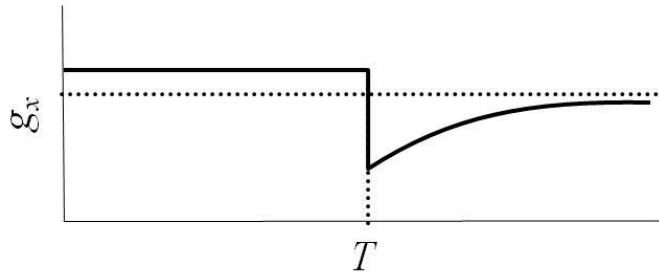

FIGURE 5. *Dynamics of abatement intensity growth, $\dot{x}_t/x_t$*

Given the growth profile for abatement intensity x_t as depicted in Figure 5, we can derive the ratio between future intensity x_{t+L} and x_t as in Figure 6.
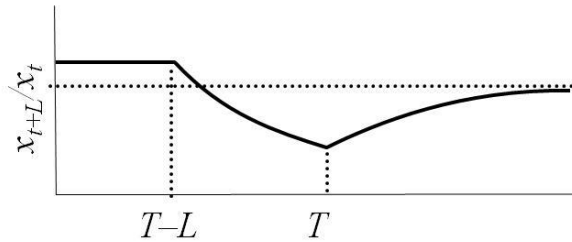

FIGURE 6. *Dynamics of x_{t+L}/x_t*

From (12) and (25) we have that

$$\frac{\dot{\eta}_t}{\eta_t} = \rho - (\beta^{-1} - 1)\frac{x_t}{\eta_t} \quad \text{and} \quad \frac{\dot{V}_t}{V_t} = \rho - \frac{x_t - e^{-\rho L}x_{t+L}}{V_t} \tag{43}$$

and from here we get the dynamic development of V_t/η_t

$$\frac{\partial}{\partial t}\frac{V_t}{\eta_t} > 0 \Leftrightarrow \frac{x_t - e^{-\rho L}x_{t+L}}{V_t} < \frac{(\beta^{-1} - 1)x_t}{\eta_t} \Leftrightarrow \frac{x_{t+L}}{x_t} > e^{\rho L}(1 - \frac{(\beta^{-1} - 1)V_t}{\eta_t}). \tag{44}$$

Thus, we can draw the dynamics of V_t/η_t in a sort of phase diagram where we consider the dynamics of V_t/η_t versus x_{t+L}/x_t.
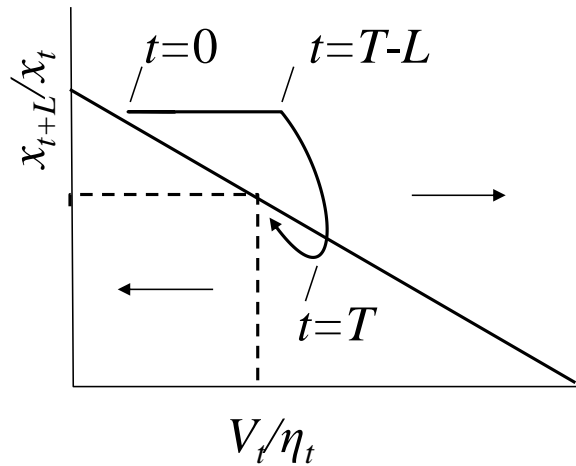

FIGURE 7. *Dynamics of* V_t/η_t

There is a downward sloping locus such that V_t/η_t is constant. Above the locus, the ratio of the private value versus the social value of innovations is increasing, below the locus, the ratio is decreasing. If the equilibrium path crosses the locus, it will cross it vertically turning clockwise. The long-term steady state will be on the locus. From the dynamics of x_{t+L}/x_t in Figure 6, we know that the steady state will be approached from below. Combining this insight with the phase dynamics, the path must approach the steady state from south-east. The path cannot cross the locus for $t>T$. At $t=T$, x_{t+L}/x_t has a minimum, and thus, going back in time, the path moves north-east. Going further back in time, from $t=T$ to $t=T-L$, the ratio x_{t+L}/x_t increases up to a level above the steady state, and thus, the path must cross the locus and move north-west. Finally, from $t=T-L$ to $t=0$, the ratio x_{t+L}/x_t is constant and the path must move horizontally. The path as drawn in Figure 7 follows. We see that the V_t/η_t increases from $t=0$ onwards until the path crosses the locus for some t^* with $T-L<t^*<T$, after which the ratio V_t/η_t decreases. The research subsidy follows an inverse pattern. Q.E.D.

APPENDIX 3: FIXED GROWTH RATES FOR THE EMISSIONS PRICES

Here we consider the case where there is no ceiling on cumulative emissions, but instead the emissions price τ is exogenous and increases at a rate equal to the income growth rate (g). In this case the emissions stock S does no longer play a role in the optimization problem. Hence, we do not need equation (1). The social optimum now becomes

$$V(H_0, S_0, Y_0) = \min \quad \int_0^\infty e^{-\rho t}[e^{gt}\tau E_t + H_t x_t + \kappa R_t]\mathrm{d}t, \tag{45}$$

subject to the knowledge accumulation equation (6), and production equations (2) and (4), with x_t, and R_t as the control variables.

The current value Hamiltonian becomes:

$$\mathcal{H}_t = e^{gt}\tau E_t + H_t x_t + \kappa R_t - \eta_t \dot{H}_t - \varepsilon_t E_t, \tag{46}$$

The first-order conditions become:

$$0 = \mathcal{H}_x = H - \beta(e^{gt}\tau - \varepsilon)H x^{\beta - 1} \tag{47}$$

$$0 = \mathcal{H}_R = \kappa - \psi\eta R^{\psi - 1} \tag{48}$$

$$\dot{\eta} = \rho\eta + \mathcal{H}_H = \rho\eta - (\beta^{-1} - 1)x \tag{49}$$

$$\varepsilon E = 0 \tag{50}$$

We use again the intensive form. Then we can show that the equation dynamics for h and p, i.e., (34) and (38), are the same as before:

$$\dot{h}_t = r_t^\psi - \frac{\psi}{\psi + \beta(1 - \psi)}gh_t \tag{51}$$

$$\dot{p} = [\rho - \frac{1 - \psi}{\psi + \beta(1 - \psi)}g]p - (\beta^{-1} - 1)\chi \tag{52}$$

In the long-term dynamics, when emissions are zero, we then have equations (40)-(41):

$$\dot{h}_t = (\psi p_t / \kappa)^{\psi/(1-\psi)} - \frac{\psi}{\psi + \beta(1 - \psi)}gh_t \tag{53}$$

$$\dot{p}_t = [\rho - \frac{1 - \psi}{\psi + \beta(1 - \psi)}g]p_t - (\beta^{-1} - 1)h_t^{-1/\beta} \tag{54}$$

Hence, the phase diagram in Figure 2 is the same, and Proposition 1 still holds.

In the short term, we consider what happens when we move backwards in time from time $t = T$, where time T is the earliest time at which emissions are zero. If the path enters balanced growth at $t = T$, so that (40) and (41) are both zero at time T, then the RHS of (38) is positive for $t < T$ since $\chi < h^{-1/\beta}$ when $E > 0$. Hence, p increases for $t < T$. Furthermore, it follows from (36) that r must increase for $t < T$. Hence, the RHS of (34) is negative for $t < T$, implying that h decreases for $t < T$. Thus, using the same arguments as in Section 2, we have that Propositions 2-3, and subsequently Propositions 4-6 also carry over to the case with an exogenously increasing emissions price.

APPENDIX 4: WHEN KNOWLEDGE DEPRECIATES

Here we present Figure 8, for the case when the social value of new technologies diminish to zero after some time
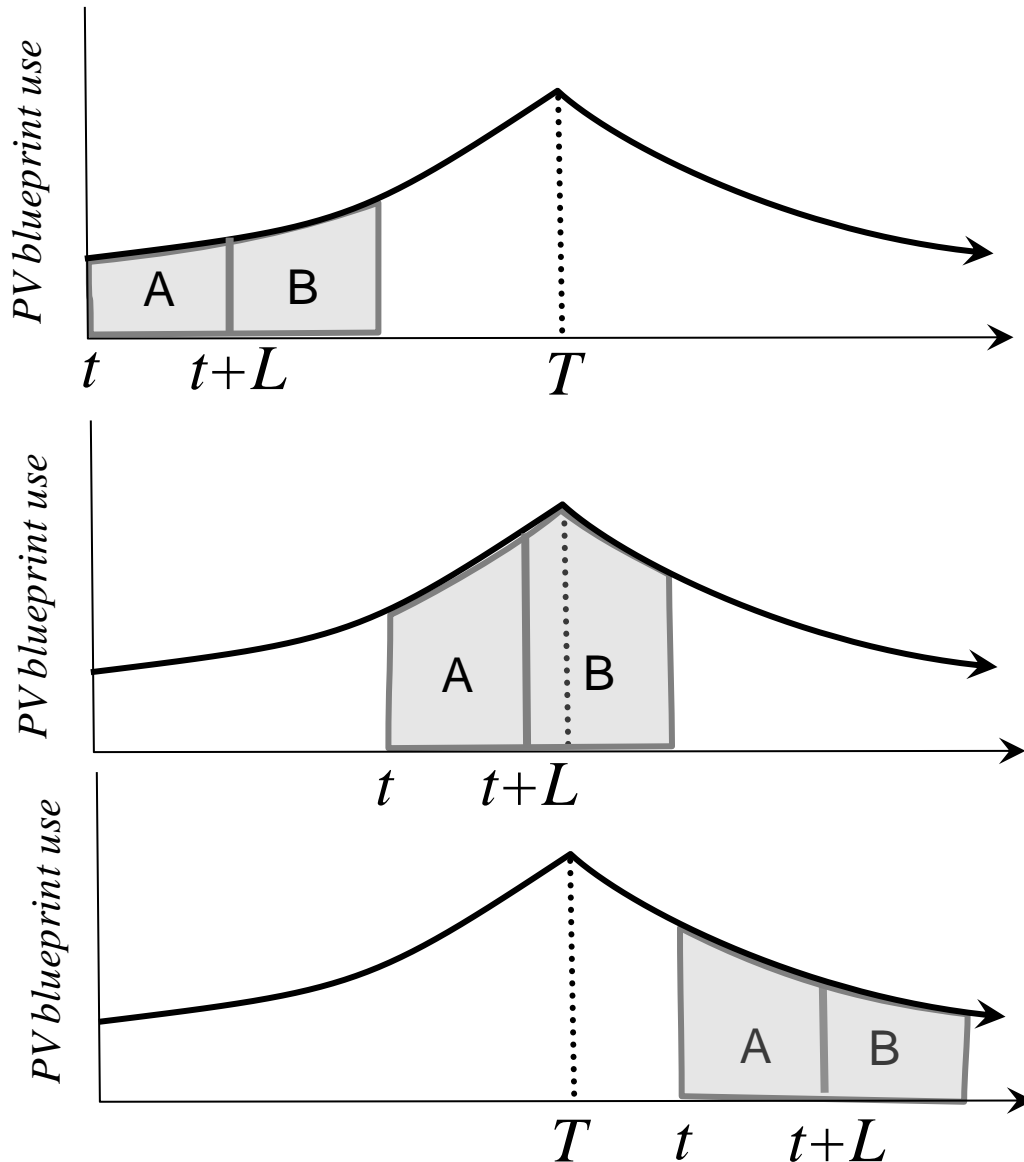
FIGURE 8. *Dynamics of Private versus Social value of blueprints*

In the figure, at time *t*, the private value of a new patent is equal to the aggregate rent value over the next *L* periods, that is, area *A*. The social value is equal to the private value plus the rent value after expiration, *A+B*. The ratio of the private value versus the social value is determined by the relative size of the areas A and B. For expositional purpose, we draw the figure for the case when patent life-time equals half the time of use of the innovation. Top diagram shows the early phase, when the private value *A* is small compared to the social value *A+B*. Consequently, the optimal subsidy should be relatively high. As time passes, and we move from the top to the next diagram, and to the bottom diagram. The share of the private value *A* in total social value *A+B* increases. One needs extra conditions on the slopes of the curves to prove monotonicity, but it is clear from the diagram that the share of the private value is larger in the top panel vis-a-vis the bottom panel. Innovations developed during the latter stage yield a high rent value to the innovators, during the lifetime of the patent, and thus the need for research subsidies diminishes.

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