



NORWEGIAN UNIVERSITY OF LIFE SCIENCES

FACULTY OF ENVIRONMENTAL SCIENCES AND
TECHNOLOGY

DEPARTMENT OF MATHEMATICAL SCIENCES AND TECHNOLOGY

Regional Sea Level Changes In Norway

Time Series Analysis of Tide Gauge Data

Author:

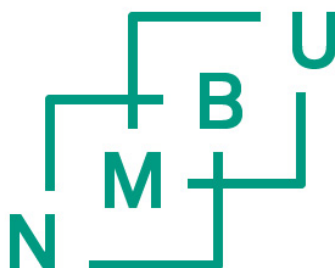
Hanna Margrethe Weng

Supervisors:

Halfdan Pascal Kierulf

Bjørn Ragnvald Pettersen

Matthew Simpson



January 15, 2015

Summary

Changes in sea level impacts the life on earth. According to the fifth assesment report, published by the International Panel on Climate Change, the global sea level is rising and the sea level is an important indicator of climate change. Recent studies states an increase of about 20 cm in global mean sea level for the past century, and an acceleration of sea-level rise in the last decades.

On order to predict future sea level changes, analysis of past sea level is needed. This master thesis is a study of regional sea level changes in Norway. Time series analysis is used on tide gauge data to find both the total sea level changes and linear sea level trends in Norway.

The study discovered an increasing sea level for the included tide gauge stations in Norway. For example, has Oslo experienced a sea level increase of 1.17 ± 0.39 mm per year, with a total sea level rise of 0.12 [0.08:0.15] meters from 1914 to 2012. Bergen has experienced a sea level increase of 1.40 ± 0.33 mm per year, with a total sea level rise of 0.14 [0.10:0.17] meters from 1915 to 2012.

The linear trend in Norwegian sea level is partly increasing, depending on the station. In nine of thirteen stations, the linear slope from the time interval 1980-2012 is significantly larger than the linear slope for the time interval 1950-1980. It was detected a significant structural change in the linear slope in Bergen, Oslo and Stavanger before and after 1980, which indicates a change in sea level. Comparisons with global studies indicates partly correspondents with Norwegian and global results.

Sammendrag

Endringer i havnivået påvirker livet på jorda. Ifølge FNs klimapanelers femte klimareport har det globale havnivået økt gjennom det siste århundret. Havnivået er en viktig indikator på globale klimaforandringer. Tidligere studier viser til en økning i havnivå for forrige århundre på omkring 20 cm og en akselererende trend i havnivået de siste tiårene.

Analyse av tidligere havnivå må til for å kunne forutsi framtidige havnivåforandringer. Denne masteroppgaven er en studie av havnivåendringer i Norge. Gjennom tidsserieanalyse av vannstandsmålerdata vil denne oppgaven se på både totale havnivåforandringer og lineære havnivåtrender i Norge.

Den lineære analysen viste en økende havnivåendring for alle de norske stasjonene inkludert i denne oppgaven. Oslo har for eksempel en havnivåøkning på 1.17 ± 0.39 mm per år, med en total havnivåøkning på 0.15 [0.08:0.15] meter fra 1914 til 2012. Bergen har en havnivåøkning på 1.40 ± 0.33 mm per år, med en total havnivåøkning på 0.14 [0.10:0.17] meter fra 1915 to 2012.

Den lineære havnivåtrenden i Norge er delvis økende. I ni av tretten stasjoner er det lineære stigningstallet for tidsintervallet 1980-2012 signifikant større enn for 1950-1980. I Bergen, Oslo og Stavanger ble det påvist signifikante strukturforskjeller i det lineære stigningstallet før og etter 1980. Dette indikerer en endring i havnivå fra før og etter 1980. Sammenligninger med studier gjort på globale data viser at resultatene fra Norge og globalt delvis samsvarer.

Acknowledgements

First I would like to thank my supervisors for help and guidance in this process. A thanks to Halfdan Pascal Kierulf, Matthew Simpson and Oddgeir Kristiansen for introducing me to the research topic and constructing a thesis formulation. A special thanks to Halfdan Pascal Kierulf for answering all my questions throughout the semester, and for advice and guidance in the analysis.

Thanks to Bjørn Ragnvald Pettersen for inspiration and to help me see the big picture in this study. Thanks for help with text improvement and language.

I would like to thank Vegard Ophaug and Siri Eikerol for help with this study. Vegard for programming and statistics guidance, and recommendation on sea level literature. Siri for helping me with language and for giving inspirational guidance.

My parents: thank you for support and interest in my work.

Finally I would like to thank my friends for guidance and support.

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1 — Introduction

1.1 Climate

Changes in climate and sea level affect the world. Approximately half of the world's populations live in coastal areas, and these areas will be directly affected by even a small rise in sea level. Higher sea level leads to floods, increased erosion and damaged freshwater due to salt supply from the sea. This will lead to people and animals losing their homes, lack of freshwater and less farming areas, which can cause food shortage, a larger unemployment rate and migrations for water. The situations will hit development countries the hardest, due to economy and the lack of infrastructure. Climate changes will also impact the weather. Higher temperature and larger water surfaces caused by melted sea-ice will cause more precipitation and extreme weather.

The sea level is continuously changing, and has done so throughout the world's history. The difference today is that the world's populations are larger than ever before. The Intergovernmental Panel on Climate Change (IPCC), concludes in their fifth assessment report that the dominant cause of the world's climate change in the last century is from human activity, anthropogenic effects [Cubasch et al.].

1.2 Sea Level

Measuring the sea level is one of the key elements to measure the effects of climate changes and it is necessary to analyse past sea level to predict the future sea level differences. A quality sea level estimation is however hard to analyse and predict. Many elements affect the sea level, such as temperature changes and the melting process of glaciers (further explained in section 2.1.3) [Church and White, 2011]. These elements also affect each other, and together it makes it hard to predict quality sea level differences.

The local sea level has been measured for hundreds of years to help humans understand the sea and to create warning systems for floods. Collection of quality relative sea level data from tide gauges have been going on for the past century, and these tide gauges are used to find past and predict future sea level, both regional and globally [Pugh and Woodworth, 2014]. However, tide gauge data are often affected by error parameters such as data gaps and short timelines [Douglas, 2001]. Tide gauges are also placed at the coast and are affected by tides, vertical land movement, weather effects and shallow waters. In order to find global sea level with tide gauges, a global distribution is needed [Pugh and

Woodworth, 2014].

Satellite altimetry has been used since the 1990's, and is one of the future methods to measure global mean sea level [Nerem, 1995]. Altimetry has the advantage of continuous global data coverage, the ability to measure at open sea [Pugh and Woodworth, 2014] and has a sea level determination with a precision of some tenth of a millimetre per year [Simpson et al., 2012]. Tide gauges and satellite altimetry data are often combined in sea level studies, to get the advantages from both measurement techniques [Nerem, 1995].

1.3 Aim of Study

In the light of IPCC's prediction on human made climate changes and sea level assumptions, the analysis will look at linear changes over time, and to see if it is possible to see a rise in sea level trend for all stations in later years. The assertion will be tested with 95 percent confidence interval differences and Chow-tests, which test for significant differences in linear slopes within a dataset.

The trends will be compared to the global mean sea level trend, to see if there is a similarity between the Norwegian trend and the global. The study will also include an analysis of annual periodic trends in the tide gauge time series.

The work will be carried out as a master study at the Norwegian University of Life Sciences (NMBU).

2 — Components that Affects the Sea-Level

2.1 Sea Surface Topography - SST

70,8 percent of the earth surface is occupied by the ocean. This equals 361 million square kilometres. The total volume is approximately 1370 million cubic meters and the mean sea depth is approximately 3800 meters [Store Norske Leksikon, 2014c]. The sea has therefore a big impact on the earth. Meteorological effects, oceanographic nature and the total water budget influence the behaviour of water on earth, and are factors that influences global and regional mean sea level. These factors will be described in the following sections. Meteorological effects, oceanographic nature and tides disturbs the distribution from the force of gravity. Sea surface topography can be defined as the sea height above the geoid [Torge and Müller, 2012]. The geoid is an equipotential surface of the earth's gravity potential, witch fits the mean ocean surface [Hofmann-Wellenhof and Moritz, 2005].

2.1.1 Meteorological Origin

The main meteorological effects are temperature, atmospheric pressure and wind. They have a great impact on the sea's movement and behaviour, and affects the sea surface topography. These effects will be described in this section.

Temperature

Heat exchange between water masses creates movement in the sea and the air. Heat has also an effect on the density of the sea's water, which is further explained bellow. The earth's position in it's orbit around the sun controls the earth's temperature, along with geothermal heat and the greenhouse effect. There are also big temperature differences on the earth's surface. Air and water have different properties under different temperatures, and therefore creates movement, such as waves and the hydrological cycle.

Atmospheric Pressure

Atmospheric pressure is created by the heat motion in the air molecules and is the weight of overlaying masses of air. The earth average air pressure is approximately 1010 hectopascal

(hPa) [Store Norske Leksikon, 2014a]. How surface pressure impacts the sea surface height is called the inverted barometer effect. The atmospheric pressure is distributed differently around the globe and is constantly changing. The inverted barometer effect has therefore an impact on sea level variations [Richter et al., 2012].

Wind

Air movement in relation to a surface, also creates a hydrodynamical pressure, also called wind pressure. Winds creates movement in the water surface, and contributes to the sea's behaviour. When the sea reaches land, it creates accumulations of water. If low pressure and onshore wind occurs, it can create a storm surge. At the southern Norwegian coast, wind from south and west often creates a higher rise in sea level than the tides [The Norwegian Mapping Authority, 2014d].

2.1.2 Oceanographic Nature

The oceanographic nature is the characteristics of the sea, and are influenced by meteorological effects such as temperature and pressure.

Salinity

The mean salinity of the sea is 35 psu (practical salinity unit) or 35 gram salt per kilo seawater. The salinity in deep waters is almost constant in open sea, but on the surface there are significant geographical differences. In addition, freshwater from precipitation, from land and calving from glaciers affects the salinity in the sea. The volume of water decreases with respect to the concentration of salt, and therefore will the sea's density increase in accordance with amount of salt [Lysaker, 2012].

Density and Pressure

Ocean currents are divided in deep and surface currents. Surface currents are created by wind on the surface, while deep currents are a result of the sea's density [Lysaker, 2012].

The pressures in air and the sea, can be explained by the same principle. Pressure from the atmosphere, high and low pressures, can help humans predict weather and it's currents. In the same way, the sea's currents can be predicted by the sea's pressure. The sea's pressure is hard to predict, but it can be calculated from the sea's density. The density is calculated from temperature and salinity of the sea [Pugh and Woodworth, 2014].

2.1.3 Water Budget

The earth has a fixed amount of water that is preserved either in the sea, as freshwater, ice or water vapour in the atmosphere. Together they make the total water budget of the earth. Freshwater, in liquid form, is preserved in lakes, rivers and as groundwater. The

amount of freshwater in liquid form or in the hydrological cycle is so small that it does not have a large impact on the global mean sea level. However, freshwater drainages from land and the hydrological cycle have an impact on perceived local sea level [Church et al., 2013].

Freshwater holds about 3 percent of the total water amount on earth. If distributed equally around the globe, the freshwater would create a 70 meter thick layer. As 90 percent of the freshwater, 34,9 mill cubic meters, is preserved as ice, ice has a big impact on the mean sea level [Store Norske Leksikon, 2014b].

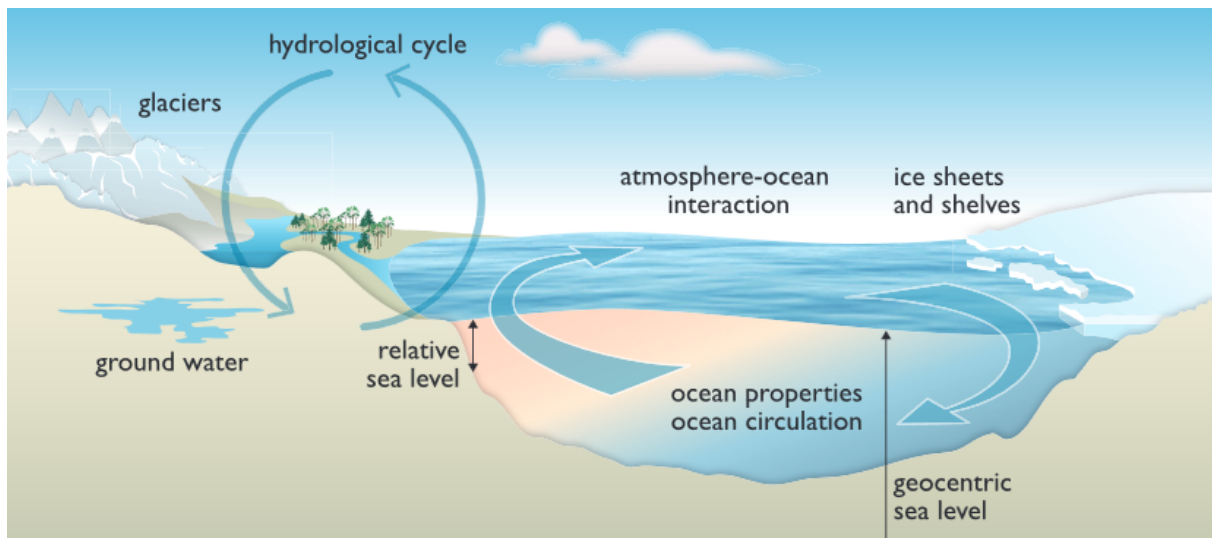


Figure 2.1: Meteorological effects, oceanographic nature and water budget [Church et al., 2013]

The recent rise in the earth's temperature from global warming has escalated the melting process on polar ice masses, in the Antarctic, Arctic and Greenland. When the sea temperature rises as a result of temperature rise in the atmosphere, it assists the ice melting proses. Antarctic and Greenland has glaciers on land, which will add water in the sea when the glacier melts. The Arctic is, on the other hand, sea-ice and will not impact the sea level due Archimedes' principle on the buoyant force.

Ice is considered to be a white mass that has a high reflection of solar radiation. Water, however, is considered a black mass, with a low reflection of solar radiation, and therefore preserves the solar heat which increases the temperature Perovich et al. [2008]. When the ice masses melts and is replaced with water the temperature increases even more, and may create an escalating effect. Polar ice melting, is therefore the key element in future mean sea level.

2.2 Ocean and Earth Tides

Astronomical object's force of attraction affects the masses of the earth. The attraction deforms the earth's shape, both liquid masses (water) and solid masses (rocks and ground) are drawn to the astronomical objects, creating a periodic deformation of the earth. This deformation is called ocean and earth tides [Torge and Müller, 2012].

The sun and specially the earth's moon are the astronomical objects that have the largest impact in this deformation. How the masses are distributed depends on where the sun and the moon is placed relative to the earth. The moon and the sun creates a deformation on both sides of the earth, due to the moons movement around the earth, the earth's movement around the sun and different force attraction on each side of the earth (figure 2.2). The force attraction is at its strongest on the earth side facing the moon, creating the first tide. However, the force at the back side is lower than the attraction force on the middle of the earth. This creates two tides per twenty-four hours [Pugh and Woodworth, 2014].

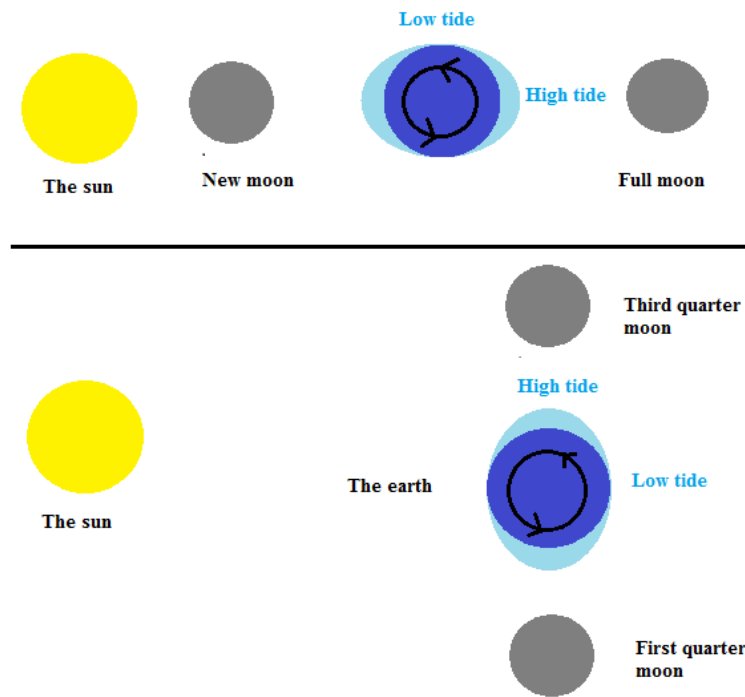


Figure 2.2: Tides

Including the earth's own rotation and the moon cycle, the forces of attraction is creating a deformation in a 12 hour period. The force is extra strong when the moon is aligned with the sun, creating spring tide, as shown in figure 2.2. The opposite when the moon is "perpendicular" to the sun, in terms of the earth [Pugh and Woodworth, 2014].

The elasticity of the earth differs on the earth's surface. This deformation, change of

potential, is described with Love numbers. Love numbers are non-denominational and are divided in vertical deformation (h), horizontal deformation (l) and deformation's potential (k) [Torge and Müller, 2012].

Earth tides and ocean tides act in different ways. Earth is a solid mass with elastic properties and the mass therefore returns to its original position when the force decreases. Water is, on the other hand, a fluid mass which moves, and will therefore not decrease to its original position, but instead create a wave. The astronomical tidal force will give the water a slope and speed; this will further be influenced by the Coriolis Effect created by the earth's rotation. The Coriolis Effect will give the sea a curvature to the east on the northern hemisphere, and the opposite on the southern hemisphere. The ocean's depth will also affect the tide wave [Pugh and Woodworth, 2014].

The tidal waves depends on the earth rotation and the depth of the water and can be several hundred kilometres long. If the length of the wave is long in relation to the depth of the ocean, the propagation velocity can be explained like this:

$$c = \sqrt{gd} \quad (2.1)$$

Where c = propagation velocity, g = the earth's gravity, d = depth of the sea [Pugh and Woodworth, 2014].

When the tidal wave meets the coast, it will continue along the coast as a Kelvin wave. A Kelvin wave will create larger wave height on shallow waters. The tides in Norway have its origin in the Atlantic and are spread around Great Britain and the Nordic sea, before it hits the Norwegian coast. It is the mixture of Kelvin waves that make the occurrence of large tidal differences in locations that are close to each other [Pugh and Woodworth, 2014].

2.3 Vertical Land Motion

The masses of ice in Fennoscandia (Norway, Finland and Sweden) during the last ice age compressed the earth masses. After the ice masses melted, the earth masses started rise to its original state. Since the soil is a viscous mass it needs longer time to rise to its original state, than the time period the glacier used to melt. Therefore there still is vertical land motion in Fennoscandia, mainly as a following effect from the last ice age Kierulf et al. [2014]. The Fennoscandian ice glacier effect is the largest contributor to the vertical land motions in Norway, and is shown in figure 2.3.

Vertical land motions have an impact on tide gauge measurements. The observed, relative, local sea level changes may vary from the actual, absolute, sea level change. For example, in Oslo, Norway, the sea level is rising, but the vertical land motion impacts the observed sea level rise, and therefore the observed sea level changes are descending [The Norwegian Mapping Authority, 2014c].

Global navigation satellite system (GNSS) and geophysical models are used to calculate the vertical land motion. In Norway the Norwegian Mapping Authority have 140 GNSS-

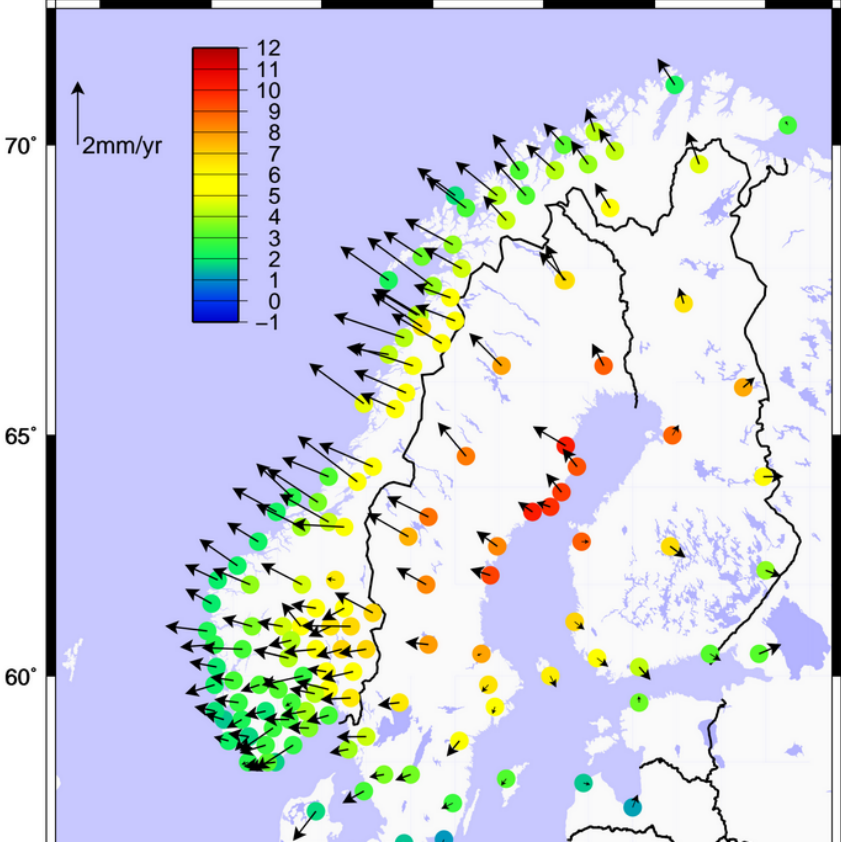


Figure 2.3: Vertical and horizontal land motion in Fennoscandia [Kierulf et al., 2014].

stations to observe the motion. Geophysical models of the ice from the last ice age in Fennoscandia are used to predict the ongoing rise in vertical land movement.

3 — Measuring Sea Level

3.1 Sea Level Measuring Systems

3.1.1 Tide Gauge Measurement

Tide gauge, or sea level gauge, measuring has been ongoing for centuries. These measurements are divided in coastal and offshore gauges. Offshore measuring is less prevalent, due to of costs and technology, but has the advantage of less influence by weather and tides. Coastal gauges are more widely spread around the world, and often have long term surveys [Pugh and Woodworth, 2014]. Tide gauges are a relative measuring technique, that measures the sea level to a reference level on the seabed or land [Simpson et al., 2014]. In Norway, the longest surveys with tide gauges records back to the nineteenth century.

Coastal tide gauges can be based on techniques pressure, radar or the more common still-well system. Pressure gauges measure the water pressure, and are used for shorter series of recordings. The radar gauges use electromagnetic waves to measure the sea surface relative to a fixed position. The still-well system (see figure 3.1) is the most common systems among coastal tide gauges in Norway. The principle is to lower a narrow tube in the sea, with a small water inlet at the bottom. This curbs decreases the impact of surface waves. A gear system holds up a weight and the tide gauge so that the tide gauge is floating on the water inside the tube. The gear is used as a optical encoder, connected to the data recorder. This data is used to calculate the local sea level changes [Tørresen, 2012]. The principle has been used for over a century, but in later years the data have been collected digitally.

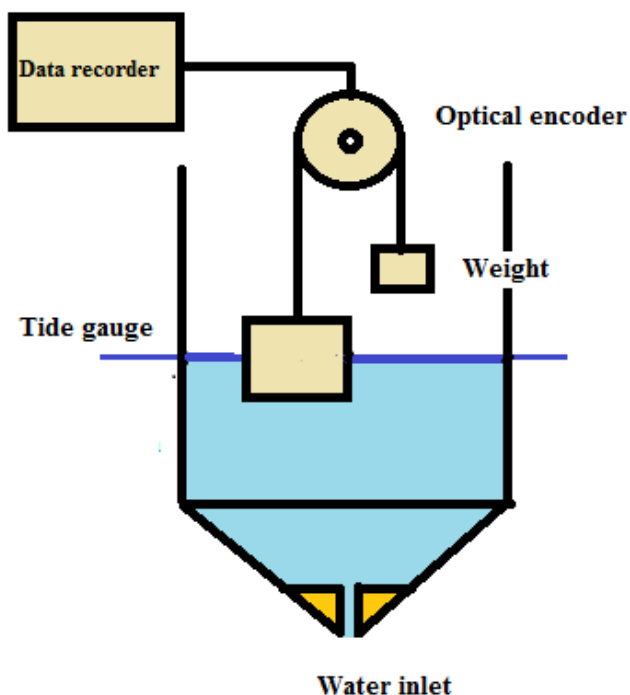


Figure 3.1: Coastal Tide Gauge - still-well system

3.1.2 Satellite Altimetry

Altimetry has been one of the main techniques to observe sea surface topography and sea level changes for the past 20 years. Satellite altimetry uses a radar pulse to measure the distance between a satellite and the ocean surface. The satellite uses time and precise satellite tracking systems like GNSS (Global navigation systems), DORIS (Doppler orbitography and radio positioning integrated by satellite) and SLR (satellite laser ranging), to determine the distance and to calculate precise orbits [Simpson et al., 2012]. The height is calculated relative to the geoid and the ellipsoid. The ellipsoid is a smooth mathematical surface of the earth [Seeber, 2003]. Figure 3.2 shows the principle of satellite altimetry, reflecting the surface of the sea, and referencing to the ellipsoid.

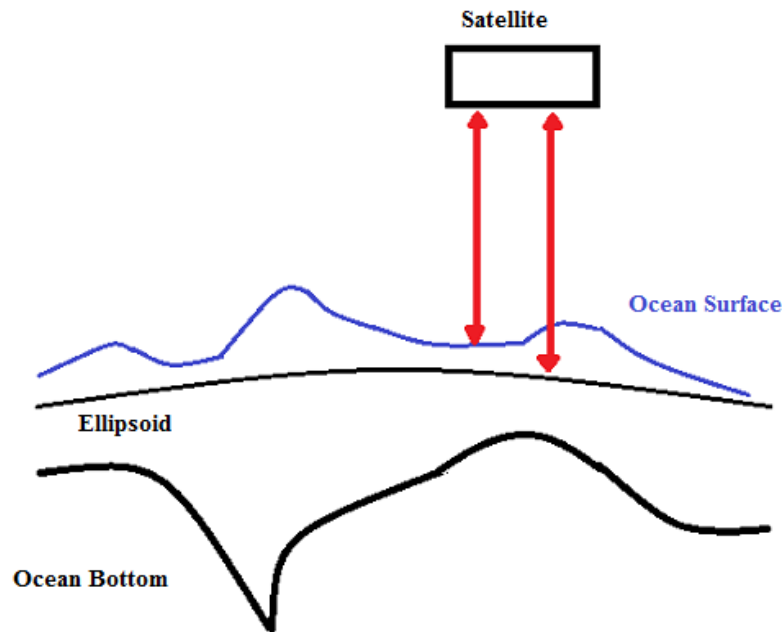


Figure 3.2: Satellite Altimetry

The wide area coverage and access to open sea, is the strength of satellite altimetry. Topex/Poseidon, Jason-1 and Jason-2 from the National Aeronautics and Space Administration (NASA) are the leading altimetry satellites, and have world wide continuous sea level data from the past 20 years [Church and White, 2011]. Satellite altimetry has a short time range on its data recordings, compared to tide gauge data. The footprint in satellite altimetry is relatively large in smaller sea level studies, such as regional and local. The islands on the Norwegian coast and the Norwegian continental shelf interferes the satellite altimetry signals and creates height errors in the dataset [Vignudelli et al., 2005]. Satellite altimetry and tide gauges is often combined in mean sea level studies to get the advantage of local sea level differences from tide gauges and the regional sea level differences from satellite altimetry (Cazenave et al. [1999] Simpson et al. [2012]).

3.1.3 Reference Level

Several reference levels exist, and can be used in measuring mean sea level (see figure 3.3). In a nautical map the height of depth is related to a given datum. In Norway the present datum is NN1954, this will be replaced by NN2000 during 2015. NN1954 is calculated from a benchmark in Tregde, and from mean sea level calculations from Oslo, Nevlunghavn, Tregde, Stavanger, Bergen, Kjølisdal and Heimsjø [Voldsund, 2014]. In NN2000 all measurements is adjusted to year 2000 and is corrected for vertical land movement since 1954 [The Norwegian Mapping Authority, 2015].

Mean sea level is calculated over a period of 19 years, from the average tide gauge data with fixed time intervals. The 19 years has its origin in the lunar cycle [Voldsund, 2014].

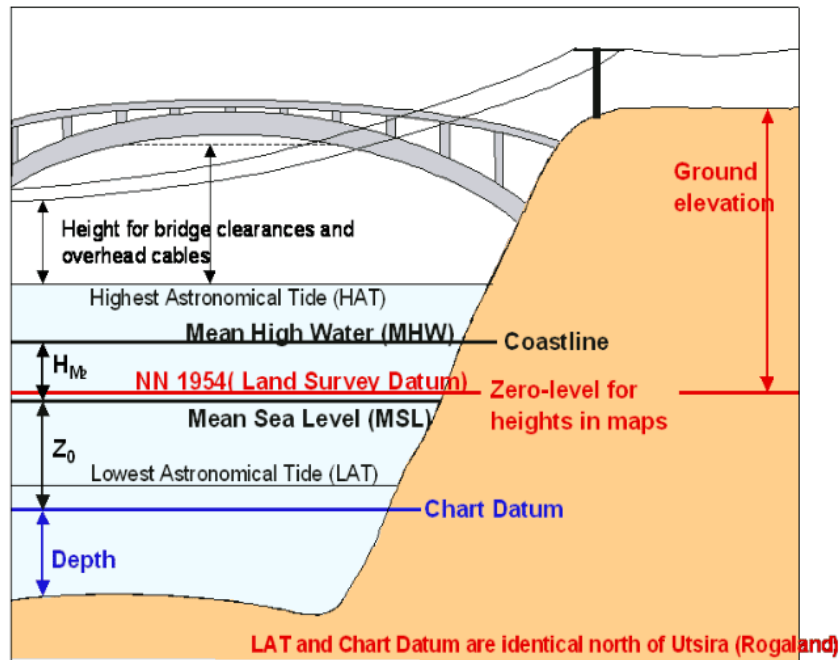


Figure 3.3: Reference level for MSL for tide gauge measurements [The Norwegian Mapping Authority, 2014a]

The reference level and tide gauge positions are calculated from supplying benchmarks.

In trend analysis the reference level is less important. Still, if the reference level has changed during the station's recording time and not been compensated, the dataset will be affected.

3.2 Studies in Mean Sea-Level

3.2.1 The Fifth Assessment Report of the Intergovernmental Panel in Climate Change

The fifth assessment report (AR5) of the Intergovernmental Panel in Climate Change (IPCC) concludes that it is very likely that the global mean sea level rate will exceed the rate observed from 1971 to 2010, in the next century [Church et al., 2013]. IPCC's *The Physical Science Basis* (2013) in the The Fifth Assessment Report, state that the global sea level rise from 1901 to 2010 is 0.19 m [0.17 to 0.21 m] (figure 3.4) and states that:

"It is very likely that the mean rate of global averaged sea level rise was 1.7 [1.5 to 1.9] mm/yr between 1901 and 2010, 2.0 [1.7 to 2.3] mm/yr between 1971 and 2010, and 3.2 [2.8 to 3.6] mm/yr between 1993 and 2010. Tide-gauge and satellite altimeter data are consistent regarding the higher rate of the latter period. It is likely that similarly high rates occurred between 1920 and 1950." [IPCC, 2013].

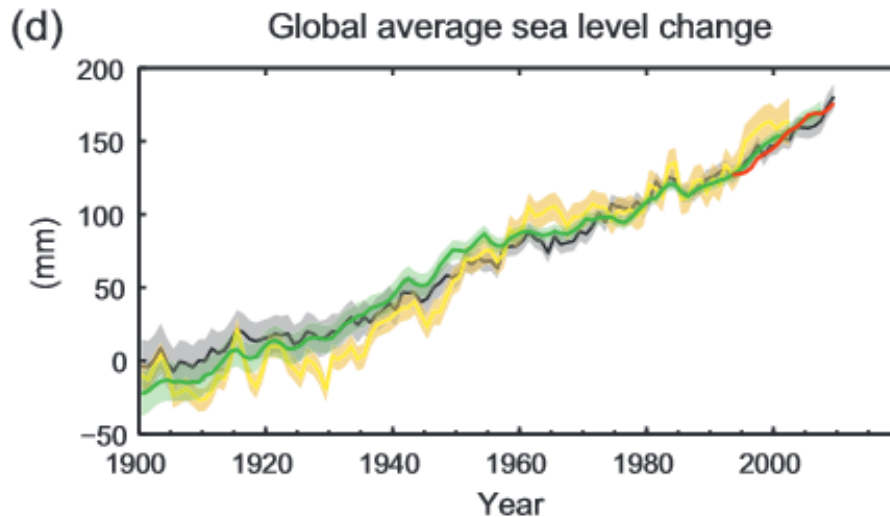


Figure 3.4: Global Mean Sea-Level [IPCC, 2013]

Figure 3.4 shows the global mean sea level relative to the 1986-2005 mean of the longest running dataset. The data are also aligned to have same values as the start year of satellite altimetry data, 1993. The datasets are; Black line - Church and White [2011] - tide gauge reconstruction 1900-2009: Yellow line - Jevrejeva et al. [2008] - tide gauge reconstruction 1900-2002: Green line - Ray and Douglas [2011] - tide gauge reconstruction 1900-2007: Red line - Nerem et al. [2010] - satellite altimetry from 1993-2009 [IPCC, 2013].

3.2.2 Permanent Service for Mean Sea Level

Permanent Service for Mean Sea Level (PSMSL) is a worldwide organisation, established in 1933, collecting sea level data from different countries. PSMSL report to the International Association for the Physical Sciences of the Oceans (IAPSO) [PSMSL.org, 2014]. The data used in this study originates from The Norwegian Mapping Authority Hydrographic Service, downloaded from PSMSL

PSMSL's analysis of linear relative sea level trends are displayed in figure 3.5, based on the same data as used in this study.

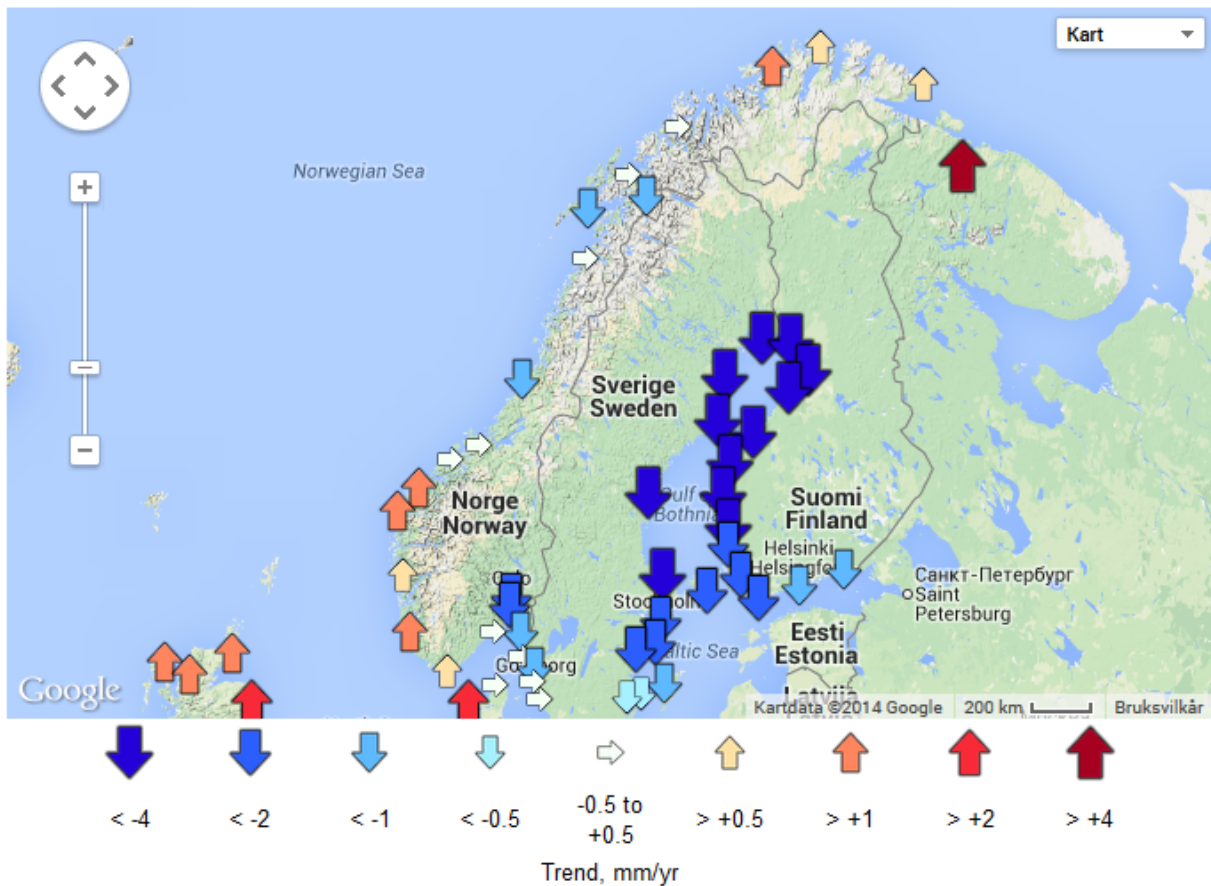


Figure 3.5: Relative Sea-Level trends for 1980-2012 from PSMSL [PSMSL.org, 2014]

3.2.3 Global Mean Sea Level

There are several studies on global mean sea level. A recent study is Church and White [2011], which also is included in the fifth assessment report from IPCC. The study have tide gauge and altimetry data from 290 locations.

Figure 3.6 shows sea level changes for the period 1860 to 2010. The blue line indicates tide gauge data, satellite altimetry data from 1993 to 2010 is indicated by a black line and the red line is a previous study by Church and White [2006]. Church and White [2011] found in their study that the global average sea level rise from 1880 to 2009 is about 210 mm and the linear trend from 1900 to 2009 is 1.7 ± 0.2 mm per year.

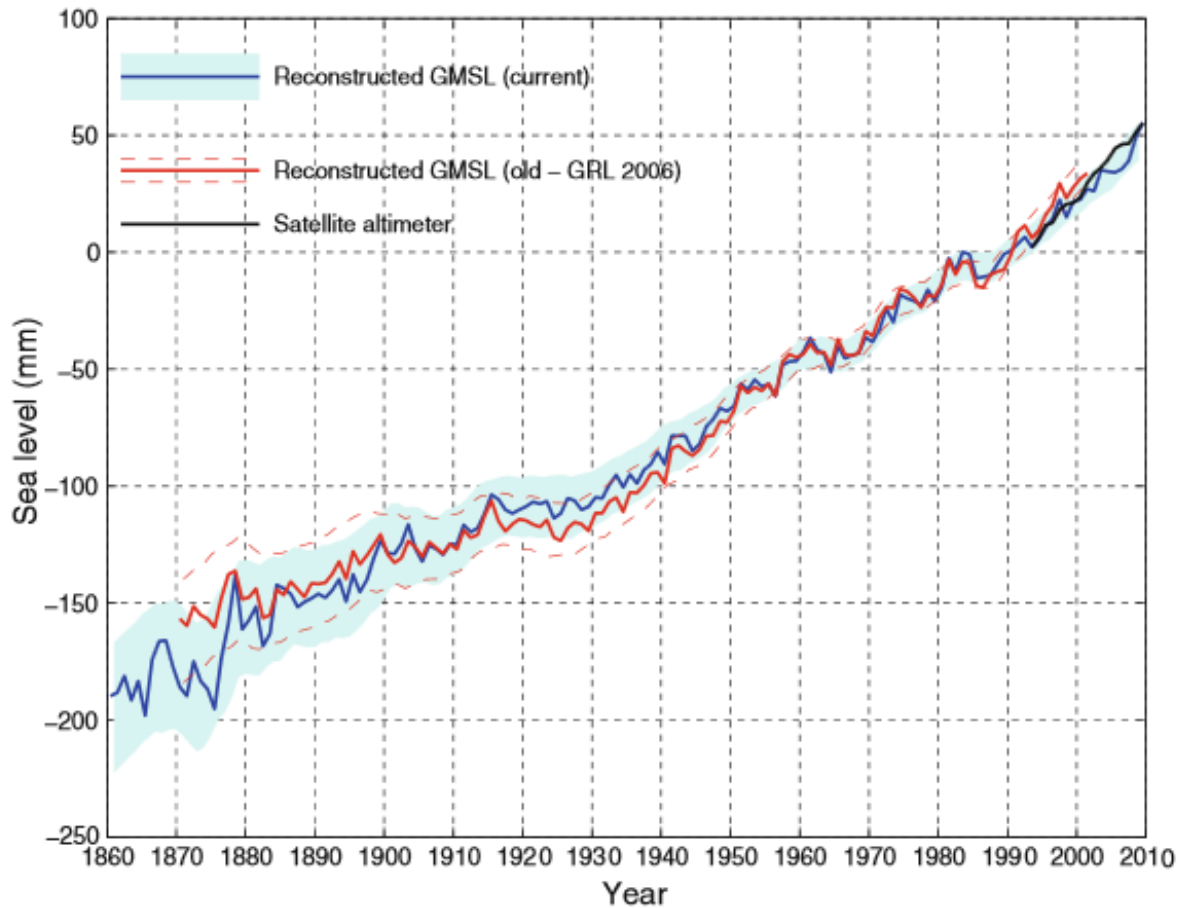


Figure 3.6: Global Mean Sea-Level for 1860-2010 Church and White [2011]

Another study of John A. Church and Neil J. White states a significant acceleration in global mean sea level rise from 1870 to 2004 of $0.013 \pm 0.006 \text{ mm yr}^{-2}$ [Church and White, 2006].

The data in Church and White [2011] was used as global mean sea level data in this study and were downloaded from PSMSL. The global mean level values are corrected for changes in temperature and inverse barometer effect.

3.2.4 Norwegian Sea-Level Changes

The Norwegian Mapping Authority reports in *Estimates of Future Sea-Level Changes for Norway* that the sea level changes in Norway will vary between -6.5 and 6 cm in the time period between 2000 and 2030, if observed rates for tide gauge records for the last 30 years continue unchanged [Simpson et al., 2012].

Figure 3.7 shows sea level rates for selected tide gauge stations in Norway, with relative data (blue), data correlated for vertical land motion (red) and data fully correlated for

Glacial Isostatic Adjustment (GIA) (open red). A fully GIA-correction includes adjustment for geoid changes. The top panel is computed from the entire time series for each station. Bottom panel has rates for the period 1980 to 2010 [Simpson et al., 2012].

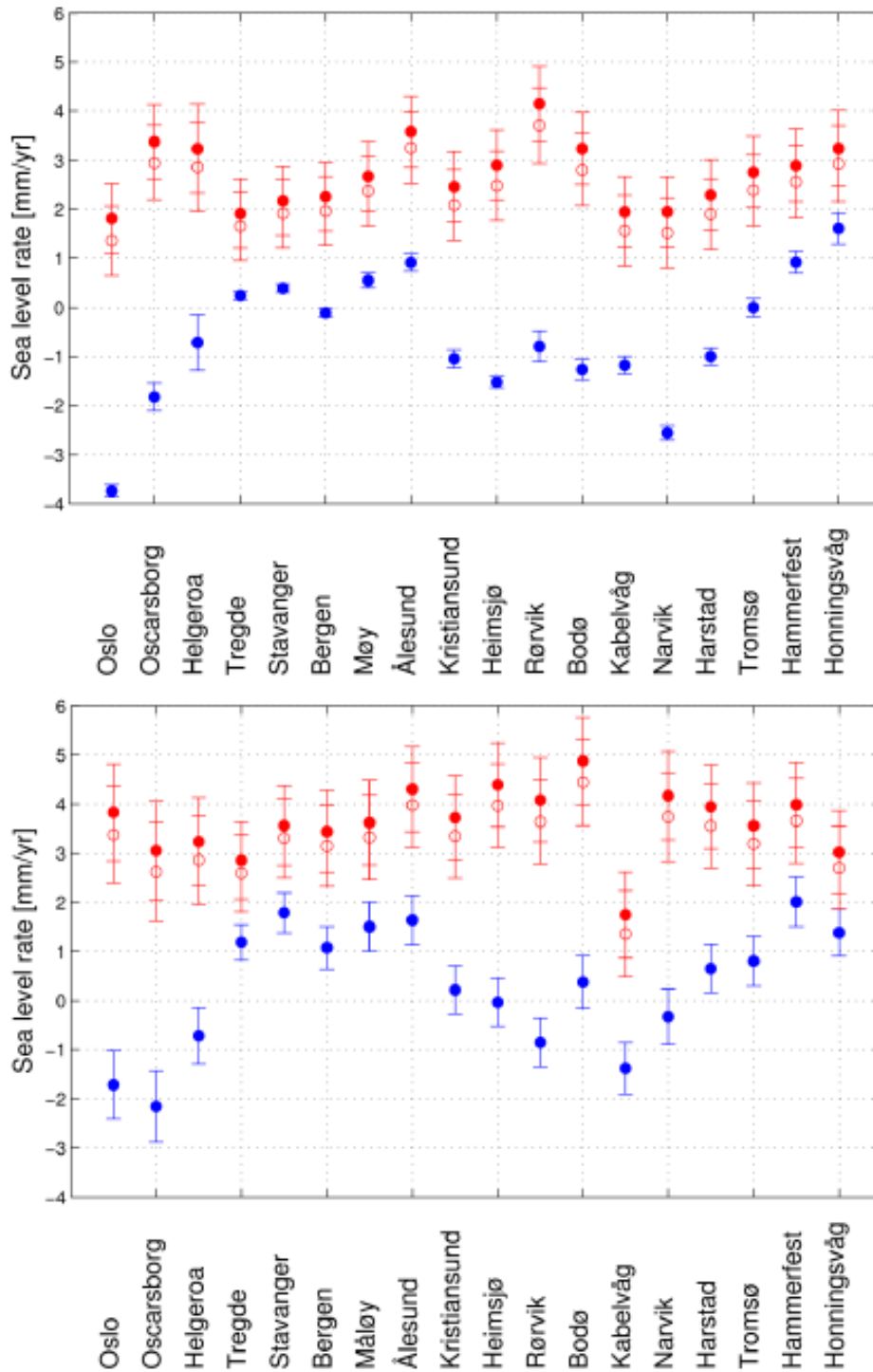


Figure 3.7: Sea Level rates from tide gauge stations in Norway [Simpson et al., 2012]

4 — Method

4.1 Time Series Analysis

Time series analysis is used to see trends in time, to even out periodic time variations and to predict future estimations [Pugh and Woodworth, 2014]. In this study, time series analysis is used to find trends in tidal gauge data from Norway.

Time series studies need long series of data. The longer the data intervals are, the better will the results approach the true values of the constants in the analysis [Pugh and Woodworth, 2014]. One of the challenges in finding trends in time series is to find time intervals that includes enough data to get a good linear estimation, but also allows to create intervals and find trends within the dataset. The data series in this thesis have maximum 98 year of data, further presented in section 4.2. In Simpson et al. [2012] a 30 year interval was used to find the linear trend from 1980-2010. A time interval of 30 years includes several Norwegian tide gauge stations and also allows to find linear trend over time for stations with more than 30 years of data. Therefore the data limit where set to minimum 30 years of data in the analysis, both station length and intervals.

For station with more than 30 years of data, a 30 year interval is used to find linear trend over time. The 30 year interval was used with five year steps in time throughout the dataset. For example, in a time series from 1970 to 2012 the trend have been calculated for 1970-2000, 1975-2005 and 1980-2012. The same time steps have been calculated for each station to make the comparison to the linear trend over time more easy.

4.2 Tide Gauge Stations

Data from 30 tide gauge stations situated on the Norwegian mainland is available from the Norwegian Mapping Authority, 24 of these are still operating. 11 of the stations were not included in this study. Vardø, Vadsø, Berlevåg, Evenskjær, Sandnessjøen, Mosjøen, Brønnøysund, Helgeroa, Nevlunghavn and Viker, have less than 30 years of data, and Andenes has bad data quality from 1940 to 1986.

The data from PSMSL and The Norwegian Mapping Authority Hydrographic Service have been corrected for tidal changes, but not for vertical land motion and weather effects. Vertical land motion and weather effects will impact the tide gauges differently due to geographical station locations. The weather effects are significantly different for a station

placed at the coast than in a fjord.

The tide gauge stations have different start time, and have several gaps in their dataset. Table 4.1 lists the stations included in this study, and identifies the gaps in the datasets. The years listed may be incomplete or entirely without data. If there is more than month missing within a year, it is listed as a incomplete year.

Table 4.1: Tide gauge station information

Station	Started year (-2012)	Years with incomplete or no data
Ålesund	1945	1951, 81, 82, 83, 86
Bergen	1915	1941-43, 45, 51, 56, 71, 88
Bodø	1949	1949, 51-56, 65-67, 69-73, 86, 88, 90
Hammerfest	1957	1962, 65-70, 76, 82, 90
Harstad	1952	1952, 66, 75-76, 80, 88, 90
Heimsjø	1928	1931-34, 38, 42-43, 59, 65, 73, 90
Honningsvåg	1970	1985, 87-89
Kabelvåg	1948	1974, 88-89
Kjølsdal	1935-88	1939, 41-44, 55-58, 63-65, 67, 72, 76, 84-85, 88
Kristiansund	1952	1952, 54, 86-87
Måløy	1943	1943, 45, 52, 59-60, 78-80, 86
Narvik	1928	1928, 30-31, 36, 38, 40-47
Oscarsborg	1872	1872, 1874-1875, 1883-1953, 1957-61, 64, 67-68, 70, 76-77, 90
Oslo	1885	1885-1914, 15, 18,28, 39,65,74,91
Rørvik	1969	1969, 88
Stavanger	1919	1920, 22, 28, 33, 40-46, 70-71, 74
Tregde	1927	1927, 41-42, 44, 56, 94
Tromsø	1952	1952, 86, 88
Trondheim1	1945-89	1945-48
Trondheim2	1990	

Figure 4.1 displays the location of the Norwegian tide gauges. The colour of the marker indicates the length of the stations operation.



Figure 4.1: Sea Level gauges in Norway[PSMSL.org, 2014]

4.3 Vertical Land Motion stations

The lands movement has an impact on the sea level measured, as tide gauge measuring systems are placed on the coast. In this study the measuring stations are analysed for trends. However, correcting for vertical land motion (VLM) allows to compare the stations.

The vertical land motion data (table 4.2) were obtain from Kierulf et al. [2014]. For some of the tide gauge stations a nearby permanent geodetic station is used, because the tide gauge and permanent GNSS network are not correlated. Also some of the permanent GNSS stations have a short operating time. The alternative station names are in parenthesis in table 4.2. The 1-sigma uncertainty is calculated by the time series analysis software CATS [Williams, 2008], which includes both white noise and flicker noise [Kierulf et al., 2014]. White noise is uncorrelated [Kristiansen, 2014], while flicker noise has some correlation in time [Mao et al., 1999].

Table 4.2: Vertical Land Motion

Station	Code	VLM (mm/y)	1-sigma confidence value
Ålesund	ALES	1,67	0,12
Bergen	BRGS	1,49	0,20
Bodø	BOD3	3,02	0,19
Hammerfest (Honningsvåg)	HONS	1,74	0,36
Harstad (Bardu, Lodningen, Bjarkøy)	BARC/LODC/BJAC	4,56	0,39
Heimsjø (Hemne)	HEMC	3,33	0,52
Honningsvåg	HONS	1,74	0,36
Kabelvåg (Svolvær)	SVOC	2,61	0,70
Kjølsdal (Stadt)	STAC	1,35	0,35
Kristiansund	HUSC	1,46	0,20
Måløy (Stadt)	STAC	1,35	0,35
Narvik (Bardu, Lodningen, Bjarkøy)	BARC/LODC/BJAC	4,56	0,39
Oscarsborg (Røyken)	OSLS	4,81	0,17
Oslo (Røyken)	OSLS	4,81	0,17
Rørvik (Vega, Roan)	VEGS/ROAC	3,00	0,34
Stavanger	STAS	1,25	0,19
Tregde	KRSS	1,55	0,11
Tromsø	TRO1	2,74	0,22
Trondheim1	TRDS	3,93	0,19
Trondheim2	TRDS	3,93	0,19

4.4 Processing

This study has four stages:

- Finding linear trends for all stations in the entire dataset and in 30 year periods.
- Correcting for vertical land movement.
- Calculating trends on global mean sea level to compare with trends from the Norwegian coast.
- Analyse for significant change in uncertainties for the linear trend and the Chow-test.

Linear regression displays the linear trend in a dataset, and determine the intersection of the y-axis and the slope of the regression line. The slope is a best adjusted trend line to the dataset and can indicate a decrease or increase in the data. The trend line can therefore indicate a sea level decrease or increase within a sea level dataset.

Harmonic equations displays periodic signals in a dataset, this shows data variations (amplitude) and peaks (phase) [Pugh and Woodworth, 2014]. In a sea level dataset, the amplitude shows the highest periodic sea level variations and the phase shows at what time the highest peak of water height accrued [Pugh and Woodworth, 2014].

There are used three methods to determine the linear trend; calculated for linear trend only (l_1), linear trend with periodic signals (l_2), and linear trend corrected for harmonic signals (l_3). The annual periodic signal for the whole station is used in the correction for l_3 . Therefore will the values for l_2 and l_3 be the same for the entire dataset. However, there will be a difference in the 30 year intervals due to different periodic signals, and therefore method l_3 is used in the rest of the study.

The first linear trend (l_1) is explained by equation 4.1.

$$MSL_{lineartrend} = MSL(t_0) + MSL_{linear} \quad (4.1)$$

The harmonic equation 4.2 is divided in three parts: linear, annual and semi-annual. The linear part (l_2) is influenced by the annual and semi-annual signals, and will therefore not have exactly the same values as the first linear trend (l_1). l_3 was also calculated by equation 4.1, after the data was corrected for periodic signals.

The annual part in the harmonic equation reveals the largest annual periodic signal. The semi-annual part is used as a parameter in the equation to improve the linear and annual results.

$$MSL_{total} = MSL(t_0) + MSL_{linear} + MSL_{annual} + MSL_{semi-annual} \quad (4.2)$$

$$\begin{aligned}
MSL_{linear} &= a(t - t_0) \\
MSL_{annual} &= A_1 \sin[\omega(t - t_0)] + A_2 \cos[\omega(t - t_0)] \\
MSL_{semi-annual} &= S_1 \sin[2\omega(t - t_0)] + S_2 \cos[2\omega(t - t_0)]
\end{aligned}$$

Where t is the observation time and $MSL(t_0)$ is the reference value [Iz et al., 2012].

Equation 4.2 was solved with a least squares fit to the observations:

$$Ax = l + V \quad (4.3)$$

x containing the unknown parameters and V is the residuals of the system. l is the observation matrix, which contains all observation data, sea level data, for each station. The design matrix A has the size $[n,m]$, where n equals numbers of observations, m equals number of unknown parameters for each tide gauge stations.

The design matrix:

$$A = [1 : t : \cos(\frac{2\pi}{1}t) : \sin(\frac{2\pi}{1}t) : \cos(\frac{2\pi}{.5}t) : \sin(\frac{2\pi}{.5}t)]$$

The x-matrix:

$$x = [X_1 : X_2 : A_1 : A_2 : S_1 : S_2]^T$$

Where X_1 and X_2 is the intersect with the y-axes and the slope for the linear trend line. A_1 A_2 S_1 S_2 is the cosine and sine parameters for amplitude and phase shown in equation 4.2.

Solving equation 4.3 for x , without V , solves the unknown parameters [Ghilani, 2010]:

$$x = (A^T P A)^{-1} A^T P l \quad (4.4)$$

P is the weight matrix. In this thesis all observations were weighted equally, and therefore equation 4.4 may be simplified to:

$$x = (A^T A)^{-1} A^T l \quad (4.5)$$

Equation 4.4 and 4.4 are transformation equations to find the amplitude and the phase.

$$\begin{aligned}
A_{annual} &= \sqrt{A_1^2 + A_2^2} \\
S_{semi-annual} &= \sqrt{S_1^2 + S_2^2}
\end{aligned}$$

$$\phi_A = \tan^{-1}\left(\frac{A_2}{A_1}\right)$$

$$\phi_S = \tan^{-1}\left(\frac{S_2}{S_1}\right)$$

To get ϕ_A and ϕ_S to return as four-quadrant inverse tangents in inverse trigonometric function, the atan2-function in Matlab was used.

The linear, annual and semi-annual trends were calculated with the least squares method. The data were then corrected for the annual and semi-annual trends and a new linear trend (l_3) was calculated with the least squares fit. The residuals (V) were calculated along with the cofactor matrix (Q_x),

$$Q_x = (A^T A)^{-1}$$

and the standard deviation (S_x) for the unknown parameters [Ghilani, 2010]. S_x was calculated by the squared root of the diagonal of the s_x matrix.

$$S_0 = \sqrt{\frac{v^T v}{n - e}}$$

$$C_x = S_0^2 * Q_x$$

S_0 is a posteriori standard deviation to the unknown and C_x is the covariance matrix.

The Quality of the Model

Calculation of the quality of the model is done by the total sum of squares (SS_T), which is found by the sum of variation in the model (SS_R) and the error sum of squares (SS_E) [Løvås, 2013]:

$$SS_T = SS_R + SS_E$$

SS_T is calculated by taking the standard deviation of the variance (v) squared and the SS_E is calculated by taking the standard deviation of the observations (l) squared.

The proportion of variation explained by the model is calculated by R^2 :

$$R^2 = \frac{SS_R}{SS_T}$$

An adjusted R^2 (\bar{R}^2) is calculated to find the the proportion of variation explained by the model, but with different number of independent variables.

$$\bar{R}^2 = 1 - \frac{SS_E/(n - e)}{SS_T/(n - 1)}$$

Where n is number of observations, and e is number of unknown parameters, where $n-e$ is the degrees of freedom.

Correlation Between the Variables

A calculation of confidence values was made. In order for the confidence values to be valid, the residuals variance have to be constant, independently of t . Normal distribution of the residuals where tested. However, if the number of observations is over 30, the observations do not have to be normally distributed according to the central limit theorem. The central limit theorem states that independent and identical random variables moves towards a normal distribution when the number approaches infinity [Løvås, 2013]. All observations (n) in this thesis is above 30.

A normal distribution test was still conducted on some of the results to test the central limit theorem. A chi-square goodness-of-fit test with a 5 percent significance level was used to find the normal distribution in the dataset.

A confidence interval is an error parameter on an observed interval, calculated from the observations, within a statistical limit. A 95 percent confidence interval was used, with $n-2$ degrees of freedom, 2-sigma values.

The confidence interval of the slope:

$$X_2 \pm t_{\alpha/2} S_x(2)$$

Where X_2 is the slope, $t_{\alpha/2}$ is the 95 percent t-value with $n-2$ degrees of freedom and $S_x(2)$ is the accompanying standard deviation to the slope in the S_x matrix.

The confidence interval of the regression line:

$$X_1 + X_2 t \pm t_{0.025} S_0 \sqrt{\frac{1}{n} + \left(\frac{t - \bar{t}}{S_x(2)}\right)^2}$$

Where X_1 is the intersect in the y-axis and S_0 is a posteriori standard deviation to the unknown.

The global mean sea level data was calculated by the same method as the Norwegian tide gauge data.

Vertical Land Motion Correction

The vertical land motion data was given i mm/y. These where added to the linear slope (l_3). The uncertainties was given in 1-sigma values (σ_l), and therefore the uncertainties,

the 2-sigma confidence value, was transformed to 1-sigma values (σ_h). Total uncertainty value (σ_d) was calculated by:

$$\sigma_d = \sqrt{\sigma_h^2 + \sigma_l^2}$$

Significant Change in Trend over Time

Confidence intervals can be used to find changes in a time series. If the confidence interval does not overlap between two parameters in an analysis, there is a significant change between the two parameters within the confidence interval percent [Gardner and Altman, 1986].

In addition to confidence intervals, a different test was used to find significant change. The Chow-test states if there is a significant change between two linear regression lines within a dataset [Chow, 1960]. The Chow-test was used on stations with a long operating time (>90 years), to find if there was a significant change between the data before and after 1980.

The Chow-test, or F-test, is designed to test against a single shift alternative and test on a structural change in a pre known change point. The Chow-test compare the chow-parameter (Ch in equation 4.6), against a 95 percent Fisher-Snedecor distribution (F-distribution, F). If $Ch > F$, then it indicates a significant structural change [Zeileis et al., 2001].

$$Ch = \frac{(SS_{E-1} - (SS_{E-2} + SS_{E-3}))/k}{(SS_{E-2} + SS_{E-3})/(n_2 + n_3 - 2k)} \quad (4.6)$$

SS_{E-1} is the error sum of squares for the whole regression line. SS_{E-2} and SS_{E-3} are the error sum of squares for the two comparing lines. n are the number of observations and k are the number of parameters.

The challenge is to find a pre known change point, and to use the Chow-test to determine if there is a significant structure change in the regression lines within a dataset.

A pre-test was made on the global mean sea-level distribution to test how sensitive the Chow-test are on tide gauge data. The annual global mean sea level dataset with accompanying regression line is shown in figure 4.2.

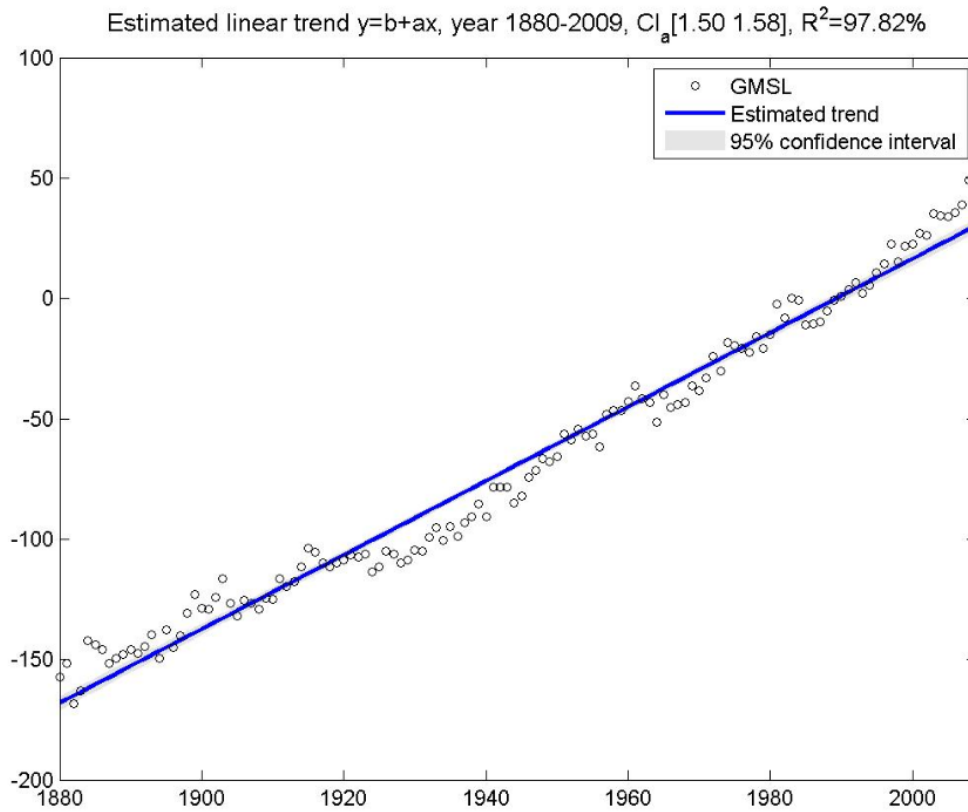


Figure 4.2: Global mean sea-level data

Studying figure 4.2 show a change in trend before and after 1930. A Chow-test was therefore applied with breakpoint on 1930 with a 50 year span on each side. The Chow-test stated a significant change in the slope before and after 1930 in the annual global mean sea level dataset (figure 4.3). The regression line from 1880-1930 had a slope on 1.14 ± 0.12 mm/year and from 1930-1980 the slope was 1.78 ± 0.09 mm/year. The chow-value was 38 against the F-distribution on 3.09.

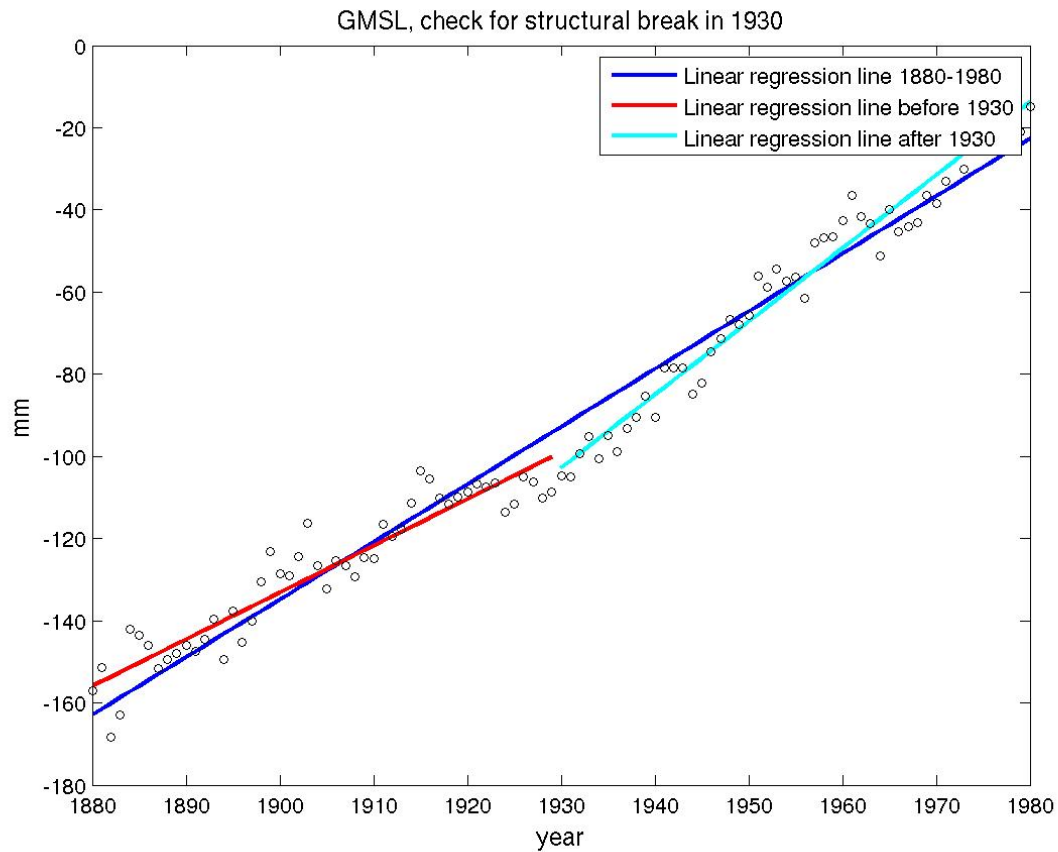


Figure 4.3: Global mean sea-level data 50

The trend in the global data scatter on figure 4.2 from 1940 to 1970 and 1970 to 2000, have similarities. The Chow-test was applied to see if there was a structure break between the two lines.

The Chow test gave a Ch-value on 0.04 and a F-value on 3.16, and the Chow-test stated that there was no significant structure change in the regression lines between 1940 to 1970 and 1970 to 2000 on the global sea level data (figure 4.4). The regression line from 1940-1970 had a slope on 1.69 ± 0.24 mm/year and from 1970-800 the slope was 1.73 ± 0.20 mm/year.

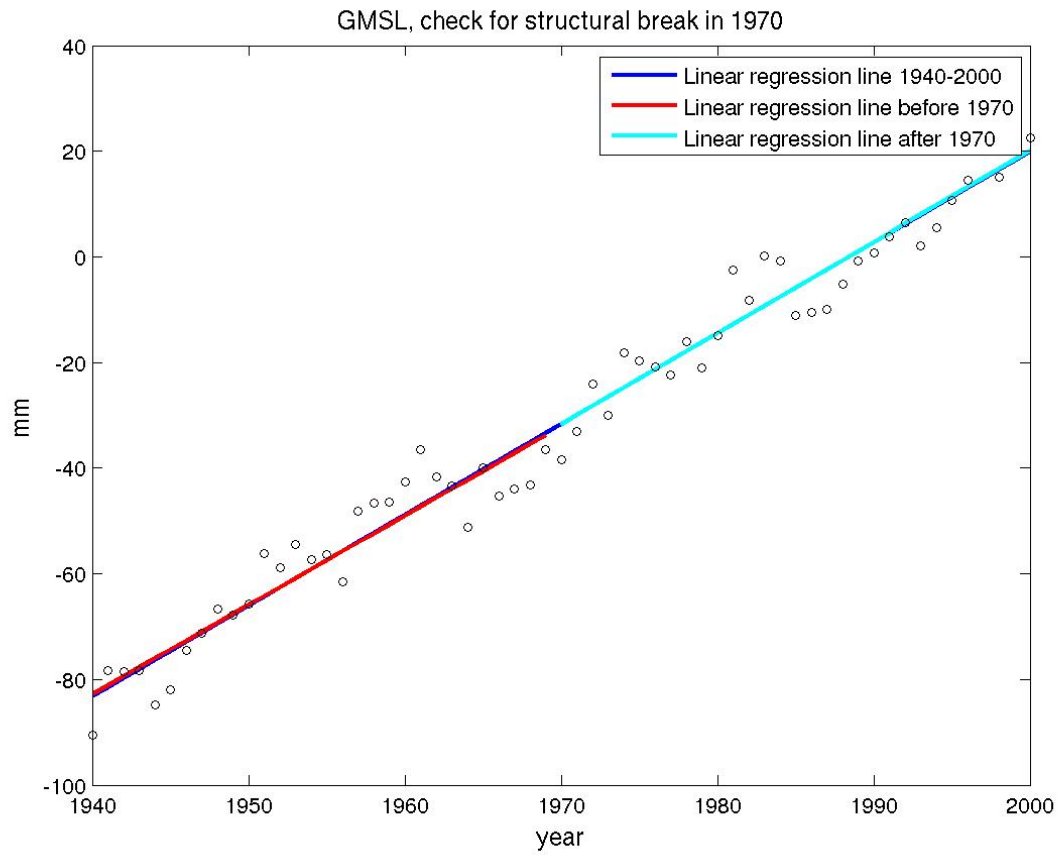


Figure 4.4: Global mean sea-level data 30

The two Chow-tests on the global mean sea level dataset shows that if the linear slope differ with 0.64 mm/year the slopes is significantly different. If the linear slope differ with 0.04 mm/year the slopes are not significantly different according to the Chow-test.

5 — Results and Analysis

5.1 Linear Regression

Linear regressions were run by three methods for the entire monthly dataset for each station. Table 5.1 shows the three methods, linear regression calculated alone (l_1), linear regression calculated with periodic signals (l_2) and linear regression corrected for periodic signals (l_3). The result from Simpson et al. [2012] is included in the table. Note that the Simpson et al. [2012] results are calculated to and including year 2010.

Table 5.1: Linear regression - three methods

Station	Simpson et al 2012 (mm/y)	l_1 (mm/y)	l_2 (mm/y)	l_3 (mm/y)
Ålesund	0,9	0,8188	0,8577	0,8577
Bergen	-0,1	-0,0793	-0,0876	-0,0876
Bodø	-1,3	-1,4900	-1,3436	-1,3436
Hammerfest	0,9	0,6886	0,7366	0,7366
Harstad	-1	-0,9486	-0,9816	-0,9816
Heimsjø	-1,5	-1,4952	-1,4808	-1,4808
Honningsvåg	1,6	1,3832	1,3435	1,3435
Kabelvåg	-1,2	-1,0367	-1,0836	-1,0836
Kjølsdal		-0,6935	-0,7749	-0,7749
Kristiansund	-1	-0,8864	-0,9462	-0,9462
Måløy	0,6	0,5922	0,5964	0,5964
Narvik	-2,6	-2,1101	-2,1341	-2,1341
Oscarsborg	-1,8	-1,9664	-1,8625	-1,8625
Oslo	-3,7	-3,5639	-3,6431	-3,6431
Rørvik	-0,8	-1,0384	-1,0101	-1,0101
Stavanger	0,4	0,4501	0,4251	0,4251
Tregde	0,2	0,2630	0,2611	0,2611
Tromsø	0	0,0255	-0,0077	-0,0077
Trondheim1		-0,8993	-0,9762	-0,9762
Trondheim2		-0,6584	-0,8477	-0,8477

Table 5.1 shows similar values between the three methods and the results from Simpson et al. [2012]. This study used the least squares method to find the linear trend. Simpson et al. [2012] used maximum likelihood estimations. Table 5.1 show that there are small

differences between the three different variations in the least squared method and the results from Simpson et al. [2012].

The three methods show differences in sub millimetre. The results for l_2 and l_3 is equal for all the stations on the entire dataset. However, when the dataset where divided in 30 year periods (see Appendix C), the results where slightly different.

Figure 5.1 shows the plot of the Bergen dataset. The plot is not corrected for vertical land motion. The estimated trend from 1915 to 2012 is shown in dark blue and the 95 percent confidence interval is shown in light blue. The slope is -0.0876 , with a confidence interval on $-0.24:0.07$ (0.155 2-sigma) and the adjusted R^2 value (\bar{R}^2) is 0.02 percent. The normal distribution for Bergen can be found in Appendix A and the chi-square goodness-of-fit test with a 5 percent significance level stated that the data was normally distributed.

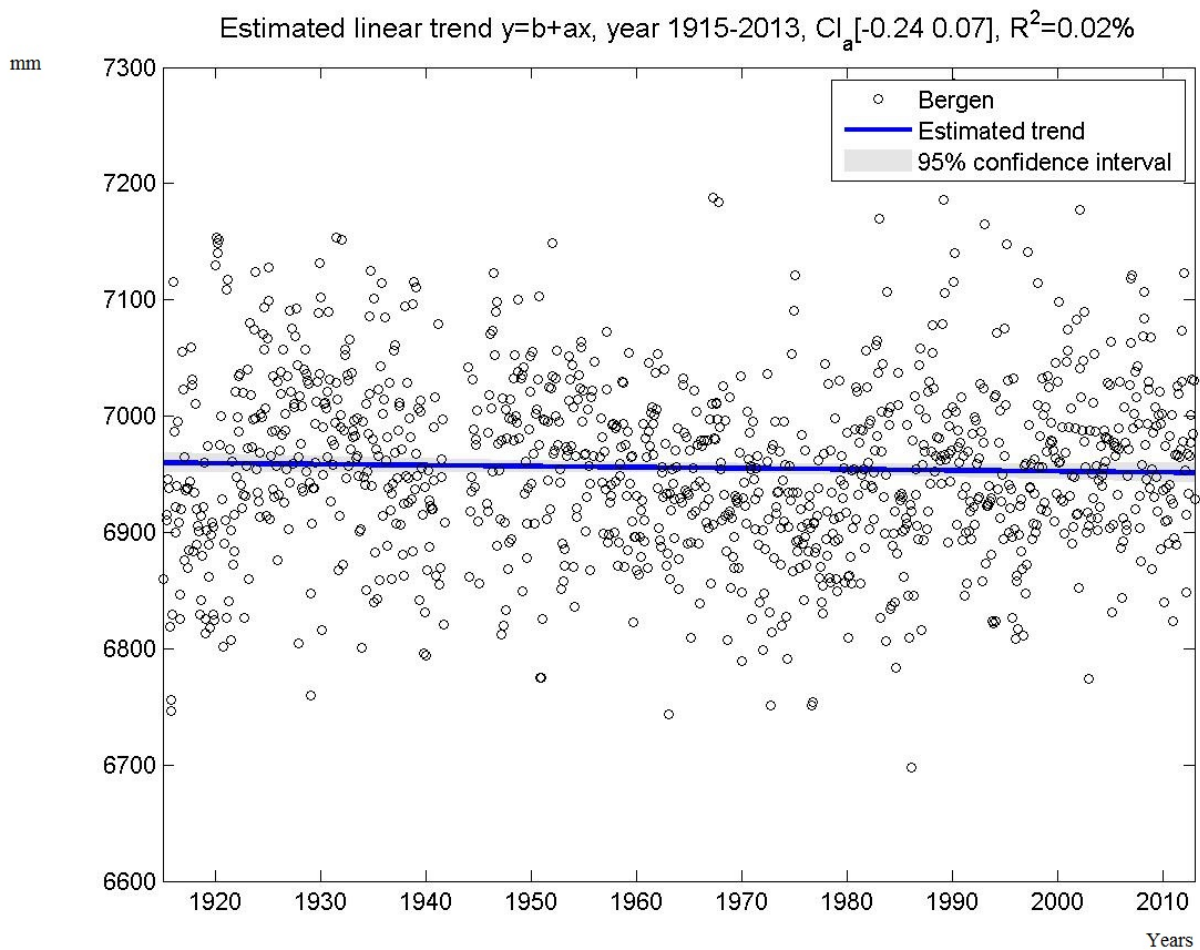


Figure 5.1: Bergen - Linear regression corrected for harmonic signals

The plot reveals the widely monthly data scatter. This is probably also why the \bar{R}^2 is only 0.02 percent, although the data amount is relatively large. Even though the wide data scatter, it is possible to see some non linear trend within the dataset. For instance there is more data density around 7000 mm around 1930, around 6900 mm around 1975 and somewhere between 7000 mm and 6900 mm in 2010. This is in agreement with the

slightly negative slope. The 95 percent confidence value is low compared to the wide data proliferation.

Table 5.2 shows the linear slope (l_3) with uncertainties (σ_h). The vertical land motion (VLM) and the accompanying uncertainties (σ_l) is included to show the results for the linear slope corrected for vertical land motion, and the new sigma values (σ_d).

Table 5.2: Linear slope corrected for vertical land motion

Station	l_3 (mm/y)	σ_h	VLM (mm/y)	σ_l	l_3 corrected for VLM (mm/y)	σ_d
Ålesund	0,86	0,17	1,67	0,19	2,53	0,25
Bergen	-0,09	0,08	1,49	0,32	1,40	0,33
Bodø	-1,34	0,20	3,02	0,31	1,68	0,37
Hammerfest	0,74	0,21	1,74	0,60	2,48	0,64
Harstad	-0,98	0,17	4,56	0,63	3,58	0,66
Heimsjø	-1,48	0,11	3,33	0,91	1,85	0,92
Honningsvåg	1,34	0,30	1,74	0,60	3,08	0,67
Kabelvåg	-1,08	0,18	2,61	1,14	1,53	1,15
Kjølsdal	-0,77	0,24	1,35	0,57	0,58	0,62
Kristiansund	-0,95	0,18	1,46	0,28	0,51	0,33
Måløy	0,60	0,14	1,35	0,57	1,95	0,59
Narvik	-2,13	0,14	4,56	0,63	2,43	0,65
Oscarsborg	-1,86	0,54	4,81	0,30	2,95	0,61
Oslo	-3,64	0,24	4,81	0,30	1,17	0,39
Rørvik	-1,01	0,30	3,00	0,78	1,98	0,83
Stavanger	0,43	0,08	1,25	0,33	1,68	0,34
Tregde	0,26	0,08	1,55	0,19	1,81	0,21
Tromsø	-0,01	0,17	2,74	0,41	2,73	0,45
Trondheim1	-0,98	0,37	3,93	0,33	2,95	0,49
Trondheim2	-0,85	0,76	3,93	0,33	3,08	0,83

After reducing for vertical land motion the slope of all stations is increasing, ranging between an increase of 0.51 mm/y to 3.58 mm/y. σ_d is between ± 0.21 and ± 1.15 . Kabelvåg has the largest uncertainty, with a value of ± 1.15 . This is mostly caused by the large uncertainty from the vertical land motion data.

Table 5.2 is plotted in figure 5.2, where both the linear slope (blue) and the linear slope corrected for vertical land motion (red) is included. All stations with the whole time series is included, and therefore is the start year of the station included in the x-axis label. The 1-sigma uncertainty is marked as a line attached to the slopes circle.

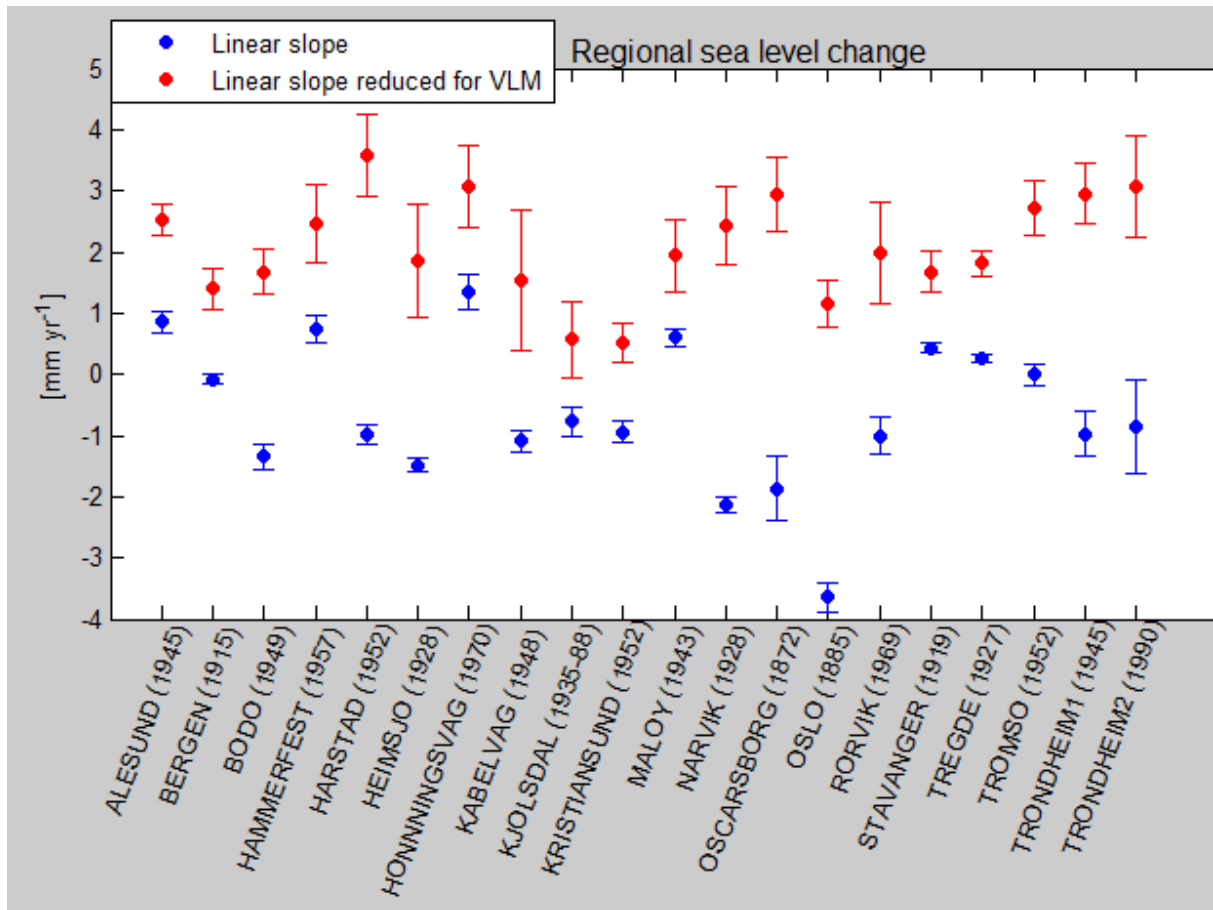


Figure 5.2: Regional sea level changes for all stations

Figure 5.2 visualizes table 5.2 and it is easier to see the difference before and after the reduction of vertical land movement. The stations in the south east and the middle part of Norway have the largest difference between relative rates and rates corrected for vertical land movement, witch figure 2.3 dictates.

The time range of 1980-2012 is shown in figure 5.3. Figure 5.2 and 5.3 is presented in the same way as the report of Simpson et al. [2012] (figure 3.7), except for the fully GIA-corrected data and the latitude sorted stations in Simpson et. al.. Figure 5.2 almost correlate with the results in Simpson et. al., as table 5.1 dictates. While the results of 1980-2012 is slightly different in some stations. For example, Narvik has a better uncertainty for the relative slope in this thesis compared to Simpson et. al., Tregde has the opposite.

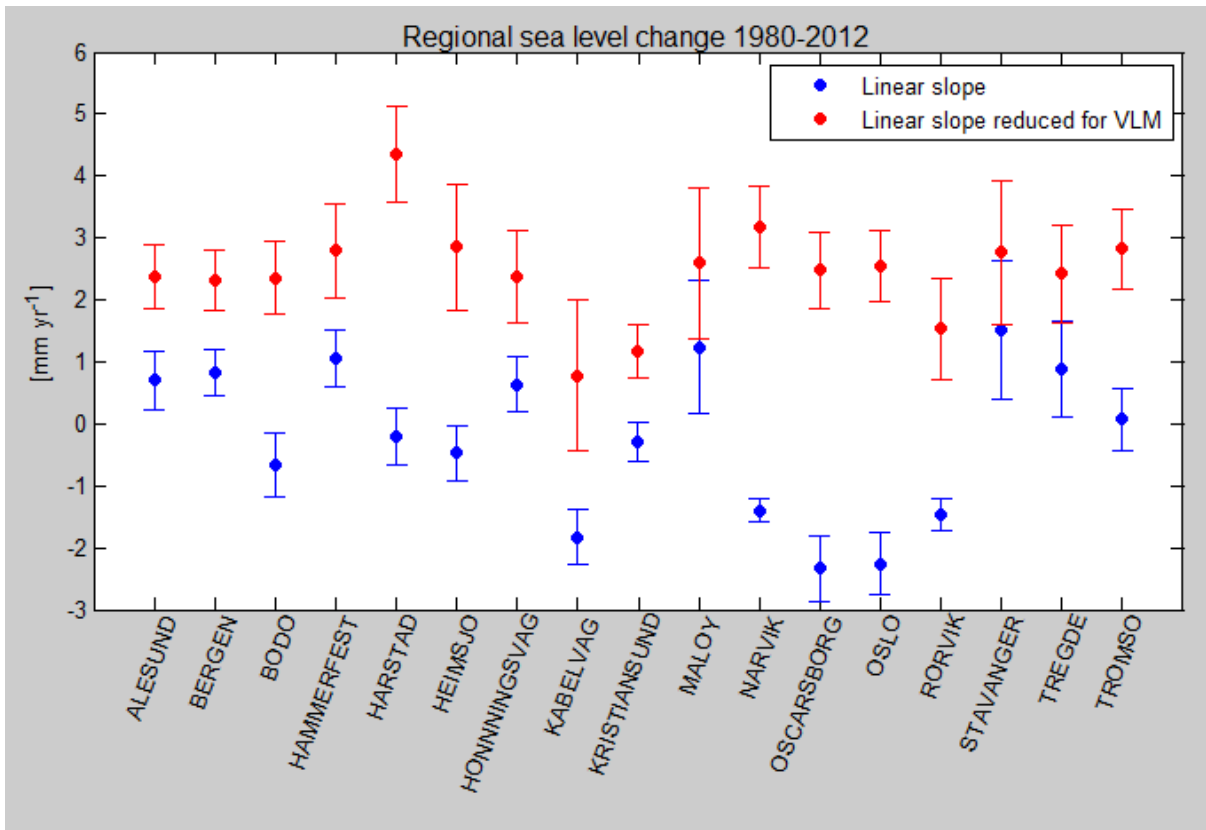


Figure 5.3: Regional sea level changes for 1980 to 2005

Sea Level Changes Over Time

The next section shows the results of the linear regression over time. The data is divided in 30 year intervals with a displacement of five year. All station data is divided in the same intervals, where the span depends on the data length of the stations. Bergen, Oslo and Narvik was chosen to represent the sea level changes over time, covering the west, south east and north of Norway. The rest of the stations are placed in Appendix C.

Figure 5.4, 5.5 and 5.6 display the changes in the linear slope over time for Bergen, Oslo and Narvik. The relative slope is marked as blue and the linear slope corrected for vertical land motion has the colour red.

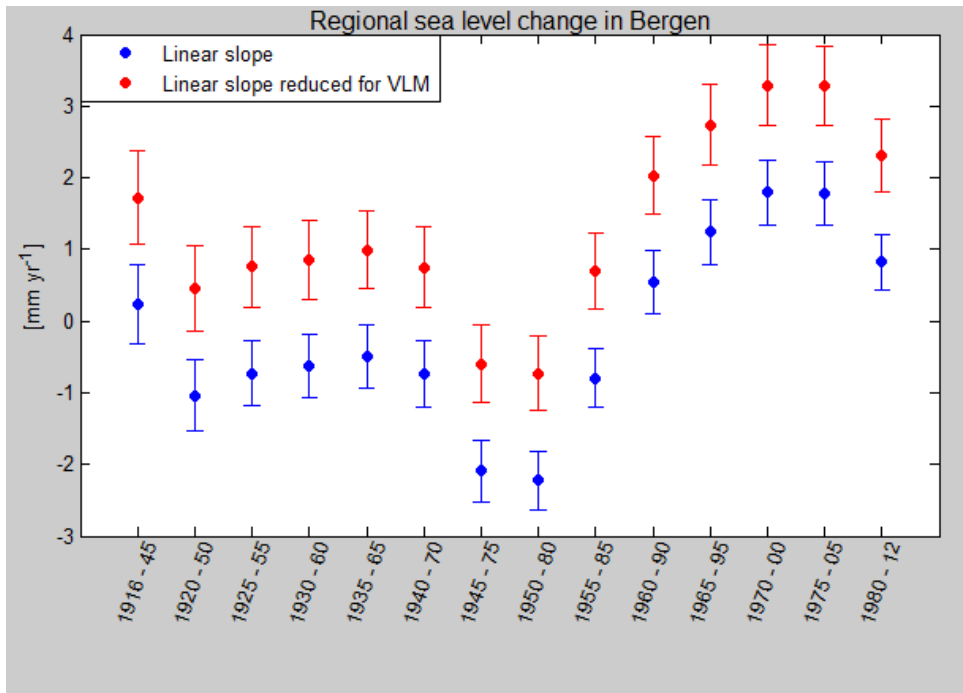


Figure 5.4: Sea level changes in Bergen in 30 year intervals

The relative slope range between -2.2 mm/y to 1.8 mm/y, and -0.7 mm/y to 3.3 mm/y for the linear slope corrected for vertical land motion in figure 5.4. The low point in the graph is around year 1945-75 and 1950-80. The peak of the graph lies around 1970-2000 and 1975-2005.

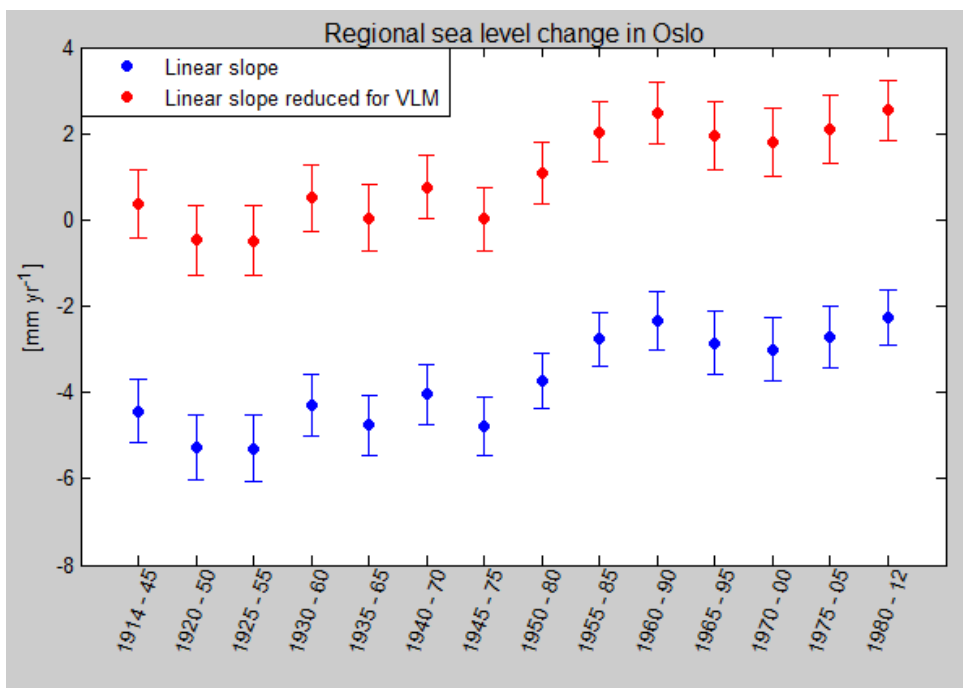


Figure 5.5: Sea level changes in Oslo in 30 year intervals

The relative slope ranges between -5.3 mm/y to -2.3 mm/y, and the linear slope corrected

for vertical land motion range -0.4 mm/y to 2.5 mm/y in figure 5.5. The low point in the graph us around year 1920-50, and the peak of the graph lies around 1980-2012.

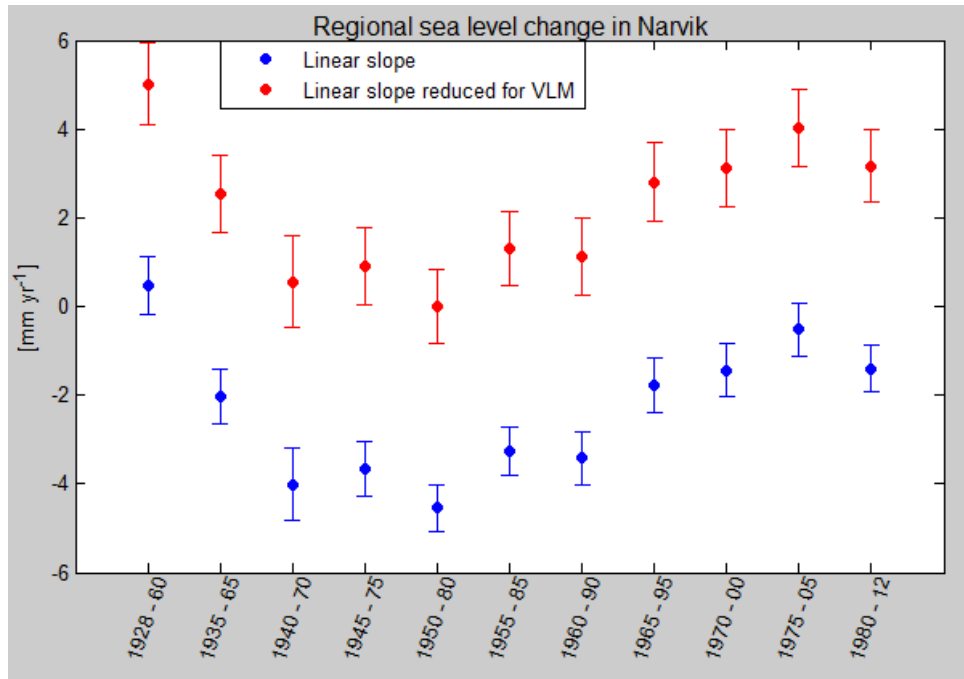


Figure 5.6: Sea level changes in Narvik in 30 year intervals

The relative slope range between -4.5 mm/y to 0.5 mm/y, and 0.0 mm/y to 5.0 mm/y for the linear slope corrected for vertical land motion in figure 5.6. The low point in the graph us around year 1950-80, and the peak of the graph lies around 1928-1960.

Figure 5.4, 5.5 and 5.6 are examples of the differences in the Norwegian tide gauge stations. Bergen has the most varying plot, with several lows and peaks. Oslo has a more overall increasing trend, while Narvik has a decreasing overall trend with it's highest point at the beginning (year 1928-60). All three stations has a increasing slope, with most variables within 2-4 mm/y in the last intervals.

The linear trend, for all stations are shown in figure 5.7. A larger print of the figure is placed in Appendix B. The rates are corrected for vertical land motion. The year is plotted as the middle year in that 30 year time interval. The global mean sea level (GMSL) data is also included (black line), indicating the global trend.

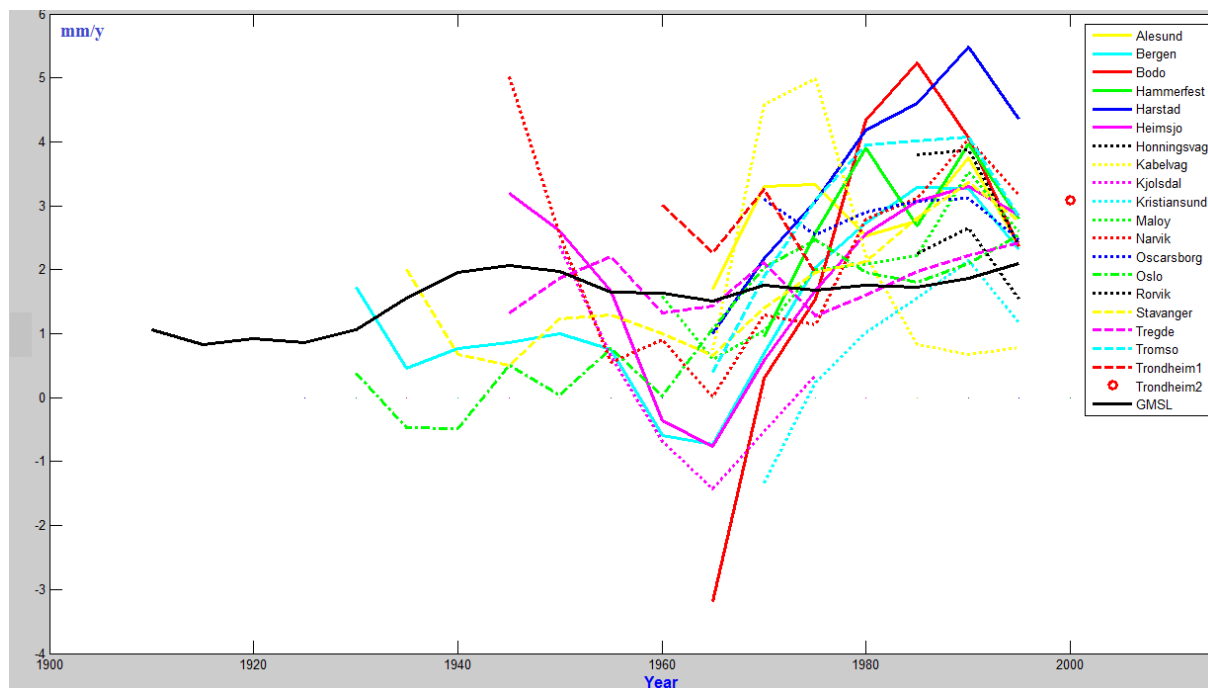


Figure 5.7: Linear trend for all station

Figure 5.7 shows a range of 9 mm/y for all the stations, most of the stations range around 5 mm/y. The curves do not show a straight increase in sea level trend, although the trends are increasing. For those stations having a data from before 1950, there is a trend that shows a small peak around 1930-1960 (presented as 1945 in table 5.7) and a decreasing trend in the interval 1945-75 (presented as 1960). All stations have an increasing slope, with a peak in the 30 year interval from 1975 to 2005, shown as the middle year 1990 in figure 5.7.

The slope decreases in all the stations for the time interval 1980 to 2012, shown as year 1995 in figure 5.7. A closer look at the last part of the figure is shown in figure 5.8, where the years from 1973 to 2012 is shown in 30 year intervals, with a displacement of one year. The decreasing trend is not as dramatic in figure 5.8 as in figure 5.7, but have a more rounded curve. Still, the trend is overall decreasing.

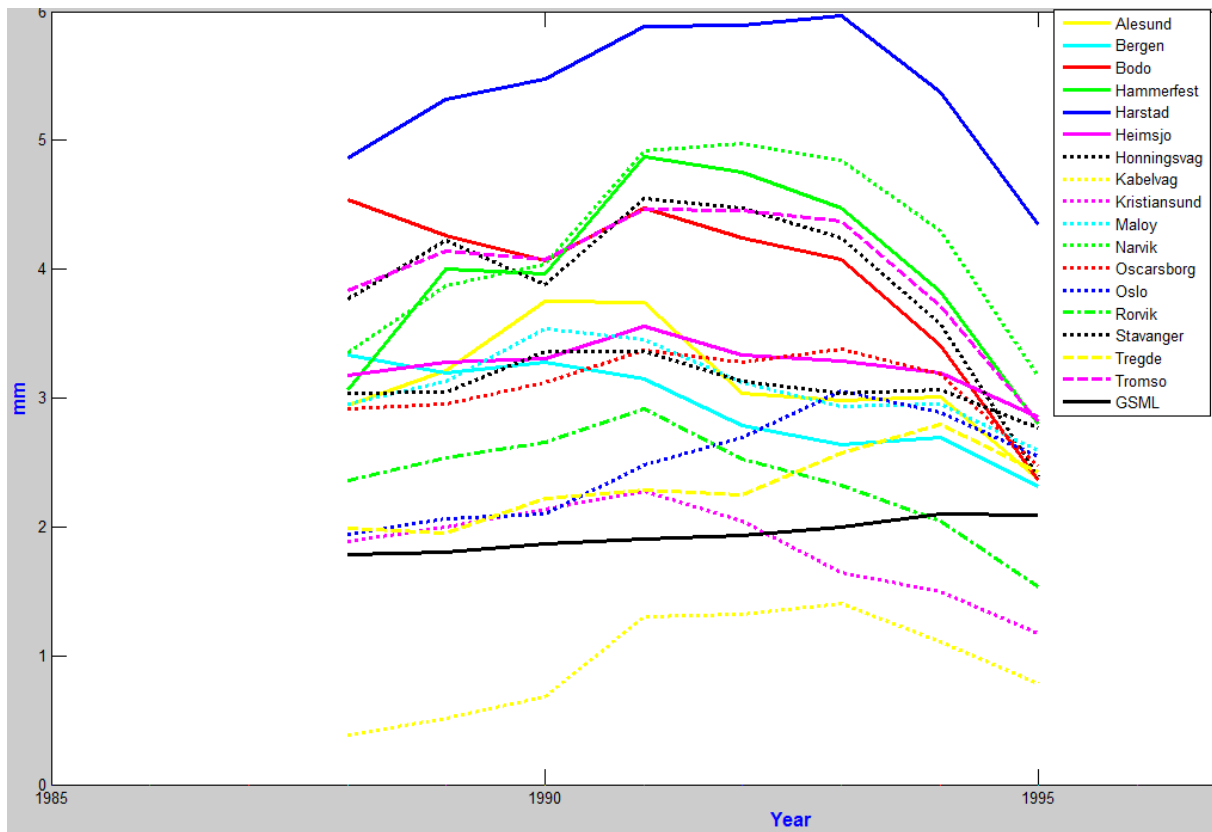


Figure 5.8: Linear trend all stations from 1973 to 2012

The global mean sea level data reveals a small slope increase in the same interval. This results differs from the hypothesis. The linear trend for the Norwegian tide gauge are not increasing over time, although the individual the slopes have an increase.

5.2 Significant Change in Linear Trend

Confidence interval

Figure 5.1, 5.5, 5.6 and 5.7 show a different linear trend for the time intervals 1950-1980 and 1980-2012. To test if the change in linear slope is significant, a confidence interval test between the two time intervals were made. Figure 5.9 plots the linear slope and 95 percent confidence interval uncertainty for the time intervals 1950-80 and 1980-2012 for stations with data from 1950-2012. A table of figure 5.9 are placed in Appendix D.

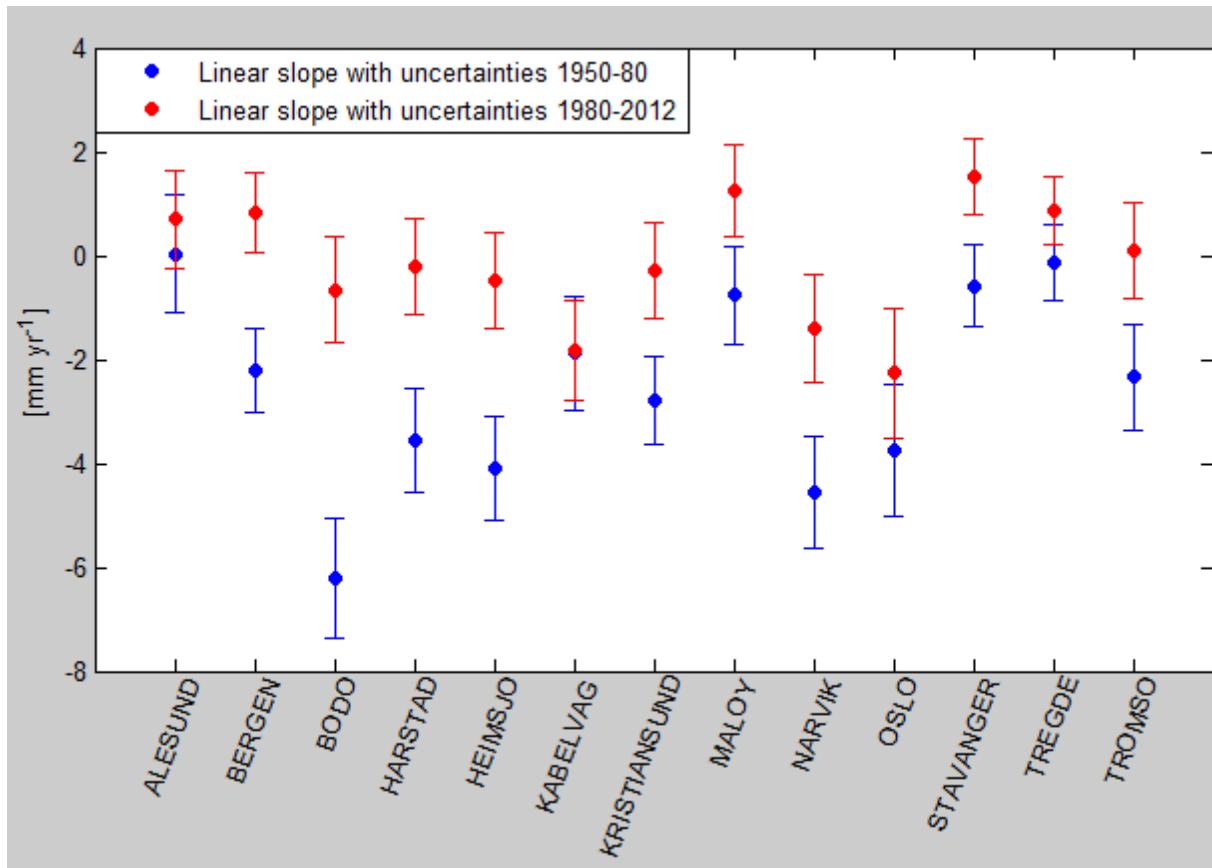


Figure 5.9: Linear trend 1950-80 and 1980-2012

Figure 5.9 shows that there are significant changes between the linear slope of 1950-80 and 1980-2010 for nine out of thirteen stations. Bergen, Bodø, Harstad, Heimsjø, Kristiansund, Måløy, Narvik, Stavanger and Tromsø have all a confidence interval that does not overlap between the two time intervals. In Ålesund, Kabelvåg, Oslo and Tregde the confidence intervals overlap and does therefore not have significant change in linear trend.

The linear trend in figure 5.7 show a decrease in the linear trend from 1975-2005 and 1980-2012. Figure 5.10 shows that this decrease is not significant, since the 95 percent confidence intervals do overlap for all stations.

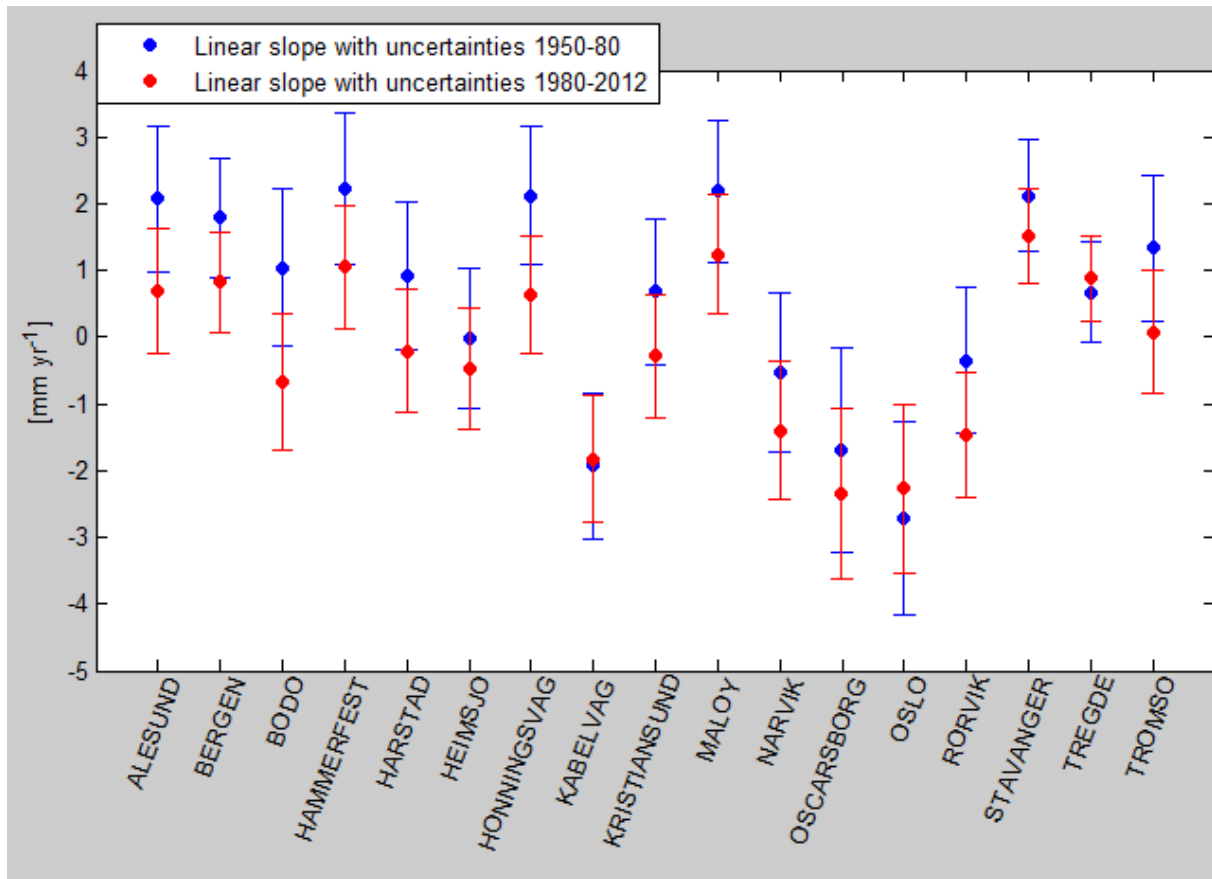


Figure 5.10: Linear trend 1975-2005 and 1980-2012

Chow-test

The Chow-test was run to see if there was a structural change in 1980 on stations with minimum 90 years of data. Figure 5.11 and 5.12 show the plot from the Chow-tests for the global mean sea level and Bergen. Oslo and Stavanger is placed in Appendix E.

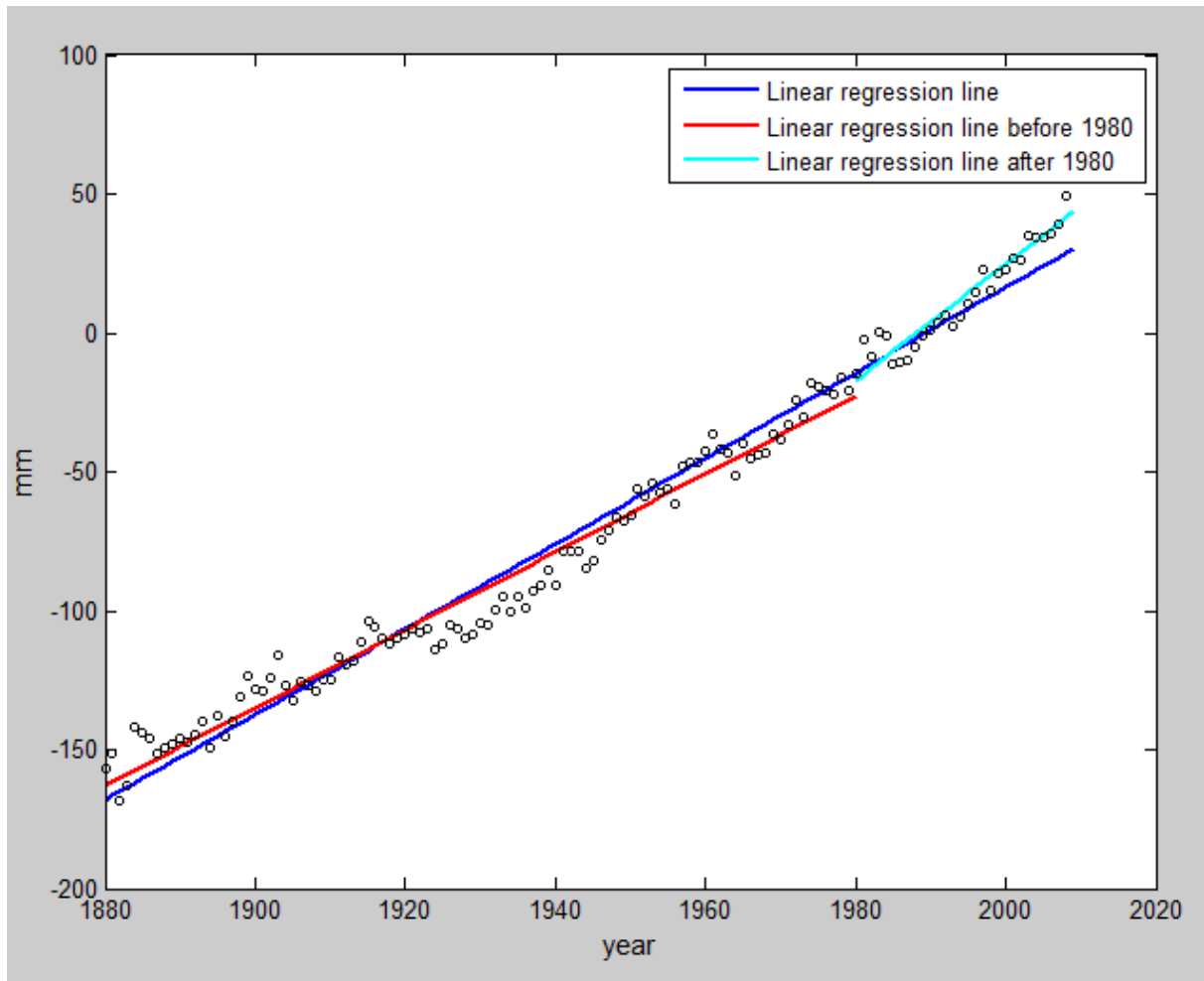


Figure 5.11: GMSL - Chow-test

The Chow-results for global mean sea level, Bergen, Oslo and Stavanger is displayed in table 5.3. All of the stations showed a significant structure change before and after 1980.

Table 5.3: GMSL - Chow-test results

Station	Interval	Intersect	Slope	CI	Ch	F
GMSL	Before 1980	-2800,28	1,40	0,05	34,342	3,068
	After 1980	-4162,70	2,09	0,25		
Bergen	Before 1980	7393,89	-0,23	0,30	9,986	3,004
	After 1980	3934,38	1,52	0,72		
Oslo	Before 1980	15795,72	-4,43	0,44	9,986	3,004
	After 1980	11529,87	-2,27	1,26		
Stavanger	Before 1980	7393,89	-0,23	0,30	15,358	3,005
	After 1980	3934,38	1,52	0,72		

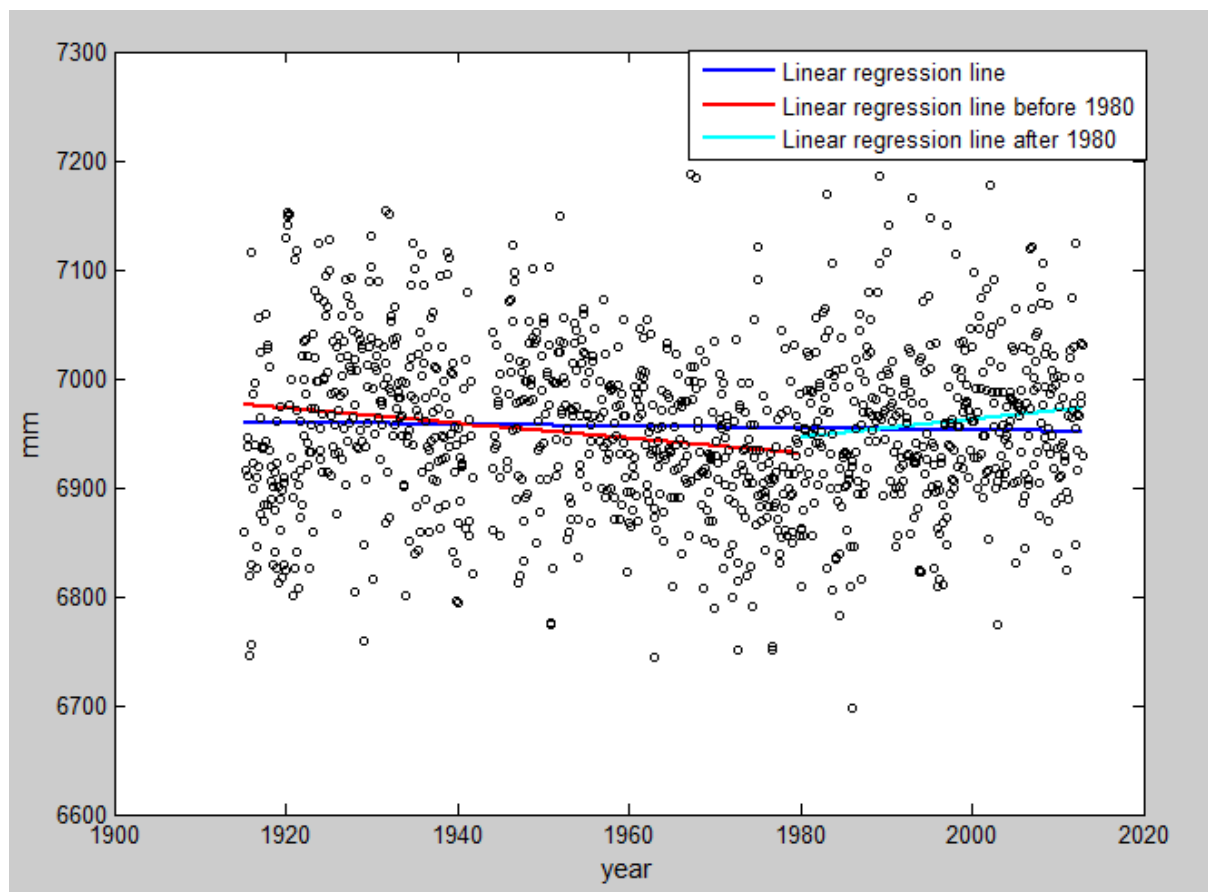


Figure 5.12: Bergen - Chow-test

5.3 Amplitude and Phase

In this section the amplitude and the accompanying phase, which was corrected from the observations, is presented.

The annual amplitude, from the harmonic equation for all stations in 30 year intervals, are shown in figure 5.13, and table 5.4 shows annual amplitude, sorted by the latitude.

The tide gauge data have values from 80 mm to 150 mm. This means that the annual variation in local sea level varies in that range. The values match previous amplitude analyses done by PSMSL [Pugh and Woodworth, 2014]. The amplitudes have a general small increase in time in most of the stations. Oslo and Oscarsborg have a decreasing amplitude from the interval 1955-65, shown as 1970 in the figure, to 1980-2012.

The annual phase accompanying the amplitude are presented in figure 5.14. The tide gauge data have values from September to December. This means that the annual peak of high waters is in autumn. The values match previous phase analyses done by PSMSL [Pugh and Woodworth, 2014]. The phase has a general small increase in time in most of the stations, but around 1980, the increase seems to be flattening or decrease.

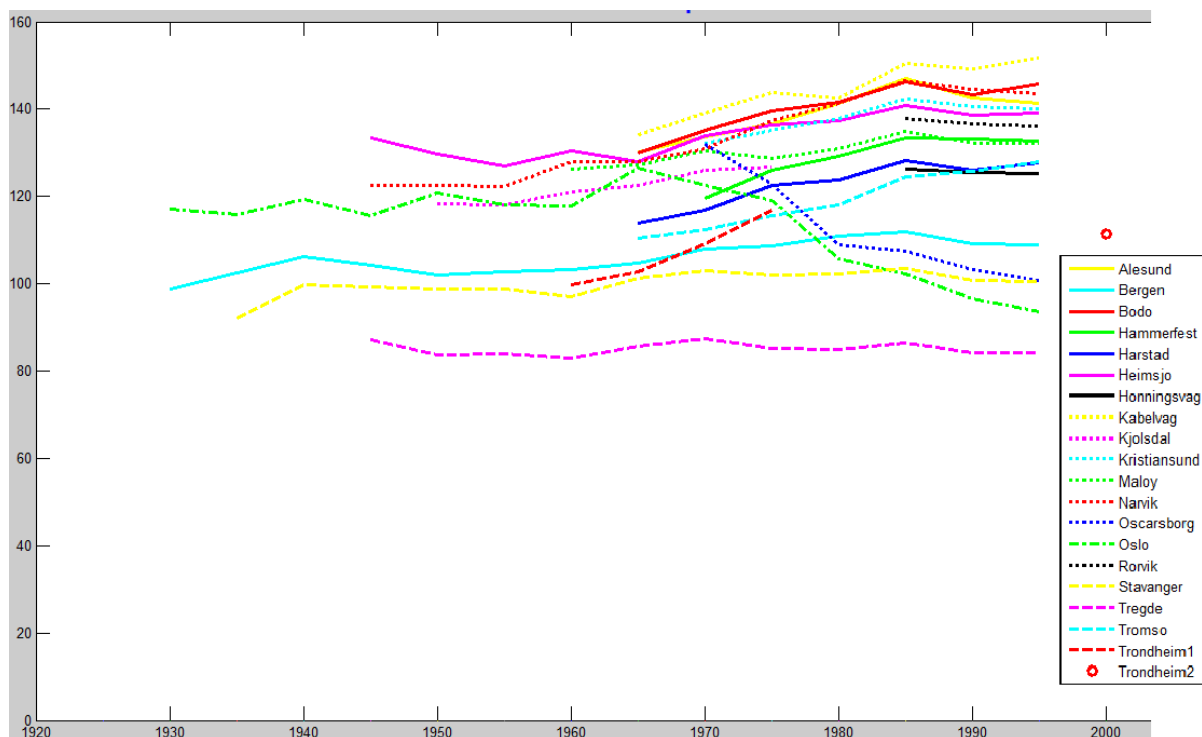


Figure 5.13: Amplitude of all stations

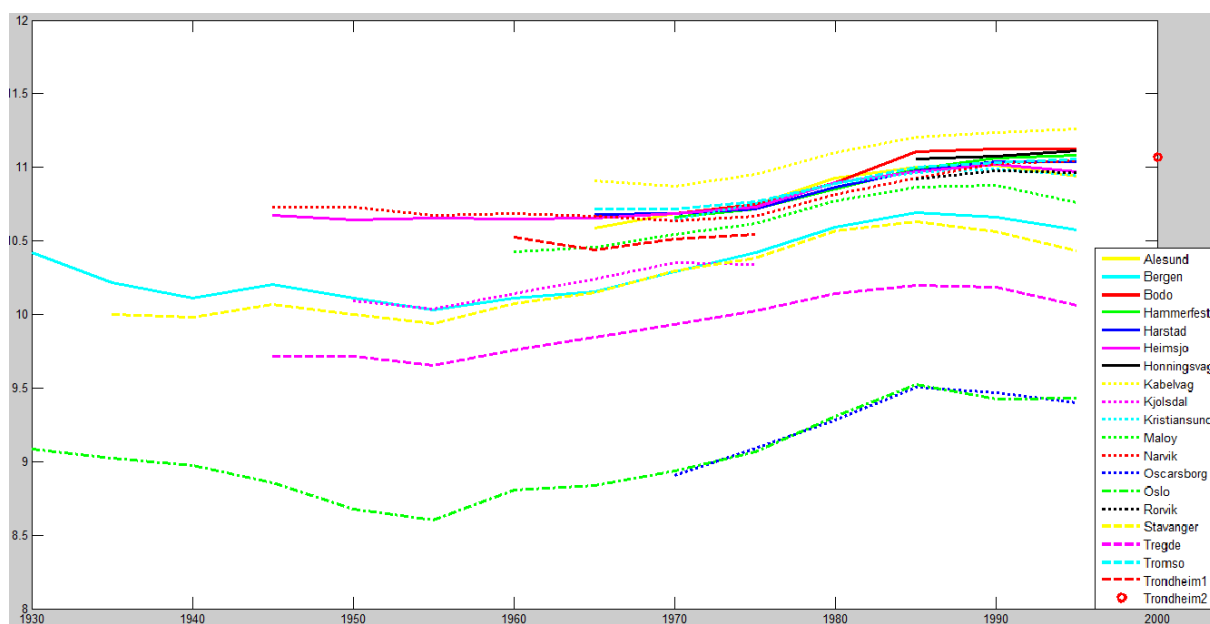


Figure 5.14: Phase of all stations

As shown in table 5.4, the amplitude decreases with the latitude. It also shows that the amplitude varies with locations. If the tide gauge is placed in a fjord, the amplitude is lower than if the tide gauge is placed near open sea. Kristiansund and Trondheim is approximately at the same latitude, but Kristiansund are placed near open sea, and Trondheim is inside a fjord. Kristiansund with larger amplitude than Trondheim.

Table 5.4: Amplitude - sorted by latitude

Station	Amplitude (mm)
Honningsvåg	124
Hammerfest	125
Tromsø	120
Harstad	120
Narvik	134
Kabelvåg	142
Bodø	138
Rørvik	134
Heimsjø	134
Trondheim1	111
Trondheim2	111
Kristiansund	135
Ålesund	135
Måløy	128
Kjølsdal	124
Bergen	104
Oslo	109
Oscarsborg	114
Stavanger	98
Tregde	84

Table 5.5: Phase - sorted by latitude

Station	Phase	Month
Honningsvåg	11,0	December
Hammerfest	10,9	November
Tromsø	10,9	November
Harstad	10,9	November
Narvik	10,8	November
Kabelvåg	11,1	December
Bodø	10,9	November
Rørvik	10,9	November
Heimsjø	10,8	November
Trondheim1	10,5	November
Trondheim2	11,1	December
Kristiansund	10,8	November
Ålesund	10,8	November
Måløy	10,6	November
Kjølsdal	10,2	November
Bergen	10,4	November
Oslo	9,1	September
Oscarsborg	9,0	September
Stavanger	10,2	November
Tregde	9,9	September

Table 5.5 shows annual phase, sorted by the latitude. The phase also decreases some with latitude, as shown in table 5.5.

6 — Discussion

6.1 Foundation

Data from long time series is a challenge, due to lack of documentation and the larger risk of errors. The tide gauges or supporting benchmarks may be broken, be moved, be influenced by outer factors such as accidents in construction work or fires, or upgraded to new reference levels. These effects may cause gaps in the data series or the dataset may be changed to fit the new reference levels. This may lead to errors in accuracy.

The tide gauge data from the end of the 19th century and the beginning of the 20th century were built with different preconditions of accuracy than the later built tide gauges [Pugh and Woodworth, 2014]. Since this study uses the Norwegian tide gauge data without further analysis of the foundation of the data, the earlier data may have a larger inaccuracy- than the data collected in recent years.

The magnitude of the annual sea level changes are in millimetres, but there is reason to question an accuracy in millimetres level for processed data from the tide gauge stations. In order to get an answer in mean sea level changes, all outer effects have to be corrected. This means waves, tidal variations, vertical land motions, annual changes in weather, pressure and salt variations. The position of the gauge also has to be accurate, or corrected for any outer changes and technical difficulties. All this summed make a lot of error sources, and a small accuracy in one or more of the error sources may cause large effects. In time series evaluations all the data are relative. This means that an exact accuracy is not necessary as long as the accuracy is the same throughout the dataset. This study only looks at trends and is not an accurate mean sea level study. However, this inaccuracy must still be considered when analysing the results in this thesis.

The tide gauge locations vary throughout the coast, from inside a fjord or near open sea. The weather can cause different impact on the local sea level inside a fjord, than near open sea. Inside a fjord wind and precipitation creates accumulations of water in heavy weather. This will impact the local sea level, and this variation from different tide gauge stations makes it not possible to directly compare the different stations, giving a mean sea level in Norway based on the tide gauge data. The stations measure local sea level changes, but not regional or global.

The continental shelf outside Norway covers most of the Norwegian sea and has a smaller sea depth than in open sea [Store Norske Leksikon, 2013]. Since the sea depth is lower on the continental shelf than in open sea, this may influence the sea level. In the north of

Norway, around Lofoten, the continental shelf has a smaller extensiveness from the main land than the rest of the coast. Kabelcåg and Harstad is placed in Lofoten and is the stations with a location nearest open sea.

6.2 Linear Analysis

The time series interval was set at 30 years, both as a minimum prerequisite for the station length and for the intervals used to find the linear slope over time. The exception was Trondheim2, witch was included to see the last years of Trondheim1 and to see if the uncertainty was different from the others. Trondheim1 and Trondheim2 could have been combined into one dataset to get the full station length. However, since there was no overlap between the datasets, the datasets where kept separate. The 95 percent confidence interval (2-sigma) of Trondheim2 was ± 1.52 from 1990-2012, witch was one of the largest uncertainties, but did not stand out from the rest of the uncertainties from the 30 year intervals. Oslo had for instance ± 1.48 in 1973-2003.

Table 6.1 show the linear slope (l_3) and the accompanying uncertainties, presented in Results and Analysis, divided in stations lengths. Dividing the station in station length, shows difference in uncertainties.

Table 6.1: Slope ranked by

Station length	Station	Station length (years)	l_3 (mm/y)	σ_h
>80	Oslo	98	-3,16	0,1
	Bergen	97	-0,09	0,08
	Stavanger	93	0,43	0,08
	Tregde	85	0,26	0,08
	Heimsjø	84	-1,48	0,11
	Narvik	84	-2,13	0,14
50-80	Måløy	69	0,6	0,14
	Ålesund	67	0,86	0,17
	Kabelvøg	64	-1,08	0,18
	Bodø	63	-1,34	0,2
	Harstad	60	-0,98	0,17
	Kristiansund	60	-0,95	0,18
	Tromsø	60	-0,01	0,17
	Oscarsborg	59	-1,86	0,54
	Hammerfest	55	0,74	0,21
	Kjølsdal	53	-0,77	0,24
30-50	Trondheim1	44	-0,98	0,37
	Rørvik	43	-1,01	0,3
	Honningsvåg	42	1,34	0,3
<30	Trondheim2	22	-0,85	0,76

The two sigma uncertainties is increasing with the station length. The calculated mean for the uncertainties is 0.1 for stations with over 80 years of data, 0.22 for 50-80 and 0.32 for stations with 30-50 years of data. Trondheim2 has an uncertainty of 0.76. This is as expected, since the longer the data intervals are the longer the results approach the true values of the constants.

Douglas [1992] states that tide gauge data with less than 60 year span are influenced by decadal variations. A prerequisite of minimum 60 years for the included stations may have been a possibility to improve the total all year rate for decadal variations. Yet, because of the relative short tide gauge data length, it is necessary to have a smaller interval than 60 years to find a linear trend in sea level. The 60 years may be more suitable if the studies that only include a study of the entire dataset, and not divided in intervals.

Ten of the GNSS stations used in the vertical land motion analysis does not correlate with the position of the tide gauge stations. Three tide gauge stations have a combination of two or three GNSS stations to find the vertical land motion parameter. This decreases the accuracy of the vertical land motions parameters. Specially where the vertical land motion differs in relative small areas, such as in the northern part of Norway, hence figure 2.3.

All the stations had years with incomplete or no data, some station with gaps as large as four years. The linear regression was run with the gaps, with two exception - the Oslo tide gauge data before 1914 and the Oscarsborg tide gauge data before 1954 was not included. This means that there is several years of missed information, and it may have influenced the linear regression. The gaps in the incomplete years could have been filled with an interpolation. In Calafat and Chambers [2013] a cubic spline interpolation have been used, and was used on gaps of one to two months. If the data had more than two months of missing data, the annual value was rejected in Calafat and Chambers [2013], whole years of incomplete data was not included This thesis operates with monthly values, not annual, and therefore a interpolation on the incomplete years may not be necessary as it evens out with the linear regression. The whole years of incomplete data may have caused errors in the dataset, especially if the missing data had extreme values.

A mean Norwegian sea level rate would have made it easier to compare with the global mean sea level trends. But, since the data have different foundation and preconditions, a Norwegian mean rate would have needed further analysis. Therefore was Bergen, Oslo and Narvik chosen to represent the east, west and north trends in Norway.

The Chow-test was used to find structural changes in the datasets with minimum 90 years of data. The challenge with the Chow-test is to find the break point within the dataset. Based on the previous results and to test the last 30 year interval against the rest of the dataset, 1980 was selected as break point. Other breakpoints

The Chow-test is originally used as a parameter in economic studies [Chow, 1960]. The Chow pre test showed that a 0.64 mm/year difference in two slopes in the global mean sea level dataset was a significant structural change, a difference of 0.04 mm/year was not. A difference in 0.04 mm/year is a relatively small difference between two slopes.

6.3 Other Sea Level Studies

The IPCC AR5 stated that the global average sea level rise was 1.7 [1.5 to 1.9] mm/y between 1901 and 2010 and 3.2 [2.8 to 3.6] mm/y between 1993 and 2010, calculated by tide gauge and altimetry data. The global rate was 1.5 mm/y \pm 0.04 in this thesis, based on data from Church and White [2011], which is in the same range as AR5. The 1980-2009 rate was 2.1 mm/y \pm 0.3, and is a bit lower than the 1993-2010 rate from AR5, which is reasonable considering the 1980's data and the altimetry data included in AR5.

The AR5 also stated that the global sea level rise from 1901 to 2010 (109 years) is 0.19 [0.17 to 0.21 m]. Church and White [2011] states that the global sea level rise from 1860 to 2010 (150 years) is about 0.21 m. The total sea level rise for a station can be calculated using the slope corrected for vertical land motion and the start point in the dataset to calculate the new intersect. The sea level rise for the longest datasets (98-85 years) in this study is presented in table 6.2. The results are slightly lower than the results in AR5, except Narvik, but the AR5 result also have several extra years. The sea level rise in AR5 and the result from this study are in the same range.

Table 6.2: Sea Level Rise

Station	Years	Sea level change (m)
Oslo	1914 - 2012	0.12 [0.08:0.15]
Bergen	1915 - 2012	0.14 [0.10:0.17]
Stavanger	1919 - 2012	0.16 [0.12:0.19]
Tregde	1927 - 2012	0.15 [0.14:1.17]
Heimsj	1928 - 2012	0.16 [0.08:0.23]
Narvik	1928 - 2012	0.20 [0.15:0.26]

The global mean sea level calculated from the data of Church and White [2011] shows a linear rate of 1.54 \pm 0.04 mm per year for year 1880-2009 and 1.63 \pm 0.05 mm per year for year 1900-2009. Church and White [2011] state that the linear trend from 1900 to 2009 is 1.7 \pm 0.2 mm per year and is in the same range as the results in this study.

The linear slopes not corrected for vertical land motion presented in figure 5.2 corresponds with the study done by PSMSL, visualized in figure 3.5.

6.4 Further Work

This study is limited to linear trends based on data pre correlated for tides, and the analysis only corrects for annual periodic variations and vertical land motion. Further work may include a study of accelerating trends on the Norwegian tide gauges. Calafat and Chambers [2013] found in their study on global tide gauge data that the acceleration is significant increasing between 1952 to 2011.

The next step is a study of how atmospheric pressure, temperature and salinity affects the sea level in Norway as demonstrated in Richter et al. [2012] and the possibility to correct for these effects. Also, see if it is possible to use shorter time series interval (< 30 years) based on a less influenced dataset.

Another study is to look closer at the foundation of the tide gauges. For example the levelling and GNSS network combined with the tide gauges, and the reference levels accompanying the tide gauges. A closer look at the tide reductions may also be relevant.

Including altimetry data for Norway, may be a way to find a better explanation for the curve in figure 5.8. Combining tide gauge data and altimetry data, also makes it possible for a study of future sea level trends in Norway, such as the study of Simpson et al. [2014].

A further analysis on using the Chow-test is necessary to find out if the difference between the chow-value and F-distribution is appropriate to determine structural changes in time series analysis on tide gauge data.

7 — Conclusion

In this master study, a time series analysis of tide gauge data from the Norwegian coast is carried out, to determine trends in past sea level changes in Norway and to compare with global mean sea level trends. The analysis is based on linear trends corrected for vertical land motion and annual periodic variations for monthly data from 21 tide gauge stations.

The sea level is increasing throughout the Norwegian coast. In Oslo, the sea level increased with 1.17 ± 0.39 mm per year, with a total sea level rise of 0.12 m [0.08:0.15 m] from 1914 to 2012. In Bergen, the sea level increased with 1.40 ± 0.33 mm per year, with a total sea level rise of 0.14 m [0.10:0.17 m] from 1915 to 2012. This is in the same range as several global sea level studies.

The Norwegian sea level trend over time is partly increasing. Each stations datasets were divided in 30 year intervals with five years steps in time throughout the dataset. In nine out of thirteen stations, the linear slope from the time interval 1950-1980 is significantly larger than the linear slope for the time interval 1980-2012. The linear trend also shows a small decrease in linear trend between the two time intervals 1975-2005 and 1980-2012. However, the 95 percent confidence intervals for all stations show that the decrease is not significant.

The Norwegian values are in the same range as the mean global sea level values, although the Norwegian stations have a larger data variations. The global mean level values are corrected for changes in temperature and inverse barometer effect, in addition to vertical land motion and tide variations.

A Chow-test was applied to find significant structural changes before and after 1980 in the linear slope for stations with minimum 90 years of data. The global mean sea level data, and the stations: Bergen, Oslo and Stavanger, all showed a significant structural change before and after 1980, according to the Chow-test.

The annual amplitude vary from 80 to 150 mm for all stations and the phase results show a peak in high waters around September to Desember. The amplitude and phase decreases with latitude.

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A — Normal Distribution

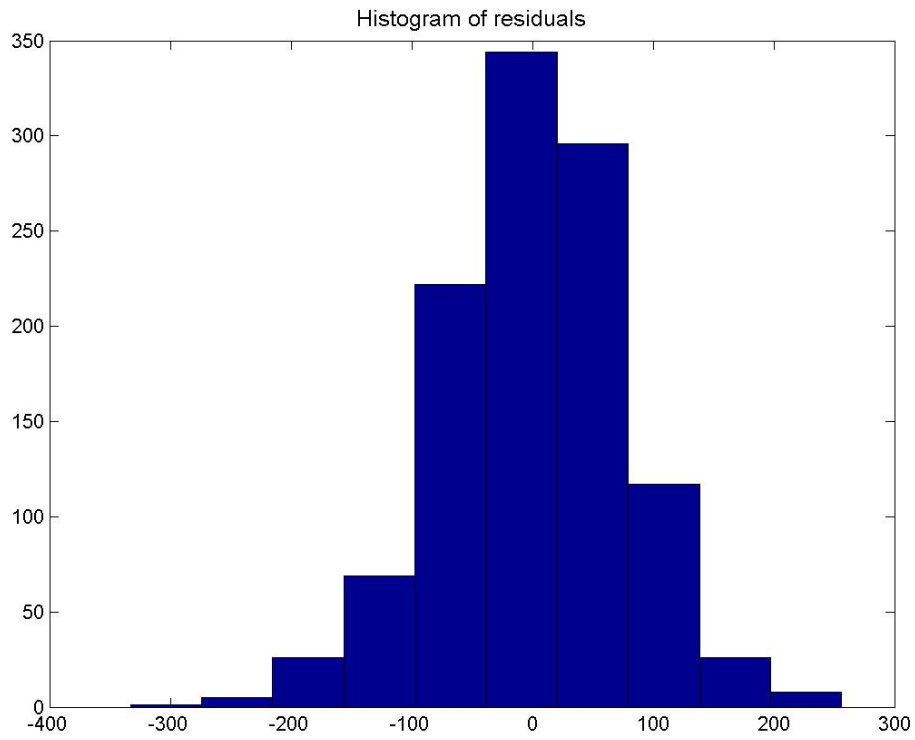
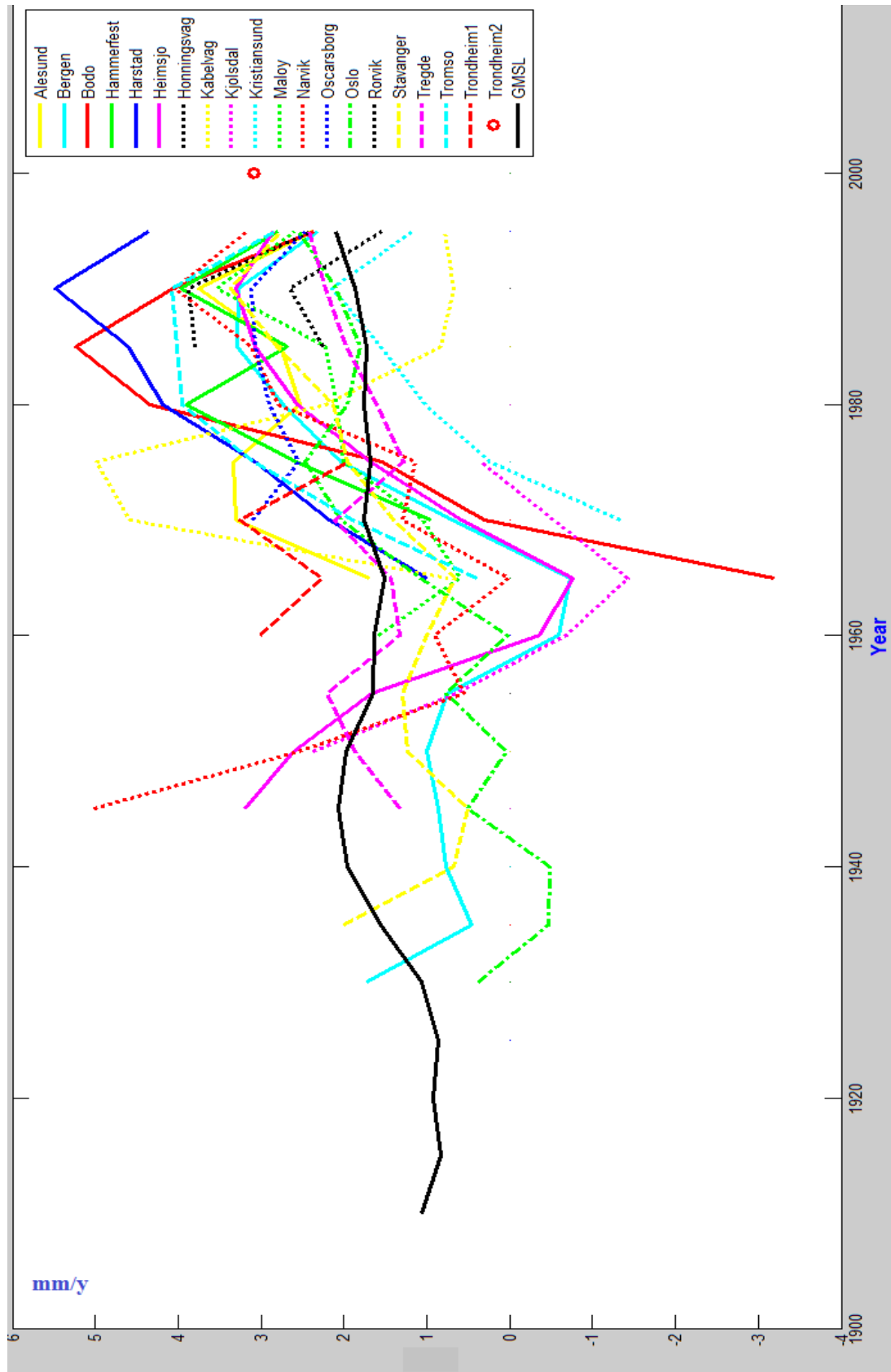


Figure A.1: Normal distribution of Bergen data

B — Figure 5.7



C — Linear Trend

Table C.1: Linear trend all stations

Ålesund

Year	l_3	95 % confidence interval
1951 - 2012	0,857720769	0,34008345
1951 - 1980	0,027225503	1,13337665
1955 - 1985	1,632909834	1,0562807
1960 - 1990	1,664499006	1,09093721
1965 - 1995	0,865069738	1,10356345
1970 - 2000	1,095246044	1,08287593
1973 - 2003	1,27419409	1,11971449
1974 - 2004	1,539372811	1,11993413
1975 - 2005	2,078852567	1,09290708
1976 - 2006	2,074285464	1,08351783
1977 - 2007	1,370954539	1,07073898
1978 - 2008	1,308607796	1,07318868
1979 - 2009	1,336628786	1,08080156
1980 - 2012	0,703478787	0,93729031

Bergen

Year	l_3	95 % confidence interval
1916-2012	-0,08760265	0,15475337
1916 - 1945	0,2328692	1,11800844
1920 - 1950	-1,03512293	0,99338498
1925 - 1955	-0,72570948	0,9130721
1930 - 1960	-0,63342179	0,88058707
1935 - 1965	-0,49416999	0,87573054
1940 - 1970	-0,73606161	0,93071088
1945 - 1975	-2,08828029	0,85939912
1950 - 1980	-2,22256289	0,80457573
1955 - 1985	-0,79700473	0,82530176
1960 - 1990	0,54245945	0,86565374
1965 - 1995	1,24878061	0,90640014
1970 - 2000	1,79765655	0,90002996
1973 - 2003	1,84607348	0,90451222
1974 - 2004	1,70414358	0,90484622
1975 - 2005	1,78812927	0,88877532
1976 - 2006	1,65402709	0,88575334
1977 - 2007	1,29364011	0,8847366
1978 - 2008	1,15233566	0,88456217
1979 - 2009	1,20413879	0,89084065
1980 - 2012	0,8228453	0,76104708

Bodø

Year	l_3	95 % confidence interval
1949 - 2012	-1,343643752	0,40602137
1949 - 1980	-6,214965548	1,15637295
1955 - 1985	-2,722558544	1,08951124
1960 - 1990	-1,483244079	1,23431817
1965 - 1995	1,325185501	1,37586728
1970 - 2000	2,222597129	1,34532958
1973 - 2003	1,515263603	1,24266136
1974 - 2004	1,242009854	1,1877893
1975 - 2005	1,045967701	1,17310822
1976 - 2006	1,457515511	1,16641768
1977 - 2007	1,22379935	1,17364334
1978 - 2008	1,05898267	1,17393741
1979 - 2009	0,3895024	1,1678603
1980 - 2012	-0,663986262	1,02023727

Hammerfest

Year	l_3	95 % confidence interval
1957 - 2012	0,736594934	0,4241712
1957 - 1985	-0,787651015	1,24116203
1960 - 1990	0,833690966	1,15417959
1965 - 1995	2,1726975	1,28322844
1970 - 2000	0,946820496	1,19639645
1973 - 2003	1,32101257	1,14500129
1974 - 2004	2,265363957	1,133349
1975 - 2005	2,220437545	1,13547251
1976 - 2006	3,134400278	1,11841046
1977 - 2007	3,01477716	1,05526167
1978 - 2008	2,738122577	1,05447051
1979 - 2009	2,088559114	1,04730765
1980 - 2012	1,05314182	0,9160733

Harstad

Year	l_3	95 % confidence interval
1952 - 2012	-0,98156091	0,3403498
1952 - 1980	-3,55481565	1,01301984
1955 - 1985	-2,37921817	0,93708077
1960 - 1990	-1,48695301	1,05025488
1965 - 1995	-0,37582163	1,13921848
1970 - 2000	0,04624385	1,08202917
1973 - 2003	0,30626974	1,10736193
1974 - 2004	0,75617561	1,10494324
1975 - 2005	0,91845927	1,09595889
1976 - 2006	1,32292581	1,08137491
1977 - 2007	1,33634381	1,06886381
1978 - 2008	1,4081067	1,07471503
1979 - 2009	0,81067454	1,06656352
1980 - 2012	-0,21257473	0,91957062

Heimsjø

Year	l_3	95 % confidence interval
1928 - 2012	-1,48081157	0,22292377
1928 - 1960	-0,13692356	0,975029
1935 - 1965	-0,7164935	1,03596407
1940 - 1970	-1,67130425	1,04000732
1945 - 1975	-3,69173847	1,01307577
1950 - 1980	-4,09283622	0,98623853
1955 - 1985	-2,74244635	0,98248192
1960 - 1990	-1,64459051	1,03435015
1965 - 1995	-0,76527881	1,10141379
1970 - 2000	-0,26620792	1,09112097
1973 - 2003	-0,14930024	1,1061918
1974 - 2004	-0,05093662	1,09174733
1975 - 2005	-0,02129138	1,06019513
1976 - 2006	0,22664877	1,06074969
1977 - 2007	0,00662803	1,06052755
1978 - 2008	-0,03830396	1,05673904
1979 - 2009	-0,13077181	1,05384801
1980 - 2012	-0,47744575	0,91099837

Honningsvåg

Year	l_3	95 % confidence interval
1972 - 2012	1,34352	0,59479608
1972 - 2000	2,05949	1,17146584
1973 - 2003	2,00428	1,04644486
1974 - 2004	2,46608	1,03918904
1975 - 2005	2,12484	1,02782574
1976 - 2006	2,80358	1,01167372
1977 - 2007	2,72966	1,01407543
1978 - 2008	2,49376	1,01224535
1979 - 2009	1,82209	0,99983028
1980 - 2012	0,63672	0,87308564

Kabelvåg

Year	l_3	95 % confidence interval
1948 - 2012	-1,083620557	0,35825572
1948 - 1980	-1,875543441	1,08760327
1955 - 1985	1,968919482	1,1228882
1960 - 1990	2,37064803	1,21256405
1965 - 1995	-0,394156192	1,1877755
1970 - 2000	-1,787299284	1,13856291
1973 - 2003	-2,222702432	1,11732461
1974 - 2004	-2,09277625	1,11720716
1975 - 2005	-1,93277191	1,09072982
1976 - 2006	-1,302937776	1,08960209
1977 - 2007	-1,28547522	1,09420028
1978 - 2008	-1,209676105	1,0968549
1979 - 2009	-1,498511401	1,09503403
1980 - 2012	-1,830144792	0,95143235

Kjølsdal

Year	l_3	95 % confidence interval
1935 - 1988	-0,774909748	0,48507415
1935 - 1965	1,019523367	1,27263295
1940 - 1970	-0,710670213	1,35183479
1945 - 1975	-2,030805458	1,10437302
1950 - 1980	-2,773911918	1,03931038
1955 - 1985	-1,881162625	1,15533007
1960 - 1988	-1,003626205	1,24040396

Kristiansund

Year	l_3	95 % confidence interval
1952 - 2012	-0,94623117	0,35616116
1952 - 1985	-2,80073052	0,84725175
1960 - 1990	-1,21082696	1,04933567
1965 - 1995	-0,44774227	1,09978551
1970 - 2000	0,09926882	1,09677678
1973 - 2003	0,42967142	1,11811373
1974 - 2004	0,53915527	1,11708846
1975 - 2005	0,67836994	1,09661541
1976 - 2006	0,81988373	1,09141077
1977 - 2007	0,58056753	1,09136759
1978 - 2008	0,18728976	1,0729286
1979 - 2009	0,0346307	1,07145959
1980 - 2012	-0,2831212	0,92352229

Måløy

Year	l_3	95 % confidence interval
1943 - 2012	0,59641331	0,28279244
1943 - 1975	0,23831888	0,85336499
1950 - 1980	-0,75938504	0,94713032
1955 - 1985	-0,30170359	0,99043604
1960 - 1990	0,64801376	1,03920894
1965 - 1995	0,72968176	1,03699406
1970 - 2000	0,86410131	1,04614702
1973 - 2003	1,60031747	1,08491521
1974 - 2004	1,78187531	1,08853454
1975 - 2005	2,19049926	1,07052324
1976 - 2006	2,10724431	1,05710153
1977 - 2007	1,77218611	1,05699992
1978 - 2008	1,58177991	1,06391136
1979 - 2009	1,60184973	1,05949387
1980 - 2012	1,24126381	0,8936573

Narvik

Year	l_3	95 % confidence interval
1928 - 2012	-2,13408354	0,28476405
1928 - 1960	0,45851948	1,3109873
1935 - 1965	-2,01812257	1,22226973
1940 - 1970	-4,00618658	1,61521614
1945 - 1975	-3,6523009	1,22873387
1950 - 1980	-4,54848568	1,07115473
1955 - 1985	-3,26379611	1,08189429
1960 - 1990	-3,42593028	1,17820551
1965 - 1995	-1,76258038	1,22887779
1970 - 2000	-1,43734425	1,20210873
1973 - 2003	-1,21190689	1,2159591
1974 - 2004	-0,68390492	1,20338578
1975 - 2005	-0,52229836	1,18949124
1976 - 2006	0,35827984	1,17006791
1977 - 2007	0,41642129	1,17700277
1978 - 2008	0,28682941	1,17509136
1979 - 2009	-0,25964175	1,17560901
1980 - 2012	-1,39606813	1,0318227

Oscarsborg

Year	l_3	95 % confidence interval
1953 - 2012	-1,86246736	0,53638002
1953 - 1985	-1,71321053	1,27987568
1960 - 1990	-2,26447354	1,46582665
1965 - 1995	-1,91044899	1,60099694
1970 - 2000	-1,7386105	1,58862919
1973 - 2003	-1,8934429	1,55151066
1974 - 2004	-1,85413802	1,55638786
1975 - 2005	-1,69013106	1,53967695
1976 - 2006	-1,44188296	1,54560715
1977 - 2007	-1,53114739	1,53314185
1978 - 2008	-1,42705438	1,50107434
1979 - 2009	-1,62371394	1,5032963
1980 - 2012	-2,33406953	1,27592208

Oslo

Year	l_3	95 % confidence interval
1953 - 2012	-3,15553494	0,20637587
1914 - 2012	-3,64270658	0,24197883
1916 - 1945	-4,43472312	1,48374151
1920 - 1950	-5,27761926	1,50504142
1925 - 1955	-5,29435183	1,51327931
1930 - 1960	-4,30277336	1,43325801
1935 - 1965	-4,76250718	1,40554624
1940 - 1970	-4,04768473	1,36134729
1945 - 1975	-4,78619507	1,33532177
1950 - 1980	-3,73113741	1,26901209
1955 - 1985	-2,76759322	1,25933099
1960 - 1990	-2,33705956	1,32128311
1965 - 1995	-2,850414	1,47582706
1970 - 2000	-3,0005314	1,47775429
1973 - 2003	-2,86879647	1,48488782
1974 - 2004	-2,74612447	1,48349239
1975 - 2005	-2,71211665	1,43740325
1976 - 2006	-2,32915467	1,43517056
1977 - 2007	-2,11899902	1,45772684
1978 - 2008	-1,75295042	1,46522066
1979 - 2009	-1,92074427	1,46334844
1980 - 2012	-2,26583001	1,26059339

Rørvik

Year	l_3	95 % confidence interval
1969 - 2012	-1,01013298	0,60399474
1969 - 2000	-0,74894701	1,06542257
1973 - 2003	-0,64020915	1,11207867
1974 - 2004	-0,46613194	1,10841851
1975 - 2005	-0,34553946	1,09448489
1976 - 2006	-0,08272496	1,09379441
1977 - 2007	-0,47628583	1,09228811
1978 - 2008	-0,67894079	1,08219421
1979 - 2009	-0,95554377	1,07682258
1980 - 2012	-1,46256341	0,93467256

Stavanger

Year	l_3	95 % confidence interval
1919 - 2012	0,425083763	0,15919077
1919 - 1950	0,749554244	1,01351087
1925 - 1955	-0,571880177	0,9230566
1930 - 1960	-0,737624336	0,87166939
1935 - 1965	-0,025163286	0,9033409
1940 - 1970	0,039210274	1,13490917
1945 - 1975	-0,247625111	0,90276122
1950 - 1980	-0,589454146	0,78867514
1955 - 1985	0,149729131	0,78485495
1960 - 1990	0,702713162	0,81532475
1965 - 1995	0,872127808	0,87411871
1970 - 2000	1,557062163	0,90908594
1973 - 2003	1,785375709	0,85989107
1974 - 2004	1,800115113	0,8600677
1975 - 2005	2,11551008	0,83566011
1976 - 2006	2,115273618	0,83425386
1977 - 2007	1,878688519	0,83304733
1978 - 2008	1,789418843	0,83194896
1979 - 2009	1,811091133	0,83900703
1980 - 2012	1,518625296	0,7208109

Tregde

Year	l_3	95 % confidence interval
1927 - 2012	0,26109472	0,15569943
1927 - 1960	-0,23068377	0,67341131
1935 - 1965	0,31892282	0,74449503
1940 - 1970	0,66061498	0,78038355
1945 - 1975	-0,22701932	0,74371428
1950 - 1980	-0,12309038	0,7314867
1955 - 1985	0,55981324	0,75844028
1960 - 1990	-0,27864654	0,74596659
1965 - 1995	0,05422552	0,78422786
1970 - 2000	0,42549613	0,75446108
1973 - 2003	0,44052035	0,76466693
1974 - 2004	0,40398197	0,75806954
1975 - 2005	0,67094093	0,74765426
1976 - 2006	0,73953145	0,74244053
1977 - 2007	0,6952955	0,75008727
1978 - 2008	1,02101049	0,75268611
1979 - 2009	1,24223347	0,76169659
1980 - 2012	0,87880712	0,65087655

Tromsø

Year	l_3	95 % confidence interval
1952 - 2012	-0,00774299	0,34713326
1952 - 1980	-2,34372513	1,02152605
1955 - 1985	-0,85113305	0,90265934
1960 - 1990	0,35114539	1,0620391
1965 - 1995	1,21600485	1,09788926
1970 - 2000	1,27420632	1,07235372
1973 - 2003	1,09615285	1,10115012
1974 - 2004	1,40472614	1,09617818
1975 - 2005	1,33287647	1,08735956
1976 - 2006	1,72573963	1,08740817
1977 - 2007	1,7194275	1,09335625
1978 - 2008	1,63665679	1,0972086
1979 - 2009	0,97428466	1,0870673
1980 - 2012	0,0785609	0,92376742

Trondheim1

Year	l_3	95 % confidence interval
1948 - 1989	-0,89926274	0,93191611
1945 - 1975	-0,92698569	1,57622832
1950 - 1980	-1,66147289	1,40321543
1955 - 1985	-0,67963639	1,53844653
1960 - 1989	-1,99326061	1,6375745

Trondheim2

Year	l_3	95 % confidence interval
1990 - 2012	-0,84771279	1,52113201

D — Linear trend 1950-80 and 1980-2012

Table D.1: Linear trend 1950-80 and 1980-2012

Station	l_3 1950-80 (mm/y)	Lower $CI_{1950-80}$	Upper $CI_{1950-80}$	l_3 1980-2012 (mm/y)	Lower $CI_{1980-2012}$	Upper $CI_{1980-2012}$
Ålesund	0,03	-1,11	1,16	0,70	-0,23	1,64
Bergen	-2,22	-3,03	-1,42	0,82	0,06	1,58
Bodø	-6,21	-7,37	-5,06	-0,66	-1,68	0,36
Harstad	-3,55	-4,57	-2,54	-0,21	-1,13	0,71
Heimsjø	-4,09	-5,08	-3,11	-0,48	-1,39	0,43
Kabelvåg	-1,88	-2,96	-0,79	-1,83	-2,78	-0,88
Kristiansund	-2,80	-3,65	-1,95	-0,28	-1,21	0,64
Måløy	-0,76	-1,71	0,19	1,24	0,35	2,13
Narvik	-4,55	-5,62	-3,48	-1,40	-2,43	-0,36
Oslo	-3,73	-5,00	-2,46	-2,27	-3,53	-1,01
Stavanger	-0,59	-1,38	0,20	1,52	0,80	2,24
Tregde	-0,12	-0,85	0,61	0,88	0,23	1,53
Tromsø	-2,34	-3,37	-1,32	0,08	-0,85	1,00

E — Chow-test

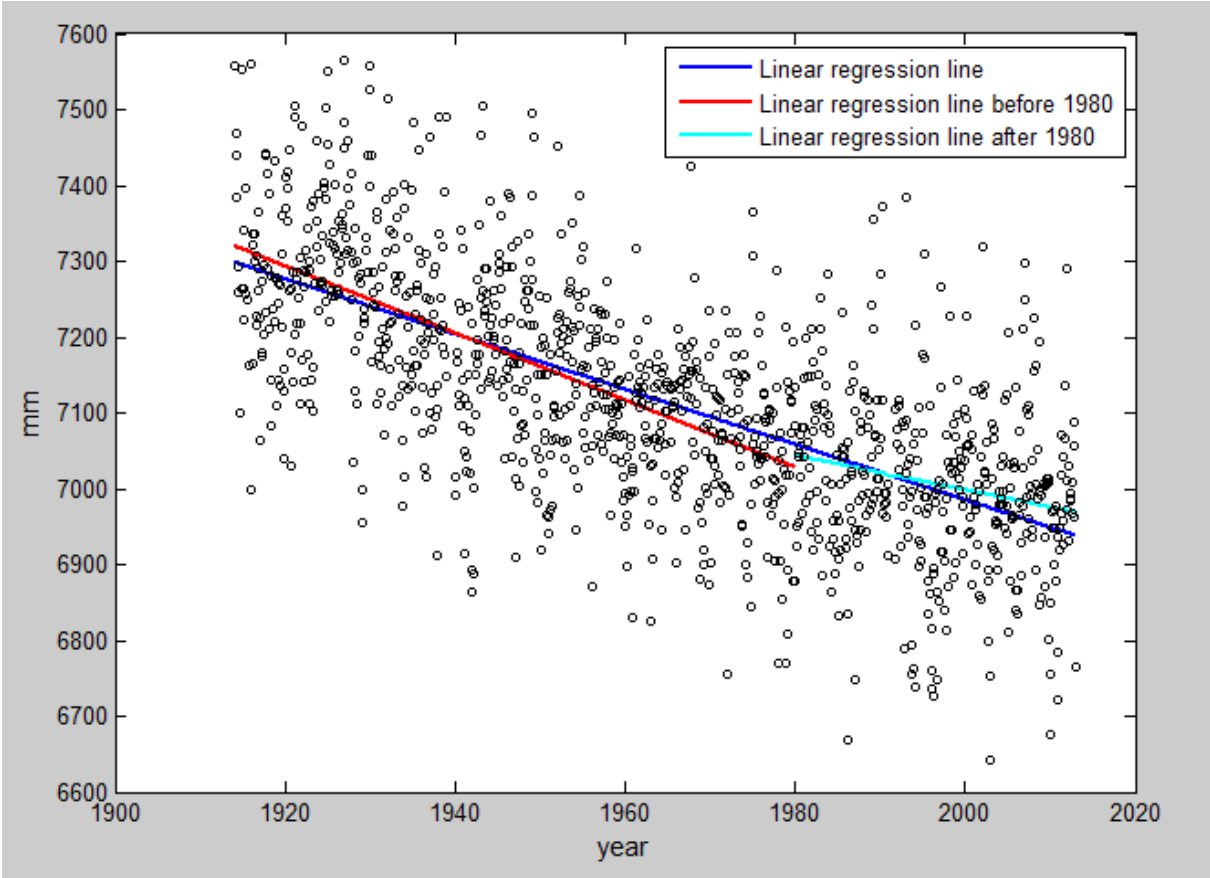


Figure E.1: Oslo - Chow-test

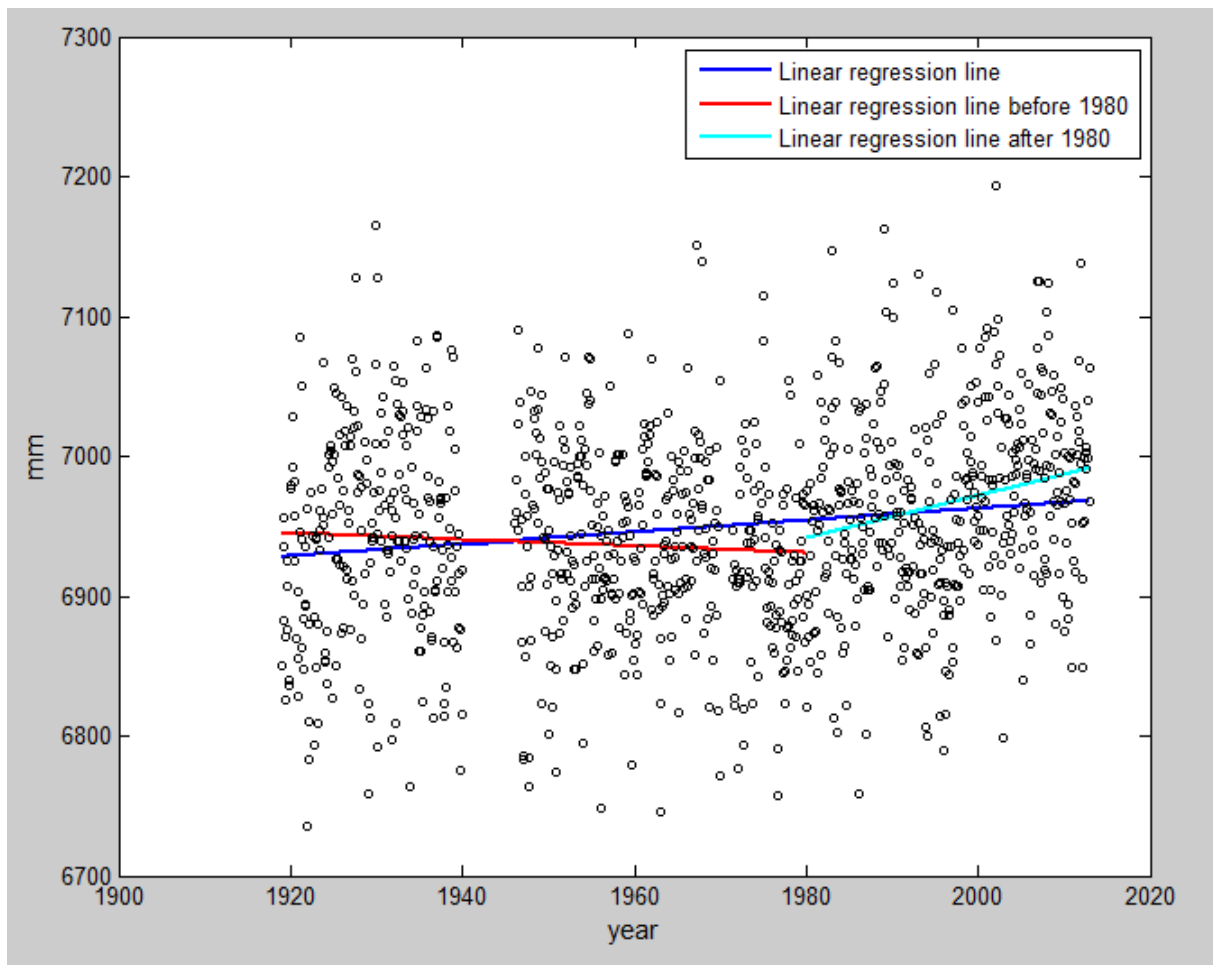


Figure E.2: Stavanger - Chow-test



Norwegian University
of Life Sciences

Postboks 5003
NO-1432 Ås, Norway
+47 67 23 00 00
www.nmbu.no