Norges miljø- og biovitenskapelige universitet
Samfunnsvitenskapelig fakultet
Handelshøyskolen

Masteroppgave 2014
30 stp

An Empirical Study of the Risk Parity Allocation Approach Using the World Equity Market

Does portfolio allocation of equity using the Risk Parity approach outperform other heuristic-, naive-, and optimized allocation strategies.

Tom Erik Sønsteng Henriksen

Abstract

The aim of this thesis is to build different Risk Parity portfolios and thereby perform an out-of-sample analysis by comparing them with other more common portfolio strategies.
The reason why this portfolio allocation is investigated is because of the increasing popularity of risk based asset allocation strategies, and especially the Risk Parity approach.

For this reason the author decided to test and compare the portfolios constructed from several MSCI World country indices based on equities during the time period 1995 to 2013. And in this thesis there will be constructed Risk Parity portfolios based on different risk measurement. These two approaches are conducted because there are really no papers that investigate this.

The tests performed are based on different measurements including characteristics of return, risk, risk-adjusted performance, diversification, and investment capacity.

The results of this investigation show that the Risk Parity approach is not superior in the measurement performed in this analysis. It seems like the Risk Parity portfolios are mediocre performing when looking at risk and return characteristics. And the portfolio is somehow not always the most diversified.

But this is a relatively new approach and need further investigation.

Contents

List of Figures

List of Tables

1. Introduction

1.1. Background and motivation

To predict the financial markets risk and return characteristics has always been the aim for both practitioners and academics. Bachelier (1900) was the first to model stock market movements as an unpredictable random walk in his dissertation "Théorie de la Spéculation", and thereby laid a foundations for modern finance. Then, in 1952, Harry Markowitz published a formal model of portfolio selection embodying diversification principles in his paper “Portfolio Selection” (Markowitz, 1952). The Harry Markowitz model laid a foundation for portfolio management with the identification of the graphical depiction of the highest expected return possible given the level of risk in the efficient set of portfolios with the assumptions for expected returns, standard deviation and correlation between assets. Then in 1958 Bill Tobin showed that the frontier can be improved by adding risk-free investments to the portfolio to create the “Capital Market Line” which represent the portfolios with the highest returns for a set level of risk than those on the efficient frontier (Tobin, 1958). The idea behind Tobins model is that when using leverage in a low-risk and highly diversified portfolio one can achieve a return-to-risk trade off superior to the unlevered traditional portfolio concentration in risky assets. Later, and based on Bacheliers work, the Efficient Market Hypothesis (EMH) was developed by Eugene Fama (1965). The EMH theory stated that it is impossible, based on the prior information available, to consistently outperform market returns on a risk adjusted basis. In other words; conditioned on the past, the expectation of asset returns will be always zero and consequently can be considered a fair game. But there is no consensus about the EMH, since some investors by using the available information, can successfully transform this to investment strategies that outperforms benchmarks or passive strategies. At the same time Sharpe (1964), Lintner (1965), and Mossin (1966) came up with the centerpiece of modern financial economics; Capital Asset Pricing Model (CAPM), which gives a prediction of the relationship that one should observe between risk of an asset and its expected return, and widely used in pricing of assets. The general idea behind CAPM is that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free rate that compensates the investors for placing money in any investment over a period of time and the other half represents risk and calculates the amount of compensation the investor needs for taking on additional risk.

These papers where milestones in portfolio management, and still are the foundation of the theory many investors today use, even though they have assumptions that are not reliable in the real world and that these approaches is very sensitive to input parameter, especially expected return (Merton, 1980). The reason why the use of these theories still obtains by practitioners is mainly due to its simplicity and that there really are no better substitutes based on theoretical justification. But there are investors that use other framework for portfolio selection that is not based on theoretical justifications, like the widely used naïve weighting schemes as the equally weighted and the “Pension fund” weighted with 60% in equity and 40% in bonds. This may be because humans are not a homogenous group with the exact same preference for risk and returns, and maybe because of the knowledge the investor holds.

When looking at these allocation strategies over the last years, one see that there are drawbacks by implement them and rely solely on the theories. This was to be seen when the dot-com crisis in 1999-2001 occurred, when institutional investors and especially pension plans , lost a great amount of its money because of its high exposure to equities (Ryan and Fabozzi, 2002). But after the crisis the equity market again restore confidence in standard financial models would continue to provide cash to the investors, and the bubble was soon forgotten and assumed not occur again. But in 2008 the financial crisis highlighted the risk inherent in many strategic asset allocation strategies. People had tried to achieve diversification in different assets classes, industries and regions, but this diversification was not enough to protect them from the rapid fall in asset prices during the crisis. The Markowitz model was fast criticized by professionals, probably because of the diversification is traditionally associated with Markowitz and his simplified statement; “Do not put all your eggs in the same basket”. Much of the failure due to the Markowitz method was caused by the input parameters, in this case, the expected returns and the covariance matrix relying on historical data, provide models with an overweight of equities. But this may not be the only allocation to blame the loss for investor, because this also happened in constant mix portfolios (equally weighted and 60/40 equity/bond). So the one thing that is for sure; as often heard *“The only thing went up during the crisis is the correlation between seemingly diversified assets” (Roncalli, 2013),* may also be considered.

There is, as on have seen, a natural reaction to look back after a crisis and the imminent danger has subsided, and evaluates what went wrong and further try to find solutions and develop strategies to mitigate the impact or avoid of future crisis with similar characteristics. When markets begin to recover, practitioners and academics come up with a seemingly endless "next generation state-of-the-art solutions" to what went wrong with the asset management industry.

The one thing that is sure in the aftermath of these two crises is that it has profoundly changed the industry of asset management by putting risk management at the heart of most investments processes. The pressure for more robustness and transparency has therefore modified the relationship between the investors and portfolio managers. It is now more focus on risk factors, risk management, diversification and information ratio, rather than promise high return and fast money. So there is no coincidence that the portfolio strategies based on risk and even more diversification is emerging and become more popular after these series of black swans (Bruder and Roncalli, 2012). Especially the Risk Parity allocation approach has been heavily discussed, which can be explained as an equally weighted portfolio in terms of risk contribution, i.e. not diversify by dollars but risk. The increasing interest in this allocation can perhaps be explained by the increasing number of papers, too numerous to cite, recently published on the subject in the practitioner's literature. In recent years Financial Times and Wall Street Journal had several articles[1], and in 2012 both Journal of Investing and Investment and Pensions Europe had special issues on the Risk Parity allocation[2]. The interest of the Risk Parity approach has also lead financial institutions and investment companies to offering risk parity funds to their clients, as one can see from Putnam Invest, Neuberger Berman, AllianceBernstein, AQR Capital, and Arbejdsmarkedets Tillægspension (ATP) which is a supplementary pension in Denmark.

But Risk only portfolio is not a new idea, since in 1952, Markowitz identifies the minimum variance portfolio (Markowitz, 1952), and when looking at the heart of risk parity, Booth and Fama may have been the first to mention the risk contribution of an asset in the context of a multi-class allocation in their paper "Diversification returns and asset contributions" (Booth and Fama, 1992). Litterman was also an economist that used the risk contribution in his study "Hot

[1] "New Allocation funds redefine Idea of Balance" (February 2012), "Risk Parity Strategy Has Its Critics as Well as Fans" (June 2012), "Rising Volatility Fuels Push to Lower Risk" (May 2013), "Pension Fund Manager Survey: Complex Investments Take a Back Seat" (July 2013). "Fashionable 'Risk Parity' Funds Hit Hard"(June 2013)
[2] Journal of Investing (Fall 2012, Vol. 21, No. 3)
Investments & Pensions Europe (June 2012 Magazine)

Spots and Hedges" (Litterman, 1996) to find the "best hedge" position for that portfolio component, and this was show to be when the risk contribution is close to zero and the position with high risk contribution was called "hot spots". The combination of diversification and the work of Markowitz, Booth and Fama, combined with Tobins leverage theory and the EMH created the fundaments for the Risk Parity allocation. But the first to mention "Risk Parity" was Edward Qian in his paper "Risk parity portfolios: Efficient portfolios through true diversification" (2005) for Panagora Asset Management, but the approach was certainly used before 2005 by some equity market neutral funds, as the All Weather fund managed by Bridgewater in the mid-1990s (Dalio, 2004). And it can also be mentioned that it has been the practice to use risk contribution analysis when calculating and analyzing Value at Risk (Hallerbach, 1999, Alexander, 2009).

To easily explain the reason why the Risk Parity portfolio is becoming a popular allocation strategy one can start looking at the heuristic 60/40 equity/bond portfolio, which is an anchor point of many Anglo-Saxon pension funds (Ambachtsheer, 1987). This portfolio, and also the 50/50 portfolio, might appear balanced in terms of capital allocation, but it is highly concentrated from the perspective of risk allocation. This concentrated risk exposure occurs because it disregards the differences in volatilities and correlations of the constituents. These constant mix portfolios have a diversified dollar contribution, but the risk contribution can be very different due to differences in the volatility so there is a lack of risk monitoring in these portfolio allocations. A seemingly well-diversified allocation in many asset classes that essentially load on the same risk factors can eventually generate a portfolio with a very concentrated risk exposure. Such a portfolio, the 60/40 or 50/50, can have about 80- to 90 percent of its risk contribution in the equity risks of the portfolio (Roncalli, 2013). But constant-mix portfolios has shown to perform well in the long term and rooted in several funds; including insurance- and pension funds[3]. The same argument of clustering risk will be made for the global minimum variance portfolio (GMV) and also the Maximum Sharpe-ratio portfolio (MSR); these portfolios may include relatively few assets, and the risk contribution will thereby be of great amount in these few assets. As one often sees, the GMV is aiming to minimizing variance, but if this is the

[3] Norges Bank Investment Management. http://www.regjeringen.no/pages/38669593/PDFS/STM201320140019000DDDPDFS.pdf

appropriate objective when it comes to reducing risk is a debatable question when focusing on risk management and diversification.

So the underlying idea of the Risk Parity portfolios is to build a balanced and diversified portfolio by identify portfolio weights such that asset classes contribute equally to the overall portfolio risk. In other words; achieve the maximum risk diversification, given by risk contribution and weights, by avoid one or few asset classes to have a dominant role in driving the portfolios risk. The weights are driven by the assets risk and correlation so that an asset with higher correlation and risk measure with other assets will have a lower weight and those with a lower correlation compared with other assets will have a higher weight. The reason why one will optimize weights with respect to these measurements is because return prediction is difficult due to the EMH, levels of dispersion as risk forecasts can be statistically modeled; this also applies to the correlation. So it does not depend on any expected return hypothesis. But when using parametric VaR and Conditional VaR (CVaR) as the objective risk measure to construct the portfolio, one use the average return as a risk measurement rather than expected returns for gaining earnings. But there is also a level of parameter uncertainty and un-stability due to inputs as variances and covariance's used in the Risk Parity portfolio, since these measurements vary over time.

But if an investor would only invest in a Risk Parity stock/bond portfolio, the fixed income part would have a weighting of approximately 90 percent and therefore it is quite common to use leverage to increase the overall portfolio volatility of the portfolio to a given target. Why ignore the return dimension ex-ante produce portfolios that are superior in ex-post risk-adjusted performance? Focusing only on risk aspects when constructing a portfolio is a perfectly sensible starting point when one has low confidence in ex-ante estimates of returns, and also because the volatility or risk forecasts can be statistically modeled and therefore are more reliable. Traditional risk measures like volatility can underestimate the true risk associated with leverage, as well as there is always risky to use leverage. Risk due to negative skew or fat tails in levered assets returns are not captured by symmetric risk measures as volatility. There is additional liquidity risk associated with levered portfolios. For example, in extreme market conditions, liquidity mismatch between the long and the short side of the portfolio may make it difficult to roll over the short term loans used to lever the fixed-income allocation. In this situation, the investor with a levered fixed-income allocation would be forced to reduce leverage aggressively by liquidating the fixed-income portfolio, which can be costly. The critics of leverage was introduced by Black (Black, 1972) by showing that the slope of the capital market line changes when there are borrowing restrictions. A more comprehensive study of drawbacks of the Risk Parity Approach is extensively well explained in Inkers 2011 paper (Inker, 2011), which later will be discussed. Keel and Ardia (2011) are also skeptic to allocate portfolios by its risk contributions. They are skeptic, mainly because the risk contribution is precise measurements for infinite small changes, and that these approximations can be poor for realistic allocations, and also that the strategy assumes changes in a single position while keeping all other position fixed, which a weak assumption and not the case in the real world of finance.

And this leads us to the main question in the risk control context is: Does it work? Will it yield a good performance in the manner of return-to-risk measurements? Does it provide better risk balance or risk profile, and can we avoid concentration and thereby achieve a greater diversified portfolio? It may be useful to examine the portfolios performance in the context of the recent history, even if it is unlikely that the future will look like the past. This type of back-testing out-of-sample analysis can provide valuable insights into the real-world risk, but is not accurately reliable.

This paper will provide an empirical analysis and an investigation of the increasingly popular risk based portfolio; Risk Parity. The paper will compare this approach to other more common allocations by constructing portfolios based on world equity indices. The reason why the paper uses this approach is due to the few papers that are investigating this portfolio based on an equity portfolio. The other reason is the heavy leverage in fixed income when included in the Risk Parity portfolio, since several investors do not have the ability to use leverage, and also the danger by using leverage in portfolios. The reason why this paper calculates different Risk Parity portfolios is because there are few papers that have compared the different results that will occur when using VaR contribution and CVaR contribution as the risk objective in constructing the Risk Parity portfolio.

From now on the Risk Parity (RP) will be stated as a generic term for all the different RP portfolios, and the different RP portfolios will be explained in detail in section 4.

The remainder of this paper is organized as follows. In Section 2 there will be given a brief literature overview on the topics previous research. Section 3 documents the data and some stylized facts are presented. Section 4 presents the methodology used in the paper, and provide a summary of the theory and methods used to calculate the findings of the paper. In Section 5 the empirical results and findings will be presented and be discussed. The final section summarizes our results and suggests some possible avenues for further research.

2. Previous Research & Literature

In the research of the risk parity approach, there will be natural to start with the work of Edward Qian, who is coined the term "Risk Parity" by his article "Risk Parity Portfolios: Efficient portfolios through true diversification" (Qian, 2005). The paper starts with a simple explanation and a theoretical justification of risk contribution. This explanation shows that in a 60/40 portfolio consisting of Russel 1000 Index and Lehman Aggregate Bond Index, with the time span from 1984 to 2004, the risk contribution to the stock index 93% and only 7% to the bond index. He states that the RP portfolio will deliver true diversification that will limits the impact of losses of individual components in the portfolio. He also states that the RP portfolio is expected to generate superior return for a given level of targeted risk. In his empirical analysis he also finds that the Sharpe ratio of both the levered and unlevered RP exceeds both the market portfolio and the 60/40 portfolio. He also shows the difference in loss contribution to the RP compared to the 60/40 portfolio. The loss contribution in the RP portfolios is more stable and weighted equally compared to the 60/40 where the average loss contribution to the stock index is about 100%. The paper also gives a brief explanation why one can set the Risk Parity on the efficient front. The assumption behind this is that when the individual asset classes have the same Sharpe ratio and are uncorrelated and by that are thereby mean-variance optimal, but as he states; these are unrealistic assumptions.

In "On the Properties of Equally-Weighted Risk Contributions portfolios" (Maillard et al., 2008) the authors looking at the Equal Risk Contribution portfolio (ERC) on an equity and commodity portfolio from the period 1973 to 2008. Thereafter they are looking at a global diversified portfolio with the time horizon from 1995 to 2008. When they compare the ERC with the equally weighted (EW) and the global minimum variance (GMV), they find that the ERC are very close to their counterpart for the EW portfolio, both in return to risk characteristics and the diversification. Both ERC and EW seems well diversified because a low Herfindal and Gini statistics. This is due to the similarities in the correlation and volatility in the equities included in the portfolio, but ERC is clearly dominant in the term of turnover, i.e. transaction costs. When comparing the ERC to the GMV portfolio, one can see that the ERC outperform the GMV in terms of diversification, as GMV have huge concentration in single equities and in a much lower turnover. When they construct the ERC on commodities the heterogeneity is also in place because of the volatilities and correlations. Also here the ERC outperforms the EW and GMV in terms of risk and return. The drawdown seems more robust in the short run of the ERC, which is an advantage with assets characterized by large tail risk. When looking at the "global diversified portfolios" there are also large heterogeneity, both in terms of individual volatilities and correlation coefficients. The ERC yield the best performance when looking at Sharpe-ratios and average returns, and ERC and GMV is largely dominating the EW when looking at the Sharpe-ratio. The paper also derive theoretical properties that shows that the ERC have a volatility between the EW and the GMV, which is later extended by Bruder and Roncalli (2012)to the risk budget approach.

"An Introduction to Risk Parity" by Kazemi (2011) concludes that the Risk Parity approach provides a close approximation of the original Harry Markowitz model. He also states that this is a suitable model for institutional and high net investors who do not face significant constraints on their asset allocation policies and are able to use leverage. The data used in his paper is HFRI Fund Weighted Composite, MSCI World index, and the Barclays Capital Global Aggregate (Hedge Fund Index/Stock Index/Bond Index) with time period from 1990 to 2011. From the data he used one can see that the ERC with leverage have higher Sharpe-ratio than both the 10/50/40

(Hedge Fund/Stock/Bond) portfolio and the Volatility Weighted portfolio, i.e. the Naïve Risk Parity portfolio (NRP).

The article “Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios” (Chaves et al., 2011) conduct an analysis on the SP500 and BarCap Aggregate in the period from 1980 to 2010, and the sub periods based on a ten year horizon. The article shows that ERC portfolio has a higher Sharpe-Ratio than well-established approaches like GMV and mean-variance (MSR), but it does not consistently outperforms the EW or 60/40 (equity/bond) portfolio. The paper discuss inclusion and exclusion of assets in the risk parity portfolio and find that the approach is very sensitive to the inclusion decision for assets, and believe that this finding needs more research. The paper also warns investors about these back-tests, because they are highly dependent on the study period and the choice of universe.

(Qian, 2011) stated in the article “Risk Parity and Diversification” that the RP allocation obvious is a good way to obtain a well-diversified portfolio. He also states that the risk parity approach, in contrast to the GMV and MSR, focuses on the maximization of diversification and assumes that risky strategies are fairly rewarded in the market equilibrium. He states that the main advantage of this relatively new method is that expected returns do not need to be estimated and incorporated into the optimization process.

In the paper “The Dangers of Risk Parity” (Inker, 2011), the author look at the bond and commodities risk premium over a longer period than most of the papers that investigates the risk parity approach, this due to the shorter time horizon in the previous research. He looks at the period from 1920 to 2010 in the U.S. market. He states that the first 41 year period between 1940 and 1981 the bond had a negative risk premium, and stating that if we were to experience that scenario once again, the ERC portfolio would long before have been abandoned by investors. He argues that looking at the ERC portfolio from the 80’s like many investigation does, will give the portfolio an incredible favorably starting point and will give the risk parity an artificially good looking return. The paper also states its concerns due to shift in volatility, as was shown by the spread of the AAA rated Asset-Backed securities (ABS) in the financial crisis of 2008. In the 2008-2009 periods the ABS truly had a paradigm shift in its volatility, with an increase in volatility by a factor of 200. This has not happened yet with bond, as he mentioning, but he states this can happen with bonds as well. The probability of that occurring in the sovereign bond is low but not zero, and if this occurs a levered portfolio with a great amount of T-notes will fail. The lesson from this is that RP portfolios rely on heavy leveraged securities with historical low volatility, and will these low volatility securities have the same low volatility in the future, we do not know, an cannot see this when performing back testing. Nothing is risk-free, even government can fail as seen recent in Europe and especial Greece. But one can ask; will another portfolio be hurt in the same manner, greater or less, as the RP portfolios when such an event occurs?

"Balancing Asset Growth and Liability Hedging through Risk Parity" (Peters, 2011) is questioning if the RP can balancing asset growth with current liability management, i.e. hedge future and current liabilities at the same time. Peters states that this is possible by a RP portfolio. In the papers author studying the following three approaches: ERC, static policy portfolio, and a partial liability-directed investment (LDI) hedge approach. He found that the 50% LDI Plan[4] has the lowest funded status volatility, followed by the ERC portfolio, and thereby states that there is an advantage to partial hedging even if the funded status depends on the return on equity market. The 50% LDI and ERC have significant correlation to both the stock and liabilities showing that they do have both elements of growth and liability hedging in them. The article also states that the ERC has less realized risk variation because its asset allocation shifts to compensate for changes in risk over time. And also the ERC portfolio outperforms the two other portfolios in terms of returns and had a higher and more consistent Sharpe ratio than the two other portfolios. Finally the paper also says that the ERC portfolio increases the liability-hedging component when you really need it and focuses on growth when liability hedging is less of an issue. This due to when there are a high volatility regime; stocks have higher volatility and lower return than the liabilities, and vice versa. The RP will balancing the liability hedging, to lock in a portion of current liabilities, with asset growth to hedge against the growth of future liabilities, and with RP these two goals can be achieved. Peters concludes that RP offer liability hedging benefits. He argues that the present durations similar to those of defined benefit pension plans, because of the leverage effect. However, this argument is counter balanced by regulatory constraints of defined benefit plans, because most of them are not allowed to use leverage.

[4] 50% LDI portfolio in this paper is to immunize a portion of the liabilities and invest the reminder in growth securities like equities, i.e. hedge 50% of liabilities and 50% risk budget in growth.

The basic principle of pension plan investing is to ensure that enough funds should be available to fulfill the liabilities when they arise and to seek the maximum accumulation when liabilities arise (Ruban and Melas, 2011). The authors state that equities arguably hold an advantage as an asset class in matching the attributes of long-term inflation-linked liabilities. Due to the volatility of equity returns, some proportion of other assets also needs to be held to diversify the risk so that the liabilities more likely can be met. When looking at a 60/40 portfolio, this portfolio will be consistent with ERC if bond volatility is 1.5 times higher than equity volatility. But historical the equities have the last 40 years have an volatility five times the fixed income in the US and Euro zone, and thereafter to get an equilibrium in the risk contribution one need only to allocate 17%-25% to equities. The drawback of implementing ERC with no leverage may reduce return below the target level since there is a significant level of fixed income with a historical lower return than the equities. When rebalancing the portfolios he found that the ERC underperforms the 60/40 portfolio in terms of return from mid-1970's to this decade. But while ERC underperforms in terms of raw return, it outperformed the 60/40 in terms of Sharpe ratios. But as the authors' states: "High risk-reward ratios do not put the money in the bank – returns do". If the cost of leverage is proportional to the Fixed Income return then the amount of leverage would be higher than shown and would also depend on the cost of leverage.

In their 2010 paper (Ruban and Melas, 2010) they also find stated that adding leverage can reduce portfolio risk only if the correlation between the asset classes is sufficiently negative and in their paper that this condition of reducing portfolio volatility is not satisfied in any of the markets considered. They also argue that there is a problem with achieving risk parity through adding leverage and not rebalancing the portfolio, since the risk of the levered portfolio may be too high. And one has to be careful in assuming linearity in the Sharpe ratio when adding a levered extension. Adding leverage in the Fixed-income allocation is most likely to lead to an enhancement in risk adjusted returns when the correlation of bonds and equities are low and the bond risk-adjusted returns are higher than the equity risk-adjusted returns. The paper also investigates two different ways of using leverage in the RP portfolio. The first is to create a RP portfolio with the same volatility forecast as the competing asset allocation, and the second is to create a RP portfolio with the same expected return as the competing portfolio (volatility versus expected return as a target for the use of leverage). The magnitude of the difference in leverage between these two methods of portfolio construction is quite large. The conclusion states that whether it is optimal to add leverage from a volatility-reduction perspective depends on the correlation between the assets, the volatility between them, and the weight of the assets in the portfolio. The paper states that when correlation is negative, adding leverage could reduce the volatility of the portfolio when the fixed income weight is low, leverage is moderate, and bonds have low risk relative to the equities. Negative correlation also, in the paper, is said to increases the likelihood that adding leverage will improve the risk-return profile of the portfolio.

In Qians 2012 article "Pension Liabilities and Risk Parity" (Qian, 2012) he suggests to use a ERC portfolio as the performance portfolio. For an overfunded pension plan, investing the surplus in the ERC portfolio decreases the overall risk and also expects extra performance, which is more certain than the 60/40 portfolio often used by Pension funds. And conversely, the ERC portfolio will be less aggressive than the current performance portfolio if the pension plan is underfunded, which implies higher recovery time.

A more theoretical and mathematical approach is the paper on efficient algorithms (Chaves et al., 2012), where the approach is shown by using matrix algebra. They find that the both NRP and ERC provide superior diversification in asset class risk contribution, i.e. each of the included asset classes does provide relatively more equal contribution strategies. They also conclude that the ERC approach in all situations provides the best ex-post and ex-ante "parity" in asset class risk contribution.

In the paper "Leverage Aversion and Risk Parity" (Asness et al., 2012) based on the paper from 2010 (Frazzini and Pedersen, 2010) tries to formalize a theory of leverage aversion. In the paper they used a broader data from 1926 to 2010 derived into the sub periods: 1973-2010 and 1986-2010. In the paper they have not only reviewed asset classes, but also the different in a global sample of different countries. They found that RP with leverage outperformed the market by a Sharpe-ratio that was 0.27 higher than the market portfolio. They conclude that this paper enhances their confidence that risk parity's superiority to traditional methods of asset allocation is real and important, and not a figment of the data. They stated that the ERC performance in funds can be explained by overweight in less volatile assets and leverage them. But as the title states, there are criticism concerning the use of leverage, but the theory of leverage aversion also applies to other portfolios that have the overweight in safer assets. The critics is also highly discussed in Inkers paper (Inker, 2011), Ruban and Melas (2011), and Sebastian (2012).

Chan-Lau (2012) compare the performance of global equity portfolios with using the weighting schemes of market capitalization and risk parity, and covering the period from 2007 to 2011 with different rebalancing in the paper “Frontier Markets: Punching Below Their Weight? A Risk Parity Perspective on Asset Allocation”. The paper states that the ERC outperforms the market cap portfolio when considering Sharpe-ratios when considering the pre-crisis of 2008. But in the aftermath, the market cap-weighting outperforms the ERC. The paper shows that Frontier equity markets have low correlation with other markets and can help diversify global equity portfolios, and thereby when overweighting the market cap portfolio with frontiers markets, this could help portfolios when equity prices are rising. This outperformance is not achieved when there are rapidly downturns relative to the market cap-weight benchmarks.

Kaya and Lee (2012) demonstrate in “Demystifying Risk Parity” that ERC, similar to GMV, by construction biased towards low beta, low idiosyncratic asset and therefore is able to capture the pricing anomalies documented by Jensen, Black and Scholes (Jensen and Scholes, 1972). The paper also conducts that the ERC allocation may better navigate a world with fat tails and noise. This can also be seen in the light of the findings Merton did (Merton, 1980). Where Merton stated that the precision of estimating risk is higher than the precision of estimating expected returns, and therefore risk parity may appear as to be a reasonable solution in allocating new strategies. They also conclude that stability of the ERC also is more robust and stable than other risk driven allocation methods, especially the when it comes to the covariance instability.

The study behind the paper “Diversifying Risk Parity” (Lohre et al., 2012) lies in an application from the framework of Meucci (2009) and his maximum diversification theory, mixed with the risk parity framework to turn these strategies into an empirical multi asset allocation framework. The paper construct uncorrelated risk sources by applying principal component analysis to the covariance matrix of the portfolio assets, and thereby use the findings of this framework to combine it with the ERC to the principal portfolio, and thereby find their “diversified risk parity” weights by maximize the “Shannon entropy”, explained in Appendix 10, section 10.8.4. The portfolio that is shown in this paper is designed to balance the risk sources in the portfolio due to the diversification measure "Shannon Entropy". The data used in the paper is based on JPM Global bond Index, MSCI World, MSCI Emerging Markets, Barclay US Aggregate Credit Index, and US 3- months U.S. T-Bills, with a time period ranging from1987 to 2011. The portfolio is shown to tracking the prevailing risk structure and to yield a better diversification effect over time, compared with other risk based portfolios. The ERC portfolio was shown to have a relatively high risk adjusted performance measures, and also had the second lowest turnover, but the GMV yield a higher Sharpe ratio than the ERC during the period and also had the lowest draw-downs. The 1/N was the portfolio with the lowest turnover, but also the lowest Sharpe ratio. The ERC also appeared to be the least affected at the outset, and that the ERC strategy decreases in its degree of diversification over time when looking at the diversification measures. It seems like the Diversified Risk Parity was superior due to the risk adjusted measures and also the diversification measurements used, but may be affected by the high turnover costs for the portfolio.

The "Least-squares approach to risk parity in portfolio selection" (Bai et al., 2013) discuss the problem of finding portfolios that satisfy RP of either individual asset or groups of assets. The paper also discusses and describes the set of all ERC solutions by using convex optimization techniques that may contain an exponential number of solutions. They later show a non-convex and a linear framework set that aims to select the most desirable risk parity portfolio. They also compare the different RP strategies against EW, GMV and the 60/40 allocation, with an investment universe of 14 asset classes including equity indices, fixed income indices, and an energy index, and a 3-month T-bill rate, and investigate the period from 2002 to 2012. When they compare the US equity portfolio with a 3 year rolling window, they found that ERC had the highest excess return, and the volatility lied between 1/N and GMV but the ERC had the highest Sharpe Ratio. When using 5 year rolling window, one see that the GMV had the highest Sharpe Ratio and that the 1/N had the highest excess return and also the highest standard deviation. When looking at the 10 year rolling window, the GMV also outperformed the ERC according to its Sharpe ratios, but the portfolios have relatively similar Sharpe ratios. The ERC portfolios outperformed the other portfolios when it comes to excess return when the 10 year rolling window was used.

The paper "Risk Parity and Beyond – From Asset Allocation to Risk Allocation Decisions" (Deguest et al., 2013) aims at analyzing whether the use of uncorrelated underlying risk factors, as opposed to correlated assets returns, can lead to a more efficient framework for measuring and managing portfolio diversification. And the paper following the Meccui (2009) paper on effective number of bets, which can be explained by the effective number of uncorrelated baskets if we use the egg-basket example. The Data used in the paper consists of 7 asset classes: US Treasury Bonds, US Corporate Bonds, US Large cap stocks, US Private Equity, and an international equity index, Real Estate, and Commodities. The time period used in the paper is from 1992 to mid-2012. The author also provide empirical evidence that incorporate constraints, or target levels, on a portfolio effective number of bets generates an improvement in out-of-sample risk-adjusted performance with respect to standard mean-variance analysis. The paper also analyzing and shows evidence that large state pension fund as an example, which even a seemingly well-diversified portfolio may end up loading on a very limited number of independent risk factors. Thereafter they aim to build a better diversified portfolio called Factor Risk Parity using a factor model, and thereby maximize the effective number of bets. This is also done previous (Lohre et al., 2012), where there also was used principal component analysis to extract the uncorrelated factors and analyze the out-of sample performance of Factor Risk Parity portfolios (see also (Roncalli, 2013)). They also states the drawback of the otherwise intuitively appealing approach of ERC, is that it disregard the fact that large portfolios may be driven by a small number of factors, this limitation of ERC can be addressed with a factor RP methodology. They conclude that a seemingly well-diversified portfolio (EW, GMV, MSR or ERC) may well result in a portfolio heavily concentrated in terms of factor exposures, and thereby important to measure and manage the effective number of bets in a portfolio. They also state the critics about the substantial overweight of bonds versus equities in the RP portfolios, and that this might be a concern in a high-bond-price-low-bond-yield environment. When looking at the risk adjusted performance of the portfolio, the MSR portfolio will outperform when short-selling is considered, but when there is a short-selling constraint, the Factor Risk Parity will yield the highest Sharpe. They also conclude that the conditional RP strategies that are designed to optimally response to changes in market conditions need further research.

"Risk-Based Allocation of Principal Portfolios" (Kind, 2013) discuss the relationship of Meucci (2009) with the concept of principal risk parity to extract uncorrelated synthetic portfolios by using Principal Component Analysis. The article discusses possible reason why the risk diversification strategies do not outperform nominal diversification in this paper. The data used in the paper is based on the time period from 1995 to 2010 with a daily data and monthly rebalancing, with US Bond and Note, S&P 500, FTSE 100, Corn, Gas oil, Japan Yen and Canadian dollar. The paper also states that the diversification strategies are outperformed by the EW portfolio, and the paper will thereby discuss possible reasons of this underperformance. They conclude that there is no reason to expect a general outperformance from risk diversification strategies since these strategies are vulnerable to estimation errors, and that diversifying bad bets will not lead to an outperformance. They also stated that it seems better to look for optimal diversification instead of maximum diversification, because there is not much to gain from diversification in an already well-diversified universe.

"Advances in Portfolio Risk Control, Risk! Parity? (Hallerbach, 2013) offer a practitioners review of techniques for the newer context of "risk control". The paper compares the EW, GMV, NRP, ERC, Maximum Diversified Portfolio (MDP) and volatility targeting. Where they discuss the strategies advantages as well as disadvantages, and compare them against the maximum Sharpe ratio criterion. The data used in the paper is monthly data selected from the US asset market, and the time period is from 2002 to 2012. The author found that when looking at the Sharpe-ratios, the MSR outperformed all the other portfolios followed by the GMV, thereafter the MDP and ERC and all the portfolios outperformed the Cap Weighted portfolio.

In the paper "Risk parity versus other µ-free strategies: a comparison in a triple view" (no author given[5]) there is conducted a comparison of two different European data sets to evaluate the three following aspects: financial efficiency, diversification, and asset allocation stability. The findings in the paper state that the ERC allocation is not consistently superior to the other allocation approaches when looking at these three aspects. The GMV portfolio outperforms the ERC portfolio both when looking risk and return characteristics. They measured the reward-to-volatility by different approaches and the only approach that ERC outperformed the GMV was during the Farinelli-Tibiletti ratio when setting the parameters for a conservative risk profile. When looking at diversification the ERC portfolio tends to have a position in the middle of the GMV and the EW, sometimes ERC outperforms the EW and vice versa. The EW is in this part of the test at bottom and clearly is the least diversified portfolio, and also shows stronger instability.

[5] Link to article: http://www.aidea2013.it/docs/244_aidea2013_banking-and-finance.pdf

Further when looking at stability and turnover the naive RP portfolio outperform both the EW and the GMV i.e. the portfolio have lower transaction cost.

In the Article “Asset Allocation with Conditional Value-at-Risk Budgets” from Journal of Risk Spring 2013 there are used ex-ante methods to evaluate the component contribution to conditional value-at-risk (CVaR) and allocate risk. They data used in the paper are from 1976 to 2010, and include a broad bond, commodity, equity, and real estate indices. They start with a static two asset analysis with bond and equity, and thereafter expand to a larger portfolio to study the effect of rebalancing under the various constraints and objectives. Their proposed Minimum concentration CVaR portfolio (MCC) is stated to be a well balanced portfolio between risk and return and diversification, with positive return potential and a low portfolio turnover when looking at the period 1984 to 2010. There are also stated as a portfolio that can easily be combined with other investor objectives and constraints. They Also found that the credit crisis the EW portfolio suffered a significant higher drawdown than the MCC portfolio (52% versus 34%), so they claim that the MCC may be an better alternative in the normal or bull market, and in a bear market they said that the minimum CVaR might be more appealing, even that this portfolio is heavy concentrated in relatively few assets. But the MCC suffer from a relatively high turnover compared with the equally weighted portfolio, but it achieves an attractive compromise between low overall risks, good upside return, high diversification, and relative low turnover. But when comparing the MCC with the ERC with standard deviation as the risk measurement, they do not find any significant difference in the performance of the two portfolios. The paper ends with a proposition for further research and they said that this methodology with risk allocation could be applied to a large-scale equity portfolio at a more aggregate level, such as the level of country or industries rather than individual stocks, which will be conducted in this thesis.

So one can conclude that; “The Risk Parity strategy has its critics as well as fans”, as the Financial Times stated.

<u>Drawbacks of the previous research:</u>

One difficulty when comparing these different studies is the heterogeneity of the portfolio construction, as some have only a few assets, while other have a broad market portfolio of different uncorrelated and correlated assets. The same will be the problems when they use different returns, i.e. annual, monthly, weekly, etc. There also is difference in the constraints in the portfolio weighting, as some have the constraints of short-selling while other do not. For instance, some studies use an annual frequency to rebalance the portfolio, while others use semi-annual, quarterly or monthly frequency. Therefore, due to the inconsistence and difference approaches to the analysis of the portfolio strategy, it is difficult to draw any conclusions as to the superiority of one methodology with respect to other portfolio constructions. The other drawback is that most of the papers take into account that RP is constructed only with standard deviation as the risk measure, and there is possible that using risk measures as VaR and CVaR will give different result, this is also debated in Boudt et al. (2013). There are also few of the papers on the topic that performs sensitivity analysis, or criticize the parameter-input-risk for the time-varying standard deviation or covariance that occurs in the Risk Parity Portfolios. These parameters are constantly changing and are very important factors for investors or risk managers as their portfolios may be heavily affected by changes in these factors, and as these RP portfolios only rely on these input parameters, they needs to be handled with caution and constantly monitored.

We can also see that the Risk Parity portfolios had a relatively good return-to-risk performance since 2000 in the previous research, but this may have been caused by the strong performance of bonds since 2000. This may be caused because of the decrease in the interest rates over the period. This is not the case today, where the interest rates are close to zero and fear of inflation there are no guarantee the bonds performance will yield the same return in the future. In this case the Risk Parity portfolios may suffer from rising interest rates.

3. Data and stylized facts

The dataset used in this thesis is extracted from Thomson Reuters Datastream[6] at BI Norwegian Business School. Datastream is a global financial and macroeconomic database covering equities, stock market indices, currencies, company fundamentals, fixed income securities and key economic indicators. Thomson Reuters Datastream is highly reliable data sources used by professionals in the financial industry and researchers.

[6] http://thomsonreuters.com/

All variables were collected with a daily resolution, but transformed to a dataset containing monthly end prices. The data is collected from the period January 1990 to January 2014, i.e. 289 observations (288 returns). The data includes 30 MSCI country equity indices and three DataStream equity indices (South Africa, India and Malaysia). The main reason why these indices are chosen is due to the countries traded volume at their stock exchanges, but and also because of diversity in the geographical location. All prices in this thesis are denominated in US Dollar ($) and are Total Return Indices. The list over the indices is provided in Appendix 8.

3.1. Benchmark Index ("The Market")

For the Benchmark there have been used the MSCI All Country World Index[7], this mainly because this index is a stock market index that contains a representative selection of approximately 2400 constituents in the large- and mid-cap segment and are based on 23 countries in developed markets and 21 countries in emerging markets. The weightings in the index are shown in Figure 1 below.

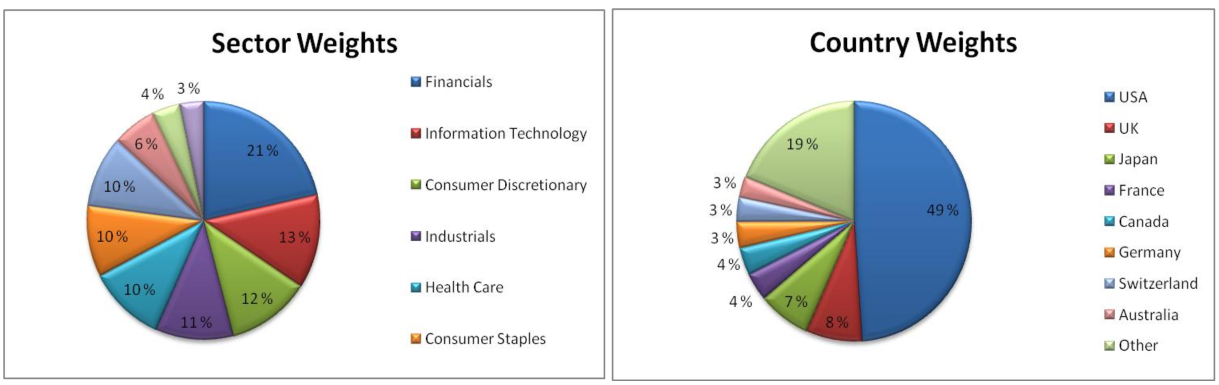

Figure 1: Sector and country weighs in MSCI All Country World Index by March 2014.
Source: www.msci.com

[7] For more information see: http://www.msci.com/resources/factsheets/index_fact_sheet/msci-acwi.pdf and https://www.agf.com/institutional/us/files/quarterly_fact_sheets/INST270_03_13_E_MSCI_ACWI_USD60672.pdf

This Index will by that cover approximately 85% of the global investable equity opportunity set and by that, it will make a good representation of the global equity indices that will be used in this thesis. But as one can see, the index is heavily weighted in the US market, and thereby the world market in this thesis will heavily be influenced by the US market and vice versa.

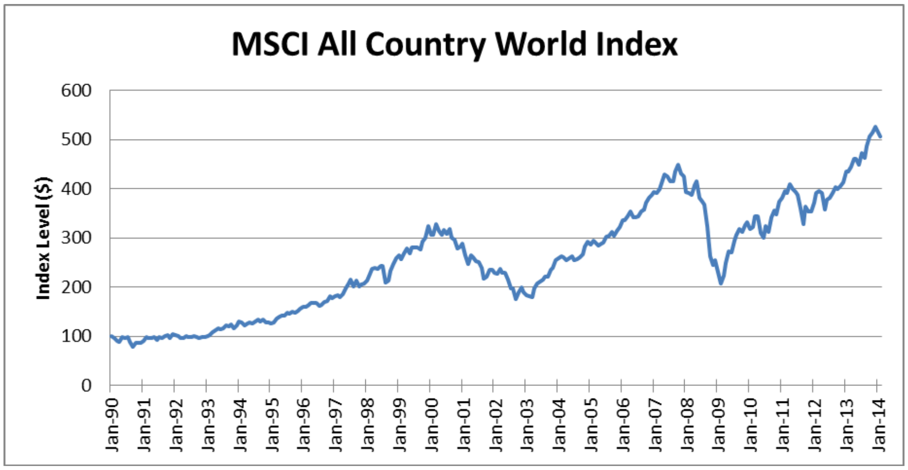

Figure 2: MSCI ACWI Index levels and price movements from January 1990 to December 2013.

Thorough the period ranging from January 1990 to December 2013, one can see the Market have increased from a level of 100 to a level of approximately 500, and as one can see from Figure 2, there have been several structural breaks due to financial crisis and stock market crashes in the world.

The first structural break was the burst of the "dot-com crisis" that occurred from 1995 to 2001. In this period the equities in internet-based investments rapidly increased and the same for the stock prices, before the bubble burst in 2001 from an index level of $300 to a level right below $200 in the end of 2002.

Then the market recovers, and had an increased to a level of approximately $450 during the pre-crisis from 2003 to the end of 2007, but fell rapidly from this $450-level to a level just above $200 in the beginning of 2009. This crisis had a huge impact on large parts of the world, and is considered to have started in the U.S. because of subprime loans that mortgage borrowers not had the ability met their obligations. Then in the aftermath of the financial crisis of 2008, one can see a more volatile price movement than earlier in the world market. These events can also be seen in Figure 3 below, where the returns of the market are illustrated.

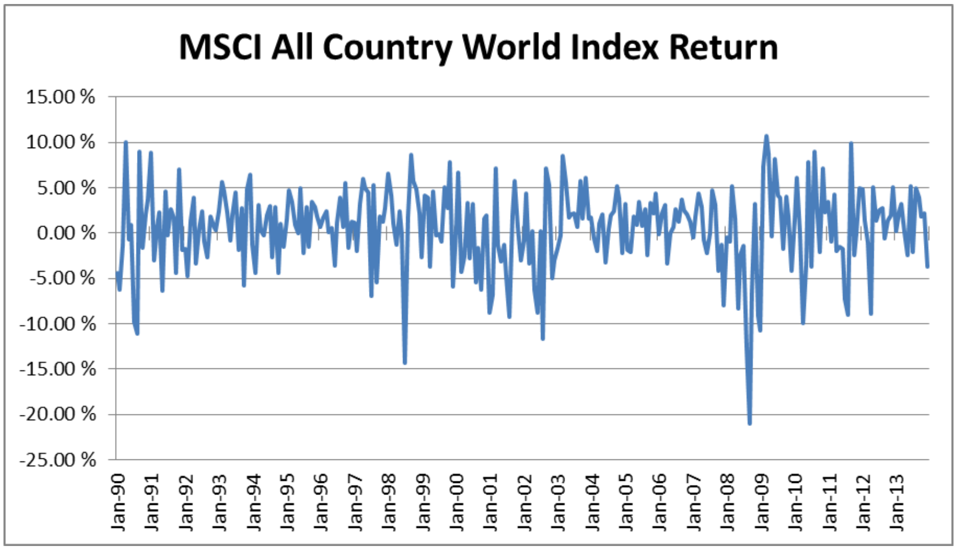

Figure 3: Historical returns for MSCI ACWI from January 1990 to December 2013

3.2. Risk-free rate

The risk-free rate taken into account is a philosophic question. But often this is a one month, three months, or a one year rate used. For the risk free rate in this thesis there have been used three moths second hand US Treasury bill (tender – middle rate) Constant Maturity Return given by the U.S. Federal Reserve[8]. One can see the development of the risk free rate in Figure 4 below, and one can see that the interest rate have declined rapidly from January 1990 to December 2013, from a level of approximately 8% to a level close to 0%. This low level is due to the aftermath of the financial crisis in 2008, and its objective is to help households and businesses finance with new spending and help support the prices of many other assets, such as stocks and houses[9].

[8] http://www.federalreserve.gov
[9] For more information see: http://www.federalreserve.gov/faqs/money_12849.htm

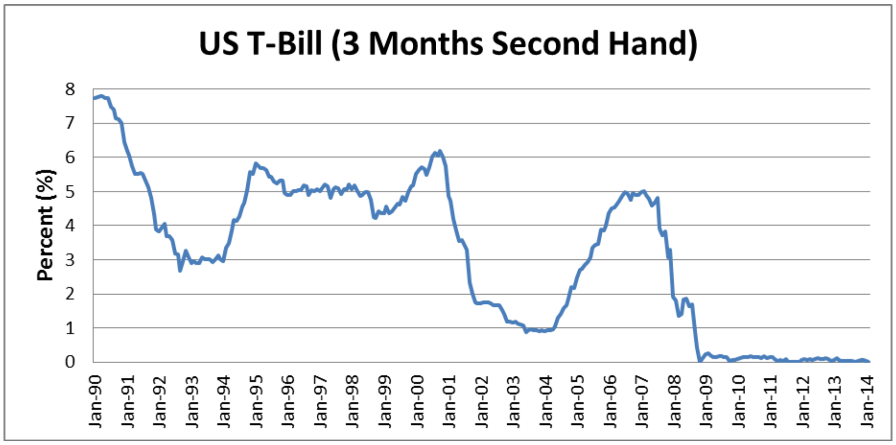

Figure 4: Three months middle rate US T-Bill, Constant Maturity Return from second hand market.

3.3. Descriptive statistics of data used (January 1990 – December 2013)

The full descriptive statistics discussed in this section will be found in Appendix 8; section 8.1, table 34.

When looking at the returns for the whole period in Figure 5 below, one can see that most of the country indices have had a positive average annual return through the whole period, ranging from 0% to 14.9%. India is the country with the highest average annual return of 14.9%, followed by Mexico with a return of 14.2%. One can see that Taiwan is the country with the lowest average return followed by Japan, but these values is not significant different from zero at a 5% level. The T-Bill had the lowest average return that is significant different from zero, with an average annual return of 3.1%, followed by Portugal with its 3.6% average annual return. One can also see that there are five other countries with non-significant return above zero. The market given by the Benchmark had an average return of 6.8% the whole period.

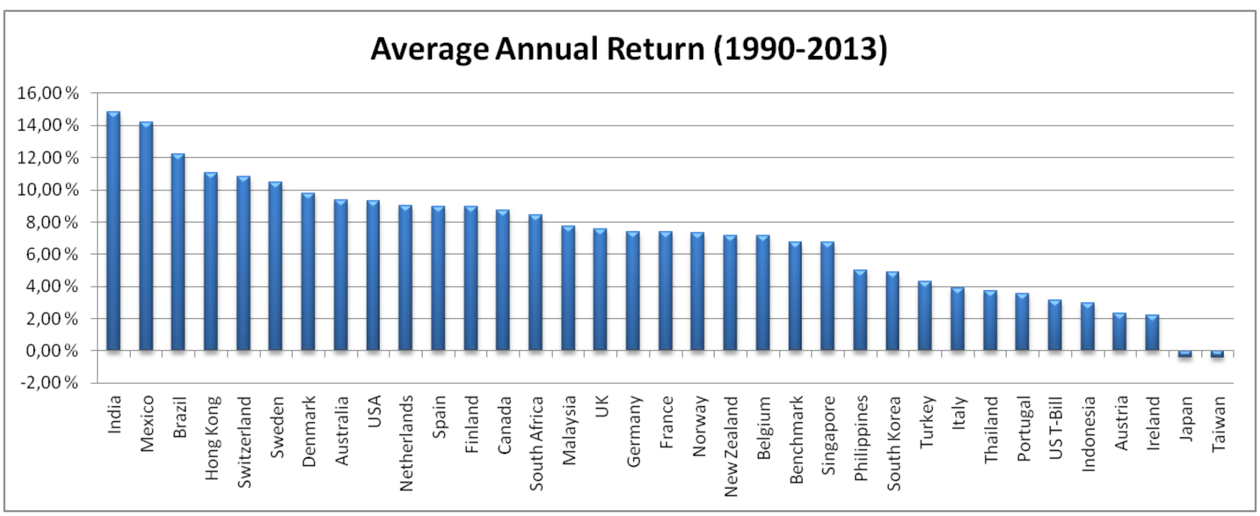

Figure 5: Average annual return of indices used in thesis (1990 to 2013).

When looking at the risk of the indices given the full period, one can see from the Figure 6 below that Turkey had the highest standard deviation, but this is not significant higher than the Brazilian index due to the F-tests p-value of 0.13, but Turkey had a significant higher standard deviation than the Indonesian index. Brazil have through the period also the biggest gap between the minimum and maximum monthly return, with values vary from -109.5% to 59.1%, and since this is in the beginning of the 1990s this may be related to the Rio de Janeiro Stock Exchange collapse that started in mid-1989.

USA is the index with the lowest standard deviation, with a measure of 15%, and is also the index with the highest proportion of systematic risk with an adjusted R^2 (R^2) of 82%. The reason why the proportion of USAs systematic risk is the highest can be seen in the context of its weight of approximately 49% in the benchmark used. But the risk measured by standard deviation for the USA index is not significant different from the standard deviation of the market, and it is debatable if it's significant lower than UK due to the p-value of 0.04 given a significance level of 5%.

One can clearly see, as expected, that the T-bill have the lowest standard deviation with an average annual standard deviation of 0.64%. When looking at the proportion of systematic risk one can see that Malaysia, with its R^2 of 0.17, had the lowest value.

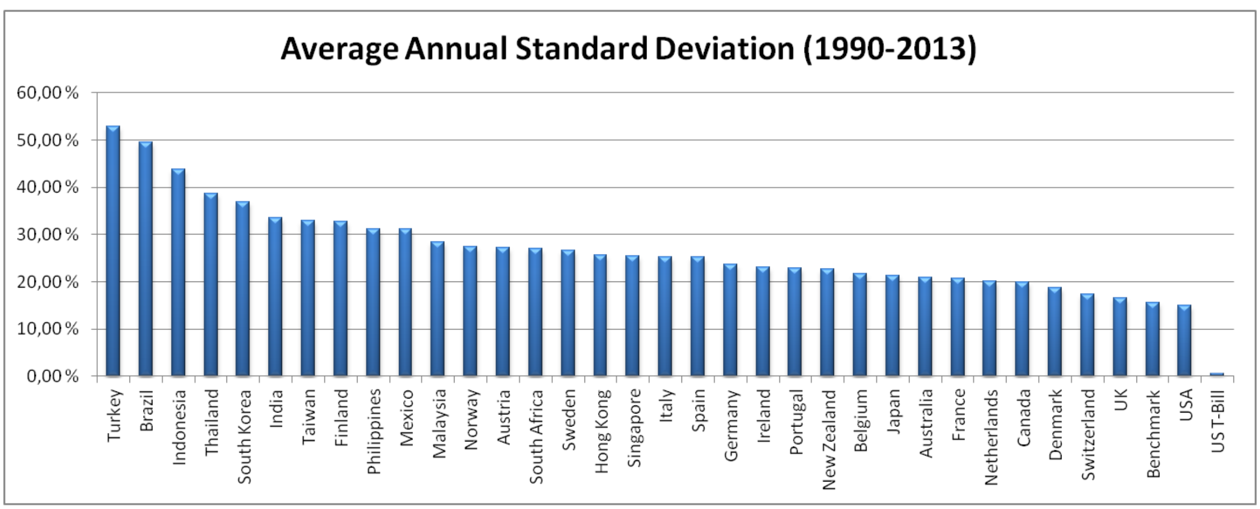

Figure 6: Average annual standard deviation of indices used in this thesis (1990 to 2013).

Then looking at the return to risk measurement, the Sharpe ratio, the index with highest value was Switzerland with a ratio of 0.45. This was followed by USA, with its Sharpe ratio of 0.41. Japan has in this time period the lowest Sharpe ratio which yields a value of -0.17, followed by Taiwan with its -0.11[10].

When considering the normality, the first to mention is that the median for all the equities is higher than the average return, something that indicates that indicates extreme values on the downside returns. This can also been seen by the minimum and maximum values for the indices in Table 34 in section 8.1. This observation leads to the discussion of the skew, where one also see that the most of the indices have a negative skew, i.e. this indicates that there are more relatively large negative deviations than large positive deviation from the mean and standard deviation will underestimate the risk, which is a crucial feature one must consider. The indices with the lowest value of skew are Belgium with its skew of -2.1, followed by Brazil with its -1.6. The only indices with a positive skew are India and South Korea. When looking at kurtosis, we see that all the indexes have fat tails, and sharp peak due the values above zero which says that it is less likely to have a mass close to the average in the normal distribution, and thus underestimates the estimated probability of extreme events. This is not the case in T-bills which have a negative kurtosis. Brazil and Belgium have the fattest tails with a kurtosis of respectively 13.4 and 11.77. Italy had the lowest kurtosis of the countries, followed by Japan. But as one can see from the Jarque Bera (JB) test; none of the Indices are normally distributed. Italy and Japan are the countries with the lowest JB and Brazil and Belgium are the two with the highest JB values.

When looking at the correlation the full period, one can see that all of the correlations are positive, ranging from a value of 0.16, between India and Japan, to the highest correlated equities France and Germany with a correlation of 0.88. Since all the correlations are positive, there are questioned how well a broad market portfolio based on equities are diversified in the same manner as an portfolio based on equities, bonds and commodities. There may be problematic to exploit the diversification effects that the RP portfolios benefits from in the weighting scheme of the RP portfolio allocation, and also the reason why leverage not are used. But as one can see from the correlation matrices in Appendix 8, the correlation matrix is not constant but it is

[10] The returns of Japan and Taiwan is not significant different from zero at a 5% level.

constantly changing, so only looking at the whole period may not be a good proxy for the diversification effects of equities, but one can see that the most of the indices are positive correlated due to the rolling window periods.

4. Calculation of the Risk Parity Portfolios

In this section it will be provided an explanation of the procedure and the methodology with a main focus on the RP calculations. For an explanation of the other calculations and theoretical background see the Technical Appendix in section 10.

4.1. Method & Constraints

In this thesis there will be conducted an in-sample calculation in the period from 1990 to 1995 (60 months) to lay the foundation for the out-of-sample testing. The out-of-sample test will range from 1995 to 2013 with a five year (60 months) rolling estimation window approach for the portfolio allocations to be analyzed. This means that the parameters we find in the in-sample period will be used in the first 60 months out-of sample period (1995-2000), and thereafter will these new 60 months parameters be used to calculate the next 60 months portfolios (2000-2005). The procedure just described will be repeated until the end of the sample period. The reason why this is done is because it provides more robust result and will be closer to the reality than doing the full period with in-sample testing of different periods.

There will be conducted continuous rebalancing, i.e. rebalancing of the portfolios every month during these rolling windows, which may be a weakness of the analysis since there will be hard to interpret the rebalancing costs, but there will be performed a test for the rebalancing cost called "Relative Investment Capacity" (RIC), which are explained in section 10.9 in the technical appendix.

There are also used constraints that do not allow short positions. This mainly because the RP portfolios do not have negative weights, and thereby using this constraint will be more robust in compare the characteristics of the portfolios constructed. Another reason for this is that the mean-variance model tends to incorporate extreme values in the asset positions when short sales are included providing portfolios of poor applicability. A third reason is that major stock exchanges have unique short sales regulations[11]. Furthermore, financial gearing by using leverage is prohibited due to the constraint some investors have, and the risk it entails. These assumptions contribute to portfolio robustness, meaning altering investment positions are comparable with changes in return and covariance estimates.

The portfolios constructed in this thesis are the Global Minimum Variance portfolio (GMV), Maximum Sharpe Ratio portfolio (MSR), Equally Weighted portfolio (EW), Maximum Diversification portfolio (MDP), and five different RP portfolios, which will be explained in the following sections in this chapter.

4.2. Naïve Risk Parity

This approach is the simplest of the Risk Parity portfolios. This mainly because this allocation technique only relies on the standard deviation as the parameter input, and not considers the covariance between the assets. In words the asset weight is given by the inverse of the standard deviation of the asset divided by the sum of the inverse of the assets standard deviations.

$$w_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{n} \frac{1}{\sigma_j}}$$

This portfolio will yield the same result as the ERC portfolio if there are only two assets, or if the correlation between the asset return are the same, and will in this thesis stated as NRP.

4.3. ERC

The marginal contribution of an asset (RC_i) to the total risk of the portfolio, when standard deviation is the risk measure, is given by the formula:

$RC_i = w_i \frac{\delta \sigma_p}{\partial w_i}$, where the last term determines the change in the total risk of the portfolio if a small change in the weights of asset occurs. Solving the derivation problem, one obtains:

[11] http://www.sec.gov/spotlight/keyregshoissues.htm

$$RC_i = w_i \frac{\sum_{j=1}^{N} w_j Cov(r_i, r_j)}{\sigma_p}$$

Thereafter, as the name states, one set the risk contribution equal to each other:

$$w_i \frac{\delta \sigma_p}{\partial w_i} = w_j \frac{\delta \sigma_p}{\partial w_j} \quad \forall i, j$$

And the Euler decomposition satisfies (Denault, 2001):

$$\sigma_p = \sum_{i=1}^{n} RC_i$$

In the presence of and full investments constraints, finding a closed-form solution for this ERC optimization weighting scheme is not possible due to an issue of endogeneity: w_i is a function of the risk contribution which, by definition, depends on w_i. And by that, the following numerical optimization algorithm provided by Teiletche et al. (2010) will in this thesis be used:

$$\underset{w^*}{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[w_i \frac{\delta \sigma_p}{\partial w_i} - w_j \frac{\delta \sigma_p}{\partial w_j} \right]^2 \qquad \text{s.t.} \quad \sum_{i=1}^{N} w_i = 1 \quad \& \quad 0 \le w_i \le 1$$

4.4. ERC VaR

The thesis also conducts an analysis of the ERC with VaR as the risk measure. In this case, the properties and optimization algorithm explained above will be the same with this risk measure. When looking at ERC when having VaR as the risk measure and the distribution is assumed Gaussian we obtain[12]:

$$RC_i = w_i \left(-\mu_i + \Phi^{-1}(\alpha) \frac{\delta R_p}{\partial w_i} \right)$$

And for the general case stated by Gouriéroux et al. (2000), this will be extended and it can be shown that the risk contribution is equal to:

---

[12] Φ^{-1} is the inverse of the CDF of the standard normal distribution, and R is the risk measure.

$RC_i = \mathrm{E}\left[L_i \middle| L_p = VaR_\alpha(L_p)\right]$, where L is loss.

There will in this thesis be constructed a portfolio based on the Gauss-distribution with a 5% percentile VaR, called ERC VaR.

4.5. ERC CVaR

In this thesis, there will be performed two different ERC portfolios based on CVaR as the risk measure. The first will be a portfolio based on the Gauss-distribution with an alpha of 95% called ERC R, and the second will be a portfolio based on a non-normal calculation of the 5% percentile CVaR and will in this thesis be called ERC CVaR.

The risk contribution when assuming Gaussian-distributed CVaR is used as the risk measure can be interpreted as[13]:

$$RC_i = w_i\left(-\mu_i + \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}\frac{\delta R_p}{\partial w_i}\right)$$

And for the general case, i.e. the non-Gaussian, this risk contribution can be generalized in the following expression stated by Tasche (2002):

$RC_i = \mathrm{E}\left[L_i \middle| L_p \geq VaR_\alpha(L_p)\right]$, and will be calculated using the method conducted by Rachev et al. (2008):

$$RC_i = \frac{\left(\frac{1}{[\alpha M]}\sum_{j=1}^{[\alpha M]} r_i^{\,j}\right)}{CVaR_\alpha(r_p)}$$, where r is return and M is the return scenarios.

[13] ϕ is the PDF of the standardized normal distribution

Also in this case, the properties and optimization algorithm explained above will be the same with this risk measure.

4.6. Drawbacks and weaknesses

Portfolio optimization based on VaR is much more difficult than the one based on the CVaR. The reason for this is that when calculated using scenarios, the portfolio VaR is not smooth as a function of portfolio positions, is not convex, and has multiple local extremal points. And by that for the non-normal ERC VaR, you have to regularize around the confidence level. This is the reason why there is only conducted Gaussian-distributed ERC VaR portfolios.

The other drawback is that there are only conducted calculations based on the 5% percentile when calculating the ERC portfolio based on non-normal CVaR. There are also only conducted analysis on the 95% percentile CVaR and 5% VaR when assuming Gaussian-distributed returns. There could also used different approaches to calculating the VaR and CVaR, as Monte Carlo simulation and the Parametric approach and thereafter compared these different and more robust results.

5. Out-of-Sample Portfolio Analysis

In this section there will be provided analysis and comments on the findings in the different portfolio allocation strategies performed during the out-of-sample periods. There will first be conducted an analysis of the sub-periods from the rolling window periods, and thereafter an aggregated full out-of-sample period analysis to see the performance of the portfolios in a longer time horizon.

The calculation discussed in this section is explained in the Appendix in section 10.

5.1. 1995-2000

When looking at the first five years of the analyzed period one can see, as shown in Figure 7 and Table 1 below, that the MSR portfolio performed well and was superior in terms of returns from the period 1994 to 2000 with an annual average return of 21.6%, closely followed by the GMV with its 20,9%. It seems like the GMV and the MSR had better performance the whole period, compared to the other portfolios that had a more flat development in the returns. The MSR and GMV portfolios were also the only two portfolios that outperformed the market this period.

In the same time period one can see that the MDP portfolio was the one that had the weakest performance in terms of returns, with its 9.9%. And the MDP portfolio had also the greatest gap in returns, from a positive return of 13.9% to a negative return of 24.7%.

During this five year period all of the portfolios had significant returns at a 5% level due to their t-value shown in Table 1.

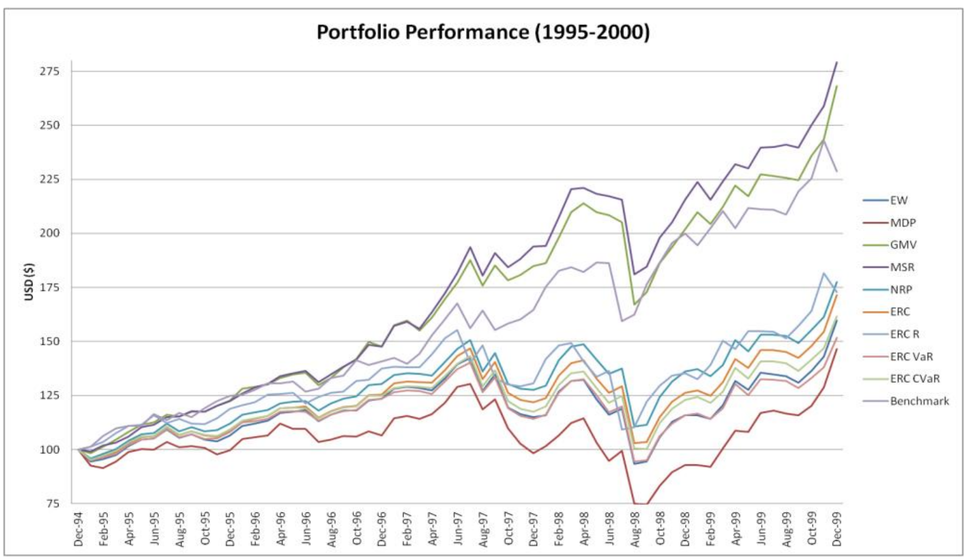

Figure 7: Performance of portfolios from 1995 to 2000

Table 1: The portfolios average annual return and the monthly minimum and maximum value during the period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
Average return	11.1 %	9.9 %	20.9 %	21.6 %	12.9 %	12.4 %	12.5 %	10.0 %	11.1 %	17.6 %	5.0 %
T-test H0=0	4.68	3.69	11.41	12.82	6.04	5.57	5.69	4.37	5.08	10.21	384.46
Monthly											
Minimum	-21.4 %	-24.7 %	-18.6 %	-16.0 %	-19.5 %	-20.4 %	-20.0 %	-21.3 %	-19.6 %	-14.3 %	0.4 %
Maximum	11.8 %	13.9 %	10.0 %	7.9 %	11.3 %	11.1 %	10.7 %	12.0 %	11.8 %	8.7 %	0.5 %

When considering the risk of the portfolios one can see from Table 2 that the MSR had the lowest annual average standard deviation with a value of 13%, just below the market with its 13.3%[14]. The MSR also was the superior when looking at tail-risk in every quantile, except the $VaR_{5\%}$. One can also see that the GMV was the second best performing portfolio when looking at these risk measurements. In contrast, the MDP portfolio had the weakest performance in all of the risk measures performed. MDP had an annual average standard deviation of 20.8% which is significant higher than the following EW portfolio with the value of 18.5%[15]. The MDP also had the highest relative risk with TESD of 26% and TEMAD of 81.6%, and also the highest tail-risk in every quantile.

One can also see that the ERC R had the lowest relative risk with a Tracking Error SD (TESD) with a value of 7.6% and Tracking Error MAD (TEMAD) of 44%, and by that had the lowest active risk when comparing the portfolios with the benchmark. It seems like the ERC is a mediocre portfolios when looking at the risk measures we have performed, and lies between the EW and GMV.

Table 2: Risk measures performed on portfolios in the time period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
SD (σ)	18.5 %	20.8 %	14.2 %	13.0 %	16.6 %	17.2 %	17.0 %	17.7 %	17.0 %	13.3 %	0.1 %
TESD	24.1 %	25.9 %	20.1 %	19.4 %	22.6 %	23.1 %	7.6 %	23.5 %	23.0 %		
TEMAD	78.5 %	81.6 %	70.9 %	69.8 %	75.9 %	77.0 %	44.0 %	77.4 %	76.7 %		
Lower tail											
10% VaR	-4.5 %	-6.7 %	-2.3 %	-1.3 %	-3.8 %	-4.2 %	-4.2 %	-4.0 %	-4.1 %	-2.8 %	0.4 %
10% CVaR	-10.3 %	-11.7 %	-6.4 %	-6.0 %	-8.8 %	-9.4 %	-9.3 %	-9.8 %	-9.2 %	-6.6 %	0.4 %
5% VaR	-7.1 %	-9.1 %	-3.6 %	-3.7 %	-5.0 %	-6.1 %	-5.9 %	-5.7 %	-5.9 %	-5.4 %	0.4 %
5% CVaR	-14.6 %	-15.2 %	-9.7 %	-8.9 %	-13.0 %	-13.4 %	-13.3 %	-13.9 %	-13.1 %	-9.0 %	0.4 %
1% VaR	-15.5 %	-16.6 %	-11.3 %	-10.6 %	-13.8 %	-14.3 %	-14.1 %	-13.8 %	-14.1 %	-9.9 %	0.4 %
1% CVaR	-21.4 %	-24.7 %	-18.6 %	-16.0 %	-19.5 %	-20.4 %	-20.0 %	-21.3 %	-19.6 %	-14.3 %	0.4 %

When splitting the risk one can see that the ERC R had the highest proportion of systematic risk, with a R^2 value of 82%, and the rest of the portfolios have negative values close to zero. One may be critical to these values as it states that the most of the portfolios cannot be explained by movements of the market. This because these low R^2 indicates that one should ignore their respective betas. One can also see that the ERC R had the highest sensitivity due to market changes by its beta-value of 1.15. The EW had the lowest beta with a value of -0.2, but as mentioned, the explanatory power of the model is close to zero.

[14] Significant different at 5% level with a p-value of 0.4
[15] p-value of 0.18 at a 5% level

If the R^2 was higher; the negative betas can be interpreted by that the indices generally moves in the opposite direction compared to the index, and the low beta value indicates that the movement of the indices is close to be uncorrelated with the movement of the benchmark.

Table 3: Systematic risk measurements during the period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
β	-0.20	-0.19	-0.19	-0.09	-0.18	-0.19	1.15	-0.19	-0.19
$\bar{R}^2$	0.02	0.00	-0.01	-0.01	0.00	0.00	0.82	0.00	0.01
t-value									
H0: β=1	-6.65	-5.83	-8.52	-8.57	-7.29	-7.11	2.08	-6.93	-7.23

From the risk performance measurements (RAPM) provided in Table 4, one can see from the most common RAPM, the Sharpe-ratio that the MSR portfolio is superior with its ratio of 1.27 and will yield a 4% relative to the market due to the M^2 value. The second highest Sharpe had the GMV with a ratio of 1.12 and a 2% M^2. The reason of these two to be the superior is that they are heavily investing in the US market by almost 50% of the dollar weight in the portfolio, and as one can see from Table 47 in Appendix 9, the US market had an relatively high average annual return of approximately 24.5% and is also one of the countries with the lowest standard deviation with its value of 14.5%. One can also see that the MSR and GMV are the only two portfolios that outperforms the market when consider the Sharpe-ratio. On the contrary, the MDP and the ERC VaR are the two portfolios with the lowest Sharpe and M^2.

When looking at the Sortino Ratio in Table 4, using the average risk-free rate this period as the target return, one can see that the ERC R is superior with a value of 6.75 followed by the MSR with its 2.37. The two portfolios with the weakest performance in this measurement are the MDP and ERC VaR.

There have also been calculated Treynor-ratios if investors will use the portfolios as an investment vehicle in another portfolio. One should be careful by using the Treynor due to the low R^2 explained above, but as one can see; the ERC R this is the only portfolio with positive Treynor-ratio and T^2, and the only reliable Treynor measure due to the R^2.

When looking at the excess return achieved when bearing risk beyond the market risk, one can see that the MSR is superior with its Information ratio (IR) of 0.57, followed by the GMV with its value of 0.41. The portfolio with the lowest value of IR is the ERC R, with a negative value of 4.62.

One can also see that the MSR and GMV had a significant positive abnormal return measured by Jensens alpha, of approximately 1%, and one can also that the ERC R had a significant negative alpha of 1%.

Table 4: RAPM performed during time period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark
Annually										
Sharpe	0.33	0.24	1.12	1.27	0.48	0.43	0.44	0.28	0.36	0.94
Treynor	-0.31	-0.26	-0.86	-1.78	-0.44	-0.39	0.06	-0.26	-0.32	
IR	-0.58	-0.56	0.41	0.57	-0.49	-0.51	-4.62	-0.72	-0.67	
Sortino	0.56	0.36	1.95	2.37	0.84	0.73	6.75	0.47	0.63	
M^2	-8.1 %	-9.4 %	2.4 %	4.4 %	-6.2 %	-6.9 %	-6.7 %	-8.8 %	-7.8 %	
T^2	-3.7 %	-3.3 %	-3.3 %	-15.9 %	-4.7 %	-4.3 %	-0.5 %	-3.2 %	-3.7 %	
Jensens	0.7 %	0.6 %	0.6 %	1.5 %	0.9 %	0.8 %	-0.6 %	0.6 %	0.7 %	
t-value										
H0: α=0	1.01	0.75	2.57	2.93	1.33	1.23	-2.05	0.90	1.09	
H0: IR=0	-4.50	-4.37	3.20	4.44	-3.76	-3.97	-35.78	-5.60	-5.16	

But one should be carefully by interpreting and trust the risk and return measurements, especially the standard deviations. One can see from Table 5 that there is a tendency in the returns of the portfolios that there are negative skew and a high positive excess kurtosis, so there will be a greater likelihood of extreme negative outcomes and the standard deviation will be underestimated.

The two seemingly best performing portfolios this period, the MSR and the GMV have, as one can see from Table 5, the highest values kurtosis and also the lowest value of skew. There is also in the normality measurements performed a tendency that the RP strategies lies between the more common strategies, i.e. it is closer to the normal distribution than the GMV and MSR, but the EW is closer to the normal-Gauss distribution than the RP portfolios. But as one can see; none of the portfolios are normally distributed when looking at the JB-test for normality, with the JB critical value of approximately 6.

Table 5: Normality measures of the portfolios in the time period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Monthly											
Median	1.2 %	0.9 %	2.4 %	2.3 %	1.3 %	1.4 %	1.2 %	1.3 %	1.3 %	1.8 %	0.4 %
Kurstosis	4.82	4.65	9.26	7.64	5.35	5.30	5.28	5.51	4.74	3.80	0.16
Skewness	-1.34	-1.32	-2.10	-1.93	-1.42	-1.44	-1.47	-1.44	-1.31	-1.34	-0.18
Jarqe Bera	365.72	342.64	1242.18	879.52	439.88	436.95	437.36	463.60	351.54	259.40	1.83

Looking at the diversification measures performed, one can see from Table 6 that the EW portfolio is the best performing when looking at almost every measurement, both in terms of risk contribution and weighs. The EW was only outperformed by the ERC when looking at the risk contribution diversification in the Gini Index and the Diversification Index. When looking at the Diversification Index the MDP should be superior since this portfolio aims to minimize this measure, but out-of-sample the ERC portfolio seems to be the best performing this sub-period.

The RP portfolios also have a great amount of diversification when looking at the Herfindahl measure, and barely outperformed by the EW.

On the other hand, the best performing portfolios, MSR and the GMV, have high proportion of both dollar-weight and risk-weight in few indices. One can see from Table 47 in Appendix 9 that these two portfolios have about 50% of its weight and risk in the US market and only small proportion of weights in a few other indices. This is the reason to their low performance due to these diversification measurements. But the US Index most likely has huge multinational corporations, and one can ask if this index is well diversified as a stand-alone investment index.

The other feature to mention is that the RP portfolios have the ERC properties only in the in-sample tests; one can for example see that in the ERC have a percentage risk contribution varying from 1.4% risk contribution in the UK index to 7.1% risk contribution in the Indonesian index.

Table 6: Diversification measures performed on portfolios during the time period 1995 to 2000

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
1995-2000	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC
Herfindahl Index	0.0 %	0.3 %	7.9 %	9.3 %	27.4 %	26.8 %	28.9 %	29.4 %	0.5 %	0.5 %	0.3 %	0.6 %	0.4 %	0.7 %	0.8 %	0.7 %	0.5 %	0.7 %
Gini Index	0.0 %	12.3 %	80.9 %	80.6 %	89.2 %	89.2 %	89.5 %	89.5 %	15.7 %	13.5 %	12.3 %	0.0 %	17.9 %	25.8 %	27.8 %	25.9 %	21.7 %	24.3 %
Shannon Entropy	3.50	3.45	2.41	2.30	1.57	1.53	1.44	1.43	3.42	3.41	3.46	3.40	3.44	3.39	3.37	3.39	3.42	3.40
Diversification Index	1.01		0.67		0.73		0.70		0.70		0.65		0.66		0.71		0.66	

When it comes to the investment capacity measurements, one can see from Table 7 that the MDP and the MSR have low values when looking at the Bottleneck RIC and the 5th percentile RIC. In contrast, the EW and the NRP have the highest values in these two measurements. But when looking at the weighted average RIC one can see that the MSR have the highest value followed by the NRP.

It seems like the optimized portfolios have a lower investment capacity than the naïve portfolios during this period, except when looking at the weighted average measurement. The reason why the weighted average RIC may be as high is because the portfolio is highly invested in the US market, since the RIC measurements answers how much capital one can deploy to an alternative approach relative to the market-value-weighted portfolio.

So these three methods in Table 7 are not consistent in which portfolio that is the superior one. But one should mention that portfolios with low RIC have high turnover and require more active management to rebalance the portfolio.

Table 7: Relative investment capacity measures performed on the portfolios in the time period 1995 to 2000

1995-2000	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
RIC^B	3.2 %	0.2 %	1.3 %	0.6 %	3.1 %	1.6 %	1.9 %	1.5 %	1.7 %
RIC^0.05	3.2 %	0.2 %	1.3 %	0.6 %	3.1 %	1.8 %	2.6 %	1.6 %	2.4 %
RIC^wa	7.0 %	1.9 %	1.6 %	50.3 %	16.9 %	10.9 %	13.4 %	13.8 %	8.7 %

5.2. 2000-2005

One can see from Figure 8 that the MDP had a weak performance the first half of the period, but when the market recovers after the dot-com crisis the MDP seems to have a greater increase in its performance due to returns. The MSR seems to start as the one with the greatest return, but during the dot-com crisis it was the portfolio with the greatest downturn. When the market starting to recover by December 2002 it seems like the risk based portfolios performed well, while the MSR and the market seem to be the two with lowest increase in returns.

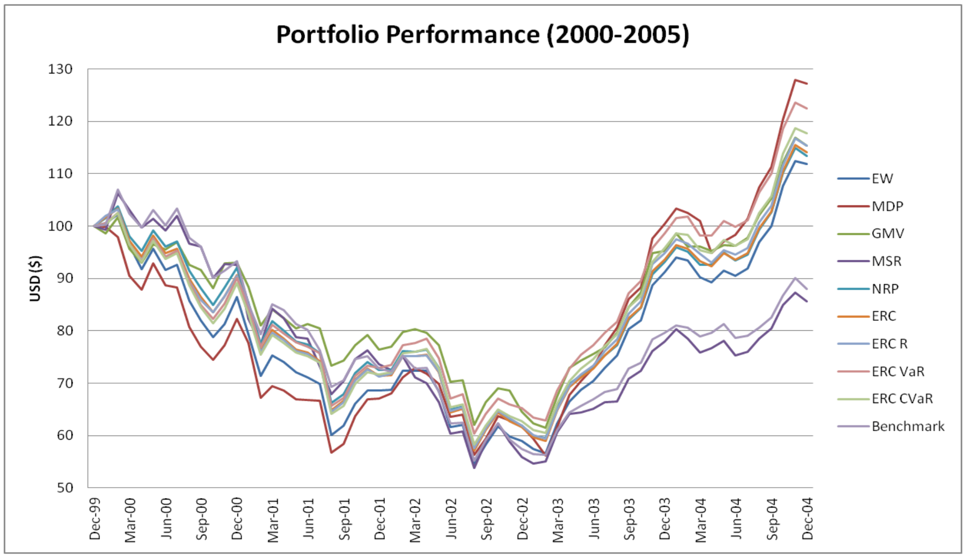

Figure 8: Performance of portfolios from 2000 to 2005

Looking at this sub-period, one can see from Table 8 that the MDP had the highest average annual return with a value of 6.8%, followed by the ERC VaR with its 5.4%. One can also see from Table 48 in section 9 that the MDP had heavy weighting on few of the country indices, but is more balanced than the MSR- and GMV portfolio. The main drivers for the MDP portfolio both in terms of risk and weights are Austria, Belgium, India, and South Korea. And all of the mentioned countries included in the MDP had a positive return this period, especially Austria and Belgium with its following returns of 21.7% and 8.7% as one can see from Table 41 in section 8.

On the other hand, the MSR was the portfolio that had the lowest return. The MSR had an annual average return of -1.7%, and was the only portfolio that underperformed the market, but as one can see from the t-value this is not significant different from zero. The MSR portfolio was mainly driven by the US market this period, with almost 50% of its weight and 46% of its risk attached to the US market, which had a negative return of 2.7% as one can see from Table 48 in Appendix 9. One can also mention that the MSR portfolio had a 10.5% dollar weight and a 19.5% risk contribution attached to the Finland index, which this period had a negative return of 15.2%.

From the monthly minimum and maximum values in Table 8, one can see that there is a tendency that the negative values are more negative than the positive in an absolute manner. And the MDP and ERC VaR had the greatest gap between the minimum and maximum values. And even thus the ERC portfolios based on tail-risk measures ought to have reduced the extreme downside risk it is performing mediocre, but this only relying on one observation and it will be more robust to look at the downside risk measures discussed in Table 9.

Table 8: The portfolios average annual return and the monthly minimum and maximum value during the period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
Average return	3.9 %	6.8 %	4.0 %	-1.7 %	4.0 %	4.1 %	4.3 %	5.4 %	4.7 %	-1.3 %	2.6 %
T-test H0=0	1.67	2.67	2.04	-0.79	1.81	1.86	1.96	2.54	2.16	-0.67	37.71
Monthly											
Minimum	-14.1 %	-15.0 %	-12.0 %	-11.4 %	-12.6 %	-12.8 %	-12.7 %	-13.4 %	-13.3 %	-11.6 %	0.1 %
Maximum	10.1 %	10.6 %	9.8 %	10.3 %	10.2 %	9.8 %	9.4 %	8.9 %	9.3 %	8.6 %	0.5 %

When looking at the standard deviation in Table 9 the GMV had, as expected, the lowest standard deviation with its value of 15.3%. One can see that in this period the ERC VaR had the second lowest standard deviation, and lies between the EW and GMV, as Maillard et al. (2008) show in their paper. The GMV also had the best performance in almost every quantile when performing VaR and CVaR, except in the $VaR_{5\%}$ and $CVaR_{10\%}$ where the MSR was the best performing.

On the contrary, one can see that the MDP had the highest risk in almost every risk measurement except the $VaR_{5\%}$. The MDP also had high values in the tracking errors, and may be caused by the weighting of the portfolio where the MDP are heavily weighted in relative low correlated indices compared with the benchmark. The low tracking errors in the MSR is consistent with high weighting in the US market and the correlation between the US market and the benchmark. And it seems like the RP portfolios lies in between the other portfolios when looking at the relative risk measurements.

From the downside risk measure, VaR, one can see from Table 9 that the ERC R had the lowest value of $VaR_{5\%}$ of about -10%. It is also a question why the ERC with VaR and CVaR do not have a better performance when looking at these downside risk-measures, and why they are outperformed by both GMV and MSR in almost every quantile.

Table 9: Risk measures performed on portfolios in the time period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
SD (σ)	17.9 %	19.6 %	15.3 %	16.7 %	16.9 %	17.0 %	17.0 %	16.6 %	16.8 %	15.6 %	0.5 %
TESD	6.8 %	11.3 %	6.9 %	3.9 %	5.5 %	6.0 %	6.4 %	6.6 %	6.4 %		
TEMAD	43.7 %	56.4 %	44.2 %	33.5 %	39.5 %	41.4 %	42.6 %	43.3 %	42.5 %		
Lower tail											
10% VaR	-5.9 %	-6.2 %	-5.0 %	-5.9 %	-5.5 %	-6.0 %	-6.2 %	-5.9 %	-6.0 %	-6.2 %	0.1 %
10% CVaR	-10.3 %	-10.9 %	-8.3 %	-8.6 %	-9.7 %	-9.7 %	-9.8 %	-9.3 %	-9.2 %	-8.6 %	0.1 %
5% VaR	-10.0 %	-9.1 %	-8.4 %	-7.4 %	-9.2 %	-9.6 %	-10.1 %	-8.9 %	-9.4 %	-8.8 %	0.1 %
5% CVaR	-12.0 %	-13.5 %	-9.9 %	-10.5 %	-12.0 %	-11.9 %	-11.8 %	-11.6 %	-11.4 %	-9.9 %	0.1 %
1% VaR	-12.8 %	-14.1 %	-10.2 %	-11.3 %	-12.6 %	-12.5 %	-12.3 %	-12.0 %	-12.4 %	-10.2 %	0.1 %
1% CVaR	-14.1 %	-15.0 %	-12.0 %	-11.4 %	-12.6 %	-12.8 %	-12.7 %	-13.4 %	-13.1 %	-11.6 %	0.1 %

When looking at the proportion of systematic risk in Table 10, the MSR have about 95% followed by the NRP with its 90%. This is not surprisingly due to the high weighting of the US market in the Benchmark used. The MDP is the portfolio with the lowest proportion of systematic risk with its 68%. The reason for the low proportion of systematic risk in the MDP could be explained by its weighting scheme, as one can see from Table 48 in Appendix 9, and their respectively correlations to the benchmark as seen in Table 42 in Appendix 8.

The GMV is the portfolio with the lowest sensitivity to the market movements with its beta value of 0.89, this together with the high R^2 implies that the GMV gains less than the benchmark in bull markets, and also have loose less than the benchmark in bear market. On the contrary the EW had a beta of 1.07, but is not statistically significant different from zero, which implies that the movement of the portfolio is generally in the same direction as, and about the same amount as the movement of the benchmark. This will also be for the rest of the portfolios due to their t-values not exceeding the critical value.

Table 10: Systematic risk measurements during the period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
β	1.07	1.05	0.89	1.04	1.03	1.03	1.02	0.99	1.01
$\bar{R}^2$	0.87	0.68	0.81	0.95	0.90	0.88	0.86	0.85	0.87
t-value									
H0: β=1	1.34	0.51	-2.04	1.35	0.77	0.58	0.35	-0.25	0.19

When looking at the RAPM in Table 11 one can see that the MDP portfolio is the best performing when it comes to the Sharpe measurements. MDP is the only with a positive Sharpe-ratio with a value of 0.09, and the least negative M^2 value by -11.4%.
On the contrary one can see that the MSR is the portfolio with weakest RAPM in every measurement performed. The MSR is also the only portfolio with a negative IR, i.e. the investment has performed worse compared with the benchmark. This is not consistent, since the MSR outperform the Benchmark when looking at the Sharpe-ratio.

But considering the RP portfolios, these perform well compared with the benchmark due to the highest values in the IR measure, i.e. the RP portfolios can achieve excess return with bearing risk beyond the risk given by the benchmark. The same will be for the Sortino-ratio where it can be seen that the RP portfolios have a better performance than the other portfolios when looking at this return-to-downside-risk measurement.

One can also see that the RP portfolios all have significant Jensens alphas at a 5% level. The MDP have the highest value of alpha by its 0.7%, but it is discussable if this is significant.

Table 11: RAPM performed during time period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark
Annually										
Sharpe	-0.06	0.09	-0.06	-0.40	-0.06	-0.05	-0.04	0.03	-0.02	-0.41
Treynor	-0.01	0.02	-0.01	-0.06	-0.01	-0.01	-0.01	0.00	0.00	
IR	8.05	4.91	9.08	-2.15	12.51	10.80	9.81	11.69	11.21	
Sortino	1.98	2.54	2.45	-25.00	3.27	3.01	3.01	4.93	3.93	
M^2	-13.4 %	-11.4 %	-13.4 %	-17.9 %	-13.4 %	-13.3 %	-13.1 %	-12.2 %	-12.8 %	
T^2	0.4 %	0.7 %	0.5 %	0.0 %	0.4 %	0.4 %	0.5 %	0.6 %	0.5 %	
Jensens α	0.5 %	0.7 %	0.4 %	0.0 %	0.5 %	0.5 %	0.5 %	0.6 %	0.5 %	
t-value										
H0: α=0	1.85	1.65	1.66	-0.11	2.25	2.10	2.02	2.35	2.17	
H0: IR=0	62.32	38.07	70.31	-16.64	96.91	83.65	75.97	90.53	86.84	

But as mention earlier, one should not rely on these measurements without looking at the distribution of the returns. And as seen from Table 12 below; the median have a higher value than the average return, all of the portfolios have a negative skew, and slightly fat tails and sharp peaks due to their positive kurtosis (except the MSR that had a negative kurtosis). This implies that there is probability that the risk will be underestimated in all the portfolios, and there is a greater probability of black swans and extreme downside tail-risk events. This can also be seen from the minimum and maximum values in Table 8, where the minimum values are more negative than the maximum is positive.

One can see from Table 12 that the MSR have the skew and kurtosis closest to zero, and one cannot reject the null hypothesis of normality due to its critical value of about 6 when performing the JB normality test, i.e. the MSR is normally distributed due to this test.

Table 12: Normality measures of the portfolios in the time period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Monthly											
Median	0.7 %	1.0 %	0.6 %	0.4 %	1.0 %	1.0 %	1.2 %	1.2 %	0.8 %	0.4 %	0.1 %
Kurstosis	0.28	0.32	0.43	-0.18	0.50	0.44	0.42	0.50	0.47	-0.19	-0.67
Skewness	-0.65	-0.62	-0.40	-0.13	-0.66	-0.67	-0.68	-0.76	-0.73	-0.35	0.96
Jarqe Bera	21.45	19.81	9.75	1.17	23.93	23.83	24.22	30.60	28.00	6.33	49.56

When looking at the diversification measures in Table 13 one can see that also in this period the EW was superior in almost every measurements performed. The EW was only outperformed when looking at the risk contribution diversification in the Gini Index, where the ERC portfolio was the best performing portfolio. But there is a close race between the ERC, NRP and EW when looking at the Shannon Entropy and the Herfindahl Index. And the ERC R is barely outperformed by the EW in the Diversification Index, with a value of 0.63 versus 0.65.

There are also as expected that the MSR and the GMV was the portfolios with the weakest performance when considering these diversification measurements. As seen from table 48 in section 9; the MSR had 49% of its weighting in the US market and 28% in the UK market, while the GMV had 55% in the UK market. But as stated earlier, these diversification measurements do not consider the great amount of multinational corporations in the US and UK, and thereby do not consider the diversification these indices may have as a standalone investment vehicle.

One interesting feature is how different the RP portfolios are performing when looking at these diversification measurements, and especially the comparison of the tail-risk-based and the two based on standard deviation.

Table 13: Diversification measures performed on portfolios during the time period 2000 to 2005

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2000-2005	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC
Herfindahl Index	0.0 %	0.4 %	16.1 %	16.3 %	36.2 %	33.5 %	31.7 %	28.6 %	0.6 %	0.6 %	0.6 %	0.6 %	1.1 %	1.0 %	0.7 %	1.2 %	0.3 %	0.8 %
Gini Index	0.0 %	28.4 %	82.8 %	80.5 %	89.2 %	89.2 %	89.5 %	89.5 %	24.3 %	0.4 %	24.2 %	0.0 %	16.0 %	22.0 %	26.0 %	34.4 %	17.4 %	28.2 %
Shannon Entropy	3.50	3.44	2.00	1.95	1.14	1.18	1.35	1.41	3.40	3.40	3.40	3.40	3.35	3.36	3.38	3.34	3.45	3.36
Diversification Index	0.63		0.71		1.10		1.08		0.73		0.72		0.65		0.70		0.70	

The investment capacity measurements from Table 14 show that also in this period that the optimized portfolios had the lowest measurements of RIC. The one with the highest value is the EW in all measurements except the weighted average RIC, followed by the NRP. So when looking at the weighted average investment capacity, the MSR also in this period had the highest value, and is caused by the same reason as in the 1995 to 2000 period. But it is clear that the MDP and the GMV had the lowest values in all the measurements, and thereby had high turnover and required more active management to rebalance the portfolio during this period.

Table 14: Relative investment capacity measures performed on the portfolios in the time period 2000 to 2005

2000-2005	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
RIC^B	3.2 %	0.0 %	0.4 %	1.8 %	2.0 %	1.6 %	1.9 %	1.5 %	1.7 %
RIC^0.05	3.2 %	0.0 %	0.4 %	1.8 %	2.3 %	1.8 %	2.6 %	1.6 %	2.4 %
RIC^wa	7.0 %	4.3 %	1.2 %	51.7 %	16.6 %	10.9 %	13.4 %	13.8 %	8.7 %

5.3. 2005-2010

When looking at the period from 2005 to 2010 in Figure 9 one can see that the MSR performed well in the beginning of the period, but from 2007 to the end of 2008 it fell drastically by about 350%. It seems like the GMV was the portfolio that handled the financial crisis of 2007-2008 the best, even it had a weak performance the two to three years before the crisis was a fact. It seems from the graph that the diversification- or risk-based allocation strategies did outperformed the market before and after the crisis, and especially the MDP can be seen as the superior from 2007 and to the end of this period.

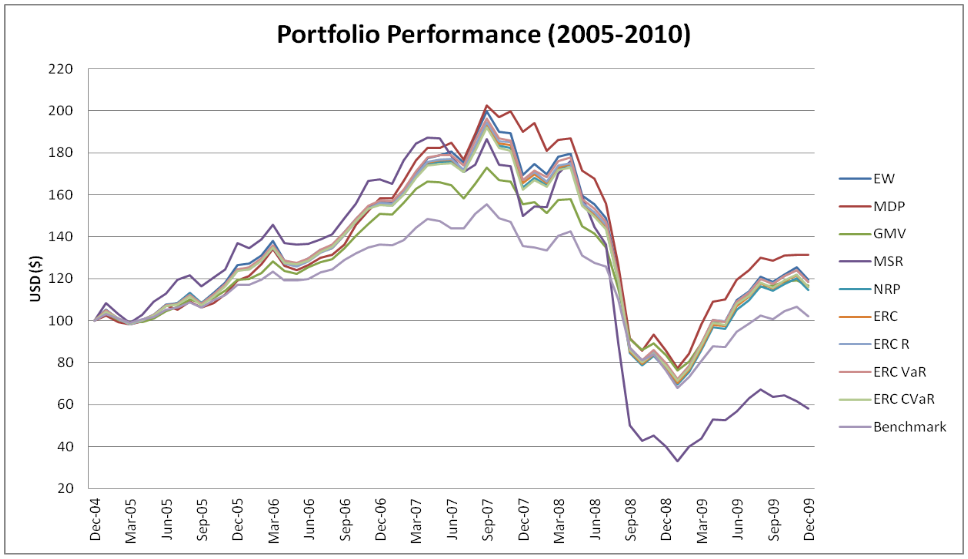

Figure 9: Performance of portfolios from 2005 to 2010

Looking at the returns in Table 15 below, one can see that during this period the MDP had the best performance when it comes to the average return. The MDP had an annual average return of 8.4% this period, followed by the EW with its 6.8%. The ERC portfolios performed better in terms of returns than the GMV during this period, as the ERC portfolios had annual average returns around 6% and the GMV had about 5%.

On the other hand, the portfolio with the lowest average return was the MSR with its average annual return of -3.3%. And the MSR is the only portfolio that was outperformed by the market this period. But the returns of the MSR and the Benchmark are not significant different from zero due to their t-values. The reason for the low performance of the MSR is that the portfolio had about 87% of its weighting in the Austrian market, as seen in Table 49 in section 9. And the Austrian index had this period an average annual return of approximately -4.7%. This period the MSR had both the lowest and highest monthly return, with the value of -43.9% and 21.3%.

Table 15: The portfolios average annual return and the monthly minimum and maximum value during the period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
Average return	6.8 %	8.4 %	4.9 %	-3.3 %	5.7 %	5.9 %	6.3 %	6.3 %	5.8 %	2.1 %	2.7 %
T-test H0=0	2.15	2.78	2.04	-0.71	1.87	1.96	2.10	2.08	1.96	0.91	38.47
Monthly											
Minimum	-29.8 %	-27.5 %	-21.1 %	-43.9 %	-28.7 %	-28.7 %	-28.3 %	-28.3 %	-27.9 %	-21.0 %	0.0 %
Maximum	14.6 %	16.7 %	11.2 %	21.3 %	13.6 %	13.7 %	13.5 %	13.9 %	13.8 %	10.7 %	0.4 %

As the MSR had the weakest performance when it comes to returns, it also had the highest risk in all measures performed during this period, as seen in Table 16. In contrast, it can be seen that the GMV had the lowest risk due to all the risk measurements performed.

One can also see from Table 16 that the ERC CVaR performed well due to the standard deviation and Tracking Error measurements. There is no consensus about which of the RP portfolios is the superior in reducing tail-risk, but they are performing relatively equal. But the RP portfolios were outperforming both the EW and MSR when looking at the tail-risk. The MDP had as mentioned the highest return, but when looking at in the risk aspect of the MDP its performance was mediocre. The MDP did not outperform the RP portfolios in standard deviations or Tracking Errors, but had lower tail-risk than some of the RP portfolios in some quantiles.

Table 16: Risk measures performed on portfolios in the time period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
SD (σ)	24.5 %	23.4 %	18.4 %	35.9 %	23.6 %	23.5 %	23.2 %	23.4 %	23.0 %	18.2 %	0.5 %
TESD	7.9 %	9.4 %	4.9 %	19.9 %	6.8 %	6.9 %	6.7 %	6.7 %	6.4 %		
TEMAD	45.9 %	50.1 %	37.2 %	66.3 %	42.2 %	42.6 %	42.4 %	42.4 %	41.4 %		
Lower tail											
10% VaR	-7.3 %	-6.9 %	-5.7 %	-9.8 %	-7.2 %	-7.0 %	-6.8 %	-6.8 %	-6.9 %	-6.7 %	0.0 %
10% CVaR	-14.4 %	-13.2 %	-10.9 %	-22.6 %	-14.1 %	-14.0 %	-13.8 %	-14.0 %	-13.6 %	-11.6 %	0.0 %
5% VaR	-10.6 %	-8.2 %	-8.1 %	-14.8 %	-10.2 %	-9.9 %	-9.8 %	-10.2 %	-10.2 %	-9.2 %	0.0 %
5% CVaR	-19.5 %	-18.7 %	-14.7 %	-31.9 %	-19.0 %	-19.0 %	-18.8 %	-18.9 %	-18.5 %	-14.8 %	0.0 %
1% VaR	-22.7 %	-22.4 %	-17.1 %	-38.3 %	-22.2 %	-22.1 %	-21.7 %	-21.3 %	-21.5 %	-16.0 %	0.0 %
1% CVaR	-29.8 %	-27.5 %	-21.1 %	-43.9 %	-28.7 %	-28.7 %	-28.3 %	-28.3 %	-27.9 %	-21.0 %	0.0 %

When looking at the R^2 in Table 17 one can see that all of the portfolios had high proportion of systematic risk, with value ranging from 86% to 96%. The RP portfolios and the EW portfolio had the highest proportion of systematic risk due to the values of about 95% to 96%. In contrast, the MDP and MSR had the lowest proportion of systematic risk, with the respectively values of 86% and 87%. The reason why all of the portfolios have a relative high R^2 may be caused by an increasing correlation between the world equities during the financial crisis, as expected during a financial turmoil and financial crises. This increasing and high correlation can be seen in the correlation matrix in Table 44 in the appendix section 8.5.

Further, as seen in Table 17, the portfolio with the highest sensitivity due to the market fluctuations is the MSR with a beta-value of 1.84, i.e. relative to the market it gains in bull market and losses in bear market. On the other hand, the GMV is the portfolio with the lowest value of 0.97, but this is not significant different than 1. This indicates that the movement of this portfolio is generally in the same direction as, and about the same amount as the movement of the benchmark.

Table 17: Systematic risk measurements during the period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
β	1.31	1.20	0.97	1.84	1.27	1.26	1.24	1.25	1.23
$\bar{R}^2$	0.95	0.86	0.93	0.87	0.96	0.95	0.95	0.96	0.96
t-value									
H0: β=1	7.77	3.17	-0.76	8.93	7.74	7.15	6.67	7.24	6.72

The RAPM performed, as one can see in Table 18 below, clearly shows that the MDP had the best performance when looking at the RAPM Sharpe and Treynor, but was outperformed by GMV when considering the IR and Sortion-ratio. The good performance of the MDP is mainly driven by the 40% weight in the Malaysian index, and 15% in the Indonesian index as one can see from Table 49 in section 9. And as seen in table 43 in section 8, these two indices had a good performance in both risk and return characteristics this period.

In contrast, the MSR is the weakest performing portfolio considering these RAPM. The MSR portfolio had the lowest values in all of the measurements calculated, and also the only portfolio with lower Sharpe than the market. The main driver in this portfolio was the Austrian index, with an 86% weight in the portfolio. And as seen from Table 43 in section 8, the Austrian index had an annual average return of -4.7% and an annual average standard deviation of 38%.

It seems like the EW and ERC portfolios have the same properties, and there are not a consistent solution to which of the portfolios is the best performing. One can see that the EW is superior over the ERC when looking at Sharpe, but when looking at the Information- and the Sortino Ratio the ERC portfolios are the best performing, and especially the ERC R.

Table 18: RAPM performed during time period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark
Annually										
Sharpe	0.07	0.15	-0.01	-0.23	0.03	0.04	0.06	0.05	0.04	-0.16
Treynor	0.01	0.03	0.00	-0.04	0.01	0.01	0.01	0.01	0.01	
IR	3.66	4.85	8.35	-0.49	3.54	3.83	4.60	4.53	4.52	
Sortino	3.22	4.42	6.65	-0.54	2.98	3.27	3.98	3.92	3.84	
M^2	-11.6 %	-10.6 %	-12.7 %	-15.6 %	-12.2 %	-12.0 %	-11.8 %	-11.8 %	-12.1 %	
T^2	0.3 %	0.4 %	0.2 %	-0.2 %	0.2 %	0.3 %	0.3 %	0.3 %	0.3 %	
Jensens α	0.4 %	0.5 %	0.2 %	-0.4 %	0.3 %	0.3 %	0.4 %	0.4 %	0.3 %	
t-value										
H0: α=0	1.94	1.65	1.22	-0.85	1.71	1.75	1.88	1.95	1.76	
H0: IR=0	28.32	37.54	64.71	-3.77	27.42	29.66	35.62	35.08	35.00	

When looking at the normality of the portfolios, one can from Table 19 below see that also in this period there are negative skew and positive kurtoses in all of the portfolios. In this period the portfolios had fatter tails and more negative skewed than the 2000-2005, but have more normally distributed returns than in the1995 to 2000 period. One can see that the MSR had the highest value of kurtosis and also lowest value of skew, and by that the highest value of JB. But none of the portfolios is normally distributed due to the JB-test. And it seem like the RP portfolios is closer a normal distribution than the MSR, but not the other portfolios. These result as seen together with the downside risk measurements, may show that the RP portfolios this period is not superior in reducing the risk of extreme events.

Table 19: Normality measures of the portfolios in the time period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Monthly											
Median	2.0 %	1.8 %	1.3 %	1.5 %	1.7 %	1.8 %	1.9 %	1.7 %	1.8 %	1.2 %	0.3 %
Kurstosis	5.15	5.00	4.25	6.05	5.22	5.32	5.31	5.19	5.18	3.93	-1.54
Skewness	-1.59	-1.47	-1.38	-1.71	-1.62	-1.63	-1.62	-1.61	-1.58	-1.43	-0.24
Jarqe Bera	440.68	404.80	308.50	580.46	452.91	466.98	465.01	447.71	441.91	283.45	31.27

From Table 20, one can see that the EW was the most diversified portfolio in all measures performed, except when looking at the Diversification Index and the Gini measure of risk contributions. Looking at the Diversification Index the ERC portfolios based on tail-risk is the seemingly most diversified portfolios. And when considering the Gini Index, the ERC portfolio is the most diversified in terms of risk contribution. But there is a close race between the RP portfolios and the EW in which is the most diversified portfolio.

As mentioned; the MSR consisting of only three indices, with about 86% of its weight in the Austria Index. This leads to the weakest performance in all the diversification measures shown in Table 20. Also the low concentration of weights in MDP and GMV make these portfolios poorly diversified, but as expected the MDP performed well when looking at the Diversification Index.

Table 20: Diversification measures performed on portfolios during the time period 2005 to 2010

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2005-2010	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC
Herfindahl Index	0.0 %	0.2 %	23.4 %	20.3 %	15.0 %	14.3 %	75.5 %	83.7 %	0.3 %	0.3 %	0.3 %	0.3 %	0.4 %	0.2 %	0.5 %	1.1 %	0.9 %	1.6 %
Gini Index	0.0 %	18.3 %	88.5 %	4.0 %	84.4 %	84.4 %	95.9 %	96.2 %	16.0 %	0.4 %	16.1 %	0.0 %	18.8 %	13.7 %	22.6 %	34.6 %	29.6 %	40.9 %
Shannon Entropy	3.50	3.47	1.50	1.54	1.87	1.89	0.48	0.34	3.45	3.46	3.45	3.46	3.44	3.46	3.41	3.22	3.31	3.25
Diversification Index	1.01		0.92		1.03		2.08		1.01		0.98		0.89		0.88		0.88	

In this period, one can see from Table 21 that the EW had the highest value when looking at the bottleneck and 5th percentile RIC. But in this period there are more diffuse which of the ERC portfolios that had the highest investment capacity. One can see that the ERC CVaR had the highest weighted average RIC compared to all the other portfolios, but the ERC VaR had slightly higher 5% RIC than the NRP portfolio. But in the overall context it seems like the RP portfolios and the EW had high RIC, and by that require less active management to rebalance compared to the MDP and MSR.

Table 21: Relative investment capacity measures performed on the portfolios in the time period 2005 to 2010

2005-2010	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
RIC^B	3.2 %	0.1 %	0.2 %	0.0 %	2.3 %	2.1 %	2.0 %	2.4 %	2.4 %
RIC^0.05	3.2 %	0.1 %	0.2 %	0.0 %	2.5 %	2.2 %	2.2 %	2.7 %	2.4 %
RIC^wa	7.0 %	5.4 %	7.6 %	0.9 %	11.3 %	10.9 %	10.8 %	10.8 %	21.1 %

5.4. 2010-2013

Looking at the most recent period, from 2010 to 2013, one can see from Figure 10 that the MDP and GMV is the best performing due to returns. One can see that the MSR had a weak performance, and that the MDP and the GMV are the only portfolios that seemingly have a better performance than the Benchmark.

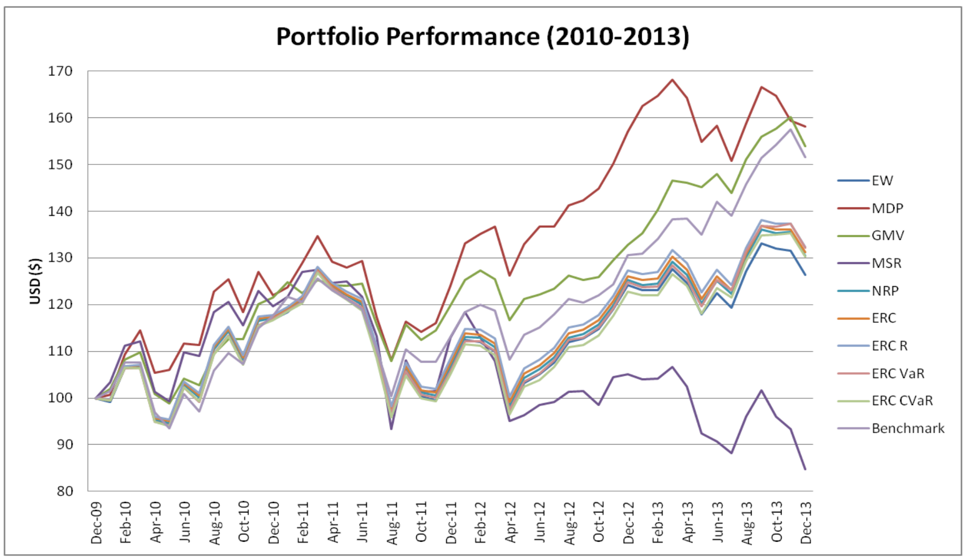

Figure 10: Performance of portfolios from 2010 to 2013

When looking at the numbers in Table 22, one can see that during this period the MDP was the best performing portfolio due to returns. The MDP had an average annual return of 12.8%, followed by the GMV with its 11.6% which had the same average return as the Benchmark. In contrast, the MSR with its annual average return of -1.8% was the portfolio with lowest annual average return, but this was not significant different from zero.

The RP portfolios only outperformed the MRS and the EW portfolio in terms of returns, and it did not outperform the market when considering returns.

One can also see that the MSR had both the lowest and the highest monthly return observation during this period, with a minimum value of -17.5% and a maximum of 15.7%. On the contrary, the MDP had both the lowest upside and downside return observation, with value of 10.3% and -9.3%.

Table 22: The portfolios average annual return and the monthly minimum and maximum value during the period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
Average return	7.5 %	12.8 %	11.6 %	-1.8 %	8.2 %	8.4 %	8.6 %	8.6 %	8.2 %	11.6 %	0.1 %
T-test H0=0	2.83	5.51	6.43	-0.56	3.20	3.27	3.37	3.33	3.19	5.23	40.54
Monthly											
Minimum	-12.0 %	-9.3 %	-8.2 %	-17.5 %	-11.6 %	-11.5 %	-11.5 %	-11.7 %	-11.8 %	-10.0 %	0.0 %
Maximum	10.8 %	10.3 %	7.3 %	15.7 %	10.4 %	10.3 %	10.3 %	10.4 %	10.2 %	9.9 %	0.0 %

When looking at the risk measurements performed in Table 23, one can see that the MSR portfolio had the highest risk in all the measurements except the $VaR_{5\%}$. But when looking at the standard deviation of the MSR it had an average annual standard deviation of 21.8%, but this was not significant higher than the EW with its 18.5%[16]. The high risk of the MSR was mainly driven by the heavy proportion of risk contribution of about 80% in the Brazilian market, which had an average annual standard deviation of about 26.4% this period.

In contrast, the GMV also in this period had the lowest risk when considering all the measures performed in Table 23. The GMV had about 43% of its risk in the US market and 30% in the Japanese market as one can see from Table 50 in section 9. And the US and the Japanese indices had a standard deviation between 14% and 15% as seen in section 8, Table 46. The GMV had a standard deviation of 12.5%. And the standard deviation of the GMV was significant lower compared with the second highest, the MDP with its 16.1%.

As also seen in Table 23, it seems like the MDP portfolio also had a lower risk compared with the other diversified and risk based portfolios, due to its relative low values in all the measurements.

[16] p-value of 0.13 at 5%-level.

Table 23: Risk measures performed on portfolios in the time period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
SD (σ)	18.5 %	16.1 %	12.5 %	21.8 %	17.9 %	17.7 %	17.6 %	17.9 %	17.9 %	15.4 %	0.0 %
TESD	6.0 %	9.4 %	4.9 %	19.9 %	6.8 %	6.9 %	6.7 %	6.7 %	6.4 %		
TEMAD	41.5 %	50.1 %	37.2 %	66.3 %	42.2 %	42.6 %	42.4 %	42.4 %	41.4 %		
Lower tail											
10% VaR	-5.5 %	-5.6 %	-3.1 %	-7.6 %	-5.0 %	-5.0 %	-5.0 %	-5.0 %	-5.0 %	-3.8 %	0.0 %
10% CVaR	-9.8 %	-7.7 %	-6.6 %	-11.6 %	-9.5 %	-9.4 %	-9.3 %	-9.5 %	-9.6 %	-7.8 %	0.0 %
5% VaR	-10.1 %	-7.8 %	-6.9 %	-9.7 %	-9.9 %	-9.8 %	-9.7 %	-9.8 %	-10.0 %	-8.3 %	0.0 %
5% CVaR	-11.6 %	-8.4 %	-7.4 %	-13.1 %	-11.2 %	-11.1 %	-11.0 %	-11.3 %	-10.8 %	-9.3 %	0.0 %
1% VaR	-11.9 %	-8.7 %	-7.6 %	-14.9 %	-11.5 %	-11.4 %	-11.3 %	-11.2 %	-11.7 %	-9.5 %	0.0 %
1% CVaR	-12.0 %	-9.3 %	-8.2 %	-17.5 %	-11.6 %	-11.5 %	-11.5 %	-11.7 %	-11.4 %	-10.0 %	0.0 %

Form the R^2 in Table 24 it can be seen that the RP portfolios had a high proportion of systematic risk during this period. The highest value was in the ERC CVaR portfolio with a value of 93.4%, closely followed by the other RP portfolios. The lowest proportion of systematic risk was found in the MSR, which had a value of 68.7%.

From Table 24 one can also see that the EW portfolio had the highest significant sensitivity due to the market movements with its beta value of 1.15, followed by ERC CVaR with its value of 1.13[17]. When looking at the properties of the GMV with its R^2 close to one and beta below one; this can be viewed as the portfolio is likely to offer better risk adjusted return i.e. gains less than the market but also have lower loss.

Table 24: Systematic risk measurements during the period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
β	1.15	0.88	0.78	1.18	1.12	1.11	1.10	1.12	1.13
$\bar{R}^2$	0.91	0.71	0.92	0.69	0.93	0.92	0.92	0.93	0.93
t-value									
H0: β=1	2.80	-1.45	-6.47	1.54	2.54	2.25	2.03	2.79	2.89

From the RAPM perspective, one can from Table 25 see that the GMV was the portfolio with the best performance when looking at all the measurements, except when looking at the Sortino-ratio. Looking at the Sortino one can see that the EW had a really high value of this downside RAPM, with a value of 19.31, followed by the GMV with its value of 14.7.

On the other hand, the MSR is the portfolio with the weakest performance in almost all the measures used. The only measure where the MSR is outperforming another portfolio is when looking at the IR, where MSR have a value of -0.49 and the EW have a value of -10.5.

[17] MSR beta is not significant different from zero

When consider the Risk Parity portfolios, it seems like the ERC R is the best performing portfolio, except when looking at the Sortino, where the ERC CVaR is the one with best performance.

Table 25: RAPM performed during time period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark
Annually										
Sharpe	0.14	0.48	0.53	-0.31	0.18	0.19	0.20	0.20	0.18	0.43
Treynor	0.02	0.09	0.08	-0.06	0.03	0.03	0.03	0.03	0.03	
IR	-10.52	4.85	8.35	-0.49	3.54	3.83	4.60	4.53	4.52	
Sortino	19.31	6.44	14.70	-0.30	5.61	5.91	6.89	6.79	7.06	
M^2	-10.7 %	-6.1 %	-5.5 %	-16.7 %	-10.1 %	-10.0 %	-9.9 %	-9.9 %	-10.2 %	
T^2	-0.4 %	0.2 %	0.3 %	-1.1 %	-0.4 %	-0.3 %	-0.3 %	-0.3 %	-0.4 %	
Jensens α	-0.5 %	0.2 %	0.2 %	-1.3 %	-0.4 %	-0.4 %	-0.3 %	-0.4 %	-0.4 %	
t-value										
H0: α=0	-2.04	0.57	1.37	-2.48	-1.88	-1.77	-1.63	-1.86	-2.05	
H0: IR=0	-72.88	33.58	57.88	-3.37	24.52	26.53	31.86	31.38	31.30	

One can see from Table 26 that also in this period there is negative skew, and values of kurtosis different from zero in all the portfolios. The MSR have the highest value of kurtosis with a value of 0.76, which indicates fat tails on the distribution. In contrast, one can see that the EW portfolio is the one with the least fat tails with its kurtosis of 0.16. Looking at the skewness of the distribution the MSR have value closest to zero, and on the other hand the GMV had a negative value of -0.69.

Looking at the JB one can see that the MSR is the closest to the normal distribution, but we reject the null-hypothesis about the normal distribution at a 5% level. One can also see that the GMV had the least normally-distributed returns in this period, with high values of skew, kurtosis, and JB. And when consider the RP portfolios, these had more normally distributed returns than the GMV but less normally distributed than the other portfolios.

Table 26: Normality measures of the portfolios in the time period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Monthly											
Median	0.2 %	1.5 %	1.5 %	0.1 %	0.4 %	0.3 %	0.3 %	0.5 %	0.4 %	1.4 %	0.0 %
Kurstosis	0.16	-0.29	0.47	0.76	0.24	0.23	0.26	0.25	0.28	0.33	-1.06
Skewness	-0.44	-0.40	-0.69	-0.07	-0.48	-0.48	-0.48	-0.50	-0.51	-0.49	0.30
Jarqe Bera	9.50	8.75	25.49	7.11	11.75	11.61	11.92	12.64	13.34	12.73	17.97

The diversification measures shown in Table 27 below indicate that when it comes to the overall diversification of the risk contribution, the ERC is the superior. The ERC portfolio had the lowest value in both Shannon and Gini. When it comes to the Herfindahl Index there are not huge differences between the EW and the NRP, ERC and ERC R in the risk contribution diversification, and this is also the result when looking at the Diversification Index. Further the MDP had as expected the greatest diversification when looking at the Diversification Index.

The least diversified portfolio this period when looking at the Herfindahl, Gini, and Shannon measures is clearly the MSR portfolio. The reason for this is that the portfolio only contains three of the indices, namely the Brazilian, the Malaysian, and the Indonesian. But when looking at the Diversification Index, the ERC VaR and ERC CVaR were the portfolios with the lowest diversification.

There is a tendency through this thesis that the plain ERC and the NRP outperforms the three other RP portfolios in all the diversification measurements performed.

Table 27: Diversification measures performed on portfolios during the time period 2010 to 2013

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2010-2013	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC	W	RC
Herfindahl Index	0.0 %	0.2 %	13.9 %	14.5 %	30.0 %	30.3 %	53.5 %	66.7 %	0.2 %	0.2 %	0.2 %	0.2 %	0.3 %	0.2 %	0.6 %	1.1 %	0.6 %	0.7 %
Gini Index	0.0 %	13.6 %	82.5 %	82.3 %	91.4 %	91.4 %	94.9 %	95.9 %	13.7 %	6.3 %	14.2 %	0.0 %	16.3 %	13.6 %	23.4 %	32.5 %	25.3 %	26.3 %
Shannon Entropy	3.50	3.46	1.97	1.95	1.21	1.21	0.69	0.54	3.47	3.46	3.46	3.47	3.45	3.46	3.39	3.11	3.39	3.40
Diversification Index	0.66		0.60		0.72		0.68		0.67		0.67		0.66		0.83		0.83	

The investment capacity for the last period, as seen in Table 28, indicates also that the EW and the NRP had the highest values when looking at the bottleneck and 5% percentile RIC. But when looking at the weighted average RIC measure the ERC CVaR have the highest value. The MSR had the lowest values in all the measurements, followed by the MDP portfolio which had a higher value than the MSR when looking at the weighted average RIC. This implies that also in this period that the MSR and the MDP required more active management to rebalance the portfolio during this period than the EW and RP portfolios.

Table 28: Relative investment capacity measures performed on the portfolios in the time period 2010 to 2013

2010-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
RIC^B	3.2 %	0.0 %	1.0 %	0.0 %	2.9 %	2.6 %	2.2 %	2.0 %	2.0 %
RIC^0.05	3.2 %	0.0 %	1.0 %	0.0 %	3.0 %	2.8 %	2.6 %	2.1 %	2.1 %
RIC^wa	7.0 %	0.5 %	4.3 %	0.0 %	10.9 %	10.2 %	7.3 %	13.8 %	17.0 %

5.5. Full Period (1995-2013)

When looking at the full out-of-sample period from 1995 to 2013, one can from Figure 11 see that the MSR had the best performance when looking at returns in the period from 1995 to about 2000. After the burst of the dot-com bubble one can see that the GMV outperformed the other portfolios in the terms of returns. The MSR portfolio also performed well during the pre-crisis of

2007-2008, but had a dramatic fall during the crisis, and never seems to recover as a well-performing portfolio. After the financial crisis in 2007-2008 one can see that the MDP had the second best performance, but this portfolio did not perform well in the 1995-2000 period. It seems like the ERC portfolios lies between the other portfolios in terms of returns the whole period.

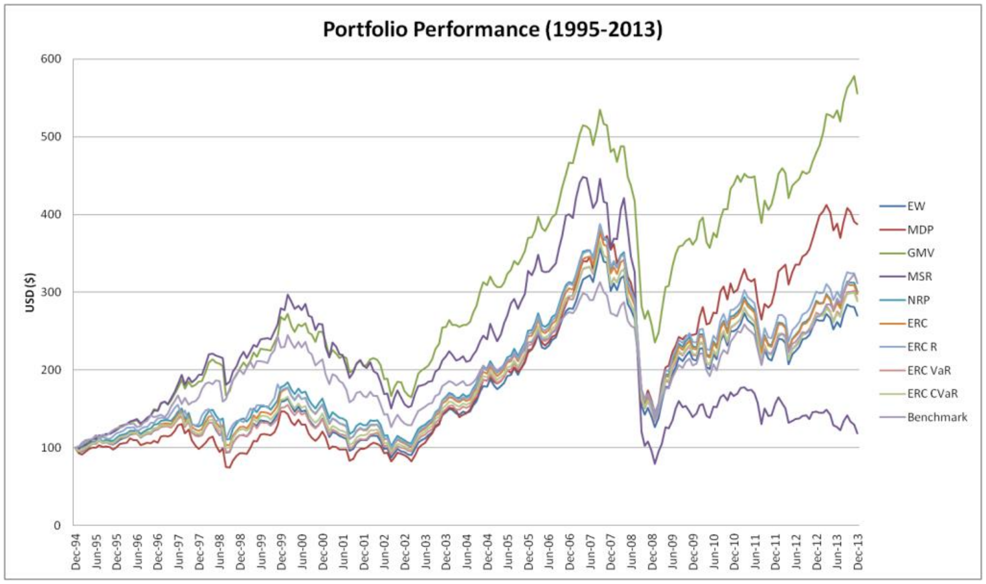

Figure 11: Performance of portfolios full period (1995 to 2013)

As one can see from Table 29 below, the portfolio with the highest return during the whole period was the GMV with an average annual return of 10.3%. The portfolio with the second highest return was the MDP with its return of 9.3%. And as seen from the full period, the MSR was the portfolio with the lowest annual average return with its value of 4%. One can also see from Table 29 that all of the portfolios have t-values above the critical t-value, and all the returns are statistically different than zero. And as one can see, the highest extreme value occurred during the time period 2005 to 2010 and not surprisingly was these values in the MSR portfolio, as seen in section 5.3.

Table 29: The portfolios average annual return and the monthly minimum and maximum value during the full period (1995 to 2013)

1995-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
Average return	7.3 %	9.3 %	10.3 %	4.0 %	7.7 %	7.7 %	7.9 %	7.5 %	7.4 %	7.3 %	2.7 %
T-test H0=0	5.54	6.93	10.08	2.55	6.13	6.08	6.31	5.97	5.97	6.96	65.84
Monthly											
Minimum	-29.8 %	-27.5 %	-21.1 %	-43.9 %	-28.7 %	-28.7 %	-28.3 %	-28.3 %	-27.9 %	-21.0 %	0.0 %
Maximum	14.6 %	16.7 %	11.2 %	21.3 %	13.6 %	13.7 %	13.5 %	13.9 %	13.8 %	10.7 %	0.5 %

When looking at the risk of the aggregated period, shown in Table 30 below, one can see that the portfolio with the highest standard deviation is the MSR portfolio with a standard deviation of 23.7%. The MSR had a significant higher standard deviation than the MDP with its standard deviation of 20.2%. The portfolio with the significant lowest standard deviation is the GMV with a standard deviation of 15.4%.

But looking at the relative risk measures, TESD and TEMAD, one can see that the MDP was the portfolio with greatest risk relative to the market, and in contrast the ERC R was the portfolio with the lowest relative risk.

When considering the tail-risk, one can see that the GMV is the portfolio with the lowest VaR and CVaR values when looking at all percentiles. It also seems like the RP portfolios performed well when looking at these tail-risk measurements. But as the GMV is the portfolio with lowest tail-risk, it only have a lower tail risk than the market when looking at the 5% and 10% percentiles, while the market had a lower VaR and CVaR when looking at the 1% percentile.

Table 30: Risk measures performed on portfolios during the full period (1995 to 2013)

1995-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Annually											
SD (σ)	20.0 %	20.2 %	15.4 %	23.7 %	18.9 %	19.0 %	18.8 %	19.0 %	18.8 %	15.8 %	0.6 %
TESD	13.8 %	15.8 %	11.4 %	15.6 %	12.7 %	13.0 %	6.6 %	13.2 %	12.9 %		
TEMAD	55.2 %	61.4 %	50.0 %	59.0 %	52.0 %	52.9 %	42.1 %	53.3 %	52.7 %		
Lower tail											
10% VaR	-5.8 %	-6.7 %	-4.0 %	-6.2 %	-5.0 %	-5.5 %	-5.4 %	-5.1 %	-5.4 %	-5.1 %	0.0 %
10% CVaR	-11.4 %	-11.1 %	-8.4 %	-13.1 %	-10.8 %	-10.8 %	-10.7 %	-10.8 %	-10.6 %	-9.0 %	0.0 %
5% VaR	-10.4 %	-8.8 %	-6.9 %	-9.7 %	-9.7 %	-9.9 %	-10.0 %	-9.7 %	-10.0 %	-8.6 %	0.0 %
5% CVaR	-14.5 %	-14.1 %	-10.9 %	-17.8 %	-13.9 %	-13.9 %	-13.7 %	-14.0 %	-13.5 %	-11.2 %	0.0 %
1% VaR	-16.8 %	-17.8 %	-13.7 %	-17.5 %	-16.3 %	-16.2 %	-15.9 %	-15.9 %	-16.1 %	-12.3 %	0.0 %
1% CVaR	-23.0 %	-23.7 %	-18.0 %	-32.0 %	-22.0 %	-22.2 %	-21.8 %	-22.3 %	-21.5 %	-16.0 %	0.0 %

When looking at the proportion of systematic risk given by R^2 in Table 31, one can see that the ERC R with its 89% clearly had the highest value, followed by the MSR with its 57%. The ERC R and the MSR also had the highest sensitivity due to market fluctuations, with their respectively betas of 1.12 and 1.13.

The MDP, on the other hand, had the lowest proportion of systematic risk with a value of 41%, and a beta of 0.82. But as stated earlier, the low R^2 indicates that the beta value is not reliable and may be ignored, this is also be a question when looking at the MSR. The only portfolio with a lower market sensitivity than the MDP was the GMV with its beta of 0.71, but also this portfolio have a relative low R^2.

Table 31: Systematic risk measurements during the full period (1995 to 2013)

1995-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR
β	0.92	0.82	0.71	1.13	0.89	0.88	1.12	0.87	0.87
$\bar{R}^2$	0.53	0.41	0.53	0.57	0.56	0.54	0.89	0.52	0.54
t-value									
H0: β=1	-1.39	-2.71	-6.48	2.02	-2.00	-2.12	4.62	-2.30	-2.35

The RAPM shown in Table 32 below shows that under the full period, the GMV had the highest values in all the RAPM performed, except when looking at the IR, and the Sortino, where the ERC R was the superior portfolio. The good performance of the GMV was, as seen previous, created by the god performance in 1995-2000 and 2010-2013. The MDP also performed well when looking at the aggregated time period, where this portfolio had a great performance when considering the RAPM from 2000 to 2013. In contrast, the MSR is the portfolio with the weakest performance during the full period when looking at all the RAPM used in this thesis, and this mainly because the weak RAPM performance of these measures after 2000.

Table 32: RAPM performed during the full period (1995 to 2013)

1995-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark
Annually										
Sharpe	0.12	0.21	0.34	-0.04	0.14	0.14	0.15	0.13	0.13	0.14
Treynor	0.03	0.05	0.07	-0.01	0.03	0.03	0.03	0.03	0.03	
IR	0.01	0.27	0.67	-0.47	0.08	0.07	0.73	0.04	0.03	
Sortino	0.74	0.88	1.68	0.18	0.93	0.87	6.37	0.82	0.86	
M^2	-11.0 %	-9.7 %	-8.0 %	-13.1 %	-10.7 %	-10.7 %	-10.5 %	-10.8 %	-10.8 %	
T^2	0.0 %	0.3 %	0.5 %	-0.3 %	0.1 %	0.1 %	0.0 %	0.1 %	0.1 %	
Jensens α	0.0 %	0.2 %	0.4 %	-0.3 %	0.1 %	0.1 %	0.0 %	0.1 %	0.1 %	
t-value										
H0: α=0	0.13	0.79	1.78	-1.09	0.31	0.30	0.02	0.27	0.25	
H0: IR=0	0.13	4.08	10.10	-7.09	1.15	1.00	11.04	0.61	0.41	

When looking at the normality measurements in Table 33 below one can see that there is, as in all the rolling window periods, a tendency to negative skew and positive kurtosis. One can see that there is both fat tails and skew due to the distribution of the portfolios returns, and especially in the MSR where the kurtosis is about 10.26 and the skew is -1.18. This is also seen in the JB-test performed where the MSR have a JB-value of 1419, compared with the second highest which is the ERC VaR with the JB-value of 287. There are really none of the portfolios which are close to the Gauss-normal distribution, but the closest is the MDP with a kurtosis of 3.6, skew of -1.14, and a JB-value of approximately 219. Looking at the monthly median, one can see that this is above the average return and by that indicates that there was a tendency of more and higher extreme values in the downside than in the upside of the returns during the whole time period.

Table 33: Normality measures of the portfolios during the full time period (1995 to 2013)

1995-2013	EW	MDP	GMV	MSR	NRP	ERC	ERC R	ERC VaR	ERC CVaR	Benchmark	US T-Bill
Monthly											
Median	1.1 %	1.4 %	1.4 %	1.0 %	1.2 %	1.3 %	1.2 %	1.2 %	1.1 %	1.3 %	0.2 %
Kurstosis	3.88	3.61	3.95	10.26	4.18	4.17	4.11	4.18	3.88	2.30	-1.68
Skewness	-1.20	-1.14	-1.25	-1.80	-1.26	-1.26	-1.26	-1.27	-1.21	-1.00	0.01
Jarqe Bera	250.52	218.76	261.71	1419.21	286.26	285.63	278.24	286.70	251.08	111.98	33.74

6. Conclusions

The main objective of this thesis was to investigate the Risk Parity approach and find the answers to the following questions: Does it work? Will it yield a good performance in the manner of return-to-risk measurements? Does it provide better risk balance or risk profile, and can we avoid concentration and thereby achieve a greater diversified portfolio? And thereby to see how good this portfolio was performing; compare it with other more common portfolio allocation approaches.

From the portfolio analysis performed in the previous section one can see that the Risk Parity portfolios were not superior when considering all these research objectives. It seems like they are mediocre performing portfolios when looking at risk and return characteristics. It seems like there are no consensus about which of the portfolio that is superior, since the performance of the portfolios changes drastically during these five year rolling windows. But it seems like the GMV and the MDP had some features that made them perform well in the risk to return characteristic after year 2000. Due to the good performance of these portfolios, and especially the GMV, it there may be a tendency to low-volatility anomaly.

It is also a surprise that none of the Risk Parity portfolios reduce tail-risk events or are superior in reducing loss when the market is decreasing, even when constructing portfolios based on tail-risk measures.

The Risk Parity portfolios are seemingly good portfolios when it comes to the diversification measures, both in terms of risk and weights, but it is not constantly outperforming the EW or the MDP portfolio when considering the measurements performed. There are somehow huge differences in the risk contribution diversification between the Risk Parity portfolios, and also a drawback is that the ERC properties are only achieved in-sample or when rebalancing rapidly.

The Risk Parity portfolios have a relatively high RIC compare to the GMV and the MDP, but it is not outperforming the EW portfolio. This implies that the EW and the Risk Parity portfolios had a lower turnover and require less active management to rebalance the portfolio compared with the GMV and the MDP. But after 2005 the EW and the Risk Parity outperformed all the other optimized portfolios when looking at investment capacity.

There may be a drawback of this thesis to not include bond- and commodities indices and leverage the low volatility positions to obtain a "true Risk Parity" allocation, and by that and take a better advantage of these less equity-correlated assets. This may have improved the Risk Parity portfolio performance, but probably also give a complete different result in the measures performed in this thesis. The reason why leverage is not used is mainly because this increase risk in several ways and some investors do not have the opportunity to use leverage, as explained earlier. Another drawback is that there are only investigated a Gauss-distributed ERC VaR portfolio, since one can see that the indices seemingly not are normally distributed.

So for further research on may include several asset classes like bonds and commodities, included leverage, and then do the same analysis again to see how well these Risk Parity portfolios then had performed. Another suggestion for further research could be to only look at Risk Parity portfolios, also with a main objective on the tail-risk-based ERC portfolios. One can by that perform a sensitivity analysis and see the difference in these portfolios based on different tail-risk percentiles both on the upper tail and the lower tail and compare these portfolios. This test may be based on single asset classes and also in a broad portfolio based on several asset classes to see the robustness of the findings.

This thesis is based on ex-post observations, and the risk of implementing strategies based on historical data will always be risky since one does not know how the future will be.

-Consensus that the future is uncertain-

7. References

ALEXANDER, C. 2009. *Market Risk Analysis, Value at Risk Models*, John Wiley & Sons.

AMBACHTSHEER, K. P. 1987. Pension fund asset allocation: In defense of a 60/40 equity/debt asset mix. *Financial Analysts Journal*, 14-24.

ASNESS, C., FRAZZINI, A. & PEDERSEN, L. H. 2012. Leverage Aversion and Risk Parity. *Financial Analysts Journal*, 68, 47-59.

BACHELIER, L. 1900. *Théorie de la Spéculation*, Gauthier-Villars.

BAI, X., SCHEINBERG, K. & TUTUNCU, R. 2013. Least-Squares Approach to Risk Parity in Portfolio Selection. *Available at SSRN*.

BLACK, F. 1972. Capital Market Equilibrium with Restricted Borrowing. *The Journal of Business*, 45, 444-455.

BOOTH, D. G. & FAMA, E. F. 1992. Diversification returns and asset contributions. *Financial Analysts Journal*, 26-32.

BOUDT, K., CARL, P. & PETERSON, B. 2013. Asset allocation with conditional value-at-risk budgets. *Journal of Risk*, 15, 39-68.

BRUDER, B. & RONCALLI, T. 2012. Managing risk exposures using the risk budgeting approach.

CHAN-LAU, J. A. 2012. Frontier Markets: Punching Below Their Weight? A Risk Parity Perspective on Asset Allocation. *Journal of Investing*, 21, 140.

CHAVES, D., HSU, J., LI, F. & SHAKERNIA, O. 2011. Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios. *Journal of Investing*, 20, 108-118.

CHAVES, D., HSU, J., LI, F. & SHAKERNIA, O. 2012. Efficient Algorithms for Computing Risk Parity Portfolio Weights. *Available at SSRN 2117303*.

CHOUEIFATY, Y. & COIGNARD, Y. 2008. Toward maximum diversification. *The Journal of Portfolio Management*, 35, 40-51.

DALIO, R. 2004. Engineering Targeted Returns and Risks. *Bridgewater Associates Working Paper*.

DEGUEST, R., MARTELLINI, L. & MEUCCI, A. 2013. Risk Parity and Beyond: From Asset Allocation to Risk Allocation Decisions.

DENAULT, M. 2001. Coherent allocation of risk capital. *Journal of risk*, 4, 1-34.

FAMA, E. F. 1965. The Behavior of Stock-Market Prices. *The journal of Business*, 38, 34-105.

FRAZZINI, A. & PEDERSEN, L. H. 2010. Betting against beta. National Bureau of Economic Research.

GOURIÉROUX, C., LAURENT, J. P. & SCAILLET, O. 2000. Sensitivity analysis of values at risk. *Journal of Empirical Finance*, 7, 225-245.

HALLERBACH, W. 2013. Advances in Portfolio Risk Control. Risk! Parity? *Parity*.

HALLERBACH, W. G. 1999. Decomposing portfolio value-at-risk: a general analysis. Tinbergen Institute Discussion Paper.

INKER, B. 2011. The Dangers of Risk Parity. *The Journal of Investing,* 20**,** 90-98.

JENSEN, M. & SCHOLES, M. 1972. The Capital Asset Pricing Model: Some Empirical Tests.

KAYA, H. & LEE, W. 2012. Demystifying Risk Parity. *Available at SSRN 1987770*.

KAZEMI, H. 2011. An Introduction to Risk Parity. 7.

KEEL, S. & ARDIA, D. 2011. Generalized marginal risk. *Journal of Asset Management,* 12**,** 123-131.

KIND, C. 2013. Risk-Based Allocation of Principal Portfolios. *Available at SSRN 2240842*.

LINTNER, J. 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics***,** 13-37.

LITTERMAN, R. 1996. Hot Spots [TM] and Hedges. *Journal of Portfolio Management,* 23**,** 52-75.

LOHRE, H., OPFER, H. & ORSZAG, G. 2012. Diversifying risk parity. *Available at SSRN 1974446*.

MAILLARD, S., RONCALLI, T. & TEILETCHE, J. 2008. On the properties of equally-weighted risk contributions portfolios. *Available at SSRN 1271972*.

MARKOWITZ, H. 1952. Portfolio Selection*. *The journal of finance,* 7**,** 77-91.

MERTON, R. C. 1980. On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics,* 8**,** 323-361.

MEUCCI, A. 2009. Managing diversification. *Risk,* 22**,** 74-79.

MOSSIN, J. 1966. Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society***,** 768-783.

PETERS, E. E. 2011. Balancing Asset Growth and Liability Hedging through Risk Parity. *The Journal of Investing,* 20**,** 128-136.

QIAN, E. 2005. Risk parity portfolios: Efficient portfolios through true diversification. *Panagora Asset Management, September*.

QIAN, E. 2011. Risk Parity and Diversification. *Journal of Investing,* 20**,** 119.

QIAN, E. 2012. Pension Liabilities and Risk Parity. *Journal of Investing,* 21**,** 93.

RACHEV, S. T., HSU, J. S., BAGASHEVA, B. S. & FABOZZI, F. J. 2008. *Bayesian methods in finance,* John Wiley & Sons.

RONCALLI, T. 2013. *Introduction to Risk Parity and Budgeting*, CRC Press.

RUBAN, O. & MELAS, D. 2011. Constructing Risk Parity Portfolios: Rebalance, Leverage, or Both? *The Journal of Investing,* 20**,** 99-107.

RYAN, R. J. & FABOZZI, F. J. 2002. Rethinking pension liabilities and asset allocation. *The Journal of Portfolio Management,* 28**,** 7-15.

SEBASTIAN, M. 2012. Risk Parity and the Limits of Leverage. *The Journal of Investing,* 21**,** 79-87.

SHARPE, W. F. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk*. *The journal of finance,* 19**,** 425-442.

TASCHE, D. 2002. Expected shortfall and beyond. *Journal of Banking & Finance,* 26**,** 1519-1533.

TASCHE, D. 2007. Capital allocation to business units and sub-portfolios: the Euler principle. *arXiv preprint arXiv:0708.2542*.

TEILETCHE, J., RONCALLI, T. & MAILLARD, S. 2010. The properties of equally-weighted risk contributions portfolios. Paris Dauphine University.

TOBIN, J. 1958. Liquidity preference as behavior towards risk. *The review of economic studies***,** 65-86.

8. Equity Appendix

Descriptive statistics for data used in thesis

8.1. Full Period Data (1990-2013)

Table 34: Descriptive Statistics for Indices (1990-2013)

	Monthly								Annually				Tests			$\alpha_i+\beta_i R_m(t)$		t-value		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test*	Annual t-test*	Jarqe Bera	α	β	H0: α=0	H0: β=1	$\bar{R}^2$
Australia	0.78 %	0.36 %	1.14 %	6.06 %	2.25	-0.75	-29.45 %	16.38 %	9.34 %	21.00 %	6.22 %	0.30	2.18	7.55	88.06	0.002	1.037	0.885	0.731	0.588
Austria	0.19 %	0.46 %	0.98 %	7.84 %	6.19	-1.41	-46.27 %	22.74 %	2.32 %	27.16 %	-0.80 %	-0.03	0.42	1.45	555.03	-0.004	1.196	-1.268	2.597	0.467
Belgium	0.60 %	0.37 %	1.30 %	6.27 %	11.77	-2.12	-45.50 %	16.71 %	7.15 %	21.74 %	4.02 %	0.19	1.61	5.58	1877.49	0.000	1.056	0.063	1.029	0.570
Brazil	1.02 %	0.84 %	1.87 %	14.31 %	13.44	-1.59	-109.52 %	59.13 %	12.21 %	49.57 %	9.09 %	0.18	1.21	4.18	2288.31	0.003	1.650	0.354	4.036	0.266
Canada	0.73 %	0.34 %	1.23 %	5.74 %	4.37	-1.07	-31.39 %	19.28 %	8.71 %	19.88 %	5.59 %	0.28	2.15	7.44	284.06	0.002	1.023	0.765	0.513	0.641
Denmark	0.82 %	0.32 %	1.33 %	5.42 %	1.21	-0.58	-19.63 %	17.04 %	9.79 %	18.76 %	6.67 %	0.36	2.56	8.85	33.73	0.003	0.776	1.306	-4.113	0.412
Finland	0.75 %	0.56 %	1.02 %	9.47 %	1.26	-0.35	-38.21 %	28.72 %	8.95 %	32.80 %	5.83 %	0.18	1.34	4.63	24.93	0.000	1.447	0.116	4.933	0.470
France	0.62 %	0.35 %	1.15 %	5.98 %	1.22	-0.67	-25.38 %	14.62 %	7.39 %	20.71 %	4.27 %	0.21	1.75	6.06	39.03	0.000	1.141	0.054	3.477	0.735
Germany	0.62 %	0.40 %	1.48 %	6.80 %	2.21	-0.91	-27.91 %	21.26 %	7.40 %	23.57 %	4.27 %	0.18	1.54	5.33	98.06	-0.000	1.265	-0.124	5.385	0.697
Hong Kong	0.92 %	0.44 %	1.02 %	7.41 %	2.51	-0.26	-34.06 %	28.69 %	11.06 %	25.66 %	7.93 %	0.31	2.11	7.31	78.96	0.003	1.049	1.014	0.654	0.403
India	1.24 %	0.57 %	1.14 %	9.65 %	4.27	0.12	-38.14 %	53.79 %	14.85 %	33.44 %	11.73 %	0.35	2.18	7.54	219.61	0.008	0.656	1.432	-2.847	0.090
Indonesia	0.25 %	0.74 %	1.13 %	12.62 %	2.99	-0.56	-51.99 %	44.20 %	2.96 %	43.72 %	-0.17 %	0.00	0.33	1.15	122.44	-0.004	1.188	-0.551	1.243	0.175
Ireland	0.19 %	0.39 %	1.28 %	6.64 %	2.91	-1.10	-30.17 %	16.91 %	2.23 %	23.02 %	-0.90 %	-0.04	0.47	1.64	159.38	-0.004	1.113	-1.593	1.963	0.565
Italy	0.33 %	0.43 %	0.79 %	7.31 %	0.47	-0.29	-26.92 %	19.61 %	3.90 %	25.32 %	0.78 %	0.03	0.76	2.62	6.82	-0.003	1.147	-0.921	2.150	0.495
Japan	-0.03 %	0.36 %	-0.13 %	6.15 %	0.90	-0.03	-21.55 %	21.72 %	-0.39 %	21.29 %	-3.51 %	-0.17	-0.09	-0.31	9.70	-0.006	0.967	-2.262	-0.580	0.494
Malaysia	0.64 %	0.48 %	1.16 %	8.18 %	4.72	-0.29	-35.76 %	37.94 %	7.72 %	28.33 %	4.60 %	0.16	1.33	4.62	271.04	0.002	0.766	0.343	-2.390	0.174
Mexico	1.18 %	0.53 %	1.96 %	9.00 %	3.86	-1.15	-41.93 %	23.87 %	14.21 %	31.18 %	11.09 %	0.36	2.23	7.74	242.01	0.005	1.240	1.312	2.578	0.381
Netherlands	0.75 %	0.34 %	1.43 %	5.82 %	3.24	-1.15	-28.92 %	13.44 %	9.02 %	20.15 %	5.90 %	0.29	2.19	7.60	189.65	0.002	1.111	0.879	2.840	0.737
New Zealand	0.60 %	0.38 %	1.32 %	6.53 %	1.03	-0.64	-25.41 %	16.58 %	7.18 %	22.62 %	4.05 %	0.18	1.55	5.38	32.22	0.000	0.951	0.168	-0.754	0.423
Norway	0.61 %	0.47 %	1.08 %	7.89 %	4.30	-1.14	-40.59 %	19.45 %	7.34 %	27.34 %	4.21 %	0.15	1.31	4.55	283.94	-0.000	1.318	-0.156	4.620	0.560
Philippines	0.42 %	0.53 %	0.82 %	9.00 %	2.02	-0.18	-34.55 %	36.04 %	5.00 %	31.18 %	1.88 %	0.06	0.79	2.72	50.69	-0.001	0.937	-0.269	-0.603	0.215
Portugal	0.30 %	0.39 %	0.61 %	6.59 %	1.64	-0.58	-30.45 %	19.69 %	3.55 %	22.84 %	0.43 %	0.02	0.76	2.64	48.40	-0.003	0.984	-0.907	-0.251	0.448
Singapore	0.56 %	0.43 %	0.94 %	7.36 %	3.00	-0.64	-34.24 %	22.99 %	6.72 %	25.49 %	3.60 %	0.14	1.29	4.47	127.84	-0.001	1.170	-0.180	2.498	0.507
South Africa	0.70 %	0.46 %	1.02 %	7.79 %	4.78	-1.22	-43.62 %	18.04 %	8.44 %	26.97 %	5.31 %	0.20	1.53	5.31	345.97	0.001	1.107	0.302	1.350	0.405
South Korea	0.41 %	0.63 %	0.09 %	10.63 %	2.97	0.19	-37.48 %	53.41 %	4.90 %	36.82 %	1.77 %	0.05	0.65	2.26	107.46	-0.003	1.328	-0.489	2.823	0.311
Spain	0.75 %	0.43 %	1.15 %	7.29 %	1.43	-0.54	-29.13 %	19.96 %	8.96 %	25.27 %	5.84 %	0.23	1.74	6.02	38.44	0.001	1.273	0.376	4.592	0.614
Sweden	0.87 %	0.45 %	1.17 %	7.70 %	1.57	-0.57	-31.00 %	22.70 %	10.46 %	26.69 %	7.33 %	0.27	1.92	6.65	44.91	0.002	1.410	0.706	7.098	0.674
Switzerland	0.90 %	0.29 %	1.29 %	4.99 %	1.00	-0.60	-17.00 %	15.43 %	10.83 %	17.29 %	7.70 %	0.45	3.07	10.63	29.54	0.004	0.867	2.054	-3.239	0.608
Taiwan	-0.04 %	0.56 %	0.15 %	9.53 %	2.17	-0.09	-41.05 %	38.14 %	-0.44 %	33.00 %	-3.56 %	-0.11	-0.06	-0.22	57.14	-0.006	1.094	-1.297	0.874	0.262
Thailand	0.31 %	0.66 %	1.05 %	11.15 %	2.31	-0.51	-41.57 %	35.93 %	3.75 %	38.63 %	0.62 %	0.02	0.48	1.65	76.74	-0.003	1.304	-0.610	2.424	0.272
Turkey	0.36 %	0.90 %	0.70 %	15.28 %	1.19	-0.24	-53.18 %	54.41 %	4.31 %	52.93 %	1.19 %	0.02	0.40	1.38	19.71	-0.004	1.537	-0.455	2.995	0.201
UK	0.63 %	0.28 %	0.70 %	4.81 %	1.51	-0.46	-21.02 %	13.89 %	7.53 %	16.65 %	4.41 %	0.26	2.22	7.68	37.72	0.001	0.934	0.610	-2.145	0.763
USA	0.78 %	0.25 %	1.29 %	4.32 %	1.67	-0.80	-18.76 %	10.82 %	9.33 %	14.97 %	6.20 %	0.41	3.05	10.57	63.79	0.003	0.872	2.350	-5.359	0.822
Benchmark	0.56 %	0.26 %	1.21 %	4.49 %	1.97	-0.86	-20.99 %	10.72 %	6.76 %	15.55 %	3.64 %	0.23	2.13	7.38	82.25					
US T-Bill	0.26 %	0.01 %	0.29 %	0.18 %	-1.23	-0.06	0.00 %	0.65 %	3.12 %	0.64 %			24.03	83.23	18.22					

Table 35: Correlation Matrix (1990-2013)

Correlation	*Australia*	*Austria*	*Belgium*	*Brazil*	*Canada*	*Denmark*	*Finland*	*France*	*Germany*	*Hong Kong*	*India*	*Indonesia*	*Ireland*	*Italy*	*Japan*	*Malaysia*	*Mexico*	*Netherlands*	*New Zealand*	*Norway*	*Philippines*	*Portugal*	*Singapore*	*South Africa*	*South Korea*	*Spain*	*Sweden*	*Switzerland*	*Taiwan*	*Thailand*	*Turkey*	*UK*	*USA*
Austria	0.63	1.00																															
Belgium	0.62	0.73	1.00																														
Brazil	0.49	0.36	0.35	1.00																													
Canada	0.75	0.62	0.59	0.48	1.00																												
Denmark	0.47	0.50	0.54	0.35	0.59	1.00																											
Finland	0.56	0.45	0.47	0.37	0.59	0.48	1.00																										
France	0.68	0.71	0.80	0.43	0.68	0.55	0.62	1.00																									
Germany	0.65	0.71	0.75	0.40	0.67	0.59	0.61	0.88	1.00																								
Hong Kong	0.61	0.52	0.46	0.40	0.65	0.45	0.45	0.55	0.55	1.00																							
India	0.38	0.34	0.31	0.30	0.35	0.24	0.22	0.37	0.35	0.31	1.00																						
Indonesia	0.42	0.43	0.38	0.27	0.47	0.31	0.25	0.36	0.37	0.53	0.31	1.00																					
Ireland	0.61	0.64	0.71	0.35	0.57	0.54	0.51	0.67	0.67	0.41	0.23	0.28	1.00																				
Italy	0.55	0.62	0.64	0.36	0.58	0.53	0.58	0.75	0.72	0.43	0.33	0.32	0.58	1.00																			
Japan	0.50	0.42	0.40	0.35	0.46	0.37	0.41	0.48	0.42	0.39	0.16	0.27	0.48	0.42	1.00																		
Malaysia	0.38	0.33	0.27	0.27	0.43	0.26	0.27	0.33	0.39	0.54	0.29	0.48	0.28	0.26	0.24	1.00																	
Mexico	0.56	0.44	0.42	0.48	0.60	0.46	0.46	0.51	0.49	0.56	0.32	0.39	0.43	0.44	0.37	0.39	1.00																
Netherlands	0.70	0.73	0.84	0.44	0.69	0.61	0.60	0.88	0.86	0.56	0.37	0.41	0.73	0.71	0.48	0.37	0.50	1.00															
New Zealand	0.75	0.56	0.53	0.38	0.57	0.40	0.48	0.56	0.55	0.52	0.27	0.46	0.50	0.47	0.48	0.34	0.46	0.60	1.00														
Norway	0.72	0.70	0.71	0.49	0.73	0.61	0.56	0.73	0.69	0.54	0.37	0.42	0.64	0.62	0.43	0.32	0.52	0.76	0.59	1.00													
Philippines	0.47	0.43	0.38	0.31	0.46	0.31	0.26	0.38	0.41	0.58	0.24	0.58	0.30	0.30	0.27	0.52	0.41	0.41	0.47	0.37	1.00												
Portugal	0.54	0.63	0.68	0.43	0.55	0.46	0.51	0.75	0.70	0.44	0.39	0.34	0.58	0.66	0.39	0.23	0.35	0.71	0.51	0.63	0.33	1.00											
Singapore	0.66	0.57	0.55	0.42	0.66	0.46	0.45	0.59	0.60	0.78	0.35	0.63	0.47	0.48	0.47	0.62	0.60	0.62	0.62	0.61	0.65	0.45	1.00										
South Africa	0.70	0.60	0.53	0.39	0.69	0.41	0.42	0.56	0.57	0.55	0.37	0.45	0.49	0.47	0.48	0.42	0.56	0.58	0.57	0.64	0.49	0.51	0.62	1.00									
South Korea	0.54	0.38	0.38	0.24	0.49	0.36	0.44	0.42	0.43	0.44	0.30	0.42	0.44	0.39	0.50	0.34	0.40	0.44	0.46	0.44	0.35	0.32	0.51	0.49	1.00								
Spain	0.64	0.64	0.68	0.48	0.59	0.52	0.57	0.82	0.76	0.52	0.30	0.34	0.63	0.76	0.49	0.29	0.53	0.75	0.56	0.66	0.37	0.76	0.56	0.52	0.40	1.00							
Sweden	0.69	0.57	0.64	0.47	0.68	0.62	0.68	0.78	0.79	0.57	0.35	0.38	0.61	0.66	0.50	0.37	0.51	0.77	0.62	0.70	0.38	0.66	0.61	0.55	0.46	0.74	1.00						
Switzerland	0.60	0.66	0.70	0.41	0.56	0.46	0.49	0.75	0.72	0.48	0.26	0.39	0.61	0.57	0.52	0.27	0.42	0.78	0.58	0.63	0.40	0.66	0.54	0.50	0.40	0.67	0.67	1.00					
Taiwan	0.45	0.44	0.38	0.35	0.47	0.36	0.34	0.46	0.49	0.55	0.25	0.34	0.41	0.37	0.34	0.46	0.40	0.44	0.39	0.42	0.47	0.31	0.56	0.46	0.46	0.41	0.43	0.33	1.00				
Thailand	0.57	0.42	0.39	0.33	0.51	0.31	0.30	0.39	0.43	0.61	0.31	0.57	0.35	0.34	0.39	0.57	0.45	0.42	0.51	0.41	0.66	0.35	0.68	0.58	0.57	0.39	0.41	0.40	0.48	1.00			
Turkey	0.38	0.32	0.33	0.36	0.44	0.32	0.39	0.43	0.43	0.33	0.31	0.28	0.30	0.36	0.26	0.23	0.33	0.41	0.39	0.37	0.27	0.36	0.41	0.38	0.28	0.37	0.48	0.35	0.26	0.29	1.00		
UK	0.73	0.70	0.76	0.44	0.70	0.58	0.60	0.82	0.77	0.59	0.28	0.34	0.74	0.64	0.53	0.34	0.52	0.83	0.58	0.76	0.36	0.65	0.63	0.57	0.45	0.74	0.72	0.76	0.39	0.41	0.39	1.00	
USA	0.68	0.56	0.67	0.46	0.78	0.61	0.64	0.75	0.75	0.59	0.27	0.40	0.68	0.58	0.45	0.38	0.61	0.77	0.54	0.66	0.44	0.55	0.64	0.54	0.48	0.66	0.72	0.66	0.45	0.48	0.41	0.77	1.00
Benchmark	0.77	0.68	0.76	0.52	0.80	0.64	0.69	0.86	0.84	0.64	0.31	0.42	0.75	0.71	0.70	0.42	0.62	0.86	0.65	0.75	0.46	0.67	0.71	0.64	0.56	0.79	0.82	0.78	0.51	0.52	0.45	0.87	0.91

Table 36: P-values from F-test for paired two variances (1990-2013)

P-values	Turkey	Brazil	Indonesia	Thailand	South Korea	India	Taiwan	Finland	Philippines	Mexico	Malaysia	Norway	Austria	South Africa	Sweden	Hong Kong	Singapore	Italy	Spain	Germany	Ireland	Portugal	New Zealand	Belgium	Japan	Australia	France	Netherlands	Canada	Denmark	Switzerland	UK	Benchmark	USA	US T-Bill
Turkey	0.50																																		
Brazil	0.13	0.50																																	
Indonesia	0.00	0.02	0.50																																
Thailand	0.00	0.00	0.02	0.50																															
South Korea	0.00	0.00	0.00	0.21	0.50																														
India	0.00	0.00	0.00	0.01	0.05	0.50																													
Taiwan	0.00	0.00	0.00	0.00	0.03	0.41	0.50																												
Finland	0.00	0.00	0.00	0.00	0.03	0.37	0.46	0.50																											
Philippines	0.00	0.00	0.00	0.00	0.00	0.12	0.17	0.20	0.50																										
Mexico	0.00	0.00	0.00	0.00	0.00	0.12	0.17	0.20	0.50	0.50																									
Malaysia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.05	0.50																								
Norway	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.27	0.50																							
Austria	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.24	0.46	0.50																						
South Africa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.20	0.41	0.45	0.50																					
Sweden	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.34	0.38	0.43	0.50																				
Hong Kong	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.14	0.17	0.20	0.25	0.50																			
Singapore	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.12	0.14	0.17	0.22	0.46	0.50																		
Italy	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.10	0.12	0.14	0.19	0.41	0.45	0.50																	
Spain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.09	0.11	0.13	0.18	0.40	0.44	0.49	0.50																
Germany	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.08	0.09	0.11	0.12	0.50															
Ireland	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.05	0.06	0.34	0.50														
Portugal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.04	0.04	0.30	0.45	0.50													
New Zealand	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.03	0.03	0.24	0.38	0.43	0.50												
Belgium	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.17	0.20	0.25	0.50											
Japan	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.09	0.12	0.15	0.36	0.50										
Australia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.06	0.08	0.10	0.28	0.41	0.50									
France	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.05	0.07	0.21	0.32	0.41	0.50								
Netherlands	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.10	0.18	0.24	0.32	0.50							
Canada	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.12	0.18	0.24	0.41	0.50						
Denmark	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.05	0.11	0.16	0.50					
Switzerland	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.50				
UK	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.26	0.50			
Benchmark	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.12	0.50		
USA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.26	0.50	
US T-Bill	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50

8.2. 1990-1995 (In-Sample)

Table 37: Descriptive Statistics (1990-1995)

	Monthly								Annually				Tests		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test	Annual t-test	Jarqe Bera
Australia	0.74 %	0.63 %	0.43 %	4.90 %	1.60	-0.90	-14.47 %	9.82 %	8.92 %	16.98 %	3.93 %	0.23	1.18	4.07	69.52
Austria	-0.19 %	0.74 %	0.62 %	5.74 %	1.11	-0.76	-19.39 %	11.66 %	-2.30 %	19.88 %	-7.29 %	-0.37	-0.26	-0.89	42.52
Belgium	1.12 %	0.62 %	1.53 %	4.83 %	5.91	-1.62	-20.61 %	10.74 %	13.47 %	16.73 %	8.47 %	0.51	1.80	6.24	544.02
Brazil	0.73 %	1.66 %	2.88 %	12.84 %	3.17	-1.14	-47.21 %	31.32 %	8.75 %	44.49 %	3.75 %	0.08	0.44	1.52	182.83
Canada	1.65 %	0.72 %	1.89 %	5.54 %	7.37	-1.67	-24.55 %	13.56 %	19.75 %	19.20 %	14.75 %	0.77	2.30	7.97	785.90
Denmark	1.76 %	0.67 %	2.06 %	5.21 %	0.49	-0.39	-12.79 %	13.10 %	21.14 %	18.06 %	16.15 %	0.89	2.62	9.07	10.18
Finland	3.75 %	1.18 %	3.70 %	9.16 %	0.50	-0.18	-21.68 %	26.62 %	45.03 %	31.75 %	40.03 %	1.26	3.17	10.99	4.56
France	1.70 %	0.63 %	1.99 %	4.85 %	0.69	-0.36	-12.14 %	12.42 %	20.36 %	16.79 %	15.37 %	0.92	2.71	9.39	12.03
Germany	1.57 %	0.67 %	2.35 %	5.20 %	3.13	-0.92	-17.91 %	15.83 %	18.89 %	18.00 %	13.89 %	0.77	2.35	8.13	157.98
Hong Kong	1.18 %	1.25 %	0.25 %	9.66 %	2.71	-0.32	-34.06 %	28.69 %	14.15 %	33.48 %	9.16 %	0.27	0.95	3.27	92.72
India	1.14 %	0.98 %	0.75 %	7.58 %	-0.42	0.19	-14.14 %	17.65 %	13.64 %	26.26 %	8.64 %	0.33	1.16	4.02	3.93
Indonesia	-1.30 %	2.48 %	-1.16 %	19.21 %	0.87	-0.21	-51.99 %	44.20 %	-15.63 %	66.56 %	-20.62 %	-0.31	-0.52	-1.82	11.24
Ireland	1.21 %	0.56 %	1.73 %	4.31 %	3.73	-1.19	-15.09 %	10.59 %	14.56 %	14.95 %	9.56 %	0.64	2.18	7.55	234.68
Italy	1.27 %	0.88 %	0.33 %	6.79 %	-0.30	0.18	-15.24 %	17.96 %	15.26 %	23.51 %	10.26 %	0.44	1.45	5.03	2.70
Japan	0.20 %	0.80 %	0.07 %	6.23 %	-0.54	0.21	-12.09 %	15.52 %	2.42 %	21.56 %	-2.58 %	-0.12	0.25	0.87	5.55
Malaysia	-0.23 %	1.79 %	0.36 %	13.90 %	1.15	-0.01	-35.76 %	37.94 %	-2.77 %	48.14 %	-7.76 %	-0.16	-0.13	-0.45	15.75
Mexico	1.16 %	1.43 %	3.00 %	11.06 %	3.06	-1.34	-41.93 %	17.51 %	13.89 %	38.31 %	8.89 %	0.23	0.81	2.81	197.80
Netherlands	1.45 %	0.64 %	2.41 %	4.92 %	1.36	-1.04	-13.07 %	9.77 %	17.42 %	17.04 %	12.42 %	0.73	2.29	7.92	73.91
New Zealand	-0.06 %	0.87 %	0.76 %	6.71 %	1.90	-0.81	-22.36 %	14.78 %	-0.69 %	23.23 %	-5.69 %	-0.24	-0.07	-0.23	74.68
Norway	0.37 %	0.90 %	1.51 %	7.01 %	7.04	-1.58	-32.49 %	15.58 %	4.43 %	24.27 %	-0.57 %	-0.02	0.41	1.41	714.85
Philippines	-1.21 %	1.49 %	-1.35 %	11.58 %	2.35	0.12	-34.55 %	36.04 %	-14.49 %	40.11 %	-19.49 %	-0.49	-0.81	-2.80	66.88
Portugal	1.57 %	0.75 %	1.80 %	5.83 %	0.18	-0.23	-14.65 %	15.29 %	18.84 %	20.21 %	13.85 %	0.69	2.08	7.22	2.84
Singapore	0.21 %	1.21 %	0.13 %	9.34 %	0.95	-0.06	-22.01 %	22.99 %	2.52 %	32.35 %	-2.47 %	-0.08	0.17	0.60	10.96
South Africa	0.48 %	1.18 %	0.76 %	9.11 %	9.00	-2.06	-43.62 %	18.04 %	5.78 %	31.57 %	0.78 %	0.02	0.41	1.42	1174.72
South Korea	0.05 %	2.00 %	-1.50 %	15.47 %	2.36	0.48	-37.48 %	53.41 %	0.63 %	53.59 %	-4.37 %	-0.08	0.03	0.09	77.80
Spain	2.06 %	0.81 %	2.50 %	6.25 %	4.37	-1.10	-24.34 %	15.02 %	24.75 %	21.64 %	19.75 %	0.91	2.56	8.86	287.45
Sweden	2.45 %	0.78 %	2.46 %	6.02 %	0.98	-0.47	-16.55 %	16.98 %	29.40 %	20.86 %	24.41 %	1.17	3.15	10.92	22.10
Switzerland	1.35 %	0.69 %	1.89 %	5.34 %	1.89	-0.84	-17.00 %	13.59 %	16.19 %	18.49 %	11.19 %	0.61	1.96	6.78	76.61
Taiwan	0.67 %	1.20 %	0.98 %	9.26 %	0.24	0.00	-24.51 %	22.31 %	8.02 %	32.07 %	3.03 %	0.09	0.56	1.94	0.68
Thailand	-2.04 %	2.06 %	-1.70 %	15.98 %	0.80	-0.09	-41.57 %	35.93 %	-24.54 %	55.36 %	-29.53 %	-0.53	-0.99	-3.43	8.10
Turkey	2.93 %	2.11 %	2.86 %	16.38 %	2.34	0.21	-49.68 %	54.41 %	35.10 %	56.73 %	30.10 %	0.53	1.38	4.79	67.78
UK	1.40 %	0.45 %	1.45 %	3.45 %	1.18	-0.73	-9.60 %	8.32 %	16.79 %	11.95 %	11.79 %	0.99	3.14	10.89	42.11
USA	2.03 %	0.54 %	2.66 %	4.19 %	3.26	-1.38	-14.97 %	7.63 %	24.39 %	14.50 %	19.40 %	1.34	3.76	13.03	218.70
Benchmark	1.46 %	0.50 %	1.77 %	3.85 %	3.80	-1.34	-14.30 %	8.67 %	17.56 %	13.33 %	12.56 %	0.94	2.95	10.21	259.40
US T-Bill	0.42 %	0.00 %	0.42 %	0.03 %	0.16	-0.18	0.35 %	0.48 %	5.00 %	0.10 %			110.98	384.46	1.83

Table 38: Correlation Matrix (1990-1995)

Correaltion	Australia	Austria	Belgium	Brazil	Canada	Denmark	Finland	France	Germany	Hong Kong	India	Indonesia	Ireland	Italy	Japan	Malaysia	Mexico	Netherlands	New Zealand	Norway	Philippines	Portugal	Singapore	South Africa	South Korea	Spain	Sweden	Switzerland	Taiwan	Thailand	Turkey	UK	USA	Benchmark
Austria	0.21	1.00																																
Belgium	0.30	0.44	1.00																															
Brazil	0.16	0.02	-0.01	1.00																														
Canada	0.62	0.29	0.29	0.11	1.00																													
Denmark	0.36	0.38	0.44	0.01	0.38	1.00																												
Finland	0.44	0.33	0.28	0.16	0.53	0.50	1.00																											
France	0.36	0.53	0.74	0.14	0.26	0.36	0.29	1.00																										
Germany	0.29	0.72	0.72	-0.04	0.32	0.43	0.37	0.76	1.00																									
Hong Kong	0.42	0.45	0.35	0.08	0.59	0.39	0.47	0.44	0.45	1.00																								
India	0.13	0.15	0.03	0.10	-0.07	-0.02	-0.08	0.14	0.06	0.03	1.00																							
Indonesia	0.11	0.39	0.21	-0.01	0.40	0.17	0.32	0.13	0.24	0.51	0.09	1.00																						
Ireland	0.35	0.43	0.72	0.11	0.42	0.52	0.46	0.64	0.61	0.32	0.08	0.15	1.00																					
Italy	0.12	0.33	0.32	0.03	0.36	0.46	0.48	0.36	0.48	0.34	0.00	0.23	0.44	1.00																				
Japan	0.29	0.27	0.37	0.25	0.28	0.33	0.40	0.40	0.34	0.18	-0.18	-0.06	0.55	0.40	1.00																			
Malaysia	0.30	0.48	0.33	0.04	0.42	0.26	0.39	0.32	0.44	0.68	0.03	0.39	0.29	0.22	0.26	1.00																		
Mexico	0.28	0.12	0.13	0.16	0.39	0.26	0.27	0.27	0.11	0.49	0.14	0.25	0.19	0.07	0.14	0.39	1.00																	
Netherlands	0.50	0.53	0.78	0.05	0.44	0.41	0.39	0.78	0.78	0.42	0.09	0.27	0.75	0.32	0.43	0.38	0.11	1.00																
New Zealand	0.77	0.25	0.33	0.19	0.50	0.41	0.54	0.40	0.40	0.38	0.01	0.21	0.38	0.26	0.41	0.36	0.27	0.53	1.00															
Norway	0.50	0.35	0.46	0.23	0.44	0.53	0.56	0.51	0.46	0.36	0.11	0.21	0.55	0.29	0.28	0.26	0.16	0.59	0.44	1.00														
Philippines	0.37	0.44	0.41	0.14	0.47	0.25	0.30	0.40	0.49	0.69	-0.02	0.47	0.26	0.27	0.13	0.62	0.23	0.40	0.36	0.19	1.00													
Portugal	0.34	0.44	0.52	0.32	0.30	0.33	0.31	0.61	0.55	0.29	0.20	0.27	0.60	0.39	0.40	0.15	0.04	0.65	0.36	0.48	0.25	1.00												
Singapore	0.43	0.58	0.52	0.09	0.53	0.51	0.55	0.50	0.59	0.70	0.01	0.44	0.53	0.44	0.46	0.84	0.40	0.58	0.51	0.41	0.67	0.39	1.00											
South Africa	0.44	0.30	0.41	-0.11	0.42	0.44	0.27	0.28	0.45	0.39	0.09	0.22	0.41	0.25	0.27	0.46	0.21	0.46	0.38	0.26	0.42	0.32	0.51	1.00										
South Korea	0.15	0.10	0.13	-0.14	0.37	0.27	0.41	0.06	0.12	0.24	0.09	0.05	0.47	0.23	0.34	0.28	0.32	0.17	0.11	0.17	0.09	0.00	0.40	0.32	1.00									
Spain	0.37	0.45	0.53	0.30	0.33	0.55	0.49	0.69	0.60	0.46	0.01	0.13	0.63	0.51	0.56	0.24	0.38	0.59	0.50	0.52	0.29	0.64	0.54	0.23	0.09	1.00								
Sweden	0.53	0.36	0.46	0.25	0.43	0.52	0.60	0.56	0.55	0.46	0.04	0.19	0.56	0.48	0.55	0.37	0.24	0.55	0.66	0.57	0.33	0.60	0.63	0.36	0.20	0.75	1.00							
Switzerland	0.40	0.56	0.60	0.28	0.42	0.35	0.32	0.61	0.58	0.43	0.05	0.29	0.63	0.26	0.55	0.39	0.24	0.70	0.45	0.46	0.37	0.57	0.60	0.30	0.24	0.56	0.60	1.00						
Taiwan	0.22	0.44	0.35	0.13	0.34	0.27	0.35	0.37	0.43	0.42	-0.04	0.20	0.36	0.23	0.24	0.44	0.20	0.32	0.19	0.18	0.56	0.14	0.52	0.42	0.31	0.27	0.21	0.19	1.00					
Thailand	0.34	0.45	0.47	0.05	0.40	0.26	0.31	0.33	0.40	0.66	0.16	0.47	0.32	0.29	0.19	0.66	0.32	0.35	0.30	0.19	0.64	0.28	0.70	0.36	0.17	0.32	0.39	0.44	0.37	1.00				
Turkey	-0.01	0.14	0.08	0.17	0.06	0.02	-0.04	0.07	0.04	0.15	0.17	0.29	0.14	0.02	0.02	0.19	-0.09	0.06	0.12	-0.01	0.20	0.19	0.22	-0.03	-0.04	0.07	0.14	0.20	0.03	0.29	1.00			
UK	0.54	0.45	0.60	0.14	0.45	0.46	0.48	0.67	0.63	0.44	0.01	0.09	0.73	0.27	0.48	0.40	0.27	0.78	0.47	0.60	0.28	0.50	0.58	0.42	0.37	0.62	0.59	0.73	0.19	0.28	0.04	1.00		
USA	0.43	0.29	0.52	0.21	0.56	0.35	0.39	0.52	0.44	0.40	-0.08	0.23	0.54	0.21	0.24	0.36	0.35	0.59	0.40	0.55	0.32	0.37	0.53	0.15	0.19	0.47	0.51	0.58	0.21	0.36	-0.05	0.64	1.00	
Benchmark	0.51	0.46	0.64	0.27	0.52	0.49	0.55	0.68	0.62	0.46	-0.10	0.12	0.76	0.46	0.84	0.47	0.31	0.73	0.57	0.55	0.37	0.57	0.70	0.38	0.37	0.74	0.74	0.78	0.35	0.41	0.05	0.79	0.67	1.00
US T-Bill	-0.10	-0.09	-0.15	-0.16	-0.05	-0.07	-0.29	-0.09	-0.11	-0.10	0.05	-0.16	-0.16	-0.13	-0.21	-0.19	0.04	-0.13	-0.25	-0.07	-0.31	-0.18	-0.22	-0.09	-0.22	-0.02	-0.09	-0.19	-0.29	-0.22	-0.09	0.02	0.01	-0.17

8.3. 1995-2000

Table 39: Descriptive Statistics (1995-2000)

	Monthly								Annually				Tests		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test	Annual t-test	Jarqe Bera
Australia	0.74 %	0.63 %	0.43 %	4.90 %	1.60	-0.90	-14.47 %	9.82 %	8.92 %	16.98 %	3.93 %	0.23	1.18	4.07	69.52
Austria	-0.19 %	0.74 %	0.62 %	5.74 %	1.11	-0.76	-19.39 %	11.66 %	-2.30 %	19.88 %	-7.29 %	-0.37	-0.26	-0.89	42.52
Belgium	1.12 %	0.62 %	1.53 %	4.83 %	5.91	-1.62	-20.61 %	10.74 %	13.47 %	16.73 %	8.47 %	0.51	1.80	6.24	544.02
Brazil	0.73 %	1.66 %	2.88 %	12.84 %	3.17	-1.14	-47.21 %	31.32 %	8.75 %	44.49 %	3.75 %	0.08	0.44	1.52	182.83
Canada	1.65 %	0.72 %	1.89 %	5.54 %	7.37	-1.67	-24.55 %	13.56 %	19.75 %	19.20 %	14.75 %	0.77	2.30	7.97	785.90
Denmark	1.76 %	0.67 %	2.06 %	5.21 %	0.49	-0.39	-12.79 %	13.10 %	21.14 %	18.06 %	16.15 %	0.89	2.62	9.07	10.18
Finland	3.75 %	1.18 %	3.70 %	9.16 %	0.50	-0.18	-21.68 %	26.62 %	45.03 %	31.75 %	40.03 %	1.26	3.17	10.99	4.56
France	1.70 %	0.63 %	1.99 %	4.85 %	0.69	-0.36	-12.14 %	12.42 %	20.36 %	16.79 %	15.37 %	0.92	2.71	9.39	12.03
Germany	1.57 %	0.67 %	2.35 %	5.20 %	3.13	-0.92	-17.91 %	15.83 %	18.89 %	18.00 %	13.89 %	0.77	2.35	8.13	157.98
Hong Kong	1.18 %	1.25 %	0.25 %	9.66 %	2.71	-0.32	-34.06 %	28.69 %	14.15 %	33.48 %	9.16 %	0.27	0.95	3.27	92.72
India	1.14 %	0.98 %	0.75 %	7.58 %	-0.42	0.19	-14.14 %	17.65 %	13.64 %	26.26 %	8.64 %	0.33	1.16	4.02	3.93
Indonesia	-1.30 %	2.48 %	-1.16 %	19.21 %	0.87	-0.21	-51.99 %	44.20 %	-15.63 %	66.56 %	-20.62 %	-0.31	-0.52	-1.82	11.24
Ireland	1.21 %	0.56 %	1.73 %	4.31 %	3.73	-1.19	-15.09 %	10.59 %	14.56 %	14.95 %	9.56 %	0.64	2.18	7.55	234.68
Italy	1.27 %	0.88 %	0.33 %	6.79 %	-0.30	0.18	-15.24 %	17.96 %	15.26 %	23.51 %	10.26 %	0.44	1.45	5.03	2.70
Japan	0.20 %	0.80 %	0.07 %	6.23 %	-0.54	0.21	-12.09 %	15.52 %	2.42 %	21.56 %	-2.58 %	-0.12	0.25	0.87	5.55
Malaysia	-0.23 %	1.79 %	0.36 %	13.90 %	1.15	-0.01	-35.76 %	37.94 %	-2.77 %	48.14 %	-7.76 %	-0.16	-0.13	-0.45	15.75
Mexico	1.16 %	1.43 %	3.00 %	11.06 %	3.06	-1.34	-41.93 %	17.51 %	13.89 %	38.31 %	8.89 %	0.23	0.81	2.81	197.80
Netherlands	1.45 %	0.64 %	2.41 %	4.92 %	1.36	-1.04	-13.07 %	9.77 %	17.42 %	17.04 %	12.42 %	0.73	2.29	7.92	73.91
New Zealand	-0.06 %	0.87 %	0.76 %	6.71 %	1.90	-0.81	-22.36 %	14.78 %	-0.69 %	23.23 %	-5.69 %	-0.24	-0.07	-0.23	74.68
Norway	0.37 %	0.90 %	1.51 %	7.01 %	7.04	-1.58	-32.49 %	15.58 %	4.43 %	24.27 %	-0.57 %	-0.02	0.41	1.41	714.85
Philippines	-1.21 %	1.49 %	-1.35 %	11.58 %	2.35	0.12	-34.55 %	36.04 %	-14.49 %	40.11 %	-19.49 %	-0.49	-0.81	-2.80	66.88
Portugal	1.57 %	0.75 %	1.80 %	5.83 %	0.18	-0.23	-14.65 %	15.29 %	18.84 %	20.21 %	13.85 %	0.69	2.08	7.22	2.84
Singapore	0.21 %	1.21 %	0.13 %	9.34 %	0.95	-0.06	-22.01 %	22.99 %	2.52 %	32.35 %	-2.47 %	-0.08	0.17	0.60	10.96
South Africa	0.48 %	1.18 %	0.76 %	9.11 %	9.00	-2.06	-43.62 %	18.04 %	5.78 %	31.57 %	0.78 %	0.02	0.41	1.42	1174.72
South Korea	0.05 %	2.00 %	-1.50 %	15.47 %	2.36	0.48	-37.48 %	53.41 %	0.63 %	53.59 %	-4.37 %	-0.08	0.03	0.09	77.80
Spain	2.06 %	0.81 %	2.50 %	6.25 %	4.37	-1.10	-24.34 %	15.02 %	24.75 %	21.64 %	19.75 %	0.91	2.56	8.86	287.45
Sweden	2.45 %	0.78 %	2.46 %	6.02 %	0.98	-0.47	-16.55 %	16.98 %	29.40 %	20.86 %	24.41 %	1.17	3.15	10.92	22.10
Switzerland	1.35 %	0.69 %	1.89 %	5.34 %	1.89	-0.84	-17.00 %	13.59 %	16.19 %	18.49 %	11.19 %	0.61	1.96	6.78	76.61
Taiwan	0.67 %	1.20 %	0.98 %	9.26 %	0.24	0.00	-24.51 %	22.31 %	8.02 %	32.07 %	3.03 %	0.09	0.56	1.94	0.68
Thailand	-2.04 %	2.06 %	-1.70 %	15.98 %	0.80	-0.09	-41.57 %	35.93 %	-24.54 %	55.36 %	-29.53 %	-0.53	-0.99	-3.43	8.10
Turkey	2.93 %	2.11 %	2.86 %	16.38 %	2.34	0.21	-49.68 %	54.41 %	35.10 %	56.73 %	30.10 %	0.53	1.38	4.79	67.78
UK	1.40 %	0.45 %	1.45 %	3.45 %	1.18	-0.73	-9.60 %	8.32 %	16.79 %	11.95 %	11.79 %	0.99	3.14	10.89	42.11
USA	2.03 %	0.54 %	2.66 %	4.19 %	3.26	-1.38	-14.97 %	7.63 %	24.39 %	14.50 %	19.40 %	1.34	3.76	13.03	218.70
Benchmark	1.46 %	0.50 %	1.77 %	3.85 %	3.80	-1.34	-14.30 %	8.67 %	17.56 %	13.33 %	12.56 %	0.94	2.95	10.21	259.40
US T-Bill	0.42 %	0.00 %	0.42 %	0.03 %	0.16	-0.18	0.35 %	0.48 %	5.00 %	0.10 %			110.98	384.46	1.83

Table 40: Correlation Matrix (1995-2000)

Correlation	Australia	Austria	Belgium	Brazil	Canada	Denmark	Finland	France	Germany	Hong Kong	India	Indonesia	Ireland	Italy	Japan	Malaysia
Austria	0.40	1.00														
Belgium	0.27	0.56	1.00													
Brazil	0.55	0.34	0.14	1.00												
Canada	0.66	0.54	0.28	0.50	1.00											
Denmark	0.45	0.50	0.39	0.49	0.56	1.00										
Finland	0.50	0.43	0.33	0.45	0.57	0.37	1.00									
France	0.46	0.57	0.66	0.34	0.61	0.55	0.46	1.00								
Germany	0.56	0.63	0.59	0.45	0.64	0.63	0.60	0.73	1.00							
Hong Kong	0.60	0.37	0.15	0.50	0.59	0.38	0.32	0.37	0.38	1.00						
India	0.02	-0.05	-0.12	0.17	0.13	0.11	0.19	0.08	-0.03	0.02	1.00					
Indonesia	0.37	0.29	0.19	0.41	0.48	0.30	0.33	0.41	0.34	0.53	0.15	1.00				
Ireland	0.57	0.64	0.53	0.28	0.57	0.37	0.47	0.48	0.53	0.34	-0.06	0.14	1.00			
Italy	0.39	0.44	0.49	0.50	0.38	0.62	0.40	0.60	0.48	0.14	0.21	0.22	0.39	1.00		
Japan	0.60	0.29	0.26	0.34	0.43	0.19	0.39	0.46	0.30	0.42	0.07	0.34	0.33	0.25	1.00	
Malaysia	0.38	0.09	-0.02	0.34	0.49	0.16	0.26	0.22	0.31	0.46	0.37	0.46	0.26	0.07	0.16	1.00
Mexico	0.57	0.35	0.21	0.73	0.59	0.49	0.53	0.37	0.42	0.49	0.17	0.30	0.42	0.47	0.39	0.30
Netherlands	0.46	0.76	0.74	0.44	0.55	0.56	0.47	0.74	0.74	0.45	0.01	0.40	0.66	0.50	0.44	0.29
New Zealand	0.65	0.53	0.40	0.39	0.59	0.40	0.42	0.51	0.53	0.46	-0.04	0.50	0.55	0.35	0.48	0.24
Norway	0.54	0.55	0.35	0.48	0.69	0.50	0.52	0.54	0.51	0.35	0.20	0.37	0.53	0.48	0.38	0.18
Philippines	0.54	0.46	0.26	0.46	0.62	0.36	0.36	0.46	0.46	0.64	0.07	0.63	0.44	0.33	0.36	0.55
Portugal	0.26	0.57	0.49	0.23	0.44	0.47	0.40	0.63	0.55	0.20	0.16	0.30	0.45	0.44	0.16	0.05
Singapore	0.59	0.31	0.19	0.54	0.64	0.34	0.38	0.45	0.40	0.81	0.15	0.68	0.36	0.24	0.45	0.58
South Africa	0.62	0.48	0.18	0.58	0.70	0.34	0.43	0.44	0.50	0.49	0.11	0.37	0.49	0.32	0.45	0.40
South Korea	0.46	0.15	0.12	0.18	0.25	0.15	0.39	0.24	0.15	0.30	0.05	0.38	0.27	0.22	0.57	0.25
Spain	0.59	0.54	0.56	0.51	0.54	0.56	0.47	0.65	0.61	0.38	0.07	0.37	0.56	0.64	0.36	0.20
Sweden	0.54	0.49	0.32	0.48	0.63	0.50	0.70	0.62	0.73	0.38	0.11	0.45	0.46	0.43	0.41	0.30
Switzerland	0.34	0.58	0.62	0.28	0.42	0.43	0.31	0.64	0.58	0.27	0.00	0.33	0.52	0.38	0.37	0.07
Taiwan	0.40	0.17	-0.07	0.45	0.40	0.16	0.18	0.26	0.26	0.55	0.29	0.37	0.19	0.21	0.35	0.47
Thailand	0.64	0.33	0.13	0.42	0.52	0.27	0.44	0.37	0.44	0.67	0.05	0.55	0.37	0.29	0.44	0.58
Turkey	0.26	0.36	0.16	0.45	0.43	0.27	0.34	0.42	0.34	0.13	0.12	0.20	0.28	0.34	0.22	0.12
UK	0.61	0.65	0.62	0.42	0.57	0.48	0.43	0.66	0.56	0.51	-0.12	0.33	0.69	0.44	0.46	0.18
USA	0.62	0.43	0.51	0.51	0.80	0.51	0.59	0.59	0.64	0.58	0.02	0.46	0.61	0.46	0.41	0.39
Benchmark	0.74	0.57	0.60	0.56	0.81	0.56	0.63	0.77	0.74	0.61	0.05	0.50	0.67	0.55	0.67	0.38
US T-Bill	-0.10	0.07	0.02	0.00	-0.02	0.05	-0.08	0.00	0.12	-0.14	-0.13	-0.17	0.02	-0.06	-0.26	-0.08

Correlation	Mexico	Netherlands	New Zealand	Norway	Philippines	Portugal	Singapore	South Africa	South Korea	Spain	Sweden	Switzerland	Taiwan	Thailand	Turkey	UK	USA	Benchmark
Mexico	1.00																	
Netherlands	0.43	1.00																
New Zealand	0.44	0.57	1.00															
Norway	0.60	0.50	0.63	1.00														
Philippines	0.46	0.48	0.50	0.44	1.00													
Portugal	0.21	0.53	0.47	0.50	0.38	1.00												
Singapore	0.58	0.45	0.58	0.49	0.73	0.24	1.00											
South Africa	0.66	0.39	0.60	0.61	0.62	0.44	0.62	1.00										
South Korea	0.16	0.18	0.43	0.21	0.29	0.18	0.32	0.36	1.00									
Spain	0.48	0.62	0.55	0.62	0.50	0.61	0.49	0.49	0.26	1.00								
Sweden	0.43	0.63	0.57	0.61	0.45	0.49	0.47	0.45	0.28	0.61	1.00							
Switzerland	0.24	0.73	0.58	0.48	0.38	0.65	0.33	0.35	0.18	0.63	0.52	1.00						
Taiwan	0.38	0.22	0.27	0.27	0.46	0.09	0.55	0.50	0.28	0.28	0.31	0.15	1.00					
Thailand	0.44	0.36	0.50	0.31	0.71	0.29	0.68	0.64	0.62	0.39	0.41	0.26	0.51	1.00				
Turkey	0.38	0.33	0.43	0.42	0.20	0.38	0.26	0.38	0.05	0.23	0.41	0.28	0.11	0.10	1.00			
UK	0.45	0.79	0.59	0.51	0.40	0.45	0.49	0.40	0.24	0.63	0.53	0.59	0.21	0.39	0.32	1.00		
USA	0.62	0.62	0.52	0.52	0.57	0.40	0.62	0.59	0.29	0.61	0.56	0.51	0.39	0.55	0.30	0.62	1.00	
Benchmark	0.64	0.78	0.65	0.62	0.61	0.49	0.66	0.64	0.40	0.72	0.68	0.64	0.43	0.59	0.37	0.76	0.91	1.00
US T-Bill	-0.14	0.05	-0.04	-0.06	-0.20	0.16	-0.28	-0.05	-0.24	0.06	0.06	0.14	-0.07	-0.18	0.09	0.03	-0.07	-0.10

8.4. 2000-2005

Table 41: Descriptive Statistics (2000-2005)

	Monthly								Annually				Tests		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test	Annual t-test	Jarqe Bera
Australia	1.11 %	0.67 %	1.14 %	5.19 %	0.46	-0.30	-13.26 %	13.63 %	13.26 %	17.96 %	10.69 %	0.60	1.65	5.72	6.86
Austria	1.76 %	0.63 %	1.70 %	4.91 %	-0.08	-0.31	-9.90 %	12.46 %	21.07 %	17.02 %	18.50 %	1.09	2.77	9.59	4.63
Belgium	0.72 %	0.78 %	1.59 %	6.05 %	1.52	-0.64	-18.15 %	16.71 %	8.66 %	20.97 %	6.08 %	0.29	0.92	3.20	47.54
Brazil	0.70 %	1.51 %	1.77 %	11.71 %	1.55	-0.84	-36.04 %	25.09 %	8.45 %	40.56 %	5.87 %	0.14	0.47	1.61	62.66
Canada	0.49 %	0.77 %	1.80 %	5.96 %	0.43	-0.79	-17.35 %	11.85 %	5.91 %	20.63 %	3.34 %	0.16	0.64	2.22	32.59
Denmark	0.22 %	0.74 %	0.97 %	5.74 %	0.12	-0.28	-15.11 %	12.03 %	2.67 %	19.89 %	0.10 %	0.00	0.30	1.04	4.05
Finland	-1.27 %	1.55 %	-0.87 %	12.00 %	1.05	-0.37	-38.21 %	28.72 %	-15.21 %	41.55 %	-17.78 %	-0.43	-0.82	-2.84	19.85
France	0.08 %	0.77 %	0.51 %	5.98 %	0.84	-0.37	-16.65 %	14.62 %	0.93 %	20.71 %	-1.64 %	-0.08	0.10	0.35	14.99
Germany	-0.27 %	1.02 %	-0.44 %	7.93 %	2.23	-0.57	-27.91 %	21.26 %	-3.27 %	27.48 %	-5.84 %	-0.21	-0.27	-0.92	75.11
Hong Kong	0.04 %	0.80 %	0.18 %	6.22 %	-0.23	-0.27	-16.77 %	12.36 %	0.42 %	21.54 %	-2.15 %	-0.10	0.04	0.15	4.16
India	0.65 %	1.12 %	1.97 %	8.70 %	0.40	-0.68	-22.59 %	16.55 %	7.79 %	30.13 %	5.22 %	0.17	0.58	2.00	24.22
Indonesia	0.82 %	1.49 %	1.19 %	11.53 %	-0.23	-0.37	-29.96 %	20.67 %	9.78 %	39.94 %	7.21 %	0.18	0.55	1.90	7.37
Ireland	0.48 %	0.78 %	1.80 %	6.05 %	0.44	-0.79	-14.85 %	11.78 %	5.72 %	20.95 %	3.14 %	0.15	0.61	2.11	32.34
Italy	0.46 %	0.77 %	1.34 %	5.99 %	0.28	-0.43	-14.83 %	12.82 %	5.48 %	20.76 %	2.91 %	0.14	0.59	2.05	9.99
Japan	-0.51 %	0.72 %	-1.13 %	5.60 %	-0.86	0.20	-12.22 %	12.63 %	-6.10 %	19.40 %	-8.68 %	-0.45	-0.70	-2.44	10.65
Malaysia	0.17 %	0.70 %	0.28 %	5.40 %	-0.30	-0.36	-11.90 %	10.76 %	2.00 %	18.70 %	-0.57 %	-0.03	0.24	0.83	7.24
Mexico	0.97 %	0.90 %	2.07 %	6.94 %	-0.26	-0.37	-16.94 %	14.05 %	11.69 %	24.04 %	9.12 %	0.38	1.09	3.77	7.37
Netherlands	-0.01 %	0.80 %	0.09 %	6.20 %	2.25	-0.98	-19.60 %	12.34 %	-0.06 %	21.46 %	-2.64 %	-0.12	-0.01	-0.02	106.89
New Zealand	1.34 %	0.76 %	2.44 %	5.91 %	0.93	-0.84	-18.15 %	11.62 %	16.11 %	20.48 %	13.54 %	0.66	1.76	6.10	44.40
Norway	1.15 %	0.80 %	0.78 %	6.21 %	0.55	-0.29	-16.03 %	14.99 %	13.84 %	21.51 %	11.27 %	0.52	1.44	4.98	7.79
Philippines	-0.58 %	1.09 %	-0.86 %	8.46 %	-0.42	0.03	-18.83 %	17.17 %	-6.97 %	29.30 %	-9.54 %	-0.33	-0.53	-1.84	2.17
Portugal	0.21 %	0.83 %	0.55 %	6.44 %	1.07	-0.68	-21.51 %	12.08 %	2.50 %	22.32 %	-0.07 %	0.00	0.25	0.87	35.80
Singapore	-0.05 %	0.87 %	0.66 %	6.77 %	2.96	-1.28	-22.86 %	12.67 %	-0.56 %	23.46 %	-3.13 %	-0.13	-0.05	-0.18	183.70
South Africa	1.17 %	0.95 %	2.84 %	7.38 %	-0.56	-0.51	-15.36 %	13.64 %	13.99 %	25.57 %	11.42 %	0.45	1.22	4.24	16.13
South Korea	0.61 %	1.31 %	1.20 %	10.12 %	-0.43	-0.06	-20.96 %	24.31 %	7.31 %	35.05 %	4.73 %	0.14	0.47	1.62	2.42
Spain	0.55 %	0.83 %	1.07 %	6.40 %	0.22	-0.29	-17.66 %	13.83 %	6.66 %	22.17 %	4.08 %	0.18	0.67	2.33	4.50
Sweden	-0.23 %	1.18 %	0.43 %	9.17 %	0.51	-0.24	-25.15 %	20.63 %	-2.75 %	31.77 %	-5.32 %	-0.17	-0.19	-0.67	5.78
Switzerland	0.42 %	0.60 %	0.51 %	4.62 %	0.40	-0.29	-12.81 %	11.16 %	5.00 %	16.00 %	2.42 %	0.15	0.70	2.42	6.04
Taiwan	-0.85 %	1.18 %	-2.50 %	9.12 %	0.60	0.26	-24.43 %	25.65 %	-10.22 %	31.60 %	-12.79 %	-0.40	-0.72	-2.50	7.57
Thailand	0.71 %	1.42 %	1.24 %	11.00 %	0.90	-0.33	-25.97 %	27.02 %	8.47 %	38.11 %	5.90 %	0.15	0.50	1.72	15.04
Turkey	-0.40 %	2.36 %	1.41 %	18.31 %	0.40	-0.49	-53.18 %	37.07 %	-4.86 %	63.42 %	-7.43 %	-0.12	-0.17	-0.59	13.44
UK	0.18 %	0.55 %	0.06 %	4.27 %	-0.08	-0.05	-10.93 %	9.90 %	2.13 %	14.81 %	-0.44 %	-0.03	0.32	1.11	0.21
USA	-0.22 %	0.61 %	0.49 %	4.75 %	-0.17	-0.20	-11.97 %	9.52 %	-2.65 %	16.44 %	-5.22 %	-0.32	-0.36	-1.25	2.34
Benchmark	-0.11 %	0.58 %	0.38 %	4.49 %	-0.19	-0.35	-11.63 %	8.55 %	-1.35 %	15.56 %	-3.92 %	-0.25	-0.19	-0.67	6.33
US T-Bill	0.21 %	0.02 %	0.14 %	0.15 %	-0.67	0.96	0.07 %	0.52 %	2.57 %	0.53 %			10.89	37.71	49.56

Table 42: Correlation Matrix (2000-2005)

Correlation	*Australia*	*Austria*	*Belgium*	*Brazil*	*Canada*	*Denmark*	*Finland*	*France*	*Germany*	*Hong Kong*	*India*	*Indonesia*	*Ireland*	*Italy*	*Japan*	*Malaysia*	*Mexico*	*Netherlands*	*New Zealand*	*Norway*	*Philippines*	*Portugal*	*Singapore*	*South Africa*	*South Korea*	*Spain*	*Sweden*	*Switzerland*	*Taiwan*	*Thailand*	*Turkey*	*UK*	*USA*	*Benchmark*
Austria	0.44	1.00																																
Belgium	0.56	0.69	1.00																															
Brazil	0.67	0.53	0.67	1.00																														
Canada	0.71	0.42	0.51	0.68	1.00																													
Denmark	0.40	0.24	0.44	0.64	0.61	1.00																												
Finland	0.45	0.06	0.27	0.41	0.52	0.38	1.00																											
France	0.67	0.52	0.79	0.75	0.72	0.59	0.61	1.00																										
Germany	0.64	0.54	0.78	0.75	0.65	0.62	0.50	0.94	1.00																									
Hong Kong	0.56	0.42	0.49	0.51	0.67	0.44	0.34	0.57	0.60	1.00																								
India	0.58	0.35	0.30	0.48	0.49	0.30	0.15	0.42	0.46	0.56	1.00																							
Indonesia	0.36	0.44	0.38	0.20	0.32	0.19	-0.16	0.15	0.23	0.40	0.51	1.00																						
Ireland	0.61	0.51	0.66	0.66	0.48	0.52	0.36	0.67	0.74	0.49	0.30	0.21	1.00																					
Italy	0.57	0.52	0.68	0.59	0.61	0.41	0.50	0.87	0.83	0.51	0.52	0.25	0.54	1.00																				
Japan	0.55	0.37	0.23	0.34	0.54	0.34	0.24	0.36	0.31	0.41	0.50	0.31	0.42	0.32	1.00																			
Malaysia	0.24	0.34	0.35	0.35	0.32	0.37	0.02	0.33	0.43	0.43	0.43	0.41	0.25	0.37	0.16	1.00																		
Mexico	0.63	0.32	0.43	0.64	0.67	0.59	0.41	0.63	0.68	0.60	0.46	0.41	0.51	0.59	0.41	0.45	1.00																	
Netherlands	0.68	0.52	0.82	0.76	0.66	0.61	0.50	0.94	0.92	0.55	0.45	0.22	0.73	0.82	0.40	0.35	0.62	1.00																
New Zealand	0.70	0.52	0.41	0.47	0.53	0.22	0.33	0.50	0.47	0.45	0.42	0.38	0.41	0.48	0.40	0.28	0.43	0.45	1.00															
Norway	0.72	0.58	0.81	0.69	0.67	0.62	0.44	0.83	0.81	0.60	0.45	0.33	0.65	0.72	0.46	0.39	0.63	0.85	0.44	1.00														
Philippines	0.42	0.30	0.20	0.20	0.27	0.19	-0.06	0.10	0.18	0.25	0.38	0.52	0.18	0.07	0.23	0.19	0.27	0.15	0.32	0.23	1.00													
Portugal	0.53	0.52	0.70	0.64	0.53	0.46	0.43	0.77	0.73	0.50	0.49	0.24	0.56	0.74	0.27	0.28	0.45	0.72	0.40	0.65	0.13	1.00												
Singapore	0.64	0.35	0.47	0.47	0.50	0.28	0.19	0.43	0.49	0.63	0.48	0.50	0.38	0.37	0.29	0.43	0.56	0.49	0.55	0.49	0.43	0.25	1.00											
South Africa	0.65	0.51	0.42	0.51	0.59	0.22	0.16	0.46	0.41	0.48	0.49	0.36	0.39	0.42	0.57	0.18	0.48	0.46	0.44	0.60	0.24	0.37	0.36	1.00										
South Korea	0.69	0.32	0.37	0.48	0.57	0.46	0.33	0.49	0.52	0.55	0.51	0.36	0.41	0.40	0.54	0.37	0.61	0.53	0.49	0.59	0.46	0.34	0.51	0.45	1.00									
Spain	0.66	0.55	0.68	0.78	0.67	0.52	0.45	0.86	0.87	0.59	0.47	0.20	0.72	0.78	0.37	0.32	0.63	0.83	0.49	0.73	0.18	0.77	0.38	0.51	0.54	1.00								
Sweden	0.67	0.37	0.59	0.66	0.73	0.61	0.61	0.87	0.86	0.62	0.52	0.19	0.60	0.78	0.34	0.39	0.65	0.80	0.46	0.73	0.17	0.69	0.46	0.42	0.56	0.81	1.00							
Switzerland	0.58	0.58	0.79	0.60	0.58	0.44	0.41	0.79	0.72	0.48	0.34	0.27	0.64	0.67	0.40	0.18	0.40	0.82	0.41	0.74	0.22	0.61	0.34	0.45	0.43	0.64	0.62	1.00						
Taiwan	0.47	0.30	0.36	0.49	0.46	0.47	0.17	0.46	0.50	0.60	0.37	0.21	0.41	0.37	0.31	0.62	0.56	0.45	0.49	0.48	0.33	0.37	0.49	0.26	0.69	0.50	0.47	0.31	1.00					
Thailand	0.61	0.30	0.31	0.37	0.41	0.23	-0.01	0.27	0.31	0.34	0.53	0.51	0.34	0.23	0.46	0.35	0.37	0.36	0.58	0.38	0.62	0.21	0.61	0.44	0.61	0.35	0.32	0.33	0.50	1.00				
Turkey	0.51	0.12	0.28	0.38	0.54	0.34	0.52	0.55	0.53	0.46	0.27	0.11	0.40	0.49	0.29	0.27	0.54	0.45	0.38	0.40	0.18	0.27	0.47	0.30	0.47	0.52	0.63	0.31	0.43	0.34	1.00			
UK	0.63	0.55	0.77	0.71	0.67	0.57	0.55	0.86	0.82	0.58	0.31	0.27	0.71	0.68	0.41	0.29	0.60	0.83	0.45	0.81	0.17	0.63	0.52	0.46	0.49	0.77	0.70	0.78	0.38	0.38	0.56	1.00		
USA	0.67	0.36	0.58	0.70	0.81	0.75	0.59	0.78	0.78	0.60	0.36	0.21	0.69	0.60	0.48	0.32	0.71	0.78	0.45	0.74	0.27	0.51	0.54	0.40	0.66	0.73	0.77	0.64	0.53	0.47	0.62	0.83	1.00	
Benchmark	0.75	0.48	0.68	0.76	0.84	0.71	0.61	0.88	0.86	0.66	0.46	0.26	0.74	0.72	0.57	0.35	0.74	0.87	0.53	0.83	0.26	0.63	0.56	0.51	0.67	0.82	0.84	0.74	0.53	0.48	0.62	0.89	0.97	1.00
US T-Bill	-0.22	-0.27	-0.13	-0.14	-0.17	0.02	-0.16	-0.14	-0.15	-0.18	-0.30	-0.32	-0.14	-0.14	-0.27	-0.28	-0.12	-0.05	-0.31	-0.14	-0.24	-0.20	-0.21	-0.24	-0.28	-0.18	-0.24	-0.07	-0.30	-0.31	-0.24	-0.18	-0.15	-0.19

8.5. 2005-2010

Table 43: Descriptive Statistics (2005-2010)

	Monthly								Annually				Tests		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test	Annual t-test	Jarqe Bera
Australia	0.84 %	1.00 %	2.01 %	7.73 %	3.31	-1.20	-29.45 %	16.38 %	10.03 %	26.76 %	7.33 %	0.27	0.84	2.90	201.11
Austria	-0.39 %	1.41 %	1.12 %	10.95 %	6.15	-1.72	-46.27 %	22.74 %	-4.69 %	37.95 %	-7.39 %	-0.19	-0.28	-0.96	596.85
Belgium	-0.38 %	1.19 %	1.28 %	9.20 %	10.35	-2.67	-45.50 %	13.46 %	-4.58 %	31.89 %	-7.28 %	-0.23	-0.32	-1.11	1627.46
Brazil	2.22 %	1.43 %	3.35 %	11.04 %	2.35	-0.99	-38.64 %	22.36 %	26.58 %	38.26 %	23.88 %	0.62	1.55	5.38	113.27
Canada	0.84 %	0.99 %	1.48 %	7.68 %	4.36	-1.18	-31.39 %	19.28 %	10.06 %	26.59 %	7.36 %	0.28	0.85	2.93	295.10
Denmark	0.77 %	0.79 %	2.13 %	6.10 %	3.12	-1.20	-19.63 %	17.04 %	9.20 %	21.12 %	6.50 %	0.31	0.97	3.37	185.54
Finland	0.37 %	1.08 %	1.38 %	8.37 %	2.71	-0.81	-27.46 %	24.92 %	4.47 %	28.98 %	1.77 %	0.06	0.34	1.19	119.92
France	0.34 %	0.88 %	1.67 %	6.79 %	2.95	-1.30	-25.38 %	12.51 %	4.10 %	23.53 %	1.40 %	0.06	0.39	1.35	184.79
Germany	0.50 %	0.97 %	1.58 %	7.51 %	2.34	-1.18	-26.10 %	15.85 %	5.96 %	26.01 %	3.26 %	0.13	0.51	1.77	133.12
Hong Kong	0.70 %	0.92 %	1.39 %	7.09 %	2.60	-0.73	-23.92 %	16.63 %	8.37 %	24.57 %	5.67 %	0.23	0.76	2.64	106.87
India	1.58 %	1.28 %	3.53 %	9.91 %	2.25	-0.71	-31.82 %	29.06 %	18.94 %	34.32 %	16.25 %	0.47	1.23	4.28	84.86
Indonesia	1.84 %	1.50 %	3.02 %	11.60 %	6.13	-1.50	-50.21 %	26.65 %	22.10 %	40.20 %	19.40 %	0.48	1.23	4.26	559.85
Ireland	-1.69 %	1.09 %	-0.91 %	8.44 %	2.62	-1.44	-30.17 %	13.03 %	-20.26 %	29.24 %	-22.96 %	-0.79	-1.55	-5.37	182.18
Italy	-0.17 %	0.92 %	0.56 %	7.11 %	3.12	-1.06	-26.92 %	17.53 %	-2.04 %	24.63 %	-4.73 %	-0.19	-0.18	-0.64	170.08
Japan	0.01 %	0.65 %	-0.13 %	5.03 %	1.67	-0.64	-16.00 %	9.86 %	0.15 %	17.42 %	-2.55 %	-0.15	0.02	0.07	53.26
Malaysia	1.00 %	0.70 %	0.84 %	5.40 %	1.97	-0.64	-18.08 %	14.40 %	11.96 %	18.72 %	9.26 %	0.49	1.43	4.95	65.95
Mexico	1.14 %	1.09 %	2.13 %	8.42 %	5.73	-1.64	-36.64 %	15.90 %	13.66 %	29.16 %	10.96 %	0.38	1.05	3.63	523.19
Netherlands	0.48 %	0.95 %	1.67 %	7.33 %	4.40	-1.60	-28.92 %	13.44 %	5.73 %	25.38 %	3.04 %	0.12	0.51	1.75	354.73
New Zealand	-0.17 %	0.95 %	0.58 %	7.35 %	1.76	-0.82	-25.41 %	16.58 %	-2.01 %	25.45 %	-4.71 %	-0.18	-0.18	-0.61	69.33
Norway	0.77 %	1.38 %	3.19 %	10.72 %	4.72	-1.78	-40.59 %	19.45 %	9.21 %	37.13 %	6.51 %	0.18	0.55	1.92	419.99
Philippines	1.06 %	0.97 %	1.22 %	7.52 %	2.96	-0.96	-27.88 %	15.41 %	12.68 %	26.04 %	9.98 %	0.38	1.09	3.77	149.45
Portugal	0.18 %	0.91 %	0.95 %	7.07 %	5.20	-1.49	-30.45 %	14.64 %	2.17 %	24.49 %	-0.53 %	-0.02	0.20	0.69	430.67
Singapore	1.00 %	1.05 %	1.84 %	8.11 %	5.70	-1.29	-34.24 %	22.20 %	12.02 %	28.10 %	9.32 %	0.33	0.96	3.31	469.81
South Africa	0.94 %	1.21 %	1.91 %	9.34 %	3.42	-1.30	-37.11 %	14.63 %	11.28 %	32.36 %	8.59 %	0.27	0.78	2.70	220.97
South Korea	0.82 %	1.27 %	0.54 %	9.83 %	1.02	-0.46	-30.28 %	23.41 %	9.80 %	34.05 %	7.10 %	0.21	0.64	2.23	22.55
Spain	0.73 %	0.96 %	1.84 %	7.43 %	3.78	-1.32	-29.13 %	15.23 %	8.71 %	25.73 %	6.01 %	0.23	0.76	2.62	254.94
Sweden	0.57 %	1.06 %	0.76 %	8.17 %	3.96	-1.03	-31.00 %	22.70 %	6.88 %	28.31 %	4.18 %	0.15	0.54	1.88	238.72
Switzerland	0.57 %	0.66 %	1.48 %	5.08 %	1.05	-0.79	-13.09 %	10.54 %	6.88 %	17.59 %	4.18 %	0.24	0.87	3.03	43.47
Taiwan	0.48 %	1.03 %	1.13 %	8.02 %	0.32	-0.40	-20.96 %	16.04 %	5.73 %	27.77 %	3.03 %	0.11	0.46	1.60	8.97
Thailand	0.66 %	1.18 %	1.61 %	9.13 %	5.44	-1.39	-40.06 %	17.38 %	7.94 %	31.62 %	5.24 %	0.17	0.56	1.94	448.51
Turkey	0.98 %	1.71 %	4.21 %	13.25 %	1.19	-0.85	-41.97 %	25.99 %	11.78 %	45.91 %	9.09 %	0.20	0.57	1.99	51.46
UK	0.13 %	0.73 %	1.14 %	5.63 %	3.41	-1.06	-21.02 %	12.99 %	1.54 %	19.49 %	-1.15 %	-0.06	0.18	0.61	193.84
USA	0.04 %	0.62 %	1.14 %	4.78 %	3.39	-1.34	-18.76 %	9.17 %	0.44 %	16.56 %	-2.26 %	-0.14	0.06	0.21	224.16
Benchmark	0.18 %	0.68 %	1.20 %	5.26 %	3.93	-1.43	-20.99 %	10.72 %	2.14 %	18.23 %	-0.55 %	-0.03	0.26	0.91	283.45
US T-Bill	0.22 %	0.02 %	0.26 %	0.16 %	-1.54	-0.24	0.00 %	0.42 %	2.70 %	0.54 %			11.11	38.47	31.27

Table 44: Correlation Matrix (2005-2010)

Correlation	*Australia*	*Austria*	*Belgium*	*Brazil*	*Canada*	*Denmark*	*Finland*	*France*	*Germany*	*Hong Kong*	*India*	*Indonesia*	*Ireland*	*Italy*	*Japan*	*Malaysia*	*Mexico*	*Netherlands*	*New Zealand*	*Norway*	*Philippines*	*Portugal*	*Singapore*	*South Africa*	*South Korea*	*Spain*	*Sweden*	*Switzerland*	*Taiwan*	*Thailand*	*Turkey*	*UK*	*USA*	*Benchmark*
Austria	0.88	1.00																																
Belgium	0.84	0.91	1.00																															
Brazil	0.84	0.81	0.77	1.00																														
Canada	0.85	0.85	0.80	0.87	1.00																													
Denmark	0.61	0.74	0.77	0.66	0.72	1.00																												
Finland	0.78	0.79	0.75	0.66	0.71	0.73	1.00																											
France	0.88	0.90	0.88	0.75	0.84	0.68	0.84	1.00																										
Germany	0.88	0.87	0.84	0.74	0.82	0.66	0.85	0.98	1.00																									
Hong Kong	0.81	0.75	0.78	0.82	0.82	0.66	0.69	0.78	0.77	1.00																								
India	0.73	0.73	0.75	0.74	0.77	0.66	0.69	0.78	0.77	0.83	1.00																							
Indonesia	0.80	0.76	0.77	0.71	0.74	0.63	0.73	0.78	0.80	0.76	0.72	1.00																						
Ireland	0.75	0.80	0.82	0.65	0.72	0.69	0.75	0.76	0.74	0.60	0.53	0.67	1.00																					
Italy	0.88	0.89	0.87	0.74	0.84	0.70	0.85	0.97	0.95	0.79	0.77	0.77	0.77	1.00																				
Japan	0.75	0.79	0.75	0.70	0.73	0.68	0.77	0.78	0.77	0.68	0.68	0.66	0.64	0.80	1.00																			
Malaysia	0.73	0.65	0.71	0.68	0.67	0.56	0.63	0.69	0.72	0.80	0.70	0.76	0.56	0.73	0.61	1.00																		
Mexico	0.81	0.84	0.81	0.77	0.81	0.64	0.73	0.85	0.84	0.70	0.69	0.77	0.72	0.82	0.75	0.66	1.00																	
Netherlands	0.86	0.89	0.93	0.76	0.82	0.77	0.81	0.94	0.91	0.82	0.78	0.79	0.78	0.92	0.78	0.71	0.82	1.00																
New Zealand	0.85	0.78	0.77	0.70	0.69	0.58	0.73	0.80	0.80	0.76	0.72	0.74	0.65	0.79	0.70	0.75	0.75	0.82	1.00															
Norway	0.84	0.89	0.86	0.86	0.87	0.76	0.72	0.84	0.81	0.82	0.72	0.71	0.72	0.81	0.70	0.67	0.72	0.85	0.73	1.00														
Philippines	0.58	0.59	0.65	0.56	0.61	0.51	0.50	0.63	0.62	0.66	0.71	0.63	0.46	0.58	0.46	0.65	0.71	0.66	0.69	0.56	1.00													
Portugal	0.78	0.81	0.85	0.70	0.73	0.61	0.71	0.86	0.84	0.76	0.79	0.71	0.68	0.87	0.74	0.74	0.76	0.83	0.76	0.74	0.67	1.00												
Singapore	0.85	0.85	0.86	0.80	0.87	0.71	0.74	0.89	0.89	0.90	0.81	0.83	0.68	0.88	0.76	0.80	0.84	0.89	0.80	0.85	0.69	0.83	1.00											
South Africa	0.89	0.84	0.80	0.80	0.81	0.61	0.75	0.85	0.84	0.77	0.79	0.77	0.63	0.83	0.82	0.71	0.81	0.81	0.77	0.80	0.59	0.79	0.83	1.00										
South Korea	0.80	0.76	0.75	0.74	0.76	0.61	0.74	0.80	0.84	0.74	0.72	0.82	0.69	0.79	0.74	0.66	0.75	0.80	0.77	0.72	0.54	0.72	0.80	0.77	1.00									
Spain	0.84	0.82	0.82	0.72	0.76	0.59	0.74	0.92	0.91	0.78	0.76	0.77	0.65	0.90	0.75	0.74	0.85	0.86	0.77	0.73	0.65	0.88	0.88	0.81	0.78	1.00								
Sweden	0.84	0.83	0.86	0.74	0.76	0.76	0.79	0.89	0.87	0.79	0.72	0.78	0.71	0.87	0.75	0.75	0.80	0.90	0.82	0.80	0.65	0.79	0.87	0.82	0.76	0.83	1.00							
Switzerland	0.82	0.83	0.79	0.66	0.71	0.59	0.81	0.91	0.91	0.68	0.66	0.75	0.70	0.89	0.75	0.64	0.81	0.87	0.78	0.73	0.57	0.75	0.80	0.78	0.79	0.85	0.83	1.00						
Taiwan	0.80	0.74	0.75	0.73	0.77	0.59	0.69	0.78	0.77	0.76	0.72	0.74	0.72	0.75	0.59	0.60	0.69	0.78	0.69	0.78	0.55	0.67	0.80	0.72	0.78	0.70	0.77	0.69	1.00					
Thailand	0.78	0.72	0.79	0.76	0.77	0.59	0.62	0.73	0.71	0.72	0.71	0.76	0.63	0.74	0.71	0.67	0.71	0.76	0.68	0.73	0.53	0.76	0.76	0.77	0.74	0.76	0.70	0.64	0.74	1.00				
Turkey	0.71	0.64	0.62	0.72	0.67	0.63	0.63	0.66	0.70	0.73	0.78	0.70	0.45	0.66	0.68	0.68	0.67	0.70	0.72	0.63	0.62	0.61	0.74	0.77	0.75	0.67	0.69	0.63	0.60	0.65	1.00			
UK	0.89	0.91	0.88	0.83	0.89	0.76	0.80	0.91	0.88	0.84	0.76	0.73	0.78	0.92	0.78	0.73	0.82	0.90	0.78	0.89	0.64	0.83	0.89	0.83	0.74	0.84	0.87	0.84	0.77	0.75	0.70	1.00		
USA	0.85	0.87	0.85	0.72	0.83	0.72	0.82	0.91	0.91	0.75	0.76	0.76	0.79	0.91	0.73	0.67	0.87	0.89	0.80	0.76	0.67	0.78	0.86	0.78	0.80	0.85	0.84	0.86	0.73	0.67	0.70	0.88	1.00	
Benchmark	0.91	0.92	0.90	0.80	0.88	0.75	0.86	0.96	0.96	0.82	0.80	0.81	0.81	0.96	0.82	0.73	0.89	0.94	0.83	0.85	0.67	0.85	0.92	0.86	0.84	0.90	0.90	0.90	0.78	0.75	0.73	0.94	0.97	1.00
US T-Bill	0.13	0.18	0.15	0.15	0.15	0.19	0.24	0.17	0.22	0.15	0.12	0.10	0.29	0.18	0.12	0.17	0.15	0.16	0.11	0.16	0.21	0.22	0.14	0.08	0.11	0.16	0.11	0.15	0.06	0.08	0.08	0.19	0.15	0.17

8.6. 2010-2013

Table 45: Descriptive Statistics (2010-2013)

	Monthly								Annually				Tests		
	Average return	Std error	Median	Std	Kurstosis	Skewness	Minimum	Maximum	Average return	Std	Risk Premium	Sharpe ratio	Monthly t-test	Annual t-test	Jarqe Bera
Australia	0.59 %	1.01 %	1.74 %	7.01 %	0.77	-0.55	-18.27 %	15.71 %	7.10 %	24.28 %	7.01 %	0.29	0.58	2.02	21.81
Austria	0.11 %	1.25 %	1.58 %	8.66 %	0.95	-0.80	-25.22 %	16.72 %	1.31 %	29.99 %	1.22 %	0.04	0.09	0.30	41.90
Belgium	0.95 %	0.82 %	1.84 %	5.69 %	-0.09	-0.20	-13.18 %	11.86 %	11.38 %	19.71 %	11.30 %	0.57	1.15	4.00	2.10
Brazil	-0.71 %	1.09 %	-1.02 %	7.57 %	0.53	0.04	-20.31 %	17.74 %	-8.55 %	26.24 %	-8.63 %	-0.33	-0.65	-2.26	3.44
Canada	0.53 %	0.70 %	0.58 %	4.87 %	1.51	-0.71	-15.16 %	9.97 %	6.36 %	16.87 %	6.28 %	0.37	0.75	2.61	51.87
Denmark	1.34 %	0.65 %	1.26 %	4.50 %	2.34	-0.65	-14.64 %	11.26 %	16.03 %	15.60 %	15.95 %	1.02	2.06	7.12	86.20
Finland	0.38 %	1.18 %	1.63 %	8.20 %	-0.49	-0.19	-18.43 %	16.13 %	4.53 %	28.40 %	4.45 %	0.16	0.32	1.11	4.65
France	0.56 %	1.01 %	1.02 %	6.96 %	-0.39	-0.37	-15.63 %	13.53 %	6.72 %	24.12 %	6.63 %	0.28	0.56	1.93	8.52
Germany	1.03 %	1.03 %	2.53 %	7.16 %	0.79	-0.71	-20.55 %	14.93 %	12.42 %	24.79 %	12.33 %	0.50	1.00	3.47	31.38
Hong Kong	0.83 %	0.80 %	1.36 %	5.52 %	2.12	-0.69	-17.90 %	13.10 %	9.97 %	19.11 %	9.89 %	0.52	1.04	3.62	76.63
India	0.14 %	0.69 %	-0.16 %	4.80 %	-0.08	0.18	-9.94 %	12.27 %	1.67 %	16.62 %	1.58 %	0.10	0.20	0.69	1.59
Indonesia	0.35 %	0.98 %	1.83 %	6.82 %	-0.33	-0.47	-16.62 %	13.11 %	4.25 %	23.62 %	4.17 %	0.18	0.36	1.25	11.75
Ireland	0.88 %	1.03 %	2.65 %	7.11 %	1.41	-1.03	-22.11 %	12.74 %	10.57 %	24.62 %	10.49 %	0.43	0.86	2.98	74.14
Italy	0.02 %	1.29 %	0.79 %	8.96 %	-0.92	-0.27	-17.26 %	14.69 %	0.25 %	31.05 %	0.16 %	0.01	0.02	0.06	13.61
Japan	0.53 %	0.62 %	0.97 %	4.30 %	0.23	-0.60	-9.56 %	8.41 %	6.40 %	14.89 %	6.31 %	0.42	0.86	2.98	17.64
Malaysia	1.04 %	0.63 %	1.63 %	4.39 %	0.81	-0.49	-11.85 %	11.82 %	12.44 %	15.20 %	12.35 %	0.81	1.64	5.67	19.25
Mexico	0.77 %	0.84 %	1.47 %	5.84 %	1.02	-0.43	-16.77 %	13.01 %	9.22 %	20.23 %	9.14 %	0.45	0.91	3.16	21.39
Netherlands	0.71 %	0.91 %	1.80 %	6.28 %	-0.45	-0.28	-12.83 %	12.34 %	8.55 %	21.75 %	8.46 %	0.39	0.79	2.72	6.17
New Zealand	1.27 %	0.84 %	1.66 %	5.79 %	0.92	-0.60	-13.91 %	15.02 %	15.20 %	20.05 %	15.11 %	0.75	1.52	5.25	27.20
Norway	0.63 %	1.14 %	0.21 %	7.89 %	0.33	0.00	-17.86 %	16.39 %	7.62 %	27.34 %	7.53 %	0.28	0.56	1.93	1.27
Philippines	1.53 %	0.94 %	2.76 %	6.53 %	0.21	-0.46	-14.08 %	17.62 %	18.41 %	22.61 %	18.32 %	0.81	1.63	5.64	10.73
Portugal	-0.23 %	1.01 %	-0.46 %	7.02 %	0.06	-0.49	-19.58 %	10.36 %	-2.79 %	24.32 %	-2.87 %	-0.12	-0.23	-0.80	11.76
Singapore	0.59 %	0.83 %	1.15 %	5.74 %	0.33	-0.48	-14.86 %	13.54 %	7.10 %	19.87 %	7.02 %	0.35	0.71	2.48	12.47
South Africa	0.37 %	0.95 %	-0.22 %	6.55 %	0.38	-0.16	-17.23 %	13.95 %	4.47 %	22.70 %	4.39 %	0.19	0.39	1.36	2.88
South Korea	0.71 %	0.96 %	1.08 %	6.68 %	0.12	-0.44	-14.38 %	14.04 %	8.47 %	23.15 %	8.39 %	0.36	0.73	2.54	9.37
Spain	0.16 %	1.34 %	-0.15 %	9.30 %	0.01	-0.12	-22.67 %	19.96 %	1.88 %	32.23 %	1.80 %	0.06	0.12	0.40	0.66
Sweden	1.17 %	1.03 %	1.17 %	7.13 %	0.27	-0.30	-15.02 %	17.00 %	14.03 %	24.70 %	13.95 %	0.56	1.14	3.94	5.11
Switzerland	1.06 %	0.71 %	2.13 %	4.93 %	0.93	-0.90	-12.10 %	9.38 %	12.77 %	17.07 %	12.68 %	0.74	1.50	5.18	49.59
Taiwan	0.56 %	0.79 %	0.98 %	5.47 %	-0.16	-0.25	-11.09 %	11.75 %	6.73 %	18.95 %	6.65 %	0.35	0.71	2.46	3.32
Thailand	1.25 %	1.04 %	2.35 %	7.20 %	0.08	-0.51	-18.77 %	13.65 %	15.02 %	24.94 %	14.94 %	0.60	1.20	4.17	12.78
Turkey	-0.44 %	1.35 %	-0.38 %	9.38 %	-0.85	0.03	-16.40 %	17.05 %	-5.34 %	32.50 %	-5.42 %	-0.17	-0.33	-1.14	8.77
UK	0.82 %	0.74 %	1.17 %	5.13 %	0.52	-0.39	-12.17 %	11.45 %	9.87 %	17.78 %	9.79 %	0.55	1.11	3.85	10.43
USA	1.24 %	0.58 %	2.02 %	4.04 %	0.20	-0.41	-8.41 %	10.42 %	14.91 %	14.00 %	14.82 %	1.06	2.13	7.38	8.48
Benchmark	0.97 %	0.64 %	1.39 %	4.44 %	0.33	-0.49	-9.96 %	9.87 %	11.61 %	15.39 %	11.52 %	0.75	1.51	5.23	12.73
US T-Bill	0.01 %	0.00 %	0.01 %	0.00 %	-1.06	0.30	0.00 %	0.02 %	0.08 %	0.01 %			11.70	40.54	17.97

Table 46: Correlation Matrix (2010-2013)

Correlation	*Australia*	*Austria*	*Belgium*	*Brazil*	*Canada*	*Denmark*	*Finland*	*France*	*Germany*	*Hong Kong*	*India*	*Indonesia*	*Ireland*	*Italy*	*Japan*	*Malaysia*	*Mexico*	*Netherlands*	*New Zealand*	*Norway*	*Philippines*	*Portugal*	*Singapore*	*South Africa*	*South Korea*	*Spain*	*Sweden*	*Switzerland*	*Taiwan*	*Thailand*	*Turkey*	*UK*	*USA*	*Benchmark*
Austria	0.83	1.00																																
Belgium	0.75	0.79	1.00																															
Brazil	0.89	0.80	0.68	1.00																														
Canada	0.85	0.81	0.70	0.87	1.00																													
Denmark	0.51	0.56	0.47	0.44	0.56	1.00																												
Finland	0.79	0.84	0.83	0.79	0.77	0.43	1.00																											
France	0.80	0.91	0.90	0.78	0.79	0.51	0.90	1.00																										
Germany	0.76	0.86	0.82	0.77	0.79	0.61	0.85	0.92	1.00																									
Hong Kong	0.80	0.77	0.67	0.82	0.80	0.47	0.74	0.75	0.76	1.00																								
India	0.55	0.57	0.53	0.65	0.49	0.35	0.60	0.57	0.65	0.63	1.00																							
Indonesia	0.62	0.47	0.46	0.56	0.48	0.33	0.35	0.44	0.47	0.49	0.51	1.00																						
Ireland	0.66	0.71	0.69	0.56	0.63	0.51	0.64	0.72	0.68	0.41	0.28	0.40	1.00																					
Italy	0.70	0.84	0.82	0.69	0.68	0.50	0.83	0.93	0.87	0.64	0.58	0.37	0.67	1.00																				
Japan	0.61	0.61	0.54	0.54	0.53	0.44	0.54	0.58	0.61	0.50	0.40	0.33	0.52	0.51	1.00																			
Malaysia	0.71	0.67	0.55	0.78	0.69	0.36	0.60	0.65	0.68	0.72	0.56	0.66	0.46	0.54	0.40	1.00																		
Mexico	0.81	0.73	0.69	0.80	0.84	0.47	0.72	0.73	0.76	0.79	0.59	0.59	0.60	0.61	0.57	0.66	1.00																	
Netherlands	0.78	0.87	0.85	0.74	0.78	0.58	0.86	0.95	0.89	0.71	0.58	0.46	0.75	0.90	0.56	0.63	0.73	1.00																
New Zealand	0.76	0.66	0.60	0.66	0.57	0.42	0.62	0.60	0.59	0.65	0.46	0.55	0.51	0.55	0.61	0.49	0.62	0.60	1.00															
Norway	0.86	0.87	0.84	0.81	0.85	0.61	0.81	0.89	0.84	0.75	0.57	0.55	0.73	0.83	0.56	0.68	0.75	0.91	0.63	1.00														
Philippines	0.66	0.55	0.60	0.65	0.51	0.32	0.51	0.52	0.50	0.61	0.64	0.78	0.38	0.43	0.34	0.62	0.61	0.52	0.57	0.59	1.00													
Portugal	0.66	0.81	0.75	0.64	0.63	0.40	0.80	0.84	0.76	0.63	0.46	0.36	0.56	0.86	0.54	0.47	0.60	0.80	0.61	0.76	0.40	1.00												
Singapore	0.83	0.76	0.69	0.83	0.75	0.50	0.70	0.72	0.79	0.86	0.69	0.63	0.47	0.62	0.56	0.80	0.80	0.72	0.75	0.76	0.71	0.55	1.00											
South Africa	0.82	0.74	0.69	0.84	0.82	0.45	0.71	0.68	0.68	0.69	0.61	0.57	0.58	0.55	0.53	0.68	0.78	0.68	0.57	0.81	0.61	0.56	0.74	1.00										
South Korea	0.79	0.79	0.66	0.79	0.75	0.53	0.74	0.72	0.79	0.79	0.63	0.54	0.62	0.62	0.45	0.78	0.72	0.72	0.61	0.75	0.60	0.52	0.86	0.73	1.00									
Spain	0.66	0.80	0.82	0.63	0.60	0.37	0.80	0.89	0.76	0.56	0.49	0.35	0.65	0.94	0.53	0.48	0.54	0.85	0.56	0.78	0.41	0.85	0.57	0.53	0.56	1.00								
Sweden	0.85	0.84	0.83	0.79	0.83	0.68	0.82	0.88	0.87	0.84	0.58	0.53	0.70	0.77	0.59	0.68	0.79	0.88	0.68	0.89	0.60	0.68	0.84	0.74	0.84	0.72	1.00							
Switzerland	0.83	0.80	0.80	0.72	0.75	0.52	0.77	0.83	0.79	0.79	0.52	0.53	0.60	0.75	0.66	0.64	0.73	0.80	0.73	0.81	0.59	0.76	0.75	0.65	0.68	0.71	0.83	1.00						
Taiwan	0.69	0.77	0.65	0.69	0.66	0.45	0.73	0.71	0.73	0.77	0.61	0.45	0.53	0.63	0.55	0.70	0.61	0.70	0.65	0.74	0.52	0.54	0.79	0.65	0.84	0.58	0.78	0.69	1.00					
Thailand	0.71	0.55	0.54	0.76	0.67	0.40	0.54	0.52	0.58	0.76	0.59	0.74	0.30	0.43	0.47	0.70	0.71	0.49	0.63	0.63	0.78	0.48	0.75	0.73	0.63	0.36	0.62	0.69	0.57	1.00				
Turkey	0.57	0.54	0.61	0.59	0.43	0.38	0.56	0.56	0.62	0.45	0.64	0.55	0.39	0.56	0.46	0.53	0.54	0.58	0.51	0.55	0.61	0.49	0.65	0.55	0.55	0.54	0.55	0.47	0.45	0.56	1.00			
UK	0.86	0.88	0.87	0.80	0.86	0.59	0.84	0.92	0.90	0.78	0.54	0.49	0.74	0.87	0.64	0.68	0.78	0.92	0.66	0.94	0.53	0.79	0.77	0.76	0.76	0.82	0.90	0.87	0.72	0.61	0.57	1.00		
USA	0.80	0.79	0.79	0.77	0.87	0.60	0.80	0.85	0.85	0.73	0.48	0.47	0.78	0.73	0.63	0.65	0.83	0.86	0.59	0.86	0.51	0.63	0.72	0.70	0.74	0.65	0.87	0.76	0.72	0.55	0.47	0.89	1.00	
Benchmark	0.87	0.88	0.86	0.83	0.89	0.61	0.87	0.92	0.92	0.80	0.56	0.52	0.78	0.83	0.70	0.70	0.84	0.92	0.68	0.92	0.56	0.75	0.80	0.76	0.78	0.77	0.92	0.86	0.76	0.62	0.56	0.96	0.97	1.00
US T-Bill	0.14	0.12	0.02	0.13	0.25	0.20	-0.02	0.03	0.10	0.25	0.07	0.27	-0.21	-0.03	0.11	0.23	0.20	0.04	0.07	0.10	0.13	0.07	0.24	0.15	0.18	-0.05	0.17	0.10	0.16	0.29	0.19	0.07	0.06	0.09

9. Portfolio Weights

Table 47: Portfolio Weights (1995-2000)

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
1995-2000	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%
Australia	3.0%	2.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.8%	3.0%	3.5%	2.9%	3.7%	2.8%	4.1%	3.1%	6.9%	4.6%
Austria	3.0%	1.9%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	2.7%	2.0%	3.5%	1.8%	2.2%	1.8%	6.5%	1.3%	5.7%	1.7%
Belgium	3.0%	1.0%	6.4%	1.5%	7.7%	3.6%	0.0%	0.0%	4.6%	1.9%	6.3%	1.5%	4.4%	1.4%	2.0%	1.7%	3.9%	2.8%
Brazil	3.0%	5.1%	7.0%	10.5%	0.0%	0.0%	0.0%	0.0%	0.9%	1.6%	1.5%	3.3%	2.0%	3.3%	1.3%	3.6%	0.6%	2.3%
Canada	3.0%	2.6%	19.1%	13.8%	21.2%	26.9%	15.0%	19.9%	5.4%	5.3%	2.8%	4.6%	4.6%	4.7%	4.8%	4.5%	4.7%	3.8%
Denmark	3.0%	1.7%	0.0%	0.0%	6.2%	4.8%	13.9%	13.1%	3.9%	2.6%	4.0%	2.5%	3.8%	2.5%	3.8%	3.7%	4.2%	3.5%
Finland	3.0%	3.4%	0.4%	0.4%	0.0%	0.0%	0.0%	0.0%	2.4%	3.1%	2.1%	2.8%	2.2%	2.9%	1.6%	3.5%	1.6%	3.4%
France	3.0%	1.9%	1.9%	1.0%	0.0%	0.0%	0.0%	0.0%	3.9%	2.8%	3.6%	2.3%	3.2%	2.2%	3.6%	4.0%	3.4%	3.3%
Germany	3.0%	2.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	2.7%	3.4%	2.2%	2.8%	2.1%	3.8%	4.1%	3.1%	3.3%
Hong Kong	3.0%	3.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	3.8%	2.1%	3.0%	2.6%	3.2%	1.7%	3.3%	2.2%	3.4%
India	3.0%	1.1%	12.4%	5.5%	4.8%	1.9%	9.1%	5.4%	1.5%	0.5%	6.0%	1.5%	5.0%	0.6%	0.6%	1.1%	2.2%	3.4%
Indonesia	3.0%	7.3%	5.6%	11.6%	0.0%	0.0%	0.0%	0.0%	2.1%	5.5%	1.1%	7.1%	2.4%	6.8%	2.2%	2.0%	3.3%	4.0%
Ireland	3.0%	1.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	1.8%	4.8%	1.3%	2.3%	1.3%	2.8%	2.1%	2.5%	1.7%
Italy	3.0%	2.0%	7.3%	4.4%	0.0%	0.0%	0.0%	0.0%	2.6%	2.0%	3.3%	2.3%	2.8%	2.3%	3.3%	2.8%	2.8%	2.3%
Japan	3.0%	2.1%	3.9%	2.5%	1.1%	0.8%	0.0%	0.0%	2.4%	1.8%	3.6%	2.1%	2.4%	2.1%	3.7%	1.3%	3.1%	1.4%
Malaysia	3.0%	4.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	4.4%	1.8%	4.0%	2.7%	4.4%	3.5%	4.5%	2.3%	2.9%
Mexico	3.0%	4.5%	1.6%	2.1%	0.0%	0.0%	0.2%	0.5%	1.8%	2.9%	1.7%	4.0%	2.8%	4.2%	3.4%	6.8%	1.7%	4.0%
Netherlands	3.0%	1.8%	0.0%	0.0%	1.2%	0.9%	0.0%	0.0%	5.2%	3.7%	3.7%	2.7%	4.8%	2.8%	6.1%	7.3%	3.7%	3.9%
New Zealand	3.0%	2.6%	2.5%	1.8%	0.0%	0.0%	0.0%	0.0%	3.1%	3.0%	2.8%	2.7%	2.9%	2.7%	5.6%	3.8%	3.2%	1.8%
Norway	3.0%	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.9%	2.9%	2.7%	2.7%	2.5%	2.6%	4.0%	2.4%	2.7%	1.6%
Philippines	3.0%	5.1%	0.1%	0.2%	0.0%	0.0%	0.0%	0.0%	2.1%	4.0%	1.6%	3.7%	2.3%	3.9%	2.9%	3.0%	1.8%	2.2%
Portugal	3.0%	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	2.3%	3.5%	2.0%	2.5%	2.0%	3.5%	2.7%	2.9%	2.1%
Singapore	3.0%	4.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	5.0%	1.9%	3.4%	2.5%	3.6%	3.7%	3.8%	2.5%	2.8%
South Africa	3.0%	4.1%	13.0%	15.9%	7.1%	12.1%	2.2%	3.7%	3.5%	5.1%	1.9%	5.5%	3.8%	5.8%	4.7%	4.5%	3.4%	3.2%
South Korea	3.0%	4.5%	12.0%	19.2%	0.0%	0.0%	0.0%	0.0%	2.4%	3.8%	1.8%	6.0%	3.4%	5.9%	4.1%	3.0%	3.9%	3.5%
Spain	3.0%	2.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	2.6%	2.9%	2.1%	2.2%	2.1%	1.9%	2.4%	2.3%	2.7%
Sweden	3.0%	2.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	2.5%	2.9%	1.9%	2.1%	1.9%	2.0%	3.3%	2.1%	3.0%
Switzerland	3.0%	1.6%	0.0%	0.0%	0.0%	0.0%	9.4%	7.5%	4.1%	2.6%	4.1%	1.9%	3.6%	1.8%	2.4%	3.0%	3.0%	3.2%
Taiwan	3.0%	3.2%	0.4%	0.4%	0.0%	0.0%	0.0%	0.0%	1.5%	1.7%	2.5%	1.9%	1.5%	2.0%	1.5%	1.4%	2.0%	2.2%
Thailand	3.0%	6.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	5.2%	1.2%	4.6%	1.9%	4.8%	1.2%	1.0%	2.2%	3.3%
Turkey	3.0%	4.6%	6.0%	9.1%	2.3%	4.6%	0.0%	0.0%	1.2%	1.9%	1.6%	4.5%	2.1%	4.2%	0.7%	1.7%	1.9%	4.5%
UK	3.0%	1.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.9%	1.8%	5.6%	1.4%	3.1%	1.2%	1.4%	1.4%	3.8%	3.1%
USA	3.0%	1.8%	0.0%	0.0%	48.3%	44.4%	50.1%	49.9%	6.0%	4.1%	4.2%	3.8%	6.9%	4.0%	1.5%	2.0%	4.4%	4.9%

Table 48: Portfolio Weights (2000-2005)

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2000-2005	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%
Australia	3.0%	2.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.1%	3.6%	3.5%	3.1%	3.7%	3.2%	4.1%	3.4%	6.9%	4.9%
Austria	3.0%	1.7%	8.1%	4.3%	0.0%	0.0%	0.0%	0.0%	3.6%	2.2%	3.5%	2.1%	2.2%	1.5%	6.5%	0.9%	5.7%	1.3%
Belgium	3.0%	2.6%	30.8%	24.3%	23.0%	27.3%	5.6%	4.7%	5.2%	5.0%	6.3%	6.0%	4.4%	3.9%	2.0%	2.1%	3.9%	3.5%
Brazil	3.0%	5.4%	0.7%	1.1%	0.0%	0.0%	0.0%	0.0%	1.6%	3.1%	1.5%	2.9%	2.0%	3.5%	1.3%	4.0%	0.6%	2.6%
Canada	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.6%	3.6%	2.8%	2.8%	4.6%	4.9%	4.8%	4.7%	4.7%	4.0%
Denmark	3.0%	2.1%	0.0%	0.0%	0.0%	0.0%	3.0%	2.5%	3.9%	3.0%	4.0%	3.1%	3.8%	3.0%	3.8%	4.0%	4.2%	3.8%
Finland	3.0%	3.6%	0.0%	0.0%	0.0%	0.0%	10.5%	19.5%	2.2%	2.9%	2.1%	2.7%	2.2%	2.9%	1.6%	3.7%	1.6%	3.4%
France	3.0%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.2%	4.7%	3.6%	4.0%	3.2%	3.4%	3.6%	4.6%	3.4%	3.9%
Germany	3.0%	4.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.9%	5.7%	3.4%	4.9%	2.8%	4.0%	3.8%	5.3%	3.1%	4.4%
Hong Kong	3.0%	2.7%	4.8%	3.9%	0.0%	0.0%	0.0%	0.0%	2.1%	1.9%	2.1%	1.9%	2.6%	2.0%	1.7%	3.0%	2.2%	3.1%
India	3.0%	3.3%	25.8%	30.4%	13.7%	15.4%	1.3%	0.8%	2.6%	2.8%	6.0%	6.7%	5.0%	4.4%	0.6%	1.5%	2.2%	5.3%
Indonesia	3.0%	3.1%	1.3%	1.4%	0.0%	0.0%	0.0%	0.0%	1.1%	1.0%	1.1%	1.1%	2.4%	2.8%	2.2%	0.0%	3.3%	0.6%
Ireland	3.0%	2.5%	0.4%	0.3%	0.0%	0.0%	0.0%	0.0%	4.8%	4.5%	4.8%	4.4%	2.3%	2.1%	2.8%	2.6%	2.5%	2.2%
Italy	3.0%	2.7%	2.3%	1.9%	0.0%	0.0%	0.0%	0.0%	3.0%	2.9%	3.3%	3.3%	2.8%	2.8%	3.3%	3.3%	2.8%	2.7%
Japan	3.0%	1.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	2.0%	3.6%	2.2%	2.4%	1.8%	3.7%	1.2%	3.1%	1.2%
Malaysia	3.0%	1.6%	2.0%	1.0%	0.0%	0.0%	0.0%	0.0%	1.5%	0.7%	1.8%	0.9%	2.7%	1.1%	3.5%	2.7%	2.3%	1.6%
Mexico	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.8%	1.9%	1.7%	1.8%	2.8%	2.6%	3.4%	6.3%	1.7%	3.4%
Netherlands	3.0%	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	5.1%	3.7%	4.2%	4.8%	4.9%	6.1%	8.1%	3.7%	4.4%
New Zealand	3.0%	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	2.4%	2.8%	2.2%	2.9%	2.1%	5.6%	3.6%	3.2%	1.7%
Norway	3.0%	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.9%	3.2%	2.7%	2.9%	2.5%	2.9%	4.0%	2.7%	2.7%	1.8%
Philippines	3.0%	2.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.7%	1.0%	1.6%	1.0%	2.3%	1.2%	2.9%	1.4%	1.8%	1.0%
Portugal	3.0%	2.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	3.3%	3.5%	3.4%	2.5%	2.5%	3.5%	3.2%	2.9%	2.4%
Singapore	3.0%	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%	1.9%	1.9%	1.7%	2.5%	2.1%	3.7%	3.2%	2.5%	2.2%
South Africa	3.0%	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%	2.0%	1.9%	1.7%	3.8%	3.6%	4.7%	3.7%	3.4%	2.5%
South Korea	3.0%	4.4%	8.7%	11.4%	0.0%	0.0%	0.0%	0.0%	1.3%	1.9%	1.8%	2.6%	3.4%	5.5%	4.1%	2.8%	3.9%	3.2%
Spain	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	3.8%	2.9%	3.2%	2.2%	2.5%	1.9%	2.7%	2.3%	3.0%
Sweden	3.0%	4.6%	0.0%	0.0%	0.0%	0.0%	2.4%	3.7%	3.3%	5.4%	2.9%	4.7%	2.1%	3.4%	2.0%	4.1%	2.1%	3.9%
Switzerland	3.0%	2.0%	3.7%	2.1%	0.0%	0.0%	0.0%	0.0%	4.0%	2.9%	4.1%	3.0%	3.6%	2.1%	2.4%	3.2%	3.0%	3.4%
Taiwan	3.0%	3.5%	4.7%	4.8%	0.0%	0.0%	0.0%	0.0%	2.2%	2.5%	2.5%	2.8%	1.5%	2.1%	1.5%	1.6%	2.0%	2.4%
Thailand	3.0%	3.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	1.5%	1.2%	1.4%	1.9%	2.4%	1.2%	0.4%	2.2%	1.5%
Turkey	3.0%	6.9%	6.8%	13.2%	0.0%	0.0%	0.0%	0.0%	1.2%	2.8%	1.6%	3.6%	2.1%	5.7%	0.7%	2.3%	1.9%	6.3%
UK	3.0%	2.1%	0.0%	0.0%	55.0%	50.1%	28.0%	22.5%	6.4%	4.9%	5.6%	4.2%	3.1%	2.1%	1.4%	1.6%	3.8%	3.5%
USA	3.0%	2.4%	0.0%	0.0%	8.2%	7.2%	49.3%	46.3%	5.0%	4.2%	4.2%	3.4%	6.9%	5.3%	1.5%	2.1%	4.4%	5.2%

Table 49: Portfolio Weights (2005-2010)

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2005-2010	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%
Australia	3.0%	3.1%	0.0%	0.0%	0.0%	0.0%	8.8%	5.9%	3.9%	4.2%	3.3%	3.5%	3.3%	3.5%	1.9%	2.9%	2.3%	3.6%
Austria	3.0%	4.3%	15.6%	22.6%	9.5%	18.1%	86.7%	91.6%	4.2%	6.3%	4.7%	7.0%	4.0%	6.6%	1.8%	4.1%	3.1%	7.0%
Belgium	3.0%	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	4.2%	3.1%	4.0%	3.1%	4.0%	4.3%	6.6%	4.0%	6.8%
Brazil	3.0%	4.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.7%	2.4%	1.5%	2.2%	1.6%	2.1%	2.3%	5.3%	2.0%	6.0%
Canada	3.0%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	3.5%	2.9%	3.0%	2.7%	2.9%	4.4%	4.9%	4.7%	5.6%
Denmark	3.0%	2.0%	0.0%	0.0%	3.9%	3.4%	0.0%	0.0%	3.6%	2.5%	3.9%	2.7%	3.6%	2.6%	4.5%	2.9%	5.3%	3.0%
Finland	3.0%	3.0%	16.0%	17.2%	0.0%	0.0%	0.0%	0.0%	1.7%	1.8%	2.5%	2.6%	2.4%	2.6%	2.8%	0.6%	0.0%	0.0%
France	3.0%	2.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	3.2%	2.8%	2.6%	2.7%	2.6%	3.7%	3.0%	4.0%	2.9%
Germany	3.0%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.6%	2.7%	2.1%	2.2%	1.9%	2.1%	4.1%	2.1%	3.6%	2.9%
Hong Kong	3.0%	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	3.0%	3.1%	2.8%	3.6%	2.9%	3.1%	1.9%	4.4%	3.2%
India	3.0%	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.3%	2.9%	2.5%	3.1%	3.0%	3.3%	1.8%	3.3%	2.2%	4.3%
Indonesia	3.0%	4.3%	15.6%	24.3%	0.0%	0.0%	0.0%	0.0%	1.8%	2.6%	2.6%	3.8%	2.1%	3.5%	1.8%	4.7%	2.0%	5.3%
Ireland	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	3.4%	3.3%	3.2%	3.0%	3.1%	3.2%	4.2%	4.1%	4.9%
Italy	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	3.4%	3.1%	3.1%	2.8%	2.9%	2.4%	2.7%	2.4%	2.7%
Japan	3.0%	1.8%	11.3%	6.9%	15.0%	12.0%	0.0%	0.0%	3.6%	2.2%	4.5%	2.7%	3.8%	2.6%	4.6%	-0.8%	4.4%	-0.4%
Malaysia	3.0%	1.8%	41.3%	28.8%	24.3%	20.8%	0.0%	0.0%	3.8%	2.4%	5.2%	3.3%	6.4%	3.6%	4.5%	2.5%	3.4%	1.8%
Mexico	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.9%	3.2%	2.7%	2.9%	3.0%	3.1%	1.9%	3.3%	0.0%	0.0%
Netherlands	3.0%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	3.4%	2.7%	2.8%	2.8%	2.8%	6.0%	3.9%	1.9%	-1.1%
New Zealand	3.0%	2.7%	0.0%	0.0%	3.1%	3.7%	4.5%	2.5%	3.5%	3.2%	3.6%	3.4%	3.7%	3.5%	2.9%	5.5%	0.3%	0.6%
Norway	3.0%	4.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	4.6%	2.7%	3.8%	2.5%	3.7%	2.2%	4.5%	0.6%	1.3%
Philippines	3.0%	2.3%	0.1%	0.0%	4.8%	4.8%	0.0%	0.0%	2.4%	1.9%	4.0%	3.1%	4.7%	3.4%	4.1%	0.7%	4.0%	1.1%
Portugal	3.0%	2.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	2.9%	3.1%	2.9%	2.8%	2.8%	3.0%	2.4%	3.3%	2.6%
Singapore	3.0%	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	3.4%	3.1%	3.5%	3.3%	3.7%	5.1%	4.4%	5.0%	6.1%
South Africa	3.0%	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	3.4%	3.1%	3.8%	3.0%	3.9%	2.9%	5.9%	2.8%	5.5%
South Korea	3.0%	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.0%	2.5%	1.9%	2.4%	1.7%	2.3%	2.9%	5.4%	2.8%	5.1%
Spain	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	3.2%	2.7%	2.6%	2.5%	2.6%	3.9%	4.5%	4.2%	5.2%
Sweden	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%	2.4%	1.9%	2.1%	1.8%	2.0%	2.6%	2.7%	3.6%	3.3%
Switzerland	3.0%	1.9%	0.0%	0.0%	15.9%	13.4%	0.0%	0.0%	4.4%	2.9%	4.2%	2.7%	4.9%	2.9%	2.8%	2.0%	3.8%	2.7%
Taiwan	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%	2.2%	2.4%	2.4%	2.0%	2.3%	2.9%	0.2%	4.8%	1.8%
Thailand	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	2.0%	2.1%	2.4%	2.2%	2.4%	0.6%	1.3%	0.6%	1.2%
Turkey	3.0%	4.5%	0.1%	0.2%	0.0%	0.0%	0.0%	0.0%	1.1%	1.7%	1.3%	2.0%	1.1%	2.0%	0.3%	0.8%	0.5%	1.5%
UK	3.0%	2.3%	0.0%	0.0%	23.4%	23.6%	0.0%	0.0%	4.8%	3.7%	4.0%	3.1%	4.1%	3.2%	2.6%	1.3%	5.7%	2.7%
USA	3.0%	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.3%	2.8%	3.6%	2.3%	3.8%	2.3%	2.4%	0.3%	4.3%	0.8%

Table 50: Portfolio Weights (2010-2013)

	EW		MDP		GMV		MSR		NRP		ERC		ERC R		ERC VaR		ERC CVaR	
2010-2013	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%	w	RC%
Australia	3.0%	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	0.2%	2.8%	3.5%	2.7%	3.4%	3.2%	4.2%	2.5%	3.5%
Austria	3.0%	4.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	0.2%	2.0%	3.1%	1.8%	3.1%	0.7%	0.5%	2.4%	3.8%
Belgium	3.0%	2.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.1%	2.4%	2.3%	2.2%	2.2%	4.8%	0.8%	4.0%	3.2%
Brazil	3.0%	3.8%	0.0%	0.0%	0.0%	0.0%	67.1%	80.1%	2.1%	0.1%	2.1%	2.8%	2.2%	2.6%	1.4%	3.9%	1.9%	5.1%
Canada	3.0%	2.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	0.1%	2.9%	2.4%	2.7%	2.4%	4.1%	4.5%	4.1%	4.5%
Denmark	3.0%	1.5%	23.7%	15.3%	0.0%	0.0%	0.0%	0.0%	3.8%	0.1%	4.3%	2.3%	4.0%	2.2%	4.8%	4.5%	4.5%	3.7%
Finland	3.0%	4.1%	1.4%	1.8%	0.0%	0.0%	0.0%	0.0%	2.8%	0.2%	2.8%	4.0%	2.7%	4.0%	3.4%	5.0%	2.9%	3.9%
France	3.0%	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	0.2%	3.1%	4.0%	3.0%	4.0%	4.4%	4.7%	3.5%	3.6%
Germany	3.0%	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	0.2%	2.8%	3.7%	2.7%	3.7%	3.5%	4.6%	3.7%	4.4%
Hong Kong	3.0%	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	0.2%	3.2%	3.0%	3.7%	2.8%	3.8%	4.1%	3.9%	4.0%
India	3.0%	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.3%	0.1%	2.4%	1.5%	2.8%	1.0%	0.9%	1.7%	1.3%	2.1%
Indonesia	3.0%	2.4%	0.0%	0.0%	0.0%	0.0%	1.5%	1.0%	2.0%	0.1%	2.0%	1.7%	1.9%	1.8%	0.3%	0.7%	0.8%	1.5%
Ireland	3.0%	2.9%	11.0%	11.3%	0.0%	0.0%	0.0%	0.0%	2.7%	0.1%	3.0%	3.0%	3.0%	3.2%	3.1%	-1.7%	4.3%	-0.9%
Italy	3.0%	4.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	0.2%	3.0%	4.5%	2.9%	4.8%	2.0%	1.8%	2.2%	1.9%
Japan	3.0%	1.6%	6.3%	3.5%	31.1%	30.2%	0.0%	0.0%	4.6%	0.1%	4.8%	2.7%	4.5%	2.9%	5.2%	1.0%	4.6%	1.2%
Malaysia	3.0%	1.9%	8.1%	5.8%	23.1%	21.7%	31.4%	18.9%	4.3%	0.1%	4.7%	3.1%	5.5%	3.0%	3.5%	4.0%	2.5%	2.3%
Mexico	3.0%	2.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	0.1%	2.7%	2.6%	3.0%	2.5%	3.2%	4.9%	1.5%	2.2%
Netherlands	3.0%	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	0.2%	2.9%	3.3%	3.1%	3.3%	4.0%	4.5%	1.2%	1.3%
New Zealand	3.0%	2.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	0.1%	3.2%	2.7%	3.2%	2.7%	4.4%	1.5%	1.2%	0.5%
Norway	3.0%	4.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.2%	0.2%	2.1%	3.1%	2.0%	3.2%	1.6%	2.8%	0.5%	1.2%
Philippines	3.0%	2.6%	25.9%	30.9%	0.0%	0.0%	0.0%	0.0%	3.1%	0.1%	3.8%	3.3%	4.3%	3.3%	3.2%	4.0%	2.6%	3.2%
Portugal	3.0%	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	0.2%	3.2%	3.6%	3.0%	3.7%	4.8%	3.4%	4.9%	4.1%
Singapore	3.0%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.1%	2.6%	2.6%	2.7%	2.6%	2.9%	4.1%	2.7%	3.7%
South Africa	3.0%	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.1%	2.4%	2.5%	2.4%	2.5%	3.3%	5.1%	3.1%	4.5%
South Korea	3.0%	3.2%	6.7%	8.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.1%	2.4%	2.6%	2.2%	2.7%	3.3%	5.0%	3.4%	4.7%
Spain	3.0%	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.1%	0.2%	3.0%	4.4%	2.8%	4.5%	3.7%	5.6%	4.0%	5.8%
Sweden	3.0%	3.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.2%	2.7%	3.5%	2.6%	3.5%	3.1%	4.0%	3.0%	4.2%
Switzerland	3.0%	2.5%	0.0%	0.0%	4.7%	5.3%	0.0%	0.0%	4.6%	0.2%	4.5%	3.9%	5.0%	3.9%	3.5%	3.3%	4.9%	3.6%
Taiwan	3.0%	2.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.9%	0.1%	3.0%	2.6%	2.6%	2.7%	2.5%	2.3%	3.4%	2.8%
Thailand	3.0%	3.0%	9.5%	12.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.1%	2.7%	2.7%	2.7%	2.7%	1.6%	1.8%	2.5%	2.9%
Turkey	3.0%	3.6%	7.6%	11.4%	0.0%	0.0%	0.0%	0.0%	1.7%	0.1%	1.9%	2.4%	1.6%	2.5%	0.5%	1.0%	1.2%	2.4%
UK	3.0%	2.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.1%	0.2%	3.8%	3.6%	3.7%	3.5%	2.8%	1.4%	5.5%	3.2%
USA	3.0%	2.0%	0.0%	0.0%	41.1%	42.7%	0.0%	0.0%	4.8%	0.2%	4.6%	3.2%	4.7%	3.0%	2.7%	0.8%	5.2%	1.7%

10. Technical Appendix

Theory and calculations used in thesis

10.1. Returns

In this thesis, the return is calculated with the use of logarithm, also called continuous return. The reason why this method is used is because it takes into account the compound interest, and one can compare returns on assets that bear interest at a similar rate or at the same time. The formula for the logarithmic returns is given by:

$r_t^L = \ln\left(\frac{P_t}{P_{t-1}}\right)$, and will be annualized when multiplied with 12 since we are using monthly data.

The portfolio return is calculated with the following formula: $r_p = \sum_{i=1}^{n} w_i r_i$, where w is the weight in the given asset and the sum of the weights equals 1, i.e. $\sum_{i=1}^{n} w_i = 1$. In this thesis one need to use leverage and thereby the sum of the weights may exceeds 1 in the Risk Parity portfolio.

To see if the returns is statistically significant different from zero there will be used an t-test, given by: $t = \frac{\bar{r}_i - H_0}{se(r_i)}$, where se(r_i) is the standard error for the return on the asset i, $\bar{r}_i$ is the average return to the asset i, and H_0 is the value 0 as the value we are testing for. The standard error is given by se(r_i)= $\frac{\sigma_i}{\sqrt{n}}$. And these calculations are also used to test portfolios returns.

10.2. Volatility and Risk Measures

10.2.1. Standard Deviation

The risk, or the volatility as it often is called, will in this thesis be measured and calculated with the use of standard deviation. This implies a measure of the spread of the observations from the mean of a given variable over time. The higher the standard deviation implies that there are more variations in the return over time for the given asset or portfolio. And by that, more risk in form of higher probability for greater changes in the value and return of the asset.

The formula for standard deviation is given by: $\sigma_t = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(r_t - \bar{r})^2}$, and given monthly observation this is annualized when multiplied by $12^{1/2}$.

The portfolio standard deviation is measured by the following formula:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

To see if the risk of the asset or portfolio is significant there will be used an F-test for paired two variances. This test is given by: $\frac{\sigma_i^2}{\sigma_j^2} \sim F - distributed$, and we will tests the null hypothesis $H_0 : \sigma_i = \sigma_j$ versus the alternative hypothesis $H_A : \sigma_i \neq \sigma_j$.

But investors are probably more interested in reducing downside risk rather than absolute risk. This for at least 2 reasons: Relevant risk measure since the downside risk is the risk investor is concern about, and because the distributions might not be normal. By that there will be used VaR and CVaR as the downside risk measures.

10.2.2. Value at Risk (VaR)

VaR is simply the quantile on the distribution and can be defined as “the potential loss one will have for a given time period with a given probability”. The advantage of this measure is its simplicity and it interprets the downside risk in a simple manner.

As illustrated in Figure 12 below one can see the for a investor that is long in the market with a 5% VaR, the investor will not lose more than 8% of the invested amount by holding this position.

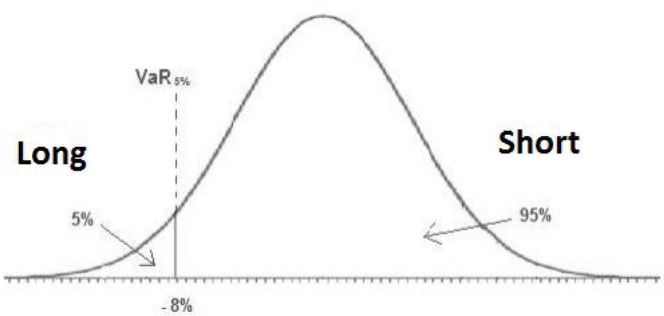

Figure 12: VaR illustration

10.2.3. Conditional Value at Risk

Conditional Value at Risk (CVaR), also called Expected Shortfall gives the average level of loss given that VaR is exceeded, i.e. the mean to the left of the VaR quantile.

As the illustrated example in Figure 13 below shows; if the 5% CVaR equals -12% is the expected tail loss with 95% probability not greater than 12% of the invested amount, if the loss will exceed the VaR value.

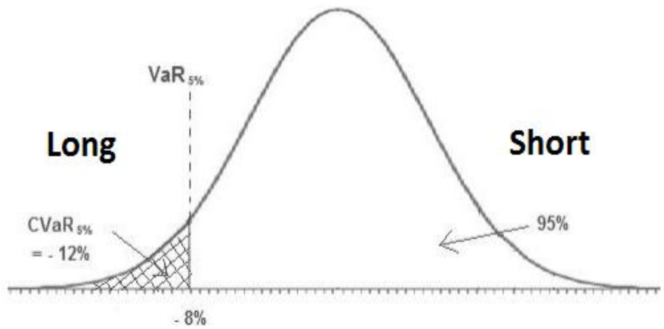

Figure 13: CVaR illustration

10.2.4. Tracking Error

Tracking Error (TE) measures the volatility to the excess return between the portfolio and the benchmark, and is often called the active risk. The Tracking Error indicates how closely a fund or portfolio follows a benchmark index, such as the MSCI Benchmark used in this thesis, but it can also be used to compare two funds. There is in this paper reported two different Tracking Error measurements: Tracking Error Standard Deviation (TESD) and Tracking Error Mean Absolute Deviation (TEMAD).

$TESD = \sqrt{\frac{\sum_{t=1}^{n}(r_{t|p} - r_{t|m})^2}{n-1}}$, where p is portfolio and m is market. Given monthly observation this is annualized when multiplied by $12^{1/2}$.

$TEMAD = \sqrt{\frac{\sum_{t=1}^{n}\left|r_{t|p} - r_{t|m}\right|}{n-1}}$, where p is portfolio and m is market. Given monthly observation this is annualized when multiplied by $12^{1/2}$.

10.3. Normality

The volatility ignores skew and kurtosis and will be a limited measure of risk because the normal distribution is non-Gaussian. Markets typically have fatter tails and higher peak than the Gaussian normal distribution. This means that there is greater likelihood of extreme positive or negative outcomes, and you will have several days where there is little or no change. Skew is a measurement on asymmetric distribution of the return. A positive skew means that there are more relatively large positive deviations than large negative deviation from the normal distribution. Conversely, when the distribution is negatively skewed, the standard deviation will underestimate the risk, and this is crucial for investors to consider. Kurtosis measures the height or peak on the probability distribution. The goodness-of-fit-test for normality is performed by a Jarque Bera test. If the data comes from a normal distribution, the JB statistic asymptotically has a chi-squared distribution with two degrees of freedom, so the statistic can be used to test the hypothesis that the data are from a normal distribution.

Skew is calculated by the following formula: $S = Average\left[\frac{(r-\bar{r})^3}{\hat{\sigma}^3}\right]$

Excess Kurtosis is calculated by the following formula: $K = Average\left[\frac{(r-\bar{r})^4}{\hat{\sigma}^4}\right] - 3$

Jarque Bera is calculated by the following formula: $JB = \frac{n}{6}\left[S^2 + \frac{1}{4}(K-3)^2\right] \sim \chi_2^2$

10.4. Single Index Model (SIM)

The regression model SIMs estimates will yield a line called Security Characteristic Line (SCL) with the intercept α_i and the slope of the line is given by the β_i in the regression.

The regression model is given by $R_i(t) = \alpha_i + \beta_i R_m(t) + e_i(t)$, where $R_m = r_m - r_f$ and $R_i = r_i - r_f$.

α_i is the securities expected excess return beyond the markets excess return, this is also called Jensens Alpha which is the "non-market" premium or the neutral market: $r_m - r_f = 0$, and is high if the security is underpriced and low if the security is overpriced. Alpha is tested with a t-test given by $\frac{\hat{\alpha} - H_0}{se(\alpha)}$. And the hypothesis for the alpha will be $H_0 : \alpha = 0$, $H_A : \alpha \neq 0$.

$\beta_i(r_m - r_f)$ where β_i is the securities sensitivity to the market or benchmark index, i.e. 1 % change in the benchmark will affect the security with the Beta value, this is also the "systematic risk premium". The formula for the Beta is given by: $\hat{\beta}_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$. The correlation in the formula is derived by: $\rho_{i,j} = \frac{Kov_{i,j}}{\sigma_i \sigma_j} \equiv \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \equiv \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \equiv Corr(r_i, r_m)xCorr(r_j, r_m)$, $-1 \leq \rho_{i,j} \leq 1$, and the diversification effect of the correlation can be illustrated as in figure 1.1:

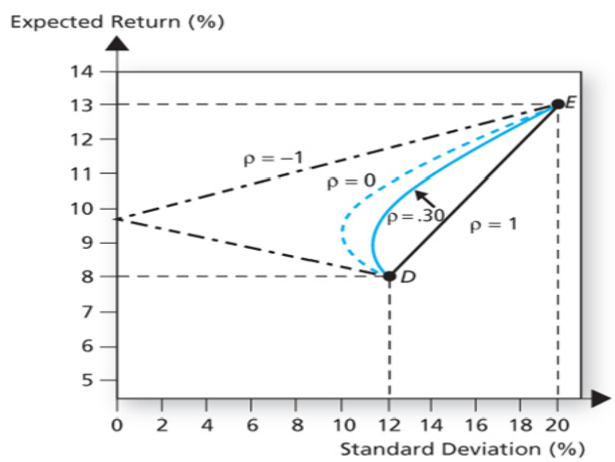

Figure 14: Correlation and the diversification effect

To see if the beta value is statistically significant there will be used an t-test given by:

$t = \frac{\hat{\beta}_i - \beta_i}{se(\hat{\beta}_{i)})}$, with the hypothesis: $H_0 : \beta = 1$, $H_A : \beta \neq 1$.

e_i in the regression model is the firm or company-specific risk or surprises in the security's return in the time period, also called the residual in the statistical language.

When looking at the risk in the SIM, one can distinguish between the different types of risk: (Total risk=Systematic risk+firm-specific risk) $\equiv(\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma^2(e_i) \equiv (TSS = RSS + USS))$, as illustrated inn figure 1.2:

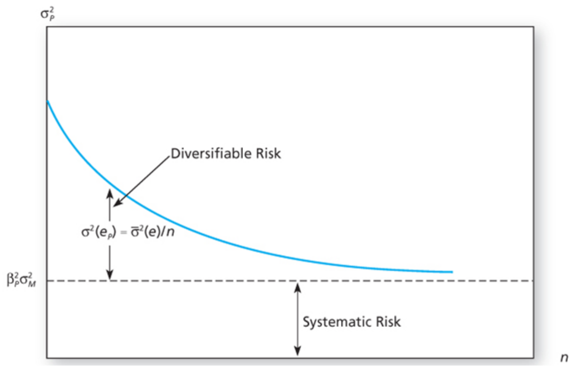

Figure 1.2: Systematic risk and firm specific risk, and diversification effect of n assets.

One can also see the proportion of the systematic risk measured by variance, measured by the: $\bar{R}^2 = 1 - \frac{RSS/(n-k)}{TSS/(n-1)} = 1-(1-R^2)\frac{n-1}{n-k}$, $0 \le \bar{R}^2 \le 1$. We using the adjusted R^2 since this is more commonly used than the R^2, since it takes care of the degrees of freedom and the numbers of regressors in the model, and therefore R^2>adjusted R^2. This is also a better measurement when comparing models. The risk will also here be tested with an F-test.

10.5. Risk adjusted performance target

10.5.1. Sharpe-Ratio

The Sharpe-ratio is estimating the portfolios average return minus the average risk free rate, i.e. the excess return (or risk premium) on the portfolio, divided by the portfolios total risk.

The Sharpe-ratio is estimated by the following formula: $S_p = \frac{(\bar{r}_p - \bar{r}_f)}{\sigma_p}$. Sharpe is also the slope coefficient to the Capital Allocation Line (CAL), shown in figure below.

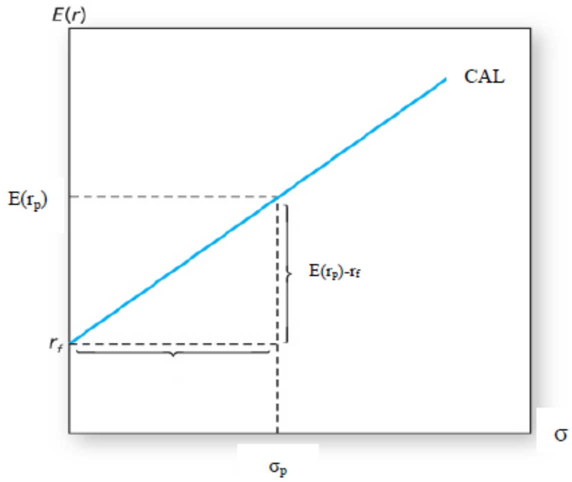

Figure 15: Capital Allocation Line (CAL)

This ratio is hard to interpret, but the portfolio with the highest ratio is the preferable portfolio. Abnormalities like kurtosis or skew on the distribution can be problematic for the ratio, as standard deviation doesn't have the same effectiveness when these problems exist, as explained in section 10.3. So there can be a problem with using this formula when returns are not normally distributed.

10.5.2. Modigliani & Modigliani (M^2)

The M^2 measures the returns of the portfolio, adjusted for the risk of the portfolio relative to the benchmark (e.g., the market). It is derived from the Sharpe Ratio, but it has the significant advantage of being in units of percent return. As the Sharpe ratio is an abstract, dimensionless ratio of limited utility to most investors. This makes the M^2 dramatically more intuitive to interpret.

$M^2 = (S_p - S_m)\sigma_m$, where S is the Sharpe ratio.

10.5.3. Treynor-Ratio

Like the Sharpe-Ratio, the Treynor's measure gives excess return per unit of risk, but uses the systematic risk instead of total risk. This measure is appealing when the one is evaluating assets or portfolios of large scale since one will thereby weight the mean excess return against its systematic risk. The measure is in best use for a fund that is just one sub-portfolio out of a large set of passively-managed portfolios. This is the slope of the Security Market Line (SML).

The Treynor measure is calculated by the following formula:

$$T_p = \frac{(\bar{r}_p - \bar{r}_f)}{\beta_p}$$

The portfolio with the highest ratio is the preferable when using this measure.

10.5.4. T^2

This measure is used to convert the Treynor measure into percentage return basis, and thereby as the M^2 easier to interpret. From the deviation of the market performance this can be interpreted as:

$$T^2 = T_p - T_m = \frac{\alpha_p}{\beta_p}$$

10.5.5. Sortino Ratio

This measurement is like the M^2; a modification of the Sharpe ratio, but penalizes only those returns falling below a user-specified target, or required rate of return[18], while the Sharpe ratio penalizes both upside and downside volatility equally. In other words; Sortino Ratio measure the incremental return of the target rate compare to the downside risk.

The Sortino ratio is a relative measure of risk adjusted performance, as the Sharpe ratio, and therefore it can be hard to interpret and see how much better a portfolio is compared with other portfolios.

The Sortino ratio is calculated by the following formula:

$SR = \frac{\bar{r}_p - \tau}{DD}$, where τ is the is the target or required rate of return for the investment strategy under consideration and DD is the target semi-deviation (the square root of target semi variance) and is termed downside deviation to the portfolios return, i.e. the standard deviation to all the returns that is lower than τ.

10.5.6. Information Ratio

The definition of Information Ratio (IR) is the amount of excess return relative to a benchmark per unit of tracking error (Sharpe (1994)). So the IR measures the excess return to the portfolio relatively to the benchmark or market, i.e. the excess return the investor achieve with bearing risk

[18] In this paper the target is set as the risk free rate.

beyond the risk given by the benchmark or the market. The higher the IR, the greater the investment has performed compared to the benchmark. If the IR is zero will this imply that the portfolio has the same return as the benchmark, and if IR is negative the investment have performed worse compared with the benchmark.

The formula for computing Information Ratio is: $IR = \frac{\bar{r}_p - \bar{r}_m}{\sigma(r_p - r_m)}$, where $\sigma(r_p - r_m)$ is the portfolios tracking error.

The IR can be tested with the null hypothesis H_0:IR=0 with a t-test given by: $t - value = IR\sqrt{n}$.

The IR when the Beta is assumed equal to one, the formula can be interpreted as: $IR_{B=1} = \frac{\alpha_p}{\sigma(e_p)}$, and measures the abnormal return per unit risk, which in principle could have been diversified away if instead holding a broad market index.

10.5.7. Jensens Alpha

$\alpha_p = \bar{r}_p - \left[\bar{r}_f + \beta_p(\bar{r}_m - \bar{r}_f)\right]$, this performance target is explained in the section 10.4 where SIM is explained. The Jensen Alpha is the intercept in the SIM regression model. Shortly explained; Alpha measures "excess return" greater or less than the market, after adjusting for systematic risk.

10.6. Traditional asset allocation

10.6.1. Global Minimum Variance Portfolio

This portfolio will give the lowest possible risk on the portfolio. And will as figure 1.4 shows this portfolio will be placed as far as possible to the left at the portfolios efficient front.

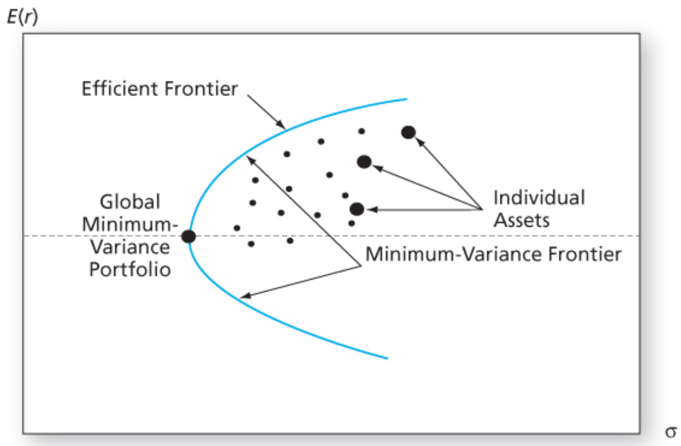

Figure 16: Minimum variance portfolio

In general this will be the variance derived on the weights, i.e. $\frac{\delta\sigma_p^2}{\delta w} = 0$

10.6.2. Maximum Sharpe-ratio Portfolio

This model can either be interpreted as minimizing the risk for a given return, or maximize the return for a given level of risk. This can be formulated mathematical as:

$$\min \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \text{ subject to } \sum_{i=1}^{N} w_i \bar{r}_i \text{ and } \sum_{i=1}^{N} w_i = 1.$$

This will yield the same result as maximizing the Sharpe-ratio explained in section 1.2.1 subject to $\sum_{i=1}^{N} w_i = 1$, and will be the tangent portfolio of the efficient front i.e. the "Optimal Risky Portfolio" (P), shown in figure 1.5. This portfolio is also called the Markowitz mean-variance portfolio.

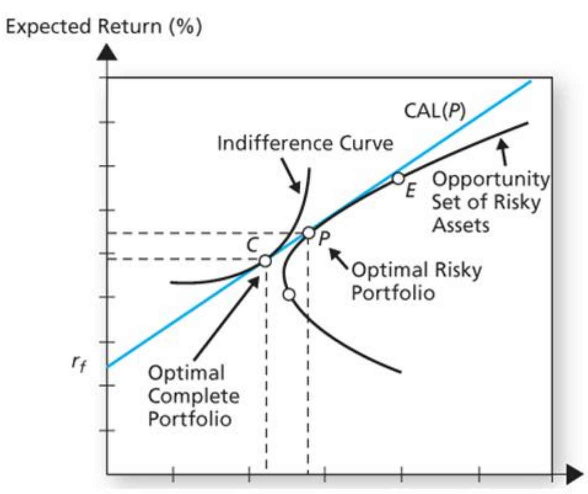

Figure 17: Optimal Risky portfolio, Maximum Sharpe-ratio.

10.6.3. Equally weighted portfolio

This is probably the most naïve portfolio, and simple portfolio one can assemble. With a great amount of asset one can get a great diversified portfolio. The portfolio will be as it name states:

$w_i = \frac{1}{n}$, where $\sum_{i=1}^{n} w_i = 1$

10.7. Risk Parity Allocations

The risk parity approach defines a portfolio where all asset classes have the same marginal contribution to the total risk of the portfolio, i.e. an equally weighted portfolio where the weights refer to the risk rather than the dollar invested in each asset. This indicates that the risk parity portfolios make relatively large allocation to low risk assets, and vice versa.

10.7.1. Volatility-weighted (Naïve Risk Parity)

This approach is the simplest of the Risk Parity portfolios. This mainly because this allocation technique only relies on the standard deviation as the parameter input, and not considers the covariance between the assets. In words the asset weight is given by the inverse of the standard deviation of the asset divided by the sum of the inverse of the assets standard deviations.

$$w_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{n} \frac{1}{\sigma_j}}$$

This portfolio will yield the same result as the ERC portfolio if there are only two assets, or if the correlation between the asset return are the same, and will in this thesis stated as NRP.

10.7.2. Equally Risk Contribution (ERC)

The marginal contribution of an asset (RC_i) to the total risk of the portfolio, when standard deviation is the risk measure, is given by the formula:

$RC_i = w_i \frac{\delta \sigma_p}{\partial w_i}$, where the last term determines the change in the total risk of the portfolio if a small change in the weights of asset occurs. Solving the derivation problem, one obtains:

$$RC_i = w_i \frac{\sum_{j=1}^{N} w_j Cov(r_i, r_j)}{\sigma_p}$$

Thereafter, as the name states, one set the risk contribution equal to each other:

$$w_i \frac{\delta\sigma_p}{\partial w_i} = w_j \frac{\delta\sigma_p}{\partial w_j} \quad \forall i, j$$

And the Euler decomposition satisfies (Denault, 2001):

$$\sigma_p = \sum_{i=1}^{n} RC_i$$

In the presence of and full investments constraints, finding a closed-form solution for this ERC optimization weighting scheme is not possible due to an issue of endogeneity: w_i is a function of the risk contribution which, by definition, depends on w_i. And by that, the following numerical optimization algorithm provided by Teiletche et al. (2010) will in this thesis be used:

$$\underset{w^*}{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[w_i \frac{\delta\sigma_p}{\partial w_i} - w_j \frac{\delta\sigma_p}{\partial w_j} \right]^2 \quad \text{s.t.} \quad \sum_{i=1}^{N} w_i = 1 \quad \& \quad 0 \leq w_i \leq 1$$

10.7.3. ERC VaR

In this case when using VaR as the risk measure, the properties and optimization algorithm explained above will be the same with this risk measure.

When looking at ERC when having VaR as the risk measure and the distribution is Gaussian we otain[19]:

$$RC_i = w_i \left(-\mu_i + \Phi^{-1}(\alpha) \frac{\delta R_p}{\partial w_i} \right)$$

And for the general case stated by Gouriéroux et al. (2000), this will be extended and it can be shown that the risk contribution is equal to:

$RC_i = \mathrm{E}\left[L_i \middle| L_p = VaR_\alpha (L_p) \right]$, where L is loss.

[19] Φ^{-1} is the inverse of the CDF of the standard normal distribution, and R is the risk measure.

10.7.4. ERC CVaR

The risk contribution when using CVaR as the risk measure can be interpreted as[20]:

$$RC_i = w_i\left(-\mu_i + \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}\frac{\delta R_p}{\partial w_i}\right)$$

And for the general case, i.e. the non-Gaussian, this risk contribution can be generalized in the following expression stated by Tasche (2002):

$RC_i = \mathrm{E}\left[L_i \middle| L_p \geq VaR_\alpha(L_p)\right]$, and can be calculated using the method conducted by Rachev et al. (2008):

$$RC_i = \frac{\left(\frac{1}{\left[\alpha M\right]}\sum_{j=1}^{\left[\alpha M\right]} r_i^{\,j}\right)}{CVaR_\alpha(r_p)}$$, where r is return and M is the return scenarios.

10.7.5. Most Diversified Portfolio

This portfolio is defined by Choueifaty and Coignard (2008), and aims to minimize the Diversification Index (DI) explained below.

The portfolio is consistent with the marginal risk contribution divided by the volatility:

$$\frac{1}{\sigma_i}\frac{\delta\sigma_p}{\partial w_i} = \frac{1}{\sigma_j}\frac{\delta\sigma_p}{\partial w_j}$$

[20] ϕ is the PDF of the standardized normal distribution

10.8. Diversification and concentration measures

To see how well the portfolio is diversified in terms of weight and risk there have in this thesis been used four different measurements of diversification. This is done because of the difficulties of defining diversification, and to provide robust results.

10.8.1. Herfindahl index

The normalized version of the Herfindahl index[21]: $\frac{n * H(\pi) - 1}{n - 1}$, where $H(\pi) = \sum_{i=1}^{n} \pi_i^2$.

This index takes the values of $[0,1]$ where 0 is the most diversified and 1 is the least diversified.

10.8.2. Gini Index

For this computation, we use follow the calculations as Chaves et al. (2012). Weights or risk contribution are sorted in ascending order and thereby computed by:

$\frac{2}{n} \sum_{i=1}^{n} i(\pi_i - \overline{\pi})$, and the lower the number, the better diversified.

10.8.3. Diversification Index

This is the ratio of the total portfolio risk divided by the weighted average of the standard deviation, and defined by Tasche (2007), which also came up with the concept of the MDP.

$$DI = \frac{\sigma_p}{\sum_{i=1}^{n} w_i \sigma_i}$$

The lower the number, the better diversified and lower concentrated is the portfolio.

10.8.4. Shannon Entropy

$$SE(\pi) = -\sum_{i=1}^{n} \pi_i \ln \pi_i$$

[21] π_i represent either an asset class weight or a risk contribution.

This reaches it maximum value of ln(n) if the asset class weights or risk contribution is identical, i.e. lowest concentration, and when there is heavy concentration in few asset the value is close to zero.

When all types in the dataset of interest are equally common, all π_i values equal 1/n, and the Shannon index hence takes the value ln(n). The more unequal the abundances of the types, the larger the weighted geometric mean of the π_i values, the smaller the corresponding Shannon entropy will be. If practically all abundance is concentrated to one type, and the other types are very rare (even if there are many of them), Shannon entropy approaches zero. When there is only one type in the dataset, Shannon entropy exactly equals zero (there is no uncertainty in predicting the type of the next randomly chosen entity).

10.9. Investment Capacity

This calculations is based on NBIMs Discussion Note #7-2012[22]

To measure the investment capacity there will in this thesis be conducted Relative Investment Capacity (RIC) as an extension for the turnover for the portfolio, this because of a high turnover and low RIC would limit the scale at which one can deploy capital. The RIC measure shows to what extent one can deploy capital to a given portfolio without any constraints.

This is a novel way to measure investment capacity, which is a distinct but complementary measure of liquidity. The measure of RIC provides an answer to how much capital one can deploy to a given portfolio without any constraints as a percentage of the market value-weighted-portfolio. NBIM studied portfolios weighted by characteristics, and found that portfolios with low RIC have high turnover and require more active management to rebalance the portfolio.

The RIC will be calculated in the three following ways due to the robustness of the test for transaction costs:

Investment capacity ratio: $ICR_i^j = \frac{w_i^{MVW}}{w_i^j}$, where MVW is the market value weight. The intuition is that if this is above 1 then the market has high capacity to absorb capital into an asset I in a portfolio j. To compare this ratio across portfolios one need to compute $SIZE_j$ of the portfolio

[22] http://www.nbim.no/globalassets/documents/dicussion-paper/2012/discussionnote_7-12_final.pdf

relative to the market value weighted portfolio as the sum of market value weights of the assets in portfolio j. I.e. $SIZE_j$ takes the stocks in portfolio j and answer what percentage these stocks make of the market value weighted, so:

$$RIC_j = ICR_j * SIZE_j$$

And there will be calculated three different RICs due to the robustness of the test for transaction costs:

10.9.1. Bottleneck RIC

$$ICR_j^B = \min_i \left(\frac{w_i^{MVW}}{w_i^j} \right) \text{ where } \left(\frac{w_i^{MVW}}{w_i^j} \right) \leq 1 \text{ and } w_i^j > 0$$

This yields how much money one can deploy to an alternative approach as a percentage of the MVM portfolio.

10.9.2. 5th Percentile RIC

Since Bottleneck can be prone to outliers, we also compute the 5th percentile RIC that focuses on other than the bottleneck assets. $RIC_j^{5\%}$ is computed as the lowest 5th percentile investment capacity ratio with the same constraints as the bottleneck, i.e. can be seen as an VaR calculation.

10.9.3. Weighted average RIC

The weighted average ICR is given by:

$$ICR_j^{WA} = \frac{\sum_{i=1}^{N} w_i^{MVW} \left(ICR_i^j \right)}{SIZE_j} \text{ where } ICR_i^j \leq 1 \text{and } w_i^j$$

Norges miljø- og
biovitenskapelige
universitet

Postboks 5003
NO-1432 Ås
67 23 00 00
www.nmbu.no