The Planck Mass Density Radius of the Universe

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Abstract

What is the size of the universe if we take the estimated mass in the observable universe and compress it until we reach the Planck mass density? We will investigate this using both the Friedmann model and the recent Haug model of the universe that takes into account Lorentz relativistic mass. The resulting size is approximately that of a proton. We will also look at a hypothetical Planck mass size universe. In the Friedmann model one needs rapid expansion of mass to maintain the initial Planck mass density or, alternatively, only space expansion with the density decreasing from its initial conditions, while in the Haug model the Planck mass universe seems to cause no such "strange" predictions.

Index Terms

Friedman model, Haug model, micro black holes, Planck density, Planck units, Planck mass universe.

I. Short historical background of the Planck units and their current standing

In 1899, Max Planck [1], [2] assumed there were three important universal constants, namely the speed of light, Newton gravitational constant and the Planck constant. Combining these with dimensional analysis he derived a fundamental length $l_p = \sqrt{\frac{G\hbar}{c^3}}$, a fundamental time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, a fundamental mass $m_p = \sqrt{\frac{\hbar c}{G}}$ and a fundamental temperature (energy) $T = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$. These are known as the Planck units.

The Planck units are often considered maximums and minimums for the properties to which they are related. For example, the Planck length and the Planck time are assumed by many physicists [3], [4], [5], [6] to be the smallest possible observable length and time, even in thought experiments. Gorelik and Ozernoi [7] suggested, in 1978, that the Planck temperature is the maximum temperature (energy) possible. In 1981, Caianiello [8] was likely the first to suggest that there is probably a maximum acceleration. Scarpetta [9], three years later, was likely the first to introduce what he called the Planck acceleration, also providing the formula (see also [10], [11])

$$a = \sqrt{\frac{c^7}{G\hbar}} \tag{1}$$

which corresponds to $a = \frac{c^2}{l_p}$. He suggested that this is the maximal proper acceleration against a vacuum, a view also presented by Fala and Landsberg [12] in 1994. Actually, the Planck acceleration is so enormous that in one Planck time it accelerates an object from zero (rest) to the speed of light. And as nothing can move faster than c, this must be the maximum acceleration if the Planck time is the minimum time[1].

In 1970 Harrisson [16] was possibly the first to describe the Planck mass density (or at least something very close to it), and he presented it as

$$\rho_{p,c} = \frac{m^*}{\frac{4}{3}\pi\lambda^{*3}} = \frac{3c^5}{16\pi G^2\hbar} \tag{2}$$

where Harrison defined $m^* = \sqrt{\frac{\hbar c}{2G}} = \frac{m_p}{\sqrt{2}}$, and λ^* simply as the reduced Compton wavelength of this mass $\lambda^* = \hbar/(m^*c)$. So it is simply the modified Planck mass of Harrison divided by the sphere with radius equal to the reduced Compton wavelength of that mass. This further corresponds to

$$\rho_{p,c} = \frac{3\frac{\hbar}{l_p}\frac{1}{c}}{16\pi l_p^3} = \frac{\frac{1}{\sqrt{2}}m_p}{\frac{4}{3}\pi(l_p\sqrt{2})^3} \tag{3}$$

This is slightly different than the Planck mass density as known today, this because Harrison defines a mass that is the Planck mass divided $\sqrt{2}$, and the reduced Compton wavelength of this mass is $l_p\sqrt{2}$. That the Harrison's Planck mass density is not simply the Planck mass divided by the Planck sphere volume is likely due to Harrison having developed it in relation to the

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[1]When we talk about maximum speed c, we are talking about it as measured with Einstein synchronised clocks; in rotating frames, for example, it could be more complicated, see discussion by [13], [14], [15].

Friedmann model of the universe and trying to get it to fit certain aspects of this. Further the Planck mass divided by $\sqrt{2}$ is the only mass that has a reduced Compton wavelength equal to the Schwarzschild radius $R_s = \frac{2GM}{c^2}$ under general relativity theory. In this paper, we will define the Planck mass density as simply the Planck mass divided by the volume of a sphere with radius l_p. We thus have the Planck mass density as

$$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3} \tag{4}$$

and naturally Planck energy density equal to this times c^2. Sometimes the Planck energy density is expressed as $m_p c^2/l_p^3$, but this would mean that the Planck mass is packed inside a cube with sides equal to the Planck length — something we think is less realistic than it being packed inside a spherical shape, though this can naturally be discussed.

Just like the Planck length is assumed to be the minimum length, the Planck time the minimum time, the Planck temperature the maximum temperature and the Planck acceleration the maximum acceleration, we conjecture that the Planck density could be the maximum possible density, and as we will see this must have implications for the hypothesis of the early universe starting in the Big Bang, as well as the Big Bang model and the interpretation of micro black holes with Planck mass size.

II. What Is The Size Of The Universe If Compressed To The Planck Density?

The Friedmann universe is a well-known solution to Einstein's [17] general relativistic field equation. The critical mass density of the universe in the Friedmann [18] model is given by

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{5}$$

where H_0 is the Hubble constant. This critical mass density is well known from standard text books (see, for example, [19], [20], [21]). And since the Hubble radius is given by $R_H = \frac{c}{H_0}$, this leads to a critical mass (see Hoyle [22] and Valev [23]) in the Friedmann universe of

$$M_c = \frac{1}{2}\frac{c^3}{GH_0} \tag{6}$$

This is an equivalent mass, as we do not distinguish how much of this is either energy or mass, as here we naturally also have $E = mc^2$. To try to find out how much of this is either energy or mass is a topic outside the scope of this paper and will not play a role in our reasoning.

The Haug [24], [25] model of the universe that we will also look at takes into account Lorentz relativistic mass, something that is ignored in general relativity theory. Already in 1899, Lorentz [26] suggested that mass should be relativistic, of the form

$$m = m_0\gamma \tag{7}$$

where γ is the standard Lorentz factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$. Equation 7 is what current text books give as relativistic mass in chapters discussing special relativity theory [27], [28], [29], [30], [31], however they typically do not seem to be aware or mention that this was an invention of Lorentz, and did not arise from special relativity theory. In the end of his famous 1905 paper on special relativity theory, Einstein [32] also provides two relativistic mass formulas, one for transverse mass $m = m_0\gamma^2$ and one for longitude mass $m = m_0\gamma^3$. His longitude mass was the same as the longitude mass given by Lorentz. Einstein's transverse mass is different from Lorentz's transverse mass and is not considered to be correct. Max Planck suggested, in 1906, relativistic momentum as we know it today $p = mv\gamma$. Some years after the invention of Minkowski [33] space-time, Einstein abandoned relativistic mass altogether and instead relied on relativistic momentum (incorporated in fourth-momentum). This also led to general relativity theory being developed without relativistic mass. The general relativity community is to this day negatively predisposed to relativistic mass (see, for example, [34], [35], [36], [37]). However, some researchers who have been supporting special relativity theory have been defending relativistic mass (see, for example, Rindler [38], [39] and Jammer [40]). Nonetheless, the implications of taking relativistic mass into account in gravity theory have never been fully investigated. With relativistic mass in gravitational theory, we are in most cases interested in what we will observe in terms of gravity when we are observing the gravitational phenomena from mass M that acts on mass m; the mass m can move relative to the observer frame M and therefore be relativistic. By also considering relativistic mass, Haug [24] has recently derived the mass of the observable universe, obtaining

$$M_H = \frac{c^3}{GH_0} \tag{8}$$

This is simply twice that of the critical mass in the Friedmann universe (Eq. 6). Assuming the mass is inside a spherical shape, the mass density is given by $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$. The radius of the Friedmann critical universe if it is compressed to Planck mass density must be

$$\begin{aligned}
\frac{m_p}{\frac{4}{3}\pi l_p^3} &= \frac{M_c}{\frac{4}{3}\pi R^3} \\
\frac{m_p}{l_p^3} &= \frac{M_c}{R^3} \\
R^3 &= \frac{M_c l_p^3}{m_p} \\
R^3 &= \frac{\frac{c^3}{2GH_0} l_p^3}{m_p} \\
R &= \frac{cl_p}{\sqrt[3]{2GH_0 m_p}}
\end{aligned} \tag{9}$$

Further, we can solve the Planck length formula with respect to G, and this gives $G = \frac{l_p^2 c^3}{\hbar}$ (see [41], [42]), and we also have $H_0 = \frac{c\bar{\lambda}_c}{2l_p^2}$, where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical Friedmann universe. In addition, we have $m_p = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_p}\frac{1}{c}$. This means that we can re-write the equation above as

$$R_{p,c} = \frac{cl_p}{\sqrt[3]{2GH_0 m_p}} = \sqrt[3]{\frac{l_p^4}{\bar{\lambda}_c}} \approx 2.59 \times 10^{-15} \text{ m} \tag{10}$$

where the calculated number is from a Hubble constant of $H_0 = 70\ (km/s)/Mpc$, which is well inside the range that has been measured in recent years [43], [44]. Be aware that in recent years we have proved that the Planck length and Planck time can be found independent of any knowledge of G, and even independent of G and h (see [45], [46], [47], [48], [49]). In the Haug universe model we get (here $H_0 = \frac{c\bar{\lambda}_u}{l_p^2}$, where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the Haug universe)

$$R_{p,h} = \frac{cl_p}{\sqrt[3]{GH_0 m_p}} = \frac{cl_p}{\sqrt[3]{c^3 \frac{\bar{\lambda}_u}{l_p}}} = \sqrt[3]{\frac{l_p^4}{\bar{\lambda}_u}} \approx 3.27 \times 10^{-15} \text{ m} \tag{11}$$

Interestingly, this is about four times the Proton radius 0.8414×10^{-15} m (CODATA 2019) value. This means that if the observable universe is compressed to the Planck mass density, it would basically be four times the size of a proton (in terms of radius). This is quite remarkable. Still, the proton radius is enormous compared to the Planck length. In the Big Bang hypothesis, it is even assumed that the universe started in a singularity, that is, in a point with no spatial dimension. It is assumed by many physicists that there can be no shorter observed length than the Planck length. We conjecture that there can also be no higher density than the Planck mass density. If so, then the universe could not have started or ever been in a singularity as suggested in the Big Bang hypothesis. The universe can at most have been inside a sphere with four times the proton radius.

That the size of a compressed Planck mass density universe has a radius equal to approximately four times the proton radius is likely a pure coincidence. Still, one can easily calculate the value of the Hubble constant that gives the radius of the Planck mass density universe exactly equal to, for example, three times the Proton radius. In the Friedmann model it is

$$H = \frac{c^3 l_p^3}{2Gm_p(3R_p)^3} \approx 75.23 \tag{12}$$

based on the fact that we are using CODATA 2019 values for the Planck mass and the proton radius, the formula above (and below) is in addition multiplied by $3.26 \times c \times 10^3 \times 365 \times 24 \times 60 \times 60$ to get the output units to (km/s)/Mpc as normally is the standard used for the Hubble constant. In the Haug model we get a Hubble constant of

$$H = \frac{c^3 l_p^3}{Gm_p(4R_p)^3} \approx 63.48, \tag{13}$$

if we set the Planck mass density radius of the universe equal to four times the proton radius. Again, we think it is a pure coincidence that the radius of a Planck mass density compressed universe is about three to four times that of the proton radius. It is also interesting that the proton radius is almost exactly four times that of the reduced Compton wavelength of the proton (see Bohr and Trinhammer [50]).

III. The Planck mass density radius for a hypotehtical universe with the Planck mass

In the previous section we looked at the compression of the universe mass to Planck mass density. We can generalise this to hold for a universe of any hypothetical mass (energy). In general relativity [17] and the Friedmann model we have

$$R_{p,gr} = \sqrt[3]{\frac{l_p^4}{\bar{\lambda}_c}} \tag{14}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical mass of the universe. Assume now for a moment that the universe only has a mass equal to the Planck mass. A Planck mass has a reduced Compton wavelength of the Planck length, and gives

$$R_{p,gr} = \sqrt[3]{\frac{l_p^4}{l_p}} = l_p \tag{15}$$

Further, in the Haug universe model we get the same result

$$R_p = \sqrt[3]{\frac{l_p^4}{\bar{\lambda}_u}} = \sqrt[3]{\frac{l_p^4}{l_p}} = l_p \tag{16}$$

We see that in the special case of the reduced Compton wavelength of the mass being the Planck length, the Planck mass density radius is the Planck length in both the Friedman and in the Haug model. This is fully consistent with that a Planck mass divided by the volume of a sphere with Planck length radius is the Planck mass density.

IV. Does the Planck mass universe indicate the incompleteness of general relativity theory?

Assume we have a Planck mass universe. According to the Friedmann [18] equation the critical universe is then described by

$$\begin{aligned} H_p^2 &= \frac{8\pi G\rho_p}{3} \\ H_p^2 &= \frac{8\pi G\frac{m_p}{\frac{4}{3}\pi l_p^3}}{3} \\ H_p^2 &= \frac{2Gm_p}{\frac{c^3}{H_p^3}} \\ m_p &= \frac{c^3}{2GH_p} \end{aligned} \tag{17}$$

This means the Hubble radius of a Planck mass universe must be given by

$$H_p = \frac{c^3}{2Gm_p} \tag{18}$$

And since the Hubble radius is equal to $\frac{c}{H_p}$, this means that the Hubble radius of a Planck mass universe is given by

$$R_H = \frac{c}{H_p} = \frac{c}{\frac{c^3}{2Gm_p}} = \frac{2Gm_p}{c^2} = 2l_p \tag{19}$$

So the density in this universe, if everything is inside the Hubble sphere, must be

$$\frac{m_p}{\frac{4}{3}\pi R_H^3} = \frac{m_p}{\frac{4}{3}\pi 8l_p^3} = \frac{1}{8}\rho_p \tag{20}$$

However, the universe cannot have the Planck mass density and 1/8 of the Planck mass density at the same time. So, either the mass must have expanded by eight times since the beginning of this universe, or the density has decreased due to the expansion of space. Both suggestions are in our view absurd. Still, this is closely related to why researchers of general relativity assume the universe is expanding and accept the Big Bang theory. It is all rooted in interpreting observations through a mathematical lens known as general relativity theory and the Friedmann solution to GRT.

Einstein and general relativity theorists have abandoned Lorentz relativistic mass [34], [36], [37], even before investigating the many things relativistic mass leads. If, on the other hand, we take into account Lorentz [26] relativistic mass, then we get the recent Haug [24], [25] model of the universe, which is

$$H_p^2 = \frac{4\pi G\rho}{3} \tag{21}$$

It looks very similar to the Friedmann critical universe model, but there is a big and important difference. First, this is not a model of the critical universe, but a model of the full observable universe, as the constant κ cancels out in the derivation when using relativistic mass. Second, the Friedmann equation has 8 on the right side rather than 4, ($H_0^2 = \frac{8\pi G\rho}{3}$), which leads to very big differences in interpretation, as we soon will see. The Hubble constant in the Haug model for a Planck mass universe must be given by

$$H_p = \frac{c^3}{Gm_p} \tag{22}$$

and the Hubble radius of the Planck mass universe is

$$\frac{c}{H_p} = \frac{Gm_p}{c^2} = l_p \tag{23}$$

The density of the universe is now the Planck mass density, and there is no need for predictions such as the expansion of mass or the expansion of space in order to have a consistent model, as is required if one builds the model on general relativity theory. Including relativistic mass also means that one needs no dark energy to get the model to fit supernova data [51]. Further, wormholes, which never have been observed, are mathematically forbidden when one includes relativistic mass [52].

V. Planck mass density radius, the escape radius, the light speed acceleration radius and the light orbital radius

The light speed acceleration radius [53] is where an object acted upon by the acceleration field is accelerated from zero (rest) to the speed of light in the Planck time. Basically solving

$$c = \frac{GM}{R^2} t_p \tag{24}$$

with respect to R. This radius is the same when derived from Newton gravity, general relativity theory and the Haug gravity theory, and is given by

$$R_l = \sqrt{\frac{GMl_p}{c^2}} = \sqrt{\frac{l_p^3}{\bar{\lambda}}} \tag{25}$$

Further, the radius where the escape velocity is c, when taking into account Lorentz relativistic mass, [24] is given by

$$R_h = \frac{GM}{c^2} = \frac{l_p^2}{\bar{\lambda}} \tag{26}$$

This is half the Schwarzschild radius $R_s = \frac{2GM}{c^2} = \frac{2l_p^2}{\bar{\lambda}}$. We also potentially have a special radius where the orbital velocity is c, which we get by solving the orbital velocity formula with respect to R

$$v_o = c = \sqrt{\frac{GM}{R}} \tag{27}$$

This gives

$$R_o = \frac{GM}{c^2} \tag{28}$$

This radius should be the same for general relativity theory and when one assumes the small mass m is relativistic. Further, we have the radius where the gravitational mass has Planck mass density, as derived in the section above. It is

$$R_p = \sqrt[3]{\frac{l_p^4}{\bar{\lambda}}} \tag{29}$$

Interestingly, for a mass with reduced Compton wavelength of $\bar{\lambda} = l_p$, that is the Planck mass, all these radiuses are identical when taking into account relativistic mass

$$R_h = l_p = R_l = R_o = R_p \tag{30}$$

This is not the case in general relativity theory where we have

$$R_s = 2l_p > l_p = R_l = R_o = R_p \tag{31}$$

That the Schwarzschild radius is different and also larger than R_l, R_p and R_0 is likely the reason why these other radiuses have not been discussed in general relativity theory. They will always be inside the black hole where the "standard" laws of physics are assumed to not hold. On the other hand, for a micro black hole, when taking into account relativistic mass, all these radiuses are the same: they are all radiuses extending to the surface of the micro black hole (the Planck mass). Table 1 sumamrises the four radiuses described above.

TABLE I
THE TABLE SHOWS FOUR DIFFERENT RADIUSES RELATED TO GENERAL RELATIVITY THEORY AS WELL AS WHEN WE TAKE INTO ACCOUNT LORENTZ RELATIVISTIC MASS.

Radius	General relativity	Taking into account relativistic mass
Acceleration to c in t_p	$R_l = \sqrt{\frac{GMl_p}{c^2}} = \sqrt{\frac{l_p^3}{\lambda}}$	$R_l = \sqrt{\frac{GMl_p}{c^2}} = \sqrt{\frac{l_p^3}{\lambda}}$
Planck mass density	$R_{p,c} = \sqrt[3]{\frac{G^2M^2\lambda}{c^4}} = \sqrt[3]{\frac{l_p^4}{\lambda}}$	$R_{p,h} = \sqrt[3]{\frac{G^2M^2\lambda}{c^4}} = \sqrt[3]{\frac{l_p^4}{\lambda}}$
Where escape velocity is c	$R_s = \frac{2GM}{c^2} = 2\frac{l_p^2}{\lambda}$	$R_h = \frac{GM}{c^2} = \frac{l_p^2}{\lambda}$
Where orbital velocity is c	$R_o = \frac{GM}{c^2} = \frac{l_p^2}{\lambda}$	$R_o = \frac{GM}{c^2} = \frac{l_p^2}{\lambda}$

Max Planck did not suggest exactly what he thought the Planck mass could represent. Eddington [54], in 1918, indicated the Planck length (and thereby likely the Planck time and Planck mass) had to play an important role in a future quantum gravity theory. In 1967, Markov [55] was likely the first to link the Planck mass to the mathematical properties of black holes, something he called Maximons. His paper was published in a Soviet journal and has received little attention in the West, perhaps also due to the Cold War that was then raging, and now unfortunately seems to be returning. Hawking [56] described, in 1971, much the same as Markov in relation to micro black holes. Hawking suggested that a mass of approximately the Planck mass would have properties similar to a black hole, and it was coined a micro black hole.

In general relativity theory, one cannot match more than one or at most two properties of the Planck scale for a given mass candidate for a micro black hole, something that has hardly been discussed. With the properties of the Planck scale, we think of such things as the Planck mass, the Planck length, the Planck time, the Planck temperature, the Planck acceleration and the Planck density. Assume that the exact Planck mass, $m_p = \sqrt{\frac{\hbar c}{G}}$, is a micro black hole. Then its Schwarzschild radius is $R_s = \frac{2Gm_p}{c^2} = 2l_p$. This is different from the Planck length, so one is not matching the Planck length. Further, the gravitational acceleration at the surface of this micro black hole is $g = \frac{Gm_p}{R_s^2} = \frac{c^2}{4l_p}$, in other words, one-fourth of the Planck acceleration. Moreover, the average density is now $\frac{m_p}{\frac{4}{3}\pi R_s^3} = \frac{1}{8}\frac{m_p}{\frac{4}{3}\pi l_p^3}$, in other words, just one-eighth of the Planck mass density, just as in the Planck mass universe under the Friedmann model. Motz and Epstein [57], in 1979, suggested a micro black hole with mass equal to half the Planck mass, which means that the Schwarzschild radius is identical to the Planck length, but it no longer matches the Planck mass nor the Planck acceleration or the Planck density. Haug [58], in 2016, and Faraoni [59], in 2017, suggested a micro black hole candidate with mass $\sqrt{\pi}m_p$. In such, the escape velocity is c at the Compton wavelength, which is now also identical to the Schwarzschild radius, but it does not match any other aspect of the Planck scale. For example, a micro black hole with mass $\frac{1}{4}m_p$ matches the Planck acceleration at the Schwarzschild radius, but no other Planck mass properties are perfectly matched; its mass is likely not able to create a micro black hole as its Compton wavelength is now outside its Schwarzschild radius, and further, the Schwarzschild radius is now $\frac{1}{2}l_p$, which is impossible if the Planck length is the smallest possible length. There is actually no mass candidate for a micro black hole in general relativity theory that matches more than one or two properties of the Planck scale. One could only have hoped that the exact Planck mass would do this, but it clearly does not do so.

One can naturally ask why a micro black hole should fit more than one or two properties of the Planck scale or why there could not be a series of different micro black holes. The latter cannot be excluded, but what is particularly interesting is that when taking into account relativistic mass, the Planck mass micro black hole fits all the aspects of the Planck scale. The Planck mass then has a radius where the escape velocity is c at the Planck length, $R_h = \frac{Gm_p}{c^2} = l_p$; it therefore has the Planck mass density; the Planck acceleration at the radius where the escape velocity is c; it takes the speed of light the Planck time to travel this radius; converted to energy, it has the Planck energy; etc. – it basically matches every single aspects of the Planck scale. We think this could be more than mere coincidence. We think this could further be related to the interpretation of the universe. When considering the relativistic mass, we have shown that for a hypothetical Planck mass universe the Hubble radius and the Planck length are the same, so there is no indication of expanding space. In the Friedman model, on the other hand, one needs expanding space, as the universe density is different than the input density. We could start with similar reasoning for micro black holes: to get them to match all the aspects of the Planck scale we could, for example, claim the Schwarzschild of the Planck mass candidate had shrunk to the Planck length after it came into existence. However, such interpretations are excluded when we take into account relativistic mass, as all properties of the Planck scale are matched for the Planck mass candidate to a micro black hole.

VI. Conclusion

We have investigated what we can call the Planck mass density radius. The magnitude of this radius is of the order of the proton radius for both the Friedmann universe and the Haug universe. In the special case of a universe with a mass equal to the Planck mass, the Hubble radius in the Haug universe is equal to the Planck length, while in general relativity theory the Hubble radius is twice the Planck length. This means that in the Freedman universe the mass must have expanded to maintain the initial Planck mass density or, alternatively, space has expanded, and the resulting density is now different than it was at the initial conditions (input). If we take into account Lorentz relativistic mass, as done in the Haug universe, there is no need for such things as expanding space or expanding mass. The Friedmann model also seems to need an expansion phase for a Planck mass size universe, though when taking relativistic mass into account, the universe seems to be consistent with a steady state. We also pointed out that in general relativity theory there can be no mass candidate that fits all the properties of the Planck scale, while when taking relativistic mass into account, all the Planck scale properties are matched for a micro black hole with mass equal to the Planck mass.

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