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Do Commodities Offer Diversification Benefits in Multi-Asset Portfolios?

An Out-of-Sample Analysis for Different Asset Allocation Strategies

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Abstract

During the last decade commodities have increased in popularity as an alternative asset class. Previous research reports the diversification benefits when adding individual commodity indices or futures to a traditional portfolio of stocks and bonds, due to low correlation to traditional asset classes. Hence, a major motive of commodity investments is to increase the performance of the portfolio. More recent research report that commodities have been financialized and that the benefits of adding commodities have virtually vanished. Most studies that report benefits of commodities are limited to in-sample mean-variance analysis, and do not consider the challenge of setting up an allocation strategy which investors are facing.

In this study, we examined the out-of-sample diversification benefits of commodities to a stock-bond portfolio, using five sector-based commodity indices. We employed different allocation strategies; Minimum Variance, Maximum Sharpe and a Fixed-weighted portfolio. In addition, we constructed four different risk parity portfolios, each based on different risk measures.

Considering the period 2000 - 2014, our results show that commodities contributed to reduced returns and increased volatility for most strategies. Moreover, commodities contributed to reduced risk-adjusted return and a higher expected tail-loss. While this period analysis do not show benefits of adding commodities, dividing the full sample-period into sub-periods indicated that benefits of commodities depend on the allocation strategy and the period studied.

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The authors have the full responsibility of the contents in this paper.

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1. Introduction

Over the last decades, there has been a strong growth of commodity investments, mainly through commodity futures and commodity index funds, which reflects an increased interest from both private and institutional investors in the commodity markets. While investors can obtain exposure to commodity markets via direct physical investments, futures and index funds are more practical because of the storage costs induced by physical investments. In earlier years, investors invested mainly in stocks and bonds due to deep and liquid markets and low transaction costs. The increased popularity of alternative investments such as commodities relates to the decreased transaction costs and increased liquidity in these markets (Stoll & Whaley 2009). The main sources of return from long-only investments in commodity futures and hence indices are (I) the risk premium that accrues to buyers of futures contracts. According to the theory of normal backwardation (see e.g. Keynes (1930) and Hicks (1946)) speculators buy futures from producers to provide price insurance, but require a price that is below the expected future spot price. Hence, the futures price will rise towards expiry. (II) The interest rate on the risk-free asset purchased as collateral against futures contracts. (III) The profit (or loss) from unexpected fluctuations in the price of the underlying asset. Some argue that commodities have positive expected return, and that it can serve as a hedge against both expected and unexpected inflation (Geman 2005).

Even though the commodity markets have existed for centuries, another reason for the growing interest might be the common perceptions that commodities have low correlation with traditional assets such as stocks and bonds. This can be related to the factors driving commodity prices i.e. the interaction of supply and demand, weather, politics and event risk (Geman 2005). Based on historical data, commodities tend to move the opposite way as to stocks and bonds. Whereas, stocks and bonds tend to performed better when the inflation is stable or slowing, commodity prices tend to rise when inflation accelerates (Bodie & Rosansky 1980; Erb & Harvey 2006; Gorton & Rouwenhorst 2006).

Several studies agree on the low correlation between commodities and traditional assets, which make commodity investments potentially beneficial for diversification when complemented with traditional assets. In addition, while commodities within one sector tend to be highly correlated, e.g. silver and gold, the correlation between different commodity sectors tend to be low. Some papers claims that there has been an increasingly co-movement between commodities, stocks and bonds (e.g. Tang and Xiong (2012)), hence reducing the benefits of adding commodities.

Most studies that examine the benefits of including commodities in a portfolio consider only in-sample analysis. In-sample analysis can only demonstrate that commodities would have improved the risk-return trade-off of the portfolio for the period, if the assets return during this period were already known in advance. The asset returns during the subsequence period are unknown and the estimates for returns often have large estimation errors (Daskalaki and Skiadopoulos (2011). Therefore, one will achieve a more realistic assessment when evaluating the portfolio benefits of commodities by performing an out-of-sample analysis. In addition, in-sample analysis does not reflect the decision environment of investors, who face the uncertainty of the future and the challenge of setting up a specified allocation decision at time t for the subsequent period.

1.1. Problem statement

The objective of this study is to analyze the out-of-sample diversification benefits of commodities to stock-bond portfolios, for different asset allocation strategies. We do this by employing the traditional allocation strategies such as Minimum Variance-, Maximum Sharpe- and 1/N portfolios. Additionally, we implement four approaches to risk parity, each based on different risk measures; standard deviation, covariance, semi-deviation and expected tail-loss. Moreover, by analyzing the contribution of commodities for different asset allocation strategies, we aim to determine whether the benefits of commodities depend on the asset allocation strategy.

Our study might be of interest for portfolio managers, risk managers and investor considering allocating part of their capital in commodity investments.

In the next chapter, we present a brief review of the related literature on commodities in the portfolio. Chapter 3 describes the asset allocation strategies used in this study. In chapters 4 and 5, we present the data and the descriptive statistics for the assets. In chapter 6, we present our empirical results. Finally, we draw conclusions based on our findings and suggest topics for further research.

2. Related Literature

Several studies have examined the benefits of investing in commodities, and especially the potential diversification benefits, i.e. reduce risk for a given level of return, when including commodities to a stock-bond portfolio. Many papers found low correlation between stocks and commodities (Bodie & Rosansky 1980; Büyükşahin et al. 2008; Erb & Harvey 2006; Gorton & Rouwenhorst 2006; Kaplan & Lummer 1997) and agreed on the potential diversification benefits of adding commodities to a stock portfolio. Erb and Harvey (2006) point out that commodities should not be analyzed as one single marked, but individual markets, because the price changes are determined differently. Therefore, many papers use individual commodity futures instead of a broad commodity index. While correlations among commodities in different groups were small, correlations within commodity groupings were substantially higher, with the exception of soft commodities (Kat 2006). Additionally, while commodity futures were relatively uncorrelated with stocks and bonds, correlation may vary between different phases of the business cycle, which suggest that commodities do not provide equally good diversification benefits compounded with these assets at all times (Gorton & Rouwenhorst 2006; Kat 2006). In the peak of the business cycle commodity prices are often high, which relates to higher demand for raw materials. However, the booming activity cause a rise in interest rates and the expectations for growth to decrease, which again cause financial assets to perform poorly.

Nevertheless, some researchers questioned the diversification benefits of commodities and emphasized that the growing presence of commodity index funds could be the reason why commodity markets might create closer integration with stocks and bonds. Many authors point to an increasing integration between commodities and financial markets, which is commonly referred to as *financialization* of commodities (Domanski and Heath (2007), Tang and Xiong (2012) and Silvennoinen and Thorp (2013)). Basak and Pavlova (2014) highlight that increased co-movements were also evident across different commodity sectors.

Bodie and Rosansky (1980), Greer (1994) and Georgiev (2001) found that the performance of a stock-only portfolio could be improved by adding commodities during the periods 1950-1976, 1970-1993 and 1995-2005, respectively. A more recent study by Conover et al. (2010), also supported this conclusion when studying the period 1970 to 2007. The authors found that investors could reduce risk without sacrificing returns when switching from a stock-only portfolio to a portfolio of stocks and commodities, and that there was diversification benefits regardless of the stock style an investor pursued.

Several papers analyzed the shift in the efficient frontier when adding commodity futures to the investment universe (Abanomey & Mathur 1999; Jensen et al. 2000; Satyanarayan & Varangis 1996). These studies concluded that commodities shifted the efficient frontier upwards indicating a better risk-adjusted performance of efficient portfolios. The same conclusion was reached in a more recent study by Idzorek (2007) for the period 1970 – 2005.

Ankrim and Hensel (1993), Anson (1999) and Laws and Thompson (2007) examined the diversification benefits of commodities under a mean-variance setting. The analysis concluded that expanding the investable universe with commodities improved the risk-return trade-off of optimal portfolios, for different risk aversion coefficients. In contrast, Cao et al. (2010) reported that the efficient frontier did not shift significantly in a mean-variance setting when adding commodities in the period 2003 to 2010, which according to the authors might be a consequence of the increased co-movement between the assets.

Furthermore, a more recent study by Lombardi and Ravazzolo (2013) examined different methods for estimating correlations, and found that the correlation between commodities and the stock market has increased after the financial crisis. In addition, they highlighted that the portfolios became substantially more volatile with commodities, and that they did not necessarily increase the Sharpe ratio.

Belousova and Dorfleitner (2012) investigated the diversification contribution of including commodities to a portfolio of traditional assets from the perspective of a euro investor. Their results showed that the diversification benefits varied greatly among different commodities, but overall their results indicated that commodities were valuable investments from the perspective of diversification.

Most of the above-mentioned literature has provided evidence that the investor is better off when including commodities in their portfolios. However, some of the literature has been conducted in a mean-variance setting, which might not reflect the right view of performance. This is due to one of the assumptions of Modern Portfolio Theory – that the asset returns are normally distributed random variables. This property is rejected by Gorton and Rouwenhorst (2006) and Kat and Omen (2006b) for commodity futures. Both papers found that commodities had positive excess kurtosis and exhibit fat-tails, but that the level of kurtosis was comparable to what is evident for US large cap stocks. Since investors prefer positive skewness and have aversion to high kurtosis, investors should consider the higher order moments in the allocation decision.

A major shortcoming of the previous research is that most studies examining the diversification benefits of commodities to a stock-bond portfolio are limited to an in-sample analysis, and do not show that commodities would actually improve the performance in an out-of-sample setting. A few papers highlight this shortcoming. You and Daigler (2013) investigated the diversification benefits of using individual futures contracts and examined the instability between in- and out-of-sample benefits, and found that there was instability. Further, the results showed that the instability was mainly driven by time-varying returns rather than the risk of the individual assets, but also that commodities improved the performance in the out-of-sample analysis. In contrast, Daskalaki and Skiadopoulos (2011) used spanning test and found that including commodities significantly shifted the efficient frontier, but the benefits of commodities found in-sample were not present in the out-of-sample analysis.

Bessler and Wolff (2014) also analyzed the out-of-sample portfolio benefits distinguishing between different commodity groups during the period 1983-2012. They employed seven different asset allocation strategies and found little or no improvements in performance by including agricultural- and livestock commodities. However, they found that industrial metals and an aggregated commodity index generated the improved performance. When the authors compared the different asset allocation strategies, they found that the risk parity allocation approach and the Black-Litterman model outperformed all other strategies. Similar to Daskalaki and Skiadopoulos (2011), the out-of-sample benefits were much smaller than indicated by the in-sample analysis. Additionally, Bessler and Wolff (2014) reported that the portfolio benefits of commodities were time varying, and that they vanished during the recent financial crisis in 2008.

2.1. Contribution to Existing Literature

Our study contributes to the existing literature by examining the out-of-sample benefits of adding commodities to different pre-specified allocation methods including the traditional methods; Minimum Variance, Maximum Sharpe and fixed-weighted portfolios, as well as the risk parity approach. Based on previous research we assume that the returns are non-normally distributed and exhibit fat tails, and therefore we find it interesting to implement alternative risk measures in the allocation decision for the risk parity approach. In addition, while most studies consider individual futures or indices' isolated contribution to a stock-bond portfolio, we consider five sector-based commodity indices employed under each strategy. In contrast to many academic papers, which optimize the portfolios once and only rely on rebalancing weights over time, we re-estimate the portfolio weights and rebalance monthly to retain the respective allocation principles. This is because the optimal weights decades ago might not be the optimal weights today.

3. Asset Allocation Models

In this section, we provide an explanation of the asset allocation techniques used in the study. We adopt four approaches to risk parity based on different risk measures such as standard deviation, covariance, semi-deviation and expected-tail-loss in addition to traditional allocation models i.e. the Minimum Variance-, Maximum Sharpe- and a Fixed-Weighted portfolio. While the risk parity and Minimum Variance portfolios only depend on volatility and correlation estimates as input variables, the Maximum Sharpe portfolio also depends on estimates for expected returns. If future returns and covariances were known in advance, the Maximum Sharpe portfolio would dominate all other strategies in terms of financial efficiency. However, estimation errors in the input parameters can lead to poor performance of Maximum Sharpe portfolio (Best & Grauer 1991). Some researchers such as Chopra and Ziemba (1993), argue that estimation errors in return estimates dominate errors in the covariance. This can partly explain the popularity of models that are not based on estimates for expected returns, such as risk parity and Minimum Variance. However, estimates for expected returns should be of great interest for investors, who prefer higher returns.

All portfolios are long-only because risk parity portfolios per definition cannot have negative weights. Additionally, the Maximum Sharpe portfolio tends to incorporate extreme values in the asset positions when short-selling is allowed. We do not use leverage as financial gearing, because of the additional dimension of risk it entails. Another argument for omitting short-selling and leverage is that many funds do not have the mandate to practice it.

The portfolios constructed were based on 60 monthly observations, from January 1995 to December 2014. We used continuous rebalancing, i.e. every month, and we re-estimated the portfolio weights annually to retain the respective allocation principles. This is in contrast of many academic papers, which only optimize the portfolio once and only rely on rebalancing the weights over time. The problem with this approach is that the optimization of the weight decades ago might not be the optimal weights today. Thus, we got estimated portfolio weights every year from 2000 to 2015, and back-testing results between 2000 and 2014. Further, we provide technical details in the Appendix 9.2.

3.1. Minimum Variance Portfolio

The Global Minimum Variance portfolio (MinVar) employs the portfolio weights that minimize the in-sample portfolio variance. In contrast to risk parity, the MinVar strategy aims at minimizing risk, rather than maximize the diversification of risk. This portfolio will allocate a substantial part to commodities if the volatilities, measured as variance, are low and/or the correlation with other assets is small or negative. The MinVar problem ignores expected return of the portfolio and the objective function is:

$$Min_w \ \sigma_p^2 = \sum_i^n \sum_j^n w_i w_j \sigma_i \sigma_j \rho_{ij} \quad s.t. \quad \sum_{i=1}^n w_i = 1 \ and \ 0 \leq w_i \leq 1$$

Where w_i is the weight of asset i, σ_i^2 is the variance of asset i, and $\rho_{ij}$is the correlation coefficient between asset i and j.

3.2. Maximum Sharpe Portfolio

The Maximum Sharpe portfolio (MaxSharpe), also called the Tangency portfolio, is where the capital allocation line (CAL) is tangent to the efficient frontier. The efficient frontier is the hyperbola that represents all allocations of the risky assets that are efficient. This means that for every given level of standard deviation, the expected return of the portfolio is maximized. The CAL intercepts the risk free rate and has a slope equal to the incremental increase in portfolio return to the incremental increase in standard deviation, which equals the Sharpe ratio. To achieve the Maximum Sharpe portfolio one maximizes the slope of the CAL:

$$Max_w \; Sharpe\; ratio = \frac{\overline{r_p} - r_f}{\sigma_p} \quad s.t. \quad \sum_{i=1}^{n} w_i = 1 \; and \; 0 \leq w_i \leq 1$$

Where $\overline{r_p}$ is the portfolio average return, r_f is the average risk-free rate, and σ_p is the portfolio volatility measured as standard deviation.

3.3. Fixed-Weighted Portfolio

The fixed-weighted portfolio, or 1/N, does not consider any parameter estimates or involve any optimization approach. It is a naïve approach to asset allocation, holding an equal and fixed capital amount in every asset class in the asset universe. In the case where we include commodities, we allocate 1/3 to all three asset classes. When we exclude commodities, we allocate 50/50 between stocks and bonds.

3.4. Risk Parity

The risk parity approach has grown in popularity over the recent years, especially after the financial crisis in 2008 when investors started to find alternative ways to allocate assets. The idea of risk parity is that the risk contribution of each asset is set equal, meaning; the investor maximizes the diversification of risk, in-sample. This allocation strategy delivers *true* diversification that limits the impact of losses of individual components to the overall portfolio (Qian 2005). Including commodities in a stock-bond portfolio contributes to spread the risk, but traditional optimization models do not necessarily include all assets in the investment universe. As with fixed-weighting strategy, risk parity ensures this feature.

In the following, we explain the risk parity portfolios we will use in this study. A full derivation of risk parity can be found in appendix 9.2.2. We adopted four different approaches to the risk parity allocation principle. The first one was based on standard deviations and covariance between the assets and is sometimes referred to as *full risk parity*, and the second is a naïve version of risk parity. According to Inker (2011) standard deviation is a dangerously limited estimate of the true risk of an asset and the risk parity model is only attractive if standard deviation is a good estimate of the true risk, which highlight the importance of using alternative risk measures. Therefore, we will conduct this strategy with other risk measures. The third and fourth risk parity strategies are also naïve approaches where standard deviation is replaced by downside deviation and expected tail-loss respectively.

The idea of risk parity is that the risk contribution of each portfolio component is made equal, mathematically defined as:

$$w_i \frac{\partial \sigma_p}{\sigma w_i} = w_j \frac{\partial \sigma_p}{\sigma w_j} \;\; \forall\, i, j \; and \; i \neq j$$

Where $w_i \frac{\partial \sigma_p}{\sigma w_i}$ is the risk contribution of asset i.

This is why risk parity portfolios are often referred to as equal risk contribution portfolios; hence, the model maximizes diversification of risk. By definition they include all[1] assets in the selected investment universe and the weight assigned to an asset class in the risk parity portfolio becomes higher (lower) the lower (higher) its volatility and correlation with other assets. Several papers have showed that the risk parity strategy performs well, and usually outperforms 1/N or a value-weighted (see e.g.(Anderson et al. 2012; Kirby & Ostdiek 2012) . In addition Fisher et al. (2012) found that risk parity also tends to outperform the tangency portfolio.

3.4.1. Risk Parity Portfolio Based on Covariance

This strategy considers both standard deviation of assets and the correlation between the assets and follows the general definition of risk parity, hereafter referred to as RPCOV. This allocation method is an optimization problem, where the objective function provided by Maillard et al. (2009) is to minimize the sum of squared differences between the assets' risk contributions, i.e. finding the weights that make the risk contributions across individual assets equal:

$$Min \;\; f(w) = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[w_i \frac{\partial \sigma_p}{\partial w_i} - w_j \frac{\partial \sigma_p}{\partial w_j} \right]^2 \forall\, i, j \; and \; i \neq j$$

$$\text{S.t.:} \;\; \textstyle\sum_{i=1}^{N} w_i = 1 \quad a \quad 0 \leq w_i \leq 1$$

Where w_i is a unique solution and the condition $f(w_i) = 0$ is ensured. Since w_i is a function of the risk contribution which again depends on w_i there is a problem of endogeneity which is taken into account in this optimization algorithm.

[1] Note that $w_i \rightarrow 0$ when $\sigma_i \rightarrow \infty$

In a hypothetical situation where the assets have the same pair-wise correlations, which in reality is uncommon, the optimal portfolio weights of the assets would be proportional to the inverse of its associated standard deviations. The strategy where the weights are given by the inverse volatility's fraction of portfolio inverse volatility is often referred to as *naïve risk parity* and this is the approach we will use for the remaining risk parity portfolios.

3.4.2. Risk Parity Portfolio Based on Standard Deviation

This is the simplest approach of the risk parity portfolios, and this strategy considers only the standard deviations as input variable, RPSTD.

The portfolio weight of each asset is calculated under the naïve approach as:

$$w_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{N} \frac{1}{\sigma_j}} \quad \forall i, j = 1, \dots, N.$$

Here, the weights of the assets are the ratio between the inverse of their volatilities and the sum of the assets' volatility reciprocals.

Even though standard deviation is a commonly used risk measure, it has important drawbacks when applied to financial analysis. One is that two different assets with the same return and volatility might have different skewness and kurtosis since many return distributions are not normal. Investors are more adverse to downside deviation compared to upside deviation with the same magnitude, rather than deviation around the mean.

3.4.3. Risk Parity Portfolio Based on Semi-Deviation

This strategy (RPSEMI) uses semi-deviation as risk measure. Semi-deviation or downside deviation is a downside risk measure and a modification of the standard deviation, a concept introduced by Markowitz (1959). In this measure, only variation below a minimum acceptable return (or target return) is considered. The minimum acceptable return (MAR) can be chosen to match specific investment objectives, and the most commonly used MAR is the risk free rate. In this study, we use risk-free rate as the MAR when calculating the portfolio weights and thereby measure the standard deviation of negative excess returns. The formula for calculating the semi-deviation is:

$$\sigma_{r_i}^{semi} = \sqrt{T^{-1} \times \sum_{t=1}^{T} \text{Min}(r_t - MAR, 0)^2}$$

Here, r_t is the period t observed return from a sample $\{r_1 \dots, r_T\}$ of T returns that is below MAR.

3.4.4. Risk Parity Portfolio Based on Expected Tail-Loss

Other popular approaches to measure downside risk are the Value-at-Risk (VaR) and conditional VaR, hereafter referred to as expected tail-loss (ETL). There are several methods to calculate these. We will adopt the historical or non-parametric method to incorporate the skewness and kurtosis in the measure of risk. The advantage with the historical approach is that we make no assumptions about the parametric form of the return distributions and consider all incidents of the distribution. However, this method assumes that all possible future loss have been experienced at some point in the past, which is an adverse assumption.

Many academics argue against VaR because it is not necessarily sub-additive[2]. This will contradict the principal of diversification and hence also the foundations of Modern Portfolio Theory. Without sub-additivity there is no incentive to hold portfolios and the metric should not be used for risk budgeting (Alexander 2008, p. 1). Instead of using VaR as risk measure in risk parity we use ETL which take into account the magnitude of losses when VaR is exceeded. ETL is however sub-additive.

The risk parity portfolio based on ETL (RPETL) is characterized by the same requirement of risk parity portfolio, meaning to assemble a portfolio composition in order to achieve equal risk contribution between assets, where the risk measure is ETL. In the historical approach, the ETL is computed by taking the average of all the losses in the tail below the VaR. The historical 5% ETL is given by:

$$ETL_{(5\%)} = E\left[r_i | r_i < -VaR_{5\%)}\right]$$

4. Data

In this study, we used monthly observations calculated as logarithmic returns of month-end prices from January 1995 to December 2014. The data was collected from Thomson Reuters Datastream, and the indices used were; MSCI All Country World Index, Barclays Global Aggregate Bonds Index, and

[2] A risk measure $\mathcal{R}$ is sub-additive if it satisfies $\mathcal{R}(X+Y) \leq \mathcal{R}(X) + \mathcal{R}(Y)$

five different commodity indices; GSCI Agricultures, GSCI Livestock, GSCI Precious Metals, GSCI Industrial Metals and GSCI Energy. All index values are total return, investable, and denominated in USD. The index decompositions are presented in Appendix 9.1, table 8.

For the stock market, we used MSCI World All Country indices. The index capture large and mid-cap representation across 23 developed markets and 23 emerging markets, which give an exposure to the global stock market. However, the US market represents a large share of this index. The index is constructed based on MSCI Global Investable Market Indexes Methodology, which aims to provide exhaustive coverage of the relevant investment opportunity set with strong emphasis on index liquidity, investability and replicability. The index is rebalanced semi-annually.

The Barclays Global Aggregate Index is a flagship measure of global investment grade debt from 24 different local currency markets. This multi-currency index includes fixed-rate treasury, government-related, corporate and securitized bonds from both developed and emerging markets issuers.

We chose five sector-based commodities to include most of the commodity market. The commodity indices are considered as the leading measures of movements in the commodity markets. These indices are sub-indices of the S&P GSCI and provide investors with a reliable and publicly available benchmark for investment performance in the different commodity groups. The indices are weighted based on world production, rebalanced annually and designed to be investable by including the most liquid commodity futures. They represent an unleveraged, long-only investment and the returns are calculated on a fully collateralized basis with full reinvestment. This provides investors with a representative and realistic picture of realizable returns attainable in the commodity markets when holding a long position. One of the benefits of using commodity indices instead of futures is that the indices deal with the problem of roll yield[3] that is present when using prolonged futures returns time series.

An issue when using commodity indices is that the constituents within that index can vary substantially over time. The outstanding value of long and short futures contracts is exactly offsetting and as a result, there is no market capitalization in commodity futures. Without a market capitalization based portfolio weighting scheme, one can think of commodity indices just as commodity portfolio strategies (Erb & Harvey 2006).

The motivation for the choice of commodity indices is derived from Bhardwaj and Dunsby (2012) who claimed that there are five commodity sectors – livestock, precious metals, industrial metals, energy and grains & oilseeds. The authors found that other soft commodities do not cohere to a common factor, but despite this, we treat all agricultural commodities as one group. Some papers use

[3] Roll yield has a demonstrated link to commodity markets in backwardation (see: Akey (2005)).

the S&P GSCI Commodity Index as a proxy for movements in the commodity markets, but the index is almost 70 percent capital weighted in the energy sector and is therefore not representative for the commodity markets.

5. Descriptive Statistics

In this chapter, we present the descriptive statistics for the indices for the full sample period, 1995-2014, and the correlations between the indices. In addition, we use several performance measures to evaluate the assets` standalone attractiveness. We use Sharpe ratio, which indicates the risk adjusted excess[4] return, non-parametric value-at-risk (VaR) and the expected tail-loss (ETL), which are commonly used measures for downside risk in financial analysis. Further, we divide the sample period into sub-periods to see if we get consistency in the descriptive statistics.

In figure 1, we present the price movement of the indices. It shows that Energy is very volatile compared with the other assets, while Agriculture and Livestock have had a flatter trend. Political interferences and the concern for long-term supply conditions have impact on commodity prices and volatility, and especially energy commodities are subject to these challenges. Before 2008, the commodity markets were booming, largely due to the increased demand from emerging countries such as China. While the prices fell sharply during the financial crisis, prices raised as demand recovered and because of low supply growth.

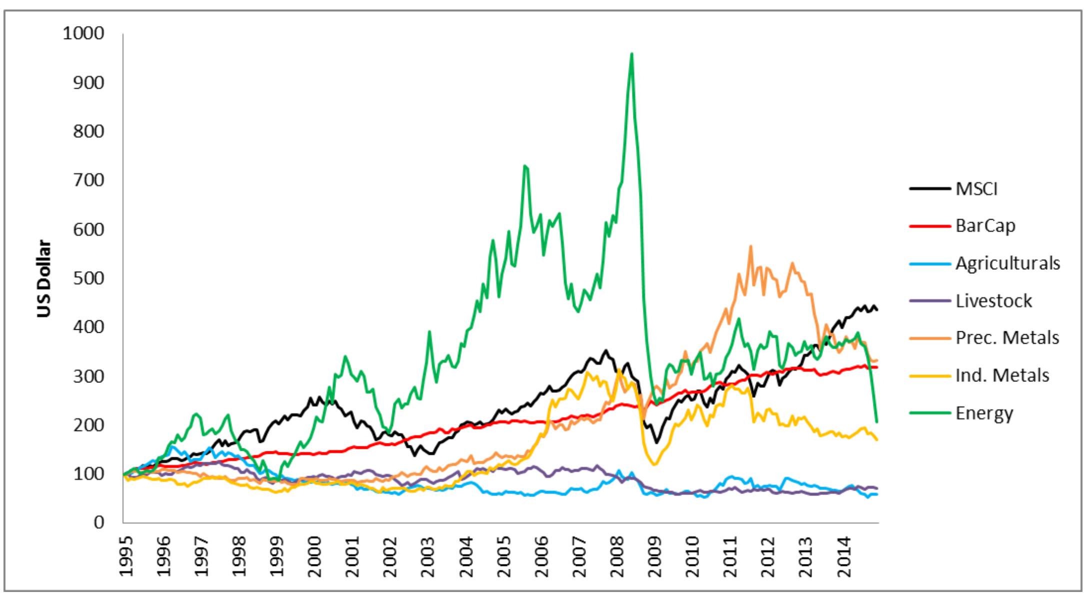

Figure 1: Cumulative wealth of $100 invested in each index in January 1995 to December 2014, (January 1995 =$100)

[4] As the risk free rate we use the interest rate on 3-month US Treasury bills.

5.1. Descriptive Statistics in the Period 1995 - 2014

In table 1 we present the descriptive statistics for the assets for the full sample, January 1995 to December 2014. In this period, MSCI had the highest return[5], while both Agriculture and Livestock had negative returns. The commodity returns vary from 5.90% for Precious Metals to -2.92% for Agriculture. Nevertheless, when considering the three risk measures volatility, measured as annualized standard deviation, non-parametric VaR and ETL, all commodities were considerably riskier than bonds. However, Energy was the most volatile asset, Livestock was less volatile than MSCI, and both Livestock and Precious Metals were less risky than MSCI when using VaR and ETL.

For this period, the average risk-free rate was 2.65% p.a., which explains the negative Sharpe ratio for both Agriculture and Livestock, which had a substantially lower return. Thus, it is hard to interpret negative Sharpe ratios. For a given level of negative excess return, higher volatility implies lower Sharpe, and for a given level of volatility, a more negative excess return implies a more negative Sharpe ratio. Then the Sharpe ratio does not provide useful information, because it is ambiguous. Therefore we do not compare the magnitude of negative Sharpe ratios, but emphasize that it is outperformed by the risk-free rate, and thereby not representing a good investment. The Sharpe ratio for all commodities were lower than for stocks and bonds, which implies that commodities as standalone investments were unattractive. BarCap has been superior over this period in terms of volatility, VaR, ETL and Sharpe ratio.

Table 1: Descriptive Statistics, Jan 1995 - Dec. 2014

1995-2014	MSCI	BarCap	Agriculturals	Livestock	Prec. Metals	Ind. Metals	Energy
Mean return	7,29% *	5,88% *	-2,92% *	-1,76% *	5,90% *	2,87% *	3,43 %
Stdev	15,48 %	4,16 %	20,55 %	13,93 %	17,34 %	20,77 %	31,28 %
Ex. Kurstosis	2,47	0,90	1,04	0,71	1,39	3,37	1,29
Skewness	-1,01	-0,02	-0,07	-0,58	-0,23	-0,61	-0,37
JB	97,53	7,31	10,16	17,63	19,91	122,07	20,74
Minimum	-20,99 %	-3,26 %	-21,03 %	-17,15 %	-20,60 %	-31,01 %	-37,39 %
Maximum	10,72 %	4,64 %	16,28 %	8,27 %	14,48 %	19,34 %	29,77 %
$VaR_{5\%}$	-8,79 %	-1,37 %	-9,74 %	-7,50 %	-6,91 %	-8,92 %	-15,55 %
$ETL_{5\%}$	-11,17 %	-2,13 %	-13,44 %	-9,40 %	-10,76 %	-13,87 %	-20,67 %
Sharpe ratio	0,30	0,79	-0,27	-0,31	0,19	0,01	0,03

Note: Mean return and standard deviation are annualized. VaR and ETL are 5% quantile of the empirical return distribution. * indicates significant at 5%. Results are based on 240 observations.

[5] When we use the term return, we refer to annualized geometric mean return.

The Jarque-Bera (JB) test rejects the hypothesis of normality in distribution of the returns for all assets, which emphasize the importance of using alternative risk and performance measures. All assets had negative skewness, which means that the left tail of the distribution was longer than the right. For investors this means that there was a greater likelihood of extremely negative outcomes and that the standard deviation underestimates the risk. In addition, the assets had positive excess kurtosis meaning that the return distributions were leptokurtic, which implied that they had heavier tails than the normal distribution. The returns did not cluster around the mean, but a higher fraction of the variance was from large but rare deviations compared with the normal distribution. When looking at the maximum and minimum returns, we see that Energy had both the highest maximum and the lowest minimum, while BarCap had the opposite.

The results so far indicated that commodities as standalone investments were not attractive. However, when considering the correlations between the commodities with stocks and bonds they might add value in a portfolio context. If the correlations between the commodities and stocks and bonds were low or negative, the commodities might be beneficial as a tool for diversification, and can hence improve the risk-adjusted return.

In table 2 we exhibit the correlation matrix for the period 1995-2014. The pair-wise correlation coefficients represent the linear statistical dependence between asset returns. The correlations between the assets were relatively low or negative; the only exception was between MSCI and Industrial Metals, which exhibited higher correlation. Livestock was the least correlated asset when comparing with the other assets, and these correlation coefficients were not statistically significant. Hence, Livestock should be best suited to complement a stock-bond portfolio. Additionally Industrial Metals and Energy were not significantly correlated with BarCap.

Table 2: Correlation Matrix, Jan. 1995- Dec. 2014

1995-2014	MSCI	BarCap	Agriculturals	Livestock	Prec. Metals	Ind. Metals
BarCap	0,11*					
Agriculturals	0,33*	0,18*				
Livestock	0,03	-0,08	0,01			
Prec. Metals	0,18*	0,33*	0,30*	-0,02		
Ind. Metals	0,53*	0,03	0,33*	0,08	0,36*	
Energy	0,28*	0,08	0,23*	0,10	0,25*	0,38*

Note: * indicates significant at 5% level.

Although most of the long-term correlations between the assets have been low, they are time-varying, which is exhibited in figure 2. The upper left graph exhibits the five-year rolling correlations between

MSCI and the other assets. MSCI was highest correlated with Industrial Metals over the full period. Between 2004 and 2008, MSCI was negatively correlated with BarCap, Livestock and Energy. As presented in the graph, we see a tendency towards increased correlation between MSCI and other assets after 2008.

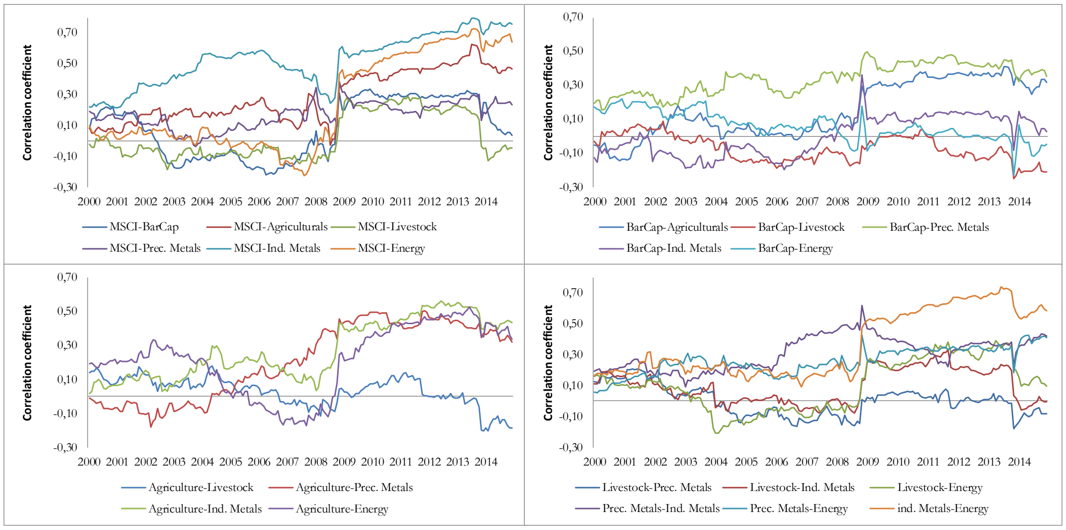

Figure 2: Rolling correlation between asset classes (Jan. 2000 – Dec. 2014). Results are based on five-year rolling window estimation (60 observations).

In the upper right graph, we exhibit the rolling correlation between BarCap and other assets. BarCap was highest correlated with Precious Metals over the full period. As illustrated in figure 2 the correlation between BarCap and other assets were more stable over time than MSCI. In the bottom graphs, we illustrate the rolling correlations between the commodities. The rolling correlations between Agriculture and Precious Metals have increased substantially over time. Also in this case we see that most correlations increased after 2008, but not between Agriculture and Livestock. The correlations between Livestock and Precious Metals, Industrial Metals and Energy have been low over the full sample-period, and the correlation between Industrial Metals and Energy increased after 2008. Although the correlations between most assets have been higher after 2008, many correlations seemed to revert in the end of 2013. When we reduce the estimation window to 24 monthly observations, the correlations decreased more in the more recent period (results reported in the appendix 9.1, figure 5) than what has been presented here.

5.2. Descriptive Statistics for Sub-Periods

In the appendix 9.1, table 8, we present descriptive statistics of four sub-periods: 1995-1999, 2000-2004, 2005-2009, and 2010-2014. When comparing different sub-periods it was evident that the statistics differ significantly over time. In differ from the full sample period the sub-samples gave other results. The first sub-period, 1995-1999, shows that while MSCI had the highest return, all commodities, except Energy had negative returns. However, Energy was the most volatile asset and had the highest downside risk, which was also the case for the subsequent sub-periods. In the period, 2000-2004, all commodity indices, except Agriculture, outperformed MSCI in terms of Sharpe ratio. The next period, 2005-2009, incorporated the period of global financial crisis but also the boom in these markets prior to the crisis. Here, Precious Metals had both the highest return and the highest Sharpe ratio, while the opposite holds for Livestock. In the last period, 2010-2014, MSCI and BarCap outperformed all commodities in terms of Sharpe ratio. Considering all sub-periods, we see that some return distributions were not rejected from normality.

Summarizing our results from this chapter, the performance of each individual asset under the full sample period implied that none of the commodities was attractive stand-alone investments compared to MSCI and BarCap. However, the correlations between the assets were low or negative, which means that they might add value in a portfolio context. Further, when studying the sub-periods we find that the statistics differed significantly over time.

6. Empirical Results

In this chapter, we present results and analyses for the different asset allocation strategies. We start by presenting the developments of weights under the different allocation methods. Further, we analyze the concentration of weights and risk in the portfolios. Finally, we present risk and return characteristics and performance evaluations for each strategy under two different investments universes; one consisting of stocks and bonds and commodities, and the other consisting of stocks and bonds.

6.1. Analysis of Portfolio Weights

In this section, we present the development of the weights for the different portfolios with commodities. The development for the portfolios in the investment universe excluding commodities can be found in the appendix 9.1, figure 6. The portfolio weights are re-estimated annually between 2000 and 2015.

On average the allocation to commodities vary greatly depending of which asset allocation strategy used. RPSTD had the highest average allocation to commodities, 47%, over the full sample period, while MinVar had the lowest average, 13%. All risk parity portfolios had a relative stable allocation to commodities with a rage from 37-53 percent when compared to the traditional portfolios. The BarCap was highly favored by all allocation strategies.

In figure 3, we present the weight decomposition of the asset allocation strategies for each year.

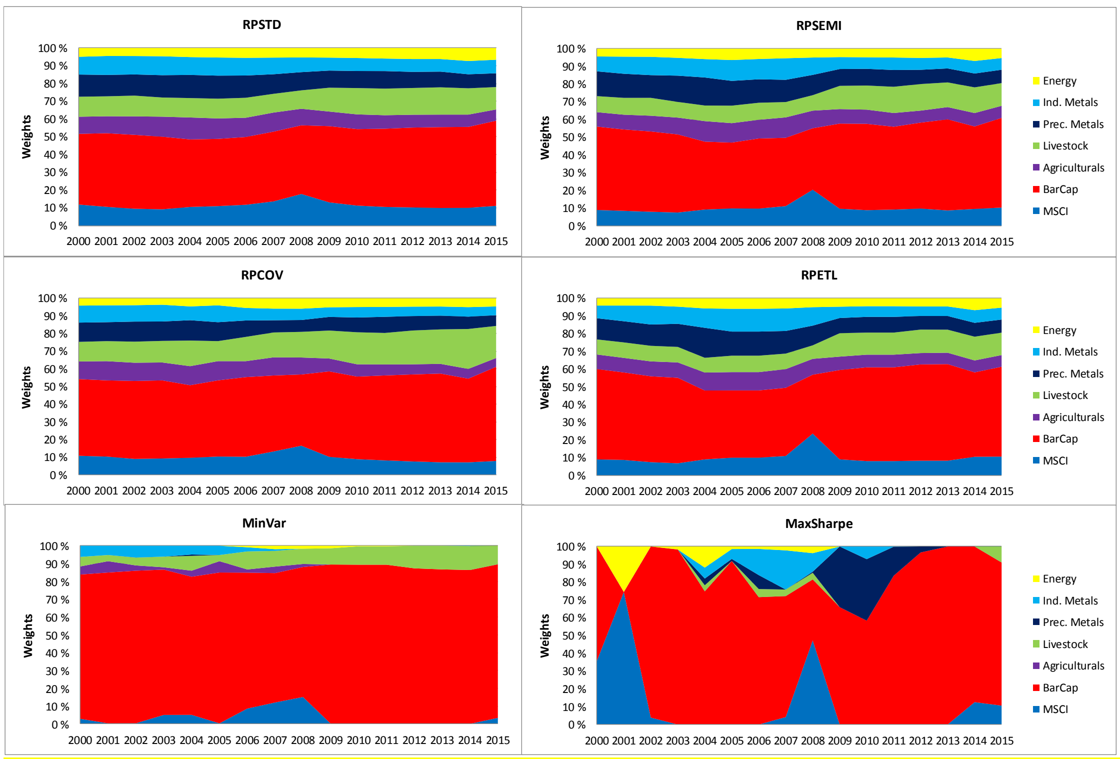

Figure 3: Annual portfolio decomposition for the different strategies, between 2000-2015.

For the risk parity portfolios the allocation of the different commodities was relatively smooth over time. Energy had the smallest share over the full period due to the high risk it entails. After 2008, Livestock was the most favored commodity for all risk parity portfolios as a result of relatively low risk. Livestock had an even greater share in RPCOV-portfolio, which also considers the low correlation it had with other asset classes. The low volatility and correlation with other assets for Livestock also made it preferable in the MinVar portfolio. However, the MinVar portfolio only allocates a small share of other commodities. The MaxSharpe portfolio was the portfolio with the greatest variations in weights. According to Michaund (1989) the large fluctuations are probably due to estimation errors, and estimation errors in returns often dominate estimation errors in the covariance matrix. These results highlight an important aspect of investing, namely the turnover-induced transaction costs[6] which will appear to be significantly higher for this allocation strategy. Since Livestock has offered a historically poor return, it was barely present under this strategy. However, the MaxSharpe portfolio included a relatively high share of Industrial Metals prior to the global financial crisis, but after 2008, Precious Metals replaced the allocation of Industrial Metals.

6.2. Analysis of Portfolio Weight and Risk Concentration

We present both Herfindahl-Hirschman Index (HHI) and the Diversification Ratio to illustrate that the risk parity portfolios are less concentrated and that they exhibit the principle of true diversification.

To get a better understanding of how diversified or, conversely, how concentrated each portfolio was in one or a few assets over time, we calculated the normalized HHI that ranges between 0 percent (for perfect equality) and 100 percent (for extreme inequality). The index value was calculated every year when the portfolio weights were re-estimated.

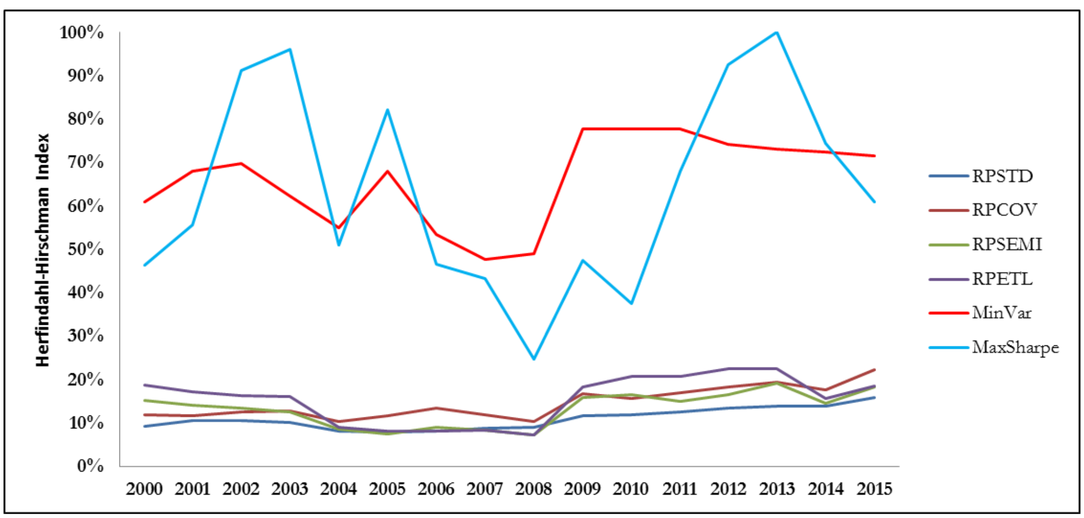

Figure 4: Development of the normalized HHI, period 2000-2015. Computational procedure is given in the appendix

As presented in figure 4, MinVar and MaxSharpe had a significantly higher concentration, while all risk parity portfolios had a low concentration, and seems to be relatively close to one another. However,

[6] Note that for a more comprehensive result, measuring transaction costs includes measuring the turnover for the portfolio (the amount of securities purchased or sold over net total value) i.e. also considering rebalancing.

the HHI measure had a great fluctuation for the MaxSharpe portfolio, from approximately 30 percent in 2008, to 100 percent in 2013 when the only asset in the MaxSharpe portfolio was the BarCap. We also notice a lower concentration for the risk parity portfolios between 2004 and 2008.

In table 3, we present the diversification ratio, which is defined in terms of the distance between portfolio volatility and the individual assets' volatility. It measures the risk concentration, where a high ratio implies a high degree of diversification[7] (Choueifaty & Coignard 2008). The ratio is calculated out-of-sample and in this analysis, we have also included the fixed-weighted portfolio.

Table 3: Diversification Ratio for the Three Sub-Periods

Period	RPSTD	RPCOV	RPSEMI	RPETL	MinVar	MaxSharpe	Fixed weight
2000-2004	2,62	2,66	2,58	2,56	1,62	1,29	1,93
2005-2009	1,76	1,83	1,72	1,70	1,62	1,36	1,64
2010-2014	1,68	1,76	1,71	1,71	1,53	1,28	1,45

Note: The computational procedure is provided in the appendix.

Further, the RPCOV was the most diversified among the portfolios in each sub-period, and the risk parity portfolios were more diversified than MinVar, MaxSharpe and the Fixed-Weight. MinVar and MaxSharpe were heavily concentrated in one asset, and were therefore highly exposed to one source of risk. The results also indicated a tendency towards an increased risk concentration for most strategies over time.

6.3. Out-of-Sample Benefits of Commodities

In this section, we evaluated the out-of-sample performance of adding commodities to the pre-specified allocation strategies consisting of stocks and bonds. Each table is divided into panel A and panel B, where panel A consists of stocks, bonds and commodities, and panel B consist of stocks and bonds. We present risk and return characteristics and use the Sharpe ratio, which is a common measure of financial efficiency, to evaluate the change in performance. In addition, we calculate the Sortino ratio, which is somewhat similar to the Sharpe ratio, but considers the excess return per unit of downside deviation. To determine whether commodities contribute to reduce tail-risk we use the non-parametric VaR and ETL for each strategy. The results are exhibited in table 4.

Over this period 2000-2014, commodities have led to a decrease in the average return in the different stock-bond portfolios. In addition, the standard deviations increased when commodities were included

[7] The minimum value for the Diversification ratio is one.

for all strategies, except MinVar that had a slightly lower volatility. The differences in volatility were only significant for the risk parity portfolios.

There was also evidence of increased excess kurtosis when we included commodities for all strategies. In panel B, we see that the skewness was negative for all portfolios. However, when commodities were included, the portfolio return distributions deviated more from the normal distribution than when commodities were excluded. This indicates that there was a greater likelihood of extremely negative outcomes compared to normal distributed returns. Additionally the range between the lowest and highest returns became greater when we included commodities in the portfolios, which can be related to the high volatility of some commodities.

Table 4: Risk and Return Characteristics of Portfolio When Adding Commodities (2000-2014)

2000-2014	RPSTD	RPCOV	RPSEMI	RPETL	MinVar	MaxSharpe	Fixed weight
			Panel A: Including commodities				
Mean return	3,56 %	3,26 %	3,63 %	3,64 %	4,26 %	1,65 %	3,68 %
Stdev	7,54 %	6,91 %	7,60 %	7,55 %	4,22 %	8,55 %	9,09 %
Ex. Kurstosis	10,13	11,28	13,12	15,37	6,41	11,94	7,38
Skewness	-1,81	-1,99	-2,14	-2,38	-1,45	-2,27	-1,52
JB	819,98	1013,72	1350,05	1835,18	350,12	1157,42	451,16
Minimum	-14,10 %	-13,24 %	-15,11 %	-15,58 %	-6,66 %	-15,84 %	-15,64 %
Maximum	5,34 %	4,65 %	4,92 %	4,99 %	3,31 %	7,08 %	6,78 %
Sharpe	0,23	0,21	0,24	0,24	0,58	-0,02	0,21
Sortino	0,20	0,18	0,20	0,20	0,49	-0,02	0,18
$VaR_{5\%}$	-3,11 %	-2,77 %	-3,01 %	-2,93 %	-1,71 %	-3,53 %	-4,04 %
$ETL_{5\%}$	-5,14 %	-4,74 %	-5,19 %	-5,19 %	-2,83 %	-7,23 %	-6,74 %
			Panel B: Excluding commodities				
Mean return	4,61 %	4,60 %	4,37 %	4,18 %	4,93 %	3,28 %	4,49 %
Stdev	5,35% *	5,35% *	5,29% *	5,39% *	4,37 %	7,75 %	8,57 %
Ex. Kurstosis	5,63	5,62	9,46	12,17	2,60	4,90	2,94
Skewness	-1,34	-1,33	-1,90	-2,26	-0,74	-0,95	-0,98
JB	274,80	274,11	736,42	1194,80	62,96	194,75	93,24
Minimum	-8,22 %	-8,22 %	-9,27 %	-10,08 %	-5,29 %	-9,21 %	-11,67 %
Maximum	4,19 %	4,19 %	4,11 %	4,04 %	4,41 %	7,15 %	5,72 %
Sharpe	0,52 *	0,52 *	0,49 *	0,44 *	0,71 *	0,19 *	0,31 *
Sortino	0,45	0,45	0,40	0,35	0,68	0,16	0,27
$VaR_{5\%}$	-2,03 %	-2,03 %	-1,92 %	-1,94 %	-1,76 %	-3,26 %	-4,26 %
$ETL_{5\%}$	-3,68 %	-3,68 %	-3,78 %	-3,94 %	-2,80 %	-6,33 %	-5,98 %

Note: Return and standard deviations are annualized. The fixed weighted portfolio is allocated 1/3rd to MSCI, 1/3rd to BarCap and 1/15th to each commodity index in panel A, and ½ to MSCI and ½ to BarCap in panel B. Sharpe ratio and Sortino ratio are based on annual data. VaR and ETL are at the lower 5% level. * indicates that the difference in mean, variance or Sharpe ratios are significant at the 5% level.

During this period, all portfolios had a higher Sharpe ratio when the investment universe only consisted of MSCI and BarCap. This means that investors would have gained a higher excess return per unit of volatility by excluding commodities from the portfolio, and these differences were significant. This result is also evident when considering the Sortino ratio. Further, RPCOV had the largest reduction in Sharpe and Sortino ratio, when comparing panel A and panel B, while the Fixed-

Weight portfolio had the lowest reduction in these measures. Additionally, commodities have contributed to increased downside risk, measured by VaR and ETL for all portfolios, with two exceptions; VaR for MinVar and the fixed-weighted portfolio.

In section 6.1, the allocations of the commodities differed substantially across the different strategies. From section 6.2, we saw that there was a higher concentration of weights and risk for the traditional strategies. Therefore, it was interesting to see whether the risk parity portfolios perform better than the traditional allocation strategies when commodities were included, meaning whether equal risk contribution across all assets have led to better performance than traditional optimizing and naïve weighting.

Among the portfolios presented in Panel A, table 4, MinVar had the highest return and lowest volatility, and the risk parity portfolios had higher average returns and lower volatilities than MaxSharpe. When evaluating the performance measures across the different strategies, the results showed that all risk parity portfolios and the Fixed-Weight portfolio had approximately the same Sharpe and Sortino ratio over the full-sample period. MinVar outperformed all other strategies in terms of these measures, and the Max Sharpe portfolio performed worst. In addition, MinVar had the lowest downside risk measured by VaR and ETL, followed by the risk parity portfolios. This means that the allocation strategies that include all commodity sectors had better performance and ETL than MaxSharpe, but not MinVar.

6.3.1. Contribution of Commodities in Different Sub-Periods

Based on our findings so far, we want to examine if the results are valid when dividing the full sample period into the same periods as studied in chapter 5; January 2000 - December 2004, January 2005 - December 2009 and January 2010 – December 2014, each of 60 monthly observations. The periods incorporate different market environments, but we do not distinguish between periods of recessionary and expansionary monetary policies.

Period 2000-2004

The average return for the portfolios in this period, presented in table 5, all portfolios except the Fixed-Weight had a lower average return when commodities were included. Commodities contributed to reduced standard deviation for the traditional strategies, but increased for the risk parity portfolios. However, these differences were not significant.

Table 5: Risk and Return Characteristics of Portfolio When Adding Commodities (2000-2004)

2000-2004	RPSTD	RPCOV	RPSEMI	RPETL	MinVar	MaxSharpe	Fixed weight
			Panel A: Including commodities				
Mean return	5,24 %	5,23 %	5,74 %	5,90 %	6,99 %	0,29 %	4,01 %
Stdev	5,31 %	5,07 %	5,22 %	5,12 %	4,04 %	9,37 %	6,92 %
Ex. Kurstosis	0,43	0,36	0,70	0,89	0,66	4,12	-0,27
Skewness	-0,47	-0,42	-0,51	-0,58	-0,45	-1,27	-0,30
JB	2,32	1,84	3,26	4,54	2,56	49,25	1,17
Minimum	-3,45 %	-3,27 %	-3,81 %	-3,98 %	-2,85 %	-10,60 %	-4,75 %
Maximum	4,22 %	4,05 %	4,26 %	4,11 %	3,31 %	7,08 %	4,74 %
Sharpe	0,50	0,52	0,60	0,64	1,09	-0,25	0,20
Sortino	0,47	0,50	0,57	0,61	1,08	-0,20	0,19
$VaR_{5\%}$	-2,92 %	-2,74 %	-2,93 %	-2,72 %	-1,66 %	-5,78 %	-2,84 %
$ETL_{5\%}$	-3,22 %	-3,01 %	-3,26 %	-3,21 %	-2,14 %	-7,84 %	-3,88 %
			Panel B: Excluding commodities				
Mean return	5,61 %	5,61 %	6,16 %	6,25 %	7,51 %	1,23 %	2,78 %
Stdev	4,75 %	4,75 %	4,48 %	4,45 %	4,40 %	9,73 %	8,09 %
Ex. Kurstosis	-0,22	-0,22	0,13	0,26	0,82	3,24	-0,50
Skewness	-0,27	-0,27	-0,33	-0,37	-0,51	-1,11	-0,28
JB	0,93	0,93	1,05	1,33	3,58	32,50	1,38
Minimum	-2,91 %	-2,91 %	-2,94 %	-2,95 %	-3,06 %	-9,21 %	-5,15 %
Maximum	3,11 %	3,11 %	3,22 %	3,29 %	3,43 %	7,15 %	4,80 %
Sharpe	0,63	0,63	0,79	0,82	1,12	-0,15	0,02 *
Sortino	0,67	0,67	0,85	0,88	1,09	-0,11	0,02
$VaR_{5\%}$	-2,15 %	-2,15 %	-1,83 %	-1,80 %	-1,54 %	-6,77 %	-4,15 %
$ETL_{5\%}$	-2,45 %	-2,45 %	-2,42 %	-2,45 %	-2,48 %	-8,27 %	-4,50 %

Note: Panel A: All assets are included. Panel B: Only MSCI and BarCap are included. Return and standard deviations are annualized. * indicates that the difference in mean, variance or Sharpe ratio are significant at the 5% level.

For most of the portfolio strategies, the excess kurtosis increased when adding commodities, which means that commodities contributed to heavier tails for these portfolios. Additionally the negative skews presented in panel B became more negatively skewed in panel A, indicating a distribution of returns with a more asymmetric tail extending toward more negative values. Commodities have contributed to lower minimum return for the risk parity portfolios over this period, but also a higher maximum return. For the MinVar portfolio, the opposite holds.

In this period, the Sharpe ratio was lower for all portfolios except the Fixed-Weight portfolio when commodities were included. The same results apply for the Sortino ratio. For the risk parity portfolios, commodities contributed to increased tail-risk measured by VaR and ETL. But for the other strategies, commodities reduced the tail-risk, except VaR for MinVar.

Among the allocation strategies in panel A, we see that all performance measures indicate that Minimum Variance was the best performing portfolio, followed by the risk parity portfolios. The Max Sharpe portfolio performed poorly, indicated by low average return and thus a negative Sharpe and Sortino ratio, as well as higher risk measured by both standard deviation, VaR and ETL. This means that the strategies that allocate to all commodity sectors, the risk parity- and Fixed-weight portfolios, outperformed MaxSharpe, but not MinVar. In addition, allocating by risk performed better than naïve allocation.

Period 2005-2009

In this period, table 6, commodities contributed to higher return for RPSTD, RPSEMI and RPETL, while the return for MinVar, MaxSharpe and Fixed-weighted decreased. Further, the standard deviation increased for all strategies when commodities were included due to the relatively high volatility of the commodities in this period. The differences in standard deviations were only significant for the risk parity portfolios. As for the previous period, the excess kurtosis increased when commodities were included and the returns became more negatively skewed.

When considering the minimum and maximum returns, the minimum returns were lower when we included commodities for each strategy, but the maximum returns were only higher for the risk parity portfolios and the Fixed-Weight portfolio.

Table 6: Risk and Return Characteristics of Portfolio When Adding Commodities (2005-2009)

2005-2009	RPSTD	RPCOV	RPSEMI	RPETL	MinVar	MaxSharpe	Fixed weight
Panel A: Including commodities							
Mean return	3,51 %	2,20 %	3,23 %	2,98 %	2,22 %	-0,73 %	3,39 %
Stdev	9,89 %	9,13 %	10,33 %	10,51 %	5,26 %	10,60 %	11,22 %
Ex. Kurstosis	10,30	10,94	11,37	12,01	6,89	12,01	9,43
Skewness	-2,31	-2,49	-2,48	-2,57	-1,94	-2,61	-2,23
JB	269,91	306,65	325,98	379,75	133,00	363,55	230,61
Minimum	-14,10 %	-13,24 %	-15,11 %	-15,58 %	-6,66 %	-15,84 %	-15,64 %
Maximum	5,34 %	4,65 %	4,92 %	4,99 %	3,31 %	5,59 %	6,78 %
Sharpe	0,08	-0,06	0,05	0,02	-0,10	-0,33	0,06
Sortino	0,07	-0,05	0,04	0,02	-0,08	-0,27	0,05
$VaR_{5\%}$	-3,37 %	-3,25 %	-3,33 %	-3,30 %	-2,14 %	-4,23 %	-4,96 %
$ETL_{5\%}$	-8,03 %	-7,73 %	-8,49 %	-8,72 %	-4,36 %	-9,63 %	-9,51 %
Panel B: Excluding commodities							
Mean return	3,39 %	3,35 %	2,33 %	1,78 %	3,63 %	4,92 %	3,72 %
Stdev	6,94% *	6,94% *	7,10% *	7,42% *	5,18 %	8,70 %	10,07 %
Ex. Kurstosis	5,40	5,39	7,78	9,02	3,24	2,59	4,72
Skewness	-1,68	-1,68	-2,15	-2,38	-0,97	-0,41	-1,56
JB	86,04	85,69	168,22	221,64	29,72	14,63	79,91
Minimum	-8,22 %	-8,22 %	-9,27 %	-10,08 %	-5,29 %	-7,15 %	-11,67 %
Maximum	4,19 %	4,19 %	4,11 %	4,04 %	4,41 %	6,86 %	5,72 %
Sharpe	0,10	0,09	-0,06	-0,13	0,17 *	0,25 *	0,10
Sortino	0,07	0,07	-0,04	-0,10	0,16	0,24	0,07
$VaR_{5\%}$	-3,62 %	-3,62 %	-3,74 %	-4,05 %	-2,26 %	-6,17 %	-6,09 %
$ETL_{5\%}$	-5,73 %	-5,72 %	-6,31 %	-6,84 %	-3,74 %	-6,57 %	-8,35 %

Note: Panel A: All assets are included. Panel B: Only MSCI and BarCap are included. Return and standard deviations are annualized. * indicates that the difference in mean, variance or Sharpe ratio are significant at the 5% level.

This sub-period differ from the first sub-period in terms of both the Sharpe ratio and Sortino ratio. Both RPSEMI and RPETL performed better when commodities were included. For these strategies, the Sharpe ratio was negative in panel B and turned positive when commodities were included, while it was the opposite for RPCOV, MinVar and MaxSharpe. Commodities contributed to a significantly reduced Sharpe in MinVar and MaxSharpe. Furthermore, when considering downside risk, VaR had a higher value for all portfolios when commodities were included, while the opposite holds for ETL. This reflects the extreme negative returns for commodities that occurred during the most recent financial crisis. In this period, MaxSharpe had the greatest change when comparing the strategies in the different investment universes in terms of performance measures, while RPSTD showed the least change.

Among the portfolios in Panel A, RPSTD had the highest return of all strategies and MinVar had the lowest risk measured by standard deviation, VaR and ETL. RPSTD was also the best performing in terms of Sharpe and Sortino when comparing the different portfolio strategies, while MaxSharpe had negative risk-adjusted return in this period. Moreover, the strategies that included all commodity indices outperformed MinVar and MaxSharpe, in terms of both Sharpe and Sortino.

Period 2010-2014

In the most recent sub-period, table 7, MaxSharpe was the only portfolio that had a higher return when commodities were included. As pictured in figure 1, Precious Metals and Industrial Metals had an increasing price in the beginning of this period, and accounted for approximately 41 percent of the portfolios share. Whereas, MinVar was the only strategy that decreased its volatility when commodities were included. This can be due to the relatively stable weighting of Livestock with a relatively low standard deviation and low correlation to BarCap, measured as a five-year average. Further, the risk parity portfolios and the Fixed-Weight portfolio had a lower return and increased volatility when commodities were included, and the only significant differences in standard deviation were for the risk parity portfolios. Contrary to the previous sub-periods, RPCOV had a reduced excess kurtosis when commodities were included. The skewness decreased for RPSTD and the Fixed-Weighted portfolio with commodities. Commodities also contributed to a higher maximum return for all strategies. And all portfolios, except MinVar, had a lower minimum return.

In this period, commodities contributed to a higher Sharpe ratio for MinVar and MaxSharpe, and also higher or equal Sortino ratio, indicating an improved risk-adjusted return. The differences in Sharpe are significant for all the risk parity portfolios and the Fixed-weighted portfolio. MaxSharpe and MinVar also had better VaR when commodities were included, but only MinVar improved the ETL.

Table 7: Risk and Return Characteristics of Portfolio When Adding Commodities (2010-2014)

2010-2014	RPSTD	RPCOV	RPSEMI	RPETL	MinVar	MaxSharpe	Fixed weight
			Panel A: Including commodities				
Mean return	1,92 %	2,35 %	1,93 %	2,05 %	3,57 %	5,40 %	3,65 %
Stdev	6,80 %	5,96 %	6,37 %	5,96 %	3,00 %	4,47 %	8,78 %
Ex. Kurstosis	0,10	-0,20	0,08	-0,02	0,27	0,11	0,95
Skewness	-0,23	-0,11	-0,25	-0,23	-0,34	0,18	-0,40
JB	0,52	0,32	0,58	0,54	1,18	0,31	3,03
Minimum	-5,28 %	-4,04 %	-5,05 %	-4,62 %	-2,07 %	-2,67 %	-7,24 %
Maximum	4,00 %	3,68 %	3,74 %	3,54 %	2,06 %	3,45 %	5,44 %
Sharpe	0,27	0,39	0,29	0,33	1,17	1,20	0,41
Sortino	0,27	0,40	0,29	0,32	1,15	1,59	0,40
$VaR_{5\%}$	-3,37 %	-2,56 %	-2,98 %	-2,62 %	-1,29 %	-1,79 %	-5,53 %
$ETL_{5\%}$	-4,11 %	-3,28 %	-3,72 %	-3,35 %	-1,69 %	-2,25 %	-6,14 %
			Panel B: Excluding commodities				
Mean return	4,83 %	4,83 %	4,63 %	4,53 %	3,64 %	3,68 %	6,95% *
Stdev	3,96% *	3,96% *	3,70% *	3,54% *	3,30 %	3,33% *	7,41 %
Ex. Kurstosis	0,00	0,00	-0,05	-0,02	0,08	0,06	0,35
Skewness	-0,20	-0,20	-0,29	-0,32	-0,42	-0,46	-0,29
JB	0,39	0,39	0,83	1,02	1,68	2,01	1,13
Minimum	-1,93 %	-1,93 %	-1,93 %	-1,97 %	-2,28 %	-2,28 %	-4,75 %
Maximum	3,14 %	3,14 %	2,83 %	2,69 %	1,97 %	1,97 %	5,17 %
Sharpe	1,21 *	1,21 *	1,24 *	1,27 *	1,09	1,09	0,94 *
Sortino	1,25	1,25	1,27	1,27	1,15	1,15	0,90
$VaR_{5\%}$	-1,85 %	-1,85 %	-1,84 %	-1,85 %	-1,66 %	-1,79 %	-4,26 %
$ETL_{5\%}$	-1,89 %	-1,89 %	-1,90 %	-1,91 %	-1,94 %	-1,98 %	-4,54 %

Note: Panel A: All assets are included. Panel B: Only MSCI and BarCap are included. Return and standard deviations are annualized. * indicates that the difference in mean, variance or Sharpe ratio are significant at the 5% level.

When comparing the portfolios in panel A, table 7, MaxSharpe had the highest return and MinVar was the least volatile portfolio. Contrary to the previous sub-period, MaxSharpe and MinVar outperformed the Fixed-Weight and the risk parity portfolios in terms of Sharpe and Sortino. In addition, MinVar and MaxSharpe had a lower downside risk than the risk parity- and the Fixed-Weight portfolios. Further, both MinVar and MaxSharpe outperformed the strategies including all commodity sectors in this period, in terms of Sharpe, Sortino and ETL.

Summarizing our results in this section, we cannot confirm that the portfolio standard deviation increased as with the exception of MinVar when commodities were included, found under the full sample period. We can neither confirm a reduction in portfolio returns for all strategies. Studying these periods, our results showed that there was always at least one strategy that benefitted of including commodities. However, there was no consistency in the results. Another finding was that when we compared the risk parity portfolios, RPCOV was always the least volatile and had the lowest ETL regardless of the period studied, but did not necessarily had the highest risk-adjusted performance.

6.4. Robustness and Drawbacks

As with any study of asset returns, the findings should be interpreted with caution. Unusual observations may occur and influence the results. It is often said that past performance is not a guarantee for future returns. This highlights that some portfolio strategies might work well in some periods and perform poorly in others. Risk parity portfolios has proven to perform well, and often outperformed other risk-based strategies over the last couple of decades. But the performance of this allocation model is highly dependent on the universe of investable assets (Chaves et al. 2011). The current economic condition with low yields is linked to a greater risk of rising yields in the future. Higher yields imply relatively higher volatility for fixed income assets, which again will lower the allocation to fixed income in risk-based portfolio strategies. Our study lacks robustness that could be enhanced by replacing the proxies for the stock and bond market with several different indices distinguishing between market sectors. Alternatively, one could reduce the estimation window to estimate the input parameters for the allocation models. Bessler et al. (2014) find that too long estimation windows of more than 48 months reduce the out-of-sample performance of many asset allocation models, as the models react too slowly to structural breaks. On the other hand, when analyzing monthly data, a short estimation window might have too few observations.

An important drawback is that we used Sharpe ratio to determine financial efficiency, but this measure is limited to investments with normally distributed returns. As we previously have shown, normal distribution of returns was often rejected. Finally, how much to allocate to commodities should be determined by individual investment goals, risk tolerance and time horizon.

7. Conclusion

The objective of this study was to analyze the out-of-sample diversification benefits of commodities to stock-bond portfolios, for different asset allocation strategies. We did this by employing the traditional allocation strategies such as Minimum Variance-, Maximum Sharpe- and 1/N portfolios. Additionally, we implemented four approaches to risk parity, each based on different risk measures; standard deviation, covariance, semi-deviation and expected tail-loss. Moreover, by analyzing the contribution of commodities for different asset allocation strategies, we aimed to determine whether the benefits of commodities depended on the asset allocation strategy. This study considered five sector-based commodity indices, in addition to one stock and one bond index under each strategy. We studied the period 2000-2014, and three sub-periods.

Our results confirmed the low correlations between the assets included in this paper. Further, we find that the correlations were time-varying, and increased after the 2008 global financial crisis.

Studying the period 2000 - 2014, the results showed that commodities contributed to a decreased mean return for all portfolios and increased portfolio volatility, with the exception of the Minimum Variance portfolio. Moreover, commodities contributed to a significant reduced risk-adjusted return and the expected tail-loss increased for all strategies.

The sub-period results were ambiguous and did not confirm our findings from the full sample period. In each sub-period, at least one strategy benefitted from commodities in terms of risk-adjusted performance measures. However, there was no single period in which all strategies benefited from commodities. Our results cannot confirm that commodities necessarily contribute to increased performance of a stock-bond portfolio in an out-of-sample analysis, as proposed by many in-sample studies. The diversification effect of including non-perfectly correlated assets does not necessarily imply diversification benefits.

When considering the performance among the allocation strategies with commodities, Minimum Variance outperformed the Risk Parity portfolios in most cases. In general, the risk parity portfolios seemed to provide lower standard deviation and expected-tail-loss than both the Fixed-Weight and Max Sharpe, but not lower than the Minimum Variance. Additionally, for the risk parity portfolios, the choice of risk measure does not seem to deviate considerably relative to one another when comparing the risk-adjusted return of these portfolios. However, the risk parity portfolio based on the covariances had both lower volatility and expected tail-loss than the other risk parity portfolios. The benefits of including commodities depended highly on the allocation strategy adopted and the time-period studied.

For further research, we suggest examining the isolated benefits of adding individual indices to a stock-bond portfolio. Additionally, one could examine commodities in other asset allocation strategies such as the Black-Litterman approach and Reward-To-Risk Timing. Furthermore, one could construct the allocation strategies with alternative risk measures than the commonly used standard deviation, which might be of great interest to the financial industry. Another aspect to consider is the sample-period. By dividing the sample period into different market environment such as expansionary and recessionary periods, one might find a greater change in the performance when using different risk measures. When evaluate out-of-sample results with long estimation windows, exponential weighting of risk and covariance estimates could also be of interest.

8. References

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9. Appendix

9.1. Statistical Appendix

Table 8: Index Decomposition

Commodities		**Bonds**		**Stocks**	
GSCI Industrial Metal	Aluminium	**BarCap**	Treasury (49,0%)	**MSCI ACWI**	Financials (21.49%)
	Copper		Securitized (21.4%)		Information Technology (13.96%)
	Zinc		Corporate (15.3%)		Consumer Discretionary (12.56%)
	Nickel		Governement-Related (14,3%)		Health Care (12.18%)
	Lead				Industrials (10.48%)
GSCI Precious Metal	Gold				Consumer Staple (9.67%)
	Silver				Energy (7.51%)
GSCI Energy	WTI light sweet crude oil				Materials (5.33%)
	Brent crude oil				Telecommunication Services (3.65%)
	Gas oil				Utilities (3.17%)
	Heating oil				
	RBOB gasoline				
	Natural gas				
GSCI Agriculture	Wheat				
	Kansas wheat				
	Corn				
	Sugar				
	Soybean				
	Coffee				
	Cocoa				
	Cotton				
GSCI Livestock	Live Cattle				
	Feeder Cattle				
	Lean Hogs				

Note: BarCap is Barclays Capital Global Agg. Bond Index (Data from 2008), MSCI ACWI is MSCI All Country World Index: (Data from 2015) and the GSCI commodity indices have data from 2015.

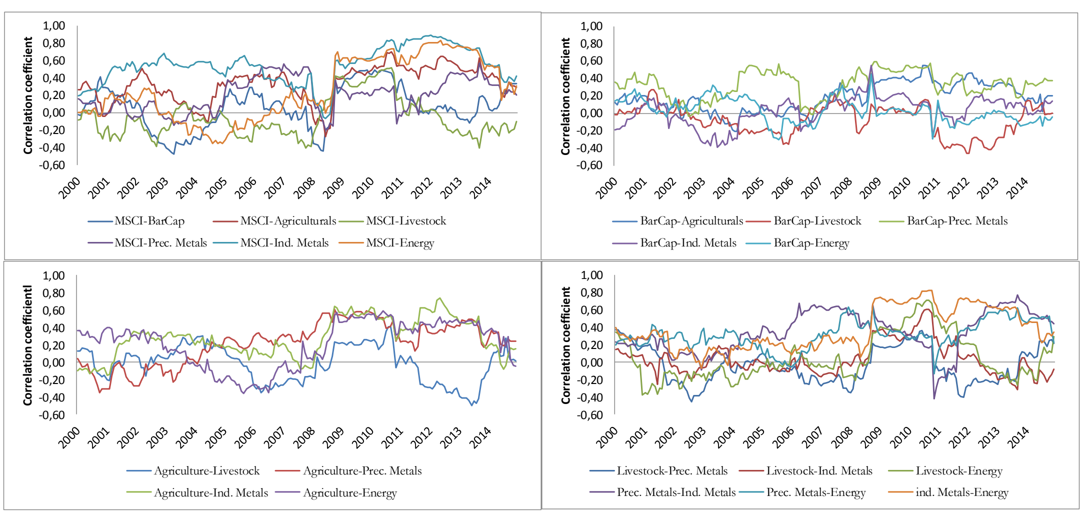

Figure 5: Rolling correlations between the assets in the period 2000-2014. Results are based on 24 months rolling window.

Table 9: Descriptive Statistics for Assets in Different Sub-Periods.

		MSCI	**BarCap**	**Agriculturals**	**Livestock**	**Prec. Metals**	**Ind. Metals**	**Energy**
1995-1999								
	Mean return	18,44% *	7,36 %	-4,48% *	-1,58% *	-2,84% *	-2,64% *	10,36 %
	Stdev	12,98 %	3,85% *	15,88 %	13,60 %	12,28 %	15,33 %	31,15 %
	Ex. Kurstosis	4,43	0,35	-0,41	0,20	4,43	1,33	1,57
	Skewness	-1,39	0,20	-0,16	-0,67	1,33	-0,29	0,04
	JB	57,74	0,50	0,79	4,29	56,22	3,96	4,55
	Minimum	-14,30 %	-2,24 %	-10,14 %	-11,00 %	-7,38 %	-13,80 %	-25,03 %
	Maximum	8,67 %	3,44 %	10,66 %	7,01 %	14,48 %	10,62 %	29,77 %
	$VaR_{5\%}$	-5,39 %	-1,21 %	-9,35 %	-8,47 %	-6,25 %	-9,72 %	-16,31 %
	$ETL_{5\%}$	-8,86 %	-1,56 %	-9,68 %	-9,36 %	-6,66 %	-11,16 %	-19,55 %
	Sharpe ratio	1,04	0,61	-0,60	-0,48	-0,64	-0,50	0,17
2000-2004								
	Mean return	-2,07 %	7,64% *	-6,54% *	3,62 %	8,71%*	7,01% *	19,44% *
	Stdev	15,75 %	4,56 %	14,96 %	15,51 %	13,29 %	16,91 %	32,31 %
	Ex. Kurstosis	-0,30	1,17	-0,51	2,51	0,63	-0,07	-0,46
	Skewness	-0,33	-0,58	-0,37	-1,18	-0,34	0,42	0,02
	JB	1,39	5,60	2,13	25,47	1,66	1,75	0,68
	Minimum	-11,6 %	-3,3 %	-11,2 %	-17,1 %	-11,7 %	-8,5 %	-20,6 %
	Maximum	8,6 %	3,7 %	6,7 %	7,2 %	8,6 %	12,6 %	21,4 %
	$VaR_{5\%}$	-8,8 %	-1,4 %	-8,3 %	-7,3 %	-4,9 %	-6,8 %	-14,6 %
	$ETL_{5\%}$	-9,9 %	-2,6 %	-9,5 %	-10,6 %	-7,2 %	-7,7 %	-16,8 %
	Sharpe ratio	-0,30	1,10	-0,61	0,06	0,46	0,26	0,52
2005-2009								
	Mean return	2,53 %	4,90% *	1,89 %	-11,96% *	17,22% *	13,09% *	-6,52 %
	Stdev	18,16 %	4,74 %	24,60 %	13,77 %	21,26 %	27,88 %	36,34 %
	Ex. Kurstosis	4,07	1,04	0,52	0,15	1,71	3,27	1,61
	Skewness	-1,46	0,39	-0,35	0,01	-0,66	-1,04	-0,79
	JB	53,46	3,30	1,54	0,00	9,59	31,30	10,65
	Minimum	-20,99 %	-2,40 %	-19,04 %	-9,98 %	-20,60 %	-31,01 %	-37,39 %
	Maximum	10,72 %	4,64 %	14,27 %	8,27 %	12,69 %	19,34 %	22,67 %
	$VaR_{5\%}$	-10,72 %	-1,98 %	-13,91 %	-8,75 %	-7,76 %	-13,71 %	-20,74 %
	$ETL_{5\%}$	-14,78 %	-2,24 %	-16,62 %	-9,53 %	-13,39 %	-19,75 %	-26,74 %
	Sharpe ratio	-0,01	0,46	-0,03	-1,07	0,68	0,37	-0,25
2010-2014								
	Mean return	10,26% *	3,63% *	-2,56 %	2,86 %	0,51 %	-5,99% *	-9,66% *
	Stdev	14,27 %	3,30 %	25,01 %	12,55 %	20,44 %	20,75 %	24,06 %
	Ex. Kurstosis	0,57	0,08	0,78	-0,30	-0,17	2,09	1,00
	Skewness	-0,44	-0,42	0,18	-0,25	-0,25	-0,56	-0,97
	JB	2,28	1,68	1,26	0,94	0,76	11,32	10,72
	Minimum	-9,96 %	-2,28 %	-21,03 %	-8,37 %	-15,16 %	-22,46 %	-21,94 %
	Maximum	9,87 %	1,97 %	16,28 %	7,31 %	11,32 %	11,72 %	11,47 %
	$VaR_{5\%}$	-8,92 %	-1,66 %	-11,45 %	-7,12 %	-11,74 %	-10,16 %	-17,12 %
	$ETL_{5\%}$	-9,29 %	-1,94 %	-14,95 %	-7,77 %	-13,31 %	-14,59 %	-18,80 %
	Sharpe ratio	0,71	1,08	-0,11	0,22	0,02	-0,29	-0,40

Note: * indicates significance at 5%.

Table 10: Risk Measures Used as Inputs in the Portfolio Allocation Decisions.

Estimation period	MSCI	BarCap	Agriculturals	Livestock	Prec. Metals	Ind. Metals	Energy	Year
Standard deviations								
1995-1999	3,71 %	1,10 %	4,55 %	3,89 %	3,52 %	4,39 %	8,92 %	2000
1996-2000	4,12 %	1,05 %	4,55 %	3,87 %	3,61 %	4,08 %	9,71 %	2001
1997-2001	4,71 %	1,07 %	4,27 %	3,82 %	3,73 %	4,30 %	9,89 %	2002
1998-2002	5,05 %	1,13 %	4,10 %	4,28 %	3,68 %	4,34 %	9,97 %	2003
1999-2003	4,69 %	1,29 %	3,97 %	4,49 %	3,76 %	4,89 %	9,62 %	2004
2000-2004	4,51 %	1,30 %	4,28 %	4,44 %	3,81 %	4,84 %	9,25 %	2005
2001-2005	4,24 %	1,30 %	4,57 %	4,41 %	3,95 %	4,99 %	8,98 %	2006
2002-2006	3,60 %	1,25 %	4,56 %	4,64 %	4,45 %	5,24 %	9,10 %	2007
2003-2007	2,59 %	1,19 %	4,95 %	4,45 %	4,50 %	5,57 %	8,72 %	2008
2004-2008	4,35 %	1,32 %	6,90 %	4,23 %	5,89 %	7,79 %	10,51 %	2009
2005-2009	5,20 %	1,36 %	7,04 %	3,94 %	6,08 %	7,98 %	10,40 %	2010
2006-2010	5,69 %	1,36 %	7,85 %	3,98 %	6,05 %	8,46 %	10,07 %	2011
2007-2011	5,99 %	1,36 %	8,44 %	4,06 %	6,73 %	8,45 %	9,86 %	2012
2008-2012	6,11 %	1,33 %	8,59 %	3,93 %	6,85 %	8,53 %	9,77 %	2013
2009-2013	4,92 %	1,08 %	7,09 %	3,32 %	6,23 %	6,57 %	6,74 %	2014
2010-2014	4,08 %	0,95 %	7,16 %	3,59 %	5,85 %	5,94 %	6,89 %	2015
Semi-deviation	**MSCI**	**BarCap**	**Agriculturals**	**Livestock**	**Prec. Metals**	**Ind. Metals**	**Energy**	
1995-1999	4,43 %	0,85 %	4,91 %	4,42 %	2,86 %	4,77 %	9,20 %	2000
1996-2000	4,81 %	0,89 %	4,92 %	4,32 %	3,02 %	4,26 %	9,20 %	2001
1997-2001	5,27 %	0,93 %	4,74 %	4,18 %	3,29 %	4,08 %	9,26 %	2002
1998-2002	5,60 %	0,97 %	4,49 %	4,81 %	2,87 %	4,24 %	8,32 %	2003
1999-2003	4,97 %	1,20 %	3,97 %	5,23 %	2,91 %	4,44 %	7,88 %	2004
2000-2004	5,13 %	1,37 %	4,64 %	5,15 %	3,64 %	4,32 %	8,00 %	2005
2001-2005	4,96 %	1,23 %	4,60 %	5,07 %	3,67 %	4,25 %	8,42 %	2006
2002-2006	4,33 %	1,26 %	4,21 %	5,60 %	3,87 %	3,99 %	9,07 %	2007
2003-2007	2,14 %	1,27 %	4,41 %	5,03 %	3,83 %	4,48 %	8,87 %	2008
2004-2008	5,97 %	1,21 %	7,08 %	4,43 %	6,12 %	8,62 %	12,33 %	2009
2005-2009	6,45 %	1,18 %	7,18 %	4,23 %	6,11 %	8,82 %	11,98 %	2010
2006-2010	6,70 %	1,33 %	8,00 %	4,19 %	6,54 %	8,81 %	12,55 %	2011
2007-2011	6,39 %	1,29 %	9,25 %	4,19 %	7,80 %	9,37 %	12,13 %	2012
2008-2012	6,86 %	1,17 %	8,70 %	4,32 %	7,64 %	9,71 %	12,27 %	2013
2009-2013	5,18 %	1,07 %	6,59 %	3,44 %	6,37 %	7,05 %	7,24 %	2014
2010-2014	4,35 %	0,89 %	6,60 %	3,52 %	5,98 %	6,82 %	8,62 %	2015
ETL	**MSCI**	**BarCap**	**Agriculturals**	**Livestock**	**Prec. Metals**	**Ind. Metals**	**Energy**	
1995-1999	-8,86 %	-1,56 %	-9,68 %	-9,36 %	-6,66 %	-11,16 %	-19,55 %	2000
1996-2000	-9,15 %	-1,61 %	-9,68 %	-9,12 %	-6,66 %	-9,00 %	-19,55 %	2001
1997-2001	-10,77 %	-1,65 %	-9,54 %	-9,12 %	-6,66 %	-7,53 %	-19,55 %	2002
1998-2002	-11,71 %	-1,65 %	-9,20 %	-9,12 %	-6,12 %	-8,25 %	-17,01 %	2003
1999-2003	-9,88 %	-2,29 %	-8,77 %	-10,70 %	-5,26 %	-8,14 %	-15,66 %	2004
2000-2004	-9,88 %	-2,60 %	-9,54 %	-10,64 %	-7,25 %	-7,66 %	-16,76 %	2005
2001-2005	-9,88 %	-2,60 %	-9,54 %	-10,64 %	-7,25 %	-7,66 %	-16,76 %	2006
2002-2006	-8,89 %	-2,53 %	-9,23 %	-11,11 %	-7,62 %	-7,66 %	-17,06 %	2007
2003-2007	-3,60 %	-2,55 %	-9,48 %	-11,13 %	-7,62 %	-8,15 %	-17,06 %	2008
2004-2008	-13,96 %	-2,50 %	-16,62 %	-9,53 %	-14,70 %	-19,75 %	-26,74 %	2009
2005-2009	-14,78 %	-2,24 %	-16,62 %	-9,53 %	-13,39 %	-19,75 %	-26,74 %	2010
2006-2010	-14,78 %	-2,24 %	-16,62 %	-9,53 %	-13,39 %	-19,75 %	-26,74 %	2011
2007-2011	-14,78 %	-2,24 %	-19,00 %	-9,21 %	-15,85 %	-22,67 %	-26,74 %	2012
2008-2012	-14,78 %	-2,24 %	-19,00 %	-9,40 %	-15,85 %	-22,67 %	-26,74 %	2013
2009-2013	-9,94 %	-2,19 %	-15,44 %	-7,77 %	-13,31 %	-14,59 %	-15,59 %	2014
2010-2014	-9,29 %	-1,94 %	-14,95 %	-7,77 %	-13,31 %	-14,59 %	-18,80 %	2015

Note: Different risk measures used as inputs for portfolio calculation each year (right column) are based on previous five-year estimation period (left column).

Table 11: Portfolio Weights of Stocks, Bonds and Commodities for the Different Portfolio Strategies

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
RPSTD															
MSCI	11,8 %	10,5 %	9,5 %	9,2 %	10,5 %	11,0 %	11,7 %	13,6 %	17,8 %	13,1 %	11,2 %	10,5 %	10,2 %	9,9 %	10,0 %
BarCap	39,8 %	41,5 %	41,6 %	40,9 %	38,1 %	37,9 %	38,2 %	39,3 %	38,7 %	42,9 %	43,1 %	44,1 %	45,0 %	45,6 %	45,6 %
Agriculturals	9,7 %	9,6 %	10,4 %	11,3 %	12,4 %	11,5 %	10,8 %	10,8 %	9,3 %	8,2 %	8,3 %	7,6 %	7,2 %	7,0 %	6,9 %
Livestock	11,3 %	11,2 %	11,7 %	10,8 %	10,9 %	11,1 %	11,3 %	10,6 %	10,4 %	13,4 %	14,8 %	15,0 %	15,1 %	15,4 %	14,8 %
Prec. Metals	12,5 %	12,1 %	11,9 %	12,6 %	13,0 %	13,0 %	12,5 %	11,0 %	10,3 %	9,6 %	9,6 %	9,9 %	9,1 %	8,8 %	7,9 %
Ind. Metals	10,0 %	10,7 %	10,4 %	10,7 %	10,0 %	10,2 %	9,9 %	9,4 %	8,3 %	7,3 %	7,3 %	7,1 %	7,2 %	7,1 %	7,5 %
Energy	4,9 %	4,5 %	4,5 %	4,6 %	5,1 %	5,3 %	5,5 %	5,4 %	5,3 %	5,4 %	5,6 %	5,9 %	6,2 %	6,2 %	7,3 %
RPCOV															
MSCI	10,8 %	10,4 %	9,0 %	9,3 %	9,7 %	10,4 %	10,3 %	13,2 %	16,5 %	10,2 %	8,9 %	8,2 %	7,6 %	7,1 %	7,1 %
BarCap	43,5 %	43,2 %	44,2 %	44,2 %	41,1 %	43,2 %	45,1 %	43,1 %	40,4 %	48,4 %	46,8 %	48,1 %	49,3 %	50,3 %	47,4 %
Agriculturals	10,0 %	10,9 %	10,3 %	10,2 %	10,8 %	10,9 %	9,0 %	10,3 %	9,6 %	7,3 %	7,0 %	6,3 %	5,7 %	5,5 %	5,5 %
Livestock	11,1 %	11,4 %	11,9 %	12,2 %	14,5 %	11,4 %	13,8 %	14,1 %	14,5 %	15,8 %	18,1 %	17,8 %	19,1 %	19,5 %	22,6 %
Prec. Metals	10,9 %	10,7 %	11,4 %	11,0 %	11,5 %	10,7 %	9,3 %	6,9 %	6,7 %	7,7 %	8,4 %	9,2 %	8,2 %	7,8 %	7,0 %
Ind. Metals	9,6 %	9,5 %	9,3 %	9,5 %	7,8 %	9,5 %	7,1 %	6,6 %	6,4 %	5,4 %	5,9 %	5,6 %	5,3 %	5,2 %	5,4 %
Energy	4,1 %	4,0 %	3,9 %	3,6 %	4,6 %	4,0 %	5,4 %	5,8 %	5,9 %	5,1 %	4,9 %	4,9 %	4,7 %	4,6 %	5,0 %
RPSEMI															
MSCI	9,0 %	8,5 %	8,0 %	7,6 %	9,3 %	9,9 %	9,8 %	11,2 %	20,5 %	9,7 %	8,9 %	9,2 %	9,8 %	8,8 %	9,6 %
BarCap	47,0 %	45,9 %	45,3 %	44,1 %	38,3 %	37,2 %	39,6 %	38,5 %	34,7 %	48,0 %	48,7 %	46,8 %	48,5 %	51,4 %	46,5 %
Agriculturals	8,1 %	8,3 %	8,9 %	9,5 %	11,6 %	11,0 %	10,6 %	11,5 %	9,9 %	8,2 %	8,0 %	7,7 %	6,8 %	6,9 %	7,6 %
Livestock	9,1 %	9,5 %	10,1 %	8,9 %	8,8 %	9,9 %	9,6 %	8,7 %	8,7 %	13,1 %	13,6 %	14,8 %	15,0 %	13,9 %	14,5 %
Prec. Metals	14,0 %	13,6 %	12,8 %	14,8 %	15,8 %	14,0 %	13,2 %	12,5 %	11,5 %	9,5 %	9,4 %	9,5 %	8,0 %	7,9 %	7,8 %
Ind. Metals	8,4 %	9,6 %	10,3 %	10,1 %	10,4 %	11,8 %	11,4 %	12,2 %	9,8 %	6,7 %	6,5 %	7,0 %	6,7 %	6,2 %	7,1 %
Energy	4,4 %	4,5 %	4,6 %	5,1 %	5,8 %	6,4 %	5,8 %	5,3 %	4,9 %	4,7 %	4,8 %	4,9 %	5,2 %	4,9 %	6,9 %
RPETL															
MSCI	9,0 %	8,7 %	7,5 %	6,8 %	9,0 %	10,0 %	10,0 %	11,0 %	23,6 %	9,0 %	8,0 %	8,0 %	8,3 %	8,3 %	10,5 %
BarCap	51,0 %	49,4 %	48,5 %	48,3 %	38,9 %	38,0 %	38,0 %	38,5 %	33,3 %	50,5 %	53,0 %	53,0 %	54,4 %	54,6 %	47,7 %
Agriculturals	8,2 %	8,2 %	8,4 %	8,7 %	10,2 %	10,3 %	10,3 %	10,6 %	9,0 %	7,6 %	7,2 %	7,2 %	6,4 %	6,4 %	6,8 %
Livestock	8,5 %	8,7 %	8,8 %	8,8 %	8,3 %	9,3 %	9,3 %	8,8 %	7,6 %	13,2 %	12,5 %	12,5 %	13,3 %	13,0 %	13,4 %
Prec. Metals	12,0 %	12,0 %	12,0 %	13,1 %	16,9 %	13,6 %	13,6 %	12,8 %	11,2 %	8,6 %	8,9 %	8,9 %	7,7 %	7,7 %	7,8 %
Ind. Metals	7,1 %	8,9 %	10,7 %	9,7 %	10,9 %	12,9 %	12,9 %	12,7 %	10,4 %	6,4 %	6,0 %	6,0 %	5,4 %	5,4 %	7,1 %
Energy	4,1 %	4,1 %	4,1 %	4,7 %	5,7 %	5,9 %	5,9 %	5,7 %	5,0 %	4,7 %	4,4 %	4,4 %	4,6 %	4,6 %	6,7 %
MinVar															
MSCI	2,8 %	0,3 %	0,3 %	5,0 %	5,0 %	0,3 %	8,6 %	12,0 %	15,1 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
BarCap	81,0 %	84,7 %	85,7 %	81,6 %	77,6 %	84,7 %	76,3 %	72,7 %	73,0 %	89,5 %	89,4 %	89,4 %	87,3 %	86,7 %	86,4 %
Agriculturals	4,6 %	6,4 %	3,0 %	1,3 %	3,4 %	6,4 %	1,8 %	3,7 %	1,7 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
Livestock	5,3 %	3,5 %	4,4 %	6,0 %	8,3 %	3,5 %	10,1 %	8,8 %	8,6 %	9,1 %	10,3 %	10,3 %	12,7 %	13,3 %	13,5 %
Prec. Metals	0,0 %	0,0 %	0,1 %	0,0 %	0,8 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
Ind. Metals	6,3 %	5,2 %	6,6 %	6,1 %	4,9 %	5,2 %	2,3 %	1,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,2 %
Energy	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,8 %	1,9 %	1,6 %	1,4 %	0,3 %	0,3 %	0,0 %	0,0 %	0,0 %
MaxSharpe															
MSCI	35,7 %	74,4 %	3,9 %	0,0 %	0,0 %	0,0 %	0,0 %	4,2 %	47,2 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	12,6 %
BarCap	64,3 %	0,0 %	96,1 %	98,3 %	74,8 %	91,8 %	71,6 %	68,0 %	34,3 %	65,9 %	58,4 %	83,6 %	96,7 %	100,0 %	87,4 %
Agriculturals	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
Livestock	0,0 %	0,0 %	0,0 %	0,0 %	3,4 %	0,0 %	4,6 %	3,7 %	3,7 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
Prec. Metals	0,0 %	0,0 %	0,0 %	0,0 %	3,8 %	1,2 %	7,8 %	0,0 %	0,6 %	34,1 %	34,5 %	16,3 %	3,3 %	0,0 %	0,0 %
Ind. Metals	0,0 %	0,0 %	0,0 %	0,0 %	6,1 %	5,6 %	14,7 %	22,0 %	10,4 %	0,0 %	7,1 %	0,1 %	0,0 %	0,0 %	0,0 %
Energy	0,0 %	25,6 %	0,0 %	1,7 %	11,8 %	1,4 %	1,4 %	2,2 %	3,7 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %	0,0 %
Fixed-weight															
MSCI	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %
BarCap	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %	33,3 %
Agriculturals	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %
Livestock	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %
Prec. Metals	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %
Ind. Metals	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %
Energy	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %	6,7 %

Table 12: Portfolio Weights of Stocks and Bonds for the Different Portfolio Strategies

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
RPSTD															
MSCI	22,89 %	20,27 %	18,53 %	18,29 %	21,58 %	22,45 %	23,46 %	25,74 %	31,49 %	23,33 %	20,70 %	19,23 %	18,49 %	17,86 %	17,98 %
BarCap	77,11 %	79,73 %	81,47 %	81,71 %	78,42 %	77,55 %	76,54 %	74,26 %	68,51 %	76,67 %	79,30 %	80,77 %	81,51 %	82,14 %	82,02 %
RPCOV															
MSCI	22,89 %	20,27 %	18,53 %	18,29 %	21,58 %	20,27 %	23,46 %	25,73 %	31,48 %	23,33 %	20,70 %	19,23 %	18,49 %	17,86 %	17,98 %
BarCap	77,11 %	79,73 %	81,47 %	81,71 %	78,42 %	79,73 %	76,54 %	74,27 %	68,52 %	76,67 %	79,30 %	80,77 %	81,51 %	82,14 %	82,02 %
RPSEMI															
MSCI	16,11 %	15,66 %	15,01 %	14,71 %	19,45 %	21,06 %	19,86 %	22,54 %	37,11 %	16,85 %	15,48 %	16,51 %	16,84 %	14,59 %	17,13 %
BarCap	83,89 %	84,34 %	84,99 %	85,29 %	80,55 %	78,94 %	80,14 %	77,46 %	62,89 %	83,15 %	84,52 %	83,49 %	83,16 %	85,41 %	82,87 %
RPETL															
MSCI	14,99 %	14,98 %	13,31 %	12,38 %	18,81 %	20,81 %	20,81 %	22,17 %	41,48 %	15,18 %	13,19 %	13,19 %	13,19 %	13,19 %	18,02 %
BarCap	85,01 %	85,02 %	86,69 %	87,62 %	81,19 %	79,19 %	79,19 %	77,83 %	58,52 %	84,82 %	86,81 %	86,81 %	86,81 %	86,81 %	81,98 %
MaxSharpe															
MSCI	35,70 %	100,00 %	3,90 %	0,00 %	0,00 %	0,00 %	0,00 %	5,99 %	21,64 %	61,40 %	0,00 %	0,00 %	0,00 %	0,00 %	12,57 %
BarCap	64,30 %	0,00 %	96,10 %	100,00 %	100,00 %	100,00 %	100,00 %	94,01 %	78,36 %	38,60 %	100,00 %	100,00 %	100,00 %	100,00 %	87,43 %
MinVar															
MSCI	6,14 %	2,28 %	3,69 %	7,84 %	9,17 %	2,28 %	12,20 %	14,02 %	15,75 %	3,21 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %
BarCap	93,86 %	97,72 %	96,31 %	92,16 %	90,83 %	97,72 %	87,80 %	85,98 %	84,25 %	96,79 %	100,00 %	100,00 %	100,00 %	100,00 %	100,00 %
Fixed-weighted															
MSCI	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %
BarCap	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %	50,00 %

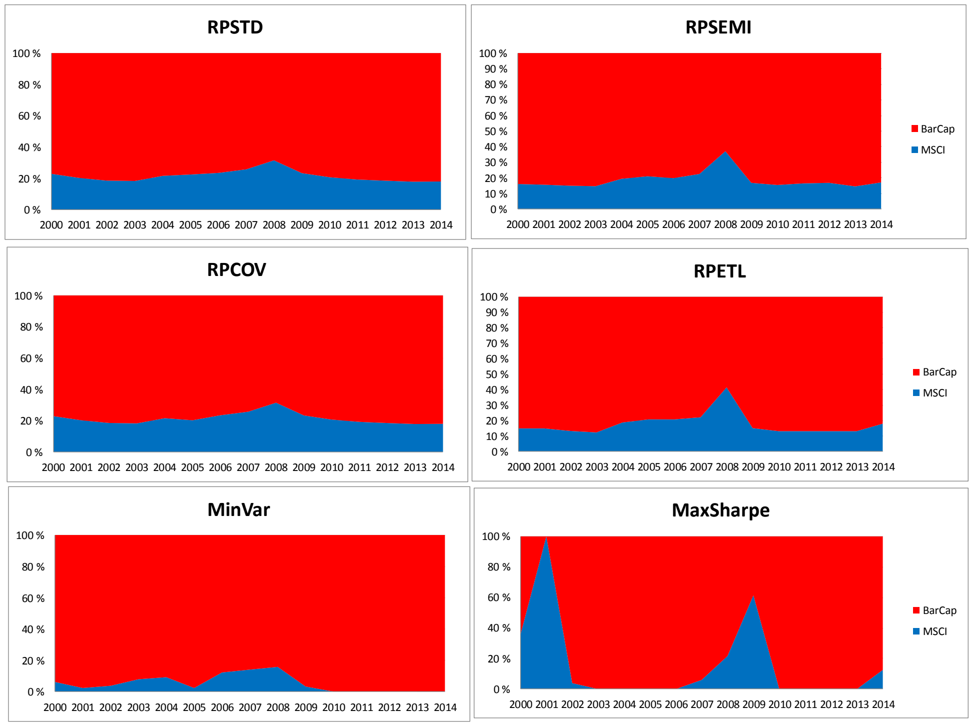

Figure 6: Portfolio weights of stocks and bonds over time.

9.2. Technical Appendix

9.2.1. Volatility and Risk Measures

Portfolio Return

The portfolio return is the weighted average of the expected returns of the individual assets of which the portfolio consists.

$$r_p = \sum_{i=1}^{n} w_i(r_i)$$

Where w_i is the weight of asset i in the portfolio.

Standard Deviation

The standard deviation is commonly used as a measure of investment risk. Standard deviation describes the variability around the mean of an investment`s returns. High values indicate an investment whose return have large spread and hence a greater risk.

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - \bar{r})^2}$$

Where r_i is the return of asset i.

Semi-Deviation

Most investors are more concerned about the downside risk rather than absolute risk. Semi standard deviation, also called downside deviation, focus on variability of returns that falls below minimum threshold or a minimum acceptable return (MAR).

$$\sigma_{semi} = \sqrt{T^{-1} \times \sum_{t=1}^{T} \mathrm{Min}(r_t - MAR, 0)^2}$$

Where r_t is the period t observed return from a sample $\{r_1 \dots, r_T\}$ of T returns that is below MAR

Value at Risk

Value at Risk (VaR) is another downside risk measure, and it is simply the quantile of the distribution. VaR is a loss that we are fairly sure will not be exceeded if the current portfolio is held over some period of time.

It then assumes that history will repeat itself, form a risk perspective. The advantage of using VaR as a risk measure is that one can incorporate skewness and kurtosis in the measure of total risk. When using historical VaR we compute the lower 5% quantile of the empirical return distribution:

$$VaR_{5\%} = \hat{F}^{-1}(5\%)$$

Where $\hat{F}^{-1}$ is the inverse of the return distribution.

.

Expected Tail-Loss

The expected tail-loss (ETL) is the expected return or loss for the lowest α percent quantile of the return distribution.

$$ETL_{(r_i)} = E\left[r_i | r_i < -VaR_{(r_i)}\right]$$

9.2.2. Derivation of Risk Parity

When standard deviations and correlations of n risky assets are assumed known, portfolio volatility σ_p is given by:

$$\sigma_p = \sqrt{\omega'\Omega\omega} = \sqrt{\sum_{i}^{n} w_i^2\sigma_i^2 + \sum_{i}^{n}\sum_{j\neq i}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j}$$

Where w_i is the portfolio weight of asset i. The n-dimensional column vector $\omega = \{w_1, \dots, w_n\}$ contains the portfolio weights of all assets, and Ω represents the covariance matrix of asset returns. The diagonal of Ω contains the variances, σ_i^2, and the off-diagonal elements correspond to the covariances between asset i and j given by $Cov(r_i, r_j) = \rho_{ij}\sigma_i\sigma_j$. All assets have a standard deviation σ_i and the correlation between asset i and j is defined as $\rho_{ij \in [-1,1]}$.

The change in standard deviation by a marginal change in portfolio weights is then given by the derivative:

$$\frac{\partial\sigma}{\partial\omega} = \frac{\Omega\omega}{\sigma}$$

The elements of this vector are the marginal risk contributions (MRC) of each asset i which can be written as:

$$MRC_i = \frac{\partial\sigma}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum_{j\neq i}^{n} w_j\rho_{ij}\sigma_i\sigma_j}{\sigma}$$

This marginal risk contribution tells us the variation caused in the portfolio standard deviation by an infinitesimal change in the weight of asset i.

The total risk contribution (TRC) or the component risk of asset i is the load on total risk contributed by the position w_i and simply the product of the marginal risk contribution and its weight:

$$TRC_i = w_i \frac{\partial \sigma}{\partial w_i} = \frac{w_i^2 \sigma_i^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j}{\sigma}$$

Since the portfolio standard deviation is a homogeneous function of degree 1, the Euler conditions[8] are satisfied and the portfolio risk is then the sum of each asset's TRC:

$$\sum_{i=1}^{n} TRC_i = \omega' \frac{\partial \sigma}{\partial \omega} = \sigma_p$$

The idea of risk parity or equal risk contribution is that the risk contribution of each portfolio component is made equal, mathematically defined as:

$$w_i \frac{\partial \sigma_p}{\sigma w_i} = w_j \frac{\partial \sigma_p}{\sigma w_j} \quad \forall\, i, j \; and \; i \neq j$$

9.2.3. Normality

When using standard deviation one ignores skewness and kurtosis and thereby standard deviation is a limited measure of risk. Financial asset returns often have heavier tails and higher peaks than the normal distribution meaning that there is a greater likelihood of extreme positive or negative outcomes, and that there is several observations clustered around the mean.

Skewness

The skewness indicates how symmetrical the distribution is around zero. In a perfect symmetric distribution, the skewness is zero. If the skewness is positive there are more observations to the right of the mean, and the right tail is longer than in a normal distribution. In this case, there is greater likelihood to get large positive returns than negative returns, while for negative skewness there is greater likelihood to have a few negative returns compared to the normal distribution. When skewness is negative, the standard deviation will underestimate the risk.

$$Skewness = \frac{\frac{1}{n}\sum_{t=1}^{n}(r_t - \bar{r})^3}{\left[\frac{1}{n}\sum_{t=1}^{n}(r_t - \bar{r})^2\right]^{3/2}}$$

Excess Kurtosis

[8] If $\mathcal{R}$ is the risk measure for the portfolio $\mathrm{P} = (w_i, \ldots, w_n)$, it verifies the Euler decomposition:
$\mathcal{R}(w_i, \ldots, w_n) = \sum_{i=1}^{n} w_i \cdot \frac{\partial \mathcal{R}(w_i, \ldots, w_n)}{\partial w_i}$

The kurtosis is a measure of how the observation is spread around the mean and characterizes the relative sharpness or flatness of a distribution. In the normal distribution, the kurtosis is 3, but we use the term excess kurtosis which is the kurtosis minus 3. If the excess kurtosis is positive, (leptokurtic) the observations is clustered around the mean, which makes the tails heavier and the peak higher compared to the normal distribution. In this case, there is a greater likelihood of extreme values to occur. Negative excess kurtosis (platykurtic) indicates less heavy tails compared to the normal distribution. The observations are less clustered around the mean. It means that there is a smaller likelihood that extreme values will occur.

$$EKurt = \frac{\frac{1}{n}\sum_{t=1}^{n}(r_t - \bar{r})^4}{\left[\frac{1}{n}\sum_{t=1}^{n}(r_t - \bar{r})^2\right]^2} - 3$$

Jarque-Bera Test

In the normal distribution the skewness and the excess kurtosis is zero. The Jarque-Bera (JB) test is a goodness of fit-test of deciding whether sample data have the skewness and kurtosis matching a normal distribution.

$$JB = \frac{T}{6}\left(Skew^2 + \frac{EKurt^2}{4}\right)$$

H_0: Data being normally distributed

H_1: Data not being normally distributed

Reject H_0 if $JB > \chi^2_{(2)}$ (Chi square distributed with 2 degrees of freedom)

Critical value at 5% level of significance is 5,99.

9.2.4. Risk-Adjusted Performance Measures

There is no correct way of measuring risk-adjusted performance. Most investors use measures together with an understanding on how the portfolios constructed as the best approach.

Sharpe Ratio

Sharpe ratio is a ratio developed by William F. Sharpe to measure risk-adjusted performance and is defined as the amount of excess return per unit of volatility. The ratio is calculated by subtracting the risk-free rate, such as 3-month U.S. Treasury bill, from the rate of return for a portfolio and by dividing the result by standard deviation of the portfolio return.

$$Sharpe\ ratio = \frac{\overline{r_p} - r_f}{\sigma_p}$$

Where $\overline{r_p}$ is the mean return, r_f is the risk-free rate and σ_p is the standard deviation.

The calculation generates a number that can be used to compare investments over a sample period. A high Sharpe ratio suggests a good investment. It is important to emphasize that there is no such thing as a good or bad absolute number for a comparative investment statistic, we only look at its relative relationship to other portfolios.

Sortino Ratio

The ratio measures the risk-adjusted return of an investment asset or portfolio. It is a modification of the Sharpe ratio but penalizes only those returns falling below a user-specified target or required rate of return, called downside deviation. It measures the incremental return of the target rate compare to the downside risk. To calculate the Sortino ratio one subtract the minimum acceptable return (MAR) from the portfolio's return, and the divides that by the downside deviation. A large Sortino ratio indicates there is a low probability of a large loss. One has to choose a MAR, usually the risk free rate. The Sortino ratio is used to compare investments over a sample period.

$$Sortino\ Ratio = \frac{\overline{r_p} - MAR}{DD_p}$$

9.2.5. Diversification Measures

Herfindahl Index

The Herfindahl index is given by the sum of the squared asset allocation weights, but we adopt a normalized version of Herfindahl index that ranges between 0% (perfect equality or diversification) and 100% (extreme inequality or concentration).

The normalized version of the Herfindahl index is:

$$\frac{n \times \sum_{i=1}^{n} w_i^2 - 1}{n - 1}$$

Diversification Ratio

For this measure we use method proposed by Choueifaty and Coignard (2008). The inferior limit for this statistic, when long-only portfolios are considered, is 1 for 100% weight in one asset, so that values far from 1 express higher diversification and lower concentration. It is calculated as the weighted average of the standard deviation divided by the portfolio standard deviation. The diversification ratio is defined as follows:

$$\frac{\sum_{i=1}^{n} w_i \, \sigma_i}{\sigma_p}$$

9.2.6. Test for Statistical Significance

T-test for Significance in Mean

We test whether the mean returns are statistical significant different from zero by the formula:

$$t = \frac{\bar{r}}{s.e.}$$

$H_0: \bar{r} = 0$

$H_1: \bar{r} \neq 0$

We reject H_0 if $|t|$ > t critical, at 95% level of significance, with 2 degrees of freedom

Test for Significance in Correlations

We test whether the correlation coefficients are statistical significant different from zero by the formula:

$$t = \frac{\rho\sqrt{n-2}}{\sqrt{1-\rho^2}} \sim t_{n-2}$$

$H_0: \rho = 0$

$H_1: \rho \neq 0$

We reject H_0 if |t| > t critical, at 95% level of significance.

Test for Difference in Means

To test whether the differences in mean returns are statistically significant we usea two-sample t-test, assuming equal variances:

$$t = \frac{\overline{r_a} - \overline{r_b}}{s.e._a + s.e._b} \sim t_{n-1}$$

$H_0: \overline{r_a} = \overline{r_b}$

$H_1: \overline{r_a} \neq \overline{r_b}$

We reject H_0 if |t| > t critical, at 95% level of significance.

Test for Difference in Variances

To test whether the differences in variances are statistically significant we use F-test for paired two variances:

$$\frac{\sigma_i^2}{\sigma_j^2} \sim F_{n-1,m-1}$$

$H_0: \sigma_i^2 = \sigma_j^2$

$H_1: \sigma_i^2 \neq \sigma_j^2$

We reject H_0 if F > F critical at the 95% level of significance.

Test for Difference in Sharpe Ratios

To test whether the differences in Sharpe ratios between portfolio A and portfolio B are significant in a two-sided test, we adopt the Z-test presented in Memmel (2003) based on Jobson and Korkie (1981):

$$Z = \frac{\widehat{SH}_A - \widehat{SH}_B}{\sqrt{V}}$$

Where V is the asymptotic variance of the difference in Sharpe ratios:

$$V = \frac{1}{T}[2 - 2\rho_{AB} + 0{,}5(SH_A^2 + SH_B^2 - 2SH_A SH_b \rho_{AB}^2)]$$

H_0: $\widehat{SH}_A = \widehat{SH}_B$

H_1: $\widehat{SH}_A \neq \widehat{SH}_B$

We reject H_0 if |Z|> Z critical at the 95% level of significance.