Physica A 493 (2018) 400–409

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Stochastic space interval as a link between quantum randomness and macroscopic randomness?☆

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HIGHLIGHTS

- Quantum Randomness and Macroscopic Randomness.
- Random behavior in trillions of photons.
- Stochastic Space Interval as possible explanation for fat-tailed distributions.
- Can quantum randomness explain fat-tailed distributions in finance?

ARTICLE INFO

Article history:
Received 30 June 2016
Received in revised form 30 October 2017
Available online 10 November 2017

Keywords:
Photons
Fat-tailed high-peaked distributions
Leptokurtic
Quantum world
Macro world
Non-Gaussian distributions
True randomness
Quantum randomness
Stock markets
Black Swans

ABSTRACT

For many stochastic phenomena, we observe statistical distributions that have fat-tails and high-peaks compared to the Gaussian distribution. In this paper, we will explain how observable statistical distributions in the macroscopic world could be related to the randomness in the subatomic world. We show that fat-tailed (leptokurtic) phenomena in our everyday macroscopic world are ultimately rooted in Gaussian – or very close to Gaussian-distributed subatomic particle randomness, but they are not, in a strict sense, Gaussian distributions.

By running a truly random experiment over a three and a half-year period, we observed a type of random behavior in trillions of photons. Combining our results with simple logic, we find that fat-tailed and high-peaked statistical distributions are exactly what we would expect to observe if the subatomic world is quantized and not continuously divisible. We extend our analysis to the fact that one typically observes fat-tails and high-peaks relative to the Gaussian distribution in stocks and commodity prices and many aspects of the natural world; these instances are all observable and documentable macro phenomena that strongly suggest that the ultimate building blocks of nature are discrete (e.g. they appear in quanta).

☆ An early working paper version of this work appeared in Haug's self-published "working paper book" – "New Fundamental Physics". This early working paper version had several errors and did not have the complete photon series from the full experimental study. There were also some logical flaws in the discussion. This final version has been significantly improved and peer-reviewed. We would like to thank two anonymous referees for very useful comments as well as a thank you to Victoria Terces for great help with editing. Any remaining errors are our responsibility.

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https://doi.org/10.1016/j.physa.2017.11.001

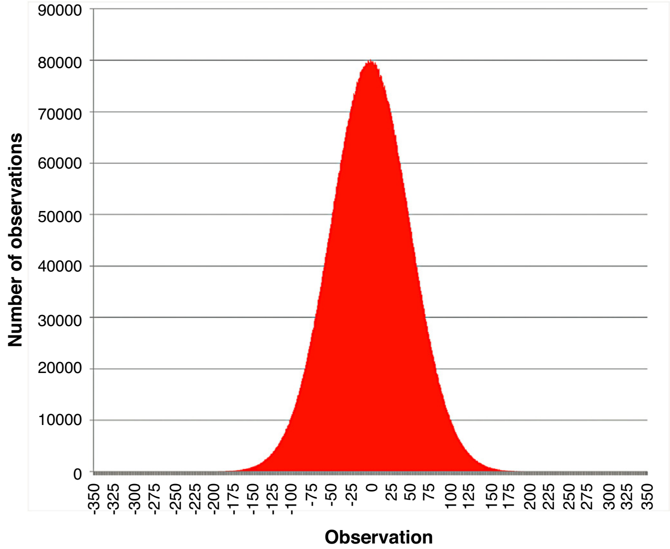

Fig. 1. Illustration of the typical Gaussian distribution, also known as the normal distribution.

1. Introduction

We focus on understanding the fat-tailed and high-peaked statistical distributions that we observe around us from the point of view of fundamental physics. In this exploration, we assess not only distinct physical events, but also phenomenon related to human behavior. Is this even possible? Remarkably, the answer is yes. We assume everything that can be observed ultimately consists of discrete subatomic particles. If we understand the randomness affecting each particle, then we should be able to evaluate the uncertainty around macroscopic phenomena more clearly by viewing them as the aggregate of a massive number of subatomic particles.

In the experiment we use only one type of "particle"–photons. An interesting question is: if we were using a different type of quantum particle, electrons, for example, would that give us different results? Based on the argumentation that is presented in the paper, we think that using a different type of particle would not be likely to change the main conclusions of the paper. It is also not that important what these particles actually are; the critical point is that the ultimate building blocks of nature are quantized. Further, a long series of experiments indicates that the subatomic world is indeed quantized, even though there are various interpretations of this concept and its implications.

2. The Gaussian curve and its failure

The normal distribution, also known as the "bell curve" or the "Gaussian distribution", is a cornerstone of modern statistics, and is actively used (and sometimes abused) across many scientific fields and social sciences, including physics, cosmology, biology, medicine, mathematical finance, and macroeconomics. Many scientists assume that the Gaussian distribution is a good description of the statistical distribution of many random phenomena. Specifically, the Gaussian curve provides information about data points that are random or at least appear to be random and describes how the phenomena in question are statistically distributed (clustered) relative to the mean. Fig. 1 shows a typical Gaussian distribution.

In order to draw the theoretical Gaussian curve for a particular phenomenon, one simply needs the mean and the standard deviation. Based on this information, one can accurately describe the entire statistical distribution of the phenomenon. Naturally this only holds true if the real distribution is normally distributed in reality.

2.1. A short history of the Gaussian curve

What is today known as the normal distribution was first described in 1733 by the French mathematician, Abraham de Moivre (1667–1754), although his theory would soon be partly forgotten. The French mathematician, Pierre-Simon Laplace (1749–1827), published a similar theory, and in many ways, extended de Moivre's result in his 1783 publication, and also in his book entitled, *Analytical Theory of Probabilities* (1812). Today, the credit for describing the normal distribution in detail often goes to the famous German mathematician, Carl Friedrich Gauss (1777–1855), even though Laplace was years ahead of him in describing such a statistical distribution. Gauss supposedly claimed that he had used the normal distribution since 1794, but he first explained it in written form in 1809. The French mathematician and artillery officer, Esprit Jouffret (1837–1904), was probably the first to use the term *bell curve* in 1872. He ascribed this name to the curve because the normal distribution has the shape of a bell when it is shown in graphical form. The term *normal distribution* was supposedly coined independently by Charles S. Peirce, Francis Galton, and Wilhelm Lexis around 1875.

Several leading researchers figured out early on that the Gaussian curve was not consistent with what they observed in practice. Carl Vilhelm Ludwig Charlier (1862–1934), a Swedish astronomer and mathematician, strongly criticized the Gaussian curve in 1920:

The responsibility for stagnation in the development of mathematical statistics until recent years rests principally upon Gauss. The great mathematician believed it possible to demonstrate that the fluctuations of the items of a statistical series – he was concerned chiefly with astronomical and geodesic observations – followed the simple law which was called after him, the Gaussian law of error. He believed the deviations from the law were accidental and would disappear if the observations increased.
In the field of non-astronomical statistics, Quetelet applied the Gaussian law with very important consequences. His theory of types is one of the most fundamental propositions of statistics if, from it, one must conclude that the mathematical type necessarily corresponds to an actual (physical, biological) type of statistical objects. However, in coming to his conclusions, Quetelet was guilty of gross exaggeration and thus, contrary to his intentions, he brought mathematical statistics into great scientific discredit; a reputation that it is not wholly free from to this day.

Despite having a valid point in his strong criticism of Gauss, Charlier was quickly ignored. Gauss was already famous and Charlier much less so. However, he was not alone in adopting a critical perspective on Gauss's work. Arne Fisher (1887–1944), a leading statistician, also criticized the Gaussian theory vigorously. In his book, published in 1922, he writes:

The great German mathematician – or rather the dogmatic faith in his authority as a mathematician – proved thus for a number of years a veritable stumbling block to a fruitful development of mathematical statistics. Gauss and his followers maintained that all statistical mass phenomena could be made to conform with the law of errors as exhibited by the so-called Gaussian Normal Error Curve. If certain statistical series exhibited discrepancies, they claimed that such deviations arise from the limited number of observations. The deviations would become less marked if the number of observations approached infinity as its ultimate value. The Gaussian dogma held sway despite the fact that the Danish actuary, Opperman, and the French mathematicians, Binemaye and Cournot, have pointed out that several statistical series, despite all efforts to the contrary, offered a persistent defiance to the Gaussian law. The first real attack on the dogma laid down so authoritatively by Gauss was delivered by the French actuary, Dormay, in certain investigations relating to the French census.[1]

Clearly there were several vocal critics of the Gaussian curve (normal distribution) by the early 1900s. What is surprising is that little has changed since then. The Gaussian curve and Gaussian thinking are still predominantly used to describe the statistical distribution of a series of phenomena by many of the sciences. However, over time the statistical apparatus has also developed into a large class of non-Gaussian statistical distributions; some of them are better at fitting empirical observations than others. In recent years, more scientists, statisticians, and people in general have acknowledged the fact that many phenomena are not Gaussian-distributed. Benoit Mandelbrot and Nassim Nicholas Taleb have been among the key figures in illuminating the weaknesses of the Gaussian distribution, see [2–6].

2.2. Non-Gaussian financial markets

Many scientists may think that finance has little or nothing to do with physics because finance is related to human behavior. Nevertheless, as we will see, even human behavior, which leads to price formation in financial markets, is governed by physics at a fundamental level. The empirical statistical distributions observed in finance are, surprisingly enough, what we would expect to discover in financial markets if everything was made of atoms and, at a deeper level, subatomic particles (photons, for example) traveling in empty space. Later we will explain why and how, but first we will take a look at the history of statistical distributions used in finance.

Early on in the fields of finance and economics, a number of researchers understood that the real market data were not Gaussian-distributed. Their work was firmly based on empirical studies. In particular, Wesley Clair Mitchell (1874–1948), who was guiding the National Bureau of Economics Research in the US, worked with a large amount of price data from the commodity market and pointed out that the data tended to have much higher peaks than the Gaussian theory predicted; see his publication from 1915 [7]. This meant that there were more days when the market hardly moved at all than the Gaussian theory would have predicted. This is known in finance as the high-peak phenomena. However, Mitchell did not state that the empirical data also had more weight in the tails than the Gaussian theory predicted, this being the so-called fat-tailed distribution. It could be that he observed fat-tails, but he did not comment on them in his 1915 publication. Fredrick Mills, who also worked at the National Bureau of Economics Research (with Mitchell as his supervisor), was probably the first to point out the fat-tails that could be seen in price data; he did so in a thick book that he published in 1927 entitled, *The Behaviour of Prices* [8]. In this book, Mills comments:

A distribution may depart widely from the Gaussian type because of the influence of one or two extreme price changes.

1 See [1] p. 149.

In other words, Mills deduced from empirical observations that extreme price swings happened more often than the Gaussian theory predicted. In finance, extreme price swings typically mean either massive losses or gains for investors. In other words, extreme price swings often have large impacts and are therefore related to a substantial amount of risk.

Stock market crashes are examples of extreme price movements that have huge impacts. Such market crashes not only have an impact on individual investors, but can sometimes cause a chain effect throughout the entire economy. In their most intense forms, a crash can even affect the world economy in dramatic ways. According to the Gaussian-type statistical models, such crashes should happen very rarely. However, by studying the empirical data, we know that extreme events occur much more often than would be expected from such models. In other words, the probability for sudden, extreme price moves is much greater than the Gaussian probability distribution tells us.

The mathematician behind much of fractal theory, Benoit Mandelbrot [9] (1924–2010), attacked the heavy reliance on Gaussian models in finance and economics in the 1960s. From empirical commodity and stock price data, he showed that it was wrong to assume that the Gaussian models were correct. Just as Mitchell and Mills had already shown, the real data had higher peaks and fatter tails than was assumed by the Gaussian models.

Even now, several of the most popular risk management tools in finance are based on the Gaussian curve. What is known as *Value-at-Risk* (VaR) has been heavily used in risk management by many financial institutions, and other common measures and models of modern finance including the *Sharpe Ratio*, the capital asset pricing model (CAPM), and the famous Black, Scholes, and Merton model [10,11] are all based on the assumption of Gaussian-distributed returns. For in-depth discussions regarding the weakness of the Gaussian distribution and its many implications for finance and beyond see [4,5,12,13], and [14].

3. Non-Gaussian galaxies?

In 2000, William C. Saslaw [15] published a book entitled *The Distribution of Galaxies*, where he pointed out that the observed distribution of the velocities of a large number of galaxies is quite different from the theoretical distribution. As we have seen before, the real distribution has higher peaks, and probably also fatter tails[2] than the theoretical model that had been developed for galaxy distribution at the time, which was a theoretical statistical distribution very similar to the normal distribution. In the same book, the author suggests that the reason for the considerable difference between the observed distribution and the theoretical distribution may be due to data error. Naturally this could be the case. However, it is reminiscent of the physicist Osborne, who rejected the high-peaks and fat-tails that he observed for stock prices in 1959 and held on to the Gaussian model, see [16]. In that instance, Osborne was committed to a theory that he believed was correct and he rejected empirical data that actually gave an accurate picture of reality. However, Osborne correctly pointed out that the motion of small particles seemed to follow geometric Brownian motion and that the statistical distribution of changes in the motion of these small particles was Gaussian – or quite close to Gaussian-distributed. In fact, many years earlier, Bachelier (1900) developed much of the mathematics for Brownian motion and its relationship to the Gaussian curve in his PhD thesis on option valuation (finance), see [17]. Five years later, Einstein [18] developed mathematics similar to Bachelier's work, but with application to physics.

In recent years, several studies have confirmed the presence of high-peaks and fat-tails in the distribution of velocities in galaxies. This merely adds to a series of phenomena in cosmology that are non-Gaussian, just as the Swedish astronomer Carl Vilhelm Ludwig Charlier had described in 1920.

4. Distributions at a quantum level

If a range of diverse phenomena, from stock market behavior to the distribution of the velocities of galaxies, clearly seem to be non-Gaussian-distributed based on the empirical evidence, one key question is: how is this related to fundamental physics? If everything is created from discrete subatomic particles, then the uncertainty at a macro level must consist of the aggregated uncertainty at a micro level. By studying the uncertainty in small observable subatomic particle phenomena carefully, we should be able to understand the uncertainty and randomness at a macro level, at least to some extent.

To better understand the link between the quantum world and the macroscopic world we have performed a simple[3] quantum physics experiment related to this question. We have looked at a basic aspect of uncertainty within the behavior of photons. As a backdrop, we note that most computers have software that simulates pseudo-randomness. Often, however, these software-based pseudo-random number generators are not truly random at all, but rather they are deterministic, which is apparent if you study the computer algorithm that is generating the numbers. And even without knowing the underlying algorithm, a series of tests can prove that most, if not all, pseudo-random number generators are not truly random. True randomness comes from the randomness in the subatomic world and our lack of knowledge about the exact positions and behavior of particles in that world.

For our experiment, we acquired a true random number generator.[4] Inside this machine is a light source that is supposed to emit photons one at a time. Based on the principles of quantum mechanics, the true random number generator should

2 Even if it is hard to say anything about the fat-tails from that graph.

3 Simple to perform with today's advanced technology, albeit unthinkable 100 years ago.

4 We used a true random number generator built by the Swiss company, ID Quantique. More precisely we used their 4-Mbits/s true quantum randomness USB generator (Product name: Quantis-USB-4M).

operate in the same way as a perfectly unbiased coin, at least in theory. This means that for each photon[5] there is a 50% probability of detection by detector A and 50% chance of detection by detector B. In other words, this is a binary random system.

Even given this process, it is still important to try to rule out possible sources of bias from the apparatus. To that end we had several conversations with the manufacturer of the true random number generator. The unit is, at production, tested for dark counts which is considered negligible,[6] something that seems to be in line with our experimental results. The unit has a self-check system to make sure it actually operates correctly, which includes testing that the deflection rate is between 3.6 and 4.4 Mbit/s. However, the observations that come out of the unit are not filtered. What we observe are the raw data (observations) from photons detected by the two detectors. The possible cosmic ray influence has not been studied, but the company assumes that it is negligible.[7]

A true random number generator is basically a perfect "coin-flipping machine", based on true quantum randomness. To test this notion, we let our true random photon generator run continuously for several years, from May 5, 2011 until October 16, 2014. The electricity grid went down for a few hours at one point, but we had prepared for this with a battery-powered back-up that kept the project running during this time. Naturally both the true random number generator as well as the computer analyzing the data need electricity to run. In a second incident, on October 16, 2014 the electricity went out for a longer period of time and the back-up battery was not enough to keep it running and, as it happened, we were both on vacation. Thus, that date marks the end point of this study, concluding a 3.5 year continuous run, which provided a massive amount of random numbers to study.

We wanted to discover what type of statistical distribution would show up in the true random quantum photon experiment over a very long time period. In theory, it should produce a "perfect" binomial distribution, or what we could call a coin-flip distribution. We observed (flipped) 427 trillion photons in total, or more precisely 426,666,400,000,000 photons collected continuously over a three and a half-year period. That is, 4.27×10^{14} number of photons, a large number of observations indeed. To put this into perspective, the theoretical probability of obtaining 47 heads in a row during a coin-flip experiment should be $0.5^{47} \approx 7 \times 10^{-15}$ percentage. In other words, the probability of getting 47 heads in a row on an unbiased coin is close to zero if you flip the coin 47 times. With this true random number generator, we effectively flipped the coins (the photons) 427 trillion times. Based on approximately 427 trillion flips, we should expect to observe 47 heads in a row $0.5^{47} \times 426{,}666{,}400{,}000{,}000 \approx 3.03$. times, or 3 times with rounding up. In fact, we observed 47 heads in a row three times – very close to what we expected theoretically. Bear in mind that theoretical probabilities are simply expected outcomes. We will not typically observe exactly the same thing as theoretically expected, but we should observe something reasonably close, if the model is good at representing reality.

Table 1 shows what we observed from flipping 426,666,400,000,000 photons, as well as the theoretically expected coin-flip (binomial) distribution for the same number of flips.

The empirical distribution basically fits the theoretical coin-flip (binomial) distribution as well as we could expect a perfectly unbiased coin (or a photon) to do. Although the very far-out tails do not match up perfectly between the theoretical and observed distributions, it is important to remember that the far-out tail in the empirical distribution will always be somewhat unstable until one has flipped enough photons (coins), and then what was the tail is no longer the tail. The tail keeps moving further and further out, the more times we flip (the more data we have). Still, we can say from statistical analysis that the true random number generator fits extremely well with what we would expect from the theory of flipping a perfectly unbiased coin.

Where is the true randomness in such an experiment? One can think of it as random for us, but it may not be random at all in the ultimate depth of reality. Each time we send out a photon, how do we know that the initial conditions are the same? On the contrary, in fact, the initial conditions are almost certainly not the same. Part of the issue lies in the physical apparatus; the photon generator itself consists of a large number of atomic particles that are constantly moving. As long as we do not have information regarding the position of every single subatomic particle that makes up the photon generator, we cannot ensure that they are at the same exact position relative to each other at each point in time that we send out a photon. Therefore, we have strong reasons to believe that the initial conditions must be different, and that this is at least partly the reason it appears to be random to us. We are constantly challenged by the fact that we have limited information about the depth of reality.

Another discussion concerns the definition of what a photon really is. In our view, a photon is a composite or collection of the most fundamental subatomic particles in the universe, see [20] for a detailed discussion of this topic. However, for the purposes of this discussion, the photon's complete nature is not of great importance as long as we agree that photons come in quanta, which is to say that energy comes in discrete packages. This view is fully consistent with mainstream quantum mechanics and a long series of physical experiments.

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5 Granted that what a photon is remains the subject of debate. See, for example, [19].

6 See also http://marketing.idquantique.com/acton/attachment/11868/f-006e/1/-/-/-/-/Photon_counting_for_Brainies.pdf for more technical information.

7 One could study this further, but our observations over three and a half years indicate that there are no detectable biases.

Table 1
This table shows the outcome of 426,666,400,000,000 (approx 4.27×10^{14}) photons (between May 25, 2011 (start time 16:54:28) and October 16, 2014 (end time 07:22:23), a total of 1240.10272 number of days. That gives 3,982,145 bits per second (approx 3.98 Mbit/s). We can think of photons that went through detector A as heads-up, and photons that went through detector B as tails-up. The first column is the number of heads or tails in a row. As expected, the theoretical binomial distribution (coin-flip distribution) describes the photon distribution as close to "perfectly" as one could expect (hope for) from true random numbers.

Heads/Tails in row	Through detector A Heads	Through detector B Tails	Expected theoretical
1	213,331,006,656,773	213,335,393,343,227	213,333,200,000,000
2	106,588,417,171,269	106,592,803,857,723	106,666,600,000,000
3	53,262,814,611,367	53,266,090,815,926	53,333,300,000,000
4	26,615,169,842,033	26,617,348,368,961	26,666,650,000,000
5	13,299,570,175,487	13,300,928,150,594	13,333,325,000,000
6	6,645,770,168,190	6,646,584,487,040	6,666,662,500,000
7	3,320,879,493,676	3,321,354,600,851	3,333,331,250,000
8	1,659,437,779,735	1,659,708,997,275	1,666,665,625,000
9	829,218,024,725	829,370,601,387	833,332,812,500
10	414,358,742,422	414,443,530,684	416,666,406,250
11	207,053,791,870	207,100,912,619	208,333,203,125
12	103,464,313,312	103,489,953,199	104,166,601,563
13	51,700,948,739	51,714,528,801	52,083,300,781
14	25,834,875,264	25,842,055,159	26,041,650,391
15	12,909,635,205	12,913,344,305	13,020,825,195
16	6,450,883,910	6,452,804,973	6,510,412,598
17	3,223,449,719	3,224,454,198	3,255,206,299
18	1,610,740,882	1,611,250,872	1,627,603,149
19	804,882,238	805,149,179	813,801,575
20	402,204,399	402,325,163	406,900,787
21	200,983,162	201,039,681	203,450,394
22	100,430,403	100,468,150	101,725,197
23	50,185,007	50,202,446	50,862,598
24	25,079,007	25,085,324	25,431,299
25	12,529,632	12,535,443	12,715,650
26	6,258,339	6,266,356	6,357,825
27	3,127,565	3,133,175	3,178,912
28	1,563,734	1,566,591	1,589,456
29	781,673	783,693	794,728
30	390,372	391,864	397,364
31	195,090	196,207	198,682
32	97,456	98,336	99,341
33	48,779	49,266	49,671
34	24,451	24,653	24,835
35	12,205	12,367	12,418
36	6,163	6,167	6,209
37	3,135	3,103	3,104
38	1,606	1,581	1,552
39	834	793	776
40	418	392	388
41	214	182	194
42	112	78	97
43	60	28	49
44	34	11	24
45	17	6	12
46	7	3	6
47	3	1	3
48	1		2

5. From quantum randomness to macro randomness

In our everyday lives, we observe macro changes that do not consist simply of a single photon or a single atom, and certainly not merely of a single subatomic particle. When we observe that something has changed in our daily lives, we are actually talking about the change in the sum of an enormous number of atoms and an even larger number of subatomic particles. The parallel to this in our photon experiment would be that we should not watch photons one by one, but we should take the sum of a large number of them, and then consider that as one single macroscopic observation. Assume for simplicity's sake that every observation consists of a short time window that involves 10,000 photons. If 5000 of them came up heads and 5000 came up tails, then we would observe this as being no change from the last period (the sum of changes). If 6000 photons came up heads (detector A) and 4000 came up tails (detector B), then we would observe this as 2000 heads up, or slightly up (or a small change from the last observation in whatever property we were actually observing). It is important to understand that even if a macro object or macroscopic phenomena appears to be unchanged, this only means that the

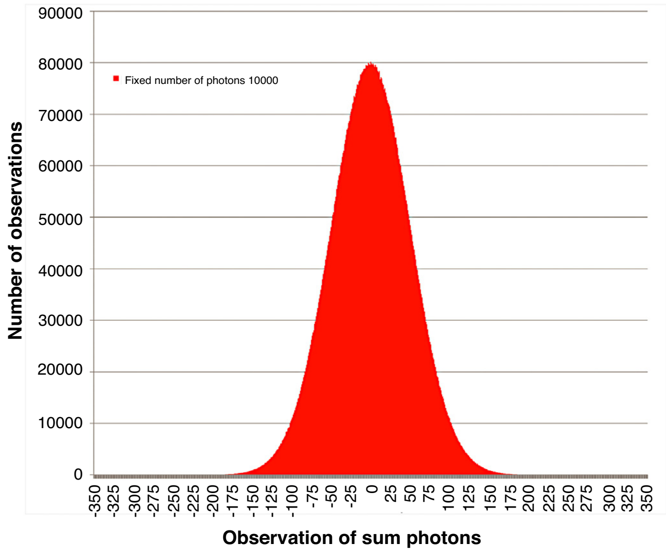

Fig. 2. Each observation consists of the sum of 10,000 photons. The sum of 10,000 photons is observed 10 million times. The distribution is, as expected, Gaussian-distributed (normally-distributed).

sum of changes in the subatomic particles was unchanged. In fact, each subatomic particle could have been undergoing significant changes in its position or direction of motion, for example.

Assume that inside a given volume, there were 5 atoms (or oxygen molecules, or photons, for example) to the far left side of the volume and 5 to the far right side of the volume. Next, the 5 atoms to the left move right, and the 5 atoms to the right move left – they exchange places. If we are not monitoring continuously, then the individual atoms would look unchanged after this move, despite the fact that each atom has actually changed position. This is related to the fact that given types of atoms or subatomic particles are indistinguishable from each other, in general, at least for us.

Consider a similar example: someone is watching a stock index that consists of 100 stocks (100 companies). Assume that the stock index is equally-weighted, which means that each share (company) at this moment comprises 1% of the index. Also assume that you are only watching the price movement in the index, and not the movement of individual stocks. If 50 shares went down in price and 50 shares went up in price by roughly the same amount, then the price of the stock index would be unchanged, or very close to unchanged. Our point is simply that every observation of change in our daily lives (macro observations) is an aggregate of the changes in a massive number of subatomic particles.

Assume that we are observing macroscopic phenomena in a small area of space and monitoring these phenomena for changes. Assume further that up to 10,000 photons could leave that area[8] and that up to 10,000 photons could also move into that area for every given unit of time. In order to eliminate the possibility that the area will become empty of photons and then negative for photons (something that naturally would be impossible in practice), we can also assume that there are already plenty of photons inside the area of space of interest at the beginning of the experiment. It is the *change in the number of photons* in the area that we are interested in, not in the absolute number of photons existing there. The area can receive or lose a maximum of 10,000 photons during each unit of time. If 10,000 photons were leaving the area, and 10,000 photons were simultaneously arriving, we would observe a change of zero. If 3000 photons left, and 1000 arrived, we would see a change of 2000 photons. We have simulated this for 10 million time units. That is, we "flipped" 10 million times 10 thousand photons, which is equal to 100,000,000,000 photons. We collected 10,000 photons for each "time unit" and looked at their aggregate as one observation. The resulting distribution is shown in Fig. 2.

In this Fig. 2, the distribution for the simulation is, as expected, very close to the normal distribution.[9] So, does this mean that apparent random phenomena in the true macro world should be assumed to be Gaussian-distributed? The answer in most cases is: No!

It is well known from the field of statistics that if one combines a few Gaussian distributions with different standard deviations, the combined distribution will have higher peaks and fatter tails than the Gaussian distribution. One common way to see this is to use stochastic time, see [12] for a short discussion on this and further references. In practice, this simply means one uses different time intervals between the observations. The idea of how stochastic clocks affect the stochastic process, and thereby, the distribution, is discussed in the 1970 Ph.D. [21] thesis of Professor Clark that was published in

8 We could assume that each time unit was a millisecond, for example.

9 We also made some statistical measurements and found that the distribution basically had a kurtosis of zero, as the Gaussian distribution should have. The simulation is not exactly Gaussian-distributed, but as close to a Gaussian as one could hope for, based on discrete units.

1973 [22]. How stochastic clocks and time potentially affect statistical distributions in finance is discussed by [23]. One reason that we observe fat-tails and high-peaks in empirical data compared to the Gaussian distribution is simply because we do not always have uniform time intervals between each observation, as recorded by a human researcher. However, in many databases, we have uniform time intervals, and the data still has high-peaks and fat-tails, so there must be more to it than stochastic time intervals and potential human error or inaccuracy.

Can quantum physics really help to explain why we typically observe higher peaks and fatter tails compared to the Gaussian distribution in stocks and commodities? Price behavior is basically driven by human behavior, which is driven in turn by news. We are now thinking about news in a wider perspective, where even a change in the price itself among many traders is analyzed by other traders and investors, and can therefore affect whether they will buy or sell. For example, some traders have stop-loss limits, which means that if the price hits a certain level, they will buy or sell a given number of shares. All human behavior in the stock and commodity markets ultimately comes from an astronomical number of interactions between subatomic particles.

Some numbers, such as the unemployment numbers, are generated from collecting data for days, or even weeks, depending on which country we are talking about. Other events are based on the aggregated information of much shorter time intervals that gets released into the market. The point is that some events are driven by aggregated information from larger time intervals than other events. Mixing a series of non-uniform intervals will lead to high-peaks and fat-tails. This is basically well known, but has received very little attention, particularly with respect to how this is ultimately linked to particle physics and space–time.[10]

Varying the time intervals (partly deterministic, partly stochastic) behind such things as news can be replaced by a stochastic space interval. We need time for uncertainty to evolve; without any time interval, there is no uncertainty. But we will claim one also needs space; without space, we can have no uncertainty. According to simple logic,[11] change is time, and without any change, there is no uncertainty. The more time that has passed, the larger the uncertainty typically is. We can also simulate this. For example, getting 40 photons in a row all going through detector A (40 heads in a row) will normally take a lot of simulation time to observe. That is, if we are only using a small volume of space in which we "flip" the photons one by one using a single true random number generator. On the other hand, if we are using many true random number generators simultaneously, then time is basically replaced with space. If we have billions of random number generators running in parallel, then the chance of one of them producing 40 heads in a row (40 photons in a row going through detector A) is considerably higher. The uncertainty is increased by increasing either the space-volume of the experiment, or the time, or both.

In finance, in practice we often mix events from different space-volumes (different number of particles) together. Some news is generated by the speech of a single person, a person takes up a small space-volume and consists of fewer particles than, for example, a whole country or a continent. Other events are generated by news from crowds of people or entire countries. The space-volume (or number of particles) used to generate the news is from different volumes (number of particles) of void-space. The bigger the void-space, the more subatomic particles such a space-volume has room for, and the more states there will appear in such a space-volume. In other words, the standard deviation of different space-volumes will be different. This means that in financial markets, we are basically mixing a series of Gaussian distributions (or something close to Gaussian), and we should expect the combination of these distributions to be fat-tailed and high-peaked relative to the Gaussian. With phenomena where we are observing a given space-volume for each observation and using the same time interval between each observation, the data will be much more normally-distributed than it would be for phenomena where we are using different space-volumes and/or different time intervals for each data point.

To better illustrate this point, we ran a photon experiment during which the random number of photons between 2000 and 21,000 randomly leaving or entering an area could vary during each time unit. This was related to the different sizes of the areas of space (volume); up to 21,000 photons could leave or enter a large area, and up to 2000 photons could leave or enter a small area. In other words, we were looking at the stochastic space-volume (or alternatively, a stochastic number of particles) between each uniform time unit. Again, what we observe in our everyday lives can be seen as the sum of changes from a large number of fundamental particles. We never observe each and every one of them individually. In addition, people often tend to mix observations from different space areas. We ran the experiment for 10 million time units, where we were looking at the sum of changes in a random number of photons between 2000 and 21,000 for each "time unit". Fig. 3 illustrates a comparison between the distribution of the random number of particles (random space-volume [in blue]) and the distribution of the constant number of particles (10,000 photons for each time step [in red]). The random number of particle distribution clearly has fatter tails and higher peaks than the bell-shaped constant space-volume distribution. In finance, for example, we are indeed not keeping track of a given number of particles, but we are "randomly" mixing different particle groups with different size. At one moment, it is the change in a single man's speech that we pay attention to (a relatively small amount of particles), and in the next moment, we are looking at the unemployment statistics for the entire country, which reflects a much larger number of particles. Based on fundamental physics rooted in discrete particles, we should expect high-peaks and fat-tailed distributions compared to the Gaussian for any phenomena where the number of particles we indirectly observe and use for each observation varies, and this is exactly what we discover.

10 See [23] for a short discussion of this.

11 See [20].

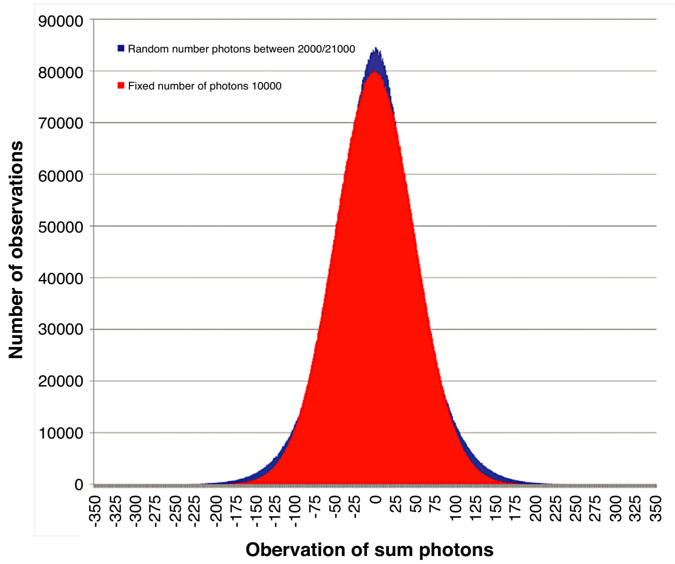

Fig. 3. Each observation consists of the sum of a random number of photons (between 2000 and 21,000). The blue graph has higher peaks and fatter tails than the fixed number of photons, 10,000, and the red graph is Gaussian-distributed. The fact that we mix observations from different space areas (or observations that consist of different numbers of subatomic particles) is likely the deeper, fundamental cause for why we have so many observed fat-tailed phenomena.

The fact that time can be partly swapped with space (number of particles) when it comes to uncertainty can easily be observed on Earth for very different phenomena. Let us say that you want to observe the number of animal or plant species in a given land area. The longer you observe this land area, the more species you will observe there. If you observe the same land area, 10 square meters, for example, over millions of years, then you will likely observe many different species passing through. Alternatively, you could move along the surface of the Earth for a few days and also observe many different species. Uncertainty (the standard deviation) will, in general, increase with increased time, but also with increased space.

For example, if we take one square meter of land and collect the rain that falls on that square meter every day, we will likely get data that is very close to being normally-distributed. (Actually, the amount of rain falling there will likely be close to log-normal-distributed, while the natural logarithm of the daily percentage changes will be very close to normally-distributed because you are using a uniform time-window and a uniform space-window in your observations.) In finance, the space-window we pay attention to (or give weight to) often changes dramatically from observation to observation, and the time-interval often varies between different observations. We could provide many more examples and formalize this theory mathematically. The important point is that many fat-tailed and high-peaked phenomena are due to the use of a stochastic space-window (or stochastic number of particles) between observations. This must lead to high-peaked and fat-tailed distribution, even if the uncertainty in the smaller building blocks should fit Gaussian theories.

6. Conclusion

Stochastic time-windows as well as stochastic space-windows that frame the actions of a stochastic number of subatomic particles will typically result in fat-tailed and high-peaked statistical distributions, as compared to the Gaussian curve. This principle holds true for financial markets, which many scientists may consider to be unrelated to physics, as they claim that the action in markets is solely related to human behavior. However, human behavior is, at the depth of reality, "nothing more" than quantum physics. In financial markets, the space areas or the number of particles that investors pay attention to for different pieces of news are of different sizes, so this is like mixing Gaussian distributions with different standard deviations. In our view, this provides a revolutionary insight into the reason that empirically we observe high-peaked and fat-tailed distributions, as opposed to the Gaussian distribution, in a wide range of phenomena.

These findings have practical important implications. Several leading researchers we have talked to in finance think that as markets become increasingly liquid they will move closer towards the Gaussian distribution. Based on the principles of quantum physics, we contend that no matter what the level of liquidity may be, we will still expect fat-tails and high-peaks in the stock and commodity markets.

References

[1] A. Fischer, The Mathematical Theory of Probabilities and its Application to Frequency Curves and Statistical Methods, The Macmillan Company, New York, 1922.

[2] B. Mandelbrot, The variation of certain speculative prices, J. Bus. 36 (1963) 394–419.
[3] B. Mandelbrot, R.L. Hudson, The (Mis) Behavior of Markets: A Fractal View of Financial Turbulence, Basic Books, New York, 2004.
[4] N. Taleb, Fooled by Randomness: The Hidden Role of Chance in Life and in the Markets, Texere, 2001.
[5] N. Taleb, The Black Swan: The Impact of the Highly Improbable, Random House, New York, 2007.
[6] N. Taleb, Antifragile: Things That Gain from Disorder, Random House, New York, 2014.
[7] C. Mitchell, Wesley, The making and using of index number: Introduction to Index Numbers and Wholesale Prices in the United States and Foreign Countries (published in 1915 as Bulletin No. 173 of the U.S. Bureau of Labor Statistics, reprinted in 1921 as Bulletin No. 284, and in 1938 as Bulletin No. 656).
[8] F.C. Mills, The Behaviour of Prices, National Bureau of Economic Research, New York, 1927. Albany: The Messenger Press.
[9] B. Mandelbrot, New methods in statistical economics, J. Political Econ. 61 (5) (1963) 421–440.
[10] F. Black, M. Scholes, The pricing of options and corporate liabilities, J. Political Econ. 81 (1973) 637–654.
[11] R.C. Merton, Theory of rational option pricing, Bell J. Econ. Manage. Sci. 4 (1973) 141–183.
[12] E.G. Haug, Derivatives: Models on Models, John Wiley & Sons, New York, 2007.
[13] E.G. Haug, N.N. Taleb, Why we never used the Black–Scholes-Merton formula, January, Wilmott Magazine.
[14] P. Triana, Lecturing Birds on Flying: Can Mathematical Theories Destroy the Financial Markets? John Wiley & Sons, London, 2009.
[15] W.C. Saslaw, The Distribution of the Galaxies: Gravitational Clustering in Cosmology, Cambridge University Press, United Kingdom, 2000.
[16] M.F.M. Osborne, Brownian motion in the stock market, Oper. Res. (1959) 145–173.
[17] L. Bachelier, Theory of speculation, in: P. Cootner (Ed.), The Random Character of Stock Market Prices, Vol. 1900, MIT Press, Cambridge, Mass, 1964.
[18] A. Einstein, On the electrodynamics of moving bodies, Annalen der Physik, English translation by George Barker Jeffery (17) (1923).
[19] C. Roychoudhuri, A.F. Kracklauer, K. Creath, The Nature of Light: What is a Photon? CRC Press, 2008.
[20] E.G. Haug, Unified Revolution: New Fundamental Physics, E.G.H. Publishing, Oslo, 2014.
[21] P.K. Clark, A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices (Ph.D. Dissertation), Harvard University.
[22] P.K. Clark, A subordinated stochastic process model with finite variance for speculative prices, Econometrica 41 (1) (1973) 135–155.
[23] E.G. Haug, Space–time finance, Wilmott Magazine, 2004.