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Assessments of vibrational properties of timber joist floors

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Abstract:

Timber joist floors, being light weight structures and having low natural frequencies, are especially vulnerable towards both structural vibration and sound transition through the structure. Vibrations are a serviceability problem, and of increasing importance as large timer structures become more common. At the same time there is a want for large open spaces and longer spans, which is a challenge for light-weight materials such as glue lam joists.

The purpose of this thesis is to investigate current approaches used in design and verification of timer joist floors. Today, no standard method is agreed upon, and several different approaches exist. The current standards investigated in this thesis mainly considers the fundamental frequency and stiffness of the floor, and only partially considers the effect of the mass of the system. This may lead to significant annoyance of the user.

The basis for the thesis is a typical timber joist floor used in residential buildings, currently being produced by a producer of element residential buildings. A reference floor based on information from this producer is investigated, as well as two other floor models, having different properties. Current methods will be compared and discussed, as well as the effect of the stiffness and mass of the floor. Lastly, measures on how to improve the vibrational properties in a timer joist floor is briefly disused.

The results of the comparison of the code based methods show that the current code based methods only to some extent are suitable for investigation of light weight, high frequency floors. Due to the high frequency, the mass requirement is disregarded. Not all of the code based methods investigated make use of a mass requirement. It is shown that transient floor response is more crucial than the steady state response, as resonance due to walking will not occur.

The vibrational performance of a floor can be increased by increasing the transversal stiffness, but the effect of this improvement is limited when investigating the floors using the code based methods. A numerical analysis should be performed to fully investigate the effect of transversal stiffening of light weight floors.

Sammendrag:

Lette konstruksjoner med lave egenfrekvenser, som etasjeskillere i tre er spesielt utsatt for både strukturvibrasjoner og at lyd overføres gjennom konstruksjonen. Vibrasjoner er et problem for bruksgrensetilstanden i en konstriksjon, som blir stadig viktigere ettersom store trekonstruksjoner blir vanligere. Store og åpne rom krever større spenn på bjelker, noe som er utfordrende for lette materialer som limtredragere.

Hensikten med denne oppgaven er å undersøke nåværende metoder som benyttes innen utforming og verifisering av etasjeskillere i tre. I dag eksisterer det ingen standard metode til dette formålet, det finnes flere forskjellige tilnærminger. De nåværende standardene som undersøkes i denne avhandlingen vurderer hovedsakelig fundamentalfrekvens og gulvets stivhet. De tar bare delvis høyde for massen til systemet, og fokuserer på faren for resonans i gulvet fremfor kortvarig akselerasjonsrespons. Dette kan føre til at akselerasjonen i gulvet på grunn av for eksempel støt fra helen under gang blir et problem.

Grunnlaget for avhandlingen er et typisk gulv av limtrebjelker, produsert som et elementgulv av en elementhusprodusent i Norge. Et referansegulv basert på informasjon fra denne produsenten undersøkes, og brukes som utgangspunkt for videre vurderinger av de ulike verifikasjonsmetodene. Nåværende metoder og effekten av stivhet og masse til gulvet blir sammenlignet og diskutert. Til slutt blir tiltak som kan bidra til å forbedre vibrasjonsegenskapene i etasjeskilleren kort diskutert.

Resultatene av sammenligningen av de kodebaserte metodene viser at nåværende metoder kun i noen grad er egnet for undersøkelse av lette gulv med høy egenfrekvens. På grunn av den høye frekvensen blir massekravet ikke tatt med i dagens metoder. Noen av de kodebaserte metodene som er undersøkt bruker ikke massekrav i det hele tatt, bland annet metodene brukt i Norge i dag. Det er vist at forbigående gulvrespons er mer avgjørende enn faren for resonans i gulvet.

Vibrasjonsytelsen til et gulv kan økes ved å øke den transversale stivheten, men effekten av denne forbedringen er begrenset når man undersøker gulvene ved hjelp av kodebaserte metoder. En numerisk analyse bør utføres for å fullt ut undersøke effekten av tverrgående avstivning av lettvektsgulv.

Acknowledgements:

I would like to thank my supervisor, Roberto Thomasi for being patient and available, shearing his knowledge and helping me navigate through the subject of this thesis. I would also like to express my deepest gratitude to Daniele Casagrande, for game changing guidance in the last part of this process.

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Ås, January 2018

Ida Nordengen Berntzen

Table of content:

1 Introduction

There is an increasing requirement for timber based structures, both for detached houses and larger buildings. This is due to the rapid development of modern construction technology, and a shift towards a more environmental conscious society. At the same time, there is an increasing demand for longer spans a want for slim and material effective constructions. This means knowledge about vibrations in timber structures are becoming increasingly important, as light weight structures such as timer floors are especially prone to vibrational problems.

Evaluation of floor vibrations are a complex matter, and no overall agreement on how this should be done. Current standards for evaluation of the vibrational properties in timber joist floors are characterised by simple expressions, relating physical parameters to limitation values for verification. These are rough methods to be used in the design phase, and no current method seem to fully cover all the important aspects. More sophisticated methods for evaluation of existing floors exist, but these are computational heavy and demand a great knowledge about the floor under consideration.

1.1 Scope

The scope of this thesis is to investigate current approaches used in design and verification of timer joist floors. These methods will be used to analyse the vibrational properties of a timber joist floor, currently in production by a producer of element houses in Norway. When the existing floor is analysed, suggestions on how to improve the vibration properties in the floor will be made.

Four analytical methods will be used, and the results compared. The methods presented will be an approach suggested by Hamm and Richter (Hamm et al. 2010), a method suggested by (Homb 2007) for use in Norway, and the recommended approach in the current Eurocode 5, with the Norwegian (EC5 Norwegian NA 2010) and Austrian National Annex (EC5 Austrian NA 2014). The numerical analysis will be carried out using the finite element software SAP2000. Results from the numerical analysis will be evaluated using the calculation of Vibration Dose Value (VDV).

1.2 Serviceability limit state

The vibration of floors is considered a serviceability issue. If a building or structure is to uphold its serviceability, it should:

- Provide acceptable human comfort
- Maintain functioning of the structure under normal use
- Uphold acceptable appearance of the construction works.

"Normal use" implies the loads that will be imposed on the structure when it is being used for its intended purpose. Ensuring serviceability is done by controlling the deformations, vibrations, and damages adversely affecting durability. The limitation of vibration falls into the category of ensuring human comfort. This means that vibrational problems primarily are related to human comfort, even though it can cause cracks or damage very sensitive equipment.

1.3 Thesis out line

This thesis is covers a broad aspect of the theory of floor vibrations and current standards of how to evaluate timber joist floors. There is a summary at the end of the major chapters 6-8, discussing the results found in the chapter. The final chapter 9 is therefore very brief. Summary. Below is given a brief introduction to the topics.

Chapter 2: Vibration in floor structures, briefly present general vibration theory, and introduces floor vibrations as a specific part of structural dynamics. Human perception of floor vibrations is also introduced.

Chapter 3: Presentation of code based methods, present approaches to evaluate a timer floor in terms of vibrational properties. Five methods - four analytical and one semi-numerical - is presented.

Chapter 4: Joist floor under consideration, introduces the reference floor, based on a type of floor currently being in production in Norway.

Chapter 5: Verification using code based methods, evaluates the reference floor and a floor having 50mm concrete screed added using the four analytical methods presented in chapter 4.

Chapter 6: Comparison of methods, closer investigates the code based methods, as they prove to give quite different results when applied to the same floor in chapter 5. Maximum span allowed according to the different methods are found.

Chapter 7: Numerical analysis of the floor, evaluate the reference floor and the 50mm concrete screed floor using the semi-numerical approach presented in chapter 3.

Chapter 8: Suggestions on how to increase the span, present different measures to increase the span of a floor construction similar to the reference floor.

Chapter 9: Conclusion and further work, summarise the thesis and gives suggestions to further work.

1.4 Limitations

Vibrational properties and behaviour is largely dependent on the damping ratio of the structure. Through this thesis, a conservative 1% damping is assumed in the two floors *without concrete screed* investigated, and 3% damping assumed in the floor *with* concrete screed included.

To fully investigate the behaviour of a floor due to human walking, a detailed hypothesis has to be presented as the details can have a great influence on the final response of the floor. For example, the material used in the floor surface influence the effect of the heel-drop, and the weight and placement of the furniture on a floor will greatly affect the mass of the system.

2 Vibration in floor structures

Structural design usually concerns with static behaviour. Vibrations are in nature different from this, as vibrations are a specific part of dynamics that considers cyclic, motion of a body. Human activities cause vertical forces on the floor surface, which leads to vertical motion of the floor perceived as vibrations.

The theory presented in the following sections is taken from (Smith et al. 2009) unless otherwise is stated.

2.1 Theory of vibrations

A vibration problem can be classified into either a continuous system or discrete system, depending on the structure under load. In a discreate system, the masses under consideration is independent of each other like in a mass-spring-system (as in Figure 2.1), whereas the masses in a continuous system will be highly dependent on each other as they are directly linked together, like in a string or a beam. The two types of systems are briefly presented below.

Continuous system:

The governing Equation for a beam in bending is:

$$m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = F(x.t) \qquad \textbf{(2.1)}$$

where:

m	is the distributed mass [kg/m]
w	is the displacement of the beam, as a function of *t* and *x* [m]
t	is the time [s]
EI	is the bending stiffness [Nm^2]
x	is the position along the beam [m]
F(x.t)	is the forcing function [N]

This equation relates the displacement, velocity, and acceleration at a certain position and time to the initial force and stiffness of the beam. That is, the response of the system is related to the force causing the response, and the properties of the system.

Problems involving continuous systems are solved by integration of the continuous equation (2.1), but as this can be troublesome, there are techniques to “discretize” a continuous system, as a discrete system is simpler to solve. The most known technique is the Finite Element Method (FEM), which is presented in more detail in section 7.1.

Discrete system:

Discrete problems are solved by finding and solving matric equations that link displacement, velocity, and acceleration of the system to the external forces. A discrete system is usually modelled as a spring-mass-damper-system as illustrated below:

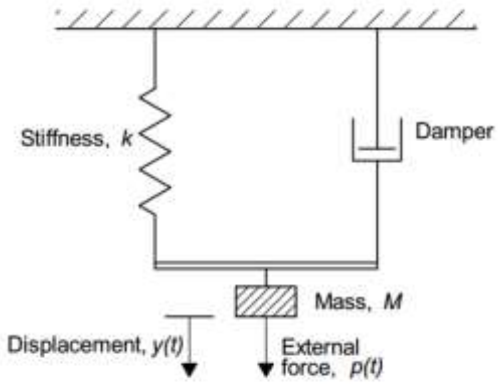


Figure 2.1: Model of a single degree of freedom (SDOF) system, (Smith et al. 2009)

The simplest form of a discrete system is the single degree of freedom-system (SDOF) which only includes one mass. SDOFs are easily solvable, while multi-degree of freedom systems (MDOF) are harder to solve as they include several masses, coupled in a variety of ways. An example of such a system is a multi-story-building where the columns are regarded as springs and the floors as masses as illustrated in Figure 2.2

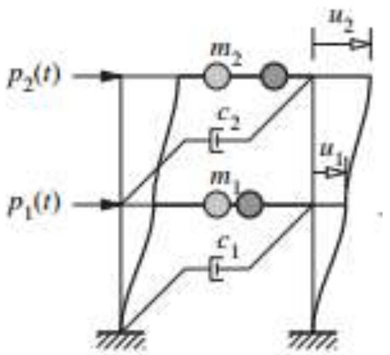


Figure 2.2: Multi degree of freedom (MDOF) system (Chopra 2012).

The illustration shows a two-story building, having two masses, horizontal displacement at each story. Two time-dependent forcing functions and the damping related to each story is also illustrated.

To find the response in a continuous system at each natural frequency, the SDOF model can be used to investigate each mode of the continuous system. For each mode different physical parameters are applied.

2.1.1 Frequency

The natural frequency of a system (given in Hz, or cycles per second) highly governs the response in a system when subjected to any excitation force. By setting the forcing function to zero and applying appropriate boundary conditions to the equation for a beam in bending (2.1), the natural frequencies of the system can be found;

$$f_n = \frac{\kappa_n}{2\pi}\sqrt{\frac{EI}{mL^4}} \tag{2.2}$$

where:

- EI is the dynamic flexural rigidity of the member [Nm^2]
- m is the effective mass [kg/m]
- L is the span of the member [m]
- κ_n is a constant representing the beam support conditions for the *n*th mode of vibration.

It is usually the first mode of vibration that is of interest when we consider human induced vibrations in floor structures. This is also called *the fundamental frequency* and corresponds to n = 1. For a simply supported (pinned/pinned) beam, standard value for κ_1 is π^2. The above equation can be simplifyed to:

$$f_1 = \frac{\pi}{2}\sqrt{\frac{EI}{mL^4}} \tag{2.3}$$

A convenient method to determine the fundamental frequency of a simply supported system is to use the maximum deflection due to a uniform mass per unit length *m*:

$$\delta = \frac{5mgL^4}{384\,EI} \tag{2.4}$$

where:

- g is the gravitational acceleration (9.81 m/s^2)
- m is mass per unit length (kg/m)

Rearranging Equation (2.4) with respect to *m* and substituting it into Equation (2.3) the following relation between fundamental frequency and maximum deflection due to self-weight (in mm) can be found:

$$f_1 = \frac{17.8}{\sqrt{\delta}} \approx \frac{18}{\sqrt{\delta}} \tag{2.5}$$

This relationship shows that the fundamental frequency of a beam will decrease with increased deflection of the beam.

Each natural frequency of a system will have a *mode shape* associated with it, meaning a system with *n* natural frequencies have *n* different mode shapes. A mode shape is the shape of the system at maximum deflection. The simplest mode shape is related to the fundamental frequency, in the form of half a sine-wave, see Figure 2.3. The second and third mode shape is in the form of one and one-and-a-half sine wave, respectively.

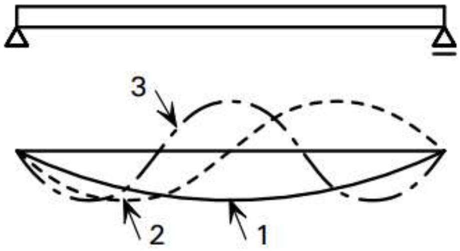


Figure 2.3: Mode shapes of a simply supported beam (Smith et al. 2009).
The mode shapes are presented with a non-dimensional amplitude of 1 (unity normalization).

The general expression for the normalized amplitude at position *x* of the *n*th mode shape of a simply supported beam, known as a shape function, is as follows:

$$\mu_n = \sin\left(\frac{n\pi x}{L}\right) \quad \textbf{(2.6)}$$

where:

n is the mode under consideration

x is the position along the beam [m]

L is total length of the beam [m]

Maximum amplitude in the first mode of a simply supported beam is at mid span, x = *L*/2, as illustrated in Figure 2.3.

To obtain the displacement of any point along the beam at any given time, the shape function is multiplied by a time-varying amplitude function:

$$g_n(t) = \sin(2\pi \cdot f_n \cdot t) \quad \textbf{(2.7)}$$

where:

f_n is the frequency of the mode under consideration [Hz]

t is time [s]

The actual displacement of the system at any given time is found by considering all the mode shapes, by *modal superposition*:

$$w_n(x.t) = \sum_{n=1}^{\infty} u_n \sin(2\pi\, f_e\, t + \phi_n) \sin\left(\frac{n\pi x}{L}\right) \tag{2.8}$$

where:

t is the time [s]

f_e is the frequency of the forcing function [Hz]

u_n is the maximum amplitude of mode n

Φ_n is the phase lag of mode n

u_n and Φ_n are determined from the initial excitement or forcing function.

Modal mass:

To express a continuous system as a series of discrete, single degree of freedom systems, the *modal mass* for each mode of the system has to be determined. The modal mass of a system is a measure of how much of the systems mass that is involved in the mode shape. It is related to how much kinetic energy there is in the system:

$$KE = \frac{1}{2} M_n v_n(t_{\text{max}})^2 = \int_{x\,min}^{x\,max} \int_{y\,min}^{y\,max} v_n(x.y.t_{max})^2\, m(x.y) dy\, dx \tag{2.9}$$

where:

M_n is the mass of the equivalent SDOF system for mode n

$v_n(t)$ is the velocity of mass M_n at time t [m/s]

t_{max} is the time at which the velocity is largest [s]

The velocity of mass M_n can be expressed as $\mu n(x.y) \times g'n(t)$, where $g'n(t)$ is the differential of $gn(t)$ with respect to time. Knowing that the maximum velocity is occurring at $g'n(t) = 1$ and rearranging Equation (2.9), the modal mass is expressed as:

$$M_d = \int_{x\,min}^{x\,max} \int_{y\,min}^{y\,max} \mu_n(x.y)^2\, m(x.y) dy\, dx \tag{2.10}$$

where:

$\mu_n(x.y)$ is the general expression for the normalized amplitude at position x (mode shape)

$m(x.y)$ is mass per square meter at position $(x.y)$.

The modal mass will indicate how much the mode under consideration will contribute to the overall response of the system; A large modal mass indicates that it takes a lot of energy to excite the mode, and this mode will not have much influence on the response.

Frequency clustering:

Because of the *inherent orthotropy* of timber joist floors (see section **Feil! Fant ikke referansekilden.**), there is a tendency towards frequency clustering of the few first modes. Frequency clustering is present if the first neighbouring frequencies are only 10-15 % apart. Closely spaced adjacent natural frequencies can cause an increase in the motion amplitude. In turn, this leads to increased acceleration and velocity levels of the floor (Glisovic & Stevanovic 2010). A measure of frequency clustering is the ratio between adjacent frequencies, called *modal separation factor*:

$$MSF_n = \frac{f_{n+1}}{f_n} \tag{2.11}$$

A high value of MSF_n indicates a low degree of frequency clustering. The level of frequency clustering is dependent on, for example floor shape and flexural rigidity across- and along joist. If the clustering of frequencies is present, it is important to include also the higher modes of vibration in the assessment of the floor. The phenomenon is more present in highly orthotropic floor constructions, so any attempt to increase the isotropic behaviour (and by that the modal separation factor) is good when considering vibration serviceability of timber floors.

Sources of excitation:

For a floor system to be set in motion, some excitation forces must be applied. Typical excitation forces on a floor are human activities such as walking, dancing or jumping, which all are dynamic forces with varying level of continuous behaviour. Synchronised dancing causes continuous forcing, while a single jump is an impulse force on the floor (Smith 2003) Normal walking has both a continuous and impulsive (or transient) nature, where the heel drop can be seen as an impulse and the consecutive steps seen as continuous. This will be investigated in more detail in section 2.1.2.The following briefly describes the two types of excitations.

Continuous and impulsive forcing function:

When a system is excited by a continuous forcing function, the function must be broken down into a series of sine waves if the response of the system is to be determined. Each of these sine waves has a frequency at an integer multiple of the forcing frequency. These multiple integers are called *harmonics*. A set of harmonics is called *a Fourier series,* an example is illustrated in Figure 2.4.

A Fourier series is a series of sine waves with decreasing amplitude. As can be seen in the figure below, the total function (bold line) does not exactly follow the half-sine shape. The reason for this is that the higher harmonic components, as well as the first harmonic component, contain energy. The decrease in amplitude reflects that most of the energy dissipates in the first few harmonics, and less in the higher harmonics.

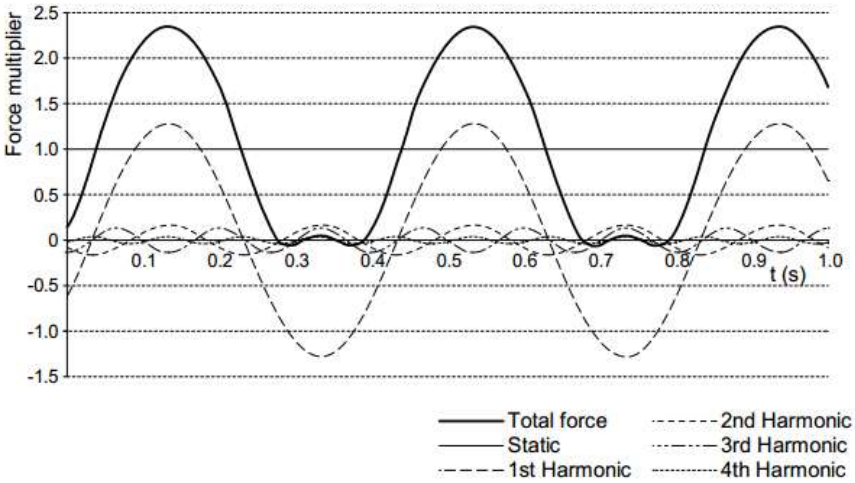


Figure 2.4: A Fourier series for low impact aerobics (Smith et al. 2009)

If the fundamental frequency of a floor is close to one or more of the first few harmonics, both resonance and of-resonance response is more likely to occur. This leads to increased response amplitude, as will be described in more detail in section 2.1.2.

The response to a series of impulses is rather different from the one to a continuous function, as will be described in section 2.1.2.

2.1.2 Response (to different excitation forces?)

Transient and steady state response:
The response of a system contains both a transient and a steady-state part, but will be dominated by one of them. The ratio between the fundamental frequency of the structure and the excitation frequency determines what part of the response that will be dominant. This ratio is called the *frequency ratio*:

$$\beta = \frac{f_e}{f_n} \qquad \textbf{(2.12)}$$

where:

f_e is the excitation frequency [Hz]

f_n is the natural frequency of mode n of the structure [Hz]

Steady-state response (Figure 2.5a) is significant if one of the natural frequencies of the structure is close to one or more of the harmonics of the exciting force.

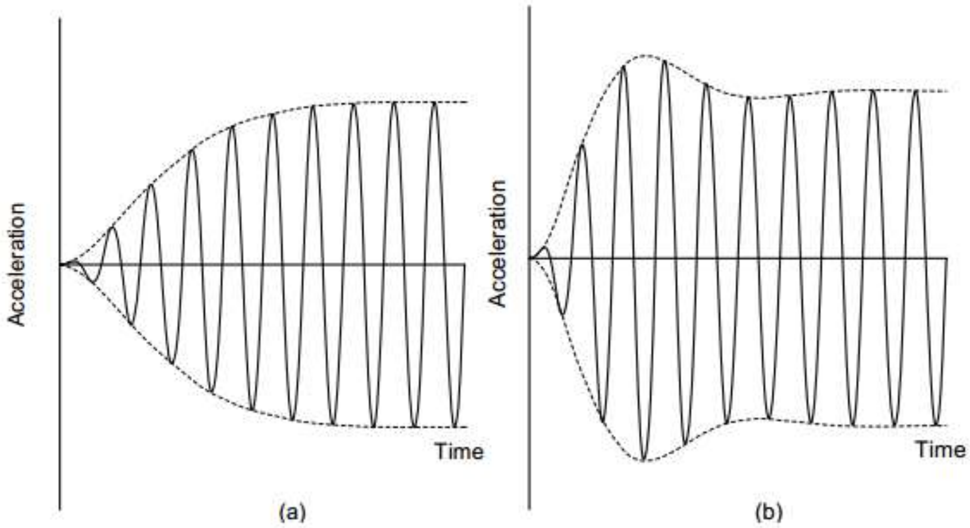


Figure 2.5: Response envelopes (Smith et al. 2009)

a) Steady state response. The wave form settles after a short transient part.
b) Transient response. The wave form is unstable for a longer period of time before settling.

If the fundamental frequency of a floor is greater than the fourth harmonic of the excitation force, the response from one footstep will die away before the next occurs. In this situation, the forcing function will appear as a series of impulses (see Figure 2.6), and the higher harmonics is of less importance.

When the fundamental frequency of a structure is high compared to the exciting frequency (frequency ratio $\ll 1$), the transient part will be dominant. A dominant transient part corresponds to the case where the applied force can be taken as a series of impulses, as illustrated in Figure 2.5b. The response to a series of impulses is illustrated in the figure below.

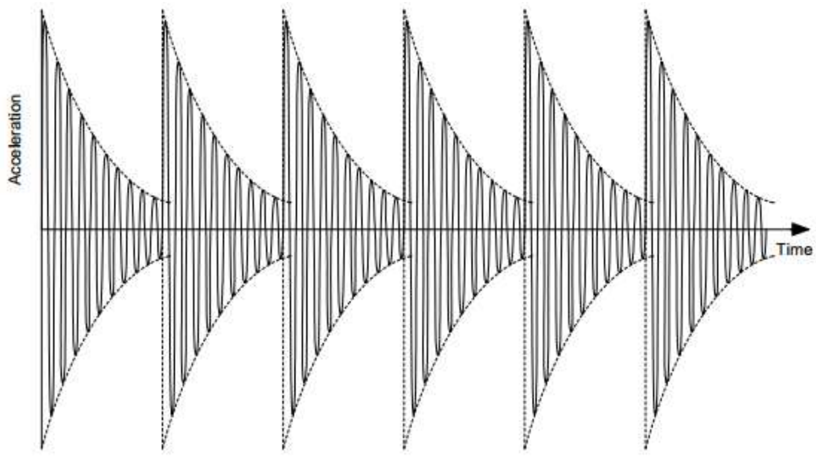


Figure 2.6: Impulsive response (Smith et al. 2009)

When $\beta = 1$, *resonance* occur, which causes large responses in the system. The excitation frequencies in between each natural frequency are called *off-resonant frequencies*. Even though the resonant frequencies result in a peak in response, off-resonant frequencies can cause a considerable response in the system. This means that avoiding resonant frequencies alone isn't enough to ensure that considerable vibrations in a system do not occur.

Dynamic magnification factor:

The *dynamic magnification factor* determines the magnitude of response of each mode at any frequency in a system. It is calculated from the ratio between the forcing frequency, the natural frequency, of the mode under consideration and the damping in the structure.

$$D_{n.h} = \frac{h^2\,\beta^2}{\sqrt{(1 - h^2\beta_{\,n}{}^2)^2 + (2\,h\xi\beta_n\,)^2}} \tag{2.13}$$

where:

- h is the number of the h^{th} harmonic [Hz]
- β is the frequency ratio
- ξ is the damping ratio

The values of the dynamic amplification factor as a result of the frequency ratio and the damping ratio is illustrated in Figure 2.7.

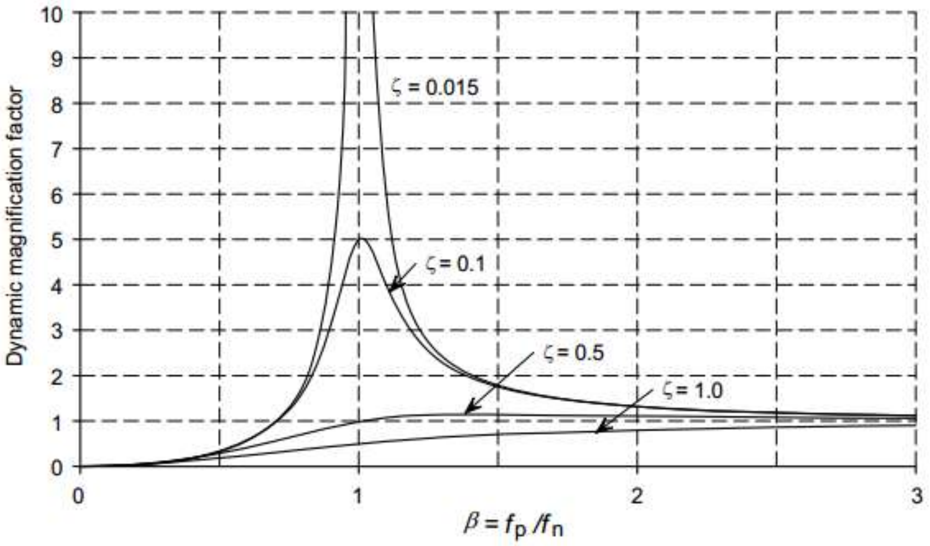


Figure 2.7: Dynamic magnification factor (Smith et al. 2009)

When the frequency ratio is 1, the dynamic magnification factor becomes very high, and in the theoretical case of no damping (ξ = 0), the dynamic magnification factor goes to infinity. As can be seen, the frequency ratio and damping in the structure is of great importance to the dynamic magnification factor, and by that the response amplitude.

2.1.3 Damping:

Damping results in more rapid decay of free vibration in a system. It is related to the conversion of mechanical energy to a form that is unavailable to the vibration (Mårtensson 2011).

In a timber floor, the total damping is a result of the damping characteristics of the wood (material damping), friction between joist and flooring (between components) and the boundary conditions at the supports in the structure. Material damping usually contributes to the smaller part of the damping, as friction between components and boundary conditions have proven to be more crucial.

Table 2.1: Different floor configurations give different damping ratios. (Hamm et al. 2010)

Type of floor	Damping ξ
Timber floors without any floor finish	0.01
Plain glued laminated timber floors with floating screed	0.02
Girder floors and nail laminated timber floors with floating screed	0.03

Table 2.2, illustrates how different floor configurations and load situations affect the damping as well. The table is concerning steel structures and only used as an example, as it illustrates how the non-structural elements influence the damping ratio of a system.

Table 2.2: Typical damping ratios for various floor types (Smith et al. 2009)

ζ	Floor finishes
0.5%	for fully welded steel structures, e.g. staircases
1.1%	for completely bare floors or floors where only a small amount of furnishings are present.
3.0%	for fully fitted out and furnished floors in normal use.
4.5%	for a floor where the designer is confident that partitions will be appropriately located to interrupt the relevant mode(s) of vibration (i.e. the partition lines are perpendicular to the main vibrating elements of the critical mode shape).

Effective damping ratio in a timber structure is in the range of 1-3 %, as it is difficult to obtain very stiff supporting conditions in practice. Imposed masses on the structure (objects, partitions) can increase the damping, especially if the system on its own is lightweight or small. In a timber floor system, the person walking across the floor will contribute with considerable mass and damping to the system, as timber floors can be light unless mass is added to the system. However, this is often not regarded in the verification methods for timber floors.

Damping, in general, is a property that is hard to determine and make use of, because of its complexity and the limited knowledge on quantification and measurements of damping in floor structures having several vibration modes.

Acceleration:

Acceleration is the second differential of displacement concerning time, meaning differentiation of the displacement equation twice gives the expression for calculating acceleration:

$$a(x.t) = \sum_{n=1}^{\infty} -4\pi^2 f_e{}^2 u_n \sin(2\pi f_e t + \phi_n) sin\left(\frac{n\pi x}{L}\right) \tag{2.14}$$

where:

t is the time [s]

f_e is the frequency of the forcing function [Hz]

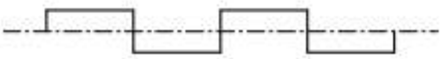
u_n is the maximum amplitude of mode n [m]

Φ_n is the phase lag of mode n [m]

u_n and Φ_n are determined from the initial excitement or forcing function.

There are several ways to present the acceleration of a system. *Peak acceleration* is a measure of the largest value of acceleration. However, it does not indicate for how long the system undergoes this maximum acceleration. To consider the wave form of the acceleration another measurement of the acceleration can be used: The *root-mean-square acceleration* is a measure of the mean value of acceleration, and widely used.

Table 2.3: Root-mean square acceleration for various wave forms, taken from (Hicks & Smith 2011)

Waveform		a_{peak}	a_{rms}
	Sine	1	$1/\sqrt{2}$
	Triangular	1	$1/\sqrt{3}$
	Square	1	1

The root-mean-square acceleration is calculated as follows:

$$a_{rms} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt} \tag{2.15}$$

where:

T is the period under consideration, needs to be taken as a time that will cover at least one complete cycle of acceleration [s]

$a(t)$ is the calculated acceleration response [m/s^2]

As the duration of high acceleration values is of great importance to human perception (see section 2.2), using the peak acceleration as a parameter in floor verification can be an overestimation of the effect of the vibration.

Human perceptibility to vibration is believed dependent on the vibration acceleration if the floor has frequencies below 8 Hz. It is believed dependent on vibration velocity in the case of frequencies above 8 Hz (Mohr 1999). This gives rise to different evaluation parameters for different floor-structures in many evaluation methods for floor serviceability, as will be discussed later.

Parameters used for evaluation:

Even though the amplitudes of the vibration are small, the effects can be severe, especially when the frequency is high. Large-amplitude, low-frequency motion can be observed visually by the maximum peak amplitude. This is rarely the case in floor motion, where the amplitudes are smaller but the frequency higher. Even though the displacement of the floor is too small to be detected visually, high-frequency, low-amplitude vibrations can contain a considerable amount of energy which is felt by the human body.

The velocity of the floor is closer related to the energy involved in the structural vibration than the displacement. The velocity of the floor vibration can be a good measure of the acceptability of the floor regarding human comfort. However, acceleration is commonly used to describe the severity of human exposure to vibrations, as it is instrumentally more convenient to measure then velocity. Root mean square acceleration is used rather than peak acceleration, as a_{rms} give a better overall indication of the vibration over time. Peak acceleration measures the sharp peaks in the acceleration, but since they are less significant regarding occurrence, it is not the best measure of the overall response of the floor.

High and low frequency floors:

Because of their different response to human walking, floors is often divided into high-frequency and low-frequency floors. Low-frequency floors are more responsive to the continuous part of human walking. High-frequency floors are more responsive to the impulsive part, i.e., the heel drop, as the response of one step dies away before the next occurs.

Table 2.4: Typical characteristics of high and low frequency floor:

	Floor type	
Characteristics	High frequency	Low Frequency
Fundamental	$f_1 > 8$ Hz	$f_1 < 8$ Hz
Dominant response:	Transient	Steady state
Mass:	Light weight (give a value)	Heavy (give a value)

Floors with a natural frequency below 7-8 Hz is classified as low-frequency, as this frequency is closer to the frequency of the excitation of the floor caused by human walking.

Short or medium spanned floors have a response consisting of both high-frequency forced vibrations and low-frequency resonant vibrations. (Smith 2003)

2.2 Human perception of vibration and human induced loads

Human activity leads to a wide range of vibration situations. The effects of human activity in a dance hall or gymnasia are very different from the ones in an office or residential building. Synchronized movement of people (dancing or exercising) is especially problematic, as this lead to approximately periodic loads, producing almost steady state structural vibration. Structures, where these kinds of activities are likely to occur, should be investigated thoroughly and designed for these load situations. The characteristics and number of persons involved in the activity affect the forces produced, along with the characteristics of the floor surface.

Soft flooring will store the energy from the foot fall and prohibit it from further distribution in the system, and reduce the effect of transient vibrations. It does not, however, significantly influence the continuous vibrations due to walking.

2.2.1 Human perception of vibrations

Low frequency vibrations are detected by humans as visual, audio and acceleration cues (Zhang et al. 2013). Visual cues can be the movement of objects resting on the structure or movement of the structure itself, relative to the observer. Audio cues can be cracking created by movement of the structure. Lastly, acceleration of the structure causes forces on the human body that are felt by the balance organs (Smith 2003).

2.2.2 Factors affecting individual perception:

The activity of the person experiencing the vibration is of great importance, as well as the proximity to and awareness of the source of the vibration. For example, vibrations are more likely to be perceived as unacceptable if the person experiencing the motion is at rest, and the source of the vibrations are in an adjacent residential unit. If the source of the vibration is known, the motion is less likely to be unacceptable (Smith 2003).

These aspects are considered in the following definitions from (Ohlsson 1984):

- *Springiness* is associated with the sensation of self-generated floor deflection and vibration from a single footstep during the time of contact between foot and the floor surface.

Springiness is usually associated with lightweight (and high frequency) floors. The response of such a floor is related to static flexibility (deformation under a static concentrated load) and impulsive velocity response.

- Vibrational disturbances are caused by foot-fall on a floor and characterized by the perception of floor vibration induced by other persons than the one that is disturbed.

Vibrational disturbances encompass stationary velocity response and impulsive velocity response. It is mostly related to heavy floors, having a low natural frequency.

The human body is especially sensitive to vibrations with frequencies in the range of 4-8 Hz since this is the frequency range of human internal organs. Structural frequencies in this range are not desired (Smith 2003).

The perception of vibration depends on the direction of the vibration compared to the direction of the human body. It also depends on the frequency of the vibration as the human sensitivity to a vibration amplitude changes with frequency. For frequencies where perception is less sensitive, this can be taken advantage of by attenuation of the calculated response or enhance the *base value of acceleration*. Enhancing the base value or attenuating the calculated response, is called *frequency weighting*, and utilized when calculating a_{RMS} and *vibration dose values* in the verification approach presented in section 3.6.

As the perception and acceptability of vibration vary from person to person, it is hard to satisfy all users of a building at all times. Level of activity of the person experiencing the vibrations affect the level of *acceptance* of vibrations as well as the perceptibility. A person will most likely take a certain level of vibration as annoying when resting in their own home, while the same level of vibration will be accepted in a public space or office building.

The *duration* of vibration also is of great importance, as short, rarely occurring vibrations are in general more acceptable than longer lasting and regularly occurring vibrations. This is investigated in detail in section 3.6.

2.2.1 Excitation forces due to walking

Walking is one of the most important sources of vertical excitation forces on a floor in a residential building, as the forces are both produced and perceived by the occupants. The effect of a single foot fall can be illustrated in a time-force diagram, as in Figure 2.8. Walking differs from running by that running causes both the feet to lift at the same time and shorter contact time between foot and ground. This results in more energy forced on the ground over a shorter period, and no overlapping steps. Walking steps overlap so that the result is both continuous and impulse-behaviour.

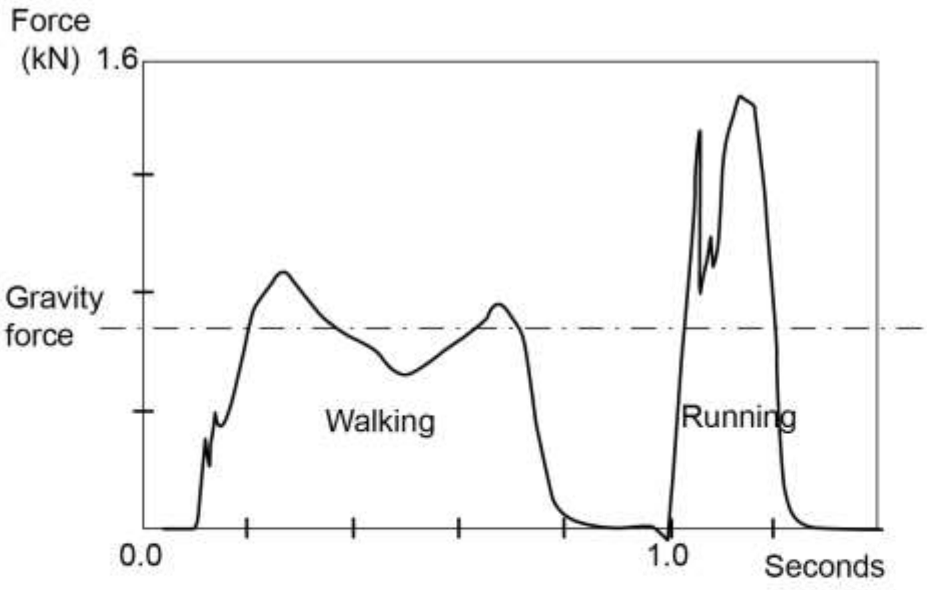


Figure 2.8: Illustration of foot fall forces, from (Smith 2003)

The vertical force of walking (and running) is characterized by two peaks: One related to the foot fall and one related to the toe uplift. The main difference between the two activities is that the force peaks form a running step is larger but has a shorter duration. The force from both running and walking extend in a very short period, less than half a second and a second, respectively (Figure 2.8).

Below is a walking activity illustrated as a series of single steps. Two successive steps overlap, resulting in the possibility of amplitude amplification.

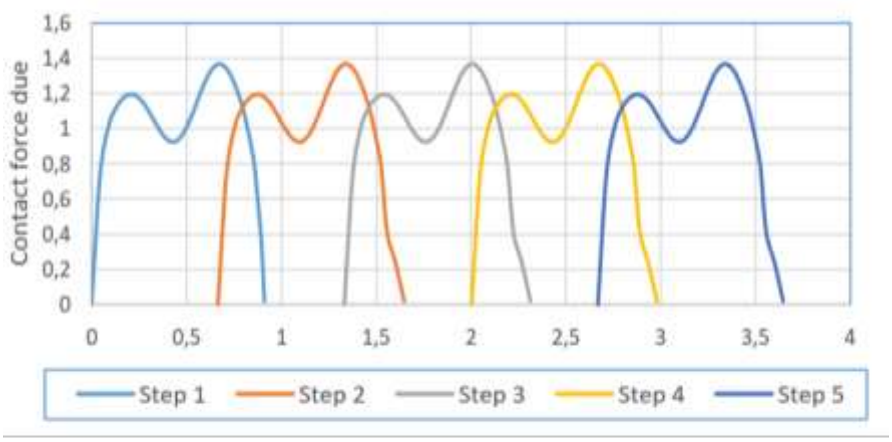


Figure 2.9: Continuous contact between foot and floor.

Common pace frequencies vary from 1.8 Hz to 2.2 Hz. Shorter walking paths give lower pace frequencies, and for further use in this thesis, a pacing frequency of 1.8 Hz is used. The velocity of a walk can be calculated using the following relation, reproduced in and taken from (Smith et al. 2009):

$$\nu = 1.67\, f_p^{\ 2} - 4.38\, f_p + 4.50 \text{ [m/s}^2\text{]} \tag{2.16}$$

where:

f_p is the pace frequency [Hz]

The duration of a single foot fall, taken from (Sedlacek et al. 2009), is found by:

$$T_p = 2.6606 - 1.757\, f_p + 0.3844\, f_p^2 \text{ [Hz]} \tag{2.17}$$

2.3 Equivalent bending stiffness and effective width

The deflection of a timber joist floor under a concentrated static force is dependent on both the stiffness in longitudinal and transversal direction; along- and across-joist, respectively. Also, the spacing between the joists is important, as a closer spacing will improve the overall longitudinal stiffness. The stiffness of the sheeting, that is the transversal (or across-joist) stiffness, dictates how well the applied load is distributed between neighbouring joists. A stiffer plate will be able to distribute the loads more effectively. Reducing joist spacing will increase the longitudinal stiffness of the floor if the subfloor is distributing the loads between neighbouring joists.

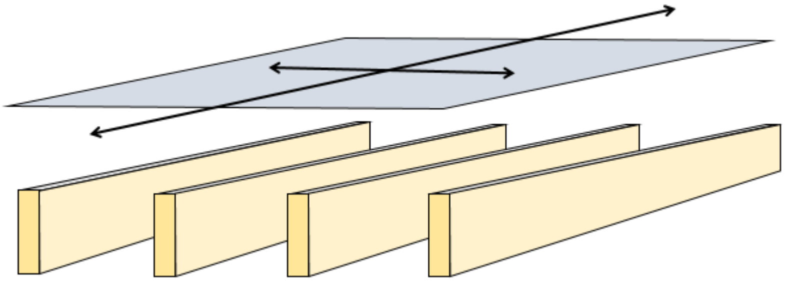


Figure 2.10: Longitudinal and transversal stiffness of a joist floor.

The stiffness of the plate will also contribute to longitudinal stiffness, even though the plate material often has a much lower modulus of elasticity (MOE) than the joist material, and has a thin cross section. When the connection between the joist and plate is stiff enough, the advantages of a combined t-cross section can be considered.

$$EI_{long} = EI_{joist} + EI_{plate} + a^2 \left(\frac{1}{EA_{joist}} + \frac{1}{EA_{plate}}\right)^{-1} \text{ [Nm}^2\text{]} \qquad \textbf{(2.18)}$$

where:

EA_{joist}	is the mean MOE of the joist, times the joist area [N]
EI_{joist}	is the mean MOE of the joist, times its second moment of area [Nm^2]
EA_{plate}	is the MOE the plate material, times the plate area [N]
EI_{plate}	is the MOE of the plate, times its second moment of area [Nm^2]
a	is the distance between the centroids of the two materials [m]

As the plate material often is quite thin, or has low MOE, it is limited how much this layer contributes to the longitudinal stiffness in a timber floor. However, as the transportation moment is included in the calculation of EI_{long}, the contribution is not neglectable.

When knowing the longitudinal stiffness of the floor configuration, it can be used to find the *equivalent bending stiffness*, EI_L, along the joist direction. EI_L is the bending stiffness of the along-joist distributed over a width equal to the centre distance between each joist in the floor:

$$EI_L = \frac{EI_{long}}{Js} \text{ [Nm}^2\text{/m]} \qquad \textbf{(2.19)}$$

where:

J_s	is the joist spacing [m]
$EI_{longitudinal}$	is the total longitudinal stiffness of the floor configuration [Nm^2]

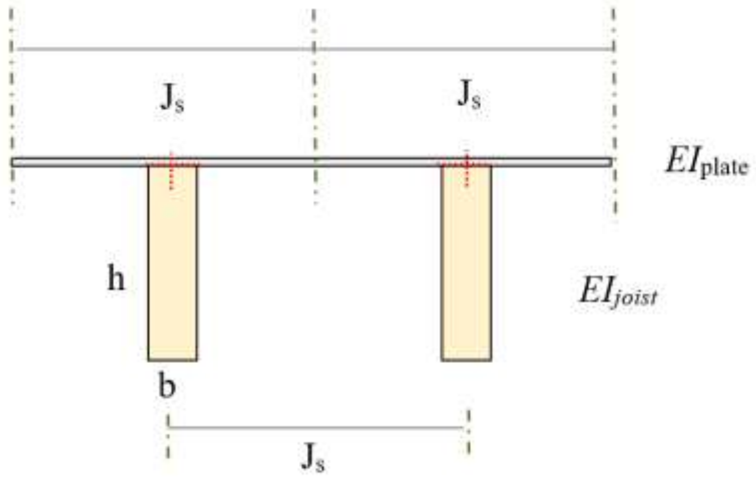


Figure 2.11: Joist spacing gives distribution of the longitudinal stiffness, of a T-cross section with a stiff connection (glued + screwed) connection.

EI_L is used to find the deflection of the floor-joist configuration under the load of a concentrated static force F:

$$w_{static} = \frac{F \cdot l^3}{48 \cdot EI_L \cdot Js} \cdot {}^{1}\!/_{1000}\ [mm] \qquad \textbf{(2.20)}$$

where:

EI_L is the equivalent bending stiffness along the joist [Nm^2/m]

F is the concentrated force applied [N]

The shear stiffness is ignored in the deflection formula, which simplifies the calculation of deflection considerably. If a more detailed calculation is desired, this is suggested in (Thiel 2012), among others. Note that in Equation (2.20 only the longitudinal stiffness of the floor is taken into account, making it a one-dimensional (1D) system. The two-dimensional (2D) behaviour of the floor is discussed in subsection 2.3.1.

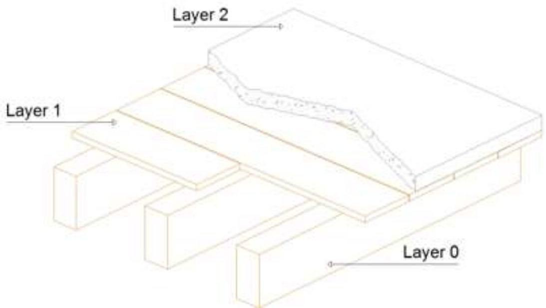


Figure 2.12: Layers in a typical timber joist floor (Timber Tech)

Figure 2.12 illustrates a typical timber joist floor, where “layer 0” is the joists, considered only to have stiffness in the longitudinal direction, “layer 1” and “layer2” are plate materials with stiffness also in the transverse direction.

2.3.1 From 1D joist to 2D floor:

The deflection formula presented in Equation (2.19 only represent a 1D system. A floor is often considered a 2D structure, and effects such as transversal stiffness have to be taken into account. To translate the 2D properties of a floor into a simple 1D-system, an “equivalent beam” is made. The width of this beam is determined depending on the transversal stiffness of the floor and called *effective width* b_{ef} (Mohr 1999). The expression for the effective width was derived numerically from a wide range of floors having different properties, using finite elements.

The value of b_{ef} is taken as:

$$b_{ef} = \frac{L}{1.1} \cdot \sqrt[4]{\frac{(EI)_T}{(EI)_L}} \tag{2.21}$$

where:

EI_T is the equivalent bending stiffness in transversal direction [Nm^2/m]

Since the effective width is dependent on the relationship between the transversal and longitudinal stiffness, there are some situations where b_{ef} is smaller than the actual joist spacing. This is when the ratio EI_T/EI_L is very small. In these cases, b_{ef} should not be used. In the following section 3, four analytical approaches are presented, and the use of b_{ef} will be further investigated. If the transversal and longitudinal stiffness is the same, as in a theoretical isotropic plate, the ratio EI_T/EI_L will be 1, giving the largest theoretical value of b_{ef}.

EI_T is found by considering the stiffness of the flooring about an axis transversal to the span direction. If the flooring consists of more than one layer, the stiffness in the layer is summed (given that the connections between the layers and the joist are stiff enough to transfer loads).

$$EI_{transversal} = EI_{0,tran.} + EI_{1.tran} \cdots + EI_{n.tran} \tag{2.22}$$

where:

$EI_{n\ tran.}$ is the bending stiffness of the n^{th} flooring element.

Where the longitudinal stiffness is divided by joist spacing to find equivalent bending stiffness, the transversal stiffness is divided by 1 m, to obtain *transversal bending stiffens per meter*.

$$EI_T = \frac{EI_{transversal}}{1\ meter} \tag{2.23}$$

where:

$EI_{transversal}$ is the total transversal stiffness of the materials in the subfloor.

It should be noted that by simply adding the transversal stiffness's of the different plate materials, and by that ignoring the transportation moment in this configuration, the total transversal stiffness is underestimated, but the error will not be severe if the plate materials are thin.

The equation for calculating the deflection of the joist floor which considers the transversal stiffness of the floor is:

$$w_{static} \frac{F \cdot l^3}{48 \cdot EI_L \cdot b_{ef}} [mm] \tag{2.24}$$

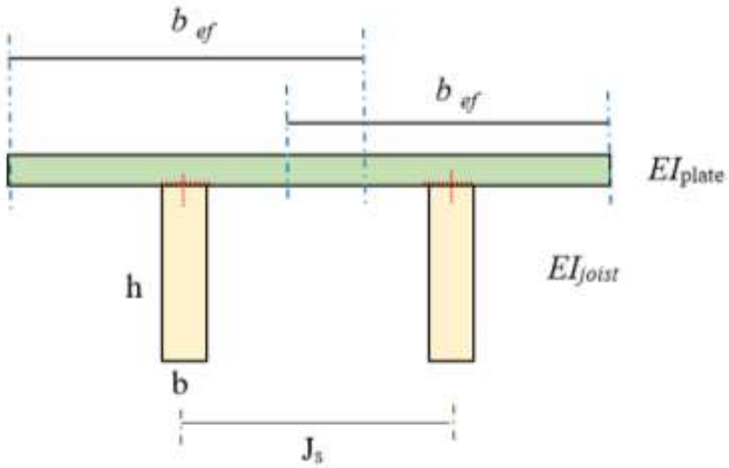


Figure 2.13: A stiffer sub floor/ plate material allows for the effect of transversal distribution to be taken into account, through the use of *effective width* b_{ef}. $b_{ef} > J_s$

The effective width, b_{ef} is used to obtain the stiffness and effective mass per unit length of the equivalent beam:

$$EI_{eq.beam} = EI_L \cdot b_{ef} \tag{2.25}$$

$$m_{eq.beam} = m_{floor} \cdot b_{ef} \tag{2.26}$$

These values are in turn used in several analytical approaches to obtain the fundamental frequency and static floor deflection in a floor.

3 Presentation of analytical methods

Analytical methods are characterised by simple expressions and are suitable for prediction of vibrational serviceability in the design phase. Physical parameters are related to limitation values for verification. The parameters used in the presented methods are fundamental frequency, vertical displacement, vibrational velocity, and acceleration.

The methods presented here, largely build on the work of Ohlsson (Ohlsson 1984) and Mohr (Mohr 1999).

The methods all have in common fundamental frequency and static deflection as design parameters. The fundamental frequency requirement is introduced to avoid a large increase in peak acceleration of the floor, due to resonance (Mohr 1999). The limitation of static deflection is used as a stiffness requirement, and the acceleration as a mass requirement.

3.1 Eurocode 5, Norwegian National Annex

The method for verifying vibrational properties according to Eurocode 5 is based on the work by Ohlsson ((Ohlsson 1991) and (Ohlsson 1984)). The human sensitivity to structural vibrations is according to Ohlsson characterized by being:

- related to vibration *velocity* when f_1 of the floor is higher than 8 Hz.
- related to vibration *acceleration* when f_1 of the floor is lower than 8 Hz.
- *increasing* by increased duration of vibration.
- *decreasing* by physical activity of the observer.
- *decreasing with awareness* of the vibration source.

These statements were the basis of systematically experimental testing and numerical analysis and led to the proposition of parameters for controlling the vibration serviceability design of timber floors. The three parameters adopted in EC 1995-1-1 is

- fundamental frequency, f_1
- static deflection under a point load applied in the centre of the floor, w
- velocity response under a unit impulse, v

National Annexes to Eurocode 5:

To make the Eurocodes more easily implemented, each country has the opportunity to adjust the codes using national annexes. The result is many different approaches on how to address the same problems throughout Europe. For more details, it is suggested to look into “Comparison of vibrational comfort assessment criteria for design of timber floors among the European countries” (Zhang et al. 2013). In this report, 13 different national annexes to Eurocode 5 has been reviewed and compared.

In this thesis, the Norwegian and Austrian national annexes will be presented. The Norwegian national annex (EC5 Norwegian NA 2010) is chosen for its relevance in Norway and the

Austrian national annex (EC5 Austrian NA 2014) because it is one of the few national annexes reporting an alternative method for the assessment of timber floors. This alternative method will also be the basis of the new proposal for Eurocode 5.

Fundamental frequency (Frequency requirement - repeated cyclic actions):

Eurocode 5, section 7.3.3 (1) states that if the fundamental frequency of the floor is greater than 8 Hz, a special investigation must be made. It does not, however, state what this investigation should be. Some national annexes give suggestions to this, for example the Austrian National Annex, which is presented later.

According to EC 1995-1-1 7.3.3(3), the calculations presented in 7.3.3(2) (that is calculation of f_1, w, and v) should be performed regarding the floor as un-loaded. This means the total self-weight of the floor, both structural and non-structural components, and other permanent loads should be regarded.

Eurocode 5, section 7.3.3 (4) gives this suggestion on how to calculate the fundamental frequency, which is a re-writing of the well-known SDOF equation for the natural frequency of vibration for simple harmonic motion 2.1.1. This value must be less than the limiting value of 8 Hz, resulting in the following criteria:

$$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI_L}{m}} \geq f_{limit} = 8\ Hz \tag{3.1}$$

where:

l is the span of the floor [m]

EI_L is the equivalent bending stiffness in a plane about an axis perpendicular to the span [Nm^2/m]

m is the mass per area [kg/m^2]

Static deflection (stiffness requirement - impulses with longer duration):

Maximum instantaneous deflection, w, caused by a static concentrated load, F, must be no greater than the limiting value, a. A value of 1 kN is used as F. No formula for w is suggested, but in (EC5 Norwegian NA 2010), pt. 7.3.3. (2), it is stated that the transversal distribution should be considered. Based on that, I have chosen to use Equation (2.24) in the calculation of w, in which the deflection is dependent on b_{ef}. The verification criteria for stiffness is:

$$\frac{w}{F} \leq a \qquad \left[\frac{mm}{kN}\right] \tag{3.2}$$

where:

a is limiting value, dependent on floor requirements.

Figure 7.2 in Eurocode 5 gives a range of values *a*, but no limiting value. Different values of *a* are given in the various national annexes (NA) of the European countries. The Norwegian NA gives two options for the limit value a, depending on the demands related to the floor:

a = 0.6 for floors demanding high stiffness.
a = 0.9 for normal floors/ all other floors.

These limiting values are applicable for floors with free span < 4.5 m, according to (EC5 Norwegian NA 2010). It's not given information on how floors with longer spans should be treated.

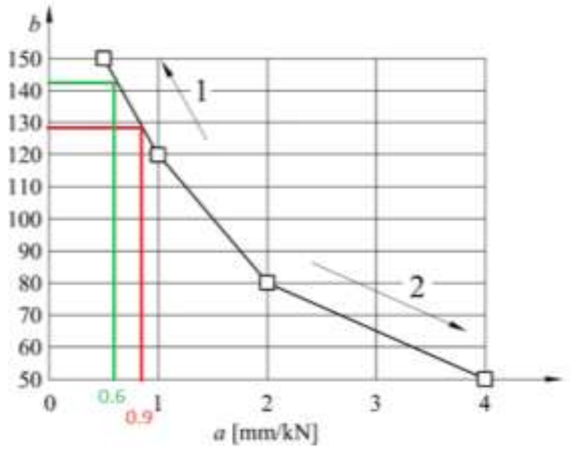


Figure 3.1: Figure 7.2 in Eurocode 5. Limiting value *a*, related to the parameter *b*. For use in Norway, limits for *a* is given in (EC5 Norwegian NA 2010)

Better performance is related to smaller values of *a*, giving larger values of *b*. The Norwegian limiting values are indicated by red and green lines.

Vibration velocity response (mass requirement – impulses with shorter duration):

The impulse velocity response, v, caused by an ideal unit impulse (1 Ns) applied at the point of the floor where it gives the maximum response. Maximum impulse velocity response needs to be smaller than a limiting value consisting of the parameter *b*, the modal damping ratio ξ and the fundamental natural frequency of the floor.

The criterion is as follows:

$$\nu \geq b^{f_1 \xi - 1} \qquad \left[\frac{m}{Ns^2}\right] \tag{3.3}$$

where:

b is found by knowing the limit value *a* (see Figure 3.1)

The value of ξ is recommended as 0.01, unless e.g. measurements of the structure under investigation give another value, according to Eurocode 5 pt. 7.3.1(3).

$$v = \frac{4(0.4 + 0.6n_{40})}{mBL + 200} \tag{3.4}$$

where:

B	is the width of the floor [m]
L	is the free span of the floor [m]
n_{40}	is the number of first order modes with natural frequency up to 40 Hz.

Frequencies above 40 Hz is considered as not affecting the human perception of structural vibrations, and so their contribution to the vibration velocity is neglected (Ohlsson 1984).

$$n_{40} = \frac{B}{L}\left\{\left(\left(\frac{40}{f_1}\right)^2 - 1\right)\frac{(EI)_L}{(EI)_B}\right\}^{1/4} \tag{3.5}$$

The unit impulse velocity response is disregarded as a vibrational parameter for serviceability limit state design in the Norwegian national annex. This is due to measurement difficulties and the theoretical complexity of the criteria (EC5 Norwegian NA 2010).

Summary:

A summary of the verification criterions according to Eurocode 5, Norwegian NA is shown in Table 3.1:

Table 3.1: Summary of the verifications according to EC5, Norwegian National annex.

	Verification:	Limit value:
Fundamental frequency	$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI_L}{m}} \geq f_{limit}$	8 Hz
Static deflection	$\frac{w}{F} \leq a$	0.9 mm/kN (high stiffness) 0.6 mm/kN (normal)

As the Norwegian national annex does not utilise the velocity response of Eurocode 5, it does not have any verification of the mass requirement at all. As timber floors are generally light weight and high frequency floors, it is a potential problem that the method fails to address the mass requirement.

3.2 Method presented by P. Hamm, A. Richter and S. Winter

What is presented here is based on the work of (Hamm et al. 2010) submitted for the World Conference on Timber Engineering in 2010. This is based on the work of (Mohr 1999). Hamm et al. performed experimental testing of the theories from Mohr. In this paper, the method presented will be called Hamm/Richter, although the basis for their work was that of Mohr.

The method focuses on the fundamental natural frequency, and the stiffness of the floor expressed as the magnitude of the deflection caused by a static point load. In addition, a criterion regarding the acceleration response of the floor is given for those cases where the fundamental frequency is below 8 Hz.

Floor requirements:

The limiting values for first natural frequency, deflection by point load, and vibration accelerations are highly dependent on the level of requirement of the floor. In (Hamm et al. 2010), the correlation between different floor parameters and subjective evaluation were sought. The floors were given marks ranging from 1 to 4, where the score 1 was given if no vibration problem were detected, 4 were given if heavy vibrational problems were detected and 2-3 represent in-between vibrational problems. Based on these evaluations, limit values for *high demand*, *normal demand*, and *no demand* floors were set (see Table 3.2).

Level of requirements is dependent on the position of the floor in the building and type of use of the floor: The highest demand is related to floor structures between different units of use, lower demands related to floors between different areas in the same unit of use. Floors under not used rooms are an example of floors with no demands.

Table 3.2 shows the limit values given from the level of requirement.

Table 3.2: Limiting values dependent on floor requirements.

	High demand floors	Lower demand floors	Floors with no demands
Evaluation	1.0-1.5	1.5-2.5	2.4-4.0
f_{limit}	8 Hz	6 Hz	-
w_{limit}	0.5 mm	1.0 mm	-
a_{limit}	0.05 m/s^2	0.10 m/s^2	-

Fundamental frequency (Frequency requirement - repeated cyclic actions):

According to (Thiel 2012), the self weight and permanent loads are regarded in the mass calculations in the Hamm/Richter-merthod. In further calculations in this thesis, the mass is taken as self weight only. This is done to have the same mass in every calculation, and because I have no information of what the permanent loads should be. An assumtion could be made for this, but I have chosen to use self weight only.

The formula for calculating the fundamental frequency considereds the transversal stiffness in the floor. According to (Thiel 2012), the fundamental frequency in this method can be calculated as:

$$f_1 = \frac{\pi}{2\, l^2}\sqrt{\frac{EI_L}{m}} \cdot \sqrt{1 + \left(\frac{L}{B}\right)^4 \cdot \frac{EI_T}{EI_L}} \geq f_{crit} \qquad \textbf{(3.6)}$$

where:

$EI_T < EI_L$ and

L is the floor span [m]

B is the width of the floor [m]

m is the mass per unit area [kg/m^2]

$(EI)_L$ is the bending stiffness in the along-joist direction [Nm^2/m]

$(EI)_T$ is the bending stiffness transverse to the floor span, per meter width [Nm^2/m] (Equal to EI_T).

Here, the twistingstiffenss of the joists are ignored, but the transversal stiffness of the floor is considered.

Again, there is a dissonance between what is presented in (Thiel 2012) and (Hamm et al. 2010), where Hamm et al. suggests using Equation (3.6) only when there is bearing on all four sides of the floor. If it's only two bearing sides, Equation (3.1) should be used.

If the fundamentaql frequency is below f_{limit}, an investigation of the vibration acceleration has to be done. The fundamental frequency cannot be less than 4.5 Hz. If this is the case, the floor is regarded as not satisfactory regardless of the outcome of the acceleration investigation.

Static deflection (stiffness requirement - impulses with longer duration):

The stiffness is controlled by looking at the vertical deflection under a concentrated static load of 2 kN at the most severe point at the beam (min-span). The deflection formula to use is the one for a simply supported beam, also for continuous beams. This substitution of original system into a single beam, as well as the use of 2kN static load, is based on better correlation between the values of deflection calculated and subjective evaluation of behaviour. (Hamm et al. 2010)

To consider the transversal capacity of the floor, the effective width of the floor, b_{ef} is used when calculating the deflection.

$$b_{w(2kN)} = min\begin{cases} b_{ef} \\ B \end{cases} \qquad \textbf{(3.7)}$$

where:

b_{ef} is calculated as in (2.23).

B is the width of the floor [m].

The formula for calculating the static deflection is same as the one of the equivalent beam, (2.24), with effective width b_{ef} as described in section 2.3.1.

Acceleration response (mass requirement – impulses with shorter duration):

In (Thiel 2012) a modified version of the Hamm/Richter approach is presented. Here the calculation of acceleration response is more complex and computational heavy. In this thesis, the acceleration response is calculated in the report "Floor vibrations -new results" (Hamm et al. 2010), where the acceleration of a single span girder is found from:

$$a = \frac{F_{dyn}}{M^* \cdot 2\,\zeta} = \frac{0.4 \cdot F(t)}{(m \cdot 0.5B \cdot 0.5L) \cdot 2\,\zeta} \quad [m/s^2] \tag{3.8}$$

where:

M^*	is generalized/modal mass [kg]
F_{dyn}	is the total dynamic force [N]
$F(t)$	is the harmonic parts of the force on the floor [N]
L	is the span [m]
B	is the minimum of *1.5*L*, and width of the floor [m]
ξ	is the modal damping ratio

The released force due to walking, *F(t)*, is reduced by a factor 0.4 to consider that the force is not always acting in the middle of the span, and only acting at a limited time. *F(t)* is dependent on the fundamental natural frequency, and the relation can be seen in Table 3.3:

Table 3.3: Relation between f_1 and *F(t)* from (Hamm et al. 2010)

Fundamental natural frequency	Released force in the course of walking
f_1 [Hz]	F(t) [N]
$4.5 < f_1 \leq 5.0$	140
$5.0 < f_1 \leq 7.5$	70

The values of modal damping ratio are dependent on the type of floor constructions, as given in Table 3.4:

Table 3.4: Values of damping ratios for different floor types, (Hamm et al. 2010)

Type of floor	Damping ξ
Timber floors without any floor finish	0.01
Plain glued laminated timber floors with floating screed	0.02
Girder floors and nail laminated timber floors with floating screed	0.03

There is an updated calculation method for acceleration response according to Hamm/Richter, presented in chapter 5 of (Thiel 2012). The modified verification also suggests a more refined calculation also for fundamental frequency. This is not presented in this thesis.

Summary:

A summary of the verification criterions according to the Hamm/Richter-approach presented in (Hamm et al. 2010) is shown in table Table 3.5:

Table 3.5: Summary of the verifications according to Hamm/Richter.

	Verification:	Limit value:
Fundamental frequency	$f_1 = \frac{\pi}{2\,l^2}\sqrt{\frac{(EI)_L}{m}}\sqrt{1+\left(\frac{l}{b}\right)^4\cdot\frac{(EI)_B}{(EI)_L}} \geq f_{crit}$ or $f_1 = \frac{\pi}{2\,l^2}\sqrt{\frac{(EI)_L}{m}} \geq f_{crit}$	8 Hz
Static deflection	$w_{static} = \frac{F\ \cdot l^3}{48\ \cdot EI_L \cdot b_{w(2kN)}} \leq w_{limit}$	0.5 mm (high demand) 1.0 mm (low demand)
Acceleration response	$a = \frac{F_{dyn}}{M^* \cdot\ 2\,\zeta} \leq a_{limit}$	0.05 m/s^2 (high demand) 0.10 m/s^2 (low demand)

A the method only considers acceleration response in the case of low frequency floors, the mass requirement is only partially adressed. High frequency, low weight floor may cause vibrational problems this approach won't be able to detect.

3.3 Eurocode 5, Austrian National Annex

The Austrian National Annex is one of the annexes that provide alternative equations and limiting values for fundamental frequency. It makes use of acceleration response criteria in the case of f_1 being lower than 8 Hz and does not make use of the velocity response criterion. The approach in the Austrian NA is, as the method presented above, highly dependent on floor requirements/classes. The method is almost identical to the one of Hamm/Richter presented in 3.2, but with some extensions to take into consideration the effect of static system and number of spans.

Table 3.6, presented below, is based on table NA.7.2-E1 in (EC5 Austrian NA 2014) showing the floor classification.

Table 3.6: Floor classes used in Austrian NA.

	Floor class 1	Floor class 2	Floor class 3
Category of use according to EN 1991-1-1	A, B, C1, C3, D		A
Type of floor construction/ utilization.	- Between different areas of utilization - Between apartments - Office floors, computer work stations or conference rooms. - Short span corridors.	- Floors within the same apartment - Floors in one-family dwellings with usual utilization	- Floors underneath rooms without residential purpose or non-developed attics - Floors without requirements regarding vibrations

Below is a summary of the criteria and limitation values associated with the floor categories. Floors in class 3 does not need further investigation, and are not relevant for this thesis.

Table 3.7: Limitation values related to floor classes

	Floor class 1	Floor class 2	Floor class 3
f_{limit}	8 Hz	6 Hz	-
w_{limit}	0.25 mm	0.5 mm	-
a_{limit}	0.05 m/s^2	0.10 m/s^2	-

Instead of verification of velocity response, the annex suggests a verification of acceleration response, similar to what is done in (Hamm et al. 2010). If the fundament frequency is below f_{crit}, an investigation of the vibration acceleration has to be done. The fundamental frequency cannot be less than 4.5 Hz. If this is the case, the floor is regarded as not satisfacroty regardless of the outcome of the acceleration investigation.

Fundamental frequency (Frequency requirement - repeated cyclic actions):

According to (Zhang et al. 2013), the mass should be determined using the quasi-permanent combination of dead loads and imposed loads, as in Equation 6.16b in Eurocode 0 (EC0 Norwegian NA 2008):

$$m = m_{Gk} + \psi_2 m_{Qk}$$

However, in (EC5 Austrian NA 2014) the mass used in vibration calculations should be taken as:

$$\sum_{j\geq 1} G_{k.j}$$

I have chosen to use the mass calculation as suggested in Austrian national annex, and so only the self-weight of the floor is regarded.

To take into account the effect of transversal stiffness, Equation (3.6) for f_1 from the Hamm/Richter-approach is presented also in the Austrian NA. Equation (3.6) can be used for "single span floor with transversal distribution". It is noted that the effect of the transversal stiffness should only be considered when $\frac{EI_T}{EI_L} \geq 0.05$. If this is not the case, the fundamental frequency is calculated as in the main document of Eurocode 5 (Equation (3.1 in section 3.1).

The equation for calculation fundamental frequency is similar to the one used in the Hamm/Richter approach, but in addition, the effect of static system and two spans are taken into account in a *modified equation for fundamental frequency*:

$$f_1 = k_{e.1} \cdot k_{e.2} \cdot f_{1\,calculated} \quad \textbf{(3.9)}$$

where:

$k_{e.1}$ is a factor considering the effect of the static system of the floor.

$k_{e.2}$ is a factor considering the effect of two floor spans.

The effect of the static system:

As different support conditions influence the overall properties of the system, this can be accounted for by multiplying the fundamental frequency by a factor $k_{e.1}$. From table NA.7.2-E2 the values of $k_{e.1}$ can be taken. This table is replicated below:

Table 3.8: Coefficients for consideration of different types of support

Support	$k_{e.1}$
Pinned - pinned	1.000
Restrained - pinned	1.562
Restrained - restrained	2.268
Restrained – free (cantilever beam)	0.356

As the floor under investigation is regarded as simply supported a value of 1.0 is used.

The effect of two spans:

The factor $k_{e.2}$ is found from table NA 7.2.-E3. This factor is dependent on the ratio between the two spans, as shown in Table 3.9

Table 3.9: Coefficients for considering the effect of two spans

l_1/l_2	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$k_{e.2}$	1.000	1.090	1.157	1.206	1.245	1.282	1.318	1.359	1.410	1.474	1.562

As the floor under investigation is a one-span floor, a value of 1.0 is used.

Static deflection (stiffness requirement - impulses with longer duration):

The Austrian National Annex gives a suggestion on how to calculate the deflection. The maximum static deflection due to a 1 kN load, positioned in the most unfavourable position of a single span joist is found as in Equation (2.21). Instead of joist spacing J_S, the effective width b_{ef} (see section 2.3.1) is used.

When calculating static deflection, a continuous beam is to be approximated as a single-span beam, having span equal to the maximum span in the continuous beam system. This means that the factors $k_{e.1}$ and $k_{e,2}$, used in the calculation of f_1, are both set to 1.0.

Acceleration response (mass requirement – impulses with shorter duration):

Austrian National Annex suggests using acceleration response instead of velocity response as a parameter in the assessment of a timber floor. This is similar to the approach suggested by Hamm/Richter.

For verification of floor class 1 and 2, the following expression for acceleration response is suggested:

$$a_{rms} = \frac{0.4 \cdot \alpha \cdot F_0}{2 \cdot \zeta \cdot M^*} \; [m/s^2] \qquad \textbf{(3.10)}$$

where:

α is the Fourier coefficient, depending on the fundamental frequency:

$\alpha = e^{-0.4 f_1}$

M^* is the modal mass [kg]:

$M^* = m \cdot \frac{l}{2} \cdot b_F$

b_F is effective width, m

L is the floor span, m

m is the mass of the floor in kg/m^2

F_0 is the vertical load of a walking person, usually taken as 700 N

where b_F is calculated as b_{ef} in Equation (2.23).

Values for damping ratio is suggested. Where Eurocode 5 only suggests a value of ξ = 0.01 (Eurocode 5 7.3.1 (3)) the Austrian National Annex suggests a number of values, depending on the floor construction type. These values are shown in the table below:

Table 3.10: Modal damping ratio for different types of floor, translated (EC5 Austrian NA 2014)

Type of floor construction	Modal damping ratio ξ
Floors with or without light ballasting	0.01
Floors with floating screed	0.02
Cross laminated timber floors with or without ballasting	0.025
Timber joist floors or nail laminated timber with unbonded screed	0.03
Cross laminated timber floors with unbonded screed and heavy ballasting	0.04

Summary:

A summary of the verification criterions according to the Austrian NA to Eurocode 5 is shown in Table 3.11:

Table 3.11: Summary of the verifications according to EC5, AU NA

	Verification:	Limit value:
Fundamental frequency	$f_1 = \frac{\pi}{2\,l^2}\sqrt{\frac{(EI)_L}{m}} \cdot \sqrt{1+\left(\frac{l}{b}\right)^4 \cdot \frac{(EI)_B}{(EI)_L}} \geq f_{crit}$ or $f_1 = \frac{\pi}{2\,l^2}\sqrt{\frac{(EI)_L}{m}} \geq f_{crit}$	8 Hz (floor class 1) 6 Hz (floor class 2)
Static deflection	$w_{static} = \frac{F \cdot l^3}{48 \cdot EI_L \cdot b_{ef}} \leq w_{limit}$	0.25 mm (floor class 1) 0.5 mm (floor class 2)
Acceleration response	$a_{rms} = \frac{0.4 \cdot \alpha \cdot F_0}{2 \cdot \zeta \cdot M^*}$	0.05 m/s^2 (floor class 1) 0.10 m/s^2 (floor class 2))

A the method only considers acceleration response in the case of low frequency floors, the mass requirement is only partially adressed. High frequency, low weight floor may cause vibrational problems this approach won’t be able to detect.

3.4 Comfort criterion / Method used in Norway

In the Byggdetaljblad 522.531 (SINTEF Byggforsk 2011a) considering design a timber joist floor, several tables for timber joists floors which contain pre-accepted cross-sections and spans are presented. There are different tables for different types of joists, and some restrictions on what type of building and load situations the tables apply to. The tables give maximum free span to the most common cross sections for each type of joist. It is assumed that the mean modulus of elasticity in the along-joist direction of the material used in the flooring is at least 3500 N/mm^2.

Bjelkedimensjon mm × mm	Lysåpning i meter Bjelkeavstand c/c (mm)		
	300	400	600
36 × 200	3,5	3,2	2,9
36 × 250	4,1	3,9	3,6
36 × 300	4,8	4,5	4,2
48 × 200	3,7	3,4	3,1
48 × 250	4,4	4,1	3,8
48 × 300	5,0	4,8	4,4
70 × 200	3,9	3,7	3,4
70 × 250	4,7	4,5	4,1
70 × 300	5,4	5,1	4,8

Figure 3.2: Table 22b from Byggdetaljblad 522.531, showing maximum span of different cross sections of glue laminated timber joists. Restrictions to the table are given in the original document. (SINTEF Byggforsk 2011a)

The method used to construct these tables are based suggestions made by (Homb 2007) in “Kriterier for opplevde vibrasjoner i etasjeskillere”, known as “the Comfort Criteria”. In this report, five approaches to handle vibrations in floor structures are presented and evaluated. This report concludes that the work of Lin Hu (Hu & Chui 2004) should be used to determine the criterion for use in Norway. In the report “Nedb, yninger og vibrasjoner til trebjelkelag”(Homb 2009), the approach is further investigated, and results from experimental tests were compared to measured results.

It is recommended by SINTEF Byggforsk to use the Comfort Criteria when evaluating timber joists floors (Glasø 2017), but this is not mentioned in the Norwegian national annex to Eurocode 5 at present date. All SINTEF Certifications for timber joists and timber floors uses the Comfort Criteria, but both the Eurocode5 and Comfort Criterion is regarded as possible to use.

It is, however, not easy to fully grasp what lies behind the tables in e.g. Byggdetaljblad 522.532 and relevant Certifications, as not enough information on how the tables were constructed is given. The tables are constructed by taking the minimum span allowed by the combined criterion of the Comfort Criterion and deflection control. In the deflection control $L/200$, the live loads are used. Since the comfort criterion is more severe in almost all cases, the effect of different live loads is not seen in the tables. If this was the case, the maximum allowed span would change between the 2.0 kN/m2 and 3.0 kN/m2 case in table 2 in Certification 2365. The maximum span changes only in a few floor configurations, meaning that the Comfort Criterion in most cases are more severe than the deflection criterion.

Bjelketype	Maksimal lysåpning i meter 1)											
	Nyttelast 2,0 kN/m² og tilleggslast fra lette skillevegger (boliger o.l.)						Nyttelast 3,0 kN/m² og tilleggslast fra lette skillevegger (kontorer ol.)1)					
	Bjelker over ett felt			Kontinuerlige bjelker over to like felt			Bjelker over ett felt			Kontinuerlige bjelker over to like felt		
	Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm		
	300	400	600	300	400	600	300	400	600	300	400	600
K-Bjelke												
36 x 200	3,45	3,25	2,95	3,60	3,40	3,10	3,45	3,25	2,85	3,60	3,30	2,85
36 x 250	4,15	3,90	3,55	4,35	4,10	3,75	4,15	3,90	3,55	4,35	4,10	3,60
36 x 300	4,80	4,50	4,15	5,00	4,75	4,35	4,80	4,50	4,15	5,00	4,75	4,35
48 x 200	3,65	3,45	3,10	3,80	3,60	3,30	3,65	3,45	3,10	3,80	3,60	3,15
48 x 250	4,35	4,15	3,80	4,60	4,35	3,95	4,35	4,15	3,80	4,60	4,35	3,95
48 x 300	5,05	4,80	4,40	5,30	5,05	4,65	5,05	4,80	4,40	5,30	5,05	4,65
70 x 200	3,90	3,70	3,40	4,10	3,90	3,55	3,90	3,70	3,40	4,10	3,90	3,55
70 x 250	4,70	4,45	4,10	4,90	4,65	4,30	4,70	4,45	4,10	4,90	4,65	4,30
70 x 300	5,40	5,15	4,75	5,70	5,40	5,00	5,40	5,15	4,75	5,70	5,40	5,00

Figure 3.3: Example of a "bjelkelagstabell", from (SINTEF Certification 2365 2017)**.** For a simply supported joist, the Comfort criterion is the most severe, except for the floor configuration marked with a red ring. For continuous joists, the deflection criterion *L*/200 is more severe for ever floor configuration.

Figure 3.3 shows larger joist spacing seems to influence weather the deflection criterion or the Comfort Criterion is the most severe. But mostly, the effect of a continues beam over two spans is of importance. The deflection criterion seems to be the limiting criterion for this configuration, as the span is shorter with increasing live loads in all joist cross sections.

Fundamental frequency (Frequency requirement - repeated cyclic actions):

The Comfort Criteria don't explicitly state what formulas to be used as it is presented in Byggdetaljblad 522.532. In (Homb 2009) it is stated that a report regarding the vibrational performance of cross laminated timber (CLT) (Homb 2008), gives the formula for calculation of fundamental frequency. What is found in the report is the expression for f_1-calculation of orthotropic and isotropic plates, simply supported along all 4 sides. A timber floor can be taken as an orthotropic plate (see section **Feil! Fant ikke referansekilden.**), and so the formula presented in section 3.1 in (Homb 2008) could be used. I have chosen not to consider this formula, as it is only given for floors being supported along 4 sides, while the floor I am to investigate is supported along two sides only.

The limit value of fundamental frequency is set to 10 Hz. In (Homb 2009), it is stated that for two-way plates, such as timber joist floors with *high transversal stiffness* (and CLT-floors), the limit for fundamental frequency should be raised to $f_1 \geq 12.5$ Hz, because the uncertainties related to the correlation between physical parameters and human discomfort when the criteria is used on two-way plates.

Static deflection (stiffness requirement - impulses with longer duration):

In (SINTEF Byggforsk 2011a) does not give a formula for calculating w, but it's noted that the software BTAB is used to find the static deflection under a 1 kN concentrated load. When constructing the "bjelkelagstabell", total deflection due to a uniformly distributed load (self-weight and live loads) was controlled to not be more than L/200 of the total span. The limit value for deflection under a concentrated static load, w_{limit} is set to 1.3 mm.

BTAB is presented in "Prosjektrapport 37: Beregning av nedb, yning i trebjelkelag» (Homb & Kolstad 2009). It takes into account the transversal stiffness of the plate in the floor. The slab- and plate stiffness of the floor must be calculated manually and inserted into the program.

More details on BTAB is can be found in (Homb & Kolstad 2009). The software is not used in this thesis, and even though the basis for the calculations are given in the mentioned report, a simplified calculation of *w*, as in Equation (2.20), is used as in the other methods presented.

Combined criteria:

As in (Hu & Chui 2004), the Comfort Criterion uses a combined criterion consisting of the ratio between fundamental frequency and static deflection:

$$\frac{f_1}{w^{0.44}} > 18.7 \tag{3.11}$$

Although it is not clearly stated in (SINTEF Byggforsk 2011a), I have in this thesis assumed all three verification checks have to be approved for the Comfort Criterion to be satisfied.

Summary:

A summary of the verification criterions according to the Comfort Criterion presented in (SINTEF Byggforsk 2011a) is shown in Table 3.12. The comfort criterion does not address the mass requirement.

Table 3.12: Summary of the verifications according to the Comfort Criterion

	Verification:	Limit value:
Fundamental frequency	$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI_L}{m}} \geq f_{limit}$	10 Hz (timber joist floor) 12.5 Hz (Floors with high transversal stiffness)
Static deflection	$w_{static} \; \frac{F \cdot l^3}{48 \cdot EI_L \, b_{ef}} \leq w_{limit}$	1.3 mm
Combined Criterion	$\frac{f_1}{w^{0.44}} > limit$	18.7

3.5 Summary of the code based methods

As will be seen in 4.1, the floor has bearings on only 2 sides, and an EI_T/EI_L-ratio < 0.05. That means that the extended expression for fundamental frequency in both Hamm/Richter and Austrian NA-method is disregarded in further calculations. Because of this, only the simple formula for f_1 is presented in the table below.

Table 3.13: Summary of formulas used in the verification methods

<table>
<tr><th></th><th>Fundamental frequency</th><th>Static deflection</th><th>Acceleration response</th><th>Combined Criterion</th></tr>
<tr><td>EC5 NO</td><td rowspan="4">$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI_L}{m}}$</td><td>$\frac{w_{st}}{F} \leq a$</td><td>-</td><td>-</td></tr>
<tr><td>HR</td><td>$w_{st} = \frac{F \cdot l^3}{48 \cdot EI_L \cdot b_{w(2kN)}}$</td><td>$a = \frac{F_{dyn}}{M^* \cdot 2\zeta}$</td><td>-</td></tr>
<tr><td>EC5 AU</td><td rowspan="2">$w_{st} = \frac{F \cdot l^3}{48 \cdot EI_L \cdot b_{ef}}$</td><td>$a_{rms} = \frac{0.4 \cdot \alpha \cdot F_0}{2 \cdot \zeta \cdot M^*}$</td><td>-</td></tr>
<tr><td>Comfort Criterion</td><td>-</td><td>$\frac{f_1}{w^{0.44}}$</td></tr>
</table>

Limit values accompanying the criteria presented in the previous chapter is summarised in Table 3.14:

Table 3.14: Summary of limitation values.

	Fundamental frequency	Static deflection	Acceleration response	Combined Criterion
EC5 NO	8 Hz	0.9 mm/kN 0.6 mm/kN	-	-
HR	8 Hz	0.5 mm 1.0 mm	0.05 m/s^2 0.10 m/s^2	-
EC5 AU	8 Hz 6 Hz	0.25 mm 0.5 mm	0.05 m/s^2 0.10 m/s^2	-
Comfort Criterion	10 Hz (12.5 Hz)	1.3 mm	-	18.7

Using these formulas and limitation values from the analytical methods, a reference floor will be investigated in section 5. The expression for calculating the fundamental frequency will be taken as the same in every method, as will the formula for static deflection. This means that it is mainly the limitation values that will give different results for the different methods.

Given another reference floor, the differences between the methods could be clearer. E.g. if the floor under investigation was a typical low frequency floor, having a heavy topping and/or longer span, the frequency could be below 8 Hz. This would call for use of the acceleration response criteria of the Hamm/Richter and Austrian National Annex. The Norwegian National Annex to Eurocode 5 and the Comfort Criterion does not give alternative criteria for such floors. None of the code based methods fully address the mass requirements, which may cause potential “problem floors” of low mass and high frequency, to be undetected.

3.6 VDV and aRMS

In this section, methods for evaluating the acceleration response in a floor is presented.

Root-mean-square acceleration (a_{rms}) and *vibration doe values* (VDVs) are two parameters that can be used to evaluate the dynamic properties of a floor. Both the British standard BS 6472 and the International Standard ISO 101 37 gives guidance in the verification of existing floors using these parameters.

“ Design of floors for vibration: A new approach” (Smith et al. 2009) proposes a simplified method for predicting the vibrational behaviour of a floor. It is an “intermediate approach”, made to be not as complex and detailed as a full simulation of a floor, but more sophisticated than the code based methods.

A strong hypothesis is needed to be able to make correct calculations when using this method. The walking path, the weight, and position of installations and partitions as well the dynamic properties of the floor should be known. This makes the method more complicated but allows for an analysis considering important aspects of human comfort about floor vibrations.

3.6.1 Perception of floor vibration:

The methods presented here focus on how vibrations are felt and evaluated as acceptable or uncomfortable. As described in section 2.2, the activity and situation of the person experiencing the vibrations highly governs the perception level. Also, the duration of and how often perceptible vibrations occur influence whether they are considered uncomfortable or acceptable.

Based on their duration and occurrence, vibrations can be categorised as:

- Continuous
- Intermittent/ periodic
- Occasional

The acceptance of occasional vibrations is higher than for the continuous ones. This can be taken advantage of by highlighting the influence of continuous vibrations and attenuate the influence of occasionally occurring vibrations in the verification of the floor.

The perception of vibrations depends on the direction of the vibrations relative to the direction of the human body. This means that the threshold or the *base value* of acceleration is dependent on the axis of vibration compared to the axis of the body. The base value of acceleration is higher for z-axis vibration (along spine) than for the other directions. This means that vibrations in the "across-spine directions" are more easily perceived.

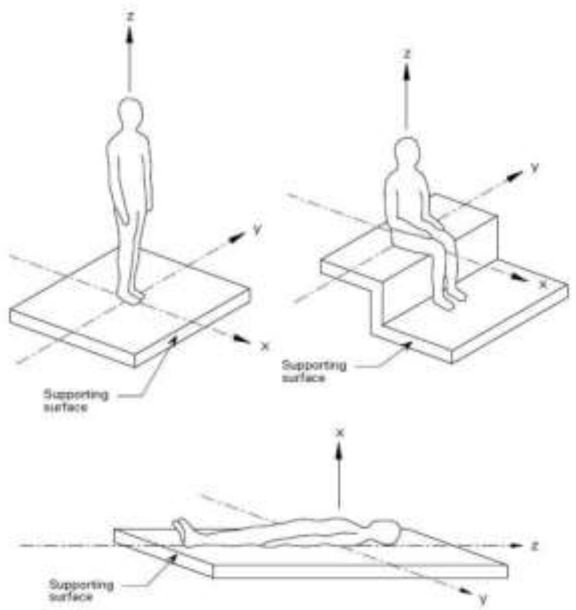


Figure 3.4: Direction of the person precepting the vibration (ISO 10137)

The acceptance level also depends on the frequency of the vibration as the human sensitivity to a vibration amplitude changes with frequency. For frequencies where perception is less sensitive, this can be taken advantage of by attenuation of the calculated response or enhance the base value. This is called *frequency weighting* and is utilized in, for example, ISO 10137.

The base values are:

- $a_{rms} = 0.005\ m/s^2$ vibration along z-axis.
- $a_{rms} = 0.00357\ m/s^2$ vibration along x- and y-axis.

These base values are taken from ISO 2631 and repeated in "Design of floors for vibration: A new approach". In this sense, a "low frequency" can be taken as the frequency of structural vibrations (Zhang et al. 2013)) is highly dependent the activity of the observer. The acceptability for low frequency vibrations is highly dependent on the activity of the observer. To summarise, it can be said that perception and acceptance of vibrations are determined by a combination of

- Frequency of vibration
- Duration and occurrence of vibration
- The direction of vibration vs. the body precepting them,

in addition to the situation of the observer. Frequency weighting takes all this into account.

3.6.2 Frequency weighting:

Frequency weighting means that the response of a floor is altered. Type of expected activity and building category, the axis of vibration and category of demand gives different *weighting curves*, which gives a *weighting factor* for each frequency. This factor is in turn multiplied by the acceleration response in the floor, as will be addressed in section 3.6.3.

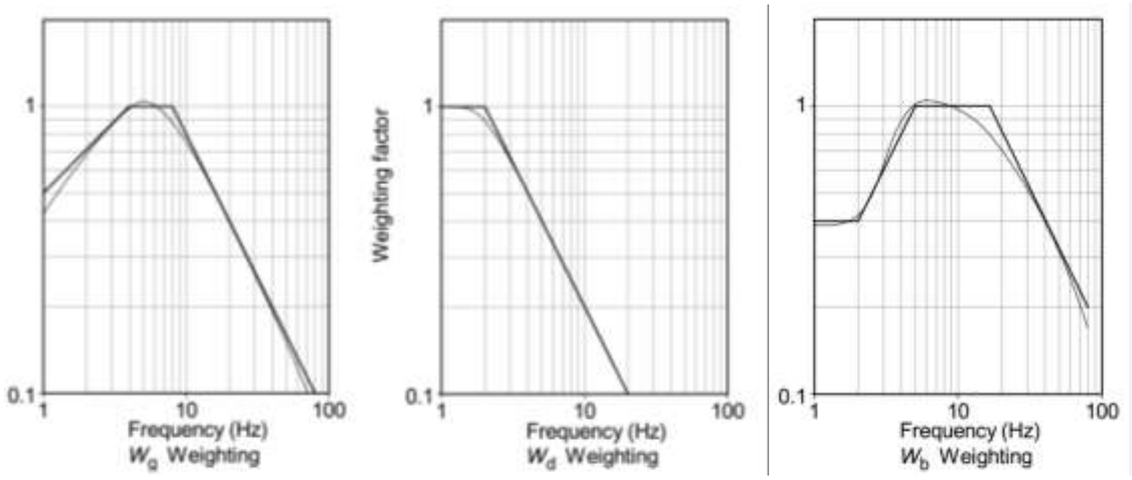


Figure 3.5: Weighting curves, W_g, W_d and W_b (BS 6841)

Different weighting curves for acceleration response in a floor can is seen in Figure 3.5. The weighting curves to use in different building type, vibration axis and category of demand, called *weighting categories*: given in Table 3.15.

Table 3.15: Weighting factors appropriate for floor design (Smith et al. 2009)

Room Type	Axis of vibration	Category	BS 6841 weighting curve
Critical working areas (e.g. hospital operating theatres, precision laboratories)	z-axis	Vision/Hand control	W_g
	x-, y-axis	Perception	W_d
Residential, offices, wards, general laboratories, consulting rooms	z-axis	Discomfort	W_b
	x-, y-axis	Discomfort	W_d
Workshop and circulation spaces	z-axis	Discomfort	W_b
	x-, y-axis	Discomfort	W_d

The British standard gives the weighting curves used in "Design of floor structures: A new Approach". The values obtained from the weighting curves can also be calculated using equation 22-23 in this document.

z-axis vibrations W_g weighting

$$W = 0.5\sqrt{f} \quad \text{for } 1\ \text{Hz} < f < 4\ \text{Hz}$$
$$W = 1.0 \quad \text{for } 4\ \text{Hz} \le f \le 8\ \text{Hz} \qquad (21)$$
$$W = \frac{8}{f} \quad \text{for } f > 8\ \text{Hz}$$

z-axis vibrations W_b weighting

$$W = 0.4 \quad \text{for } 1\ \text{Hz} < f < 2\ \text{Hz}$$
$$W = \frac{f}{5} \quad \text{for } 2\ \text{Hz} \le f < 5\ \text{Hz}$$
$$W = 1.0 \quad \text{for } 5\ \text{Hz} \le f \le 16\ \text{Hz} \qquad (22)$$
$$W = \frac{16}{f} \quad \text{for } f > 16\ \text{Hz}$$

x- and y-axis vibrations W_d weighting

$$W = 1.0 \quad \text{for } 1\ \text{Hz} < f < 2\ \text{Hz} \qquad (23)$$
$$W = \frac{2}{f} \quad \text{for } f \ge 2\ \text{Hz}$$

Figure 3.6: Equations used to determine weighting factors (Smith et al. 2009)

Note that the weighing factor is found from the frequency of the mode under consideration when the transient response is calculated, and from the frequency of the harmonic under consideration when steady state response is investigated. This is specified in the expressions for calculating the two acceleration responses (section 3.6.3.)

The calculated acceleration response a_{rms} of a floor due to an excitation force is multiplied with the appropriate weighting factor to obtain the weighted acceleration response. This acceleration response is in turn divided by the base value of acceleration to obtain the *response factor*, *R*. Section 3.6.3 will give an overview of how the acceleration response is calculated, and what limit values are used to compare to the response factor.

3.6.3 Calculation of acceleration response

As seen in section 2.1.2, floor response to vibrations can be categorised as steady state or transient, where steady state response is associated with low frequency floors. Continuous vibrations are assumed. However, continuous vibrations due to walking are rare in residential buildings, as continuous vibrations have to develop over time. To assume a floor in a residential building to be exposed to this kind of vibrations are to be on the conservative side, as the human walk is random and occur periodically rather than continuous. The intermittent nature of the vibrations can be considered through the use of *vibration dose values*, or VDVs (Smith et al. 2009), which will be investigated in section 3.6.4.

The methods presented here assume that the force is only applied in one point on the floor, even though the force due to walking only will pass this point for a short period. For practical use, only the excitation and response point that gives the most severe acceleration must be investigated. This is usually when the point where the response is to be taken, *r*, and the excitation point, *e* coincides at the mid span. An analysis based on walking paths are more complex and computational heavy, and will not be presented here.

Figure 3.7 illustrates the assumed

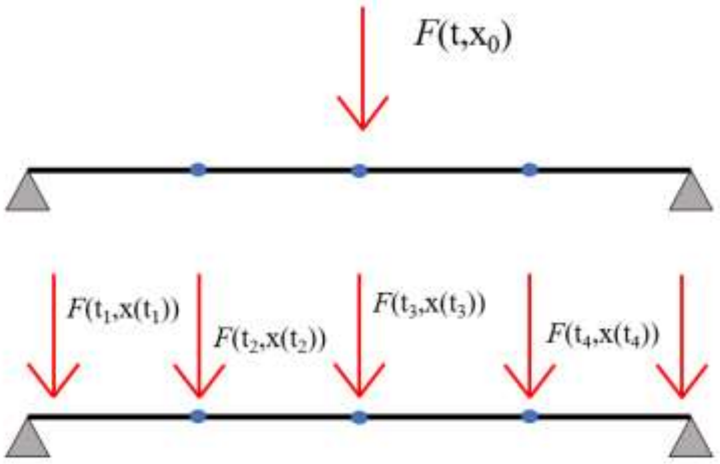


Figure 3.7: Illustration of the assumed fixed force (above) and how the force moves along the walking path (below).

When the fundamental frequency of a timber floor is lower than its *cut-off frequency*, it is regarded as low frequency. In this case, both steady state floor response and transient floor response must be investigated. In a high frequency floor, only the transient floor response needs to be investigated. Table 3.16 shows the cut-off frequency for floors in different building types.

Table 3.16: Low- to high frequency floor cut-off values (Smith et al. 2009)

Floor type	Low to high frequency cut-off
General floors, open plan offices etc.	10Hz
Enclosed spaces, e.g. operating theatre, residential	8Hz
Staircases	12Hz
Floors subject to rhythmic activities	24Hz

The cut-off frequency for floors subjected to rhythmic activities is considerably higher than the other floor types because of the increased probability of development of steady state floor response and resonance.

Steady state acceleration response:

When one or more of the harmonic components of the excitation is close to one of the first few natural frequencies of the floor, steady state response of the floor can occur (as seen in section 2.1.2). From (Smith et al. 2009) it is recommended that all vibration modes having natural frequencies up to 2 Hz above the cut-off frequency should be considered in the calculation of steady state acceleration response. The weighted root-mean-square acceleration must be calculated for each mode and harmonic under consideration:

$$a_{w,rms.e.r.n.m} = \mu_{e.n}\,\mu_{r.n}\frac{F_h}{M_n\sqrt{2}}D_{n.h}W_h \tag{3.12}$$

where:

$\mu_{e.n}$	is the mode shape amplitude from the unity or mass normalised FE output, at the point on the floor, were the excitation force F_h is applied
$\mu_{r.n}$	is the mode shape amplitude from the unity or mass normalised FE output, at the point on the floor, were the response is to be calculated
F_h	is the excitation force of the h^{th} harmonic: $F_h = \alpha_h Q$, where α_h is given in Table 3.17 and Q is the static force exerted by an "average person" (746 N), [N]
M_n	is the modal mass of mode n (if the mode shapes are mass normalised, this is equal to 1 kg), [kg]
$D_{n.h}$	is the dynamic magnification factor (as in Equation (2.13)
W_h	is the weighting factor, found from the appropriate weighting curve (Figure 3.5 and Table 3.15)

This calculation gives the acceleration response in point r of the floor, in mode n of vibration to a forcing frequency of the h^{th} harmonic, when the force is applied in one specific point, e.

Table 3.17: Fourier coefficients α_h for walking activities (Smith et al. 2009)

Harmonic h	Excitation frequency range hf_p (Hz)	Design value of coefficient α_h	Phase angle ϕ_h
1	1.8 to 2.2	$0.436(hf_p - 0.95)$	0
2	3.6 to 4.4	$0.006(hf_p + 12.3)$	$-\pi/2$
3	5.4 to 6.6	$0.007(hf_p + 5.2)$	π
4	7.2 to 8.8	$0.007(hf_p + 2.0)$	$\pi/2$

To find the total response of point e due to excitation in point r is found by summing the responses of each mode of vibration at each harmonic of the forcing function. There are more ways to do this, but here this is done by the *square-root sum of squares* method (SRSS), as this is the easiest of the alternatives presented in (Smith et al. 2009), that provides sufficient accuracy.

$$a_{w.rms.e.r} = \frac{1}{\sqrt{2}}\sqrt{\sum_{h=1}^{H}\left(\sum_{n=}^{N}\left(\mu_{e.n}\,\mu_{r.n}\,\frac{F_h}{M_n}D_{n.h}W_n\right)\right)^2} \quad \textbf{(3.13)}$$

Also, the *resonance build-up factor* may be applied to the steady state rms acceleration. This is done to consider the fact that steady state condition may not be reached for short spans. When resonance doesn't have time to build up, the calculated acceleration response can be decreased.

$$\rho = 1 - e^{\left(\frac{-2\pi\xi L_p f_p}{v}\right)} \quad \textbf{(3.14)}$$

where:

f_p	is the pace frequency [Hz]
ξ	is the critical damping ratio
L_n	is the length of the walking path [m]
v	is the walking velocity, as in Equation (2.16 [m/s^2]

Transient response:

When the fundamental frequency of the floor is sufficiently larger than the excitation frequency, transient floor response is considered. All modes up to two times the fundamental mode should be included in the calculation.

The weighted acceleration response is taken as:

$$a_{w,peak.e.r.n} = 2\pi f_n\sqrt{1-\xi^2}\mu_{e.n}\mu_{r.n}\frac{F_I}{M_n}W_n \quad \textbf{(3.15)}$$

where:

$\mu_{e.n}$	is the mode shape amplitude from the unity or mass normalised FE output, at the point on the floor, were the excitation force F_I is applied.
$\mu_{r.n}$	is the mode shape amplitude from the unity or mass normalised FE output, at the point on the floor, were the response is to be calculated.
F_I	is the excitation force given in Equation 3.15, [Ns]
M_n	is the modal mass of mode *n* (if the mode shapes are mass normalised, this is equal to 1 kg), [kg]

W_n	is the weighting factor, found from the appropriate weighting curve (Table 3.15), dependent on the direction of the vibrations on the human body and the frequency of the mode under consideration.

The mode shape amplitudes must be taken from a finite element model, while the excitation force is calculated from:

$$F_I = 60 \frac{f_p^{1.43}}{f_n^{1.3}} \frac{Q}{700} \text{ [Ns]} \tag{3.16}$$

where:

f_p	is the pace frequency [Hz]
f_n	is the frequency of the mode under consideration [Hz]
Q	is the static force exerted by an "average person" (746 N), [N]

The weighted acceleration response of all the modes under consideration, due to one impulse, is found using the following formula is used:

$$a_{w.e.r} = \sum_{n=1}^{N} 2\pi f_n \sqrt{1-\xi^2}\, \mu_{e.n}\mu_{r.n} \frac{F_1}{M_n} \sin\left(2\pi\sqrt{1-\xi^2}\, t\right) \cdot e^{-\xi 2\pi f_n t} W_n \tag{3.17}$$

The *root-mean square* acceleration is obtained by using the expression for total acceleration response in Equation (2.15, repeated here:

$$a_{rms} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt}$$

Response factor:

When the weighted *rms* acceleration is found, it is used to obtain the *response factor*. The response factor is the ratio between the calculated acceleration response and the base value for acceleration (see section 3.6.1) As can be seen, the response factor is dependent on the direction of the vibration about the human body, to take into account the difference in perception level in the different directions.

$$R = \frac{a_{w,rms}}{base\ value_{z-axis}} \tag{3.18}$$

where:

the base value for z-axis = 0.005 m/s^2

$$R = \frac{a_{w,rms}}{base\ value_{x-and\ y-axis}} \quad \textbf{(3.19)}$$

where:

the base value for x- and y-axis = 0.00357 m/s^2

The response factor must be compared to some limit value to verify the floor. For steady state and transient response (to continuous vibrations), the values presented in Table 3.18. The response factor must be lower than the limit value if the verification is to be satisfied.

Table 3.18: Multiplying factors specified in BS 6472 for "low probability of adverse comment", used as limit values for the steady state and transient acceleration response factors. (Smith et al. 2009)

Place	Time	Multiplying factor for exposure to continuous vibration 16 h day 8 h night	Impulsive vibration excitation with up to 3 occurrences
Critical working areas (e.g., hospital operating theatres	Day	1	1
	Night	1	1
Residential	Day	2 to 4	60 to 90
	Night	1.4	20
Office	Day	4	128
	Night	4	128
Workshops	Day	8	128
	Night	8	128

If the response exceeds the conservative limits of continuous vibrations, *vibration dose values* (VDVs) can be used to consider the intermittent nature of the dynamic forces (given that the design specifications permit the use of VDVs.). The transient response should always be evaluated using VDVs.

3.6.4 Vibration dose value (VDV)

Human activities are often random and cause occasional short-duration vibrations (the exceptions are for example gymnastic halls or walking corridors, where continuous vibrations are more likely to occur). This can be considered using VDVs. These allow the level of vibration to be higher for short-duration vibrations than for continuous ones, but only for a short period.

VDV limits are dependent on how often the vibrations occur, and the acceptance of probability of adverse comments. The British standard (BS 6472) give guidance in the use of VDVs. However, the method for calculating VDVs used in this thesis is based on the work of Ellis (2004).

The general expression for finding the VDV is found through Equation (3.20, while the expression in Equation (3.21 is used further in this thesis. The latter equation takes into account the number of times a day/night a vibration can occur, and also the duration of the vibration. These are important factors for vibration perception and acceptability (see section 2.2)

$$VDV = \left(\int_0^T a_w(t)^4 \, dt\right)^{1/4} \quad [\mathrm{m/s^{1.75}}] \tag{3.20}$$

where:

$a_w(t)$ is the weighted acceleration response, m/s^2

T is the total period of the day during which vibration may occur, s

$$VDV = 0.68 \, a_{w,rms} \sqrt[4]{n_a T_a} \tag{3.21}$$

where:

$a_{w,\ rms}$ is the frequency weighted rms acceleration, [m/s^2]

n_a is the number of times the activity will take place during an exposure period (day or night)

T is the duration of an activity, for esample the time it takes to cross a floor [s]

The value obtained from Equation (3.21 can be compared directly to the limitation values in Table 3.19.

Table 3.19: Limit values for intermittent vibrations (Smith et al. 2009). Multiplying factors specified in BS 6472.

Place	Low probability of adverse comment	Adverse comment possible	Adverse comment probable
buildings 16 h day	0.2 to 0.4	0.4 to 0.8	0.8 to 1.6
buildings 8 h night	0.13	0.26	0.51

Summary of section 3.6 in short terms:

Acceleration response is an important aspect of the vibrational performance of a structure. It is highly relevant in timber floors, as these often are light weight structures. The low mass causes in most cases a high fundamental frequency. In most standards to day, acceleration response is verified only in the case of low frequency floors. It is, therefore, useful to be able to investigate leigh weight floors using a_{rms} and VDVs. When using these methods, it is important to know as much as possible about the use of the floor, and preferably also how heavy installations and partitions will be arranged, as these objects affect the properties used to predict the acceleration response of the floor. A finite element model should be made to obtain mode shapes and frequencies of multiple modes.

4 Joist floor under consideration

In this chapter, the properties of a typical timber joist floor will be discussed. The reference floor is introduced. This floor is very similar to a typical floor element produced by the element house producer. As will be seen, this reference floor is a very light, so an additional floor is introduced, having 50 mm concrete screed added. These floors represent two different floor types, that is expected to have different vibrational properties.

From a structural point of view, the timber joist floor can be treated as a rib-stiffened plate. Depending on the material used in the flooring, the platelayer can be seen as either iso- or orthotropic. Floor boards is an example of materials that will have an orthotropic behaviour, while particle boards or OSBs are isotropic materials. To increase the transversal (across joist) stiffness in the floor, blocking and cross bracing are commonly used. Still, even with measures like these, the overall behaviour of a timber floor is orthotropic. (Smith 2003)

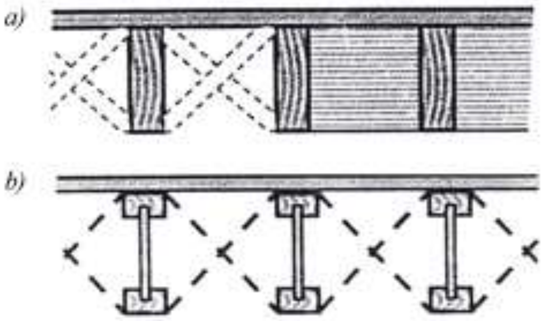


Figure 4.1: Measures to improve the transversal stiffness in a floor (Smith 2003).

a) Bracing and blocking of a floor with rectangular joist cross sections.
b) I-joist braced with metal straps.

A timber joist floor is most often regarded as having “simply supported” support conditions, even if the floor is somewhat “clamped” between two supporting walls as an intermediate floor in a platform construction (Smith 2003). This is due to the typical material properties of wood, with a low modulus of elasticity (MOE) perpendicular to the grain and high MOE longitudinal to the grain. This result in deformation of the wood at the supports, for example crunching of fibres under the weight above stories. These small deflections make it very difficult to obtain a fully fixed/moment resistant support condition.

In floors of moderate width, having support on all four sides can increase the overall stiffness considerably compared to a two-side-supported floor. The fundamental frequency of a four-side supported floor will not change significantly, but other important parameters such as deflection under concentrated load and reduction of vibration amplitude under impact load will be improved.

In this thesis, only the plate layer directly attached to the joists are regarded as a part of the structural system. The layers above will influence the vibrational serviceability of the floor, for example, transient vibrations in the floor might be affected, as a soft flooring will be able to obtain more of the energy from the heel impact. As this thesis investigates the vibrations in the structural system, the upper layers are not considered here. However, the serviceability of the floor is highly dependent on details such as the upper layers of the floor, as they influence the sound insulations properties. According to (Smith 2003) floors with only one plate or one layer of sheeting have objectionable dynamic responses.

4.1 Geometrical and material properties of the reference floor:

The reference floor is a timber joist floor designed to have good sound-insulating properties. Figure 3.2 taken form SINTEF Certification 2232 illustrates the floor:

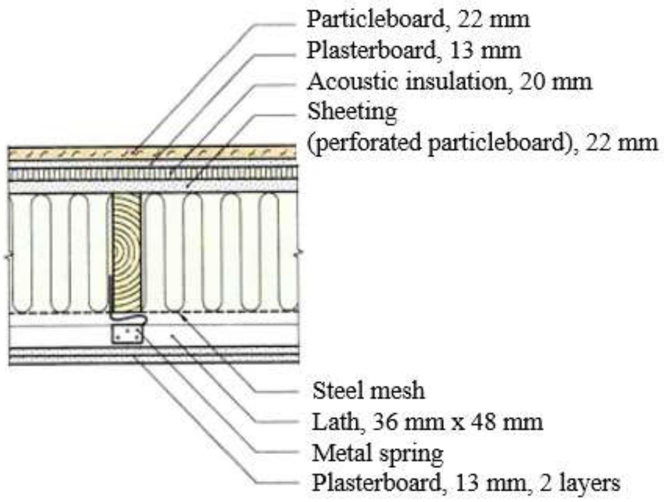


Figure 4.2: Schematic plan of the floor (SINTEF Certification 2232 2015).

The perforated plaster board is glued and screwed to the timber joist. Due to this configuration, the joist and sheeting are assumed to interact as a composite structure stiff enough to transfer stresses without being deformed. The other layers in the floating floor system do not contribute to lateral or transversal stiffness in the floor. The support conditions is assumed to be simply supported. An overview of the properties of the reference floor is given in Appendix B.

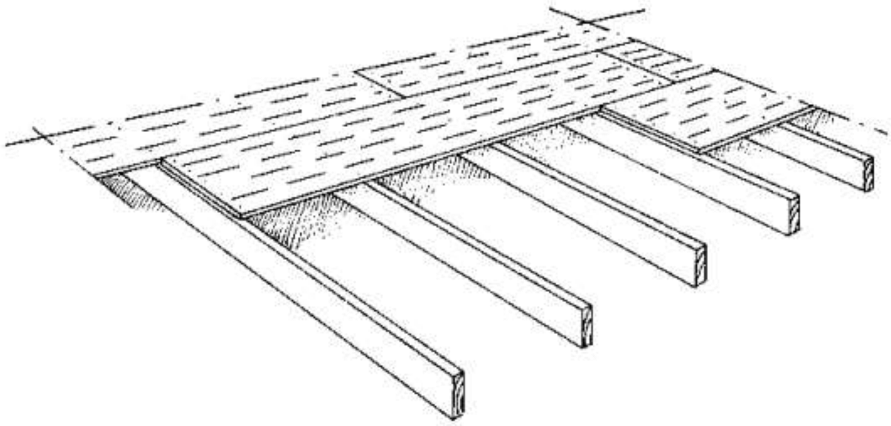


Figure 4.3: Perforated particle board in use, directly fastened to the joists, directed transversal of the span. (Forestia.no 2016)

The plates are directed transversal to the joist span. They must be glued together along all edges, according to (SINTEF Byggforsk 2011b).

The perforated particle board has a smaller cross-section area due to the holes. The manufacturer of the particle board used as sheeting informs that they assume 10-20% volume reduction after the holes has been cut out in the plate (Forestia v. Christian Sørlie 2017). This causes a reduction in overall strength and stiffness of the plate. By subtracting the areas of the holes from the plate cross section, this is assumed considered. This reduction will be different when considering the along joist or across joist direction.

Table 4.1: Reduction of plate width due to holes, along-joist and across-joist direction.

	Along joist	Across joist
b x h, holes	12 mm x 129 mm	129 mm x 12 mm
N	7 per 0.6 m	10 per 2.4 m
b_{holes} x n	12 x 7	10 x 129
$b_{new} = b - \sum b_{holes}$	600 – 84 = 516 mm	2400 – 1290 = 1110 mm
% reduction	14 %	46 %

The reduced width of the plate is used for further calculation of moment of inertia (I_{plate}) and mass of the plate.

The weight of the materials above the sheeting is regarded in the calculation of the total weight of the reference floor. Table 0.1 in Appendix A gives the mass calculation. Table 4.2 gives an overview of what is regarded in the mass calculation.

Table 4.2: Assumed mass used for further calculation

Elements included in the mass calculation:	Calculated mass of the floor
- 5 x Glue Lam timber joists (3.925 m each) - 2 x Particle board (subfloor) - 3 x Plaster boards (part of the sound insulating system and sub-floor) - Insulation (mineral wool) - 6 x 36x40 mm^2 lath (2.4 m each)	63 kg/m^2

Table 4.3 shows characteristics of the structural elements used in the analysis of the floor.

Table 4.3: Floor properties used in calculations and modelling.

Overall geometry:		
Span/length of floor	3925	mm
Element width	2400	mm
Joist spacing	600	mm
Joist section properties, K-bjelken Pluss:		
Depth/height	300	mm
Width	48	mm
Density (mean)	460	kg/m^3
Modulus of elasticity, $E_{L\ joist}$	14000	N/mm^2
Flooring properties, “slisseplate”:		
Thickness	22	mm
Density	685	kg/m^3
Modulus of elasticity. $E_{T\ flooring}$	2250	N/mm^2
Perforation of plate considered: $b_{flooring}$ has to be reduced by 14% and 46 %		
Connection joist/plate:		
Regarded as “infinity stiff”	glue + screw	

As the damping is of great importance to the vibrational behaviour of the floor, it is important to choose a value as realistic as possible. As described in section 2.1.2, higher modal damping is beneficial. It is therefore conservative, and regarded as being on the safe side, to assume a lower damping ratio than what is occurring. Since the reference floor is based on a simple floor element, a damping ratio of 0.01 % is assumed.

$\xi = 0.01\ \%$

5 Verifications using code based methods

In this section, two floors will be evaluated using the four analytical methods presented in section 3, the reference floor and a floor having 50 mm concrete screed. This has been done using Matlab. Tables and graphs are presented using Excel. A script calculating the needed parameters, and comparing the outcome of each method was written.

The first floor is the reference floor. As seen in section 4.1, it has a very weight, and a plate used as sub-floor with a quite low MOE. Because of this, there will be no effect of transversal distribution (see section 5.1.2). The second configuration is a floor having a reinforced concrete screed, adding both stiffnesses to the platelayer, and mass to the overall system.

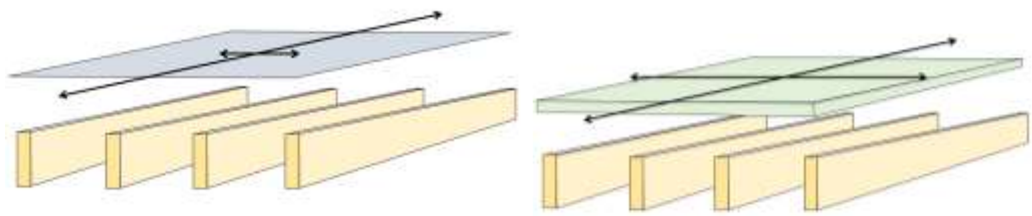


Figure 5.1: Floor configuration, reference floor and floor with concrete screed.
To the left is an illustration of the reference floor, to the right is a floor similar floor, but with added concrete screed. The transversal stiffness is higher in the floor with added screed.

The requirements for the floor is set to the highest level. This means floor class A in Austrian National Annex and Hamm/Richter, and “stiff floor-requirements” in Norwegian National Annex to Eurocode 5.

5.1.1 Mass considered:

In the calculations in the following sections, the same mass is used for all the methods. This is done so that the limitations values of the methods are compared, and because it is in some instances are dissonance between the different literature on what mass should be included in the calculations within the same method. One example is the mass used in the Austrian national annex approach, where (Zhang et al. 2013) and (Thiel 2012) both states that the quasi-permanent load combination of Eurocode 0 should be applied, but this is not found in Austrian national annex. Only the self-weight and other permanent loads are used in the following calculations.

As the self-weight of the floor is quite small (only 63 kg/m^2), the calculations are also done on the floor with concrete screed. In SINTEF Certification 2365, it stated that the values in the “bjelkelagstabell” also applies to joist floors having heavy ballasting, such as a concrete screed, up to 2.6 kN/m^2 self-weight. The masses used for the two floor calculations can be found in Table 4.2.

Table 5.1: Mass used in the analytical analysis, simple self-weight and self-weight + added mass (concrete screed).

Method	$m = \sum g_k$	$m = \sum g_k + "added\ mass"$
EC5 NO	63 kg/m^2	265 kg/m^2
Hamm/Richter		
EC5 AU		
Comfort Criteria		

As can be seen from Table 5.1, it is a big difference in the two masses used. In the first column, no mass has been added to consider other permanent loads (such as permanent installations or partitions). This could have been done, by assuming a reasonable value. In SINTEF publication 522.351 it looks like a mass of 0.7 kN/m^2 dead load + 0.5 kN/m^2 from partitions is used when finding maximum span for different joist cross section. The low self-weight of the reference floor may indicate an error in the assumptions made, or that a value for partitions should have been assigned when the mass was calculated. See table 0.1 in Appendix A- for the whole calculation of the self-weight of the floor.

As the mass of the reference floor is as low as in the reference floor, that the weight of the person walking is a considerable amount of the overall mass in the human/floor-system. If this is not considered in the analysis of the floor, an important influence on the vibrational properties of the system is neglected.

5.1.2 Finding EI_T, EI_L and b_{ef}:

In this subsection, the transversal and longitudinal stiffness of the reference floor is calculated, as well as the effective width of the 1D system that represents the floor.

Using equations presented in section 2.3, and material properties as presented in Table 4.3, the results of the calculations are shown in Table 6.3.

Equation (2.18 used to calculate the longitudinal stiffness of the reference floor is repeated below. The transversal stiffness is simply the stiffness of the plate, as there is only one layer of the sub-floor directly fastened to the joists. This configuration is only stiff in compression, so that it will be an over Equation 5.1 will be an over estimation of the capacity.

$$EI_{long} = EI_{joist} + EI_{plate} + a^2 \left(\frac{1}{EA_{joist}} + \frac{1}{EA_{plate}} \right)^{-1} \tag{5.1}$$

Table 5.2: Calculated longitudinal and transversal stiffness of reference floor:

Longitudinal, 0			Transversal, 90		
I joist,	108 x10^6	[mm^4]	I plate.90	887.3 x10^3	[mm^4]
E joist	14 000	[Nmm^2]	E plate	2550	[Nmm^2]
I plate, 0	457.8 x10^3	[mm^4]			
E plate	2250	[Nmm^2]			
Js	600	[mm]	per m. flooring	1000	[mm]
EI_L	3.07	[MNm^2/m]	EI_T	1717	[Nm^2/m]

When calculating the longitudinal stiffness of the floor with 50 mm concrete screed, the contribution of the screed is simply added to the stiffness of the joist/plate-configuration. This is done as the concrete is not rigidly fastened.

$$EI_{longitudinal} = EI_{joist} + EI_{plate} + a^2 \left(\frac{1}{EA_{joist}} + \frac{1}{EA_{plate}} \right)^{-1} + EI_{concrete} \tag{5.2}$$

Table 5.3: Calculated longitudinal and transversal stiffness, of concrete screed floor

Longitudinal, 0			Transversal, 90		
I joist,	108 x10^6	[mm^4]	I plate.90	887.3 x10^3	[mm^4]
E joist	14 000	[Nmm^2]	E plate	2550	[Nmm^2]
I plate, 0	457.8 x10^3	[mm^4]	I concrete, 90	6250 x10^3	[mm^4]
E plate	2250	[Nmm^2]	E concrete	26 000	[Nmm^2]
I concrete, 0	6250 x10^3	[mm^4]			
E concrete	26 000	[Nmm^2]			
Js	600	[mm]	per m. flooring	1000	[mm]
EI_L	3.35	[MNm^2/m]	EI_T	0.272	[MNm^2/m]

Using Equation (2.21), the effective width b_{ef} of the equivalent beam can be calculated.

Table 5.4: Calculated b_{ef} of reference floor and concrete screed floor:

Floor configuration	EI_T [MNm^2/m]	EI_L [MNm^2/m]	b_{ef} [m]
Reference floor	1.72 x10^-3	3.07	0.55
Concrete screed floor	0.272	3.35	1.91

As shown in Table 5.4, b_{ef} is lower than the actual joist distance. Using it won't be beneficial when calculating the vertical deflection due to a point load (Equation (2.24)) of the reference floor. Because of this, the actual joist spacing is used instead of b_{ef}. The floor with concrete screed, however, obtain a much larger b_{ef}, as higher transversal stiffness gives a larger value of the ratio EI_T/EI_L. For this floor configuration, b_{ef} is used.

An overview of the values used in the calculations on the two floor configurations is seen in Table 5.5

Table 5.5: Floor configurations being analysed using analytical methods

	Reference floor	**Reference floor, with screed**
Mass	63 [kg/m^2]	265 [kg/m^2]
EI_T	1717 [Nm^2/m]	272 550 [Nm^2/m]
EI_L	3 076 498 [Nm^2/m]	3 347 331[Nm^2/m]
b_{ef}	Not used, b_{ef} taken as J_s= 0.6 [m]	1.91 [m]

As can be seen from the table above, the concrete slab influences the transversal and longitudinal stiffness, as well as increasing the mass of the floor. Using b_{ef} in the calculations on the reference floor will have a negative effect. Comparing the two floor types, it is clear that they represent two very opposite floor examples.

5.2 Verification of the floors

5.2.1 According to Eurocode 5 Norwegian National Annex:

The results presented here are from calculations done using expressions from section 3.1.

Fundamental frequency:

Using Equation (3.1) the fundamental frequency of the floor joists is found:

Table 5.6: Verification of fundamental frequency according to Eurocode 5, Norwegian NA

	f_{limit} [Hz]	EI_L [MNm^2/m]	m [kg/m^2]	f_1 [Hz]
Reference floor				
EC NO	8	3.07	63	22.5
Floor with concrete screed				
EC NO	8	3.35	265	9.62

The verification is ok when using both stripped-down self-weight and the added mass. It is, however, a drastic reduction in fundamental frequency when the mass of concrete screed is added.

Deflection /stiffness criteria:

Using Equation (2.24) the static deflection due to a 1 kN concentrated load is found:

Table 5.7: Static deflection according to Eurocode 5, Norwegian NA

	b_{ef} [m]	EI_L [MNm^2/m]	F [kN]	w [mm]	a [kN/mm]
	Reference floor				
EC NO	Js = 0.6	3.07	1	0.68	0.6
	Floor with concrete screed				
EC NO	1.91	3.35	1	0.08	0.24

As the ratio *w/F* is lower than the limit value *a*, the verification of static floor deflection of the reference floor is within the limit value of the Comfort Criterion, but not the other methods. The floor with concrete screed has acceptable floor deflection according to every method.

As the Norwegian national annex has disregarded the vibration velocity as a criterion (EC5 Norwegian NA 2010), this will not be presented here.

5.2.2 According to Hamm/Richter:

The results presented here are based on what is presented in section 3.2

Fundamental frequency:

Using Equation (3.6) the fundamental frequency of the floor is found.

Table 5.8: Verification of fundamental frequency according to Hamm/Richter

	f_{limit} [Hz]	EI_L [MNm^2/m]	m [kg/m^2]	f_1 [Hz]
	Reference floor			
Hamm/Richter	8	3.07	63	22.5
	Floor with concrete screed			
Hamm/Richter	8	3.35	265	9.62

As the fundamental frequency of the joists is higher than the limit value, the verification is satisfied. As seen from the similar calculations in section 3.1, it is a large difference in fundamental frequency dependent on the mass used.

Deflection /stiffness criterion:

Using Equation (2.24) the static deflection due to a 2 kN concentrated load is found:

Table 5.9: Static deflection verification, Hamm/Richter

	b_{ef} [m]	EI_L [MNm²/m]	F [kN]	w [mm]	w limit [mm]
	Reference floor				
Hamm/Richter	Js = 0.6	3.07	2	1.36	0.5
	Floor with concrete screed				
Hamm/Richter	1.91	3.35	2	0.47	0.5

As the deflection value is higher than the limit value, the verification of static floor deflection is not satisfied for the reference floor. The floor with added screed, on the other hand, gives a deflection well below the limit value, and the verification of this floor is satisfied.

Acceleration response:

The acceleration response criterion is set to take care of low-frequency floor. As the floor under consideration is not low-frequency (since f_1 is higher than the limit value of 8 Hz) this verification should not be performed. It is, however, done to illustrate that the criterion will fail for high frequency floors.

The acceleration response is found from Equation (3.8), and is presented in the Table 6.11, even though not possible to follow the calculation approach correctly when investigating the floor configurations in this thesis. The reason for this is that Table 3.3 does not give values of *F(t)* when f_1 is higher than 7.5 Hz.

Table 5.10: Acceleration response according to Hamm/Richter

	a_{limit} $[m/s^2]$	*B* *[m]*	*L* *[m]*	ξ	*F(t)* *[N]*	*m* $[kg/m^2]$	a $[m/s^2]$
	Reference floor						
Hamm/Richter	*0.05*	*2.4*	*3.925*	*0.01*	*70*	*63*	*9.51*
	Floor with concrete screed						
Hamm/Richter	*0.05*	*2.4*	*3.925*	*0.03*	*70*	*265*	*0.74*

The verification is not satisfied, neither when using only self-weight nor when using added mass. Due to the increased weight and damping ratio in the floor with concrete screed (the value is taken from Table 3.10), acceleration response is much lower in this floor configuration.

Since the fundamental frequency is above limit value, this verification is not supposed to be done. That the verification is not satisfied is no surprise.

5.2.3 According to Eurocode 5, Austrian National Annex:

The result presented here are based on what is presented in section 3.3.

Fundamental frequency:

Using Equation (3.1) the fundamental frequency of the floor joists is found:

Table 5.11: Verification of fundamental frequency according to Eurocode 5, Austrian NA

	f_{limit} [Hz]	EI_L [MNm^2/m]	m [kg/m^2]	f_1 [Hz]
	Reference floor			
Hamm/Richter	8	3.07	63	22.5
	Floor with concrete screed			
Hamm/Richter	8	3.35	265	9.62

Again, as the fundamental frequency of the joists is higher than the limit value, the verification is satisfied. As seen from the similar calculations in section 3.1, also here it is a large difference in fundamental frequency dependent on the mass used.

Deflection /stiffness criterion:

Using Equation (2.24) the static deflection due to a 1 kN concentrated load is found:

T**able 5.12:** Static deflection, EC5 AU

	b_{ef} [m]	EI_L [MNm^2/m]	F [kN]	w [mm]	w_{limit} [mm]
	Reference floor				
Hamm/Richter	Js = 0.6	3.07	1	0.68	0.25
	Floor with concrete screed				
Hamm/Richter	1.91	3.35	1	0.24	0.25

As the deflection value is higher than the limit value, the verification of static floor deflection is not satisfied for the reference floor. The verification of the floor with concrete screed is satisfied.

Acceleration response:

The acceleration response criterion is set to take care of low-frequency floor. As the floor under consideration is not low-frequency (since f_1 is higher than the limit value of 8 Hz) this verification should not be performed. It is, however, done to illustrate that the criterion will fail for high frequency floors.

Table 5.13: Acceleration response according to Eurocode 5, Austrian NA

	a_{limit} [m/s²]	*B* [m]	*L* [m]	ξ	F_0 [N]	α	*m* [kg/m²]	*a* [m/s²]
	Reference floor							
EC5 AU	*0.05*	*2.4*	*3.925*	*0.01*	*700*	*0.27 E-03*	*63*	*0.057*
	Floor with concrete screed							
EC5 AU	*0.05*	*2.4*	*3.925*	*0.03*	*700*	*0.27 E-03*	*265*	*0.30*

As the calculated acceleration is higher than the limit value, the verification is not satisfied, neither for the simple self-weight nor the added mass. The damping coefficient changes as the concrete screed are added, see Table 3.10.

5.2.4 According to the Comfort Criterion:

As mentioned in section 3.4, the verification of the reference floor according to the Comfort Criterion will not give the same results as for example the table 2 in SINTEF Certification 2635 (SINTEF Certification 2365 2017), as not enough information on how the "bjelkelagstabeller" were constructed are available. What is presented here is based on section 3.4, and the properties of the reference floor.

Fundamental frequency:

Using Equation (3.1) the fundamental frequency of the floor joists is found:

Table 5.14: Verification of fundamental frequency according to Comfort Criterion

	f_{limit} [Hz]	EI_L [MNm²/m]	m [kg/m²]	f_1 [Hz]
	Reference floor			
Hamm/Richter	10	3.07	63	22.5
	Floor with concrete screed			
Hamm/Richter	10	3.35	265	9.62

The fundamental frequency verifications are satisfied for the reference floor. The heavier floor, however, is not verified, as the limit value of the Comfort Criterion is 10 Hz.

Deflection /stiffness criterion:

Using Equation (2.24) the static deflection due to a 1 kN concentrated load is found:

Table 5.15: Static deflection according to the Comfort criterion

	b_{ef} [m]	EI_L [MNm^2/m]	F [kN]	w [mm]	w_{limit} [mm]
	Reference floor				
Hamm/Richter	Js = 0.6	3.07	1	0.68	1.3
	Floor with concrete screed				
Hamm/Richter	1.91	3.35	1	0.24	1.3

As the deflection calculated in lower than the limiting value, the static deflection criterion is satisfied for both the floor configuration. Neither of the floor deflections is close to the relatively generous limit of 1.3 mm deflection.

Combined criterion:

Combining the results from Table 6.15 and Table 5.15 as in Equation (3.11), the result from the combined criteria are presented in the table below:

Table 5.16: Combined criterion, Comfort criterion

	f_1/w-limit	w [mm]	f_1 [Hz]	$f_1/w^{0.44}$
	Reference floor			
Comfort Criterion	18.7	0.68	22.5	26.6
	Floor with concrete screed			
Comfort Criterion	18.7	0.24	9.62	18.2

The combined criterion is satisfied when the self-weight of the reference floor, as the ratio $\frac{f_1}{w_{0.44}}$ is higher than the limiting value for both the floor configuration. The concrete floor is not satisfied.

5.3 Summary

Table 5.17 is an overview of the calculations performed on the reference floor, while Table 5.18 shows the results of the verification of the floor with concrete screed.

Table 5.17: Summary of reference floor. Mass used is 63 kg/m^2.

<table>
<tr><td></td><td></td><td colspan="3">Calculated values</td></tr>
<tr><td></td><td>Limit values</td><td>Frequency requirement
$f1 > f_{limit}$</td><td>Stiffness requirement
$w < w_{limit}$</td><td>Mass requirement
$a < a_{limit}$</td></tr>
<tr><td rowspan="2">EC5 NO</td><td>f1 ≥ 8Hz</td><td rowspan="10">22.5 Hz
OK</td><td rowspan="2">0.68 mm
OK</td><td rowspan="2">-</td></tr>
<tr><td>w ≤ 0.6 mm</td></tr>
<tr><td rowspan="3">EC5 AU</td><td>f1 ≥ 8Hz</td><td rowspan="3">0.68 mm
Not OK</td><td rowspan="3">Not evaluated</td></tr>
<tr><td>w ≤ 0.25 mm</td></tr>
<tr><td>a ≤ 0.05 m/s^2</td></tr>
<tr><td rowspan="3">Hamm/
Richter</td><td>f1 ≥ 8Hz</td><td rowspan="3">1.36 mm
Not OK</td><td rowspan="3">Not evaluated</td></tr>
<tr><td>$w_{2 kN}$ ≤ 0.5 mm</td></tr>
<tr><td>a ≤ 0.05 m/s^2</td></tr>
<tr><td rowspan="3">Comfort
Criterion</td><td>f1 ≥ 10 Hz</td><td rowspan="3">0.68 mm
OK</td><td>Combined criteria</td></tr>
<tr><td>w ≤ 1.3 mm</td><td rowspan="2">21.7
OK</td></tr>
<tr><td>f1/ $w^{0.44}$ ≥ 18.7</td></tr>
</table>

The verification of the reference floor is satisfied according to the methods used in Norway; Norwegian National Annex and the Comfort Criterion. Due to the stricter limitation values of the European methods, the floor verification is not satisfied using the Hamm/Richter approach and the Austrian National Annex.

Table 5.18: Summary of the floor with concrete screed. Mass used is 265 kg/m^2.

		Calculated values		
	Limit values	Frequency requirement $f1 > f_{limit}$	Stiffness requirement $w < w_{limit}$	Mass requirement $a < a_{limit}$
EC5 NO	$f1 \geq 8$Hz	9.62 Hz OK	0.24 mm OK	-
	$w \leq 0.6$ mm			
EC5 AU	$f1 \geq 8$Hz		0.24 mm OK	Not evaluated
	$w \leq 0.25$ mm			
	$a \leq 0.05$ m/s^2			
Hamm/ Richter	$f1 \geq 8$Hz		0.47 mm OK	Not evaluated
	$w_{2\,kN} \leq 0.5$ mm			
	$a \leq 0.05$ m/s^2			
Comfort Criterion	$f1 \geq 10$ Hz	9.62 Hz Not OK	0.24 mm OK	Combined criteria
	$w \leq 1.3$ mm			18.2 Not OK
	$f1/w^{0.44} \geq 18.7$			

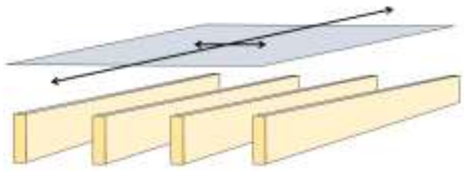
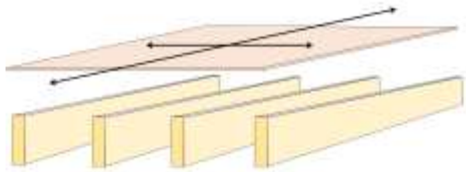
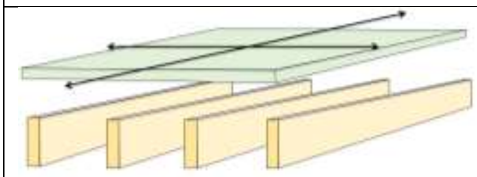
As the fundamental frequency is above the limit in the Austrian NA and Hamm/Richter approach, the acceleration response is not evaluated for the floors. The Norwegian NA has disregarded the vibration velocity response. This means that none of the methods considers any mass requirement. The method for calculating acceleration response in the Austrian National Annex makes it possible to perform the calculations even if the fundamental frequency is higher than the limit value. The calculation according to the Hamm/Richer-approach is dependent on a value for *F(t)*, taken form figure 20 in (Hamm et al. 2010), or Table 3.3in this document. This table does not give values of *F(t)* when f_1 is higher than 7.5 Hz. It is therefore not possible to follow the calculation approach correctly when investigating the floor configurations in this thesis.

6 Comparison of methods

In this section, the different criteria in the methods are compared and illustrated by looking at a maximum allowed span, L_{max} dependent on bending stiffness EI [MNm^2]. This is done to see what criterion in each method is the most severe, that is, what criterion dictates the outcome of the verification of the different methods.

Three floor configurations are considered, introduced in Table 6.1

Table 6.1: Floor types used to compare the code based methods.

	1) The reference floor: Very low transversal stiffness, low mass, assumed no effect of the transversal distribution.
	2) 22 mm OSB floor: Low transversal stiffness, live loads are taken into account (a bit higher mass), effect of transversal load distribution is considered.
	3) 50 mm concrete screed floor: Higher transversal stiffness, higher mass, effect of transversal load distribution is considered.

Floor 1) and 3) is identical to the floors investigated in section 5, and represent in a way two different floor types. The second floor is introduced to represent something in between, a floor in which the transversal distribution is assumed to be effective, and considered in the calculation, but with relatively low stiffness. An OSB plate having MOE of 3500 N/mm2 is chosen. This is the assumed floor configuration in the joist tables in (SINTEF Byggforsk 2011a), in which table 22b gives maximum allowed spans for glue-laminated beams. From the sub script of this table, I assume that the mass used when making this table is 0.7 kN/m^2 (maximum self-weight of the floor) + 0.5 kN/m^2 (from light partition / non-structural walls).

The same expressions for f_1 are used in all the methods when comparing them in the graphs, as well as the same expression for w. Joist spacing is taken as 600 mm. When using the same expression for f_1 and w in every method, only the difference in limitation values is considered. However, as seen in the above sections, there are differences also in proposed expressions for f_1 and w in the methods compared.

The strictest criterion in each of the verification methods is compared in the three different floors. Also, the deflection criterion from Eurocode 5 is compared to these results. This is done to briefly compare the limitations due to vibration verifications and those due to the deflection verification in serviceability limit state (SLS) design. Eurocode 5, pt. 7.2 is used, and the limitation value for final deflection, w_{fin} is used:

$w_{fin} \leq L/300$

The highest floor restrictions are set in every method: Floor class A in Hamm/Richter and Austrian NA, and "high demand floor" in the Norwegian National Annex.

Investigation of acceleration response criterion:

According to current code based methods, the acceleration response *does not have to be considered unless the fundamental frequency is below a limit value* (Hamm et al. 2010) and (EC5 Austrian NA 2014). This indicates that if the span is shorter than the span that gives $f_1 > f_{limit}$, the acceleration response will not be a problem. In other words, if the deflection criterion gives the lower value of L_{max} than the fundamental frequency criterion, there is no need to investigate the maximum allowed span according to the acceleration criterion. However, this statement will be tested by investigating L_{max} from the acceleration criterion from the Austrian National Annex to Eurocode 5.

The acceleration response criteria in Hamm/Richter is not considered in the comparison. This approach is dependent on *F(t)*, described as the harmonic parts of the force on the floor which is found from figure 20 in HR. The figure is dependent on the natural frequency being below 7.5 Hz. As the goal for investigating the acceleration criterion in the case of f_1 being equal to or above the limit value of 8 Hz, no value of *F(t)* can be found. Because of this, the acceleration response criterion will only be investigated in section 6.1.3, 6.2.3 and 6.3.2, where maximum span according to the Austrian National Annex is found.

Maximum span from the acceleration response criterion is found using the goal seek function in Excel. Since many of the parameters in the acceleration response criterion is dependent on *L*, using goal seek is a good approach. Figure 6.1 shows a screen shot from the Excel sheet used to find maximum span.

b x h	48 x 200		b x h	48 x 250		b x h	48 x 300	
m	63,5	kg/m²	m	63,5	kg/m²	m	63,5	kg/m²
EI L	1003508,04	Nm2/m (input)	EI L	1850694,25	Nm2/m (input)	EI L	3076497,58	Nm2/m (input)
L	3,05169533	m (output)	L	3,59092778	m (output)	L	4,11597575	m (output)
EI T	1716,99	Nm2/m	EI T	1716,99	Nm2/m	EI T	1716,99	Nm2/m
Js	0,6	m	Js	0,6	m	Js	0,6	m
b_{ef}	0,60	m	b_{ef}	0,60	m	b_{ef}	0,60	m
M*	58,13	kg	M*	68,41	kg	M*	78,41	kg
ξ	0,01		ξ	0,01		ξ	0,01	
f_1	21,204	Hz	f_1	20,796	Hz	f_1	20,409	Hz
α	0,0002		α	0,0002		α	0,0003	
F_0	700	N	F_0	700	N	F_0	700	N
Accel	0,050	m/s²	Accel	0,050	m/s²	Accel	0,051	m/s²
Accel limit	0,05	m/s²	Accel limit	0,05	m/s²	Accel limit	0,05	m/s²
Accel - Acce	0,000	(put to zero)	Accel - Acce	0,000	(put to zero)	Accel - Acce	0,001	(put to zero)

Figure 6.1: Screen shot form Excel work sheet were the maximum span from acceleration limit of 0.05 m/s^2 is found.

6.1 Without effect of transversal distribution

In this section, the maximum span according to each method is found by *neglecting* the effect of transversal load distribution. It is shown in section 5.1.2 that the effective width of the equivalent 1D beam representing the reference floor, is less that the actual joist spacing. The reason for this is the low stiffness of the plate material, causing the effect of transversal distribution to be neglectable or even negative. Because of this, the reference floor is used to illustrate the maximum span of the different methods for *a floor without effective transversal distribution of loads* is compared.

A summary of the material properties used to find L_{max} is found in Table 6.2.

Table 6.2: Properties for investigating L_{max} of reference floor.

MOE joist	14 000 N/mm^2
EI_L	Depend on joist cross section
MOE particle board	2550 N/mm^2
EI_T	1717 Nm^2/m
Joist spacing	600 mm
Sub-floor dimensions:	22 x 600 mm^2
Mass	64 kg /m^2

Total stiffness in the longitudinal direction is calculated as in section 2.3. The expression is repeated below:

$$EI_{tot} = EI_{joist} + EI_{plate} + \left(x_{joist} - x_{plate}\right)^2 * \left(\frac{1}{EA_{joist}} + \frac{1}{EA_{plate}}\right)^{-1} \quad \textbf{(6.1)}$$

The expressions used for calculating L_{max}, obtained by rearranging the different criterion Equations, is presented in Table 6.3:

Table 6.3: Expressions used to determine L_{max}, without transversal stiffness.

Criterion used:	Expression	Expression for *L [m]*	Units:
Frequency	$f_1 = \frac{\pi}{2\,l^2}\sqrt{\frac{EI}{m}}$	$L_{\max} = \sqrt{\frac{\pi}{2\,f1min}\sqrt{\frac{EI}{m}}}$	*EI* is in Nm^2 *m* is in kg/m f_1 is in Hz
Deflection for point load	$w = \frac{F \cdot l^3}{48 \cdot EI_L \cdot Js}$	$L_{\max} = \sqrt[3]{w_{max}\,\frac{48}{10^6 \cdot F}\,EI}$	*EI* is in Nm^2 *w* is in mm F is 1 kN
Comfort criteria	$\frac{f_1}{w^{0.44}} = 18.7$	$L_{max} = \frac{1}{7.9} \cdot \frac{EI^{\,0.283}}{m^{0.15}}$	*EI* is in Nm^2 *m* is in kg/m

6.1.1 Maximum allowed span according to EC5 Norwegian National Annex:

Norwegian National annex to Eurocode 5 has limitation values give in section 3.1, and repeated here:

$f_{1\ limit} = 8$ Hz, $w_{\ limit} = 0.6$ mm

Table 6.4 gives maximum span calculated using the expressions in Table 6.3.

Table 6.4: Maximum span for each cross section. Calculated from f_1 and *w*-criterion in EC5 Norwegian National Annex.

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]	
no.	b x h	[MNm^2/m]	f1 criterion	w criterion
1	36 x 200	0.486	4.71	2.40
2	36 x 250	0.886	5.48	2.94
3	36 x 300	1.461	6.21	3.47
4	48 x 200	0.602	4.97	2.58
5	48 x 250	1.110	5.80	3.17
6	48 x 300	1.846	6.58	3.76
7	70 x 200	0.812	5.36	2.85
8	70 x 250	1.517	6.27	3.52
9	70 x 300	2.546	7.13	4.18

The deflection criterion is clearly the most severe of the two.

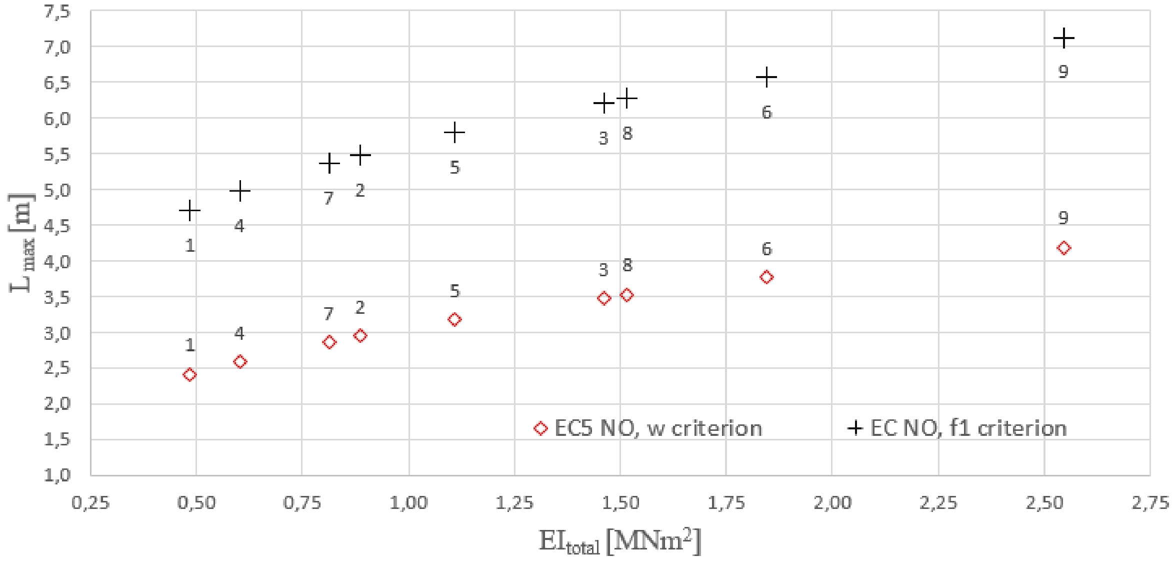


Figure 6.2 illustrates this.

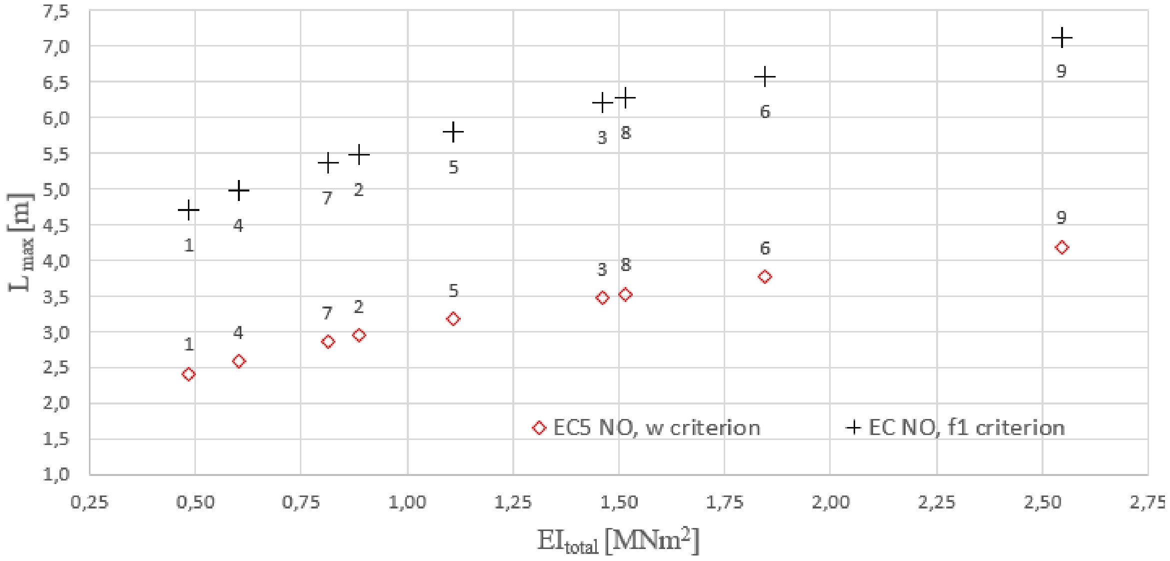


Figure 6.2: Maximum allowed span of the reference floor according to EC5 Norwegian NA

The vibration velocity is not regarded in this section, as it is not used in the Norwegian national annex to Eurocode 5.

6.1.2 Maximum allowed span according to Hamm/Richter approach:

Norwegian National annex to Eurocode 5 has limitation values give in section 3.2, and repeated here:

$f_{1\ limit}$ = 8 Hz, w_{limit} = 0.5 mm

As seen in Table 6.5, the deflection verification is the most severe, and the acceleration response verification is not considered. Figure 6.3 illustrates the results.

Table 6.5: Maximum span for each cross section. Calculated from f_1 and *w*-criterion in Hamm/Richer approach.

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]	
no.	b x h	[MNm²/m]	f1 criterion	w criterion
1	36 x 200	0.486	4.71	1.79
2	36 x 250	0.886	5.48	2.19
3	36 x 300	1.461	6.21	2.59
4	48 x 200	0.602	4.97	1.93
5	48 x 250	1.110	5.80	2.37
6	48 x 300	1.846	6.58	2.80
7	70 x 200	0.812	5.36	2.13
8	70 x 250	1.517	6.27	2.63
9	70 x 300	2.546	7.13	3.12

The deflection criterion is much more severe than the fundamental frequency criterion. Meaning the span needed to obtain a fundamental frequency *below limit value* for f_1 will not be accepted anyways. It is limited by the deflection criterion. This shows that including the acceleration response in this part of the thesis is not necessary for this floor configuration.

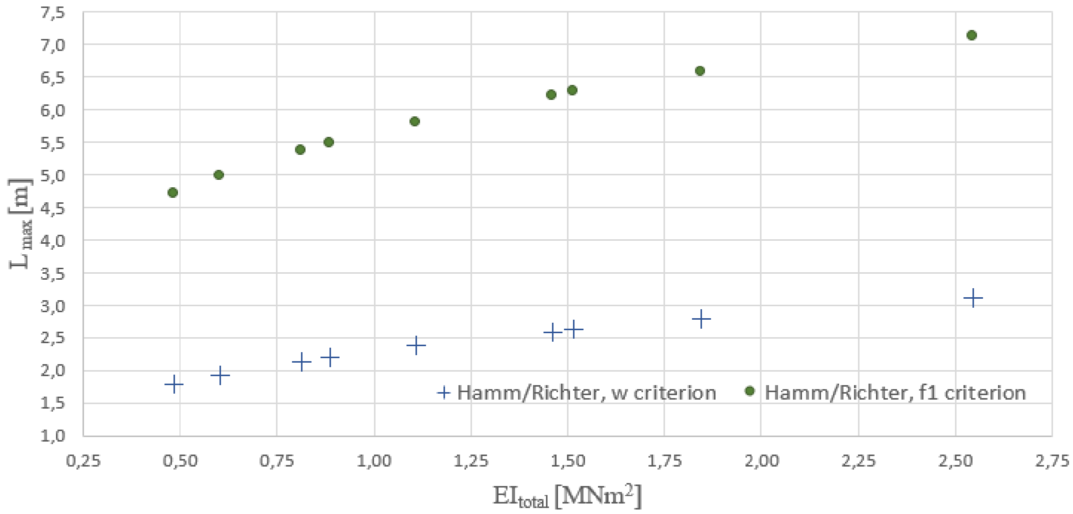


Figure 6.3: Maximum allowed span of the reference floor according to Hamm/Richter

6.1.3 Maximum allowed span according to EC5 Austrian National Annex:

Norwegian National annex to Eurocode 5 has limitation values give in section 3.3:

$f_{1\ limit}$ = 8 Hz, w_{limit} = 0.25 mm, a_{limit} = 0.05 m/s²

When finding the maximum span from the Austrian NA, the results from fundamental frequency and deflection criterion should be identical to the ones from the Hamm/Richter approach. This is because the *w* limit value of the Hamm/Richter approach is twice the one in the Austrian NA, but the applied point load in the *w* calculation is also twice the value (see 3.2 – Static deflection). Table 6.6 shows the results.

Table 6.6: Maximum span for each cross section of the reference floor, EC5 AU

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]		
no.	b x h	[MNm^2/m]	f1 criterion	w criterion	a criterion
1	36 x 200	0.486	4.71	1.79	2.88
2	36 x 250	0.886	5.48	2.19	3.39
3	36 x 300	1.461	6.21	2.59	3.87
4	48 x 200	0.602	4.97	1.93	3.05
5	48 x 250	1.110	5.80	2.37	3.59
6	48 x 300	1.846	6.58	2.80	4.12
7	70 x 200	0.812	5.36	2.13	3.31
8	70 x 250	1.517	6.27	2.63	3.90
9	70 x 300	2.546	7.13	3.12	4.47

It should be noted that the maximum span allowed by the acceleration criterion is much lower than maximum allowed span from the fundamental frequency criterion. This means that using L_{max} from the f_1 will cause the acceleration response verification to fail. However, the deflection verification is the most severe, so the span used will not cause the acceleration response verification to fail. Figure 6.4 illustrates the results.

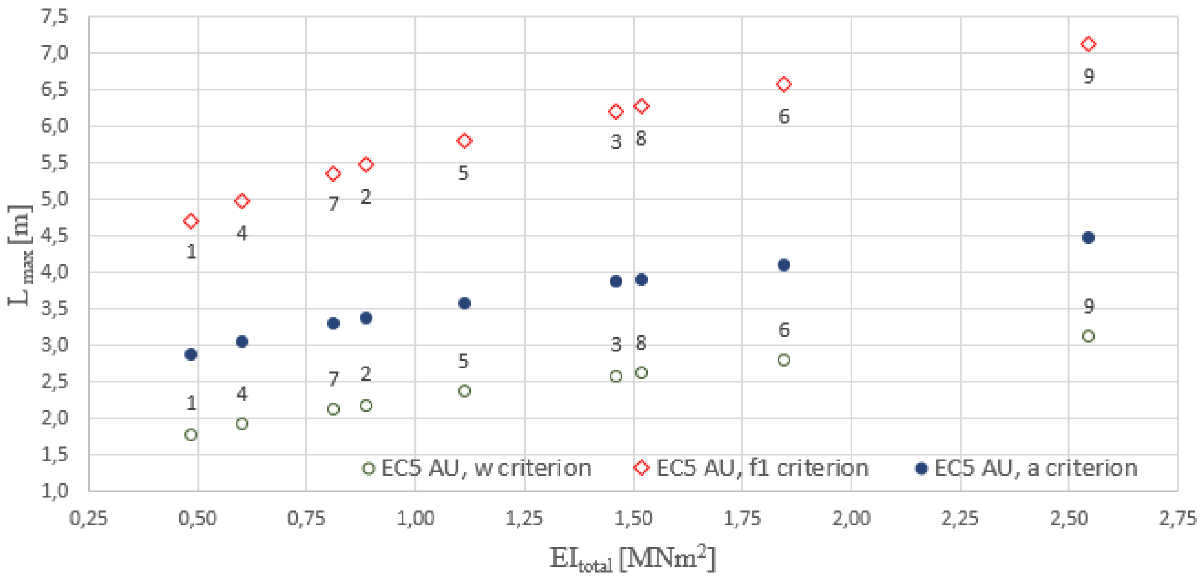


Figure 6.4: Maximum allowed span of the reference floor according to EC5 Austrian NA

6.1.4 Maximum allowed span according to Comfort Criterion:

The Comfort Criterion has limitation values give in section 3.4, and repeated here:

$f_{1\ limit}$ = 10 Hz, $w_{\ limit}$ = 1.3 mm, $f_1 / w^{0.44} \geq 18.7$

Table 6.7 gives maximum span according to the Comfort Criterion, calculated using the expressions in Table 6.3.

Table 6.7: Maximum span for each cross section. Calculated from f1-, w- and combined criterion in Comfort Criterion.

no.	b x h	Stiffness EI_{tot} [MNm²/m]	Maximum span L [m] f1 criterion	w criterion	combined [m]
1	36 x 200	0.554	4,21	3,11	2,98
2	36 x 250	0.995	4,90	3,80	3,53
3	36 x 300	1.620	5,55	4,50	4,07
4	48 x 200	0.676	4,45	3,34	3,17
5	48 x 250	1.226	5,18	4,10	3,77
6	48 x 300	2.014	5,89	4,86	4,35
7	70 x 200	0.892	4,79	3,70	3,45
8	70 x 250	1.640	5,60	4,55	4,12
9	70 x 300	2.722	6,38	5,41	4,77

The maximum span determined from the three criteria are closer spaced according to the Comfort Criterion than the other methods. Compared to the other methods, the limit value for f_1 is more severe in the Comfort Criterion, and the w criterion is less severe.

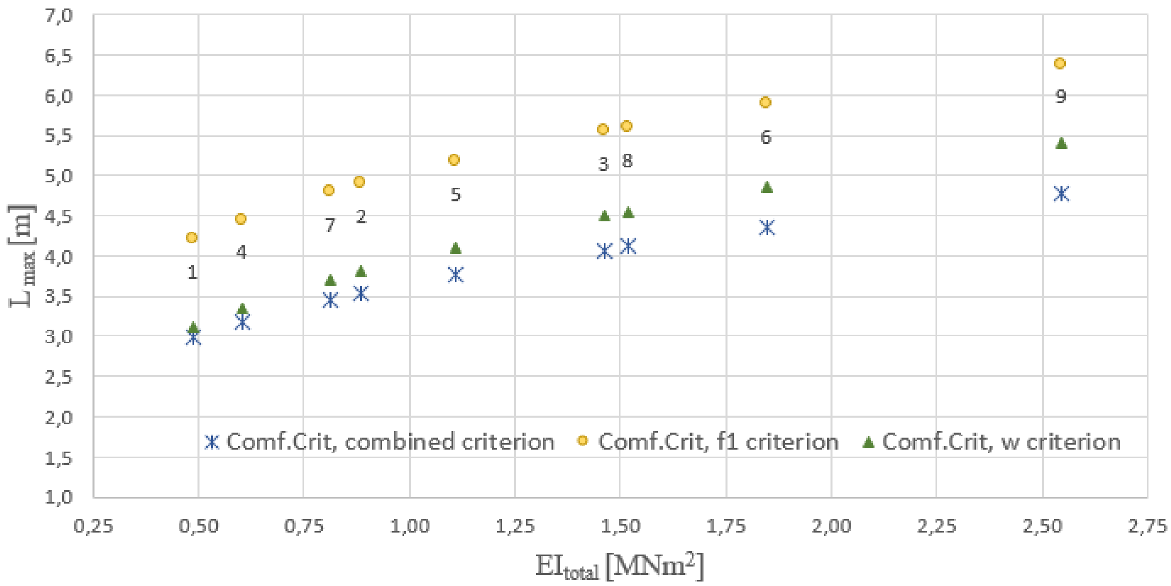


Figure 6.5: Maximum allowed span of the reference floor according to the Comfort Criterion

6.1.5 Overall strictest criterion:

The strictest criterion in each method is gathered, to more easily compare the methods. Also, the deflection criterion from Eurocode 5, pt. 7.2 is compared to the different vibration criteria. The results are shown in Table 6.8 and Figure 6.6.

Table 6.8: Maximum span of the reference floor, using the overall strictest criteria

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
EC5 NO [m]*	2.40	2.94	3.47	2.58	3.17	3.76	2.85	3.52	4.18
HR [m]*	1.79	2.19	2.59	1.93	2.37	2.80	2.13	2.63	3.12
EC5 AU [m]*	1.79	2.19	2.59	1.93	2.37	2.80	2.13	2.63	3.12
Comfort C. [m]**	2.98	3.53	4.07	3.17	3.77	4.35	3.45	4.12	4.77
δ criterion	4.28	5.23	6.18	4.60	5.64	6.68	5.08	6.26	7.44

* Deflection criterion, ** Fundamental frequency criterion

In this floor configuration, the vibration verifications are more severe than the verification of deflection according to EC5, pt.7.2.

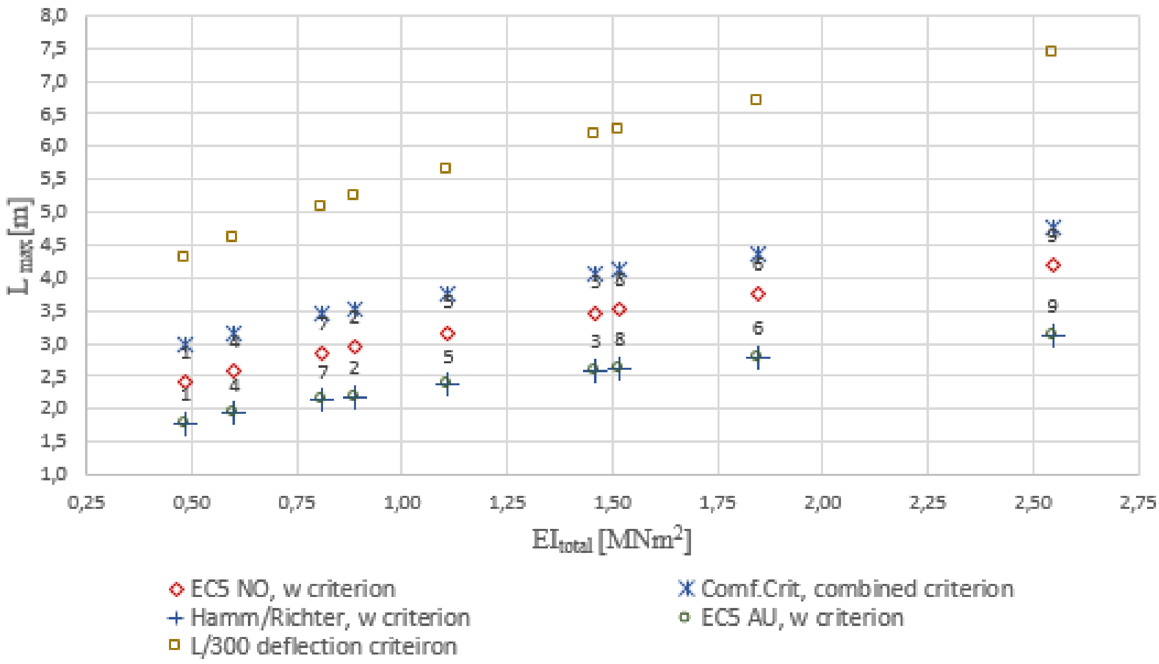


Figure 6.6: Comparison of the strictest criteria in each method for the reference floor. Joist spacing is taken as 600 mm. The effect of transversal distribution ignored.

From Figure 6.6 it can be seen that the maximum span according to the deflection criteria in Hamm/Richter and Austrian NA to Eurocode 5 coincide. From this graph, it is clear that these two are the most sevre methods.

When considering a light floor, such as the reference floor, the deflection criteria is the most severe in every method except in the Comfort Criteiron.

6.2 With the effect of transversal distribution, using 22 mm OSB

In this section, a floor configuration using an OSB plate instead of a perforated particle board is investigated. The properties are given in Table 6.9. The OSB plate is assumed having high enough stiffness so that the effective width of the equivalent beam should be wider than the joist spacing. I had chosen to use a mass of 1.2 kN/m^2 (or 122 kg/m^2) when investigating this floor configuration, as I assume this is the load used when the tables in (SINTEF Byggforsk 2011a) were made.

Table 6.9: Properties for investigating L_{max} of a floor 22 mm OSB

MOE joist	14 000 N/mm^2
EI_L	Depend on joist cross section
MOE OSB	3500 N/mm^2
EI_T	2670.9 Nm^2/m
Joist spacing	600 mm
Sub-floor dimensions:	22 x 600 mm^2
Mass	122 kg /m^2

The 22 mm OSB floor has a bit higher stiffness than the reference floor. Calculation of b_{ef} of the OSB floor, with the reference length of 3.925 m, is seen in the table below.

Table 6.10: b_{ef} depending on J_s, L = 3.925 m.

Joist cross section			b_{ef} depending on Js		
nr.	b x h	EI tot [MNm2]	0.3	0.4	0.6
1	36 x 200	0.554	0.70	0.75	0.83
2	36 x 250	0.995	0.60	0.65	0.71
3	36 x 300	1.620	0.53	0.57	0.63
4	48 x 200	0.676	0.66	0.71	0.79
5	48 x 250	1.226	0.57	0.61	0.68
6	48 x 300	2.014	0.50	0.54	0.60
7	70 x 200	0.892	0.62	0.66	0.73
8	70 x 250	1.640	0.53	0.57	0.63
9	70 x 300	2.722	0.47	0.50	0.56

As can be seen from Table 6.10, b_{ef} is larger than the joist spacing in most of the floor configurations, but not all. This shows that if the maximum allowed is lower than some limit length, b_{ef} won't be applicable to that floor configuration. In this case, the joist spacing has to be used instead.

Expressions used to calculate the maximum span, *with* the effect of transversal load distribution, is presented in Table 6.11:

Table 6.11: Expressions used to determine L_{max}, with transversal stiffness.

Criterion used:	Expression	Expression for *L [m]*	Units:
Frequency	$f_1 = \frac{\pi}{2\, l^2}\sqrt{\frac{EI}{m}}$	$L_{\text{max}} = \sqrt{\frac{\pi}{2\, f1min}\sqrt{\frac{EI}{m_{(line)}}}}$	EI is in Nm^2 m is in kg/m f_1 is in Hz
Deflection for point load	$w = \frac{F\, \cdot l^3}{48\, \cdot EI_L \cdot b_{ef}}$	$L_{\text{max}} = \sqrt[2]{w_{max}\, \frac{48}{10^6 \cdot\, F}\, EI_L\, \frac{1}{1.1}\sqrt[4]{\frac{EI_T}{EI_L}}}$	$EI_{T \text{ and } L}$ is in Nm^2/m w is in mm F is 1 kN
Comfort criteria	$\frac{f_1}{w^{0.44}} = 18.7$	$L_{max} = \frac{1}{11} \cdot \frac{{EI_L}^{0.288} \cdot {EI_T}^{0.0382}}{M^{0.1736}}$	EI_L is in Nm^2/m EI_T is in Nm^2/m M is in kg/m^2

6.2.1 Maximum allowed span according to EC5 Norwegian National Annex:

Table 6.12 shows the maximum according to the Norwegian National Annex to Eurocode 5.

Table 6.12: Maximum span for each cross section. Calculated from f_1 and *w*-criterion in EC5 Norwegian National Annex.

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]	
no.	b x h	[MNm^2/m]	f1 criterion	w criterion
1	36 x 200	0.554	4.13	2.36
2	36 x 250	0.995	4.78	2.94
3	36 x 300	1.620	5.40	3.54
4	48 x 200	0.676	4.34	2.55
5	48 x 250	1.226	5.03	3.18
6	48 x 300	2.014	5.70	3.84
7	70 x 200	0.892	4.65	2.83
8	70 x 250	1.640	5.41	3.55
9	70 x 300	2.722	6.14	4.30

Figure 6.7 illustrates the result. The deflection criterion is the most severe also for this floor configuration.

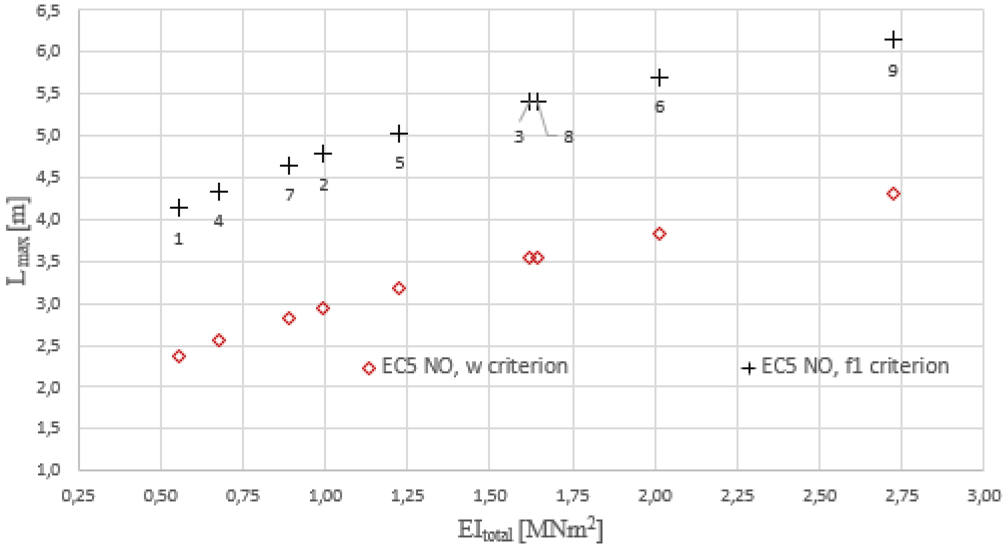


Figure 6.7: Maximum allowed span of the 22mm OSB- floor according to EC5 Norwegian NA

6.2.2 Maximum allowed span according to EC5 Austrian National Annex and Hamm/Richter approach:

Since both the floor verifications in section 5.2 and the comparison of the methods in section 6.1 shows that the Hamm/Richter approach and Austrian National Annex gives the same results for calculated deflection and fundamental frequency, results from the methods will be presented in one single table and graph. Note: Maximum span according to the acceleration criterion is only valid for the Austrian National Annex, but presented in the same table.

Table 6.13: Maximum span for each cross section of the 22mm OSB floor. Calculated from f_1 and *w*-criterion in EC5 Austrian National Annex and Hamm/Richter.

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]		
no.	b x h	[MNm^2/m]	f1 criterion	w criterion	a criterion*
1	36 x 200	0.554	4.13	1.52	2,62
2	36 x 250	0.995	4.78	1.90	3,06
3	36 x 300	1.620	5.40	2.28	3,48
4	48 x 200	0.676	4.34	1.64	2,76
5	48 x 250	1.226	5.03	2.05	3,23
6	48 x 300	2.014	5.70	2.47	3,69
7	70 x 200	0.892	4.65	1.82	2,97
8	70 x 250	1.640	5.41	2.29	3,50
9	70 x 300	2.722	6.14	2.77	4,00

* Austrian National Annex criterion only

The static deflection criterion is the most severe, as seen in Table 6.13 and Figure 6.8. Again the maximum span allowed from the acceleration criterion is much lower than maximum allowed span from the fundamental frequency criterion.

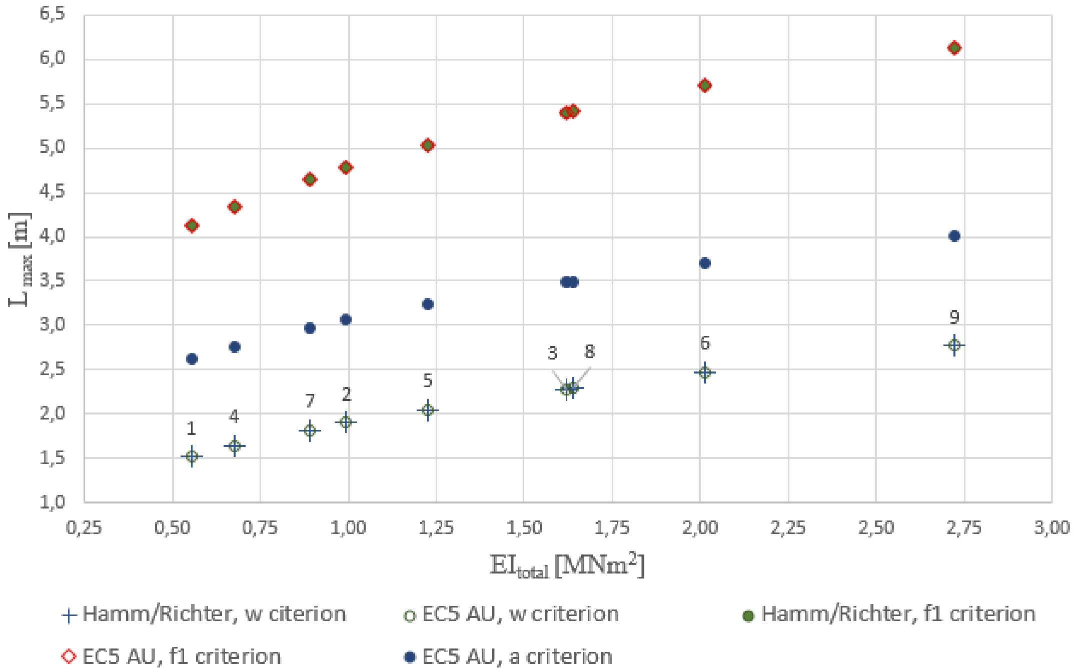


Figure 6.8: Maximum allowed span of the 22mm OSB floor according to Hamm/Richter and Austrian NA

6.2.3 Maximum allowed span according to Comfort Criterion:

The most severe criterion is the combined criterion, as seen in Table 6.14 and Figure 6.9. For the cross sections with lower frequency (cross section no. 1,2.4.5 and 7), the deflection criterion is more severe than the fundamental frequency criterion. However, when the stiffness increases the opposite is the case, illustrating that in this floor configuration, higher stiffness is beneficial for the deflection due to a point load.

Table 6.14: Maximum span for each cross section. Calculated from f_1-, w- and combined criterion in Comfort Criterion.

no.	b x h	Stiffness EI_{tot} [MNm²/m]	Maximum span L [m] f1 criterion	w criterion	combined [m]
1	36 x 200	0.554	3,69	3,48	2,80
2	36 x 250	0.995	4,27	4,34	3,31
3	36 x 300	1.620	4,83	5,21	3,80
4	48 x 200	0.676	3,88	3,75	2,96
5	48 x 250	1.226	4,50	4,69	3,51
6	48 x 300	2.014	5,10	5,65	4,04
7	70 x 200	0.892	4,16	4,16	3,20
8	70 x 250	1.640	4,84	5,23	3,81
9	70 x 300	2.722	5,50	6,33	4,40

Figure 6.9 shows the results.

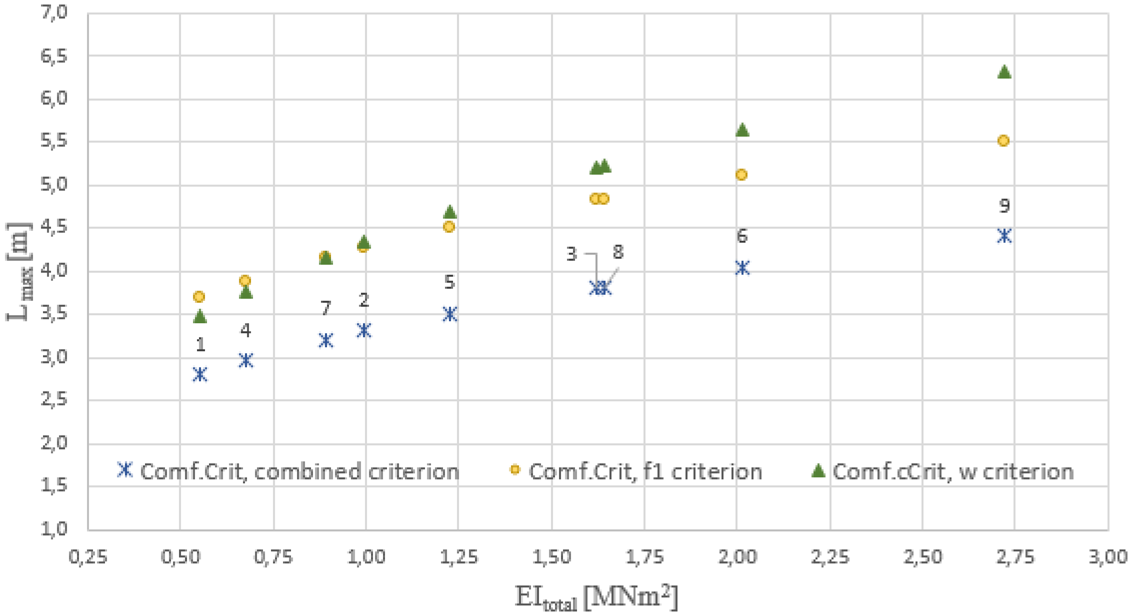


Figure 6.9: Maximum allowed span of the 22mm OSB floor according to the Comfort Criterion

6.2.4 Overall strictest criterion:

The overall strictest criterion in each of the vibration verification methods is presented in Table 6.15 and Figure 6.10, along with the deflection verification of Eurocode 5. There is no difference between the maximum span according to Hamm/Richter and Austrian NA, and again these are the most severe verification methods due to their strict limitation values for deflection.

Table 6.15: Maximum span of the 22mm OSB floor, using the overall strictest criteria

Cross section:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
EC5 NO [m]*	2,36	2,94	3,54	2,55	3,18	3,84	2,83	3,55	4,30
HR [m]*	1,52	1,90	2,28	1,64	2,05	2,47	1,82	2,29	2,77
EC5 AU [m]*	1,52	1,90	2,28	1,64	2,05	2,47	1,82	2,29	2,77
Comfort C. [m]**	2,80	3,31	3,80	2,96	3,51	4,04	3,20	3,81	4,40
δ criterion	4,47	5,44	6,40	4,78	5,83	6,88	5,24	6,42	7,60

* Deflection criterion, ** Fundamental frequency criterion

Again, the deflection verification is less severe than any of the vibration verification criteria. Using the maximum span form Hamm/Richter and Austrian NA, the effective width b_{ef} is shown in the **Feil! Fant ikke referansekilden.**. These two are used to illustrate, as the use of b_{ef} is introduced in these two methods.

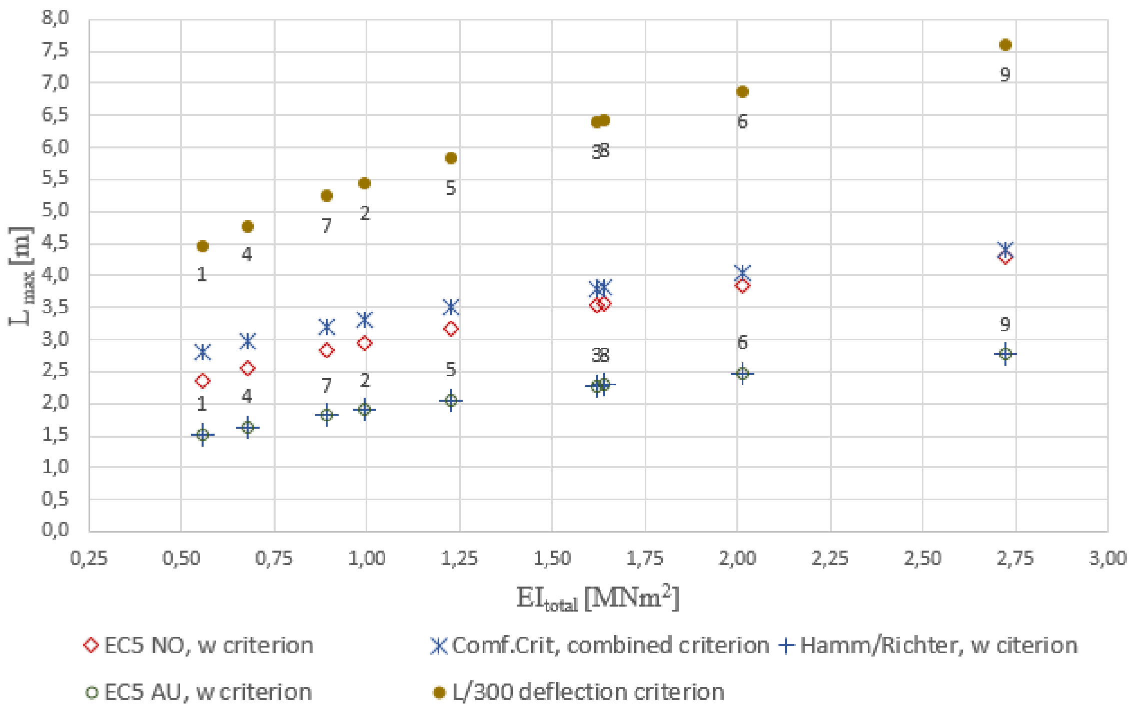


Figure 6.10: Comparison of the strictest criteria in each method for the 22mm OSB floor. Joist spacing is taken as 600 mm. The effect of transversal distribution considered.

6.3 With the effect of transversal distribution using 50 mm concrete.

Equation (6.2 is used to find the longitudinal stiffness. Now the 50mm concrete screed is included in the calculation. However, since it is not rigidly fastened to the particle board, its contribution is simply added to the stiffness of the joist and plate configuration.

$$EI_{tot} = EI_{joist} + EI_{plate} + \left(x_{joist} - x_{plate}\right)^2 * \left(\frac{1}{EA_{joist}} + \frac{1}{EA_{plate}}\right)^{-1} + EI_{con.} \quad \textbf{(6.2)}$$

The stiffness of the concrete screed is constant, while the overall stiffness changes with the different joist cross section. This gives that the contribution of the concrete screed varies from between 25 % to the overall stiffness in the smallest joist cross section, to only 6 % in the largest one.

Table 6.16: Properties for investigating L_{max} of a floor 50 mm concrete screed

MOE joist	14 000 N/mm^2
MOE particle board	2550 N/mm^2
$EI_{L\ joist+plate}$	Depend on joist cross section
MOE concrete	26 000 N/mm^2
$EI_{concrete}$	162 500 N/mm^2
$EI_{L\ tot}$	Depend on joist cross section
EI_T	272 550 Nm^2/m
Joist spacing	600 mm
Sub-floor dimensions:	22 x 600 mm^2
Mass	265 kg /m^2

The mass is taken from the sub script of table 22b in (SINTEF Byggforsk 2011a).

6.3.1 Maximum allowed span according to EC5 Norwegian National Annex:

As seen in Table 6.17 and Figure 6.11, it is now the fundamental frequency criterion that is most severe. This is due to the heavy mass, since this reduces the fundamental frequency, but is not considered in the deflection criterion.

Table 6.17: Maximum span for each cross section. Calculated from f_1 and w-criterion in EC5 Norwegian National Annex.

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]	
no.	b x h	[MNm^2/m]	f1 criterion	w criterion
1	36 x 200	0,65	3,54	4,47
2	36 x 250	1,05	3,99	5,36
3	36 x 300	1,62	4,45	6,31
4	48 x 200	0,76	3,68	4,76
5	48 x 250	1,27	4,19	5,76
6	48 x 300	2,01	4,69	6,84
7	70 x 200	0,97	3,92	5,21
8	70 x 250	1,68	4,49	6,39
9	70 x 300	2,71	5,06	7,65

Figure 6.11 illustrates the result.

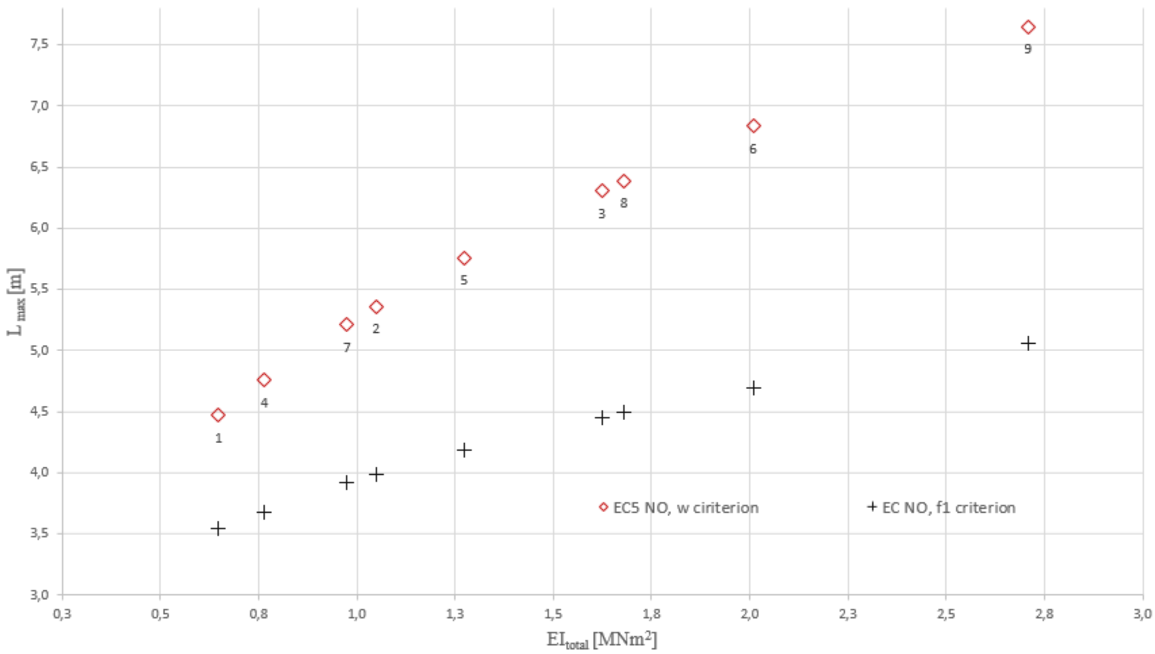


Figure 6.11: Maximum allowed span of the 50mm concrete screed floor according to EC5 Norwegian NA

6.3.2 Maximum allowed span according to Hamm/Richter and EC5 Austrian National Annex:

As in section 6.2.2, the results from Hamm/Richter and Austrian National Annex will be presented in one table and one graph. Note: Maximum span according to the acceleration criterion is only valid for the Austrian National Annex, but presented in the same table.

Table 6.18: Maximum span for each cross section. Calculated from f_1 and *w*-criterion in EC5 Austrian National Annex ana Hamm/Richter**.**

Joist cross section		Stiffness EI_{tot}	Maximum span, L_{max} [m]		
no.	b x h	[MNm^2/m]	f1 criterion	w criterion	a criterion*
1	36 x 200	0,65	3,54	2,88	2,86
2	36 x 250	1,05	3,99	3,46	3,28
3	36 x 300	1,62	4,45	4,07	3,71
4	48 x 200	0,76	3,68	3,07	3,00
5	48 x 250	1,27	4,19	3,72	3,47
6	48 x 300	2,01	4,69	4,41	3,95
7	70 x 200	0,97	3,92	3,36	3,22
8	70 x 250	1,68	4,49	4,13	3,75
9	70 x 300	2,71	5,06	4,94	4,30

* Only Austrian National Annex

The acceleration response is now the most severe criterion. This means using spans according to any other verification will cause the acceleration response verification to fail. Figure 6.12 illustrates the results.

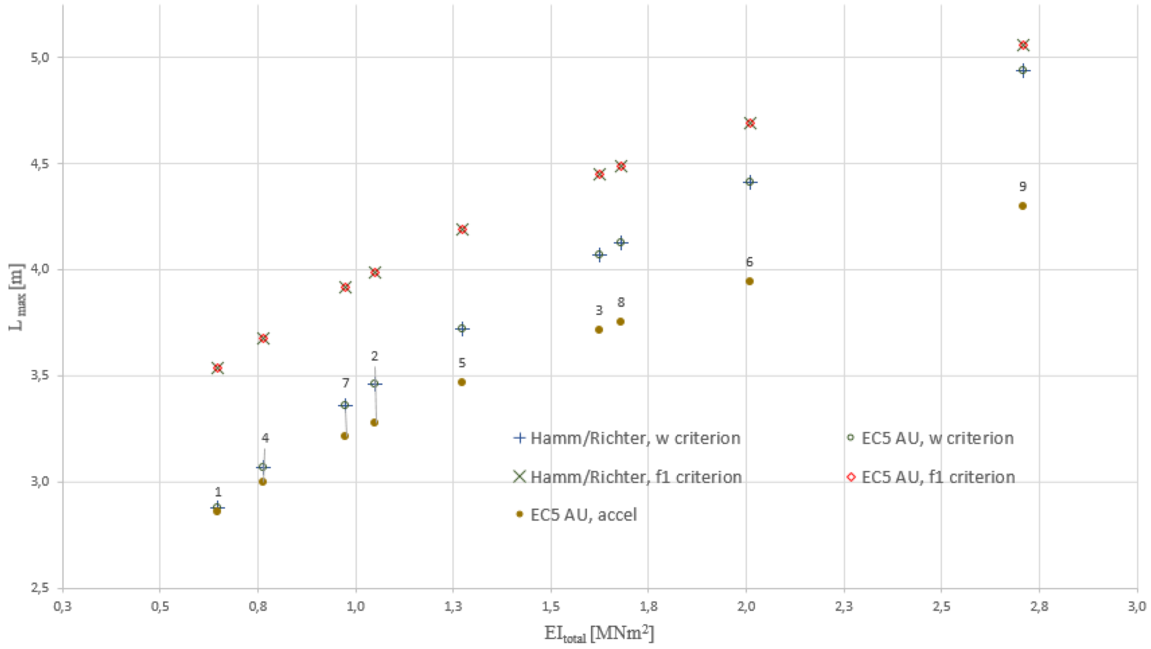


Figure 6.12: Maximum allowed span of the 50mm concrete screed floor according to EC5 Austrian NA and Hamm/Richter

6.3.3 Maximum allowed span according to Comfort Criterion:

Because of the generous limit value for deflection in the Comfort Criterion (w_{limit} = 1.3 mm), the stiff floor with concrete slab obtain much longer spans than the frequency criterion, which is more affected by the increased mass. The result is a large difference in the maximum allowed spans from the two criteria.

Table 6.19: Maximum span for each cross section. Calculated from f_1-, w- and combined criterion in Comfort Criterion.

		Stiffness EI_{tot}	Maximum span L [m]		
no.	b x h	[MNm²/m]	f1 criterion	w criterion	combined [m]
1	36 x 200	0,65	3,16	6,58	3,04
2	36 x 250	1,05	3,57	7,89	3,49
3	36 x 300	1,62	3,98	9,29	3,96
4	48 x 200	0,76	3,30	7,01	3,19
5	48 x 250	1,27	3,74	8,48	3,69
6	48 x 300	2,01	4,20	10,07	4,21
7	70 x 200	0,97	3,50	7,67	3,42
8	70 x 250	1,68	4,01	9,41	4,00
9	70 x 300	2,71	4,52	11,26	4,59

The combined criterion is the most severe for the smaller (and less stiff) cross sections, while the fundamental frequency is barely stricter for the stiffer cross sections.

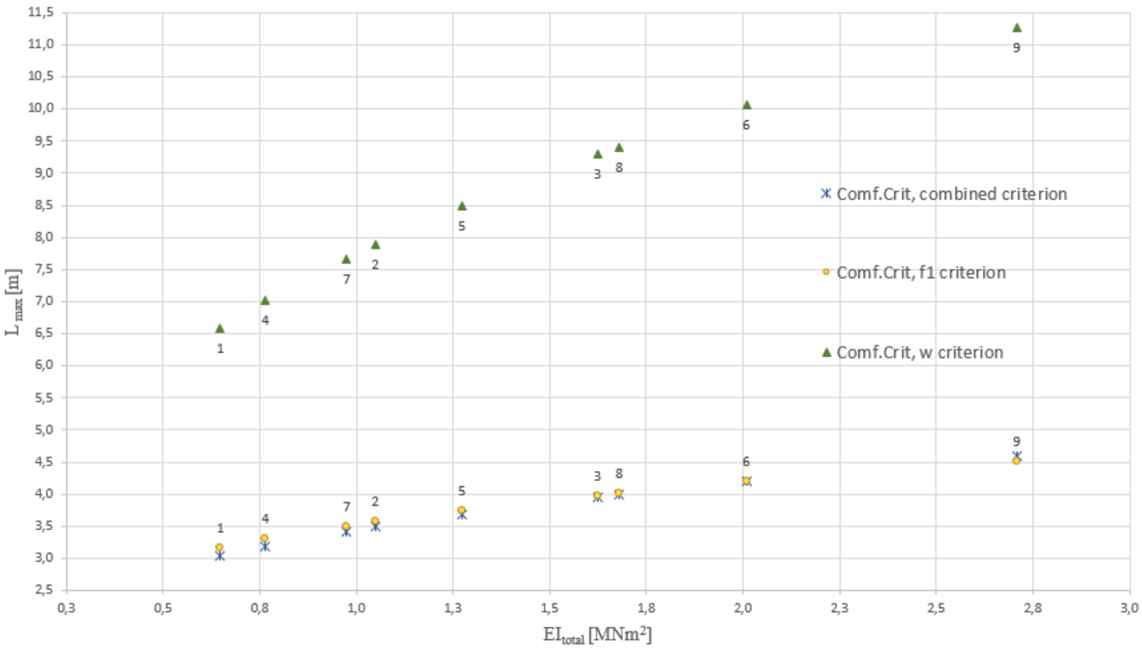


Figure 6.13: Maximum allowed span of the 50mm concrete screed floor according to the Comfort Criterion

6.3.4 Overall strictest criterion:

From Figure 6.14, it is the fundamental frequency criterion from the Comfort Criterion that is the most severe. This is due to the high mass of the floor, and the stricter limitation values for f_1 of the Comfort Criterion compared to the other methods.

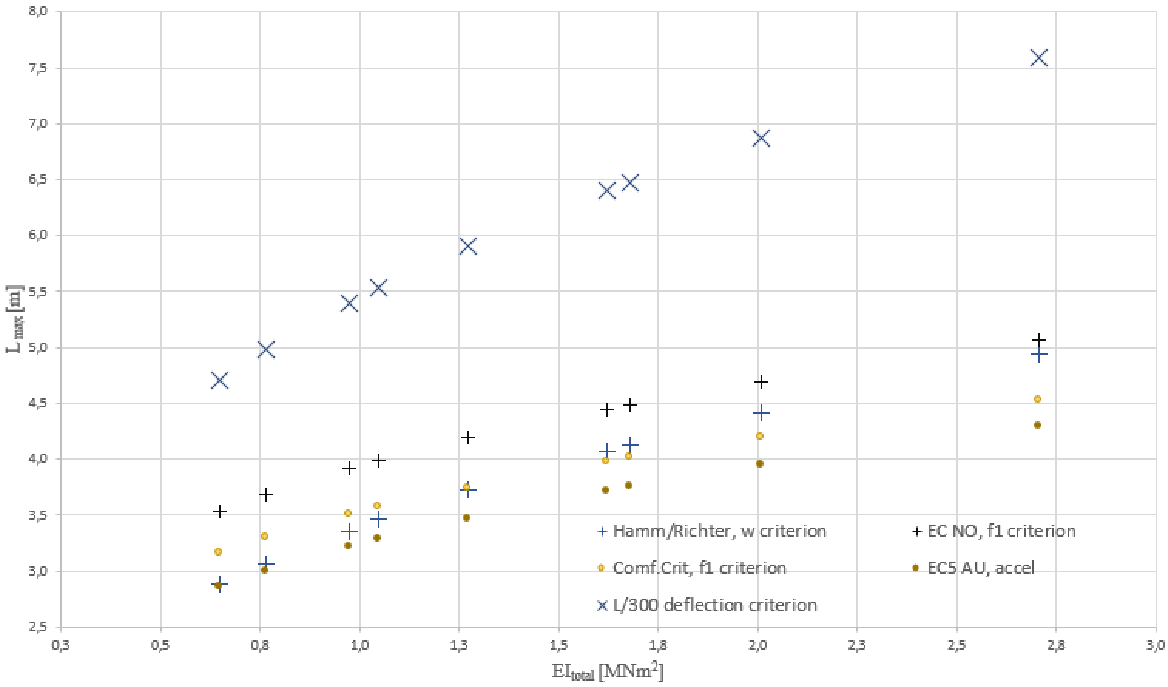


Figure 6.14: Comparison of the strictest criteria in each method for the 50mm concrete screed floor. Joist spacing is taken as 600 mm. The effect of transversal distribution considered.

The deflection criterion from the Norwegian National Annex and the SLS deflection verification from EC5 gives much more generous spans. The acceleration response verification is the most severe out of all the criteria. The values can also be seen in Table 6.20:

Table 6.20: Maximum span of the 50mm concrete screed floor, from the overall strictest criteria in each method

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
EC5 NO [m]*	3,54	3,99	4,45	3,68	4,19	4,69	3,92	4,49	5,06
HR [m]*	2,86	3,28	3,71	3,00	3,47	3,95	3,22	3,75	4,30
EC5 AU [m]*	2,88	3,46	4,07	3,07	3,72	4,41	3,36	4,13	4,94
Comfort C. [m]**	3,04	3,49	3,96	3,19	3,69	4,20	3,42	4,00	4,52
δ criterion	4,71	5,53	6,40	4,98	5,90	6,87	5,40	6,47	7,59

* Deflection criterion, ** Fundamental frequency criterion, *** Acceleration criterion

6.4 Summary:

The goal of this chapter was to compare the different code based methods. This was done by finding the maximum allowed span according to each criterion within the methods, regarding longitudinal stiffness of the floor. When investigating the 22mm OSB floor and the 50mm concrete screed floor, transversal distribution was considered through the use of the *equivalent width,* b_{ef} (see section 2.3.1). However, Table 6.10 shows that the effect of b_{ef} cannot always be taken into account.

Summarised, it can be said that the deflection criterion is the most severe in the light floors (the exception is the Comfort Criterion, where the fundamental frequency is more severe due to higher f_1 limit value). In the heavier floor with concrete screed, the acceleration response criterion was the most severe according to the Austrian National Annex, the other results were as in the light floors. The maximum span from the $w \leq L/300$ criterion of Eurocode 5, pt. 7.2 was also found. The results showed that this deflection verification criterion is less severe than any of the vibration verification criteria.

Another goal was to verify the statement "If the deflection criterion gives a lower value of L_{max} than the fundamental frequency criterion, there is no need to investigate the maximum allowed span according to the acceleration criterion". This is interesting as the current code based methods only investigate the acceleration response in a floor if the fundamental frequency is below a certain limit value if it is investigated at all. In turn, this means that the effect of systems mass is not fully considered, which means that problematic vibrations may occur due to transient acceleration response.

Table 6.21 shows that the statement holds for the reference floor, as the maximum span from the deflection criterion is shorter than for the acceleration response criterion (see Table 6.6). As can be seen, the fundamental frequency is way above the limit value of 8 Hz, when the short span from the deflection limitation is used. Acceleration response will not be a problem.

Table 6.21: Calculated acceleration response of the reference floor form L_{max} from the deflection criterion of EC5 Austrian NA.

b x h	L_{max} from w [m]	EI_L [Nm2/m]	f_1 [Hz]	M^* [kg]	a [m/s^2]
36 x 200	1,79	809168,356	55,34	34,1	0,0000001
36 x 250	2,19	1476491,22	49,94	41,7	0,0000007
36 x 300	2,59	2434930,22	45,85	49,3	0,0000031
48 x 200	1,93	1003508,04	53,01	36,8	0,0000002
48 x 250	2,37	1850694,25	47,74	45,1	0,0000016
48 x 300	2,80	3076497,58	44,10	53,3	0,0000057
70 x 200	2,13	1353412,31	50,55	40,6	0,0000006
70 x 250	2,63	2528621,81	45,32	50,1	0,0000037
70 x 300	3,12	4242863,72	41,71	59,4	0,0000134

This is also the case in the 22mm OSB floor (see Table 6.13)

In the heavier floor with 50mm concrete screed, however, maximum span from deflection exceeds the maximum span form acceleration response. Table 6.22 shows that using L_{max} from the deflection criterion gives fundamental frequencies above the limit value. As an example, this means that the acceleration response will be too high in these floors when using L_{max} from deflection criterion, but the verification of acceleration won't be done, as the fundamental frequency is above the limit value.

Table 6.22: Calculated acceleration response of the 50mm concrete screed floor form L_{max} from the deflection criterion of EC5 Austrian NA.

b x h	L_{max} from w [m]	EI_L [Nm2/m]	f_1 [Hz]	M^* [kg]	a [m/s^2]
36 x 200	2,88	1080002	12,09	229,0	0,162
36 x 250	3,46	1747325	10,65	127,2	0,517
36 x 300	4,07	2705764	9,58	149,6	0,675
48 x 200	3,07	1274341	11,56	112,8	0,406
48 x 250	3,72	2121528	10,16	136,7	0,587
48 x 300	4,41	3347331	9,08	162,1	0,763
70 x 200	3,36	1624246	10,89	123,5	0,484
70 x 250	4,13	2799455	9,47	151,8	0,697
70 x 300	4,94	4513697	8,40	181,5	0,893

7 Semi-numerical analysis of the floor

This chapter will introduce a more advanced investigation, as the a_{rms} and VDV method presented in section 3.6 is used to investigate the reference and the 50mm concrete screed floor. In brief introduction to the finite element method is given, and a description of how a simplified model is used to represent the two-dimensional floor is obtained. Due to these simplifications, a full model of the floor is not made, and the analysis is called “semi-numerical”.

7.1 Introduction to the finite element method

Numerical analysis is concerned with obtaining approximate solutions, within a reasonable range of error, not exact answers. These are often impossible to obtain in practice, and it is in most cases important to find the solution that is “correct enough”.

The finite element method an example of numerical analysis. When dealing with physics and mechanics, the problems encountered are commonly modelled as a set of differential equations. These are often too complex to be solved by using analytical methods, and numerical methods must be applied. The finite element method seeks to derive an approximate solution to general differential problems, by numerical means. It transforms a continuous system with an infinite number of unknowns into a discrete system with a finite number of unknown.

Structural problems often involve parameters/variables that vary over the different regions in the structure in a non-linear manner (continuous systems). To investigate the behaviour of a region or the entire structure, it is divided into smaller elements. The behaviour of one element is determined using the approximation that the variable under consideration varies in a certain manner over the entire element. The element is often approximated to a polynomial, is linear, quadratic, cubic, etc. The solution for the entire region in the structure is obtained by assembling the behaviour of all the elements into a global linear equation system. The number of elements needed to obtain a “correct enough” solution will depend on the complexity of the structure and load situation. More and smaller elements (also called a finer mesh) results in a more accurate approximation but also increased computation time. An approximation to how fine a mesh should be, is that when doubling the number of elements do not significantly alter the variables, the mesh is fine enough.

A full model of the floor will not be made here, due to the limitations of this thesis. Instead, *equivalent beam* models of the floor will be made, using the *effective width* b_{ef} as a parameter taking into account the transversal stiffness of the floor, and by that the two-dimensional properties. The equivalent beam is a one-dimensional system, which can be investigated using the approach presented in section 3.6.This is a simplification, and cannot replace a full model of the floor. However, it will give indications on how the dynamic properties of the floor, when the equivalent beam models are used to performed an modal analysis.

The properties of the equivalent beam model can be obtained through a 2D model of the floor, or through an analytical approach. This will briefly be discussed in the following section.

7.2 Analysis procedure

Since only smaller loads are applied, only linear material and geometrical behaviour are assumed. The materials are thus modelled as linear elastic. Here the modelling process will be explained for the 2D floor, and the 1D equivalent beam.

7.2.1 2D floor

This sub section gives a summary of the modelling process. To model the floor as a two-dimensional structure, frame elements were used to represent the timber joists, and shell elements used to represent the sheeting in the sub-floor.

As the connection between the joist and sheeting is regarded as very stiff (screwed and glued connection), it was modelled without the use of link-elements with a lateral and transversal stiffness that otherwise could have been used to adjust the stiffness of the connection. The joist and sheeting were modelled using the method of Frame insertion point and joint offset, illustrated in Figure 7.1. This ensures composite action between the elements.

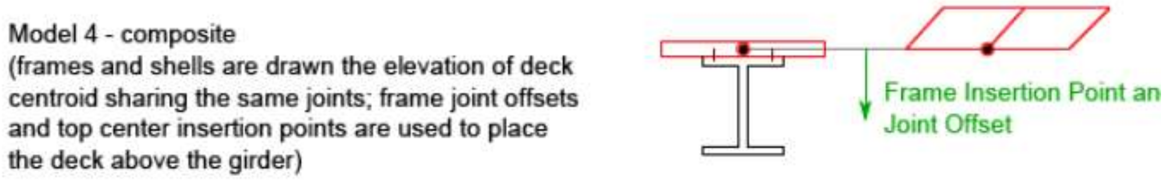


Figure 7.1: Connection between frame and shell element (Kalny 2016)

The reference floor is modelled using the same geometrical properties as in Table 4.3 and a joist spacing of 0.6 m. An illustration can be seen in Figure 7.2

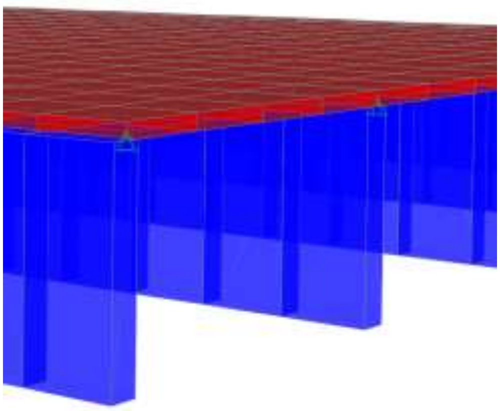


Figure 7.2: Illustration of floor model in SAP2000

The mass multiplier in the model is set to 0, meaning that the weight of the floor has to be applied separately. This is done by applying an area load. Now it is easy to adjust whether only the self-weight of the floor structure that should be applied, or if the weight from e.g. partitions should be added.

The 2D model can be used to obtain properties for the 1D equivalent beam. This is done by using the deflection under 1 kN static load, applied mid-span in the floor model. By re-arranging Equation (3.1), b_{ef} from the modelled floor can be found:

$$b_{ef\ SAP} = \frac{1}{48} \frac{1\ kN\ L^3}{EI_{L,SAP}\ w_{SAP}} \tag{7.1}$$

w_{SAP} is the deflection in the model due to a 1 kN concentrated load mid-span

$EI_{L.SAP}$ is the longitudinal stiffness of the floor in the model.

By running a modal analysis of the floor, $EI_{L.SAP}$ is obtained from the fundamental frequency, as the fundamental frequency is dependent on the stiffness of the floor through equation

The longitudinal stiffness of the floor per meter was found through the fundamental frequency obtained from the "infinite floor"-model:

$$EI_{L,SAP} = f_{1,SAP} \frac{4\ m\ L^4}{\pi^2} \tag{7.2}$$

To find the longitudinal stiffness of the equivalent beam, the longitudinal stiffness of the two-dimensional floor is multiplied by the effective width found in Equation (7.1:

$$EI_{1D,eq} = EI_L \cdot b_{ef\ SAP} \tag{7.3}$$

The mass of the one-dimensional equivalent beam can be found by following the same approach:

$$m_{1D,eq} = m \cdot b_{ef\ SAP} \tag{7.4}$$

NOTE:

When using this approach to obtain the stiffness of the 1D equivalent beam, it was found that b_{ef} was very big, giving an extremely high value of $EI_{1D.eq}$. Because of this, it was decided to model the 1D beam using only a frame element, having a t-section of uniform material and by that also MOE. This approach is presented in sub section 7.2.2.

7.2.2 1D equivalent beam

Instead of modelling the cross section as a frame and shell element, an equivalent frame section is made. This is now only one (frame) element having one material and is easier to investigate. It is assumed that the connection between the plate material and the beam is rigid.

Equation (2.25) is used to obtain the stiffness of the equivalent beam. The longitudinal stiffness of the floor under consideration is used and multiplied by the effective width b_{ef} to include the effect of transversal distribution.

$$EI_{1D,eq} = EI_{L\,floor} \cdot b_{ef} \tag{7.5}$$

Equation (2.26) is used to obtain the mass of the equivalent beam.

$$m_{1D,eq} = m_{floor} \cdot b_{ef} \tag{7.6}$$

The dimensions of the equivalent beam are found by dividing $EI_{1D,\,eq}$ by the modulus of elasticity of the material the equivalent beam will consist of, obtaining the moment of inertia $I_{eq.section}$.

$$I_{eq.section} = \frac{EI_{1D,eq}}{E_{eq.section}} \tag{7.7}$$

The new cross section is set to be rectangular, having uniform material. The height of the new cross section is set to be equal to the total height of the original cross section, called $h_{teq.sectionl}$. The width of the new cross section is found:

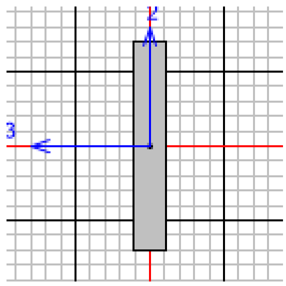
$$b_{eq.section} = I_{eq.con} \cdot \frac{12}{h^3_{eq.section}} \tag{7.8}$$

Now, the equivalent cross section can be modelled as a frame element in SAP2000 as a rectangular section. The two types of floors investigated in section 5 will also be investigated in this section. This is because of their different properties, with the reference floor being very light and having a plate material with very low stiffness. The floor with concrete screed has higher stiffness in the platelayer and is much heavier.

Reference floor:

In section 5, it is showed that the sub floor/plate material of the reference floor has a low level of contribution to the transversal stiffness of the floor. This means that b_{ef} is not used, and the plate width of the cross section is taken as the joist spacing. Table 7.1 shows the values used to obtain the equivalent beam, using Equation (7.5-(7.8.

Table 7.1: Equivalent cross section from reference floor:

Property		
$EI_{L,\ \text{reference floor}}$	3 076 497[Nm^2/m]	
$b_{\text{ef reference floor}} = J_s$	0,6 [m]	
$EI_{\text{1D equivalent}}$	3 076 497 [Nm^2]	
$I_{\text{1D equivalent}}$	0,1318 x10^3 [m^4]	
$E_{\text{K-beam Pluss}}$	14 000 [N/mm^2]	
Hight, *h*	322[mm]	
New beam width, $b_{j,\ \text{new}}$	47 [mm]	

The mass of the equivalent beam is taken as the weight of the reference floor, multiplied by the joist spacing.

$m_{\text{line}} = 65\ kg/m^2 * 0.6\ m * 9.81\ m/s^2 = 0.384\ kN/m$

In addition, a floor model where the approximate weight of one person is added mid span. This is done to consider the effect of the full "human/floor-system", as the mass of one person is a considerable part of the overall mass of the system.

$m_{\text{person}} = 75\ kg * 9.81\ m/s^2 = 0.73\ kN$

This point load is applied mid span, as a joint load.

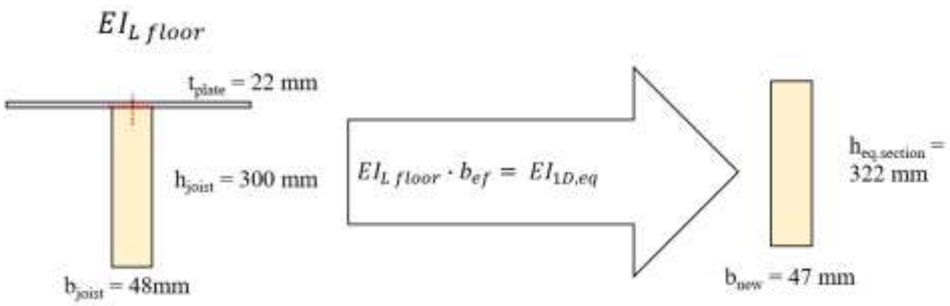


Figure 7.3: Transformation of the reference floor into a rectangular glulam beam.

Floor with 50 mm concrete screed:

The floor with 50mm concrete screed has higher transversal stiffness than the reference floor, and so the *effective with* is used. This found using Equation (2.21, values for EI_T and EI_L is taken from Table 6.16.

$$b_{ef} = \frac{3.925\ m}{1.1} \cdot \sqrt[4]{\frac{272\ 550\ \ Nm^2/m}{3\ 347\ 331 Nm^2/m}} = 1.91\ m$$

Figure 7.4 illustrates the original and new cross section:

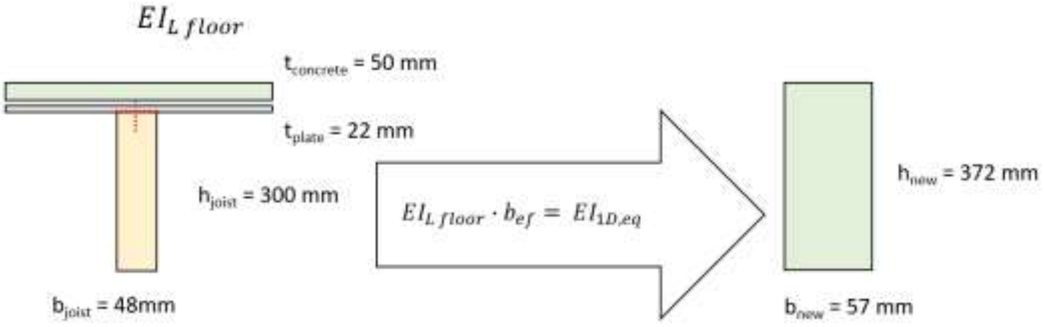
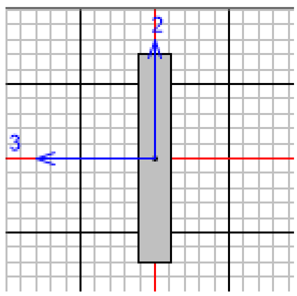


Figure 7.4: Transformation of a timber joist floor with 50mm concrete screed into concrete beam.

The result of the calculation is shown in Table 7.2

Table 7.2: Equivalent cross section from the floor having 50 mm concrete screed:

Property		
$EI_{L,\ concrete\ floor}$	3 347 331[Nm^2/m]	
$b_{ef\ concrete\ floor}$	1,91 [m]	
$EI_{1D\ equivalent}$	6 359 928 [Nm^2]	
$I_{1D\ equivalent}$	0,2445 x10^{-3} [m^4]	
$E_{concrete}$	26 000 [N/m^2]	
Hight, h	372 [mm]	
New beam width, $b_{j,\ new}$	57 [mm]	

The mass of the equivalent concrete beam is taken as the weight of the floor with 50 mm concrete screed, multiplied by the joist spacing.

$m_{line} = 265\ kg/m^2 * 1.91\ m * 9.81\ m/s^2 = 4{,}96\ kN/m$

There is also made a model including the mass of one person standing mid span, applied as a joint load.

The two equivalent beams are modelled in SAP2000 using the properties from Table 7.1and Table 7.2. The frame elements are divided into 16 sub elements. The fundamental frequency and mode shapes are found and used to investigate the acceleration response of the two equivalent beams.

Modal mass from SAP2000:

The modal mass taken form SAP is unity normalised. When the load applied is in kN, the modal mass is 1000 kg.

7.3 Analysis of 1D equivalent beams

Here the equivalent beams modelled in SAP2000 will be used to obtain mode shapes and frequencies, which in turn will be used in the methods presented in section 3.6.

First, the fundamental frequency will decide if both the steady state and the transient acceleration response has to be found (in the case of $f_1 < 8$ Hz), or if it is sufficient to investigate the transient acceleration response (if the case of $f_1 > 8$Hz). Then, the mode shapes and natural frequencies related to the three first modes will be found and used to calculate the acceleration response according to what is presented in section 3.

Only the situation where both the excitation force and response point are in mid span is investigated. In (Smith et al. 2009) it is stated that in practice only the most responsive point has to be checked, which in this case (simply supported beam) is in mid span.

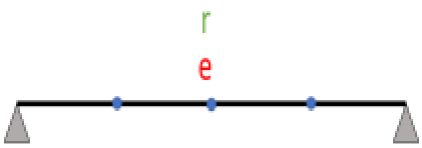


Figure 7.5: Response point and excitation point coincide mid pan as the most severe situation.

The natural frequency of the three first modes of the reference floor can floor with concrete screed from the SAP models are shown in Table 7.3.

Table 7.3: Frequencies of equivalent beams from SAP2000

	Reference floor:	**Floor w/ concrete screed:**
	m = 0.384 kN/m	*m = 1.56 kN/m*
f_1	19.7 Hz	10.7 Hz
f_2	76.6 Hz	41.5 Hz
f_3	165,6 Hz	88.7 Hz
	m = 0.384 kN/m + 0.73 kN mid span	*m = 1.56 kN/m + 0.73 kN mid span*
f_1	14.6 Hz	10.3 Hz
f_2	76.6 Hz	41.5 Hz
f_3	136 Hz	86.0 Hz

The difference in frequency between successive modes is large for both floors, and the fundamental frequency is above the *cut-off* frequency of 8 Hz (see Table 3.16). Because of the large difference in excitation frequency (1.8 Hz) and fundamental frequency, the applied force can be seen as a train of impulses. This means that only the transient response has to be investigated. The steady state response will be insignificant compared to the transient response, and only the transient response has to be investigated.

7.3.1 Reference floor:

The output form SAP-model of the equivalent beam representing the reference floor can be seen in Table 7.4. In this section, a summary of the calculation process is presented.

Table 7.4: Mode shapes of reference floor from SAP2000

Mode shape number	f_n **[Hz]**	μ_n, **value at mind span**
	m = 0.384 kN/m	
1	19.7	3.24
2	76.6	0
3	165,6	3.24
	m = 0.384 kN/m + 0.73 kN mid span	
1	14.6	2.43
2	76.6	0
3	136	1.59

As expected, the frequencies decrease a bit when the weight of "an average person" is applied at mid span, as the mass increases. The mode shapes also decrease. According to the method for calculating transient response (see section 3.6.3), all modes having frequencies up to two times that of the fundamental mode must be investigated. In this case, the frequency of the second mode is almost four time that of the fundamental frequency. This means that only the fundamental mode has to be considered. However, the second and third mode of vibration will be included to illustrate the method.

An *equivalent impulsive force* F_I has to be calculated according to Equation(3.16 and used as the representation of a single foot fall.

Table 7.5: F_I used in calculation of transient acceleration response of reference floor

Mode *n*	**f_p [Hz]**	**f_n [Hz]**	***Q* [N]**	**F_I [Ns]**
		m = 0.384 kN/m		
1	1.8 Hz	19.7	The weight of "average person": 746 N	3.08
2		76.6		0.52
3		165,6		0.19
		m = 0.384 kN/m + 0.73 kN mid span		
1	1.8 Hz	14.6	The weight of "average person": 746 N	4.45
2		76.6		0.52
3		136		0.24

The impulsive force of the fundamental mode is much larger than in mode 2 and 3. This indicates that the fundamental mode contributes the most to the total acceleration response.

Weighting factors W_n are found using Equation 22 in (Smith et al. 2009), corresponding to weighting curve W_b in Figure 3.6. The mode dependent frequency of the floor used to find weighting factors when calculating the transient acceleration response.

Table 7.6: Weighting factors used in the calculation of $a_{w, rms}$ of the reference floor

Mode *n*	**f_n [Hz]**	***Wn***
	m = 0.384 kN/m	
1	19.7	0.81
2	76.6	0.21
3	165,6	0.09
	m = 0.384 kN/m + 0.73 kN mid span	
1	14.6	1.00
2	76.6	0.21
3	136	0.12

The weighted transient acceleration response at time *t* is found from Equation (3.17, repeated below:

$$a_{w.e.r} = \sum_{n=1}^{N} 2\pi f_n \sqrt{1-\xi^2}\, \mu_{e.n}\mu_{r.n} \frac{F_1}{M_n} \sin\left(2\pi\sqrt{1-\xi^2}\; t\right) \cdot e^{-\xi 2\pi f_n t} W_n \quad \textbf{(7.9)}$$

To obtain the weighted a_{rms}, each $a_{w.e.r}$ summed using Equation (2.15, repeated here:

$$a_{rms} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt} \quad \textbf{(7.10)}$$

Each time step is taken as 0.01 s, and it is assumed that 500 time-steps is enough to cover the vibration. This leads to a total duration of 5 seconds. T is found from $1/f_p$ = 1/1,8 Hz = 0,56 s. Table 7.7 shows the result.

Table 7.7: Calculation of transient acceleration response, reference floor

Mode *n*	f_n	μ_e	μ_r	F_I	W_n	$a_{w.e.r}$
			m = 0.384 kN/m			
1	19.7	3.24	3.24	3.08	0.81	
2	76.6	0	0	0.52	0.21	1.96 m/s^2
3	165,6	3.24	3.24	0.19	0.09	
			m = 0.384 kN/m + 0.73 kN mid span			
1	14.6	2.43	2.43	4.45	1.00	
2	76.6	0	0	0.52	0.21	1.72 m/s^2
3	136	1.59	1.59	0.24	0.12	

Time step, t = 0.01 s $\quad$ M_n = 1000 kg

T = 0.56 $\quad$ ξ = 0.01

There is a reduction in frequency when the point load applied in mid span is introduced, but the added mass does not seem to affect the final acceleration response too much. For further calculations, the acceleration response calculated *including* the person standing mid-span is chosen, as the weight of the person is a considerable part of the all over the weight of the system.

Calculation of VDV:

When calculating the VDV, one has to know the duration of the activity causing the vibrations. In this example, the activity is a person walking across the floor, along the joist. This gives a walking path of 3.925m long. The step frequency f_p is 1.8 Hz. Using equation 2.18 for calculating walking velocity, gives:

$$v = 1.67\, f_p{}^2 - 4.38\, f_p + 4.50 = 1.2\frac{m}{s}$$

This gives a total duration of the activity of:

$$T_a = \frac{L_a}{v} = \frac{3.925\, m}{1.2\frac{m}{s}} = 3.22\, s$$

Using Equation (3.21, the vibration dose value is found. The number of times a day or night the vibration occurs n_a, has to be included in the calculation. A value for n_a =32 for daytime, and n_a =16 for night time is chosen. The limit values are found in Table 3.19, using the highest demands “low probability of adverse comments”.

Table 7.8: Calculation of VDV of the reference floor

$VDV = 0.68\, a_{w,rms} \sqrt[4]{n_a T_a}$	$a_{w,\,rms}$	T_a	n_a	**VDV**	**VDV** $_{limit}$
Daytime	1.72 m/s^2	3.22 s	32	3.73 m/s$^{1.75}$	0.2-0.4
Night time			16	3.13 m/s$^{1.75}$	0.13

The difference between daytime and night time calculated VDV is not big. They are both out of range of the limit values. The VDV-verification of the reference floor is not satisfied, not even when using the lowest demands "Adverse comment probable, where the limit values range from 0.8-1.6 (daytime) to 0.51 (night time)

7.3.2 Floor with concrete screed. (work in progress)

The output form SAP-model of the equivalent beam representing the reference floor can be seen in Table 7.4. In this section, the calculation process is presented.

Table 7.9: Mode shapes of concrete screed floor from SAP2000

Mode shape number	f_n **[Hz]**	μ_n, **value at mind span**
	m = 4.96 kN/m	
1		0.95
2		0
3		0.95
	m = 4.96 kN/m + 0.73 kN mid span	
1		0.92
2		0
3		0.88

As expected, the frequencies decrease a bit when the weight of "an average person" is applied at mid span, as the mass increases. The mode shapes also decrease. The frequency of the second mode is approximately four times higher than the fundamental frequency of the floor with concrete screed. Again, only the first mode has to be considered according to the method from section 3.6.3, but the three first modes are included to illustrate the method.

An *equivalent impulsive force* F_I is calculated in Table 7.10.

Table 7.10: F_I used in calculation of transient acceleration response of the 50mm concrete screed floor

Mode *n*	**f_p [Hz]**	**f_n [Hz]**	***Q* [N]**	**F_I [Ns]**
		m = 4.96 kN/m		
1	1.8 Hz	10.7	The weight of "average person": 746 N	6.80
2		41.5		1.16
3		88.7		0.43
		m = 4.96 kN/m + 0.73 kN mid span		
1	1.8 Hz	10.3	The weight of "average person": 746 N	7.14
2		41.5		1.16
3		86.0		0.45

The impulsive force of the fundamental mode is much larger than in mode 2 and 3. This indicates that the fundamental mode contributes the most to the total acceleration response.

The mode dependent frequency of the floor used to find weighting factors when calculating the transient acceleration response.

Table 7.11: Weighting factors, transient acceleration response

Mode *n*	**f_n [Hz]**	***Wn***
	m = 0.384 kN/m	
1	10.7	1.00
2	41.5	0.38
3	88.7	0.18
	m = 0.384 kN/m + 0.73 kN mid span	
1	10.3	1.00
2	41.5	0.38
3	86.0	0.18

The weighted transient acceleration response at time *t* is found from Equation (3.17, repeated below:

$$a_{w.e.r} = \sum_{n=1}^{N} 2\pi f_n \sqrt{1-\xi^2}\, \mu_{e.n}\mu_{r.n} \frac{F_1}{M_n} \sin\left(2\pi\sqrt{1-\xi^2}\, t\right) \cdot e^{-\xi 2\pi f_n t} W_n \tag{7.11}$$

To obtain the weighted a_{rms}, each $a_{w.e.r}$ summed using Equation (2.15, repeated here:

$$a_{rms} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt} \tag{7.12}$$

Each time step is taken as 0.01 s, and it is assumed that 500 time-steps is enough to cover the vibration. This leads to a total duration of 5 seconds. *T* is found from $1/f_p$ = 1/1,8 Hz = 0,56 s. Table 7.7 shows the result.

Table 7.12: Calculation of transient acceleration response, concrete floor

Mode *n*	f_n **[Hz]**	μ_e	μ_r	F_I	W_n	$a_{w.rms}$
			m = 4.96 kN/m			
1	10.7	0.95	0.95	6.80	1.00	
2	41.5	0	0	1.16	0.38	0.191 m/s^2
3	88.7	0.95	0.95	0.43	0.18	
			m = 4.96 kN/m + 0.73 kN mid span			
1	10.3	0.92	0.92	7.14	1.00	
2	41.5	0	0	1.16	0.38	0.187 m/s^2
3	86	0.88	0.88	0.45	0.18	

Time step, t = 0.01 s M_n = 1000 kg

T = 0.56 ξ = 0.03

There is a reduction in frequency when the point load applied in mid span is introduced, but the added mass does not seem to affect the final acceleration response too much. For further calculations, the acceleration response calculated *including* the person standing mid-span is chosen, as the weight of the person is a considerable part of the all over the weight of the system.

Calculation of VDV:

The length of the walking path, walking velocity and total duration of the activity is the same as in section 7.3.1

L_a = 3.925m,

f_p = 1.8 Hz,

$$v = 1.67\, f_p{}^2 - 4.38\, f_p + 4.50 = 1.2\frac{m}{s}$$

$$T_a = \frac{L_a}{v} = \frac{3.925\ m}{1.2\frac{m}{s}} = 3.22\ s$$

Using Equation (3.21, the vibration dose value is found, and the results presented in Table 7.13

Table 7.13: Calculation of VDV of the 50mm concrete screed floor

$VDV = 0.68\, a_{w,rms} \sqrt[4]{n_a T_a}$	$a_{w,\,rms}$	T_a	n_a	VDV	VDV $_{limit}$
Daytime	0.187 m/s^2	3.22 s	32	0.41 $m/s^{1.75}$	0.2-0.4
Night time			16	0.34 $m/s^{1.75}$	0.13

The difference between daytime and night time calculated VDV is not big. Both values are higher than the limitation value, although the vibration dose value for vibrations occurring at daytime is only just out of range. The VDV-verification of the 50mm concrete floor is not satisfied for the high demand of "low probability of adverse comments", but is satisfied for the limit values of the less strict demands (see Table 3.19).

7.4 Summary

A key point when using the a_{rms} and VDV method is to have a strong hypothesis, and a final element model that represent the floor in a realistic manner. This semi-numerical approach represents a more refined and sophisticated approach, giving more a more realistic picture of the floor behaviour that the rougher code based methods. But this demands good input values. As the basis for this method is *verification of existing floors*, having rough estimations and assumptions as parameters is not a good enough approach.

In this thesis, a final element model was made. However, the basis for the model was a simplification of the floor, since the equivalent beam were made using an analytical approach and not a full model of the floor. The results still give an indication of the floors behaviour, and the properties equivalent beam model were based on the same properties used in the evaluation of the code based methods.

As the fundamental frequency of both the reference floor, and the floor with 50mm concrete screed were found to be above the cut off-frequency for residential buildings, only transient acceleration response was investigated. The weighted acceleration response, $a_{w\,rms}$ were used to obtain a VDV-value, which in turn was compared against accompanying limit values. Neither the reference floor nor the 50mm concrete screed floor were found within the limits of the VDV, with high demands. However, as expected, the heavier floor was closer to the limit.

8 Suggestions on how to increase the span

The producer of the floor elements used as a template for this thesis reference floor, want to increase the maximum span of their floor elements. In the previous chapters, it is seen the mass and stiffness is of great importance to the dynamic behaviour of a floor, especially the transversal stiffness. This part of the thesis presents some measures that can be applied to increase the transversal stiffness in a floor. Increase in span due to one of these measures will then be further investigated.

The existing floor is made according to SINTEF Certification 2232. SINTEF Certification 2365 gives the details of the timber joist used, the K-beam. Table 2 in this Certification gives maximum spans for different joist cross section. The table is made according to the demands of the Comfort Criterion (see section 3.4 and 5.2.4). Table 8.1 gives this table. The relevant spans for the reference floor cross section is highlighted.

Table 8.1: Table 2 in SINTEF Certification 2365, a timber joist table for the K-beam and K-beam Pluss.

Bjelketype	Maksimal lysåpning i meter 1)											
	Nyttelast 2,0 kN/m² og tilleggslast fra lette skillevegger (boliger o.l.)						Nyttelast 3,0 kN/m² og tilleggslast fra lette skillevegger (kontorer ol.)1)					
	Bjelker over ett felt			Kontinuerlige bjelker over to like felt			Bjelker over ett felt			Kontinuerlige bjelker over to like felt		
	Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm			Bjelkeavstand mm		
	300	400	600	300	400	600	300	400	600	300	400	600
K-Bjelke												
36 x 200	3,45	3,25	2,95	3,60	3,40	3,10	3,45	3,25	2,85	3,60	3,30	2,85
36 x 250	4,15	3,90	3,55	4,35	4,10	3,75	4,15	3,90	3,55	4,35	4,10	3,60
36 x 300	4,80	4,50	4,15	5,00	4,75	4,35	4,80	4,50	4,15	5,00	4,75	4,35
48 x 200	3,65	3,45	3,10	3,80	3,60	3,30	3,65	3,45	3,10	3,80	3,60	3,15
48 x 250	4,35	4,15	3,80	4,60	4,35	3,95	4,35	4,15	3,80	4,60	4,35	3,95
48 x 300	5,05	4,80	4,40	5,30	5,05	4,65	5,05	4,80	4,40	5,30	5,05	4,65
70 x 200	3,90	3,70	3,40	4,10	3,90	3,55	3,90	3,70	3,40	4,10	3,90	3,55
70 x 250	4,70	4,45	4,10	4,90	4,65	4,30	4,70	4,45	4,10	4,90	4,65	4,30
70 x 300	5,40	5,15	4,75	5,70	5,40	5,00	5,40	5,15	4,75	5,70	5,40	5,00
K-Bjelke Plus												
36 x 200	3,65	3,45	3,10	3,85	3,60	3,25	3,65	3,45	3,10	3,85	3,55	3,10
36 x 250	4,40	4,15	3,75	4,60	4,35	3,95	4,40	4,15	3,75	4,60	4,35	3,90
36 x 300	5,05	4,80	4,40	5,30	5,05	4,65	5,05	4,80	4,40	5,30	5,05	4,65
48 x 200	3,85	3,65	3,30	4,05	3,80	3,50	3,85	3,65	3,35	4,05	3,80	3,40
48 x 250	4,65	4,40	4,00	4,85	4,60	4,20	4,65	4,40	4,00	4,85	4,65	4,20
48 x 300	5,35	5,10	4,70	5,65	5,35	4,90	5,35	5,10	4,70	5,65	5,35	4,90
70 x 200	4,15	3,95	3,60	4,35	4,15	3,80	4,15	3,95	3,60	4,35	4,15	3,80
70 x 250	5,00	4,75	4,35	5,25	4,95	4,55	5,00	4,75	4,35	5,25	4,95	4,55
70 x 300	5,75	5,50	5,05	6,05	5,75	5,30	5,75	5,50	5,05	6,05	5,75	5,30

The maximum span of a timber joist floor equal to the reference floor, having joist cross section 48 x 300 mm^2 is taken as 4.40 m when using "K-bjelken", and 4.7 m when using "K-bjelken Pluss": Material properties for the two joist types are shown in Table 8.2.

Table 8.2: K-bjelken and K-bjelken Pluss, material properties

		K-Bjelke	K-Bjelke Plus
Fastheter			
Bøyefasthet			
- på kant og på flasken	$f_{m,k}$	24,0	33,0
Strekkfasthet			
- i bjelkens lengderetning	$f_{t,0,k}$	14,0	14,0
- tvers på fiberretningen	$f_{t,90,k}$	0,4	0,4
Trykkfasthet			
- i bjelkens lengderetning	$f_{c,0,k}$	21,0	21,0
- tvers på fiberretningen	$f_{c,90,k}$	5,3 [1)]	5,3 [1)]
Skjærfasthet	$f_{v,k}$	3,5	3,5
Stivheter for stabilitetsberegninger			
Elastisitetsmodul			
- bøyning og aksiallast	$E_{0,05}$	7400	9400
Stivheter for deformasjonsberegninger			
Elastisitetsmodul			
- bøyning og aksiallast	$E_{0,m}$	11000	14000
- tvers på bjelkekant	$E_{90,m}$	370	370
Skjærmodul	$G_{0,m}$	690	690

The producer intend to use K-bjelken Pluss in the future (Tørum 2017), so this joist type is used in the calculations in this chapter.

Table 2 in SINTEF Certification 2365 is meant for a regular floor, i.e. a floor without the sound insulation system described in 4.1. According to Byggdetaljblad 522.532, pt. 232 (SINTEF Byggforsk 2011a), the spans in the timber joist tables has to be reduced by a factor 0.89 due to this sound insulation system. This is to consider the increased mass of the system, and that a stiff ceiling is not present, causing the floor to have less transversal stiffness.

This section will suggest measures that will increase the transversal stiffness, and make it possible to have a sound insulated floor of longer spans. According to table 2 in Certification 2365, a floor configuration using K-bjelken Pluss should allow a maximum span of 4.70 m, and the regular K-bjelken should support a span of 4.4 m. In section 6.1.4, the maximum span of different joist cross sections of reference floor was found. The maximum span of the 48x300-cross section was 4.35 m, indicating that obtaining spans of 4.4 or 4.7 should be possible. However, using the other code based methods, the maximum spans is much shorter (see Table 6.8).

8.1 Different measures to obtain larger transversal stiffness.

Some ways to improve the stiffness, and by that hopefully the vibrational properties of the floor, is presented in this section. Unless otherwise is stated, the information in the sections is taken from chapter 14.5.3 in the book Timber Engineering (Smith 2003). This book is recommended for an introduction to vibration of timber floors in general, and to read more about construction detail in timber floors.

8.1.1 Strapping

The material used for strapping is most often either thin timber battens or light gauge steel. In this work, the suggestion is to fasten *n* number of 36x98's underneath the joists in the floor, as illustrated in Figure 8.1:

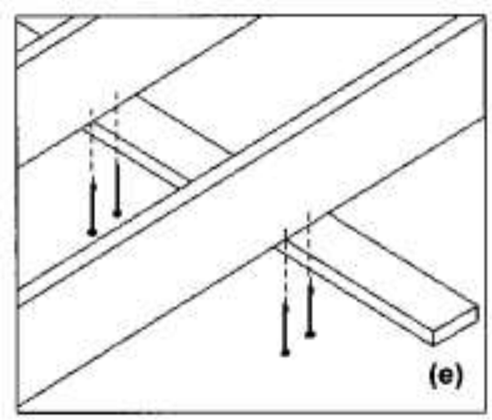


Figure 8.1: Strapping. (Jiang et al. 2004)

According to Smith (2003), strapping is usually nailed to the joists. The stiffness of this connection is crucial for the success of the strapping. Knowing the capacity of the connectors is not enough; for example, minor crushing of the fibres around the nails in this connection, can affect the overall stiffness of the floor. This solution need to be investigated more in detail to know the stiffness. Due to the limitations of this thesis, this will not be done, and the solution will not be looked further into.

8.1.2 Shorter joist centre distance

Reducing the centre distance between adjacent joists will improve the fundamental frequency f_1 as it increases the longitudinal stiffness of the floor, but can result in modal clustering as it increases the orthotropy of the floor (increase the along joist stiffness more than the across joist stiffens). As mentioned in section 2.1.2, the closely spaced vibration modes may interact and produce motion with relatively high amplitudes, modal clustering can cause increased acceleration and velocity levels (Glisovic & Stevanovic 2010).

Reduction of joist spacing may therefore not always result in overall improved vibrational performance, and the effect of this solution has to be closer investigated before it can be recommended. Shorter joist distance also causes an increase in used materials, and if it's shown that adding joists mainly contribute to increase the *longitudinal stiffness*, other measures might be more effective.

8.1.3 Thicker plate in sub-floor/sheeting

A thicker plate in the sub-floor/sheeting can highly reduce the static deflection due to a concentrated load, and increase the transversal stiffness (depending on the plate material). In a parametric study of a timber joist floor, Glisovic and Stevanovic (2010) found that increasing the plate thickness, reduces the fundamental frequency of the floor. The degree of the frequency reduction depends on the relationship between modulus of elasticity of the plate in the two orthogonal directions; Larger degree of isotropy causes less reduction in natural frequency, meaning it is preferable to use a plate having isotropic material properties.

Like increasing the plate thickness, adding a layer of heavy topping (e.g. a layer of concrete) for ballasting will decrease the static deflection of the floor under a concentrated load, but the natural frequencies will decrease due to the added weight. According to Smith (2003), this leads to less influence of springiness in the floor, but increase the probability for vibrational disturbances.

8.1.4 Bridging, blocking and cross bracing.

Bridging, blocking and cross bracing are very beneficial measures to increase the transversal stiffness in a floor (Smith 2003). It reduces the overall orthotropy in the floor, as the across joist stiffness is increased. Still, the floor is regarded as orthotropic, but the modal separation factor will increase. The frequency clustering and associated increase of motion amplitude is less significant. Blocking assist the floor plate in distributing the force from the floor above between the longitudinal joists, which reduces the static deflection.

Figure 8.2 illustrate blocking, bridging and cross bracing. In the illustration of blocking, each element is installed with an offset relative to each other. This is done to have easy access when installing the blocking elements. Angle brackets can also be used, eliminating the need for element offset.

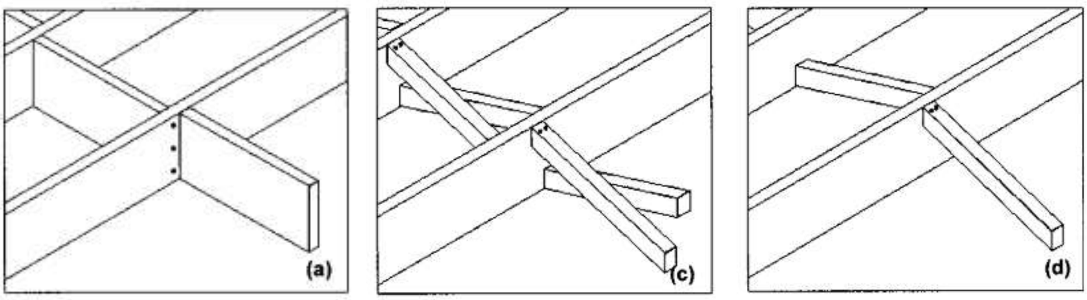


Figure 8.2: Left to right: Blocking, cross bracing and bridging. (Jiang et al. 2004)

According to Smith (2003), at least one row of bridging or blocking at mid span should be present in a timber floor. In many floors, one of these measures is necessary to prevent torsional movement in the joists. It is important to be aware of the moisture content of the elements when installed. Shrinkage can lead to cracks between longitudinal joists and blocking elements, which will degrade the efficiency of the configuration as well as lead to unwanted creaking in the floor. It should be noted that the angle of the bridging elements can give especially less effective connections due to uneven shrinkage of the bridging element. Bridging and cross bracing is not further investigated in this thesis. Blocking provides the same benefits as the other measures, and it is less uncertainties involved in the assumptions made as the details are less complicated in the blocking elements.

8.2 Investigation of added rows of blocking

Out of the four measures presented, only one will be further investigated. This is due to the limitations of this thesis. The method of using blocking seems the most promising, as it is the measure where there are the least uncertainties regarding the stiffness of the connection, and since it addresses increasing the transversal directly. For this reason, only increased stiffness and spans due to adding rows of blocking will be calculated.

When investigating the effect of possible alterations of the reference floor, only one criteria from two of the analytical methods has been considered. The criteria are

- *combined criterion* from the Comfort Criterion approach, and
- *static deflection criterion* from the Austrian national annex.

The first criterion is chosen for its relevance in Norway today, and since this is the approach most similar to the basis for the timber joist tables used in the SINTEF Certifications for joist floors. The Austrian National Annex is chosen due to its similarities with the expected new proposal to Eurocode 5. When investigating maximum span of the reference floor in section 6.1, the deflection criterion in the Austrian National Annex was found to be the most severe. For this reason, the deflection criterion will be used to investigate maximum span of the new floor configurations.

Since adding blocking will increase the transversal stiffness, it is assumed that the effect of transversal distribution is present. That is, $b_{ef} > J_s$. The floor is reduced to a 1D equivalent beam as in the above sections. The equation for calculating the stiffness of this equivalent beam is repeated below:

$$EI_{1D,eq} = EI_{L\,floor} * b_{ef} = \frac{EI_{joist}}{Js} * \frac{L}{1.1} * \sqrt[4]{\frac{EI_T}{EI_L}}\ [Nm^2] \tag{8.1}$$

where:

EI_T	is the total stiffness provided by the different measures, see Equation (8.2) [Nm^2/m]
EI_L is	calculated as in 2.3.1. [Nm^2/m]
b_{ef}	is taken as: $J_s \leq$ calculated $b_{ef} \leq$ floor width B

The mass of the floor will also increase when adding rows of blocking. New mass calculation for the floor can be seen in Table 0.2 in Appendix A

The transversal stiffness due to blocking is found by adding the longitudinal stiffness of the blocking elements to the transversal stiffness of the plate:

$$EI_{\mathrm{T}} = \frac{EI_{plate\ transversal}}{1\ m} + \frac{EI_{blocking}}{B_s} \left[\frac{Nm^2}{m}\right] \tag{8.2}$$

Where:

EI_{blocking} is the longitudinal stiffness of the blocking element [Nm^2]

B_s is the spacing of the blocking elements, found from Equation (8.3), [m]

$$B_s = \frac{L}{n_{blocking} + 1} \tag{8.3}$$

where:

L is the floor span [m]

$n_{blocking}$ is the number of blocking elements

Figure 8.3 shows the example of 1 row of blocking, giving a B_s of $L/2$. The stiffness of the blocking (green solid line) is added to the transversal stiffness of the plate material (red solid line), giving an increased total transversal stiffness of the floor. The longitudinal stiffness (dotted line) is unchanged by the introduction of the blocking.

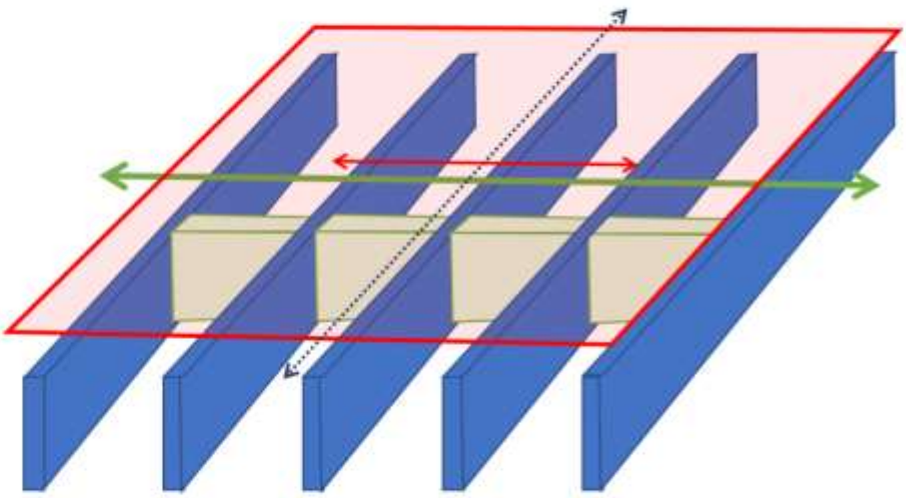


Figure 8.3: EI $_{\text{blocking}}$ (green solid line) is added to the transversal stiffness of the plate EI $_{\text{transversal plate}}$ (red solid line). Dotted line represents the longitudinal stiffness of the floor.

This is an overestimation, as the blocking elements now is regarded as one continuous joist, having length equal to the width of the floor. I assume the error is not too big, as it is possible to obtain very stiff connections using angle brackets. When using angle brackets the blocking element can be aligned. It’s important to have control over the moisture in the blocking elements when they are inserted, as creep in the blockings longitudinal direction will cause a less effective connection as well as unwanted creaking in the floor.

8.2.1 Using comfort criterion

Using the combined criterion was found to be the most severe in the Comfort Criterion-method, the maximum span when adding *n* rows of blocking is found. This was done by using the "goal seek" function in Excel, as in section 7.3. The span *L* is altered until the difference between the limit value and calculated value of the combined criterion is zero. Input values are:

Plate stiffness, $EI_{T\ plate}$ = 1717 Nm²/m.
Longitudinal stiffness of the floor, $EI_{long.}$ = 3 076 497 Nm^2. L_{max} calculated from the deflection criterion from Austrian National Annex. Joist spacing is 600 mm.

Figure 8.4 shows the example of 0-2 rows of blocking added. As can be seen, the transversal stiffness EI_T increases drastically when introducing the blocking. The fundamental frequency is reduced, as well as the static deflection.

Row of bloking 1	0		Row of blocki	1		Row of blokir	2	
m	62,51	kg/m²	m	64,02	kg/m²	m	65,52	kg/m²
EI L	3076497,58	Nm²/m (inpu	EI L	3076497,58	Nm²/m (inpu	EI L	3076497,58	Nm²/m (input
L	4,38451862	m (output)	L	5,04809589	m (output)	L	5,16042861	m (output)
Bs	0,00	m	Bs	2,52	m	Bs	1,72	m
EI T	1717	Nm2/m	EI T	76597	Nm2/m	EI T	151476	Nm2/m
bef	0,61		bef	1,82		bef	2,21	
f1	18,126638	Hz	f1	13,5125879	Hz	f1	12,7812965	Hz
w	0,93166742	mm	w	0,4778721	mm	w	0,42110878	mm
lim	18,7		lim	18,7		lim	18,7	
calc	18,700		calc	18,700		calc	18,700	
difference	0,000	(put to zero)	difference	0,000	(put to zero)	difference	0,000	(put to zero)

Figure 8.4: Maximum span found from the combined criterion, using goal seek.

Table 8.5 gives the values calculated. The stiffness of the 1D equivalent beam is found. The longitudinal stiffness of the blocking, $EI_{blocking}$ is added to the transversal stiffness of the floor. The dimensions of the rows of blocking is taken as 48x150mm. That is, half of the cross-section height of the main joists in the floor.

Table 8.3: Maximum span due to adding *n* rows of blocking, from the combined criterion of Comb.Crit. No restriction of b_{ef}

n rows	EI blocking	B_s [m]	EI_T [Nm²/m]	b_{ef} [m]	$EI_{1D\ equivalent}$	L_{max} [m]
0	0	-	1716,99	0,61	1884792,8	4,38
1	189000	2,52	76596,71	1,82	5608282,4	5,05
2	378000	1,72	151476,43	2,21	6798635,9	5,16
3	567000	1,34	425870,67	2,97	9121977,4	5,35
4	756000	1,09	698100,67	3,41	10477857	5,43
5	945000	0,91	1034602,98	3,80	11691597	5,49

From Table 8.3 it looks like the maximum span is almost 5.5 m when adding five rows of blocking. However, the effective width of this floor configuration is 3.8 m, which is larger than the width of the floor element. According to the Austrian National Annex to Eurocode 5, the effective width has to be smaller than the floor width, which in this example is the element width *B* = 2.4 m. Taking this into account, the maximum allowed span is found in Table 8.4:

Table 8.4: Maximum span due to adding *n* rows of blocking, from the combined criterion of Comb.Crit. Restrictions of b_{ef} included.

n rows	EI blocking	B_s [m]	EI_T [Nm²/m]	b_{ef} [m]	$EI_{1D\ equivalent}$	L_{max} [m]
0	0	-	1716,99	0,61	1884792,8	4,38
1	189000	2,52	76596,71	1,82	5608282,4	5,05
2	378000	1,72	151476,43	2,21	6798635,9	5,16
3	567000	1,30	437923,69	2,40	7383594,2	5,20
4	756000	1,04	731162,22	2,40	7383594,2	5,18
5	945000	0,86	1099459,29	2,40	7383594,2	5,17

There is actually a small reduction in maximum span when the effective width is set to 2.4 m. This might be because of the increase in mass, reducing the fundamental frequency, while the "freezing of the ratio between EIL and EIT" causes the static deflection to stay the same.

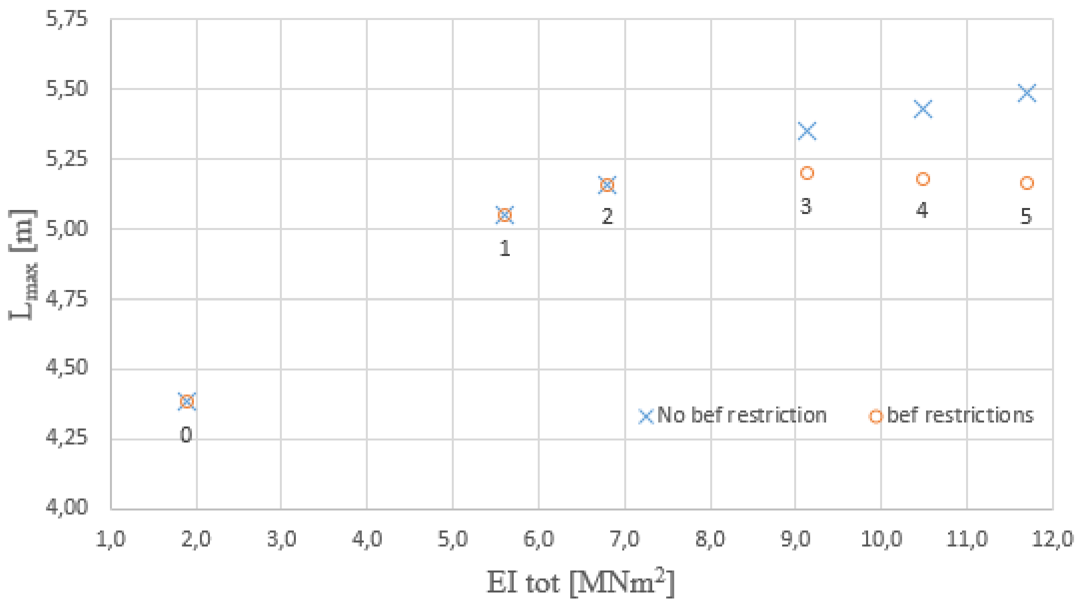


Figure 8.5: Maximum span according to the Comfort Criterion, adding 0-5 rows of blocking.

8.2.2 Using the Austrian National Annex

The same approach is used when finding maximum allowed span according to the deflection criterion in the Austrian National Annex. Table 8.5 gives the result:

Table 8.5: Maximum span due to *n* rows of blocking. No restriction of b_{ef}

n rows	EI $_{blocking}$	B_s [m]	EI_T [Nm^2/m]	b_{ef} [m]	$EI_{1D\ equivalent}$	L_{max} [m]
0	0	-	1716,99	0,60	1845898,5	2,81
1	189000	1,89	101619,88	1,47	4511367	3,78
2	378000	1,42	267296,11	2,11	6483612,3	4,27
3	567000	1,15	493663,73	2,65	8160795,9	4,61
4	756000	0,98	776495,98	3,14	9671604,2	4,88
5	945000	0,85	1112771,96	3,60	11068756	5,10

When increasing the number of rows of blocking, the effective transversal stiffness EI_T increases. The overall stiffness in the system, and by that the maximum allowed span, increases as well. However, the effect of adding more rows of blocking is decreasing with as a larger number of blocking rows are introduced. When the restrictions of b_{ef} from the Austrian National Annex is applied, the maximum span is prevented from increasing above 4.46 m-.

Table 8.6: Maximum span due to *n* rows of blocking. Restrictions of b_{ef} included

n rows	EI blocking	B_s [m]	EI_T [Nm²/m]	b_{ef} [m]	$EI_{1D\ equivalent}$	L_{max} [m]
0	0	-	1717	0,60	1845898	2,81
1	189000	1,89	101619	1,47	4511367	3,78
2	378000	1,42	267296	2,11	6483612	4,27
3	567000	1,12	510056	2,40	7383594	4,46
4	756000	0,89	849118	2,40	7383594	4,46
5	945000	0,74	1273505	2,40	7383594	4,46

The stiffness of the equivalent beam, and by that the maximum allowed span, stagnates at approximate 4.5 m when the restrictions of b_{ef} is considered. This shows adding more than 3 rows of blocking won't have any effect.

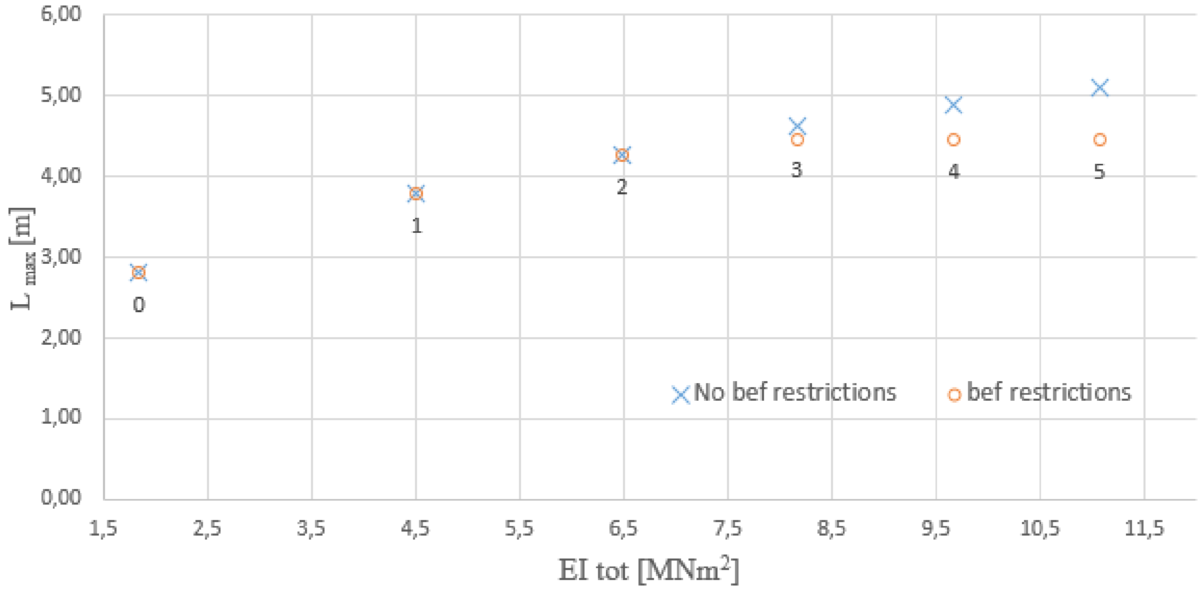


Figure 8.6: Maximum span according to the Austrian NA, adding 0-5 rows of blocking.

8.3 VDV analysis of reference floor with 3 rows of blocking

Since the effect of adding more than 3 rows of blocking is limited by the floor element width, only 3 rows of blocking are added in the updated version of the reference floor. The span is taken as 4.4m, as this is within the limits of L_{max} according to the deflection criterion of EC5 Austrian National Annex.

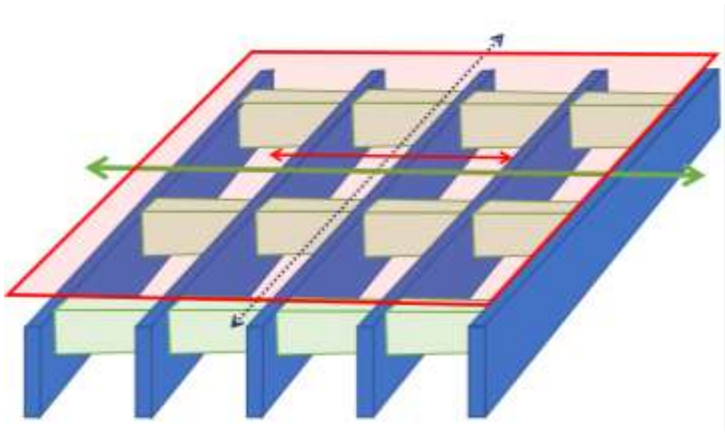
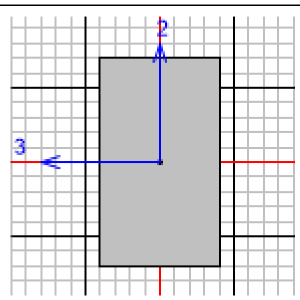


Figure 8.7: Illustration of the reference floor reinforced with 3 rows of 48x150mm blocking elements.

This "Reinforced floor" will be investigated through the use of the VDV-method presented in section 3.6 and applied to the reference floor and concrete screed floor in section 7.3. Adding three rows of blocking will greatly improve the transversal stiffness, and slightly increase the mass of the floor. The stiffness of the blocking elements is EI_{blocking} = 567 000 Nm^2, and the new mass 67 kg/m^2.

Table 8.7: Floor properties of reinforced floor, with 3 rows of blocking

Property		
$EI_{L,\ \text{blocking}}$	3 076 497[Nm^2/m]	
$b_{\text{ef blocking}}$	2.4 [m]	
$EI_{\text{1D equivalent}}$	7 383 594 [Nm^2]	
$I_{\text{1D equivalent}}$	0,5273 x10^3 [m^4]	
$E_{\text{K-beam Pluss}}$	14 000 [N/mm^2]	
Hight, h	322[mm]	
New beam width, $b_{j,\ \text{new}}$	189 [mm]	

The mass of the equivalent beam is taken as the weight of the reinforced floor, multiplied by the effective width:

$m_{\text{line}} = 75\ kg/m^2 * 2.4\ m * 9.81\ m/s^2 = 1.57\ kN/m.$

The output form SAP-model of the equivalent beam representing the reinforced floor is summarised in Table 8.8. floor. Also in this calculation, each time step is taken as 0.01 s, and it is assumed that 500 time-steps is enough to cover the vibration.

Table 8.8: Calculation of transient acceleration response, reinforced floor

Mode *n*	f_n **[Hz]**	μ_e	μ_r	F_I	W_n	$a_{w.e.r}$
			m = 0.384 kN/m			
1	19.5	1.61	1.61	3.11	0.82	
2	76.1	0	0	0.53	0.21	0.492 m/s^2
3	164.5	1.61	1.61	0.19	0.09	
Time step, t = 0.01 s			M_n = 1000 kg			
T = 0.56			ξ = 0.01			

As in the reference and concrete screed floor, there is a large difference between successive frequencies of the reinforced.

Calculation of VDV:

The reinforced floor has a span of 4.4m (taken from Table 8.6), which is taken as the walking path L_a. The step frequency f_p is 1.8 Hz, resulting in a walking velocity of 1.2 m/s, as in section 7.3. This gives a total duration of the activity of:

$$T_a = \frac{L_a}{v} = \frac{4.4\ m}{1.2\frac{m}{s}} = 3.6\ s$$

Using Equation (3.21, the vibration dose value is found. The number of times a day or night the vibration occurs n_a, has to be included in the calculation. The values of n_a is taken as 32 for daytime, and 16 for night time. The limit values are found in Table 3.19, using the highest demands “low probability of adverse comments”.

Table 8.9: Calculation of VDV of the reference floor

$VDV = 0.68\, a_{w,rms} \sqrt[4]{n_a T_a}$	$a_{w,rms}$	T_a	n_a	**VDV**	**VDV limit**
Daytime			32	1.09 m/s$^{1.75}$	0.2-0.4
	0.492 m/s^2	3.6 s			
Night time			16	0.92 m/s$^{1.75}$	0.13

The difference between daytime and night time calculated VDV is not big. They are both out of range of the limit values. The VDV-verification of the reference floor is not satisfied, not even when using the lowest demands “Adverse comment probable, where the limit values range from 0.8-1.6 (daytime) to 0.51 (night time) (see Table 3.19).

8.4 Summary

When using the maximum allowed span according to the Comfort Criterion, it is possible to obtain spans of both 4.4 and 4.7m. However, the deflection criterion of the Austrian National Annex is much stricter, and does not allow such spans. This is illustrated in Figure 8.8. *Note*: The two methods give different stiffness for the same number of added rows of blocking. This is because the transversal stiffness EI_T [Nm2/m] is dependent on the distance between the rows of blocking, B_s (Equation (8.3), which in turn is dependent on the span. Using the goal seek function in Excel will therefor adjust EI_T according to the span.

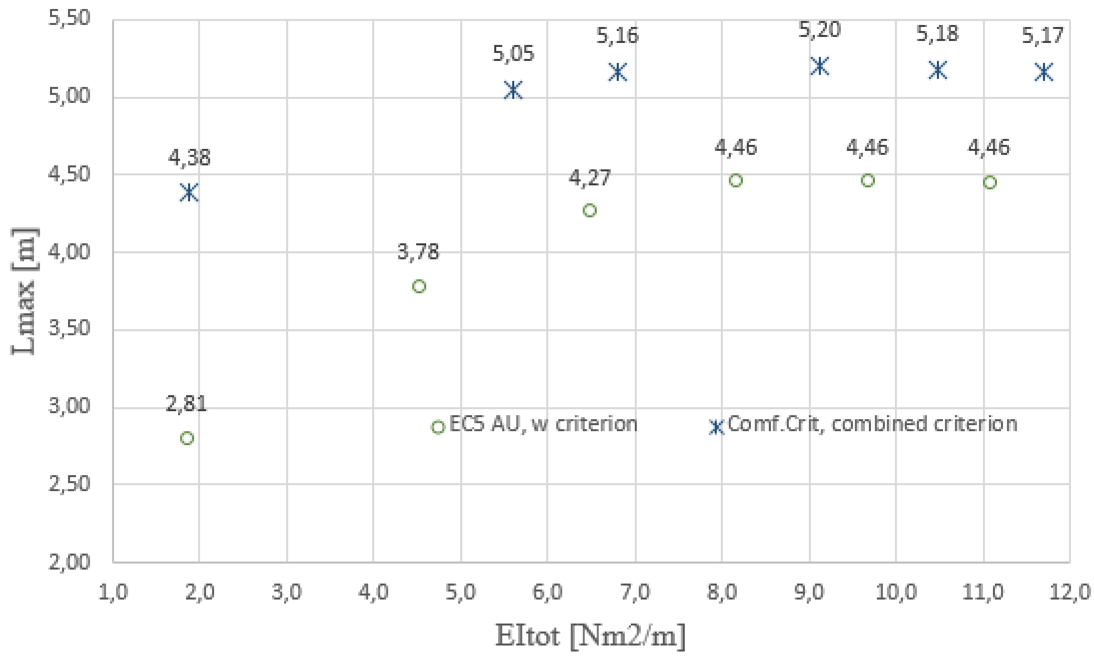


Figure 8.8: Comparing maximum span according to Comfort criterion and Austrian National Annex, deflection criterion. Restriction of b_{ef} considered.

In Figure 8.8 the b_{ef} is taken as the floor element width, 2.4m when 3, 4 and 5 rows of blocking are added. When considering this limitation, it is not possible to obtain a span of 4.7m, according to the Austrian National Annex. The Comfort Criterion, however, is less strict and allows for spans up to 5.2 m when adding 3 rows of blocking.

In section 8.3 it is found that a span of 4.4m does not satisfy the VDV verification of the reinforced floor. This is as expected, as VDV verification is found to be more severe than any of the code based methods.

8.4.1 Other possible measures

Other measures to increase the transversal stiffness won't be beneficial to add, as the limit of b_{ef} is reached already. If done so, the "extra transversal stiffness" cannot be regarded, according to current standards. Adding more mass can be a solution, but also increasing the longitudinal stiffness along with the transversal. An isotropic layer of screed or sheeting can be an option.

It should be noted that the reinforced floor is very light, as the mass considered is only the calculated self-weight of the floor itself. In practice, the floor will be heavier due to partitions and furnishing, and the people on it (both causing and experiencing the vibrations).

Measures to obtain longer spans should be investigated more thoroughly. This could be done by using the code based methods, or the semi-numerical approach presented in section 3.6 with a stronger hypothesis that what was used here.

9 Discussion

As seen in chapter 5, none of the code based methods considers contains any mass requirements. The Hamm/Richter approach and Austrian National Annex to Eurocode 5 considers acceleration response when the fundamental frequency is below a limit value of 8 Hz. This does not cover the transient acceleration response of low weight, high frequency floor. The Norwegian National Annex has disregarded the *vibrational velocity response*-criterion, proposed in the main document of EC5, and the Comfort Criterion does not have a specific mass requirement.

In chapter 6, the analytical code based methods were compared, in terms of maximum allowed span for different joist cross sections. The maximum span from the SLS deflection criterion $w \leq L/300$ was also included in the results. The chapter showed that the vibration verifications were more serve than the SLS deflection verification. Three floors were investigated, and depending on weight and stiffness in the plate layer, different criteria would be the most severe. IN the light floor, the deflection criterion of the Austrian NA (and Hamm/Richter) approach was the most severe. In the heavy floor, the acceleration response criterion of Austrian NA was the most severe.

Chapter 7 investigated the reference floor and the 50mm concrete screed floor, using the VDV method. This shows that the transient acceleration response verification is more severe than the code based methods (as expected) as none of the floors is verified.

9.1 Conclusions

Chapter 5-7 illustrates the importance of the analysis method; one approach can indicate that a certain span is ok, while the same span is not satisfied when using a different method of analysis.

Current codes assume that acceleration response of a floor is a problem when resonance occur. This can only happen when the excitation frequency is close to the natural frequency of the floor. For this reason, the verification of acceleration response is only considered when f_1 is below a certain limit value (8 Hz is widely used). In the case of walking on a light weigh flor, this will not be an issue, as low mass and relatively high stiffness causes the fundamental frequency of to be out of range of the first (and most energetic) harmonics of the excitation.

Numerical models and advanced methods (VDV) show how the transient response is the most critical. This acceleration response is highly governed by the low mass of the floor, and the high frequency, and not covered by any of the current standards.

Due to this, the current conde based methods does not seem suitable for light floors. More thorough and sophisticated methods, as the VDV, can be used to consider transient acceleration responses, but are not as easy to use and demand a strong hypothesis and knowledge about the floor.

Increase of transversal increase the performance of the floor. Increasing the transversal stiffness alone is not enough to obtain long spans, as seen in chapter 8. When investigating a floor with span 4.4m, found acceptable by the strictest code based method, using VDV, the floor was not accepted.

9.2 Suggestions to further work

As the main focus of this thesis is the assessment of vibrational properties of timber joist floors, the main focus has been on the investigation and evaluation of the different code based methods and the more advanced approach of VDV.

The following further work is suggested:

1) Successfully model two-dimensional joist floor, investigate the dynamic properties: The next step from the modal analysis of an equivalent beam, is to model a full two-dimensional floor, and obtaining dynamic properties form this model. These properties can be used in the vibration dose value-analysis. A strong hypothesis is needed, and enough information about the floor structure and intended use.
2) Successfully model two-dimensional joist floor, and performed a time- history analysis: This includes a applying a dynamic load, moving across the floor model and varying in time. This will give a full simulation, and a very good indication of the vibrational behaviour of the floor.
3) Parametric study of measures to improve the stiffness of the floor: A further investigation of the effect of different measures to improve the vibrational properties of the floor.
4) Experimental testing: This could be done to both verify a numerical model of the floor, or to further investigate and discuss the analytical approaches.

10 List of figures

11 List of tables

APPENDIX A: Mass calculation reference floor

Table 0.1: Mass calculation of reference floor

REFERENCE FLOOR						
	K-beam	Particle board	Gypsum	30x48 lekt	Insulation	"Trinnlydplate"
Desity [kg/m3]	460	685	--	380	16,5	16,5
Thickness/hight [mm]	0,3	0,022	--	0,03	0,3	0,02
Width [mm]	0,048	2,4	--	0,048	2,4	2,4
Lentgh [m]	3,925	3,925	--	2,4	3,925	3,925
mass [kg] per element	25,9992	120,66549	--	1,31328	42,89868	3,1086
Floor width [m]	2,4	2,4	--	2,4	2,4	2,4
Floor lenght [m]	3,925	3,925	--	3,925	3,925	3,925
Number of elements	5	2	3	6		1
5 joist in 2,4 m:	129,996		6 lekter	7,87968		
Area mass [kg/m2]	13,800	25,619	20,1	0,836	4,554	0,33
			Total area mass of floor:		**65,24**	**kg/m2**

Table 0.2: Mass of floor with added rows of blocking

			Blocking	Blocking	Blocking	Blocking	Blocking	Blocking
Desity [kg/m3]			460	kg/m3	kg/m3	kg/m3	kg/m3	kg/m3
Thickness/hight [mm]			0,3	0,3	0,3	0,3	0,3	0,3
Width [mm]			0,048	0,048	0,048	0,048	0,048	0,048
Lentgh [m]			2,4	2,4	2,4	2,4	2,4	2,4
mass [kg] per element	ref.floor		15,8976	15,8976	15,8976	15,8976	15,8976	15,8976
Floor width [m]			4,4	4,4	4,4	4,4	4,4	4,4
Floor lenght [m]			2,4	2,4	2,4	2,4	2,4	2,4
Number of elements			1	2	3	4	5	10
4 joist in 2,4 m:			15,8976	31,7952	47,6928	63,5904	79,488	158,976
Area mass [kg/m2]			1,51	3,01	4,52	6,02	7,53	15,05
	area mass	**62,51**	**64,02**	**65,52**	**67,03**	**68,54**	**70,04**	**77,57**

APPENDIX B: Properties of the reference floor.

Floor property	Value		Taken from
Total mass of the floor structure	63	kg/m^2	Appendix A
Bridging	No		
Support conditions	Simply supported at two sides		
Overall geometry:			
Span length	4410	mm	TG 2232 [1)]
Element width	2400	mm	Saa.
Joist spacing	600	mm	Saa.
Joist section properties, K-beam (C24)			
Depth/height	300	mm	TG 2232 [1)]
Width	48	mm	saa.
Density (mean)	460	kg/m^3	TG 2365 [2)]
Modulus of elasticity	14000	N/mm^2	saa.
Flooring properties, “slisseplate”:			
Thickness	22	mm	TG 2232 [1)]
Density	685	kg/m^3	Assumed
Modulus of elasticity E_L, E_T, E_R	2250	N/mm^2	YT-11 [4)]
Area reduction taken into account	1935	N/mm^2	Excel sheet
Connection between flooring and joists:			
Assumed stiff / full connection because of screw + glue connection.			
Notes:			
1) Technical approval 2232: “Støren Treindustri Trehuselementer”			
2) Technical approval 2365: “K-Bjelke og K-Bjelke Plus”			
3) Technical approval 2280: “Forestia gulv sponplater”			
4) Declaration of performance YT-11: “Ytelseserklæring, 22 mm Forestia Slissegulv Ekstra”			

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