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The Norwegian Electric Vehicle Policy and the Excess Burden of Taxation

Is the Norwegian electric vehicle policy economically efficient?



The Norwegian Electric Vehicle Policy and the Excess Burden of Taxation

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Abstract

In Norway the sale of electric vehicles per capita is higher than in any other country. The high share of electric vehicles has been achieved by a broad range of government incentives which have been introduced to help Norway reduce GHG emissions. The electric vehicle is more environmentally friendly, but produces approximately the same amount of road use externalities as the conventional vehicle. According to literature on optimal environmental taxation the electric vehicle should therefore be levied with a tax equal to its social marginal damage instead of being subjected to a subsidy. The incentives have distorted consumer choices in the market and created a range of adverse effects. Previous studies find that even though the policy is goal effective, it is not necessarily cost-effective. This thesis examines the economic efficiency of the incentives by using traffic count data from the Oslo region to estimate the excess burden of the Norwegian electric vehicle policy in a second best world.

My findings agree with previous research that suggest that the electric vehicle is attractive as long as the incentives are strong enough. My results also suggest that there has been a substitution away from public transportation as a result of the incentives.

Given the current taxation on conventional vehicle my estimations suggest that it is efficient to reduce the benefits for the electric vehicle user by 10-20%. This corresponds to a approximately 65% decrease in the predicted market share for electric vehicles, and a 2.20% decrease in estimated excess burden compared to the current situation. On the other hand, keeping the incentives fixed, the results suggest that it is efficient to reduce the current taxation of the conventional vehicle by 7.3%. This corresponds to only a 49.54% decrease of the predicted market share for electric vehicles and a 0.72% decrease in estimated excess burden compared to the current in the predicted market share for electric vehicles and a 0.72% decrease in estimated excess burden compared to the current situation.

In addition, the estimated optimal taxation of the conventional vehicle and the corresponding changes in the estimated excess burden compared to the current situation is predicted for different changes in the Norwegian electric vehicle policy. I find that there is a positive correlation between a decrease in benefits and increase in the estimated optimal taxation on conventional vehicles. As benefits decrease the estimated optimal taxation increases until it reaches a upper limit where the taxation is so close to external cost that changes in demand no longer matter.

My results also suggest that there exist a similar lower limit for the estimated excess burden. Implying that after this limit is reached further decreasing the benefits have no effect on the estimated excess burden. The limit is a 2.87% decrease in excess burden and corresponds to the reduction in excess burden from introducing Pigovian tax. This implies that second best optima can be reached without introducing a Pigovian tax. An optimal Pigovian tax would result in a 99.8% lower predicted traffic market share for the electric vehicle. For what level of benefits the lower limit of excess burden is reached depends on the estimated cost of externalities. For the estimates used in this analysis the limit is reached for a 50-75% removal of benefits. This corresponds to a 95-99% decrease in predicted traffic market share for electric vehicle. For comparison a 2.4% decrease in estimated excess burden can be reached by a 25% decrease in benefits. This results in a 80% decrease in the predicted market share for electric vehicles.

My estimations show that there are only a small percentage decrease in estimated excess burden to gain by decreasing the electric vehicle benefits and that this corresponds to a large predicted decrease in the market share for electric vehicle.

Sammendrag

I Norge er salget av el-biler per innbygger høyere enn i noe annet land i verden. Den høye andelen av el-biler kan forklares av det eksisterer et bredt spekter av statlige insentiver for bruk av el-bil. Insentivene har blitt introdusert som en del av Norges forpliktelse til å redusere CO_2 utslippet. El-biler er miljøvennlige men produserer, per bil, omtrent en like stor andel av negative eksternaliteter fra bruk på vei som en vanlig bil. I henhold til litteratur om optimal skatt i nærvær av eksternaliteter bør el-bilen dermed underlegges en skatt lik kostnaden av sosial marginal skade og ikke en subsidie. Insentivene har forvridd konsumentenes valg i markedet og er assosiert med en rekke uheldige effekter.

Tidligere studier finner at selv om tiltakene er måleffektive er de ikke nødvendigvis kostnadseffektive. Denne oppgaven undersøker insentivenes økonomiske effektivitet ved å estimere insentivenes dødvektstap i en nest best verden. Alle prediksjoner og estimeringer er gjort på bakgrunn av trafikk data fra Oslo området.

Funnene i min oppgave samsvarer med tidligere undersøkelser som påstår at el-bilen er attraktiv så lenge insentivene er sterke nok. I tillegg viser resultatene mine at det har vært en substituering vekk fra offentlig transport. Gitt dagens skattenivå på vanlige biler viser oppgaven at det er effektivt å redusere el-bil fordelene med omtrent 10-20%. Dette korresponderer med en predikert reduksjon i trafikk markedsandel for el-bilen på 65% og en 2.2% reduksjon i estimert dødvektstap sammenlignet med dagens situasjon. På den andre siden, gitt dagens fordeler, antyder funnene i oppgaven at det er optimalt å redusere dagens skattenivå for vanlige biler med 7.3%. Dette gir en predikert nedgang på 49.25% i trafikk markedsandel for el-bil og en 0.72% reduksjon i estimert dødvektstap sammen lignet med dagens situasjon.

I tillegg estimerer jeg forskjellige optimale skattenivåer for vanlig bil og tilhørende endring i dødvekts tap, gitt forskjellige endringer i el-bil fordeler. Mine funn viser at det er en positiv korrelasjon mellom en nedgang i el-bil fordelene og en økning i estimert optimal skatt på vanlig bil. Samtidig som fordelene reduseres økes den estimerte optimale skatten på vanlig bil inntil den når en øvre grense der ytterligere reduksjon i fordelene ikke lenger spiller noen rolle for estimert optimalt skattenivå. Resultatene mine viser at det eksiterer en lignende nedre grense for reduksjon i estimert dødvektstap. Når denne grensen er nådd vil en ytterligere reduksjon i el-bil fordelene ikke lenger påvirke det estimerte dødvektstapet. Grensen tilsvarer en 2.87% reduksjon i estimert dødvektstap sammenlignet med dagen situasjon og gir samme reduksjon som Pigovian skatt. Pigovian skatt resulterer i en predikert nedgang på 99.8% i trafikk markedsandel for el-bil. For hvilket intensiv intensitet grensen for estimert dødvektstap blir nådd er avhengig av de kostnaden på de estimerte negative eksternalitetene. For estimater brukt i denne analysen nåes denne grenses ved en 50-75% reduksjon i el-bil fordeler. Dette gir en predikert 95-99% reduksjon i trafikk markedsandel for el-bilen. Til sammenligning kan en 2.4% reduksjon i estimert dødvektstap oppnås med en 25% reduksjon i fordeler, noe som gir en predikert reduksjon i markedsandel for elbiler på 80%.

Estimatene mine viser at det er en liten prosentvis nedgang i dødvektstap å tjene på en reduksjon i insentivene på bekostning av en høy predikert nedgang i markedsandel el-biler.

Preface

Working with my master thesis have been a rewarding and frustrating experience. It is scary, for the first time, to attempt solving a open ended problem and produce a sound scientific product. I have made it out of the process alive and with a deepen knowledge of my field of study. (And in the end what matters the most is that I love what I do, and for some reason, I do really love economics.)

I would like to thank Fjellinjen AS for providing me with quantity data on the number of toll passing for the Oslo toll ring. Thanks to my thesis supervisor, Eirik Romstad, for his guidance and for believing in me. Also thanks to Olvar Bergland and Elise Caspersen for providing feedback on my work.

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All errors in this thesis are my own. If you, after reading this thesis, and have any feedback or comments please feel free to contact me.

Kine

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1 Introduction

In Norway the sale of electric vehicles per capita is higher than in any other country (European Energy Review, 2016). This has been achieved by using a broad range of incentives, including access to bus lanes, exemption from VAT and toll charges, which have been added one at a time until the market for electric vehicles responded. The importance of the different government incentives vary and are not equal across geographical locations. The VAT exemptions, exemption from toll charges and access to bus lanes are the two most important incentives listed by users (Figenbaum & Kolbenstvedt, 2013). For Oslo and Akershus, with the long bus lanes and extensive rush hour delays, the time saving from access to bus lanes can be up to 30 minutes (Figenbaum *et al.*, 2015). In addition the large number of toll-passes, low parking coverage, high parking costs and the access to charging station are important explanatory factors behind the high share of electric vehicles in the Oslo area. The full range of incentives can be seen in table 1.

Table 1: The importance of electric vehicle incentives in Norway as listed by users and year for introduction

Incentive	Introduced	Importance
VAT exemption	2001	++
Exemption from registration tax	1996	+
Reduced annual vehicle license fee	1996/2004	+
Free public parking (often with free charging)	1999	+
Free toll roads	1997	++
Reduced rate on ferries	2009	0
Access to bus lanes	2003/2005	++

Source: Figenbaum & Kolbenstvedt (2013)

The electric vehicle incentives have been introduced by the government as a measure to reduce Norway's GHG emissions and encourage users to make the transition from a «dirty» to a «clean» technology. In Norway the largest source of GHG emissions is transport, and approximately 60% of the emissions comes from road traffic (Miljøstatus, 2016).

Norway has the advantage that its electricity is largely produced by hydro-power, and therefore is considered virtually GHG emission free. The electric vehicle incentives contribute to lowering GHG emissions by distorting consumer choice between electric vehicles and conventional vehicles.

1.1 Background

When the first incentives were introduced in 1996, the market for electric vehicles was immature and the incentives had little effect. (Figenbaum & Kolbenstvedt, 2013). Since then there has been a rapid development in electric vehicle technology resulting in longer range, longer life expectancy, better quality and reliability. The charging infrastructure have also improved, and together these developments have helped decrease the disadvantages of the electric vehicle. In addition the government incentives have helped to reduce the risk connected with buying, and using a electric vehicle, and compensated users for the disadvantages of electric vehicle. This has been necessary in order to push the electric market out of the early development phase. Now the electric vehicle market is no longer a niche market, and the large vehicle manufacturers are selling electric vehicles as a standard part of their supply.



Figure 1: Share of electric vehicle in daily traffic in the Oslo area for the period 2011-2015. Data provided by Fjelllinjen AS

The success of an innovation process depends on the knowledge of the technology being spread and the number of people willing to be early adopters. Lack of knowledge of the electric vehicle technology among consumers has been, and still is, a barrier for purchasing an electric vehicle. The widespread use of electric vehicles is also still hindered by the limited battery capacity and the lack of a sufficient charging network (Jing *et al.*, 2016). Common factors listed by non electric vehicle owners as a barrier for purchasing a electric vehicle is vehicle range, access to charging stations, charging time, uncertainty about the incentives, and the second-hand value of the vehicle (Figenbaum *et al.*, 2014; Nygaard, 2015; Figenbaum & Kolbenstvedt, 2013).

Another problem is that parts of Norway suffer from cold weather in the winter and the low temperature is a challenge from the electric vehicle. Users report a 20-50% loss of range in winter (Figenbaum & Kolbenstvedt, 2013). In addition, many people do long vehicle trips for holidays, leisure and business that exceed the upper battery range (Hjorthol *et al.*, 2014). The second-hand value for the electric vehicle is uncertain due to lack of information about battery life time expectancy (Figenbaum & Kolbenstvedt, 2013), the rapid development of technology and price fall of new electric vehicles. Thus, the second hand market for electric vehicles is under developed.

The result has been that most Norwegians buy a electric vehicle as an addition to the household vehicle fleet. For example, Figenbaum & Kolbenstvedt (2013) estimated the number electric vehicle owners with an addition vehicle in its fleet to be as high as 90% for the period between 2006-2013. This makes it possible to replace the conventional vehicle with the electric vehicle in a multi-vehicle household from a mobility perspective. The typical electric vehicle owner uses the electric vehicle for daily travel to and from the work place and trips to the store (Hjorthol *et al.*, 2014).. Even in households with multiple vehicles the electric vehicle is used for a large proportion of all trips. It is therefor possible that ownership of electric vehicles leads to increased vehicle use (Figenbaum *et al.*, 2014), and hence an increase in other relative to technology than the average vehicle user and they view themselves as more environmentally minded (Figenbaum & Kolbenstvedt, 2013).

Initially the incentives were to be kept until the number of electric vehicles reached 50 000, which it did in April 2015 (Dagens Næringsliv, 2015). According to a new political agreement in 2015 the tax incentives will now be kept until 2018 and gradually decreased. There are differences in opinion on how the electric vehicle market will handle phasing out incentives. For example Figenbaum *et al.* (2014) and Rasmussen

& Ekhaugen (2015) argue that the incentives are so extensive that to remove all of them would destroy the electric vehicle market completely.

1.2 Discussion of incentives

The electric vehicle incentives have caused a range of adverse effects. First, substitution away from the higher taxed conventional vehicle reduce tax revenues. The electric vehicle exemption from toll charges has led to a sizable loss in toll-revenues (Aasness & Odeck, 2015). The exemption from parking fees also cause a reduction in government revenue. The electric vehicles access to bus lanes can be seen as beneficiary to society as long as the spare capacity in the bus lanes is used without delaying public transportation. A recent study shows that this capacity has reached its limit and that the travel time on bus lanes has increased proportionally to the increase in electric vehicles using the bus lanes (Aasness & Odeck, 2015). In addition the solution chosen by Oslo where the municipality stands for the operation of free charging stations can hinder private operators from establishing themselves in the market.

Substitution away from public transportation, bicycling and walking towards the electric vehicle are other adverse effects. For example, Nygaard (2015)» finds that the effects on public and manual transportation are minor and only significant when consumers are commuting. In another study 14-20% of electric vehicle users report to using less public transportation after purchasing a electric vehicle. 20-31% of electric vehicle owners also state that they drive more. (Figenbaum *et al.*, 2014).

The range of externalities connected with traffic that justifies taxation include pollution, road wear, accidents, queue and noise. It is often difficult to quantify all the elements of the external costs of traffic because the cost vary by vehicle type, geographical location and the traffic amount. Electric vehicles are usually perceived to be more environmental friendly than conventional vehicles. The electric vehicle does not emit any tail-pipe pollution and electric vehicles using Norwegian energy are virtually GHG emission free. There are uncertainties considering the actual life cycle GHG emissions from electric vehicles. For example, Singh & Strømman (2013) finds that the production of electric vehicles releases more GHG emissions compared to the production of a conventional vehicle. In addition the electric vehicle battery cause toxic impacts . The contribution to particulate matter through the use of spike tires during winter season is the same for both vehicle types. It is also a common perception that the electric vehicle produces less noise than the conventional vehicle. Electric vehicles do have potentials to reduce traffic noise for speed levels below 30 km/h, but when the driving speed is increased above 30 km/h the difference in noise between the electric and conventional is not significant (Iversen *et al.*, 2013).

A recent study finds that the marginal external cost of the electric vehicles road use such as congestion, accidents and road wear is approximate the same as for the conventional vehicle (Aasness & Odeck, 2015), yet the electric vehicle is not subjected to any taxation except the reduced yearly fee. The literature on optimal environmental taxation, see for example Sandmo (1975), shows that taxes should be levied on goods that generate externalities. Several studies argue that the tax rates should be set equal to social marginal damage (Bovenberg, 1999; Jacobs & De Mooij, 2011). Hence the tax for both the electric vehicle and conventional vehicle should be set equal to their marginal external cost. Electric and conventional vehicles will also probably to become closer substitutes as the technology matures, the range and infrastructure improves and perceptions change. Taxing close substitutes with different rates is likely to generate substantial welfare costs and considerable distortions as consumers substitute towards the low tax good (Hatta & Haltiwanger, 1986).

On the other hand the electric vehicle market suffer from network externalities that are caused by a insufficient charging infrastructure. This market distortion is an argument in favor of a subsidy for the electric vehicle. Even though the charging infrastructure has improved as municipalities now offer parking with free charging it is still perceived to be insufficient by users (Figenbaum *et al.*, 2014), making it difficult for the electric vehicle to compete with the conventional vehicle.

The 'effectiveness' of the policy depends on how 'effectiveness' is defined. Goal effectiveness is a measure of the extent to which the incentives achieve their objective. If the objective is to increase the electric vehicle share, the policy have been very successful. There is a very strong and clear relationship between the size of the benefits on the one side and the market share of electric vehicles on the other side (Figenbaum *et al.*, 2015). The Norwegian incentives suggest that the electric vehicle is attractive as long as the benefits are large enough. The main problem with using goal achievement as a measure of effectiveness is that it says nothing about whether or not the goal it self benefits society.

If the objective is a reduction in GHG and other local emissions, the policy also been successful. In addition a reduction in GHG emissions is beneficiary to society, but this reduction may have possible been achieved at a lower cost. Cost-effectiveness is a measure of the extent to which objectives are achieved at the lowest possible cost. Bjertnæs (2013) finds that the welfare gain, excluding environmental effects, generated by increasing the Norwegian tax rate on purchase for electric vehicle from 8-37% amounts to approximately 5500-6500 NOK¹ per ton increase in GHG emission. This suggest that the policy may not have been cost effective.

If a reduction in local pollution is the main goal, then promoting a switch from diesel to gasoline vehicles may possibly be a simpler and cheaper remedy (Holtsmark & Skonhoft, 2014). Holtsmark & Skonhoft (2014) concludes that "the electric vehicle policy is extremely costly and should not be adopted by other countries". The policy is also suggested removed in a recent Norwegian report on green tax (Finansdepartementet, 2015).

1.3 My Contribution

In economics we often talk about efficiency as a measure of 'effectiveness'. A deadweight loss or excess burden represents the loss of efficiency caused by distortions in the market. As discussed, previous research has used other measures of 'effectiveness' when evaluating policy. Hence, they are not in agreement on whether or not the Norwegian electric vehicle policy was a good idea, and if it should be adopted by other countries. The main objective of this thesis is to use the measure of excess burden to examine the economic efficiency of the Norwegian Electric Vehicle Policy in a second best world. Together with previous research this will hopefully provide some additional guidelines for the policy makers.

The thesis research objectives are stated as

- Is the Norwegian electric vehicle policy economically efficient?
- If not, what policy structure would minimize the excess burden and what are the effects of this on the electric vehicle traffic market?

The rest of the thesis is organized as follows: First I introduce the reader to the theoretical framework that form the foundation for the model. Then the models demand, welfare measures, excess burden and optimal commodity taxation criteria are formulated. I continue by giving a overview of the empirical data used in the numerical analysis and specifying the demand equations. The numerical results are presented and discussed in section five. I finish the thesis by concluding my findings relating it to the research objectives and relevant literature, and discussing the models short comings and future research.

¹NOK: Norwegian Krone

2 Model Formulation

This chapter starts with an introduction to the theoretical foundation of the model. This includes the microfundation of representative consumer, the properties needed for a representative consumer to exist, functional form and duality in consumer theory. The supply side is explained. Then the demand equations is formulated using a nested constant elasticity of substitution (CES) utility structure, including the CES price index. Since the compensated elasticities have a slightly different form for the nested CES demand functions than for the non-nested CES demand functions these will be stated and explained.

2.1 Theoretical Foundation

Travel behavior, discrete choice and the representative consumer

The analysis of travel behavior is typically disaggregated and discrete. The individual consumer chooses whether or not to travel, when to travel, the destination, route and mode of transport. Discrete choice models describe the behavior of consumers when they are faced with these types of mutually exclusive choices. The consumers are assumed to be heterogeneous and not completely observable such that their behavior can only be properly described using a probability function. This approach makes analytical analysis difficult and empirical analysis is only possible with panel data. Anderson *et al.* (1988) explores the linkages between discrete choice models and the representative consumer by taking the logit model and showing how this demand system is associated with the a representative consumer.

The representative consumer is a fictional character, who when facing a aggregated budget constraint, generates the economy's aggregated demand functions by either maximizing utility or minimizing expenditure. The representative consumer approach allows for modeling the problem as a single minimization or maximization problem. The representative consumer does not rule out heterogeneity, it only requires that the potential sources of heterogeneity have a structure such that the sum of all consumers behave as if they where a single consumer. Heterogeneity can be seen as an aggregated preference for diversity and is typically captured by some parameters in the representative consumer's utility function. When preferences can be represented as «Gorman preferences» (Gorman, 1953), aggregated behavior can be represented by a representative consumer. Gorman preferences imply that the Engel curve for each consumer for each commodity is linear and parallel across consumers. The Engel curve represents the relationship between expenditure on a particular commodity and income for a given set of prices.

In general in an economy with heterogeneous agents the behavior of average quantities depends on how the quantities are distributed across consumers. When using the representative consumer approach these averages depends on the same averages and ignore the distribution (Caselli & Ventura, 2000). Giving rise to the question «How representative is the representative consumer?» which is important to keep in mind when generalizing on the background of the representative consumers behavior.

Functional Form

The Constant Elasticity of Substitution (CES) utility function has the feature that the price of all goods influence the demand of each good, which is consistent with modeling demand for transportation. The indirect utility form of the CES utility function also satisfies the Gorman form, making it usable as a utility function for the representative consumer. In addition, Anderson *et al.* (1988) show that the CES representative consumer can be micro-founded using a discrete choice model. This equivalence breakdown in the presence of asymmetric price changes (Tito, 2016), but that is not relevant for this thesis. Another weakness of the CES utility function is that the income elasticity is fixed at one, making

the model unsuited for long term forecasting. The CES utility function implies that the elasticity of substitution is identical for all goods. This is not the case for all transportation modes. In order to solve this challenge the CES utility function can be divided into sub CES utility functions (Keller, 1976). This requires that the initial utility function is weakly separable in its arguments. A utility function is weakly separable in its arguments. A utility function is weakly separable in its arguments if the goods can be partitioned into subsets in such a away that every marginal rate of substitution involving the same goods from the same subset depends only on the goods in the subset. This structure is often referred to as a utility tree, and all goods located in the same branch of the tree will react identically to a price change situated in another branch of the tree. Each subset of goods have their own constant elasticity of substitution. The nested CES can be viewed as a multistage decision process where expenditure or utility is allocated between groups using price indices. The different price indices provides a summary statistic of the different prices in the underlying nest.

Duality in Consumer Theory

Marshallian demand functions are derived by maximizing utility holding budget constant and are often referred to as uncompensated demand. The dual approach is to minimize expenditure by holding utility constant, resulting in the Hicksian (compensated) demand functions. The fundamental difference between the two is that, when looking at changes in demand as a response to changes in income, the Hicksian demand curve compensates the price change through a change in income leaving only the substitution effect. The relationship between them can be shown using the Slutsky equation. The demand observed in markets is the Marshallian demand. Inserting Marshallian demand into the direct utility function gives the indirect utility function. The inversion of the indirect utility function is the expenditure function and the use Shepard's Lemma gives the Hicksian demand.

2.2 Supply

The supply side consist of the marginal private cost faced by the consumer. This section also includes a brief discussion of the marginal social cost of traffic and how these two together with demand form a market. Since the equilibrium in the traffic market depends on the behavior of all consumers, it is also referred is also identified as a Nash equilibrium.

Marginal Private Cost

The marginal private cost of transportation is the generalized price *G*. The generalized price usually consists of resource cost, time cost and transport related taxes Because consumers perceive there is sizable disadvantages connected to purchasing and using the electric vehicle, the generalized price for the electric vehicle will also consist of a disadvantage cost. The resource cost for the private vehicle consists of purchase, maintenance and user costs. The resource cost of public transportation is the fare. The time cost is often included to capture the time losses associated with congestion and the use of different transport modes. This model does not include congestion and the time cost is therefore set to be the average time cost. Hence, the time cost does not increase with quantity and neither does the marginal private cost.

As mentioned, the resource and time cost alone cannot explain the consumer preferences in the motorized vehicle market. Information costs for the electric vehicle are the cost of acquiring the necessary information for purchasing and buying a electric vehicle. Previously mentioned research (Figenbaum & Kolbenstvedt, 2013) suggest that electric vehicle owners consider themselves to be more technological competent than the average sole conventional vehicle owner. This cost is for simplicity assumed to be zero for the conventional vehicle. There is also more uncertainty surrounding the purchase and use of the electric vehicle compared to the conventional vehicle, particular related to the second hand value, future policy and uncertainty of range and access to necessary charging network. Together these form, what I in the rest of the thesis will refer to as, the disadvantage costs of the electric vehicle.

Marginal Social Cost

Externalities associated with traffic is pollution, congestion, road wear, accidents and noise. The external costs are costs generated by the consumer, but not paid by the consumer. The marginal social cost is the cost of these externalities in addition to the marginal private cost. The optimal traffic volume is reached when the marginal social cost is equal to demand as shown in Figure 2.



Figure 2: The relationship between marginal social cost, marginal private cost and demand

The marginal external congestion cost is the cost of the extra time loss for all road users when one extra vehicle is added to the traffic flow. It is a product of three elements, the average time loss due to speed reduction caused by the extra vehicle, the number of vehicles on the road, and the monetary value of time.

2.3 Hicksian demand, Marshallian Demand and the Price index

The utility function $U(\cdot)$ is a nested CES utility function with three levels. Each nest consists of a pair of goods. All goods are mutually exclusive and the individual consumer can consume only one good for a given time frame. (The given time frame is simplified to weekday in my model formulation without expecting much impact on estimated results.) The utility function is weakly separable and homothetic in its arguments. The representative consumers problem can be solved in two steps. The representative consumer has already decided to travel, where to travel and the time for the travel and now has to make the choice of how to travel.



Figure 3: The Nested-CES utility structure for motorized passenger transport

The quantity of goods at level two is denoted by $x_{2,j}$, where j = 1 is motorized vehicle $(x_{2,1})$ and j = 2 is public transportation $(x_{2,2})$. The quantity of goods at level one is denoted by $x_{1,i}$ where i = 1 is electric vehicle $(x_{1,1})$ and i = 2 is conventional vehicle $(x_{1,2})$. The generalized price for level two is denoted as $G_{2,j}$ for level two and $G_{1,i}$ for level one. U_2 is the total utility the consumer received from consuming transportation goods. The total utility received from consuming goods at level one is $U_{2,1}$. The direct utility for level two is

$$U_2 = (\sum_{j=1}^2 x_{2,j} \rho_2)^{1/\rho_2}$$

where ρ_2 can be interepretated the taste parameter and corresponds to the constant elasticity of substitution σ_2 , where $\sigma_2 = \frac{1}{(1-\rho_2)}$. The taste parameter is set to be between zero and one, $\rho_2 \varepsilon(0,1)$, to allow for zero quantities and ensure the concavity of $U(\cdot)$. U_2 is the utility constraint for the dual problem. M_2 is the budget the consumer is willing to spend on transportation goods and the budget constraint for level two is

$$M_2 = \sum_{j=1}^2 G_{2,j} x_{2,j}$$

where $\sum_{j=1}^{2} G_{2,j} x_{2,j}$ is the expenditure.

Marshallian Demand

The Marshallian demand function for level two is

$$x_{2,j}^{M} = \frac{M_2 \left(G_{2,j}\right)^{\frac{1}{p_2 - 1}}}{\sum_{j} \left(G_{2,j}\right)^{\frac{\rho_2}{p_2 - 1}}} \tag{1}$$

The budget share that is allocated to motorized vehicle is

$$M_{2,1} = \frac{M\left(G_{2,j}\right)^{\frac{P_2}{P_2 - 1}}}{\sum_{j=1}^2 \left(G_{2,j}\right)^{\frac{P_2}{P_2 - 1}}}$$

The Marshallian demand for level one

$$x_{1,i}^{M} = \frac{M_{2,1}(G_{1,i})^{\frac{1}{p_{1}-1}}}{\sum_{i\in j,i=1}^{2} (G_{1,i})^{\frac{\rho_{1}}{p_{1}-1}}}$$
(2)

Indirect utility and expenditure function

The corresponding indirect utility function for the top nest is

$$W_2(G_{2,1}, G_{2,2}, M_2) = \frac{M_2}{\left[\sum_{j=1}^2 \left(G_{2,j}\right)^{\frac{\rho_2}{\rho_2 - 1}}\right]^{\frac{\rho_2 - 1}{\rho_2}}}$$

which when inverted gives the expenditure function

$$M_2(G_{2,1}, G_{2,2}, U_2) = U_2 \left[\sum_{j=1}^2 \left(G_{2,j} \right)^{\frac{\rho_2}{\rho_2 - 1}} \right]^{\frac{\rho_2 - 1}{\rho_2}}$$

Hicksian Demand

The Hicksian demand function for level two is

$$x_{2,j}^{H} = \frac{U_2(G_{2,j})^{\frac{1}{p_2 - 1}}}{\left[\sum_{j=1}^{2} (G_{2,j})^{\frac{\rho_2}{p_2 - 1}}\right]^{\frac{1}{\rho_2}}}$$
(3)

The utility share allocated to level one is $x_{2,1}^H$, such that $U_{2,1} = x_{2,1}^H$. The Hicksian demand functions for level one

$$x_{1,i}^{H} = \frac{U_{2,1}[G_{1,i}]^{\frac{1}{p_{1}-1}}}{\left[\sum_{i \in j, i=1}^{2} (G_{1,i})^{\frac{\rho_{1}}{p_{1}-1}}\right]^{\frac{1}{p_{1}}}}$$
(4)

CES Price Index

The CES price index is identical for both Marshallian and Hicksian demand.

$$G_{2,1} = \left[\sum_{i \in j, i=1}^{2} (G_{1,i})^{\frac{\rho_1}{\rho_1 - 1}}\right]^{\frac{\rho_1 - 1}{\rho_1}}$$
(5)

Compensated elasticities

The constant elasticity of substitution can be found by solving

$$MRS = \frac{x_{1,1}^{\rho_1 - 1}}{x_{1,2}^{\rho_1 - 1}} = \frac{G_{1,1}}{G_{1,2}}$$

for ρ_1 . The constant elasticity of substitution between vehicle types, σ_1 , is

$$\sigma_1 = \frac{1}{(1-\rho_1)}$$

The larger σ_1 is the closer substitutes the conventional and electric vehicle are. When σ_1 approaches infinity ρ_1 approaches one and the goods are perfect substitutes. The goods are perfect complements when σ_1 approaches zero making ρ_1 approach zero as well. The compensated own-price and cross-price elasticities for level two, the top nest, have the same structure as non-nested elasticities. The compensated own-price elasticity for $x_{1,i}$ is given by

$$\varepsilon_{1,i} = \frac{\partial x_{1,i}}{\partial G_{1,i}} \frac{G_{1,i}}{x_{1,i}}$$

and is explicitly stated as

$$\boldsymbol{\varepsilon}_{(1,i)}^{H} = -\boldsymbol{\sigma}_{1} \left[1 - \frac{\boldsymbol{G}_{1,i}^{\rho_{1}-1}}{\sum_{i \in j, i=1}^{2} \boldsymbol{G}_{1,i}^{\rho_{1}-1}} \right] - \boldsymbol{\sigma}_{2} \left[1 - \frac{\boldsymbol{G}_{2,1}^{\rho_{2}-1}}{\sum_{j=1}^{2} \boldsymbol{G}_{2,j}^{\rho_{2}-2}} \right] \left[\frac{\boldsymbol{G}_{1,i}^{\rho_{1}-1}}{\boldsymbol{G}_{2,1}^{\rho_{2}-1}} \right]$$
(6)

The uncompensated own-price elasticity is explicitly stated as

$$\boldsymbol{\varepsilon}_{(1,i)}^{H} = -\boldsymbol{\sigma}_{1} + [\boldsymbol{\sigma}_{1} - 1] \left[\frac{\boldsymbol{\sigma}_{1}^{\rho_{1}}}{\boldsymbol{\Sigma}_{1i}^{2} \boldsymbol{\sigma}_{1,i}} \right] + \left[-\boldsymbol{\sigma}_{2} + [\boldsymbol{\sigma}_{1} - 1] \left[1 - \frac{\boldsymbol{\sigma}_{2,1}^{\rho_{2}}}{\boldsymbol{\Sigma}_{j=1}^{2} \boldsymbol{\sigma}_{2,j}^{\rho_{2}}} \right] \right] \left[\frac{\boldsymbol{\sigma}_{1,i}^{\rho_{1}}}{\boldsymbol{\sigma}_{1,i}} \right]$$
(7)

The compensated cross-price elasticity is

$$\varepsilon_{(1,i),(1,k)} = \frac{\partial x_{1,k}}{\partial G_{1,i}} \frac{G_{1,i}}{x_{1,k}}$$

and is explicitly stated as

$$\varepsilon_{[1,i],[1,k]}^{H} = \sigma_1 \left[\frac{G_{1,i}^{\frac{\rho_1}{p_1-1}}}{\sum_{i \in j, i=1}^2 G_{1,i}^{\frac{\rho_1}{p_1-1}}} \right] + \sigma_2 \left[1 - \frac{G_{2,1}^{\frac{\rho_2}{p_2-1}}}{\sum_{j=1}^2 G_{2,j}^{\frac{\rho_2}{p_2-2}}} \right] \left[\frac{G_{1,i}^{\frac{\rho_1}{p_1-1}}}{G_{2,1}^{\frac{\rho_2}{p_2-1}}} \right]$$
(8)

$$\boldsymbol{\varepsilon}_{[1,i],[1,k]}^{M} = [\boldsymbol{\sigma}_{1}-1] \left[\frac{G_{1,i}^{\frac{\rho_{1}}{p_{1}-1}}}{\sum_{i \in j, i=1}^{2} G_{1,i}^{\frac{\rho_{1}}{p_{1}-1}}} \right] + [\boldsymbol{\sigma}_{2}-1] \left[1 - \frac{G_{2,1}^{\frac{\rho_{2}}{p_{2}-1}}}{\sum_{j=1}^{2} G_{2,j}^{\frac{\rho_{2}}{p_{2}-2}}} \right] \left[\frac{G_{1,i}^{\frac{\rho_{1}}{p_{1}-1}}}{G_{2,1}^{\frac{\rho_{2}}{p_{2}-1}}} \right]$$
(9)

Because of the nested structure the both elasticities have an extra term. The first term is the standard non-nested elasticities. The second term consist of the non-nested elasticity of the nest above weighted by changing price's share of the CES price index multiplied with the compensated own-price elasticity of $x_{2,1}$. This captures the effect of changes in allocation originating at the upper level.

3 Welfare

This chapter starts with brief discussion on the possible measures of welfare for Hicksian and Marshallian demand. The general formula for excess burden using equivalent variation is stated and adjusted for the presence of externalities. Since the economy does not fulfill the requirements for a first best tax policy a formulation for optimal commodity taxation is stated and discussed in relation to the presence of externalities.

3.1 Defining Welfare Changes

The measurement of welfare when using the Marshallian demand curve as a reference is called consumer surplus. Consumer surplus measures the area to the left of the Marshallian demand curve and is path dependent under multiple price changes and does not ensure a unique solution. This is known as the integrability problem of demand (Hurwicz & Uzawa, 1971). The welfare measures using the Hicksian demand curve as a reference does not have this problem and provide exact measures with correct ranking. The two measures of welfare using the Hicksian demand curve is compensated and equivalent variation and are both based on the expenditure function. The expenditure function gives the minimum cost of achieving a utility U_2 level for a set of prices. The two Hicksian demand measures of welfare differ in which utility curve they use as a reference. The compensated variation uses the initial level of utility as a reference and it measures how much the government must compensate the consumer in order to restore the consumer to her initial indifference curve. In other words the amount of money that is required for the consumer to accept the price change. The compensated variation is

$$V_2(\mathbf{G^0}, M_2) = V_2(\mathbf{G^1}, M_2 + CV)$$

or

$$CV = E(\mathbf{G}^1, V_2(\mathbf{G}^0, M_2)) - E(\mathbf{G}^0, V_3^0)$$

where G^0 is the initial price vector and G^1 is the price vector after the price change. The reference for the equivalent variation is the final level of utility and it measures the amount consumer is willing to pay as for example a lump-sum tax in order to avoid the price change and still be at her initial indifference curve.

$$V_2(\mathbf{G^0}, M_2 - EV) = V_2(\mathbf{G^1}, M_2))$$

or

$$EV = E(\mathbf{G}^1, V_2(\mathbf{G}^1, M_2)) - E(\mathbf{G}^0, V_2^1)$$

Remember that $E(\mathbf{G}^0, V_2(\mathbf{G}^0, M_2)) = E(\mathbf{G}^1, V_2(\mathbf{G}^1, M_2))) = M_2$ for a partial equilibrium. The relationship between the two measures is shown in Figure 4. Both measures are exact and provide a correct ranking, but the size of the measurements may differ because they use different demand curves as a reference. For a normal good the equivalent variation will give a higher measure than the compensated variation. For a change in tax on electric vehicle the equivalent variation for the derived Hicksian demand curves is explicitly stated as

$$EV = M_2 \left[1 - \frac{\left[\sum_{j=1}^2 \left(G_{2,j}^0 \right)^{\frac{\rho_2}{\rho_2 - 1}} \right]^{\frac{\rho_2 - 1}{\rho_2}}}{\left[\sum_{j=1}^2 \left(G_{2,j}^1 \right)^{\frac{\rho_2}{\rho_2 - 1}} \right]^{\frac{\rho_2 - 1}{\rho_2}}} \right]$$
(10)

where $M_2 = x_{2,1}^0 G_{2,1}^0 + x_{2,2}^0 + G_{2,2}^0$ represents the budget constraint. $\left[\sum_{j=1}^2 (G_{2,j})^{\frac{\rho_2}{\rho_2-1}}\right]^{\frac{\rho_2-1}{\rho_2}}$ is interpreted as the price index for all transportation goods.

3.2 The Excess Burden of Taxation

Taxes have multiple purposes. The government levies taxes on goods in order to collect enough revenue to cover public expenditures, and to improve allocation and distribution in society. One way of collecting the taxes needed is through lump-sum taxes. Since the consumer has to pay a lump-sum tax regardless of their behavior, lump-sum taxes do not distort consumer choice and does not cause ineffeciencies provided that collecting taxes is without other costs.

Taxes can also be used as a measure to correct for market failure and improve market efficiency. This is referred to as a Pigovian tax. Pigovian taxes create distortions that correct for market failure by shifting the marginal private cost curve to be equal to the marginal social cost curve. If the economy is characterizes by a complete set of competetive markets, and Pigovian and lump-sum taxes are available then the first best socially efficient allocation is attainable.

If the world was so simple, the discussion of the electric vehicle policy should end here, the electric vehicle should be levied with a tax equal to its marginal external costs. In practice lump-sum taxes are rarely used. If taxes are to have a redistributional role, lump-sum taxes need to be differentiated. Collecting information to decide the size of the lump-sum tax levied on each individual in sociaty may not be possible due to the large amount of information needed being both private and costly to gather. If there exists one efficiency condition that cannot be fulfilled, then the other efficiency conditions, altough still attainable in general are no longer desirable (Lipsey & Lancaster, 1956).

The second best optimum can then only be acvhided by departing from all other Pareto conditions. This result indicates that the well-known recepie for calculating Pigovian taxes is likely to be sensetive to the existance of other distortions than what the distortions they where intended to fix (Sandemo, 2000). Allthough second-best taxes may be Pareto improving they have the harmfull side-effect of creating distortions because the taxes violate conditions of social efficiency and will therefore have an excess burden.

In general the deadweight loss measures the economic cost of distortions in the market. When these distortions are caused by taxation, the deadweight loss is usually refered to as the excess burden of taxation. The excess burden is the consumers loss, in excess of the tax revenue collected (Creedy, 1999) and measures the cost of not being able to impose non-distorting taxes. The excess burden allows for the compareson of different tax systems, where the most efficient tax system is the one that carries the lowest excess burden. The excess burden can be defined in terms of the equivalent variation (Mohring, 1971), or the compensated variation (Diamond & McFadden, 1974). Without any restrictions on preferences the two measures will differ. The equivalent variation is the money metric utility function for the representative consumer an ensures duality for the problem of optimal commodity taxation (Kay, 1980). This is the measure I will use in my estimations.



Figure 4: The two different measures of excess burden.

The measure of excess burden using the compensated variation is results in the triangle area *DEF*. The triangle area *ABC* is the equivalent variation measure of excess burden. The area to the left of the Hicksian demand curve, $x^H(V(G^1, M))$ from G^0 to G^1 is the equivalent variation. The tax revenue is the square G^1G^0AB .

The excess burden when there exists no initial tax-distortions using equivalent variation can be stated as

$$EB_{EV} = EV - R(\mathbf{G}^{0}, \mathbf{G}^{1}, V_{2}(\mathbf{G}^{1}, M_{2})))$$
(11)

where G^1 is the new price vector, G^0 is the old price vector and $R(G^1, V_2(G^1, M_2)))$ is the tax-revenue. Since the both electric and conventional vehicle are associated with a negative marginal externality, I correct the measure of tax-revenue for the cost of externalities

$$R(\mathbf{G^0}, \mathbf{G^1}, V_2(\mathbf{G^1}, M_2)) = \sum_{i \in j, i=1}^2 (G_{1,i}^1 - G_{1,i}^0 - e_{1,i}) x_{1,i}^H(\mathbf{G^1}, V_2^1))$$

Where $e_{1,i}$ is the externality for the good $x_{1,i}$. Because $x_{1,i}^H(G_{1,1}^1, G_{1,2}^1, V_2(\mathbf{G}^1, M_2)) = x_{1,i}^M(G_{1,1}^1, G_{1,2}^1, M_{2,1})$ this can be written as

$$R(\mathbf{G^0}, \mathbf{G^1}, V_2(\mathbf{G^1}, M_2)) = \sum_{i \in j, i=1}^2 (G_{1,i}^1 - G_{1,i}^0 - e_{1,i}) x_{1,i}^M(\mathbf{G^1}, M_2))$$

If the initial equilibrium is already distorted due to pre-existing distorting taxes or subsidies the changes in excess burden can be calculated. This if often referred to as the marginal excess burden.

$$MEB_{EV} = EV - \left[R(\mathbf{G}^{0}, \mathbf{G}^{1}, V_{2}(\mathbf{G}^{1}, M_{2})) - R(\mathbf{G}^{0}, V(\mathbf{G}^{1}, M_{2})) \right]$$
(12)

$$\left[R(\mathbf{G}^{\mathbf{0}},\mathbf{G}^{\mathbf{1}},V_{2}(\mathbf{G}^{\mathbf{1}},M_{2}))-R(\mathbf{G}^{\mathbf{0}},V(\mathbf{G}^{\mathbf{1}},M_{2}))\right] = \sum_{i\in j,i=1}^{2} \left[t_{1,i}^{1}-e_{1,i}\right] x_{1,i}^{M}(\mathbf{G}^{\mathbf{1}},M_{2}) - \sum_{i}^{2} \left[t_{1,i}^{0}-e_{1,i}\right] x_{1,i}^{M}(\mathbf{G}^{\mathbf{0}},M_{2})$$

where $t_{1,i}^1 = G_{i,1}^1 - G_{i,1}^0$, the new tax.

3.3 The Second Best

Optimal Commodity Taxation and Ramsey's Rule

The theory of optimal commodity taxation defines the optimal linear taxes on goods and services. Ramsey (1927) showed that a uniform commodity tax system, which alters none of the relative prices of goods, is in general not optimal. Instead, efficiency cost minimizing commodity taxes will in general differ by commodity. The Ramsey rule states that for a commodity tax structure to be optimal the proportional tax-induced reduction in the quantities demanded of a taxed commodity as measured along its compensated demand curve should be the same for all taxable commodities. The basic problem of optimal taxation is to choose a tax that minimizes the excess burden subject to a required amount of tax revenue to be raised by the government (for full derivation of the rules see Appendix)

$$\min_{G^1} EB$$

subject to a revenue constraint

$$\sum_{i \in j} t_{1,i} x_{1,i}^M \ge C$$

solving for the first order conditions gives

$$\frac{\left[t_{1,1}\frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{1}} + t_{1,2}\frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{1}}\right]}{x_{1,1}^{M}} = \frac{\left[t_{1,1}\frac{\partial x_{1,2}^{M}}{\partial G_{1,1}^{1}} + t_{1,2}\frac{\partial x_{1,2}^{M}}{\partial G_{1,2}^{1}}\right]}{x_{1,2}^{M}}$$
(13)

which is Ramsey's basic rule.

For small changes in the tax system it can be convenient to use a approximation which results in the equi-proportional rule.

$$\frac{\Delta x_{1,1}^M}{x_{1,1}^M} = \frac{\triangle x_{1,2}^M}{x_{1,2}^M}$$

The proportional tax induced reduction in the quantities demanded of a taxed commodity measured along its compensated demand curve should be the same for all taxable commodities. The validity of the Ramsey rule is general and does not require any special assumptions of the demand curve. The basic insight from Ramsey is that taxes should be set such that they reduce the consumption of each good (along its compensated demand curve) equi-proportionatly. We should tax goods with high compensated demand elasticities with low tax-rates and goods with low compensated demand elasticities with high rates.

Adaption of Ramsey's Rule for the Case with Externalities

As discussed, when the conditions for first best holds, taxing the externality generting goods directly with a Pigovian tax such that the marginal private cost equals marignal social cost is opimal.

The additivity property suggest that the presence of externalities only affects the tax formula for the externality generating goods and that the tax formula for all other goods are uneffected (Sandmo, 1975; Cremer *et al.*, 1998). Optimal taxes can be expressed as the sum of optimal Pigovian taxes and optimal commodity taxes in a related problem without externalities. Externalities should be targeted directly even when other distortionary taxes are used.

Remember that since the negative external costs from traffic depend on local conditions as time, population density and geographic factors, the optimal Pigovian tax will also differ with local conditions. An ideal tax system would be spatiotemporal based on GPS locations, as has been suggested by a recent Norwegian government report on green taxes (Finansdepartementet, 2015).

In a general-equilibrium setting the level of the optimal pollution tax depends on the level of other taxes. The optimal general equilibrium pollution tax is likely to differ from a partial equilibrium Pigovian tax. The Pigovian tax, in addition to reduce pollution, have the added effect of raising revenue. If this revenue can be used to replace other taxes there is a potential for a «double dividend». If there exists other distortionary taxes in the economy 'the double dividend hypothesis' (Sandmo, 1975) states that the increase in tax revenue from a introduction of environmental taxes can allow the government to reduce other distortionary taxes. The double dividend term comes from the two benefits of environmental gain and reduction of overall efficiency loss from other taxes needed to collect the necessary revenues to the government. The hypothesis has strong theoretical foundation but has been difficult to verify empirically. Research still shows that in the presence of preexisting distortionary taxes the optimal pollution tax typically lies below the Pigovian tax (Bovenberg & De Mooij, 1994; Bovenberg & Goulder, 1996).

If the government are not interested in setting Pigovian taxes but decides on a fixed tax-rate for the electric vehicle the second best optimal taxation on the conventional vehicle is the tax that minimizes the excess burden.

$$\min_{G^1} EB$$

where

$$EB = EV - \sum_{i \in j, i=1}^{2} (t_{1,i} - e_{1,i}) x_{1,i}^{\mathcal{M}}$$

solving for first order conditions give

$$\frac{(t_{1,1}-e_{1,1})\frac{\partial x_{1,1}^M}{\partial G_{1,1}^1} + (t_{1,2}-e_{1,2})\frac{\partial x_{1,2}^M}{\partial G_{1,1}^1}}{x_{1,1}^M} = \frac{(t_{1,1}-e_{1,1})\frac{\partial x_{1,1}^M}{\partial G_{1,2}^1} + (t_{1,2}-e_{1,2})\frac{\partial x_{1,2}^M}{\partial G_{1,2}^1}}{x_{1,2}^M}$$

Because of the CES nature of demand this has no unique solutions. From the equation one can see that one possible solution is Pigovian taxes. The results gives the same instinct as Ramsey's rule, but in addition takes into consideration the effects of the externalites.

4 Data

In this chapter I will state and present the data sources used for the numerical analysis and estimate the parameters needed for the Marshallian demand functions.

4.1 Data Sources

Quantity

Data on daily traffic count for the electric and conventional vehicle in the Oslo area is provided by Fjellinjen for a period between 2011-2015. The data include hourly data on traffic count for six representative days for 2011-2015 for all the toll rings in the Oslo area. The representative day is a mid-weekday not in direct relation to any holidays. The data have been compressed to a single count for each day.

Year	Vehicle Type	Obs.	Mean	Standard Deviation
2011	Electric Vehicle	4	1418	174.1
2011	Conventional Vehicle	4	329538	10024.1
2012	Electric Vehicle	6	2778.5	821.64
2012	Conventional Vehicle	6	329405.5	13883.6
2013	Electric Vehicle	6	5577.167	1248.4
2015	Conventional Vehicle	6	329916	10626.2
2014	Electric Vehicle	6	11877	2880.9
2011	Conventional Vehicle	6	319490	10789.2
2015	Electric Vehicle	6	20066.2	3449.4
2015	Conventional Vehicle	6	309539.2	12062.9

Table 2: Summary statistics of traffic count data for different years

The market share between motorized vehicle and public transit is collected from Ruters Årsrapport (2014). The quantity data is from the urban Oslo region and is not representative for the whole of Norway.

Price

The price data used for the numerical analysis have been collected from various sources. The generalized price for the conventional vehicle include purchase cost, maintenance cost, tax, fuel cost, additional parking cost and time cost compared to the electric vehicle. The generalized price for the electric vehicle is purchase cost, maintenance cost, tax, fuel cost and a unknown disadvantage cost as discussed in section 3.2. The generalized price is calculated on a trip basis. The average distance per trip is set to be 26 kilometers (Figenbaum *et al.*, 2014), and the number of trips for one year is set equal to the number of working days (260). The average life time for both vehicles is assumed to be 12 years. The conventional vehicle is a combination of the average diesel and gasoline vehicle weighted by the actual shares of vehicle types in the region for Oslo and Akershus for each year. The purchase price is for simplicity assumed to be the same for all years. For gasoline this is $0.074 \ l/km$, diesel $0.042 \ l/km$ and $0.25 \ kWh/km$ for the electric vehicle. The fuel prices for each month is collected from Statistics Norway (table 09654). The electric vehicle benefits of free parking and use of bus lane are collected from Aasness & Odeck (2015). All prices have been converted to 2015 NOK. The generalized price for public transportation is

the transit fare which is calculated to be the per trip cost for three zone monthly traveling card for Ruter in the Oslo and Akershus area. The generalized cost using public transportation is also assumed to include a disadvantage cost.

An example of the cost structure used for 2015 can be seen in table 2.

Cost	Electric Vehicle	Conventional Vehicle
Purchase cost	254400 NOK	290500 NOK
Maintenance	1967 NOK	2514 NOK
Annual License Fee	435 NOK	3060 NOK
Additional Parking	0 NOK	3425 NOK
Additional time cost	0 NOK	8090 NOK
Fuel cost per trip	4.7 NOK	48.5 NOK
Toll	0 NOK	28.8 NOK
Total cost per trip	94.6 NOK	199,7 NOK

Table 3: Purchase, user and time cost for electric and conventional vehicle, NOK 2015.

Externalities

The abatement cost of CO_2 is based on estimates from the report Klimakur 2020 (2010). The average conventional vehicle emitted 99.75 g/km, 2593.5 g/trip in 2015, collected from "Opplysningsrådet for Veitrafikken AS" (2016). Estimates used for external cost of traffic for the electric and conventional vehicle can be seen in table 4.

Table 4: External cost per trip for conventional and electric vehicle for different geographic locations, 2015 NOK

	Congested Urban	Large Urban	Small Urban
Conventional Vehicle	203.27	51.81	27.39
Electric Vehicle	173.63	36.93	23.90
0	(TT) I	. 1 0014)	

Source: (Thune-Larsen et al., 2014)

The external costs in the numerical analysis consist of 80% large urban and 20% congested urban. These estimates include local air pollution, noise, accidents, congestion, road wear and winter operation. There are multiple ways of estimating the cost of externalities. Ideally these estimates would reflect the true willingness to pay for a reduction of externalities. The external cost in table 3 calculated using a damage function approach. For local air pollution this includes for example valuation of health and well being impacts, and international abatement costs.

Substitution between motorized vehicle and public transit

The substitution parameter between public transit and motorized vehicle is set to 0.9. For comparison the Trenen-model (De Borger & Proost, 2001) uses a value of 0.8 between private and public transport in their inter regional model and 0.98 for urban.

4.2 Estimation

Method

The method used to fit the data to the equation of Marshallian demand (1) and (2) is non-linear regression. Non-linear regression uses a iterative algorithm which estimates the parameters by systematically adjusting the parameters estimates to reduce the sum of squares of the residual error. The pair of Marshallian demand functions is fitted simultaneously.

Estimation

Here the parameter $\beta_{1,1}$ is included as a part of the generalized price $G_{1,1}$ such that $G_{1,1} = \beta_{1,1} + p_{1,1}$, where $p_{1,1}$ is the cost structure in table 2. When fitting the data to the equation I estimate the disadvantage for the electric vehicle.

The parameter estimates can be seen in Table 4.

Table 5: Non-linear regression estimates for the substitution parameter ($\hat{\rho}_1$) between electric and conventional vehicle, and disadvantage cost ($\hat{\beta}_{1,1}$) for electric vehicle, using data from different periods as basis for estimation.

	2011-2015	2012-2015	2013-2015	2014-2015	2015
Number of observations	28	18	9	7	6
$\hat{oldsymbol{eta}}_{1,1}$	124.85	128.54	131.34	125.71	121.62
Standard error	3.254	5.19	14.04	10.86	3.32
$\hat{ ho}_1$	0.966	0.959	0.956	0.964	0.969
Standard error	0.0044	0.00697	0.0192154	0.016088	0.0057

The estimates for data from 2013-2015, 2014-2015 and 2015 where used to examine how accurately the fitted Marshallian demand functions replicate the true trend for electric and conventional vehicle.



Figure 5: Comparison of predicted demand and observed demand for electric and conventional vehicle using estimations from 2013-2015, 2014-2015 and 2015.

Figure 5 shows that the estimates capture the general trend in demand. The estimated demand deviates from true demand by approximately 1-2% at the end of 2015. The main reason the estimated demand functions deviate is that demand does not seem to respond to fluctuations in the price of fuel.

The disadvantage cost $\beta_{2,2}$ for public transportation is calculated by adding $\beta_{2,2}$ to the generalized price of public transportation such that $G_{2,2} = p_{2,2} + \beta_{2,2}$ where $p_{2,2}$ is the fare. Then, given price, quantity and demand, the marginal rates of substitution equal to each other and I solve for the unknown, the disadvantage cost $\beta_{2,2}$.

5 Numerical Analysis

The implementation of the model and calculations are done using MATLAB. All calculations are made on a trip basis for the representative consumer and monetary values are stated in 2015 NOK. This means that in order to for example predict the changes in excess burden for a given year, the monetary predictions needs to be multiplied with the number of working days and the daily traffic count.

5.1 Demand

This section examines the predicted transportation market share structure for different intensities of the Norwegian electric vehicle incentives The results for demand is reported using the Marshallian demand function derived in section 3.3 and estimated in chapter 4. The market share is the percentage market share of the total market for motorized transportation for weekday travel in the Oslo area.



Figure 6: Predicted traffic market shares for electric and conventional vehicles as a response to a decrease in electric vehicle benefits

Figure 6 represents the predicted market share for the electric and conventional vehicle for given changes in the electric vehicle benefits. The result suggest that the predicted demand for electric vehicle is very sensitive to even small changes in benefits. This confirms previous findings discussed in section 2.2 that suggest that as long as benefits are large enough the electric vehicle is a attractive choice for the consumer (Figenbaum *et al.*, 2015). It also suggest that if the electric vehicle policy was not introduced the conventional vehicle would have had a larger market share.

It is very unlikely that this result predicts the actual consumer behavior in the market if benefits where to be removed today. If benefits where to be removed the consumer who prefers to travel my motorized vehicle is now facing the same time, parking and toll costs for both vehicle types. Since the removal does not affect fuel cost, the electric vehicle is still superior to the conventional vehicle when it comes to user costs. Therefore for a consumer who already owns an electric vehicle, the removal of benefits does not give a incentive to switch vehicle type.

Remember that the estimate of the perceived disadvantage cost $\hat{\beta}_{1,1}$ is an average value for both the conventional and electric vehicle owner. The perceived disadvantage cost for the electric vehicle is likely to be lower for a consumer who has experience with the electric vehicle than for a consumer with no experience of the electric vehicle. The perceived disadvantage cost will continue to decrease as the electric vehicle users share their experiences with the electric vehicle. It is also likely that the market and technology will continue to develop such that the constant elasticity of substitution will increase making the vehicle types even closer substitutes. If this happens simultaneously as a slow removal of the benefits, the generalized price for the electric vehicle will not be subject to a large change and neither will the traffic market share.

Figure 7 shows the predicted market shares for motorized vehicle traffic and public transportation. The predictions suggest that the current policy has to a degree distorted the consumers choice between motorized vehicle and public transportation.



Figure 7: Predicted market shares for motorized vehicle and public transportation as a response to a decrease in electric vehicle benefits

This is in agreement with previous research that suggest that there has been a substitution away from public transportation (Figenbaum *et al.*, 2014; Nygaard, 2015). Remember that predicted demand are based on quantity data from the urban Oslo region and that the degree of substitution between motorized vehicle and public transportation depends on the access to public transportation. For the Oslo region public transportation is a real option to the motorized vehicle when it comes to weekday travel. This is not the case for Norway's more rural areas that often suffer from a poor supply of public transportation and the motorized vehicle is the only transport mode option.

5.2 Excess Burden of Taxation for the Current Policy

The measure of changes in excess burden of taxation can be divided into three components. The equivalent variation, changes in government revenue and changes in externalities. The equivalent variation is a money metric utility measure of how much the consumer is willing to pay for a given change in the incentives.



Figure 8: Estimated equivalent variation as a response to a decrease in electric vehicle benefits

Figure 8 show that the estimated equivalent variation increases rapidly as the consumer rapidly substitutes away from the electric vehicle and over to other modes of transportation. The estimated equivalent variation is strictly increasing as the generalized price for electric vehicle increases. Remember the CES price index in section 2.3, as the generalized price for electric vehicle increases the CES price index for motorized vehicle will increase. At one point, when the consumers have all substituted away from the electric vehicle, the CES price index for motorized vehicle will be equal to the generalized price for conventional vehicle. Any price changes for electric vehicle after this point will have no effect on the estimated equivalent variation.



Figure 9: Estimated changes in government revenue as a response to a decrease in electric vehicle benefits

The second component in the excess burden of taxation is changes in government revenue and is predicted in figure 9. The estimates shown in Figure 9 show that there is a steep increase in estimated changes in government revenue as long as the removal benefits have a strong effects on the predicted traffic market share for electric vehicle. When the predicted market share for electric vehicle approaches 1.5% the changes are decreasing.



Figure 10: Estimated changes in external cost as a response to a decrease in electric vehicle benefits

The third component in the excess burden of taxation is the changes in external cost, the estimates are shown in figure 10. Since the electric vehicle has a lower external cost than the conventional vehicle the changes are strictly increasing as more consumer substitute towards the conventional vehicle. The external cost are given endogenously and the model assumes that consumers behavior in the traffic market is not affected by changes in externalities. This is in contrast with what is empirically observed. For example, a study by Figenbaum & Kolbenstvedt (2013) shows that electric vehicle owners consider themselves more environmentally friendly and that their choice of vehicle is affected by this.



Figure 11: Predicted changes in excess burden as a response to a decrease in electric vehicle benefits, holding the current taxation on conventional vehicle fixed

Together these three components form the changes in excess burden, these estimates are shown in figure 11. The estimated the changes in excess burden are strictly decreasing for approximately a 0-20% removal of benefits. The estimated changes in excess burden go from decreasing to increasing at approximately the same point as the estimated growth in external cost and estimated equivalent variation slows down, and the estimated changes in government revenue start to decrease.

Remember that the most efficient policy is the policy with the lowest estimated excess burden, therefore the most efficient policy is found at the bottom of the curve in Figure 11. The estimated changes in excess burden suggests that, given the current taxation of conventional vehicle, the Norwegian electric vehicle policy is close to being efficient. When converted in to monetary values a 0-20% removal of benefits is approximately a 5-13 NOK increase in the generalized price of the electric vehicle. Taking a closer look at the graph of the predicted electric vehicle market share gives a suggestion as to what would be, given the current taxation of conventional vehicle, the efficient traffic market share for electric vehicle. All predictions are in agreement that this value is approximately 1.5% and this corresponds to a 2.20% decrease in the estimated excess burden, for the 2014-2015 estimates, compared to the current policy.

Another way to evaluate the policy is by holding the electric vehicle benefits fixed at the current level and examine the changes in estimated excess burden as the taxation on conventional vehicle changes. The current tax as a percentage of price on the conventional vehicle is approximately 70%, and is located close to the bottom of the curve. The estimated changes in excess burden for a change in taxation on conventional vehicle suggest, that given the current electric vehicle policy, the taxation on

conventional vehicle is close to being efficient.



Figure 12: Estimated changes in excess burden as a response to changes in taxation on conventional vehicle holding the current electric vehicle incentives fixed.

5.3 Minimizing the Excess Burden of Taxation

The previous section examined the estimated changes in excess burden when one either the electric vehicle benefits or taxation on conventional vehicle is fixed. This section examines the efficiency of different levels of electric vehicle benefits given that the estimated taxation on conventional vehicle is optimal. The benchmark for comparison is the current incentives and taxation of electric and conventional vehicle. For simplicity, the calculations are based on the 2014-2015 estimates for constant elasticity of substitution and perceived disadvantage costs. All changes are reported as percentage change compared to benchmark. As mentioned in section 3.3, because of the CES nature of the demand functions ,the conditions for optimal taxation gives no unique solutions. The solution space for taxation on conventional vehicle has only been evaluated for the interval 0-150% of the current price for conventional vehicle. The Pigovian taxation for electric vehicle corresponds to a 80% decrease in electric vehicle benefits. The external cost estimates used for sensitivity analyses, low, medium and high correspond to a 50% decrease of the cost estimates used in the previous section, the cost estimates used in the previous section and a 50% increase in the cost estimates used in the previous section.

	<i>t</i> _{1,2}	x _{1,1}	X _{1,2}	X _{2,2}
Current policy (benchmark)	82.28	0.043	0.625	0.325
Pigovian Taxation	6.07%	-99.77%	-4.17%	16.86%
0 % decrease in electric vehicle benefits	-7.29%	-49.54%	16.46%	-16.43%
25 % decrease in electric vehicle benefits	2.73%	-80.18%	0.38%	8.09%
50 % decrease in electric vehicle benefits	6.38%	-95.39%	-5.04%	17.57%
75 % decrease in electric vehicle benefits	7.11%	-99.08%	-6.06%	19.51%
100 % decrease in electric vehicle benefits	7.11%	-99.9%	-6.00%	19.51%
150 % decrease in electric vehicle benefits	7.11%	-100.0%	-6.00%	19.51%

Table 6: Estimated optimal taxation on conventional vehicle and predicted demand for motorized transportation for different intensities of electric vehicle policy

 $t_{1,2}$ is taxation of conventional vehicle, $x_{1,1}$ is market share for electric vehicle, $x_{1,2}$ is market share for conventional vehicle and $x_{2,2}$ is market share for public transportation.

The predicted traffic market share for electric vehicle with Pigovian taxation will be close to zero. The estimated optimal taxation of conventional vehicle for the current incentives is 7.29% lower than what it is today. This small change in the price for the conventional vehicle corresponds to a 50% decrease in the predicted demand for electric vehicle.



Figure 13: Estimated changes in optimal taxation of conventional vehicle as a response to decrease in electric vehicle benefits

Figure 13 show the estimated changes in optimal taxation of conventional vehicle for different estimates of external cost. The estimates show that as the electric vehicle benefits decrease the estimated optimal taxation on conventional vehicle increases. Remember the equation for optimal commodity taxation de-

rived in section 3.3. The closer the tax lies to the external cost the less changes in demand due to price changes matter. When taxation is equal to the external cost, the changes in demand does not matter at all. For the low external cost estimate, the taxation, is set at the optimal level even for a zero percentage decrease in the electric vehicle benefits. The estimated optimal tax on conventional vehicle, for the medium and high estimates follow the same convergence towards this level, but because the cost externality is higher the limit is not reached as fast. Still, the connection between the estimated optimal taxation on conventional vehicle, percentage decrease in electric vehicle benefits and cost of externality is clear. There is a positive correlation between the percentage decrease in electric vehicle benefits and the estimated optimal taxation on conventional vehicle.

	<i>t</i> _{1,2}	Government Revenue	External cost	EV	Excess Burden
Current policy (benchmark)	82.28	51.45	57.365	0	84.00
Pigovian taxation	6.07%	1.66%	-8,82%	4.49%	-2.87%
0 % decrease in electric vehicle benefits	-7.29%	7.95%	13.24%	-5.27%	-0.72%
25 % decrease in electric vehicle benefits	2.73%	3.40%	-3.53%	2.25%	-2.40%
50 % decrease in electric vehicle benefits	6.38%	1.14%	-9,44%	4.66%	-2.81%
75 % decrease in electric vehicle benefits	7.11%	0.66%	-10.58%	5.13%	-2.87%
100 % decrease in electric vehicle benefits	7.11%	0.69%	-10.57%	5.13%	-2.87%
150 % decrease in electric vehicle benefits	7.11%	0.70%	-10.56%	5.13%	-2.87%

Table 7: Estimated percentage change in government revenue, external cost, equivalent variation and excess burden for different electric vehicle policies when taxation on conventional vehicle is optimal.

My estimates that as the electric vehicle benefits decrease, the optimal taxation on conventional vehicle increases and the excess burden decreases. Figure 14 shows the estimated changes in excess burden for different estimates of external cost.



Figure 14: Estimated changes in excess burden as the electric vehicle benefits decrease, given the estimated taxation on conventional vehicle is optimal

For the medium external cost estimate the estimated excess burden approaches its lower limit for a 50-75% decrease in electric vehicle benefits. This limit corresponds to Pigovian tax, which in according to economic theory for a partial equilibrium analyses reflect the optimal situation (Sandmo, 1975; Sandemo, 2000; Kopczuk, 2003). This limit gives the same level of estimated excess burden as the Pigovian tax but at a slightly lower decrease of benefits. The estimations in table 7 and figure 14 show that the optimal taxation on also approaches its upper limit at the same time. Meaning that beyond this point there is no point in decreasing the incentives further. One can argue that the difference in a change in excess burden from -2.40% to -2.87%, which corresponds to a 25% decrease and 75% decrease in benefits, is an acceptable loss of efficiency. A 25% decrease in benefits would not abolish the electric vehicle market, but a 75% is close to doing so.

Figure 14 show that the for what level of benefits when the estimated excess burden reaches its limit is highly dependent on external cost estimates. For a low external cost estimate this limit is reached immeditatly and for a medium external cost estimate this limit is reached at approximately 50%. For a high external cost estimate this limit is not reached until a 100% decrease of benefits. Again, the closer the benchmark taxation is to the external cost, the less adjustment is needed to reach the limit.

6 Discussion and Conclusion

6.1 Conclusion

Previous research that have used measures of goal effectiveness and cost effectiveness to evaluate the Norwegian electric vehicle policy. Figenbaum *et al.* (2015) finds that the policy is goal effective because it is a clear correlation between the incentives and the high market share for electric vehicles. On the other side, Holtsmark & Skonhoft (2014) and Bjertnæs (2013) argue that the policy is to costly.

A deadweight loss represents the loss of economic efficiency caused by distortions in the market. When the deadweight loss is caused by tax it is often referred to as the excess burden of taxation. The main objective of this thesis is to analyze the Norwegian electric vehicle policy economical efficiency using the measure of excess burden. The thesis research objectives have been stated as

- Is the Norwegian electric vehicle policy economically efficient?
- If not, what policy structure would minimize the excess burden and what are the effects of this on the electric vehicle traffic market?

The numerical analysis of demand is based on a nested CES utility structure for a representative consumer and the measure of excess burden has been formulated to take both changes in government revenue and externalities into consideration. The changes in tax revenue and externalities are estimated on the basis of Marshallian demand. The estimation of Marshallian demand is done using traffic count data for the Oslo area. The welfare measure used to quantify the welfare changes is equivalent variation.

My findings agree with Figenbaum *et al.* (2015) that the electric vehicle is attractive as long as the incentives are strong enough. As little as a 20% decrease in the benefits would have lowered the predicted market share for the electric vehicle with more than a half. The results also suggest that there has been a substitution away from public transportation as a result of the electric vehicle incentives.

Given the current taxation on conventional vehicle my estimations show that it is efficient to reduce the benefits for the electric vehicle user by 10-20%. This corresponds to a approximately 65% decrease in the predicted market share for the electric vehicle, pushing it down from a total market share of 4,5% to 1.5%. The estimated decrease in excess burden compared to the current situation is 2.2%. On the other hand, given the current incentives, my estimations show that it is efficient to reduce the current taxation of the conventional vehicle by 7.3%. This corresponds to only a 49.54% predicted decrease of the market share of the electric vehicle and a 0.76% decrease in estimated excess burden.

When estimating the optimal taxation on conventional vehicle and the corresponding changes in excess burden compared to the current situation I find that there is a positive correlation between the decrease in benefits and increase optimal taxation on conventional vehicle. As benefits decrease the estimated optimal taxation increases until it reaches a upper limit, where the taxation is so close to external cost that changes in demand does no longer matter. There exists a similar limit for the estimated excess burden. The lower limit corresponds to a 2.87% decrease in estimated excess burden compared to today's structure. This is the same reduction in estimated excess burden that the Pigovian tax gives, and this in agreement with standard tax theory (Sandmo, 1975). My estimations show that this limit can be reached without using Pigovian taxation. The Pigovian taxation would result in a 99.8% lower predicted market share for the electric vehicle. For what level of incentives and taxation this limit is reached depends on the estimates of external costs. For the estimates used in this analysis, this is reached for a 50-75% removal of benefits. This corresponds to a 95-99% decrease in predicted market share for electric vehicle. For comparison a 2.4% decrease in estimated excess burden can be reached by a 25% decrease in benefits. The predicted market share of the electric vehicle is then decreased by approximately 80%.

The result from my thesis can be summed up in this sentences: My estimations show that there are only a small percentage decrease in estimated excess burden to gain by decreasing the electric vehicle benefits and that this corresponds to a large predicted decrease in the market share for electric vehicle.

6.2 Discussion

All initial values for generalized cost, external cost, taxation and quantity are based on averages and ignore distributional issues. All estimates and predictions are based on a representative consumer approach and ignore distributional issues. Anybody with a basic course in statistics knows how flawed this method is in truly explaining the world (unless you are true frequentist). This is not the only distribution this thesis ignores, it also ignores the distribution effects of the tax among the consumers.

All the estimates are based on a small data sample. The estimate of constant substitution of elasticity between the electric and conventional vehicle and the perceived disadvantage cost are suggestive at best. It is unfortunately not within the scope and time limit given for this thesis possible to provide more accurate estimates. In order to provide a more accurate analysis of the subject better estimates for these parameters are needed. In addition I believe a estimation for these parameters are interesting and informative in it self. The development of the constant elasticity of substitution and the disadvantage cost can tell us something about how the attitudes towards the electric vehicle have changes over the period the incentives have been in effect. In addition if the constant elasticity of substitution is sufficiently high and the perceived disadvantage cost sufficiently low there is no longer a need for the incentives. If I where to propose any further research within this matter I would suggest a more accurate estimation of these parameters.

My approach to quantifying welfare and estimating demand is just one of many possibilities. This thesis has taken a more theoretical approach when formulating demand, welfare and the excess burden. A more empirical approach would be to more freely estimate demand and use for example the work of Hausman (1981) to find the formulation of the welfare measures needed. The excess burden can also be empirically estimated using a Taylor approximation as explained in Auerbach (1982). This would probably result in a more accurate measure of the "real world" and it would be interesting to see how it compares to my more theoretical approach. I also think it would be interesting to look at the distributional issues of the electric vehicle policy, who is this policy really subsidizing and how does this relate to concepts of fairness and equity.

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A Mathematical Derivation

CES Price Index

The CES price index is derived by setting the expenditure on motorized vehicle in level two $G_{2,1}x_{2,1}^H$ equal to the budget constraint in level one, $\sum_i G_{1,i}x_{1,i}^H$, and solving for the price index $G_{2,1}$.

$$G_{2,1}x_{2,1}^{H} = \frac{U_{2}(G_{2,1})^{\frac{\rho_{2}}{p_{2}-1}}}{\left[\sum_{j} (G_{n,j})^{\frac{\rho_{2}}{p_{2}-1}}\right]^{\frac{1}{p_{2}}}}$$
$$\sum_{i} G_{1,i}x_{1,i}^{H} = \frac{U_{2}\left(\frac{(G_{2,1})^{\frac{\rho_{2}}{p_{2}-1}}}{\left[\sum_{j} (G_{n,j})^{\frac{\rho_{2}}{p_{2}-1}}\right]}\right) * [G_{1,i}]^{\frac{\rho_{1}}{p_{1}-1}}}{\left[\sum_{i \in j, i=1}^{2} (G_{1,i})^{\frac{\rho_{1}}{p_{1}-1}}\right]^{\frac{1}{p_{1}}}}$$
$$\frac{U_{2}(G_{2,1})^{\frac{\rho_{2}}{p_{2}-1}}}{\left[\sum_{j} (G_{n,j})^{\frac{\rho_{2}}{p_{2}-1}}\right]^{\frac{1}{p_{2}}}} = \frac{U_{2}\left(\frac{(G_{2,1})^{\frac{\rho_{2}}{p_{2}-1}}}{\left[\sum_{j} (G_{n,j})^{\frac{\rho_{2}}{p_{2}-1}}\right]}\right) * \sum_{i} [G_{1,i}]^{\frac{\rho_{1}}{p_{1}-1}}}{\left[\sum_{j} (G_{n,j})^{\frac{\rho_{2}}{p_{2}-1}}\right]^{\frac{1}{p_{1}}}}$$
$$G_{2,1} = \left[\sum_{i \in j, i=1}^{2} (G_{1,i})^{\frac{\rho_{1}}{p_{1}-1}}\right]^{\frac{\rho_{1}-1}{p_{1}}}$$

Ramsey's Rule

Maximizing the indirect utility

$$\max V_{2,1}(G_{1,1}^1, G_{1,2}^1)$$

subject to a revenue constraint

$$R = \sum_{i=1}^{2} t_{1,i} x_{1,i}^{M}$$

The Lagrangian is

$$\mathscr{L} = V_{2,1}(G_{1,1}^{1}, G_{1,2}^{1}) + \lambda \left[\sum_{i=1}^{2} t_{1,i} x_{1,i}^{M} - R\right]$$

and the first order conditions are

$$\frac{\partial \mathscr{L}}{\partial G_{1,1}^{1}} = \frac{\partial V_{2,1}}{\partial G_{1,1}^{1}} + \lambda \left[x_{1,1}^{M} + \sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,1}^{1}} \right] = 0$$
$$\frac{\partial \mathscr{L}}{\partial G_{1,2}^{1}} = \frac{\partial V_{2,1}}{\partial G_{1,2}^{1}} + \lambda \left[x_{1,2}^{M} + \sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,2}^{1}} \right] = 0$$

The first part of the first order conditions corresponds to the marginal effect on private surplus for the

individual, the last part is the marginal effect on government revenue. From Roy's Identity I get

$$x_{1,i}^M = -\frac{\partial V_{2,1}}{\partial G_{1,i}^1} / \frac{\partial V_{2,1}}{\partial M_{2,1}}$$

or

$$-x_{1,i}^M \frac{\partial V_{2,1}}{\partial M_{2,1}} = \frac{\partial V_{2,1}}{\partial G_{1,i}^1}$$

Letting $\frac{\partial V_{2,1}}{\partial M_{2,1}} = \alpha$, the first order conditions become

$$\frac{\partial \mathscr{L}}{\partial G_{1,1}^{l}} = -x_{1,1}^{M} \alpha + \lambda \left[x_{1,1}^{M} + \sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,1}^{l}} \right] = 0$$
$$\frac{\partial \mathscr{L}}{\partial G_{1,2}^{l}} = -x_{1,2}^{M} \alpha + \lambda \left[x_{1,2}^{M} + \sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,2}^{l}} \right] = 0$$

this is rewritten to

$$(\lambda - \alpha) x_{1,1}^{M} \alpha + \lambda \left[\sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,1}^{1}} \right] = 0$$
$$(\lambda - \alpha) x_{1,2}^{M} \alpha + \lambda \left[\sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,1}^{1}} \right] = 0$$

Dividing them by each other gives

$$\frac{x_{1,1}^{M}}{x_{1,2}^{M}} = \frac{\sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,2}^{1}}}{\sum_{i=1}^{2} t_{1,i} \frac{\partial x_{1,i}^{M}}{\partial G_{1,1}^{1}}}$$

From the symmetry of the Slutsky matrix $\frac{\partial x_{1,1}^M}{\partial G_{1,2}^I} = \frac{\partial x_{1,2}^M}{\partial G_{1,1}^I}$

$$\frac{\left[t_{1,1}\frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{1}} + t_{1,2}\frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{1}}\right]}{x_{1,1}^{M}} = \frac{\left[t_{1,1}\frac{\partial x_{1,2}^{M}}{\partial G_{1,1}^{1}} + t_{1,2}\frac{\partial x_{1,2}^{M}}{\partial G_{1,2}^{1}}\right]}{x_{1,2}^{M}}$$

which is Ramsey's basic rule. The dual approach is to minimize Excess Burden.

$$\min_{G^1} EB$$

subject to a revenue constraint

$$\sum_{i \in j} t_{i,1} x_{i,1}^M \ge R$$

The equi-proportional rule

Since $EB = E(G^1, V_2^1) - E(G^0, V_2^1) - R$, minimizing *EB* is equivalent to maximizing $V_2^1(G^1, M)$ (the indirect utility function).

For small changes in the tax system it is convenient to use a approximation

$$t_{1,1}\frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{l}} + t_{1,2}\frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{l}} \approx \Delta x_{1,1}$$

$$t_{1,1} \frac{\partial x_{1,2}^M}{\partial G_{1,1}^1} + t_{1,2} \frac{\partial x_{1,2}^M}{\partial G_{1,2}^1} = \triangle x_{1,2}$$
$$\frac{\Delta x_{1,1}^M}{x_{1,1}^M} = \frac{\triangle x_{1,2}^M}{x_{1,2}^M}$$

such that

Adapting Ramsey's rule to the case with externalities

Want to minimize the excess burden

$$\min_{G^1} EB$$

where

$$EB = EV - \sum_{i \in j, i=1}^{2} (t_{1,i} - e_{1,i}) x_{1,i}^{\mathcal{M}}$$

where $EV = E(G_{2,1}^1, G_{2,2}^0, V_2^1) - E(G_{2,1}^0, G_{2,2}^0, V_2^1)$. The Lagrangian is

$$\mathscr{L} = -EV + \sum_{i \in j, i=1}^{2} (t_{1,i} - e_{1,i}) x_{1,i}^{\mathcal{M}}$$

and the first order conditions

$$\begin{aligned} \frac{\partial EB}{\partial G_{1,1}^{l}} &= -\frac{\partial EV}{\partial G_{1,1}^{l}} + (t_{1,1} - e_{1,1}) \frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{l}} + (t_{1,2} - e_{1,2}) \frac{\partial x_{1,2}^{M}}{\partial G_{1,1}^{l}} + x_{1,1}^{M} &= 0\\ \frac{\partial EB}{\partial G_{1,2}^{l}} &= -\frac{\partial EV}{\partial G_{1,2}^{l}} + (t_{1,1} - e_{1,1}) \frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{l}} + (t_{1,2} - e_{1,2}) \frac{\partial x_{1,2}^{M}}{\partial G_{1,2}^{l}} + x_{1,2}^{M} &= 0 \end{aligned}$$

Let

$$\frac{\partial EV}{\partial G_{1,2}^{1}} + \frac{\partial EV}{\partial G_{1,1}^{1}} = \frac{\partial EV}{\partial G_{2,1}^{1}}$$

such that

$$\frac{\partial EV}{\partial G_{2,1}^1} = \frac{\partial E(G_{2,1}^1, G_{2,2}^0, V_2^1)}{\partial G_{2,1}^1} - \frac{\partial E(G_{2,1}^0, G_{2,2}^0, V_2^1)}{\partial G_{2,1}^1}$$

where $V_2^1 = V_2(\mathbf{G^1}, M_2)$. Solving this in two parts. First $\frac{\partial E(G_{2,1}^1, G_{2,2}^0, V_2^1)}{\partial G_{2,1}^1}$. Using Shepard's Lemma

$$\frac{\partial E(G_{2,1}^1,G_{2,2}^0,V_2^1)}{\partial G_{2,1}^1} = x_{2,1}^H(G_{2,1}^1,G_{2,2}^0,V_2^1)$$

since $x_{2,1}^H(G_{2,1}^1, G_{2,2}^0, V_2^1) = x_{2,1}^M(G_{2,1}^1, G_{2,2}^0)$

$$\frac{\partial E(G_{2,1}^1, G_{2,2}^0, V_2^1)}{\partial G_{2,1}^1} = x_{2,1}^M(G_{2,1}^1, G_{2,2}^0)$$

Then decomposing $E(G_{2,1}^0, G_{2,2}^0, V_2^1), E(G_{2,1}^0, G_{2,2}^0, V_2^1) = x_{2,1}^H(G_{2,1}^0, G_{2,2}^0, V_2^1)G_{2,1}^0 + x_{2,2}^H(G_{2,1}^0, G_{2,2}^0, V_2^1)G_{2,2}^0.$

Writing it out gives

$$E(G_{2,1}^{0}, G_{2,2}^{0}, V_{2}^{1}) = V_{2}^{1} \frac{\left[\sum_{j=1}^{2} \left(G_{2,j}^{0}\right)^{\frac{\rho_{2}}{\rho_{2}-1}}\right]}{\left[\sum_{j=1}^{2} \left(G_{2,j}^{0}\right)^{\frac{\rho_{2}}{\rho_{2}-1}}\right]^{\frac{1}{\rho_{2}}}} = V_{2}^{1} \left[\sum_{j=1}^{2} \left(G_{2,j}^{0}\right)^{\frac{\rho_{2}}{\rho_{2}-1}}\right]^{\frac{\rho_{2}-1}{\rho_{2}}}$$

where the term $\left[\sum_{j=1}^{2} \left(G_{2,j}^{0}\right)^{\frac{\rho_{2}}{p_{2}-1}}\right]^{\frac{\rho_{2}-1}{\rho_{2}}}$ is the old price index for transportation. Let $\left[\sum_{j=1}^{2} \left(G_{2,j}^{0}\right)^{\frac{\rho_{2}}{\rho_{2}-1}}\right]^{\frac{\rho_{2}-1}{\rho_{2}}} = G_{t}^{0}$, such that $\frac{\partial E(G_{2,1}^{0}, G_{2,2}^{0}, V_{2}^{1})}{\partial C_{t}} = \frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial C_{t}}G_{t}^{0}$

$$\frac{\partial E(G_{2,1}^{0}, G_{2,2}^{0}, V_{2}^{1})}{\partial G_{2,1}^{1}} = \frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial G_{2,1}^{1}}G_{t}^{0}$$

From Roy's Identity $x_{2,1}^M(G_{2,1}^1, G_{2,2}^0, M_2) = -\frac{\partial V_2^1(G_{2,1}^1, G_{2,2}^1)}{\partial G_{2,1}^1} / \frac{\partial V_2^1(G_{2,1}^1, G_{2,2}^1)}{\partial M_2}.$

$$\frac{\partial E(G_{2,1}^0, G_{2,2}^0, V_2^1)}{\partial G_{2,1}^1} = -x_{2,1}^M(G_{2,1}^1, G_{2,2}^0) \frac{\partial V_2^1(G_{2,1}^1, G_{2,2}^1)}{\partial M_2} G_t^0$$

putting all this together gives -

$$-\frac{\partial EV}{\partial G_{2,1}^{1}} = -x_{2,1}^{M}(G_{2,1}^{1}, G_{2,2}^{0}) + x_{2,1}^{M}(G_{2,1}^{1}, G_{2,2}^{0})\frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial M_{2}}G_{t}^{0}$$
$$-\frac{\partial EV}{\partial G_{2,1}^{1}} = x_{2,1}^{M}(G_{2,1}^{1}, G_{2,2}^{0})\left[\frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial M_{2}}G_{t}^{0} - 1\right]$$

Remember that $x_{2,1}^M(G_{2,1}^1, G_{2,2}^0) = x_{1,1}^M(G_{1,1}^1, G_{1,2}^1) + x_{1,2}^M(G_{1,1}^1, G_{1,2}^1)$. Insert this into the FOCs and get

$$\frac{\partial EB}{\partial G_{1,1}^{1}} = x_{1,1}^{M}(G_{1,1}^{1}, G_{1,2}^{1}) \left[\frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial M_{2}} G_{t}^{0} \right] + (t_{1,1} - e_{1,1}) \frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{1}} + (t_{1,2} - e_{1,2}) \frac{\partial x_{1,2}^{M}}{\partial G_{1,1}^{1}} = 0$$

$$\frac{\partial EB}{\partial G_{1,2}^{1}} = x_{1,2}^{M}(G_{1,1}^{1}, G_{1,2}^{1}) \left[\frac{\partial V_{2}^{1}(G_{2,1}^{1}, G_{2,2}^{1})}{\partial M_{2}} G_{t}^{0} \right] + (t_{1,1} - e_{1,1}) \frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{1}} + (t_{1,2} - e_{1,2}) \frac{\partial x_{1,2}^{M}}{\partial G_{1,2}^{1}} = 0$$

divide, the result is

$$\frac{(t_{1,1} - e_{1,1})\frac{\partial x_{1,1}^M}{\partial G_{1,1}^1} + (t_{1,2} - e_{1,2})\frac{\partial x_{1,2}^M}{\partial G_{1,1}^1}}{(t_{1,1} - e_{1,1})\frac{\partial x_{1,1}^M}{\partial G_{1,2}^1} + (t_{1,2} - e_{1,2})\frac{\partial x_{1,2}^M}{\partial G_{1,2}^1}} = \frac{x_{1,1}^M}{x_{1,2}^M}$$

rewrite to

$$\frac{(t_{1,1}-e_{1,1})\frac{\partial x_{1,1}^{M}}{\partial G_{1,1}^{1}} + (t_{1,2}-e_{1,2})\frac{\partial x_{1,2}^{M}}{\partial G_{1,1}^{1}}}{x_{1,1}^{M}} = \frac{(t_{1,1}-e_{1,1})\frac{\partial x_{1,1}^{M}}{\partial G_{1,2}^{1}} + (t_{1,2}-e_{1,2})\frac{\partial x_{1,2}^{M}}{\partial G_{1,2}^{1}}}{x_{1,2}^{M}}$$

which is the result used in section 3.3



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