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Principal Component Analysis of Swap Curve Movements in Two Different Swap Markets: The Norwegian -and the Euro Swap Market

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Abstract

Analyzing and modeling the risk inherent in term structure of interest rates is crucial for participant in financial markets, e.g. banks and investors with portfolios containing a large portion of interest rate sensitive securities. This thesis aims to identify what factors that have historically driven the shape of the swap curves in the Norwegian –and euro swap market from 2000-2014, with corresponding sub-periods analyzed. By applying Principal Component Analysis (PCA) on basis point changes for swap rates with different maturities in both swap curves I was able to identify several interesting features from the sample period(s), 1) movements in the Norwegian swap curve has been more volatile than in the case of the euro swap curve, thus indicating that more emphasis on the second principal component is needed for reaching a sufficient level of explanatory power, 2) various interest rate regimes yields different results. Again, movements in the Norwegian swap curve are more volatile; outputs from PCA indicate that the shape of the Norwegian swap curve fluctuates more given the prevailing interest rate regime.

Content

1. Introduction

Value and pricing of fixed-income securities, such as bonds, swaps, loans etc., are closely linked to standard market interest rate(s). Further the term structures of interest rates are often few defined benchmarks in financial markets. Example of such term structures could be treasury notes, bills and/or yield on bonds with different credit ratings, or interest rate swaps. The initial price of a fixed income security is usually derived from the prevailing term structure of interest rates (also called discount rates). The uncertainty on how these rates will develop from today until maturity leads to speculation and volatility for fixed income securities, and influence the price of a security. How the individual interest rate change in the future is a major factor that influence the future value of a fixed income security. However, the term structure movements, and how the shape of the term structure develops in the future is even more influential on the value of a fixed income security with several cash-flows. The risk associated with term structure movements are often referred to as yield curve risk.

Modeling yield curve risk is a crucial task for owners of fixed income securities. A significant part of the economic value of a bank's assets and its income are closely linked to movements in interest rates and the corresponding term structure. This means that a portfolio is now exposed to changes in several market interest rates simultaneously. The changes on individual rates in the yield curve may shift and twist the shape of the curve, exposing the securities to different types of yield curve risk. Yield curves can contain as much as 60 different interest rates of different maturities. Even if a fixed income portfolio is not mapped to all of the maturities, the amount of risk factors that could potentially influence the P&L distribution of the portfolio could be very high indeed. So measuring interest rate -or yield curve risk at a high frequency (daily) is crucial, but also a resource - intense exercise. In addition to internal system and incentives for measuring interest rate risk, financial regulators also require that financial institutions measure and report their interest rate risk. The purpose is to make sure that these institutions incorporate interest rate risk measurements in their risk managements system, and that the level of accuracy in their interest rate risk-modeling is within prudent levels to ensure the safe and soundness of the individual bank (Basel Committee on Banking Supervision, July 2004). Besides the individual interest rates and term structures, other risk factors may be spreads between interest rate with different maturities and term structure of interest rates denoted in different currencies. So an exhaustive list of risk factors could be very high, making it computational demanding to incorporate in a risk management context.

For a risk manager who oversees a portfolio containing interest rate sensitive securities it is important to isolate the effect each risk factors has on the portfolio (Alexander 2008). The task could be challenging with many risk factors and thousands of different securities. However, risk factors inherent in term structure systems like swap curves very often show a high degree of correlation. So it is possible to turn a set of highly correlated risk factors in to a set of uncorrelated (orthogonal) risk factors; each risk factor has a unique interpretation on how it historically has influenced the movements in the term-structure.

Principal component analysis (PCA hereafter) is a statistical method that provides such a possibility. PCA offers two major advantages; 1) identifying the true sources of variations in term structures, hence risk, in the historical time series, and 2) reducing the number of risk factors down to a more manageable size. From PCA it is often possible extract 2-3 uncorrelated factors that together explain about 95 % of the total variation in term structure movements during the sample period.

Specifically, in the thesis I will perform PCA on the following research questions:

1. What kind of movements has historically driven the shape of the swap curves in the Norwegian -and the Euro swap market? In addition, are there any similarities or perhaps more interesting, differences between the two swap curves?

2. Are the findings consistent through various regimes of swap rate levels and movements? The aim is to identify two sub-periods of swap rate evolution; 1) a sustained period of falling trend in the swap rates, and 2) a sustained period of rising trend in swap rates. After applying PCA on the sub-samples is it possible to identify if the swap curves behave differently during various interest rate regimes.

The thesis it organized as follows. Section two gives an overview of the literature. Section three presents the data and further the descriptive statistics for the data sample. Section four describes the methodology applied, namely PCA. Section five is the main section, and contains PCA of the swap curves. In this section I will also elaborate on the key findings from the PCA; linking these against the research questions presented above. Lastly, in section six I draw conclusions.

2. Literature on the application of PCA in analyzing term structures of interest rates.

There is a vast amount of literature on issues regarding term structure of interest rates, and efforts on trying to explain the movements that are inherent in these systems by using PCA. In this section I will present a selection of the articles that has laid the basis for this thesis. They are presented in chronological order, and starts with the classical article by Litterman & Scheinkman (1991).

The aim for this article is to identify common factors that have affected returns on Treasury-based securities in the years between 1984 and 1988. Analysis conducted by the authors' show that returns on fixed-income securities can be explained by three distinct movements in yield curves. These movements are referred to as factors, and each factor have its unique interpretation on how the yield curve moves. The factors are called level, steepness and curvature.[1] The findings in the article have a lot in common with other articles regarding identifying factors that determine movements in a yield curve. In addition the authors point out that risk measures solely based on duration-hedging (only parallel shifts in a yield curve) would undermine the true risk of a fixed-income portfolio. So by taking into account additional measures of sensitivity a fixed-income portfolio has on its risk factors, the authors suggest that risk further can be eliminated through more accurate hedging.

Bertocchi et al. (2005) seek to capture changes in yields for corporate bonds with different credit ratings. They do so by applying PCA on term structures of corporate bond yields with different credit ratings. As a result the models they develop are able to capture nearly 98 % of all the variations in the corporate bond market. In addition, when similar calculations are obtained on several credit classes simultaneously the result seems to be somewhat consistent with calculating one credit rating class at a time. This could very well be due to high correlation

[1] These findings are confirmed in the article by Jones (1991). Jones divides Treasury yields in different maturity ranges, and finds that purely shift/twist interpretation of yield curve changes is not sufficient in explaining important historical movements. The author also argues that one must take into consideration the butterfly (curvature) movement in the yield curve to achieve a realistic approach.

between the different credit rating classes and their corresponding term structure, and further indicating that it could be beneficial for risk management purposes to include multiple term structures in a PCA.

Pérignon et al. (2007) analyzes the common factor structure of US, German and Japanese government bond returns. They use weekly data from the Meryl Lynch Government Bond Indices in the period January 1990 to October 1999. They argue that understanding commonalities between different currencies term structures may lead to more accurate hedging and risk management of international bond portfolios. However, they point out that it can be challenging to separate a common factor from a local factor, example; a local factor that contributes to most of the variations in one of the term structure may also be interpreted as a common factor that drives movements in the other term structures as well.

Novosyolov & Satchkov (2008) uses daily data on ten different term structures from the period August 2006 – August 2007. Among the selected term structures are both government and LIBOR rates from developed economies like US, UK and Euro. They purpose three different approaches for jointly describing global term structures of interest rates. The approach they find preferable is first performing a PCA on the local term structures. In step two they suggest decomposing the local PCs into a joint structure. As the authors put it, this approach preserves the following features; "important economic interpretations as shift, twist and butterfly moves in the yield curve" (p.49). They conclude that a global yield curve structure can be described by 15-20 factors. This approach is far less parsimonious than PCA on an individual yield curve.

One of the primary goals in the article by Barber & Copper (2012) is to determine whether the shape of the first principal component is persistence through time, and can the first principal component be looked upon as a level shift indicator for a yield curve. They do so by splitting their 10-year dataset into two sub-sets of five years each. Their analyses provide the following results;

a) The hypothesis that the first principal component can be interpreted as a level shift is rejected. In reality, the authors then proclaim that the first principal component cannot be regarded as shifting all maturities on a yield curve in a parallel manner. This further enhances the view that duration-based methods may not capture the true interest rate risk inherent in yield curve systems.

b) Anyhow, the shape of the first principal component seems to be persistence over time. This is true when all but the first six principal components are disregarded due to the notion that they contribute very little to the total variation.

What distinguishes this article is that they provide statistical tests to determine the robustness of the results from a PCA of historical yield curve changes. One drawback of this study is that they use data consisting of monthly observations. For the purpose of monitoring interest rate risk such a low frequency on observation of the risk factors may undermine the true risk. Correlations between low-frequency (weekly or monthly) data are likely to be higher than high-frequency (daily) data. This may lead to seemingly better outputs from PCA. The reason being that higher correlation generally leads to a more parsimonious model; a model where fewer factors describe more of the historical movements in the term structure. So by using monthly observations as input the results from PCA may be overestimated.

The aim of the article by Stelmach (2010) is to examine whether we can assume that there is only "one true" interest rate in the market, hence homogeneous interest rates. The author applies PCA to euro yield curve data in the period January 2007 – April 2010. To verify if such an assumption is valid through different interest rate regimes the author applies PCA to yield curve data on different time sub-periods. The main findings from his research are the following; a) only in fairly tranquil periods can the assumption of one true interest rate prevail. This is measured by the relative importance of the first principal component in the yield curve system; and b) in more turmoil periods, e.g. 2008-financial crisis, the relative importance of the first principal component decreases. During this period the second principal component increases its relative explanatory power, and adds to the notion that one true interest rate is less true in turmoil periods. PCA in this article is based on the correlation matrices for basis point changes in the different maturities. This approach utilizes only co-movements between maturities and ignores the volatility of the individual interest rates in the term structure. By considering the volatility of each interest rate during the sample period, an approach based on a covariance matrix may have lead to different and more realistic results.

Abdymomunov & Gerlach (2014) point at some limitations of PCA as a method, especially in the area of stress testing interest rate exposures. They highlight the importance of stress testing interest rate exposure in the prevailing low interest rate environment, where a sudden increase in interest rates may have serious implications for financial stability. Like other studies their data set[2] can also be explained by the first three principal components. However, they are reluctant to apply PCA as a method of generating stochastic scenarios on yield curve movements. The main argument being that PCA generates economical implausible scenarios.

Some of the above-mentioned studies, Novosyolov & Satchkov (2008) and Pérignon et al. (2007), aim to explain common factors in yield curves denoted in different currencies by applying PCA on joint covariance or correlation matrices. This approach may not be adequate in supervising multi-currency interest rate risk. It may be challenging to separate local –and common factors, and the element of parsimonious in the models is now less influential. Differently, this thesis uses another approach when determining multi-currency interest rate risk, which may be more realistic and easier to understand; namely to keep PCA on the two swap curves separated.

Further, what separates this thesis from the above-mentioned studies is the use of sub-periods. Relying on which principal components that drives movements in the swap curve, and how many components that is appropriate to explain most of the variation, solely based on one examined period may not reflect the true risk inherent in these interest rate systems. It is plausible that analyzing sub-periods with different characteristics will reinforce the impression on which components that has driven changes in the swap curves.

[2] Monthly zero-coupon Treasury yields. Sample period November 1985 – December 2012.

3. Data and descriptive statistics

3.1 What is an interest rate swap, and what characterizes a swap curve?

Trading with interest rate swaps has had an incredible growth the last decades. Today interest rate swaps are one of the most traded types of derivatives in financial markets. In 2014 The Bank for International Settlements reported that the total notional amount of outstanding OTC interest rate swaps where $421 trillion and the gross value of the contracts amounted to $17 trillion.[3] The market for interest rate swaps denoted in EUR is by far the largest in the world, followed by USD. Markets for interest rate swaps are highly liquid and work as an important benchmark for interest rates measurement in financial markets, both short and long-term. In most economies interest rate swaps are applied as benchmark rates for determining the value and pricing of fixed-income securities. In contrast to governmental securities, which often exhibit poor liquidity and low outstanding volume (e.g. Norwegian Gov. securities), swap rates works as an alternative for this purpose (Rakkestad & Hein 2004). Below follows a brief presentation on what an interest rate swap is.

Interest rate swaps have zero value at inception, like most derivatives. However, during the life of a swap contract the value will most likely change. To see why this is the case we start by looking at how a swap is initially valued. An interest rate swap is an agreement between two parties to swap interest rate payments- one party pays a fixed rate and the other pays a floating rate in a pre-determined period. The buyer of a swap pays a fixed rate and receives floating, while the seller of a swap has the opposite position. The reasons for engaging in a swap contract may be numerous, such as; hedging interest rate risk, speculating on the future movements of interest rates and to transform liabilities and/or assets to suit the investors' preferences. An interest rate swap has two interest rate legs, fixed and floating. The floating leg is based upon a short-term money-market interest rate, usually a 6-month rate (e.g. NIBOR, LIBOR etc). For the purpose of calculating the future interest rate cash-flows between the parties in the contract a principal notional value is set, e.g. NOK 1 million.

[3] https://www.bis.org/statistics/dt21a21b.pdf

If we assume that the notional value is exchanged by inception and at the end of the swap contract we can regard the two interest leg payments as holding a fixed-rate bond and a floating-rate bond. By assuming this we can value the swap contract in terms of fixed -and floating-rate bonds. Thus, at inception the discounted present value of both cash-flows must be equal, meaning that the net present value is zero for the swap contract. The fixed rate, or swap rate, is set so that this is true. A swap rate reflects market participant view on the future path of interest rate, and given the size of the interest rate market, we can also assume that the swap rates are highly efficient.

Banks are the biggest participants in the interest rate swap market, so the swap rates will to some extent reflect the credit risk in the banking sector (Rakkestad & Hein 2004). As illustrated in figure 2 and 4 below the swap rates, particularly the short-term rates, skyrocketed during the impact of the financial crisis; reflecting the surge in perceived credit risk of lending money between banks.

A swap curve contains swap rates of maturities ranging from 3-month to 10-years. In most economies a swap curve is widely used as a proxy for interest rate analysis. Since the swap curve contains information on the both the future expectation of short-term money-market rates and the credit risk of the banking sector, these curves works as an alternative to yield curves derived from governmental securities. Lastly, an argument that a swap rate may reflect information that makes it relevant for the use as a discounting rate for other fixed-income securities, that shares many of the same risk elements incorporated in the swap rate, can explain the popularity for interest rate swaps.

3.2 Descriptive statistics on the Norwegian swap rates

2000 - 2014	NIBOR 3m	NIBOR 6m	NOR 1y	NOR 2y	NOR 3y	NOR 4y	NOR 5y	NOR 6y	NOR 7y	NOR 8y	NOR 9y	NOR 10y
Mean	-0,11	-0,12	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13
Median	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Std.Dev.	4,79	4,87	4,15	4,58	4,60	4,50	4,33	4,38	4,25	4,24	4,27	4,23
Min	-60	-68	-47	-44	-42	-37	-33	-33	-30	-28	-28	-29
Max	57	45	26	26	28	28	27	26	23	22	25	24
Kurtosis	37,72	48,35	17,66	9,72	7,47	6,05	4,70	4,81	3,96	3,76	3,85	3,81
Skewness	-1,09	-3,29	-1,86	-1,06	-0,67	-0,36	-0,16	-0,23	-0,11	-0,14	-0,06	-0,04
Count	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912

Table 1: Descriptive statistics on basis point changes for Norwegian swap rates with different maturities. Period; 2000-2014.

Table 1 presents the descriptive statistics for Norwegian swap rates measured in basis point changes for the whole sample period. Mean and standard deviation are shown in daily values. The mean basis point changes for each rate are negative, and the level of the changes is fairly similar for each rate. Standard deviation, a measure of volatility, indicates that the short-term rates are a bit more volatile. Annualized the standard deviation for the money-market rates (3m and 6m) are 76bp and 77bp, and for the long-term rates (3y-10y) it is in the range of 67bp-73bp. However, these differences may not be regarded as significant. In figure 1 below the volatility of each swap rate for both markets are graphed (annualized).

If we look at the higher moments - skewness and kurtosis - they indicate a non-normal distribution of the basis point changes. This is confirmed when applying a Jarque Bera test[4]. A non-normal distribution does indeed make risk management a more demanding task.

A brief look at descriptive statistics from the two sub-samples, January 2006 - July 2008 (upward trending swap rates) and June 2012 – December 2014 (downward trending swap rates), yields some deviations from the general picture we observed during the whole sample period:

a) The characteristics from the first sub-period are fairly in line with the whole period – short-term rates more volatile than long-term rates and indications of non-normality is

[4] Results from the Jarque Bera test are not presented in the thesis. The values of the test scores are way above the critical score. Thus, one can reject the null hypothesis that the data is normally distributed.

present in the data. A closer look at the standard deviation for this sub-samples reveals that in the period of 2006-July 2008 the values are quite stable, between 47bp and 60bp. This is in a period where the swap rates are ascending due to higher policy rates set by the central bank, and a general belief in the market of economic growth. Low values of skewness and kurtosis for swap rates with maturities over three years indicate that the data in this sub-period exhibits more elements of a normal distribution.

b) In the more recent sub-period standard deviation is actually higher for long-term rates than for the short-term rates. Indeed, Money-markets rates are very stable with an annualized standard deviation of only 27bp and 30bp (3m and 6m). In contrast the long-term rates (3y-10y) are far more volatile; ranging in annualized standard deviation from 41bp to 53bp. Long-term rates where relative stable until September 2013, then they decreased rather sharply, making the spread between short –and long-term rates historically low. This decline in long-term rates reflects the perception from market participants that both credit –and market risk is low. In addition, it seems that the general view among participants in the Norwegian swap markets is a future of low money-market rates, hence a stable monetary policy.
A plausible explanation is that the policy rate has been low and stable during the period, short-term rates which are more reactive to changes in the policy rate also exhibits the same characteristics. This decline in long-term rates narrowed the spreads between short –and long-term rates down to levels not seen since before the financial crisis.

3.3 Descriptive statistics on euro swap rates

2000 - 2014	Euro 3m	Euro 6m	Euro 1y	Euro 2y	Euro 3y	Euro 4y	Euro 5y	Euro 6y	Euro 7y	Euro 8y	Euro 9y	Euro 10y
Mean	-0,08	-0,09	-0,10	-0,11	-0,12	-0,12	-0,13	-0,13	-0,13	-0,13	-0,13	-0,13
Median	0	0	0	-0,05	-0,1	-0,15	-0,1	-0,1	-0,1	-0,085	-0,05	-0,1
Std.Dev.	2,41	2,84	3,75	4,17	5,08	4,47	4,56	4,47	4,29	4,31	4,28	4,23
Min	-26	-25,5	-37,5	-29,5	-37,7	-29,1	-30,1	-28	-26,9	-26	-26,5	-27,5
Max	24	28,5	39	22,7	44,8	20,5	22,8	22,5	23	27,5	29,5	24,6
Kurtosis	19,80	19,61	12,91	3,21	5,79	2,47	2,81	2,23	2,29	2,46	2,90	2,69
Skewness	-0,91	-0,40	0,09	0,10	0,08	0,15	0,05	0,13	0,15	0,15	0,12	0,04
Count	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912	3912

Table 2: Descriptive statistics on basis point changes for Norwegian swap rates with different maturities. The period is from 2000-2014.

Table 2 above displays descriptive statistics for basis point changes in euro swap rates. The mean changes of the different rates are all negative. Further, euro swap rates seem to be closer to what we can regard as a normal distribution (evident from the lower values of skewness and kurtosis). The pattern for standard deviation however shows that in the case of euro swap rates, short-term rates are less volatile than long-term rates. This is the opposite pattern from what we witnessed in the Norwegian swap market. Figure 1 below highlights these differences visually.

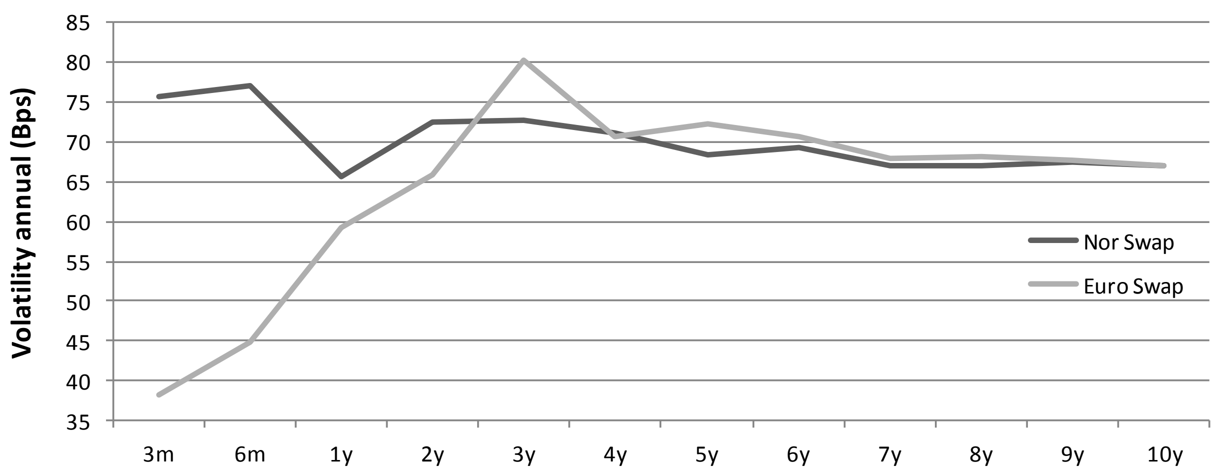

Figure 1: Standard deviation on basis point changes for different maturities of both Norwegian -and euro swap rates. Annualized according to 250 days.

The darkest line represents the Norwegian swap rates. As mentioned above the pattern for the Norwegian case is that short-term rates exhibit more volatility than long-term rates. In addition, the 1-year Norwegian swap rate deviates from its adjacent maturities with a significantly lower volatility. Another pattern appears when we examine the euro rates and their respective volatility. Overall the volatility is somewhat lower than in the Norwegian swap market. However, volatility on long-term rates (from the 4-year swap rate) is remarkably similar between the two markets. The most interesting feature of the volatility curve of the euro swap rates is the differences between short –and long-term rates. Annualized volatility is nearly twice as much for the 3-year rate compared to the 3-months rate, respectively 80bp and 38bp. This seemingly lack of linkage between volatility (changes) for the different rates in the euro swap curves could very well indicate that the shape of the swap curve may exhibit some volatile movements itself. A principal component analysis may be suitable for handling substantial movements in the swap curve – since it extracts the most common factors that could help quantifying these movements and hence the risk associated with them.

A closer look at the sub-samples for the euro area reveals that the differences between volatility of euro swap rates is more stable than in the Norwegian case. The short-term rates are less volatile in both sub-periods, with increasing volatility as a function of maturity. As witnessed in the long-term Norwegian rates, the long-term euro rates also started to decline during autumn 2013; narrowing the spreads and hence flattening the swap curve.

3.4 Correlation

Correlation between interest rates at different maturities in a term structure is usually relatively high. This means that their movements are in general driven by the same factors, and/or they are influenced by each other. In practice this means that it should be possible to extract some key risk factors that cumulative explains most of the variation in the system being analyzed (swap curve in this instance). Generally the following is true; to what extend the factors in the system is correlated to each other determines how successful such an extraction may be.

Factors in this case being swap rates and a system equivalent to a swap curve. Below in table 3 the correlation matrix for the Norwegian swap market is presented:

2000 - 2014	NIBOR 3m	NIBOR 6m	NOR 1y	NOR 2y	NOR 3y	NOR 4y	NOR 5y	NOR 6y	NOR 7y	NOR 8y	NOR 9y	NOR 10y
NIBOR 3m	1,00	0,77	0,51	0,35	0,28	0,23	0,19	0,17	0,15	0,15	0,14	0,12
NIBOR 6m	0,77	1,00	0,56	0,48	0,39	0,33	0,28	0,26	0,24	0,24	0,23	0,20
NOR 1y	0,51	0,56	1,00	0,77	0,72	0,66	0,63	0,58	0,55	0,53	0,51	0,50
NOR 2y	0,35	0,48	0,77	1,00	0,93	0,88	0,84	0,80	0,78	0,76	0,74	0,71
NOR 3y	0,28	0,39	0,72	0,93	1,00	0,95	0,90	0,87	0,85	0,83	0,81	0,79
NOR 4y	0,23	0,33	0,66	0,88	0,95	1,00	0,94	0,92	0,91	0,89	0,87	0,85
NOR 5y	0,19	0,28	0,63	0,84	0,90	0,94	1,00	0,93	0,92	0,90	0,89	0,87
NOR 6y	0,17	0,26	0,58	0,80	0,87	0,92	0,93	1,00	0,94	0,95	0,93	0,89
NOR 7y	0,15	0,24	0,55	0,78	0,85	0,91	0,92	0,94	1,00	0,95	0,93	0,92
NOR 8y	0,15	0,24	0,53	0,76	0,83	0,89	0,90	0,95	0,95	1,00	0,97	0,93
NOR 9y	0,14	0,23	0,51	0,74	0,81	0,87	0,89	0,93	0,93	0,97	1,00	0,95
NOR 10y	0,12	0,20	0,50	0,71	0,79	0,85	0,87	0,89	0,92	0,93	0,95	1,00

Table 3: Correlation matrix of Norwegian swap rates, 2000-2014.

As expected correlation is highest for adjacent rates, and the further away the rates are of each other the lower the correlation. Another common feature is that longer-term rates tend to be more correlated with each other than shorter-term rates are. Again, this is likely due to the “disturbance-effect” from monetary policies, which is more interfering for short-term rates.

Table 4 below shows the correlation for the euro swap rates. The correlation coefficients do show the same pattern as the Norwegian rates.

2000 - 2014	Euro 3m	Euro 6m	Euro 1y	Euro 2y	Euro 3y	Euro 4y	Euro 5y	Euro 6y	Euro 7y	Euro 8y	Euro 9y	Euro 10y
Euro 3m	1,00	0,74	0,58	0,39	0,31	0,31	0,29	0,27	0,26	0,24	0,23	0,22
Euro 6m	0,74	1,00	0,76	0,56	0,45	0,48	0,44	0,42	0,40	0,37	0,36	0,36
Euro 1y	0,58	0,76	1,00	0,75	0,62	0,67	0,63	0,61	0,59	0,55	0,53	0,53
Euro 2y	0,39	0,56	0,75	1,00	0,81	0,89	0,84	0,82	0,80	0,76	0,74	0,70
Euro 3y	0,31	0,45	0,62	0,81	1,00	0,82	0,78	0,77	0,75	0,72	0,71	0,67
Euro 4y	0,31	0,48	0,67	0,89	0,82	1,00	0,94	0,93	0,91	0,88	0,87	0,82
Euro 5y	0,29	0,44	0,63	0,84	0,78	0,94	1,00	0,93	0,92	0,89	0,88	0,84
Euro 6y	0,27	0,42	0,61	0,82	0,77	0,93	0,93	1,00	0,95	0,94	0,93	0,89
Euro 7y	0,26	0,40	0,59	0,80	0,75	0,91	0,92	0,95	1,00	0,96	0,96	0,92
Euro 8y	0,24	0,37	0,55	0,76	0,72	0,88	0,89	0,94	0,96	1,00	0,97	0,93
Euro 9y	0,23	0,36	0,53	0,74	0,71	0,87	0,88	0,93	0,96	0,97	1,00	0,94
Euro 10y	0,22	0,36	0,53	0,70	0,67	0,82	0,84	0,89	0,92	0,93	0,94	1,00

Table 4: Correlation matrix of euro swap rates, 2000-2014.

3.5 Historical swap rate movements

In this section I will present and comment the historical swap rate movements chosen for this thesis, namely the swap rates in the Norwegian –and euro market. The discussion focuses on identifying key events that has driven significant movements in the swap rate, and hence different shapes of the respective swap curves. To further elaborate on this issue I have added the two respective central bank's policy rates in figure 2 and 4 below (solid red line). This is useful when trying to understand how the different swap rates relates to monetary policy. Also the similarities and differences between the two central banks policies are discusses for the purpose of trying to explain the historical movements in the swap rates. This section is meant to create a link between the upcoming sections which will focus on quantitative methods in analyzing movements in the swap rates and the corresponding curves. The section begins with a discussion on historical movements in the Norwegian swap market. Further the similar process is repeated for the euro swap market.

Norwegian swap curve:

Figure 2 below shows the movements in selected swap rates with maturities ranging 3 months to 10 years. In addition the Norwegian Central Bank policy rate is included. The observations are daily and runs in the period 2000-2014. On January 1st 2000 we can observe a common feature of a swap curve, namely a upward sloping curve, although the slope is almost flat. During the year 2000 the central bank raised their benchmark rate from 5,5 % to 7 %. This action made the short-term swap rates, or money-market rates, move in the same direction as the policy rate, while the longer-term rates where failry stable. This resulted in a downward sloping swap curve, indicating that the market expected interest rates to fall. This scenario was apparent until late 2002. Then the central bank lowered the policy rate from 6,5 % in the beginning of 2003 down to 2,25 % at the end of the year. The main reason for this relativlely sharp reduction was due to the fact that inflation was lower that the aim set by the central bank.[5] Since short-term swap

[5] http://www.magma.no/konsekvensene-av-et-inflasjonsmaal

rates are more attached to the policy rate than longer-term swap rates this lead to a normalization of the swap curve (upward sloping).

The swap rates during the period 2004-2006 were relativly stable, hence the swap curve exhibited an upward sloping tendency. The policy rate hoovering around 2 % in this period contributed to the relatively stable interest rate enviroment. From figure 2 we can also observe that during this period the spreads between short –and long-term swap rates are historically high. The spread between the 3-month NIBOR and the 10-year swap was well over 300bp during may 2004. These high spreads lead to a steep swap curve during this two-year period.

So from early 2006 till mid 2008 the central bank raised the policy rate in high frequency. This is visual from the step-formation that the solid red line represent. In the same period the spreads between the short –and long-term swap rates narrowed. Indeed, from late 2007 the swap curve was downward sloping. At its height the 3-month NIBOR peaked at 7,91 % on October 1st (10y swap rate 5,48% on same day) - mirroring the imense volatility and turbulence in the relationship between financial institutions in the wake of the sub-prime crisis. During the same period the Norwegian central bank raised their policy rate from 2,25 % to 5,75 %. These actions where in large part due to several factors; a) after a couple of years with low interest rates a normalization of interest rates where regarded as a way of controlling the growing activity in the Norwegian economy, and b) parallel with the higher economical activity in Norway the Norwegian Central Bank saw the same tendency within their trading partners; mainly countries in the euro area and other Nordic countries.[6] However, this was not a distinctively Norwegian phenomenon; interest rates where set higher and higher by central banks in the western world - trying to cool down an increasingly hotter economy, bring down inflation and adjusting interest rates back to normal levels in a historical perspective.

Then the unexpected happened; a financial melt-down starting in the US sweeped all over the financial-world. Leading to a stagnation in economic output and worsening outlooks for the global economy. Credit was now a scarce “resource” compared to only a few months ago. A

[6] Several press releases through 2006, 2007 and 2008 from the Norwegian Central Bank states these arguments.

quote from a press release after a meeting at the Norwegian Central Bank highlights the challenges (Norwegian Central Bank, October 15th 2008):

> Also domestically there is less access to capital and financing for banks, businesses and households are more expensive and somewhat more difficult. Banks have increased their lending rates. Inflation remains high, but the forces that have contributed to the high inflation have now diminished.

On the 17th of December 2008 the Norwegian Central Bank decides to lower the policy rate from 4.75 % to 3 %. Falling oil price as demand weakened, decreasing inflation, worsening outlooks for the Norwegian economy than first assessed just months ago was some of the arguments for lowering the policy rate down to 3%. During this period the volatility in swap rates were high and especially short-term rates. As a result of volatility in individual swap rates the shape of the swap curve had some volatility itself. From June 2007 to late 2008 the swap curve was downward sloping. A curvature shape is also apparent from late 2008 to approximately April 2009. The relative low levels of the 1, 2 and 3-year swap rates forms a curvature shape.

If we take a closer look at what happened with the policy rate in the last months of 2011 and the beginning of 2012, we can find a descriptive example on how markets are connected together. From the solid red line we can observe that the Norwegian central bank reduced the policy rate twice in these turbulent months for especially European markets. The main reason for the turmoil in the financial markets was now concern regarding huge amount of outstanding debt of some European countries, and the uncertainty if these countries where able to manage and repay their debts to creditors. Also, what would the consequences' be in a scenario where these nations defaulted on their payments? In this highly uncertain context the vice president of the Norwegian Central Bank, Jan F. Qvigstad, had the following statement after the meeting in the General Council on December 2011 (Norwegian Central Bank, December 14th 2011):

> The turmoil in the financial markets has intensified and we are now expecting a clearly weaker development abroad, particularly in the euro area. To mitigate the effects on the Norwegian economy the benchmark rate is adjusted down 50 basis points to 1.75%.

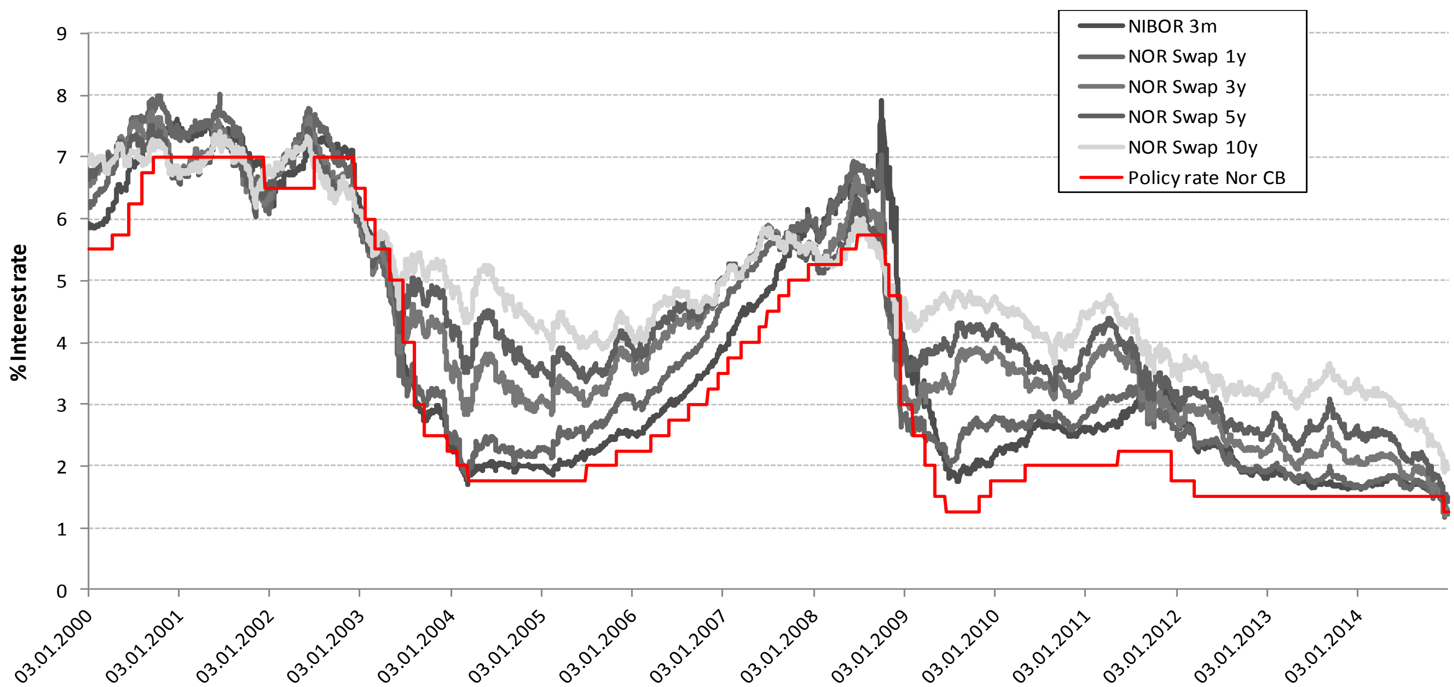

Figure 2: Historical daily observations on Norwegian swap rates with maturities 3m, 1y, 3y, 5y and 10 years. The solid red line is the Norwegian Central Banks policy rate. Period: 2000-2014. Source: Reuters.

After 2009 the swap curve has been downward sloping. It is clearly visible from figure 3 below that the spreads between short -and long-term rates started to increase from the end of 2008. During the European sovereign debt-crisis the spread narrowed, due to a combination of higher short-term rates and increasingly lower long-term rates. From figure 3 below it is clear that the swap curve changes its shape many times during a large time span. Such changes may occur in a slow fashion, or more sudden and drastic. Although changes in the shape of a swap curve is possible to visualize from a chart, the outputs from a PCA on the same data sample can contribute with a quantification of the most common movements in the swap curve. These outputs serve as an objective way of measuring the degree of common movements; either they are parallel, tilted or curvature, or a combination of the three (Alexander 2008).

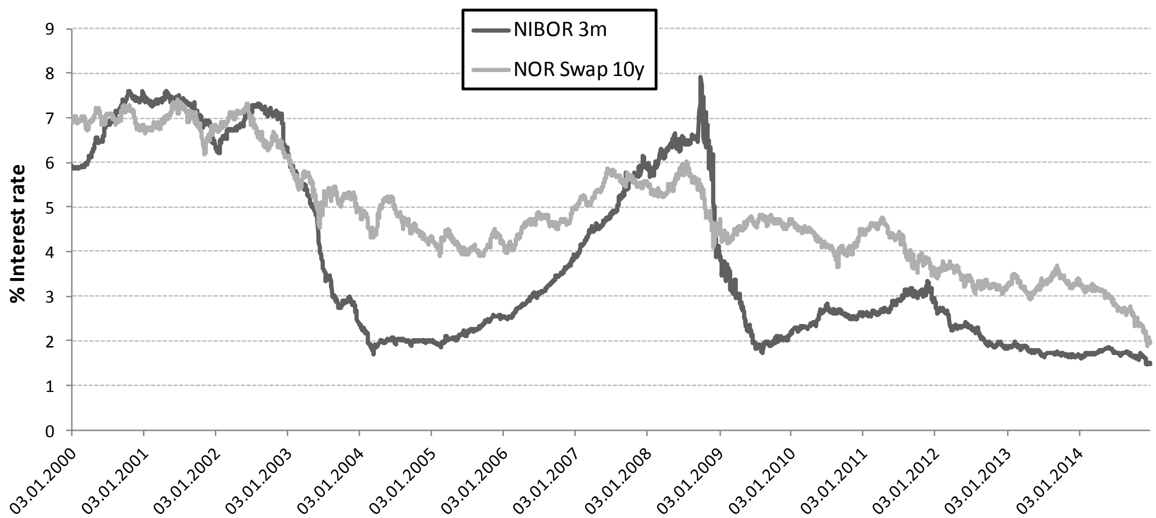

Figure 3: Spread between the NIBOR 3-month rate and the 10-year swap rate during the period 2000-2014. Changes in the spreads are a sign of volatility and change in market conditions.

Euro swap curve:

Figure 4 below shows swap rates in the euro area, with maturities similar to that of the Norwegian market. The key policy rate in the euro area, or ECB's (European Central Bank) main

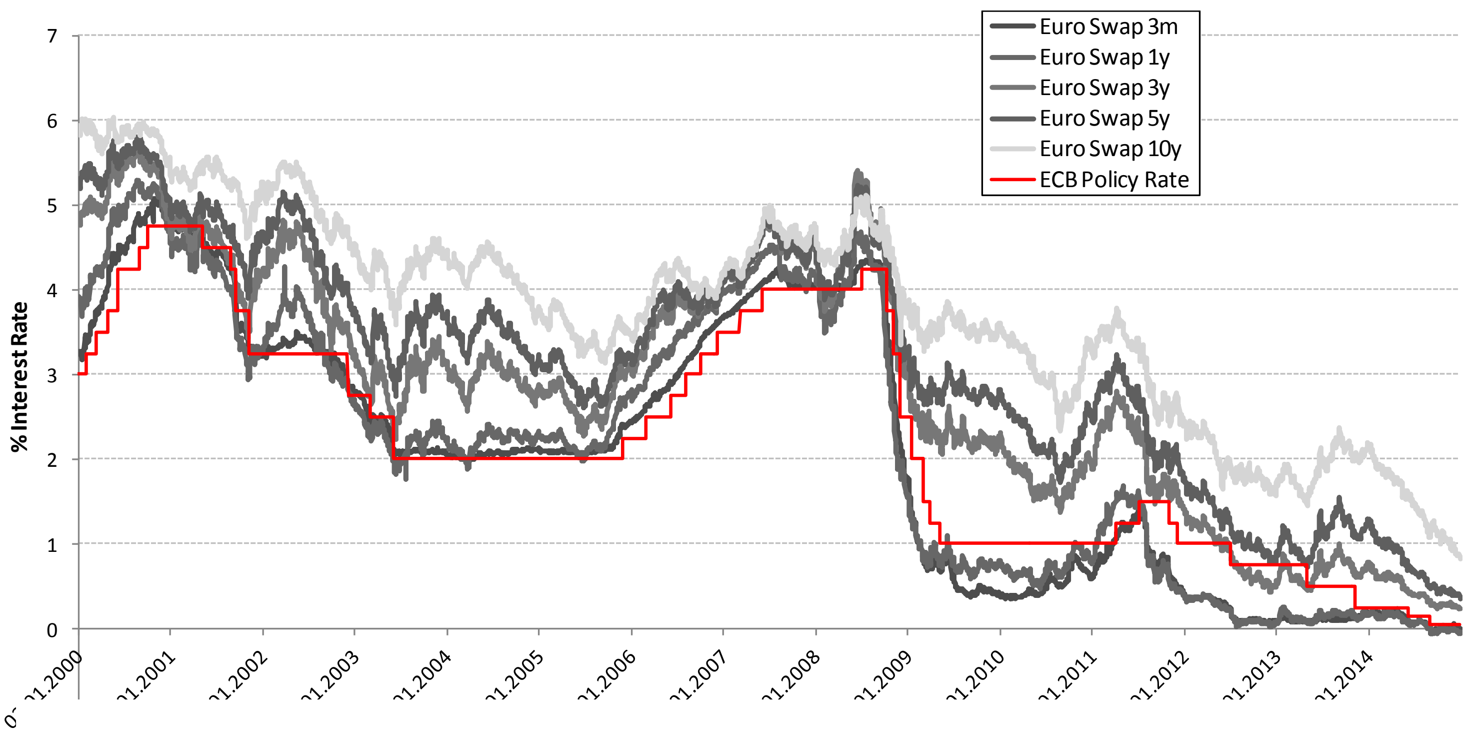

Figure 4: Historical daily observations on Euro swap rates with maturities 3m, 1y, 3y, 5y and 10 years. The solid red line is the ECB policy rate. Period: 2000-2014. Source: Reuters.

refinancing rate, is highlighted with a solid red line. This rate is equivalent to the policy rate set by the Norwegian Central Bank.

During year 2000 the key policy rate rose from 3% to 4.75%. This surge was mainly driven by what the ECB called "a prospect of cyclical upswing in the euro area ". Also concerns regarding depreciation of the euro, contributing to increasing import prices, and a jump in credit aggregate[7]. Together these factors, along with others, set the bases for adjusting the policy rate to 4.75%. Along with the policy rate being adjusted to higher levels, the swap rates also followed, although, spreads between short –and long-term rates narrowed. This is illustrated in figure 5, where the swap curves in the beginning and end of year 2000 are displayed. Cleary we can see that the swap curves is less steep in December compared to January. From the figure it

[7] Press releases regarding monetary policy decisions from ECB trough 2000.

is also apparent that it is the short-term rates that moved the most, as they are more attached to the policy rate and guidance on further monetary policy published ECB.

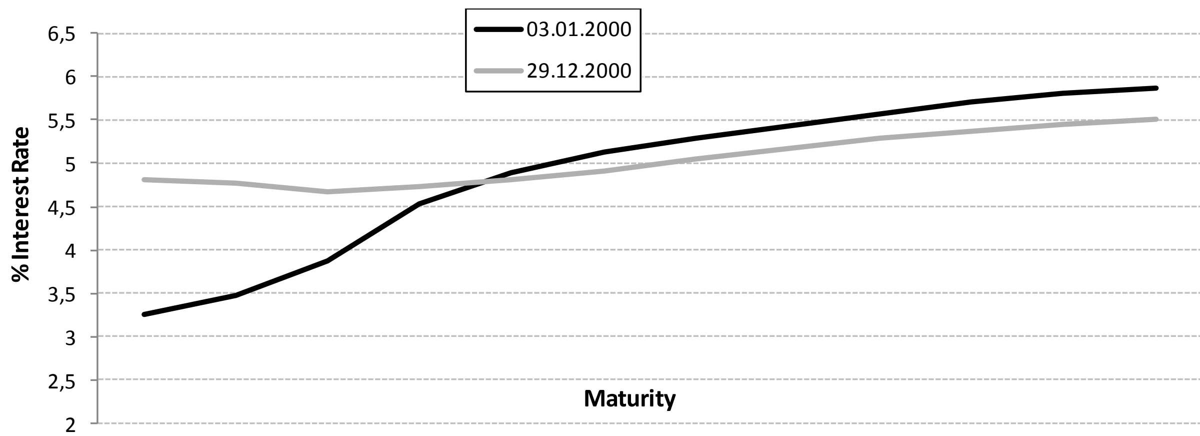

Figure 5: Euro swap curve on the 3rd of January 2000 and 29th of December 2000. Maturities ranging from 3 month rate to 10 year rate.

Developments like illustrated above gives rise to so-called yield curve risk. Portfolios containing securities that are initially priced against a set of interest rate inherent in a term structure are exposed to unanticipated shifts in the shape of the respective term structure. Principal component analysis may help to shed some additional light on what kind of movements that is most common in a term structure of interest rates (i.e. swap curves). This issue will be thoroughly treated in section 5 - “*PCA and discussion on key findings*”.

In the following years ECB was forced to take a U-turn on their view of the underlying factors that drove the economical and financial development in the euro area. In the period from April 2001 till June 2003 the policy rate was lowered from 4.45% to 2%. In a series of press conferences following the Governing Council of the ECB’s meetings, the arguments for adjusting the policy rate down to 2% had the following key arguments; a) after the terrorist attacks on 11th of September 2001 ECB saw it appropriate to lower the policy rate to meet the expected decreasing lower economic activity. However, ECB had confidence in the resilience and robustness of the US financial system, and they expected that this period of slow economic activity would be short-lived. But it would take five years before the policy rates where back to the same level, b) inflationary pressure diminished in the period, in addition the credit supply to the private sector showed signs of slowing down, and c) economic activity in the euro area - measured by real GDP growth - showed that economic activity remained subdued during the first part or 2003. The war in Iraq contributed to lower global economic activity, thus affecting European exports. In this period of falling interest rates the spreads again widened between the short –and long-term swap rates; resulting in a steeper upward sloping swap curve.

Then a period of steady monetary policy from ECB followed. ECB did not move the policy rate at all between June 2003 and December 2005. Figure 6 below illustrates the different shape of the swap curve on June 2003 and December 2005. The curve flattened gradually in the period. Indeed, we observe that short-term rates actually rose while long-term rates declined.

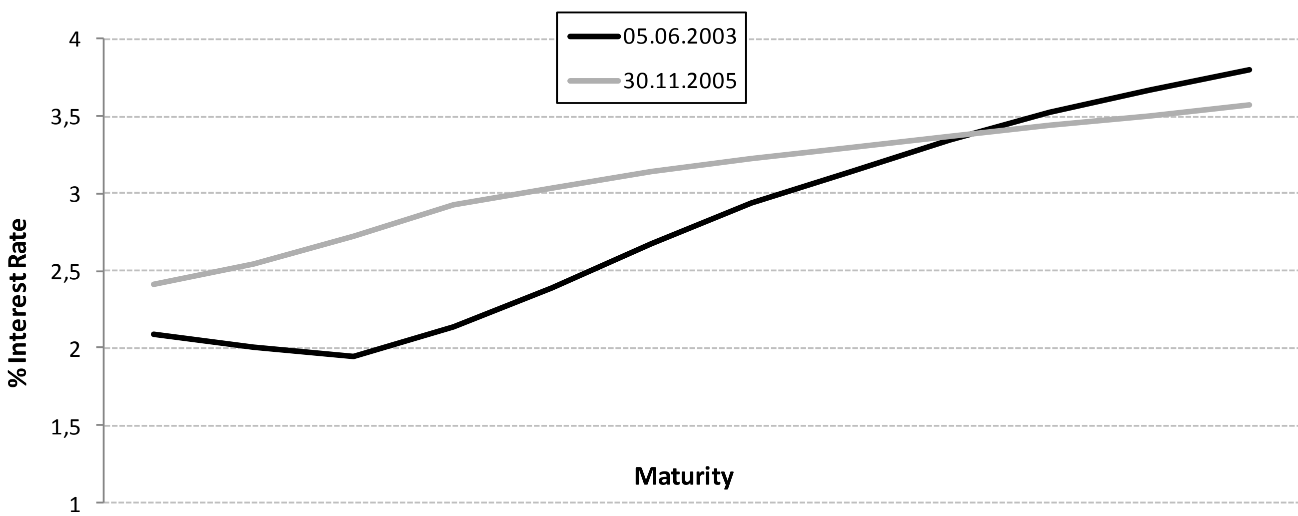

Figure 6: Euro swap curve on the 5th of June 2003 and 30th of November 2005. Maturities ranging from 3 month rate to 10 year rate.

As we observed in the case of the Norwegian swap market the same pattern of rising policy rates ahead of the financial crisis in 2008 is also apparent in the euro area. The arguments from the ECB are very much in line with that of the Norwegian Central Bank. Some of them are highlighted below:

- Quote from ECB press conference on 7th of December 2006:

  After today's increase, our monetary policy continues to be accommodative, with the key ECB interest rates remaining at low levels, money and credit growth strong, and liquidity in the euro area ample by all plausible measures.

- Significant higher inflationary rate. Even if labor participation in 2008 is the highest in 25 years, the ECB is concerned that the real income disposal will be weakened due to the sharp increase in e.g. commodity prices.
- Strong growth of money -and credit supply. Paired with absence of constraints on bank loan supplies, ECB sees this as an upside risk for price stability.
- Simultaneous growth in economic strength for trading partners with the European nations indicates positive outlooks for euro area exports, thus possibly resulting in a rapid growth of GDP.

In the period between September 2007 and September 2008 the European swap curve fluctuates a lot. This is evident from figure 7 below. This figure displays swap curves that inhabits more or less relative extreme shapes during the abovementioned period. On February 8th 2008 the swap curve exhibits a curvature shape- a scenario where the medium-term rates are lower than the short – and long-term rates. Compared to the swap curve on September 14th 2007 this shape results in a different environment for investors in fixed income securities.

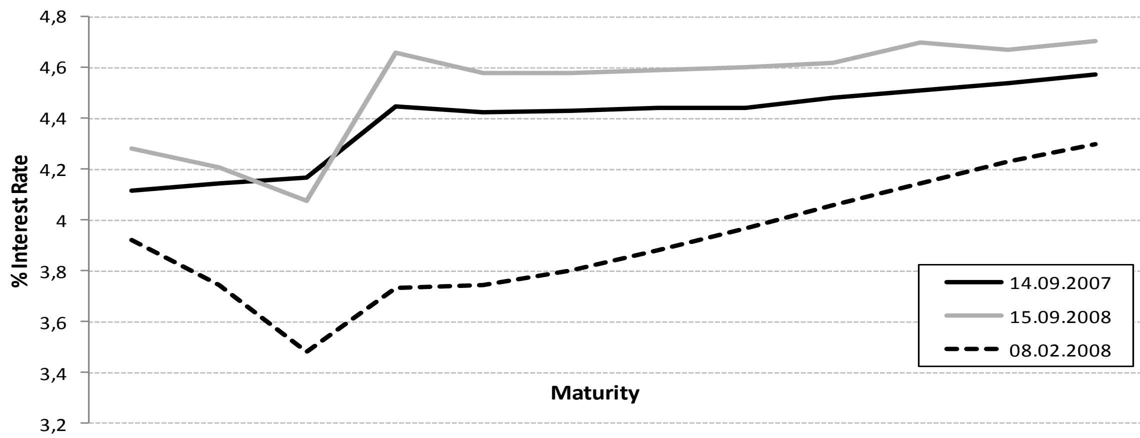

Figure 7: Euro swap curve on the 14th of September 2007, 8th of February 2008 and 15th of September 2008. Maturities ranging from 3 month rate to 10 year rate.

ECB did not adjust the policy rate at all between May 2009 and April 2011. In the same period the short-term swap rates very low and stable, they were actually below the policy rate level. And for most of the time being since 2009 they have stayed below the ECB's policy rate. This can be interpreted that the market believes that money market rates will continue to stay low. The spreads between short –and long-term rates exhibited in this period a large proportion of volatility, as shown in figure 8 below. Although the swap curve seemingly is upward sloping between 2009 and 2011, the volatility in the spread between the 3-month and the 10-year swap rate is very high indeed.

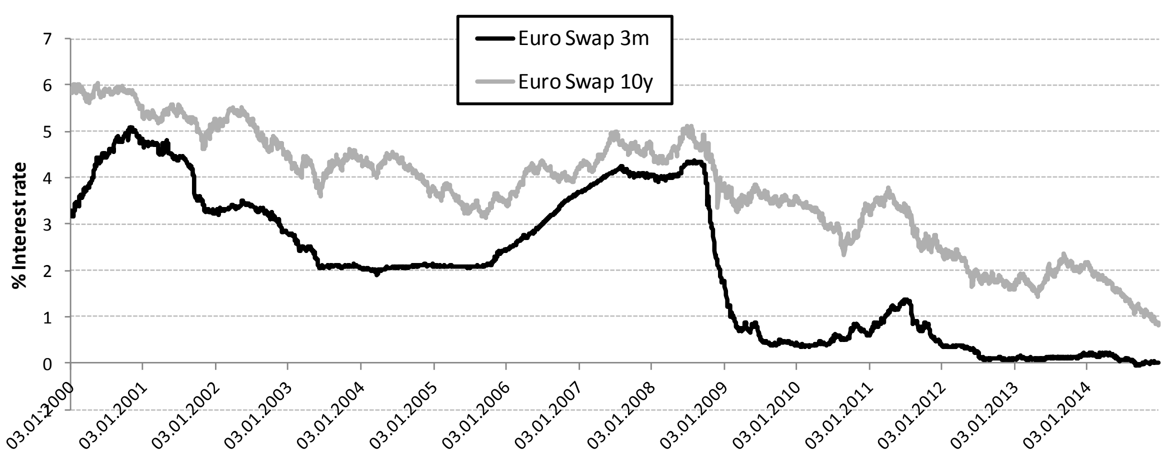

Figure 8: Spread between the euro swap 3-month rate and the 10-year swap rate during the period 2000-2014. Changes in the spreads are a sign of volatility and change in market conditions

Early December 2011 the ECB started lowering the policy rate to face the challenges that rose from the so-called European sovereign debt crisis. By lowering the policy rate the ECB hoped e.g. to achieving higher economic activity, restoring the confidence in the financial system in a situation where credit supply where scarce, meet challenges from the lack of availability for credit to investment-projects in private –and governmental sector that threatened to undermine economic development in the euro area. The euro area still struggles with low economic growth, low inflation, and some member states of the European Union holds the whole union in an “economic pincer”.

Meanwhile swap rates has moved to historical low levels. The shape of the swap curve has during the last years been somewhat normal with short rates lower than long rates. However, the abnormality is the historically low interest rates. Investors have been used to the low interest rates for a long time now. Fixed income securities has been priced and are valued compared to these low interest rates. In a scenario where central bank increases their policy rates or inflation unexpectedly rise the risk and possible loss of holding securities that are sensitive to such factors may provoke financial disturbance. Section five quantifies this risk in the historical period of 2000-2014, with sub-periods reflecting different market conditions.

4. Methodology

PCA is a statistical method that is part of the factor model family. The uniqueness of PCA is that it contributes to a parsimonious analysis of the risk factors inherent in a highly complex correlated system, e.g. term structure of interest rates. This feature makes the process of risk management a more manageable task by reducing the number of risk factors for a particular security or a portfolio. Example: In a term structure consisting of 50 different maturities, from 3 months to 30 years, the individual risk factors are the interest rates separated by maturity. Movements in these risk factors are influenced by many of the same factors, and they also influence each other, hence within these systems there are a high degree of correlation between the risk factors (interest rate).[8] PCA extracts the variations from each risk factor and the co-movements between them and transform this into an uncorrelated system, or orthogonal. This uncorrelated system makes the process of risk management a less formidable task, and it also has the advantage of identifying the key sources of risk; as each principal component do have its own unique interpretation (Alexander 2008). The technicality of the method is described in the section below.

1. Choosing a data sample on the term structure one would like to perform a PCA on. This provides us with an *T x n* data matrix,

   $$\mathbf{S = [S_{ij}]}$$

   where,

   S_{ij} is the j^{th} swap rate of the i^{th} swap curve. In matrix S each row contains a swap curve on a specific date and each column a set of values of a particular swap rate.

2. Transforming the values in the matrix from levels to changes. Since we are dealing with interest rates the common practice is to convert to basis point changes.

3. The next step in the process is to calculate a correlation or covariance matrix of the data matrix,

   $$\mathbf{C} = \boldsymbol{Cov}\ \mathbf{(S)}$$

   The difference between applying a correlation or a covariance matrix can be summarized as followed; using the correlation matrix as an input in the PCA, the outputs from the analysis will only capture the co movements between the basis point changes of the respective swap rates. In addition to that, a covariance matrix also has the ability to handle the individual volatility of each swap rate. So in instances where the volatility of

[8] We observed this in the correlation matrices presented in the section "Data and descriptive statistics" above.

the swap rates deviates significant from each other it may be more suitable to apply a covariance matrix as an input to the PCA.

4. Based on the covariance matrix we can now compute the eigenvalue – eigenvector decomposition. The eigenvalues, denoted λ, are to be presented in descending order, from highest to smallest. The eigenvalue for each principal component represents their share of the total variations present in the historical data sample, i.e. swap curve. There are as many eigenvalues as there are risk factors. This means that the sum of the all the eigenvalues are equal to the total variations in the swap curve. However, in a highly correlated system like a swap curve typically the eigenvalues of the first 2-3 principal components makes a significant contribution to the overall variations.
   Part of the eigenvalue – eigenvector decomposition calculations involves finding the eigenvectors. Each eigenvalue has a corresponding eigenvector. An eigenvector is a vector that neither expands or contracts by the matrix transformation, in other words it is almost invariant. Figuratively explained the vector lies on the same line in a two – dimensional space. When calculating PCA from a covariance matrix an eigenvector represents the coefficient in the linear combination of the original risk factors (interest rate) that together make up the n^{th} principal component (Jorion 2007).

Eigenvector – general definition:

i. $\mathbf{A\,w = \lambda\,w}$

   where,

   A, a square matrix, e.g. covariance matrix

   w, eigenvector

   λ, eigenvalue belonging to the eigenvector x.

5. Preliminary summary: The input for the PCA is a covariance matrix (or correlation matrix) of S. S is a *T x n* matrix on basis point changes for individual swap rates at different points in time. The time series of the individual swap rates are normally highly correlated, and are therefore ideal in the purpose of calculating parsimonious factor models like PCA.

6. If the covariance matrix, C, is symmetric, so is the matrix on the eigenvectors, W. Thus meaning that the eigenvectors are orthogonal. The matrix of the eigenvectors, W, is ordered so that the first eigenvector, w, corresponds to the largest eigenvalue of S, and so on (Alexander 2008).

7. A transformation of the random and correlated variables *n* from the original data matrix S into orthogonal random variables can be done by performing a linear transformation defined by W. The new matrix P is called the principal components of S. Algebraically it can be written as P = SW. The columns P are now orthogonal to each other.

From the procedure presented above we now have reduced the number of risk factors for further use in risk management of e.g. our portfolios containing fixed income securities. We are now left with the decision on how many principal components that is appropriate to our further analysis. It is common in a system like term structure on interest rates that the first three principal components explain roughly 95 % of the total variations. Each of these three principal components has a unique influence on movements in the shape of a term structure. The interpretation of the outputs from a PCA is the core issue in this thesis. Do the outputs from a PCA differ in the case of the Norwegian –and euro swap curve? And are the outputs stable through time.

What attributes can we assign to the respective principal components? PCA helps us to reduce the number of risk factors down to possibly three main factors. In addition the principal components have an intuitive interpretation (Alexander 2008);

- Normally the first principal component captures a common trend in interest rates. Plotting the eigenvectors as a function of maturity would yield a linear line in a perfect correlated system. However, a perfect correlated swap curve system is never observed, so we would never witness a flat line. Instead the line will deviate from a flat line, how much depends on co movements between the interest rates and the individual volatility of each rate. The first principal component is often referred to as the trend component.
- The second principal component normally represents changes in the slope of the term structure. The steepness of the slope and if it is upward –or downward sloping is determined by the eigenvectors of each interest rate. The second principal component is often referred to as either the slope or tilt component.
- The third principal component captures elements of convexity. By plotting the eigenvectors as a function of maturity would results in a convex line. This principal component is therefore referred to as the curvature or convexity component.

5. PCA and discussion of key findings

The data sample consists of daily basis point changes for swap rates of maturities ranging in tenors from 3 month to 10-year. The first section starts with a PCA on the whole dataset (2000-2014) for the Norwegian swap market. Then a PCA on the two sub-samples are performed. These two sub-samples represents periods where the swap rates exhibit a distinctive trend, either upward –or downward. The purpose is to identify any deviations in the results from PCA on the whole period versus periods where swap rates are trending in either direction. Then the same procedure is performed on the data sample containing euro swap rates. Lastly, a comparison between outputs for the two swap markets are presented and discussed.

The input matrices in the PCA are the covariance matrices. Based on descriptive statistics presented in the section “Data and descriptive statistics” above it is clear that the volatility in each swap rate is so unique that it should be taken into consideration.

5.1 PCA on the Norwegian swap curve

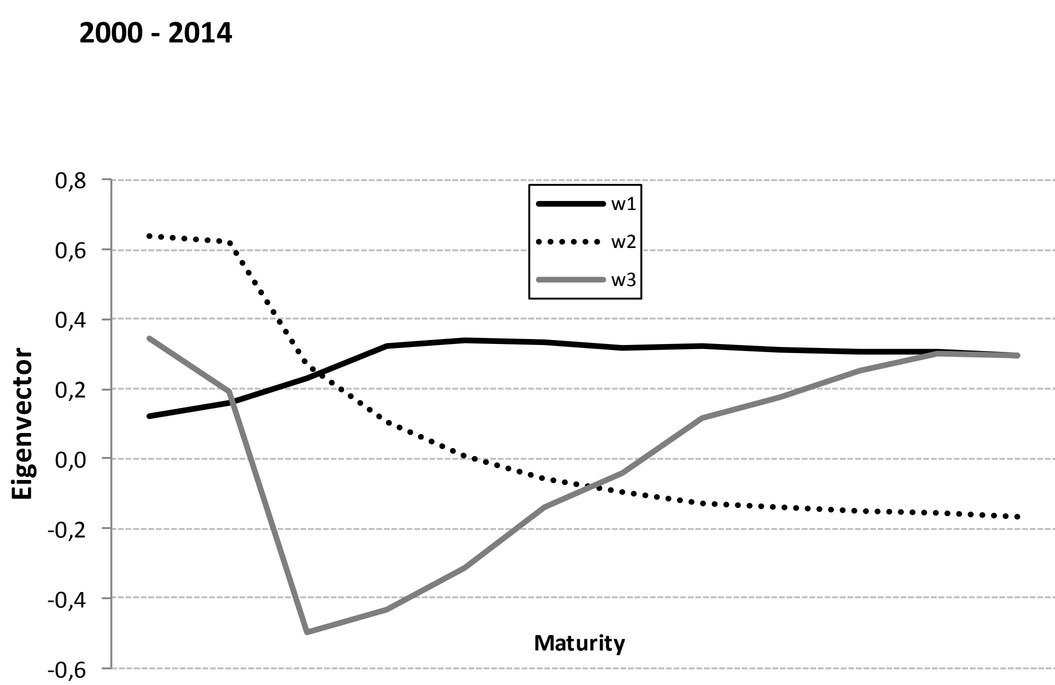

Figure 9: Eigenvectors of the Norwegian daily swap rate covariance matrix, 2000-2014.

2000 - 2014	1	2	3
Eigenvalues	166,90	41,70	10,12
% Variation	71,02 %	17,74 %	4,31 %
Cumulative %	71,02 %	88,76 %	93,07 %

Table 5: Eigenvalues of the Norwegian swap rate covariance matrix. 2000-2014.

From figure 9 above we can observe that the first principal component cannot be interpreted as a parallel shift in all rate simultaneously. It is upward trending up to the 2-year rate, from that point it stabilizes; meaning that the short-term rates have a lower correlation with the first component than the long-term rates. If the first principal component increases and the other two components are fixed then we would observe that the swap curve moved less in the short end than in the long end. The eigenvalue of the first component indicates that this component explains 71.02 % of total variations in the historical data sample. The explanatory power of the first component is relatively low. This means that other movements have a significant influence on the shape of the swap curve during the sample period.

The second principal component, also called the tilt component, captures the slope of the swap curve. In this case the short-term rates have a positive correlation with the second component (signaled by positive eigenvectors). Long-term rates have an increasingly lower correlation with the second component, however from the 7-year swap rate it stabilizes. Relatively high values for the eigenvectors of the 3 –and 6-month rates, respectively 0.64 and 0.62, indicates that monetary policy from the central bank influence the tilt movement in a downward fashion. Historically this component explains 17.74 % of the total variation in the Norwegian swap curve. This amount is fairly high, thus ignoring this kind of swap curve movements may have lead to challenges in risk managing a portfolio of fixed income securities. The high degree of fluctuations in the spread between short –and long-term swap rates (see figure 8 page 26) during the sample period is a contributor to the fact that the second component explains as much as 17.74 %.

As mentioned above the third component can be interpreted as yielding a convex swap curve. In this sample period the short and long rates have positive correlation with the component, and the rates in the middle are negatively correlated. The third component explains 4.31 % of the total variation. This relatively high level of variation explained by the third component may be attributable for the same reasons as mentioned above – the swap curve fluctuated greatly in the sample period.

Together the first three components explain 93.07 % of all the variations in the Norwegian swap curve during the sample period. Considering the different market conditions, shapes of the swap curve, and by how much individual rates fluctuated during the sample period, the outputs serves us with actual measures on the main drivers of changes in the shapes of the swap curve. A practical application of the results tells us that we now can relate the risk in a portfolio of swap rate sensitive securities to movements in these three factors, rather than taking into account movements in all twelve swap rates. This makes interest-rate risk management a more parsimonious process (Hull 2012).

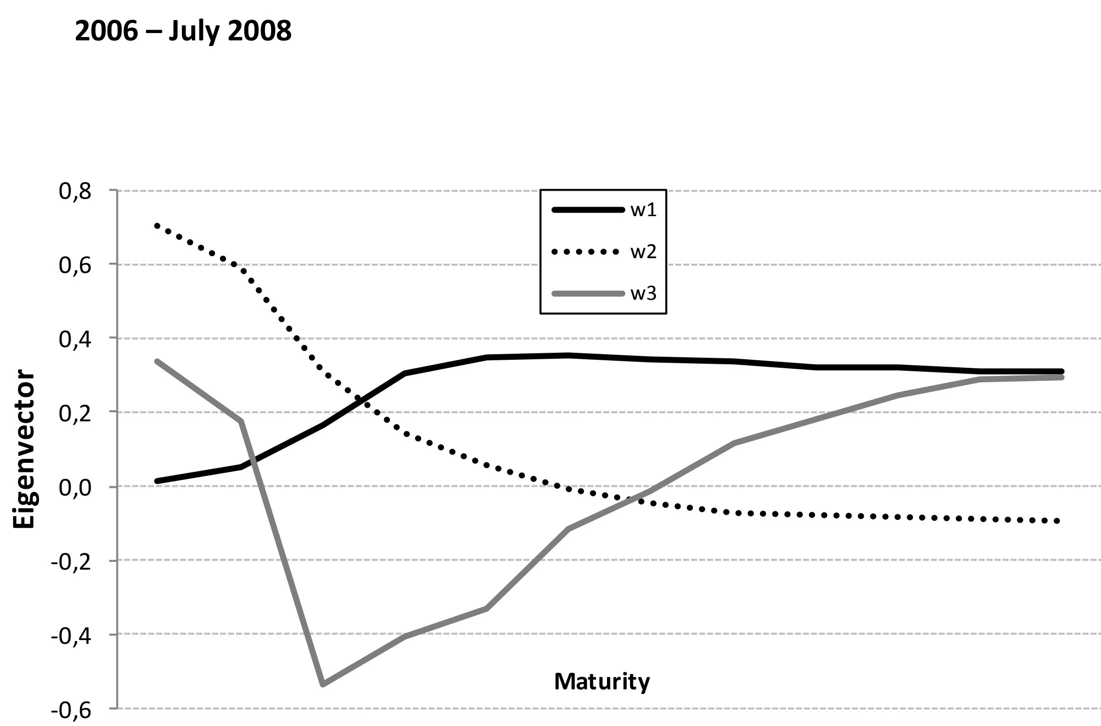

Figure 10: Eigenvectors of the Norwegian daily swap rate covariance matrix, 2006 – 07.2008.

2006 - 07.2008	1	2	3
Eigenvalues	110,66	24,44	6,65
% Variation	73,59 %	16,25 %	4,42 %
Cumulative %	73,59 %	89,84 %	94,26 %

Table 6: Eigenvalues of the Norwegian swap rates covariance matrix. 2006 – July 2008.

In this period, characterized by upward trending swap rates, the results from a PCA is fairly in line with that of the whole sample period. The most significant difference is the eigenvectors for the 3 –and 6-month NIBOR rates. In this period the eigenvectors are respectively, 0.02 and 0.05. Compared to the eigenvectors of the whole sample period (0.12 and 0.16) these rates are during this period even less correlated with the first principal component.

If the second component increases the swap curve shifts up at the short end and down at the long end. Although, from the 4-year rate it is fairly stable. During this period the slope of the swap curve changed frequently (evident from figure 3 page 19), so that the second component explains such a relative high amount, 16.25 %, of the total variations during this period is no surprise.

Together the first three components explain 94.26 % of the total variation; slightly more compared to the whole sample period.

June 2012 - 2014

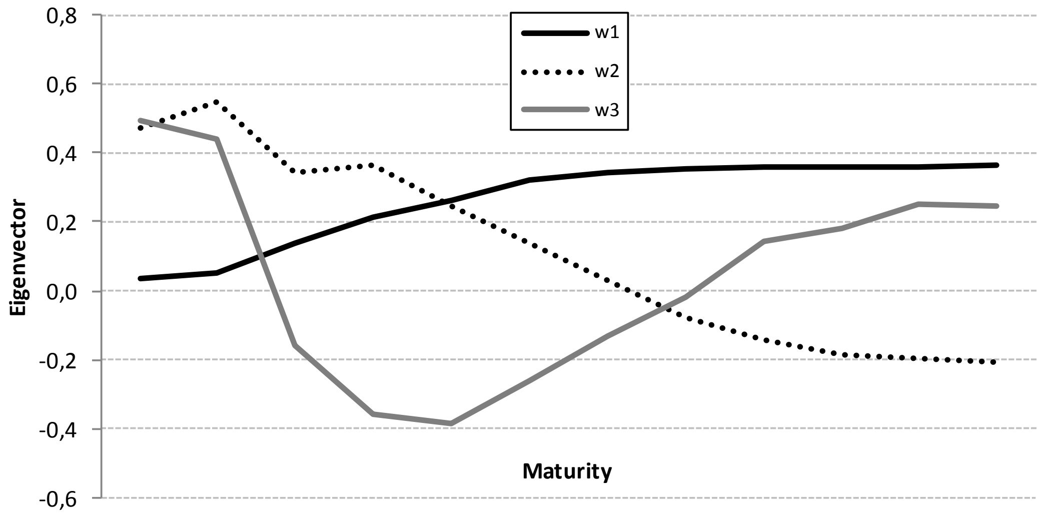

Figure 11: Eigenvectors of the Norwegian daily swap rate covariance matrix, 06.2012 – 2014.

06.2012 - 2014	1	2	3
Eigenvalues	77,32	7,98	2,81
% Variation	82,80 %	8,55 %	3,01 %
Cumulative %	82,80 %	91,34 %	94,36 %

Table 7: Eigenvalues of the Norwegian swap rate covariance matrix. June 2012-2014.

This period, characterized by downward trending swap rates, has some main differences compared to the previous two periods.

The first component does to a much higher extend explain total variations than in the two sample periods mentioned above. During the period June 2012 to 2014 the first component explains 82.8 %. This is more in line with general findings from PCA on term structures of interest rates. We can also observe that the eigenvalues for the components, indicating the amount of volatility during the period, is far lower than in the previous sample periods. In large part this is due to the fact that the Norwegian Central Bank did adjusted its policy rate only twice, and both times they lowered the policy rate. Compared to the fifteen adjustments in the period 2006 – 07.2008, this stability contributes to less volatility in the swap curve.

Still we can see that changes in the first component will lead to a smaller shift in the short rates compared to the long rates. So assuming that the first principal component can be interpreted as a parallel shift in all rates (duration-based methods) seems to be a simplification of the true movement.

The total amount explained by the first three principal components is equal for all three sample periods. However, the variations explained by the respective principal components differ from sample periods to sample period.

5.2 PCA on the euro swap curve

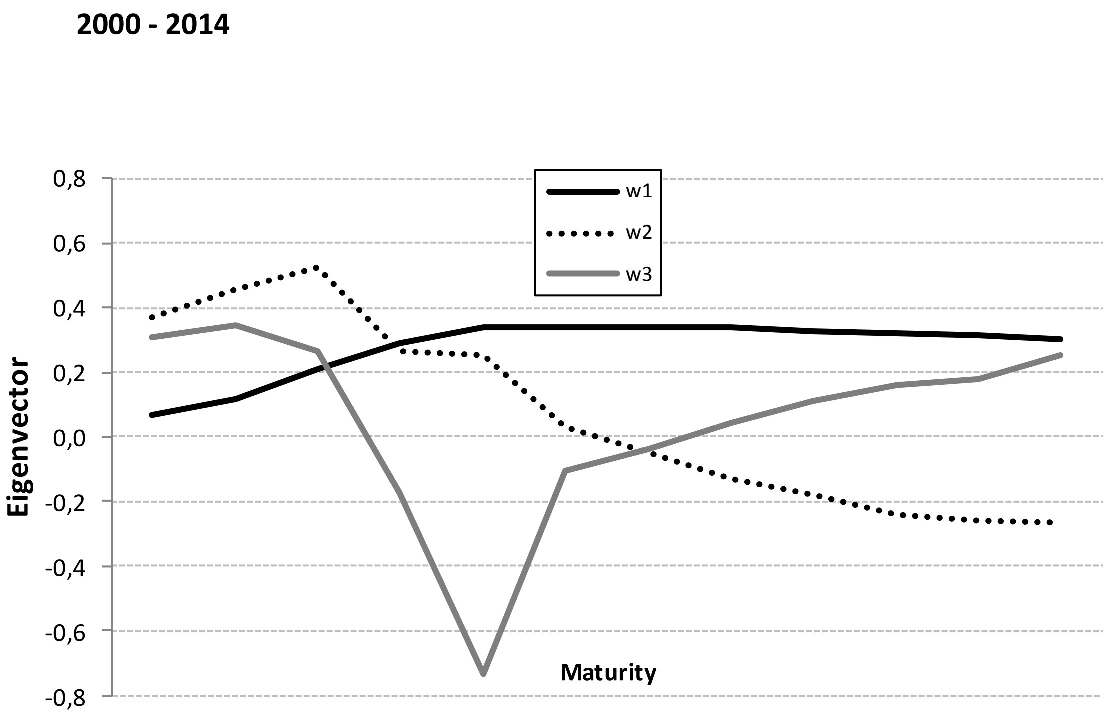

Figure 12: Eigenvectors of the euro daily swap rate covariance matrix, 2000 – 2014.

2000 - 2014	1	2	3
Eigenvalues	162,92	17,70	9,08
% Variation	79,83 %	8,67 %	4,45 %
Cumulative %	79,83 %	88,50 %	92,95 %

Table 8: Eigenvalues of the euro swap rate covariance matrix. 2000-2014.

The first principal component explains by far most of the historical variations in the euro swap curve. Almost 80 % can be assigned to a movement in the swap curve interpreted as an approximately parallel shift in all rates with maturities above 1-year. The less volatile money market rates, 3 and 6-month, are not correlated to the same extent with the first component as the long-term rates. This is displayed in figure 12 above with a dip at the short end of the solid black line.

The second component is downward tilting. However, the line (dotted line) representing the eigenvectors as a function of maturity is not linear. This gives us an indication that euro short-term swap rates have historically had unique sensitivity to the second component. The second component explains 8.67 % of the total variations. Compared to the Norwegian swap curve in the same period this is about half.

The solid grey line representing the eigenvectors of the third component shows that there is a significant dip in the 3-year rate; contributing to a high degree of convexity. The eigenvector for the 3-year rate is -0.74. While the eigenvectors for the adjacent rates, namely 2 and 4-year, is respectively -0.17 and -0.10. This means that in a scenario where the second component increases with a unit of 1, the 3-year rate will decline with 74bp (using the sample period as basis).

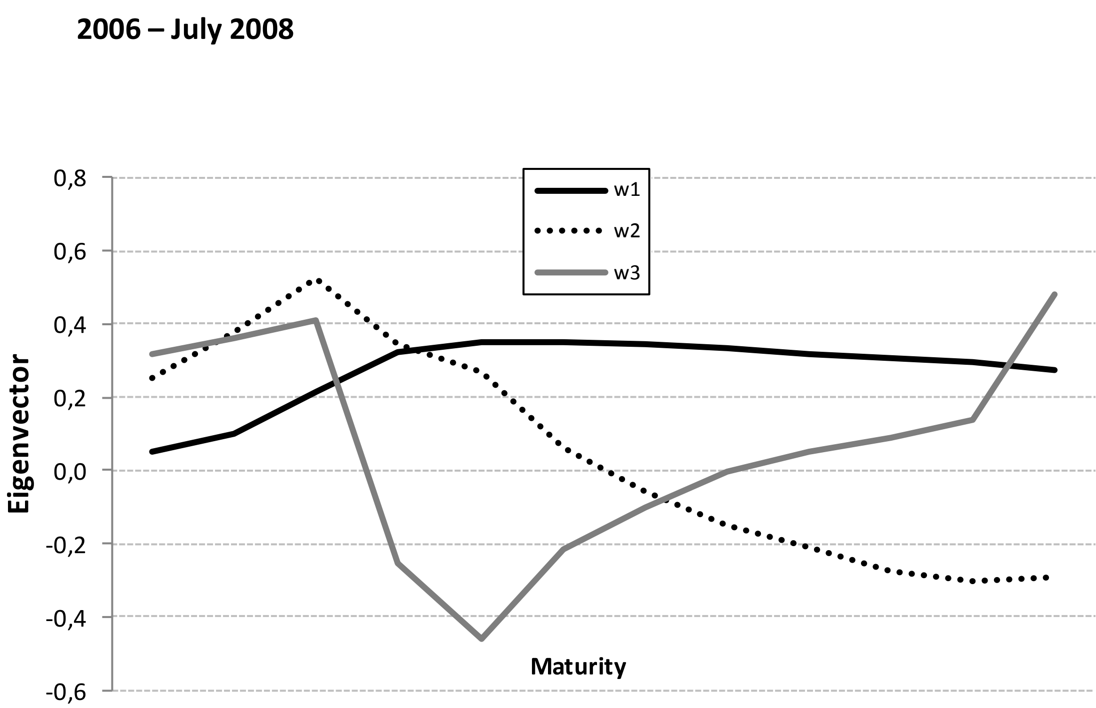

Figure 13: Eigenvectors of the euro daily swap rate covariance matrix, 2006 – 07.2008.

2006 - 07.2008	1	2	3
Eigenvalues	132,77	11,04	5,08
% Variation	84,91 %	7,06 %	3,25 %
Cumulative %	84,91 %	91,97 %	95,22 %

Table 9: Eigenvalues of the euro swap rate covariance matrix. 2006 - July 2008.

Movements in the first component during this period accounts for almost 85 % of total variations. The period from 2006 to June 2008 is less volatile than the whole sample period (2000-2014); this may explain that an approximately parallel shift in the swap curve is more influential than in periods with higher volatility. We can also observe that the shape of the first component (solid black line) is almost equal to that of the whole period presented above.

The second component describes 7 % of the total variations in the swap curve during the period. From the dotted line in figure 13 above we can observe a hump in the shape of the second eigenvector. The eigenvector of the 1-year rate, 0.53, contributes to this shape.

Third component; eigenvector for the 3-year rate is not as high relative to its adjacent rates as mentioned in the case above (-0.46 vs. -0.74).

All in all the first three components explain 95.22 %. This is about two and a half percentage points more than for the whole period.

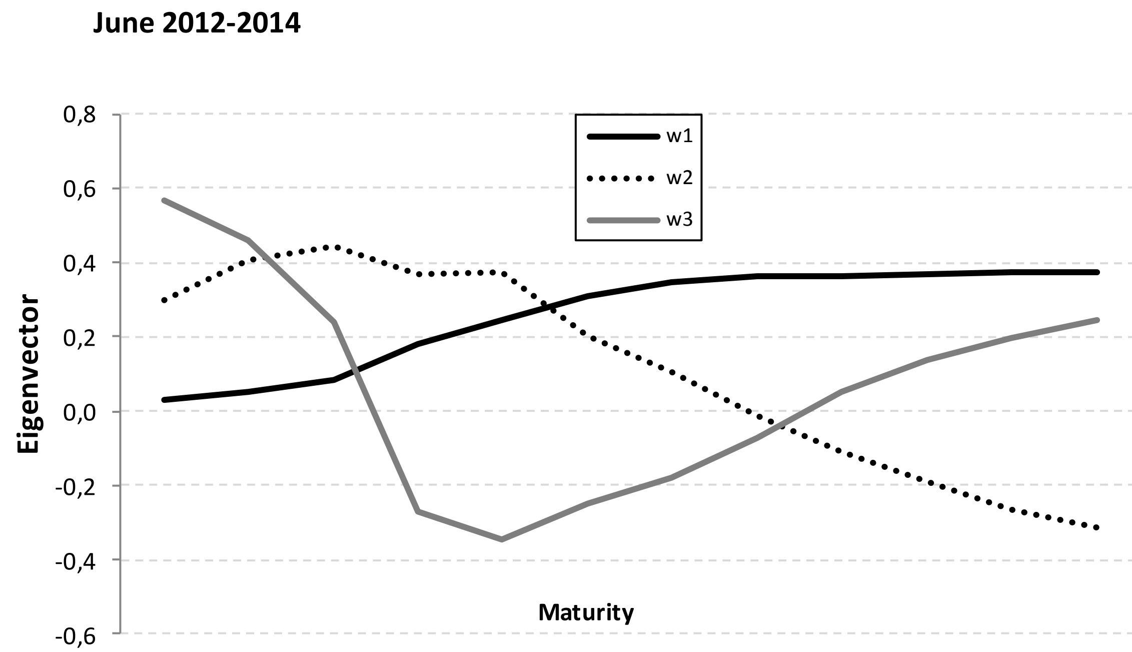

Figure 14: Eigenvectors of the euro daily swap rate covariance matrix, 06.2012 – 2014.

06.2012 - 2014	1	2	3
Eigenvalues	75,29	5,75	2,59
% Variation	85,06 %	6,50 %	2,93 %
Cumulative %	85,06 %	91,56 %	94,49 %

Table 10: Eigenvalues of the euro swap rate covariance matrix. June 2012-2014.

In the period between June 2012 and December 2014 the policy rate in the euro-zone declined from 1 % to 0.05%. From figure 15 below we can observe that the short-term rates (3m, 6m and 1y) exhibits a high degree of stability and the swap rates are indeed historically low. They are actually below the policy rate during the whole period. The long-term rates are more volatile and downward trending. The downward trending long-term rates flattened the swap curve during this period.

From the eigenvectors of the 3 -and 6-month rates and the 1-year rate (0.03, 0.05 and 0.09) we can summarize that these rates had very low correlation with shifts in the first component. A unit change in the first principal component would not have changed the short-term rates, but the long-term rates would have a more significant change upwards. It is this movement that stood for 85 % of the total variations in movements of the euro swap curve during the selected period. Compared to the first two periods the volatility of the first component (eigenvalue) is far lower than in this period. This is also true for the two other principal components.

The second component in this period explains about 6 %. Like in the other two periods the eigenvectors is downward sloping, meaning that a positive one-unit shift in the second component would have made the short-term rates move up, and the long-term rates down. This is apparent in figure 15 below, where we can see that the swap curve is indeed flattening (spreads between short –and long-term rates are narrowing).

Principal component number three explains about 3 % of the swap curve movements in this period. This is approximately equal to the other two periods.

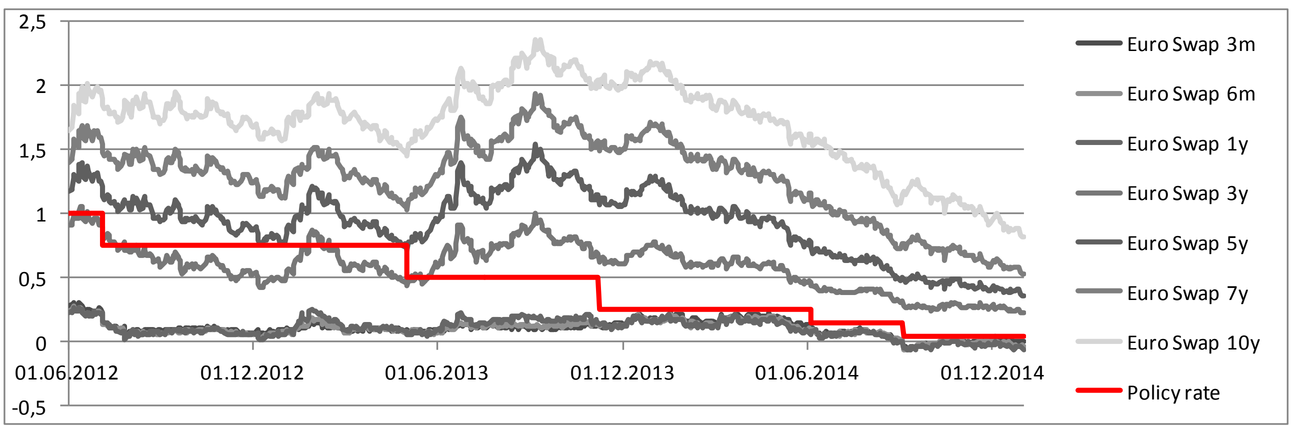

Figure 15: Level of selected swap rates and the ECB policy rate, period June 2012 to end of 2014.

5.3 Comparison of PCA-outputs from the two swap rate markets

2000-2014

There are a couple of interesting differences between the PCA-outputs from this period;

1) It is a significant difference in the explanatory power of the first component between the two swap curves, respectively 71% for the Norwegian swap curve and 80% for the euro swap curve. This means that movements in the euro swap curve during this sample period is more driven by the so-called trend shifts in the swap curve. However, the eigenvectors for the short-term swap rates in both swap curves indicates that the first principal components cannot be directly interpreted as a component yielding a parallel shift in the whole swap curve simultaneously. But if we discard the money-market rates (3m and 6m) from the swap curve the first principal component looks more like a component yielding a parallel shift. This is in line with most of the literature. It clearly indicates that money-markets rates in both markets are somewhat disconnected from movements in the swap curve as a whole.
2) The findings in the second component also yield some interesting deviations between the two swap rate markets. The eigenvalues presented in table 5 and 8 told us that the second components explained the following amount of the total variations in the two swap curves; Norwegian swaps market 17.74% and the euro swaps market 8.67%. This difference is clearly significant, and indicates that the shape of the Norwegian swap curve fluctuates more than the euro swap curve. Figure 16 and 17 below visualizes this finding in a nice way. Here the basis point changes for the principal component representations[9] are plotted as a function of time. This figure gives us a good indication on the relative importance for each of the two principal components during the sample period 2000-2014. Figure 16 illustrates that the relative importance of movements in the second components (grey line) for the Norwegian swap curve is highly volatile.

[9] Principal component representation is by construction uncorrelated time series corresponding to the original time series used as inputs in the PCA.

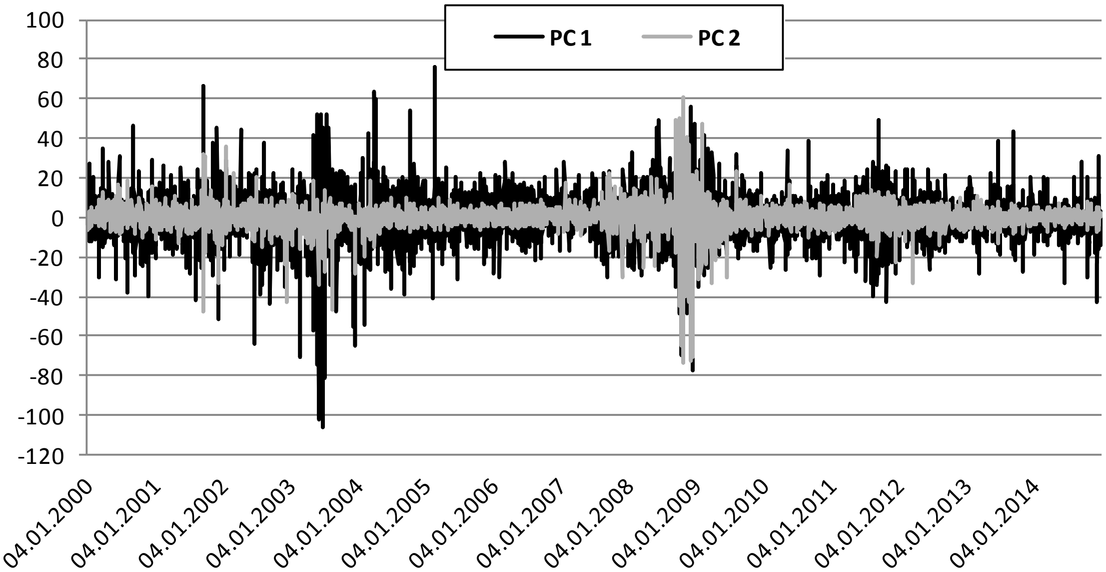

Figure 16: First and second principal component for Norwegian swap rates, 2000-2014. The black line represents the first component and the grey line the second component.

One key observation from figure 16 is that changes in the second principal component during the financial crisis of 2008-2009 are remarkably more influential on movements in the swap curve compared to the other periods in the sample. Indeed, performing a PCA during 2008 underpins this finding. During 2008 the second principal component explains 26% of the total variations in the Norwegian swap curve. In the euro case this number is 10%, i.e. not significant higher than the other periods. In figure 17 below we can observe that the relative importance of changes in the second principal component in the euro swap curve is not as volatile as we observed in the Norwegian swap curve.

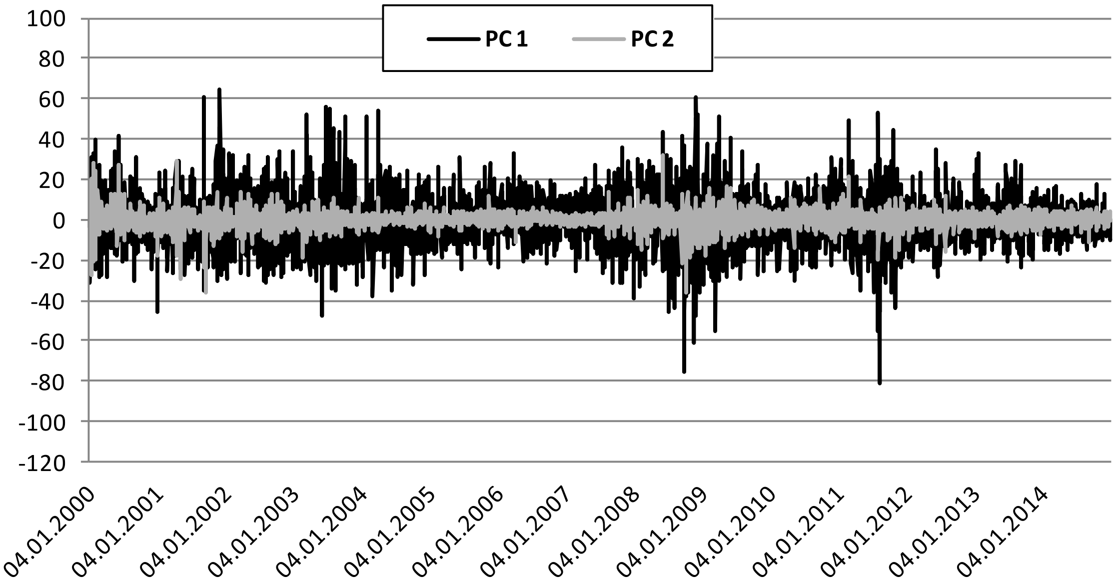

Figure 17: First and second principal component for euro swap rates, 2000-2014. The black line represents the first component and the grey line the second component.

The volatility for the swap rates in 2008 also contains some significant differences and contributes to major divergence in the outputs from the PCA. However, the volatility is almost similar for the 2-year rate and longer maturities. This has an impact on the outputs from PCA; due to the high volatility of the money-market rates compared to the longer-term rates the shape of the eigenvectors is now downward sloping with maturity. Unlike other periods where the shape has been upward sloping. This finding is only apparent in the Norwegian swap curve. For the euro swap curve the PCA outputs from 2008 are in line with the other periods.

2006-July 2008

The differences in the amount explained by the first principal component are during this period a bit higher than the period presented above. This means that during this period, a period where rates in both markets trended upwards, movements in the euro swap curve had more elements of a parallel shift than in the Norwegian swap curve. Further, the eigenvectors for the second component in the Norwegian case indicate that the swap curve where more driven by changes in the slope, compared to the euro swap curve. This is evident from the relative importance of the eigenvalues for the Norwegian and the euro swap curve, respectively 16% and 7% (table 6 and 9). Identifying the source of this difference we can take a closer look at charts showing movements in the swap rates during this period, they are displayed in figure 18 and 19 below. Movements in the policy rates seem to be a determining factor for what drives the swap curves. The policy rates start of at the same level, 2.25%, in 2006. Until June 2007 the policy rates follows approximately the same pattern, and the same is true for the swap curves- which are upward sloping. But after June 2007 the policy rates take different paths – the Norwegian policy rate steady increases at a relative high frequency, but the ECB policy rate does not move at all before a year later, on July 2008. This seems to transform the swap curves in different ways. From figure 19 we can see that the short-term rates (3m and 1y) exceeds the long-term rates during autumn 2008. This leads to a downward sloping swap curve; indicated by the PCA discussed above. The euro swap curve seems to fluctuate to a certain degree, but not nearly as much as the Norwegian swap curve.

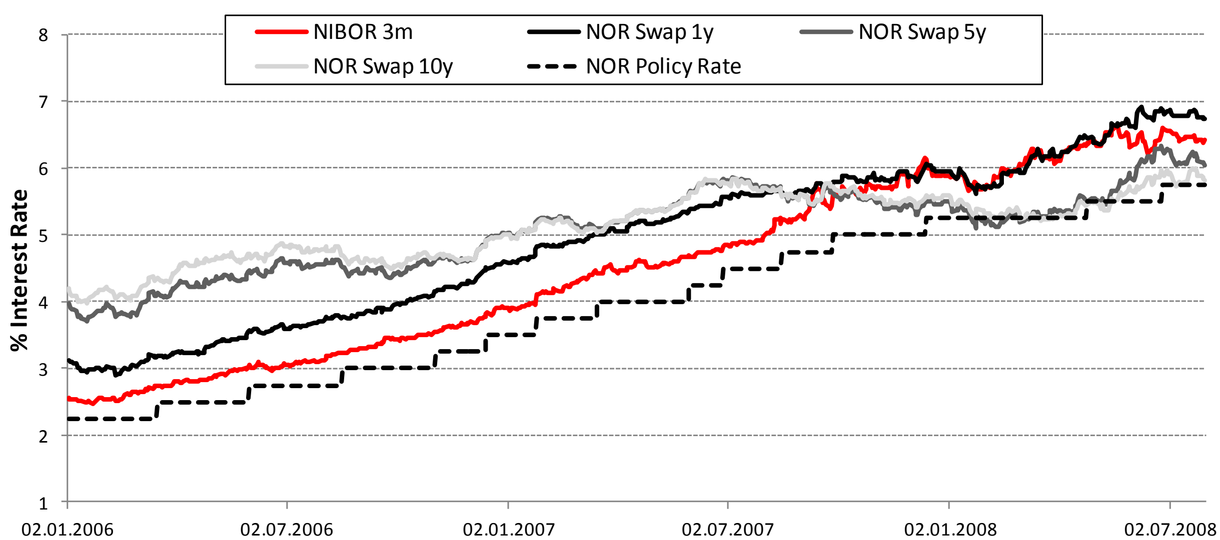

Figure 18: Selected Norwegian swap rates and the Norwegian policy rate, 2006-July 2008.

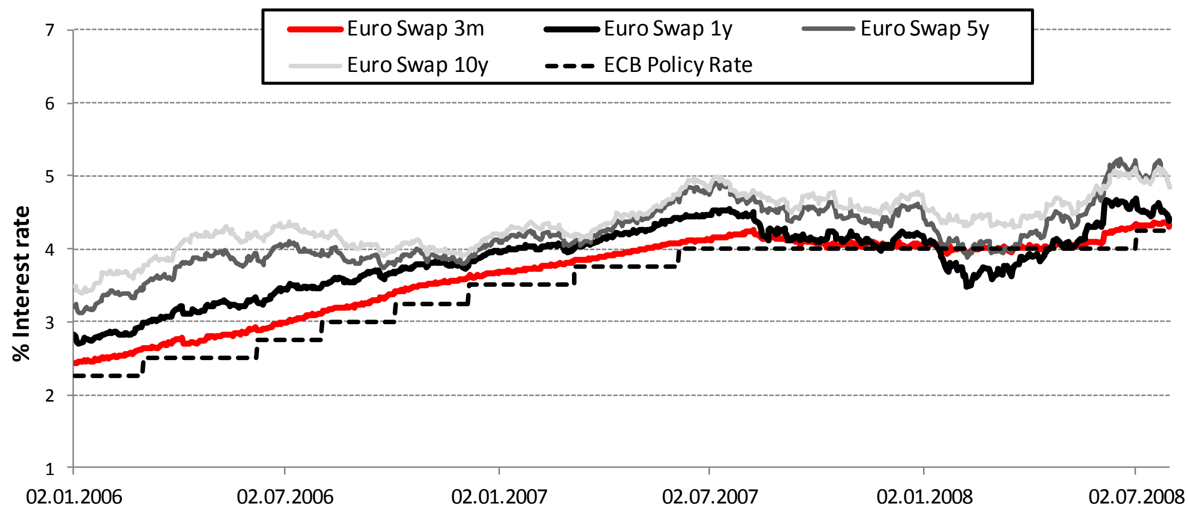

Figure 19: Selected Norwegian swap rates and the Norwegian policy rate, 2006-July 2008.

June 2012-2014

During this period, which are characterized by downward trending swap rates in both markets, the outputs from PCA indicates that the movements in the two swap curves are most alike compared to the other sub-periods analyzed. Figure 2 (page 20) above, shows that the Norwegian policy rate during this period is very stable; with only one adjustment being made. In addition, the movement explained by the first principal component during this period is as much as 83% for the Norwegian swap curve. This is significant more than during the other periods analyzed. Compared to the euro swap curve the outputs from PCA are now more similar than ever between the two swap markets. A closer look at the eigenvectors confirms this view; the corresponding eigenvectors for each rate are remarkably similar.

The second components which explain respectively 8.55% and 6.5% of the Norwegian -and euro swap curve movements is historically low. But if we peek into the details of the second component we find some differences. The eigenvectors for the money-market rates in each curve indicates that these rates influence the slope of the respective swap curve with different magnitude. The eigenvectors for the 3-month and 6-mont rates in the Norwegian case are respectively 0.47 and 0.55. In the euro case the corresponding eigenvectors are 0.3 and 0.41. So in a scenario where the second component increases with one unit the money-market rates in the two markets will react a bit differently; the Norwegian money-market rates will increases by a larger amount than the euro money-market rates - contributing to a steeper swap curve in the short-end.

The third components are almost identical in the two swap curves, both in terms of relative explanatory power and slope of the eigenvectors.

6. Conclusion

In the sections presented above I have analyzed and discussed the different properties and similarities of the Norwegian –and euro swap curves. In this section the essential findings regarding the research questions are summarized. As a reminder the research questions are repeated below, together with the main findings and concluding remarks.

- **What kind of movements has historically driven the shape of the swap curve in the Norwegian -and the euro swap market?**
- **In addition, are there any similarities and/or differences between the evolutions of the two swap curves?**
- **Are the findings consistent through various regimes of swap rate levels and movements? The aim is to identify two sub-periods of swap rate evolution; 1) a sustained period of falling trend in the swap rates, and 2) a sustained period of rising trend in swap rates.**

Some of the findings in this thesis are coherent with other studies on term structures of interest rates, a) the first component is the component that historically has driven most of the movements in the two swap curves, b) movements in the short-term rates or money-market rates, namely 3 –and 6-month rate, are to a large degree independent from the rest of the swap curve. This results in the fact that the first principal component cannot be interpreted as a component yielding a parallel shift in the entire swap curve. This finding is similar to what Barber & Cooper (2012) found in their research. The only time this finding deviates from the general view is on the Norwegian swap curve during the sample period of 2008. The money-market rates surged well above the other rates in this one-year period, leaving the Norwegian swap curve downward sloping.

The relative importance of the first principal component is different in the two swap curves. Evidence from PCA suggests that movements in the first principal component have historically driven more of the total variations in the euro swap curve compared to the Norwegian swap curve. These findings are generally consistent through different time periods and various interest rate regimes. In the period from June 2012-2014, a period with historically low volatility for Norwegian swap rates, the first principal component explains to a larger degree the total variations in the Norwegian swap curve. This finding is not surprising, but settles in a nice way that in periods of less volatility in swap rates we can expect to see that movements in the swap curve are more driven by the first principal component, i.e. an approximately parallel shift in the swap curve.

The outputs from PCA are in the euro swap curve fairly stable through the different time periods analyzed. Even in the turmoil period of 2008 evidence from PCA suggest that movements in the swap curve do not deviate in a significant way from other periods. For the Norwegian swap curve the evidence suggest differently. Higher volatility in the Norwegian swap rates leads to more deviations in the outputs from PCA; hence we can observe more volatility in the shape of the Norwegian swap curve. Such deviations during turmoil markets did Stelmach (2010) also find in his article.

In the section “Historical swap curve movements” we discussed that the policy rates set by respectively the Norwegian Central Bank and ECB has historically been economical factors that have driven the shape of the swap curves. The policy rate in the Norwegian case have fluctuated a bit more than the corresponding euro rate; which may have lead to the more unstable shapes of the Norwegian swap curve. Another plausible explanation for the higher volatility in the Norwegian swap curve could be the size of the swap market and the amount of market participants. A market with fewer participants and less trade has less liquidity. This may have lead to a higher risk-premium in the Norwegian swap market during certain periods, contributing to more volatile swap rates, especially short-term rates (money-market rates).

The third principal component, which is referred to as the curvature component, is fairly stable in both swap curves and during different time periods. The explanatory power of this component lies in the range of 3-5%. In periods with relative high volatility in swap rates and thus swap curve movements, changes in the third component explains more of the total variations than in periods with more stable swap rates and less frequent swap curve movements. Even though the importance of the third component is not significantly high, the finding that the relative importance of the third component is increasing during periods of more volatility is a useful knowledge.

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