Emissions Trading with Offset Markets and Free Quota Allocations

1. Introduction

Free emissions rights or quotas are a standard feature of most existing emission trading schemes. In particular, within the EU's Emissions Trading System (EU ETS) for greenhouse gases (GHGs), 99.9 % of emissions rights were on average handed out for free to participating entities from the start (Convery et al., 2008). This share was reduced somewhat in the second period, but has been well above the minimum requirement of 90 %. From 2013 on, the share is scheduled to fall below 50%; a more complete phase-out is however not yet on the horizon.

Free allocation of quotas has four major impacts in our context, three negative and one potentially positive. The first, well recognized, is that substantial revenue is foregone for governments (see, e.g., Goulder et al., 1999); this includes the fact that the polluter pays principle is more or less abandoned. Two further issues have until recently been less recognized, but are no less detrimental. One is that free allocations may reduce firms' incentives to abate. This follows mainly because current activity (emissions or production) may serve as the basis for future free allocations, thus acting as an effective premium on emissions (directly, or indirectly as a premium on output). This "raises the bar" with respect to abatement that is privately efficient for emitters, since more abatement may reduce the extent of future free allocations.

The second issue, the main focus of this paper, is that free allocations can make offset markets less efficient. In fact, when firms' gains from free allocations are sufficiently high, we find that governments may choose to ban the offset market completely. Even when the offset market is allowed to operate, it will do so inefficiently. The fourth and potentially positive effect of free allocation is that it may alleviate carbon leakage and improve the competitiveness of trade-exposed sectors. In addition to the third issue, we will also touch upon the second and the fourth issue.

Two alternative mechanisms for allocating free emissions quotas are here relevant. The first is based on *updating of quota allocations according to past emissions*, as these may be taken as an indication of future quota "needs". This issue has been treated in the literature, e.g. by Böhringer and Lange (2005), Rosendahl (2008), Harstad and Eskeland (2010) and Rosendahl and Storrøsten (2011), who have shown that when free allocations are updated in such a fashion, much of the incentive to abate could be removed from emitters. Furthermore, the quota price will exceed firms' marginal mitigation costs.[1] The second allocation mechanism entails *free*

[1] As shown by Böhringer and Lange (2005), such allocations can still in principle be cost-effective, given a "closed system" with no offset market and identical updating rules and price expectations across emitting firms. The quota price of carbon will then however be (perhaps substantially) higher than firms' marginal abatement costs.

allocations as based on past output, using a "benchmark" emissions intensity index for the industry. Higher output then secures more free allocations (the "need" for free allocations depends on output). In this case output will be excessive, and in consequence also emissions.[2]

We find that the perhaps most basic incentive problem created by updated free allocations is to weaken the link between the carbon price and incentives to reduce emissions within the policy bloc, as the carbon price necessary to implement an efficient carbon abatement effort is raised, perhaps drastically. Efficient allocations can still in some cases be implemented, but this requires a (much) higher carbon price than otherwise.

This paper discusses effects of free quota allocations in an emission trading system comprising a "policy bloc" of countries, which faces a "fringe" of (non-policy) countries. We assume that an offset market is established, whereby emissions in policy bloc countries can be offset through emissions reductions executed in the fringe, and purchased by entities in the policy bloc. In the context of the Kyoto Protocol (under which the EU ETS is established), the CDM serves such a role; it will be useful for the reader to have this mechanism in mind in the following. A main purpose of this paper is to study how free quota allocations to firms in policy-bloc countries interact with the working of the offset market.

We study two separate cases by which emissions are limited through a market of tradable emissions quotas in policy bloc countries, and where a fraction of these quotas are given away for free by governments to emitters. In the first case, we assume that emitters buy offsets directly from the fringe, which mimics the current situation with respect to the EU ETS and the CDM. Further, offsets substitute perfectly with domestic quotas. In this case, the price of offsets must be equal to the quota price in the policy bloc.[3]

In the second case, the quota markets in the policy bloc and fringe are kept apart. As in the first case, there is free trading of emissions quotas within the policy bloc. The difference is that free trading with offsets among market participants is now prohibited, and offsets are instead purchased directly from the fringe by policy countries' governments. The offset price turns out to be below the quota price in the policy bloc countries. Strand (2013) has recently shown, in a model with similarly separate markets (but without free quota allocations), that such a price structure is optimal for the policy bloc.

[2] Output-based allocations can however be favorable by reducing leakage; see e.g. Böhringer et al. (2010). Strand and Rosendahl (2012) show that the Clean Development Mechanism (CDM), the key "offset" mechanism under the Kyoto protocol, may create similar incentives for excessive production.

[3] In our analysis we disregard factors such as uncertainty and restrictions on the use of CDM credits, which may explain why prices of CDM credits are usually below the quota price in the EU ETS.

The outline of the paper is as follows. We first consider a single quota market with updating of free allocations, free trading of emissions quotas among firms, and no fringe and thus no offset market. We replicate, in Section 2, some key results from earlier studies for this case (see above). Sections 3-4 extend this model to include an offset market with free quota trading, and thus a unified trading price for offsets and quotas in the policy bloc.

In Section 3 we study free updated quota allocations based on emissions. When the quota allocation is "not too beneficial" to firms, it is optimal for the policy bloc not to constrain the use of offsets. Due to the updating rule, the marginal mitigation cost of policy-bloc firms will be below that of the fringe. This will lead to an excessive share of offsets. However, we show that the optimal emissions price at the same time is below (the policy bloc's value of) marginal environmental costs, meaning that there will be inefficiently low volumes of abatement both in the policy bloc and in the fringe. As the quota allocation rule becomes sufficiently generous, it is optimal for the policy-bloc to switch, discretely, from free use of offsets to banning offsets completely. This implies no abatement in the fringe whatsoever, which is also inefficient.

In Section 4 we assume instead that free allocations are based on firms' past outputs. This also leads to inefficiency in both the offset market and the policy bloc. With no carbon leakage, output of policy bloc firms is always inefficiently high. Leakage can however change this conclusion. The policy bloc could here, as in Section 3, choose to include an offset market with emission prices below marginal environmental costs (given moderate output subsidy); or exclude it and implement a higher quota price when the output subsidy is larger.

A basic assumption in Sections 3-4 is that all emissions quotas must be traded at a common quota price for the policy bloc and offset market. This is the principal policy applied to the CDM today. In Section 5 we instead assume that the policy-bloc countries behave in a unified manner in the offset market, as a non-discriminating monopsonist with no knowledge of project-specific abatement costs in the fringe. We show that the (unified) offset price is then set *below* marginal environmental damage cost. The optimal quota price within the policy bloc is set to equalize marginal damage costs to marginal abatement cost (as with no offset market). The consequence could be a large difference between the internal quota price and the external offset price. The use of offsets will be inefficiently low, but suboptimal from the policy bloc perspective, due to the monopsonistic behavior of the policy countries versus the fringe.

Section 6 studies maximization of global (and not as in sections 3-5 only the policy bloc's) welfare, given that free allocations depend on past emissions and assuming that the quota price cannot be differentiated (as in section 3). We then show that the (constrained optimally) chosen quota price will be higher as global and not regional valuation of the global externality is considered. Also, free offset purchases are chosen more often than in section 3, as such purchases are no longer a net fiscal cost (instead, a transfer from the policy bloc to the fringe).

Section 7 concludes, and indicates scope and topics for future research.

2. Basic Model with Emissions Trading and Updated Allocations

Consider a "policy bloc" of countries which initiates and issues emissions quotas for GHGs, to a large extent given out for free to emitters within the bloc. It is also possible to purchase emissions rights from "non-policy countries" (the "fringe"). Assume that the offset market works perfectly in the sense that all offsets are additional and efficient. In the policy bloc, there are a given number of firms with aggregate revenue function $R_1(E_1, X_1)$, where E_1 and X_1 denote respectively emissions and production from the policy bloc. We have $R_{1E}' > 0$ for $E_1(t) < E_{10}(t)$ and $R_{1X}' > 0$ for $X_1(t) < X_{10}(t)$, where $E_{10}(t)$ and $X_{10}(t)$ are the BaU emissions and production levels. The probability that any given firm survives to the next period is β (same for all firms and such that firm exits are random events).

Assume that free allocation of emissions rights follow an "updated grandfather" rule whereby the number of free quotas awarded to firms with emissions $E(t\text{-}1)$ and production $X(t\text{-}1)$ in period $t\text{-}1$ equals $\alpha E(t\text{-}1) + \gamma X(t\text{-}1)$ in period t. That is, allocation of quotas is based on firms' past emissions and/or production, where the updating parameters α and γ are assumed to lie between zero and unity. In the two first phases of the EU ETS (2005-2012), α has been closer to one while γ has been mostly zero. In the third phase (2013-2020), however, α is mostly zero except in some sectors,[4] while γ is close to one in exposed sectors but smaller (or zero) in other sectors (cf. the discussion in Section 4).

Denote the discount factor between periods by δ, and assume that firms have a potentially infinite life span. The discounted value of net returns for a representative firm in the policy bloc, $V_1(t)$, can then be expressed as

(1) $$V_1(t) = W_1(t) + \beta\delta W_1(t+1) + \beta^2\delta^2 W_1(t+2) + ...$$

where $W_1(t)$ denotes net returns in period t. $W_1(t)$ is in turn given by[5]

(2) $$W_1(t) = R_1(E(t), X(t)) - q(t)(E_1(t) - \alpha E_1(t-1) - \gamma X_1(t-1)),$$

where $q(t)$ is the quota price in period t. We note that $\alpha E_1(t\text{-}1) + \gamma X_1(t\text{-}1)$ represents the amounts of free allocations of emissions rights available to the representative firm in period t. This amount is exogenous to the firm when period t arrives. However, the firm looks ahead to future

[4] Although output-based allocation will be the main allocation rule in the third phase (75-80% of freely allocated quotas), for several products the allocation will be based on either past energy input or past (process) emissions for the individual firm. Energy input is closely related to emissions, except for the possibility of fuel switching. For further discussion see Lecourt (2012).

[5] A condition for (2) to hold is $E_1(t) \geq \alpha E_1(t\text{-}1)$, which we assume to hold (in particular, it holds in steady state).

periods, in which the payoff will be affected by current emissions through the updating mechanism. Inserting from (2) into (1) we may write

(1a)
$$\begin{aligned} V_1(t) &= R_1(E_1(t), X_1(t)) - q(t)(E_1(t) - \alpha E_1(t-1) - \gamma X_1(t-1)) \\ &+ \beta\delta[R_1(E_1(t+1), X_1(t+1)) - q(t+1)(E_1(t+1) - \alpha E_1(t) - \gamma X_1(t))] + \beta^2\delta^2 V_1(t+2) \end{aligned}.$$

The representative firm seeks to maximize $V_1(t)$ with respect to current emissions and production levels, $E_1(t)$ and $X_1(t)$. This yields the following first-order conditions:

(3)
$$\frac{dV_1(t)}{dE_1(t)} = R_{1E}\text{'}(E_1(t), X_1(t)) - q(t) + \alpha\beta\delta q(t+1) = 0$$

and

(4)
$$\frac{dV_1(t)}{dX_1(t)} = R_{1X}\text{'}(E_1(t), X_1(t)) + \gamma\beta\delta q(t+1) = 0$$

and thus

(3a)
$$R_{1E}\text{'}(E_1(t), X_1(t)) = q(t) - \alpha\beta\delta q(t+1)\ .$$

(4a)
$$R_{1X}\text{'}(E_1(t), X_1(t)) = -\gamma\beta\delta q(t+1)\ .$$

Because of the sequential (Bellman-type) nature of each firm's decision problem, $E_1(t)$ and $X_1(t)$ here enter into $V_1(t+1)$, but not into $V_1(t+2)$. This drastically simplifies the problem as we do not need to explicitly consider $V_1(t+2)$ or higher-order value function terms in deriving the optimal solution. To simplify the analysis, we focus on steady-state cases, with constant revenue functions, constant number of firms (so that a fraction β of all firms are replaced by entering firms in any given period), and a constant overall emissions cap. Hence, we have $q(t) = q(t+1) = q$, and (3a) and (4a) may be written as

(3b)
$$R_{1E}\text{'}(E_1(t), X_1(t)) = (1 - \alpha\beta\delta)q = (1-a)q\ .$$

(4b)
$$R_{1X}\text{'}(E_1(t), X_1(t)) = -\gamma\beta\delta q = -bq\ .$$

where $a=\alpha\beta\delta$ and $b=\gamma\beta\delta$. The parenthesis on the right-hand side of (3b) expresses the "net price" paid for emissions quotas by policy country firms in a steady state. This price is lower than the "gross" price q since the free quota allocation is an increasing function of past emissions. The difference between the "net" and the "gross" price depends on the product of three parameters: the updating share (α); the probability of firm survival to the next period (β); and the discount factor (δ). All these three parameters might be close to unity; in that case their product will also be relatively close to unity. The effective quota price could then be much lower than the statutory price, q.

Similarly, the right-hand side of (4b) expresses the implicit subsidy per unit production by an output-based allocation where $\gamma>0$.[6] Note that this subsidy is proportional to the quota price.

Our analysis so far simply restates already known results. However, we notice that the deviation between "net" and "gross" prices can create inefficiencies when the quota market in the policy bloc is linked with an offsets market, where the "statutory" quota price, q, is also the effective price of emission reductions. This implies an asymmetry between the regular (internal) quota market, and the (external) offset market; with a favoring of the former types of emitters.

In the next section we will assume that $\gamma=0$, and focus on the case with emissions-based allocation ($\alpha>0$). In Section 4 we consider output-based allocation and set $\alpha=0$ (and $\gamma>0$). To simplify notation, we skip $X(t)$ in the expressions in Section 3.

3. Offset Policies with Emissions-Based Allocation of Quotas

Consider the offset market in the fringe countries. This market has an aggregate revenue function $R_2(E_2(t))$ in period t, and can be viewed as operating on a period-by-period basis. We assume (conservatively) that all offsets represent real emissions reductions in the offsetting region, where the comparison benchmark is overall emissions in the absence of offsets.[7] Define this benchmark by $E_{20}(t)$ in period t, given by:[8]

$$R_2'(E_{20}(t)) = 0 . \tag{5}$$

We assume that quotas and offsets can be traded freely by all actors in the carbon markets, both within the policy bloc, within the fringe, and between the policy bloc and the fringe. Such free trading implies that there exists a single trading price q for all quotas (including offsets). Fringe market participants have no incentives to buy quotas except for resale; this we can ignore here.

---

[6] To be precise, the right-hand side expresses the implicit tax. However, since this is negative in our case, we have an implicit subsidy.

[7] There are several reasons why not all offsets need to reduce global net emissions. One reason is leakage (see Rosendahl and Strand (2011)). Another reason is baseline manipulation and output inflation under "relative baselines" with incentives to increase emissions (see Fischer (2005), Germain et al (2007), Strand and Rosendahl (2012)).

[8] E_{20} is assumed given and unaffected by the model parameters. This requires that there is no (positive or negative) emissions "leakage" from the policy bloc to the fringe. In Section 4 we return to this issue.

In Section 5 we consider alternative assumptions with different quota trading prices within the policy bloc and in the offset market.

Define next the maximal (potential) supply of offsets from the fringe, for a given offset price *q*, by $\hat{Q}_2(t)$. This supply corresponds to the difference between the benchmark emissions $E_{20}(t)$ and the emissions level $\hat{E}_2(t)$ given by

(6) $$R_2'(\hat{E}_2(t)) = q.$$

As shown below, it may be optimal for the policy bloc to restrict the number of offsets. Let *k* denote the share of offset supply from the fringe that is utilized in the policy bloc, and let Q_{2k} denote the corresponding offset purchases. We then have

(7) $$Q_{2k}(t) = k\hat{Q}_2(t) = k(E_{20}(t) - \hat{E}_2(t)).$$

We assume that the sales of offsets are regulated on a first-come-first-served basis, meaning that realized offsets are a random draw among all potential offset suppliers (so that each has probability *k* of successfully selling offsets, and the cost distribution for realized offsets is the same as for all potential offsets).[9] We show below that it is optimal for policy-bloc country governments to choose either $k = 0$ or $k = 1$, so this potential challenge turns out to be irrelevant.

It is clear that mitigation cannot be overall optimal, under our assumptions. The reason is that policy bloc firms and fringe firms face different effective mitigation costs, with lower costs for policy bloc firms than for fringe firms (compare (3b) with (6)). Thus, there exist some firms in the fringe that mitigate to the level where marginal mitigation cost equals *q*, whereas no mitigation options in the policy bloc with marginal cost between $(1-a)q$ and *q* are realized. Hence, mitigation through offsets is on average more costly (and inefficiently so) than mitigation by the policy bloc. This inefficiency can however to some degree be counteracted by reducing the overall volume of offsets, given by (7), by lowering the "purchase rate" parameter *k*. The distribution of mitigation within the fringe will still remain inefficient, since abatement costs in the fringe are then not minimized for given abatement.

We now search for optimal combinations of *q* and *k*, i.e., the quota price and the share of potential offsets to be purchased, and study how these depend on the parameter *a* ($=\alpha\beta\delta$) which

[9] This need not be the case. Since low-cost projects imply more rent to project sponsors, these will have greater incentives than others to promote their projects thus attracting more attention from policy bloc firms. Rent sharing between contracting parties, ignored here but studied by Brechet, Meniere and Picard (2011), would also make low cost projects more directly attractive to policy bloc parties.

we treat as exogenously given.[10] Equivalently, we could search for optimal combinations of E^* and k, where E^* is the emissions cap level in the policy bloc, i.e.:[11]

(8) $$E_1 - Q_{2k} = E_1 - k(E_{20}(t) - \hat{E}_2(t)) \leq E^*$$

What is an "optimal" offset policy is not obvious. We postulate in sections 3-5 the following simple objective function, defined by the policy bloc only (defining all relevant variables as functions of the policy variables q and k):

(9) $$B_1(q,k) = R_1(E_1(q)) - cE(q,k) - qk(E_{20} - \hat{E}_2(q)).$$

The first term is simply the aggregate revenue function, the second term accounts for the environmental damages from global emissions (E), valued at a constant unit cost c, and the last term represents costs of buying offsets from the fringe.[12]

E_1 and $\hat{E}_2$ are here both simple functions of q only (from (3b) and (6) respectively). For E we have the following accounting definition:

(10) $$E = E_1 + E_{20} - k(E_{20} - \hat{E}_2) = E_1 + (1-k)E_{20} + k\hat{E}_2.$$

We can now insert into (9) from (10) for E, which yields

(9a) $$B_1(q,k) = R_1(E_1(q)) - cE_1(q) - (c-q)k\hat{E}_2(q) - [c(1-k) + qk]E_{20}.$$

This expression can be maximized with respect to q and k, yielding the following general first-order conditions for internal solution:

(11) $$\frac{\partial B_1(q,k)}{\partial q} = (R_1' - c)\frac{\partial E_1}{\partial q} - k(c-q)\frac{\partial \hat{E}_2}{\partial q} - k(E_{20} - \hat{E}_2) = 0$$

(12) $$\frac{\partial B_1(q,k)}{\partial k} = (c-q)(E_{20} - \hat{E}_2) = 0,$$

[10] β and δ are non-policy parameters, whereas α is clearly a policy parameter.

[11] For a given combination of E^* and k, a corresponding level of q follows (and vice versa for a given combination of q and k). Hence, although we do not use (8) in the following analysis, (8) may be used to derive the corresponding value of E^*. Furthermore, instead of regulating k directly, which is difficult, the bloc could regulate the total number of offsets Q_{2k}.

[12] Note that only welfare for the policy bloc is considered, so that welfare for the fringe is excluded. In section 6 below we will instead take a "welfare maximizing" view, where also the fringe's welfare is included.

(11) and (12) determine optimal levels of q and k. Consider first how the optimal k depends on q. From (12), since $E_{20} - \hat{E}_2 > 0$, an internal solution for k is not feasible unless $q = c$, in which case any value of k fulfills (12).[13] Without loss of generality we assume that k is set to zero whenever $q = c$, as B_1 is then independent of k. If $q < c$ is optimal, $k = 1$ as B_1 then increases in k for any k. On the other hand, if $q > c$ is optimal, $k = 0$ (B_1 is then decreasing in k). Intuitively, we know that E_1 only depends on q, i.e., emissions in the policy bloc are independent of k, for given q. Hence, k only determines how many offsets, or emissions reductions, the policy bloc purchases from the fringe. Consequently, it is optimal for the policy bloc to buy offsets from the fringe if and only if the costs of buying offsets (q) are lower than the damage costs of emissions (c), which then represents the benefits of buying offsets.

What about the optimal level of q? At first glance, one would expect the optimal q to be set equal to the marginal damage costs c. However, there are two reasons why it may be optimal to deviate from this standard result, which we come back to below.

From the reasoning above, we must either have $q \geq c$ and $k = 0$, or $q < c$ and $k = 1$. Let us characterize these two potential outcomes of the policy bloc's optimization.

In the first case there is no offset market available for the firms since $k = 0$. From (11) we then have the standard optimality condition $R_1' = c$. Then there is no inefficiency within the policy bloc, but not using offsets at all is inefficient since cheap abatement options are foregone. Still, it may be a second-best solution for the policy bloc. This outcome implies, from (3b) and (11):

$$(13) \qquad q = \frac{c}{1-a} \geq c .$$

Thus, an optimal solution with $k = 0$ requires that the quota price be set higher than the marginal damage cost of emissions, c, as long as $a > 0$. This is just as in Böhringer and Lange (2005) and Rosendahl (2008), who show that the quota price is driven up by the updating rule (for a given emissions constraint). A high quota price in this case makes it too expensive for the policy bloc to purchase offsets. Note that when a is relatively close to unity, the mark-up relative to c could be large.

Consider next the outcome where $q < c$ and $k = 1$. $q = 0$ cannot be optimal (cf. (11)), so only internal solutions are feasible. We may write (11) as follows, inserting from (3b):

[13] We have assumed linear environmental damage costs, represented by the marginal costs c. If we rather assume convex damage costs $C(E)$, we would still have $q = C'(E)$ as the only internal solution. However, in this case E and thus $C'(E)$ is a function of both q and k, and so the choice of k is no longer irrelevant under an internal solution.

(11b) $$-aq\frac{\partial E_1}{\partial q} = (E_{20} - \hat{E}_2(q)) + (c-q)\left(\frac{\partial E_1}{\partial q} + \frac{\partial \hat{E}_2}{\partial q}\right)$$

The LHS and the first term on the RHS are here both positive for $a, q > 0$, whereas the second term on the RHS is negative for $q < c$. We distinguish between the following three cases:

a) $-aq\frac{\partial E_1}{\partial q} > E_{20} - \hat{E}_2(q)$ for any $q < c$. In this case, equation (11b) has no solution when $q < c$, and thus it is optimal to increase q until $q \geq c$ (cf. (11)). But then we are back to the case with $k = 0$ and q determined by (13), as described above. The intuition here is that the ability of the offset market to profitably deliver offsets to the policy bloc (RHS of the inequality) is relatively small. The solution is then determined out of a concern for the domestic mitigation market. Notice that the higher a is (e.g., the higher the allocation rate α is), the more likely this case is. Further, without the updating rule (i.e., $a = 0$) this case can never occur.

b) $-aq\frac{\partial E_1}{\partial q} < E_{20} - \hat{E}_2(q)$ for $q = c$. In this case, equation (11b) must have (at least one) internal solution with $q < c$.[14] The ability of the offset market to profitably deliver offsets is now greater. This implies that the quota price may be determined more out of a direct concern for the offset market, and less out of a concern for the domestic mitigation market in the policy bloc. However, we cannot conclude in general whether or not this solution with $q < c$ and $k = 1$ is preferred over the solution with $k = 0$ and q determined by (13). The former case utilizes relatively cheap abatement options in the offset market, but abatement in the policy bloc is far too low as $R_1'(E_1) < q < c$. In the latter case, mitigation in the policy bloc is optimized, but none of the abatement options in the offset market are utilized.

c) $-aq\frac{\partial E_1}{\partial q} < E_{20} - \hat{E}_2(q)$ for some $q < c$ (but not for $q = c$). In this case we may or may not have an internal solution of (11b). For instance, the fringe may be quite able to deliver profitable offsets relative to emissions reductions in the policy bloc at low levels of q, but not at higher levels of q. The policy bloc does not want the quota price to be too low, however, due to the environmental concern. Hence, it may be optimal to increase q until $q \geq c$, i.e., similar to a). However, we may also have an internal solution similar to b).

[14] If emissions in the two regions are convex (or linear) in q, we can show that the second order condition of (12) is fulfilled for $q \leq c$. Thus, there is just one internal solution which is optimal given $q \leq c$.

To sum up so far, it is optimal for the policy bloc to either ban offsets completely, and let the quota price be given by (13), or to not restrict offsets, implying $R_1'(E_1) < q = R_2'(E_2) < c$. Further, the higher is *a*, and the lower is the offset potential relative to domestic emissions reductions, the more likely it is that offsets are banned.

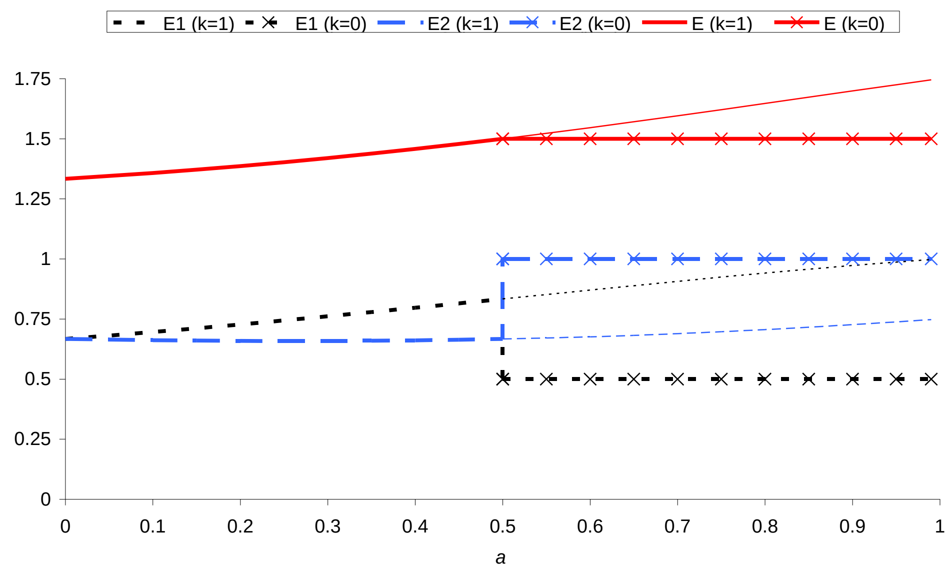
In considering b) further, note first that if $a = 0$, i.e., no updating, $q < c$, and $k = 1$ is preferred over the alternative solution $q = c$ and $k = 0$. The reason is that the latter outcome is equivalent with $q = c$ and $k = 1$ (see above). But then we know from the investigation of case b) that reducing *q* below *c* will be beneficial. This case, i.e., without updating, has already been analyzed in Strand (2012). In other words, without updating it is never optimal to restrict offset purchases (given our model assumptions), and the optimal quota price should be below marginal damage cost. The latter conclusion is seen by replacing $R_1'(E_1)$ by *q* and setting $k = 1$ in (11). At $q = c$ the RHS is negative as the policy bloc, acting as a monopsonist, benefits from reducing *q* due to lower costs of importing quotas.

When *a* is marginally increased from zero, i.e., a mild updating rule, the optimal *q* will increase. This is shown in the appendix for quadratic mitigation costs. Updating creates a difference between the marginal costs of abatement, from (3b), and the marginal costs of emissions, *c*, for given *q*. Thus, it is optimal to increase *q* even though the policy bloc's costs of buying offsets increase. Given $k = 1$, updating will then make it optimal with more offsets at low levels of *a*. However, beyond a certain level, a further increase in *a* will reduce the optimal *q* (cf. the appendix), thus also reducing the use of offsets. The policy bloc would ideally prefer a lower *q* in the offset market in order to reduce the costs of buying offsets. This matter becomes more important when *a* is high, and hence the optimal *q* declines. Finally, when *a* becomes sufficiently high, it becomes optimal to prohibit offsets, i.e., switch from $k = 1$ to $k = 0$.

Although the optimal quota price increases when *a* is increased from zero, abatement in the policy bloc declines (at least, this is the case with quadratic mitigation costs, cf. the appendix). The reason is that for abatement to stay constant within the policy bloc when *a* increases, *q* must increase so that the RHS of (3b) does not change; while we find that *q* increases less rapidly. In other words, the effects of higher *a* dominate the effects of higher (optimal) *q* in equation (3b), so that E_1 increases. Moreover, global emissions increase, too, when *a* is increased, as the higher emissions in the policy bloc ("region 1") will always dominate the (initially) lower emissions in the fringe ("region 2"; see the appendix). This holds only as long as $k = 1$, however, as when *a* becomes so high that $k = 0$ is optimal, *q* jumps to the level given by (13). Then emissions in both regions are insensitive to a further increase in *a* (which then only affects *q*, from (13)).

In the appendix we show that, given quadratic mitigation costs, welfare in region 1 decreases monotonically in the level of *a* as long as offsets are used. Thus, introducing (or intensifying the level of) updating in a quota system with access to offsets will unambiguously reduce welfare in the policy bloc. Updating then increases the deviation between the desired domestic quota price and the desired offset price. It also becomes optimal to switch to no offsets exactly when global emissions are the same with and without offsets (see the appendix). It follows that increasing the level of *a* will always increase global emissions as long as offsets are used initially. A partial explanation for this result, which may not hold with other specifications than quadratic mitigation costs, is that the policy bloc wants to minimize global emissions, and hence has incentives to pick the alternative where these are lowest.

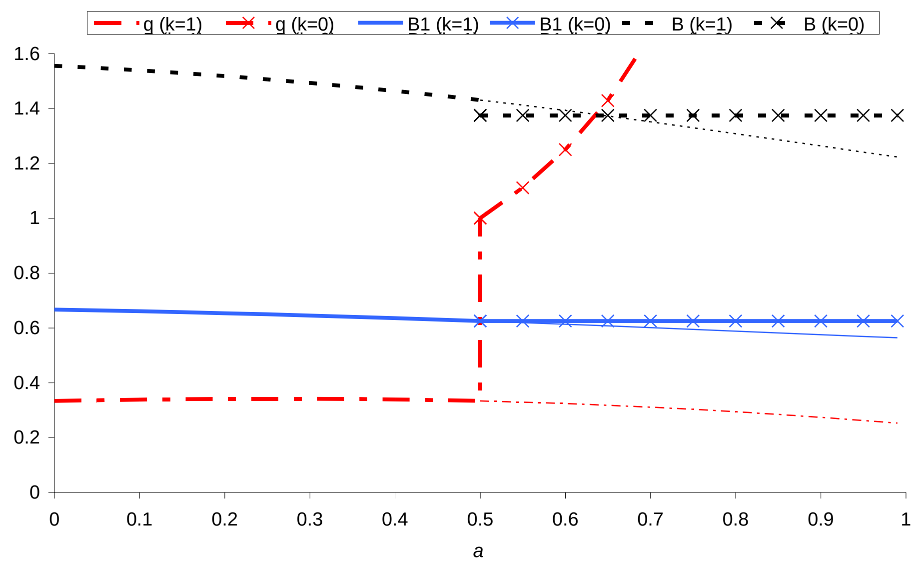
Figure 1. Emissions in the policy bloc, the fringe and global emissions as a function of *a*



Figures 1 and 2 illustrate the cases discussed in this section, assuming identical and quadratic mitigation costs in the two regions.[15] The thick curves show the outcome of region 1's optimization as a function of *a*. In addition, the figures show the (hypothetical) outcome given $k = 1$ also when $k = 0$ would be optimal (see the thin curves). With the chosen parameters, it is optimal to switch from $k = 1$ to $k = 0$ at $a = 0.5$. From Figure 2 we notice that the quota price is almost constant up to $a = 0.5$ (first slightly increasing, then slightly decreasing), but jumps substantially when offsets are no longer utilized. From Figure 1, this implies almost constant emissions in region 2 up to $a = 0.5$, whereas emissions in region 1 are steadily increasing. Hence, global emissions increase, consistent with the analytical findings. At $a = 0.5$ emissions in region 2 jumps to its BaU-level, whereas emissions in region 1 fall due to the much higher quota price. As indicated above, the changes in the two regions exactly cancel each other out so that global emissions neither jump nor fall at $a = 0.5$.

[15] Referring to the quadratic model specification in the appendix, parameters are as follows: $\mu_{jA} = \mu_{jB} = \mu_{jC} = 1$; $c = 0.5$.

Figure 2. Quota price and welfare in the policy bloc and global welfare as a function of *a*



Welfare in region 1 declines as *a* increases (see Figure 2) until it reaches the welfare level with *k* = 0 (in which case it is unaffected by *a*). We also notice that global welfare (*B*) decreases.[16]

How large are these welfare losses compared to other potential welfare losses from emission-based updating? As shown in e.g. Böhringer and Lange (2005), emission-based updating can be cost-effective in a closed emission trading system, but not in an open system, e.g., with a fixed emission price. Calculating the welfare losses in region 1 from emission-based updating, given a fixed emissions price and no emission changes outside this region, these are up to 11 percent when *a* = 1. The corresponding welfare losses in Figure 2 amount to 6 percent for region 1 (B_1) and 12 percent for both regions combined (*B*).

So far we have assumed that all offsets are additional, i.e., reflect real emissions reductions (vis-à-vis BaU). If some offsets are not additional, it becomes optimal for the policy bloc to ban offsets at lower levels of *a*. In the numerical examples in Figures 1-2, the switching point drops from *a* = 0.5 to *a* = 0.22 if only half of the offsets are real emissions reductions.

We sum up our main findings in the following proposition:

Proposition 1. *Consider a policy bloc, maximizing policy-bloc welfare, with an emissions trading system, free quota allocations based on firms' past-period emissions, and an offset market with free trading among market participants. Then:*

[16] Here we have assumed that Region 2 values global emissions by the same price as Region 1, i.e., by *c*.

i) If free allocations are sufficiently generous, it is optimal for the policy bloc to ban offsets. If banning offsets is not optimal, the use of offsets should be unrestricted.

ii) If offsets are used, marginal damage costs strictly exceed the quota price and marginal abatement costs in the fringe, which strictly exceed marginal abatement costs in the policy bloc.

iii) As long as using offsets is optimal for the policy bloc, increasing the free allocations of quotas leads to higher emissions and lower welfare in the policy bloc, and higher global emissions (given that mitigation costs are quadratic).

Proof: Follows from the discussion above, and from the appendix.

4. Offset Policies with Output-Based Allocation of Quotas

In the previous section we assumed that free quotas are based only on firms' past emissions. As explained in Section 2, the EU ETS is now moving more towards output-based allocations of quotas, although emissions-based allocations will still be used in some sectors (see footnote 4). Output-based allocations are also highly relevant for regions such as Australia, New Zealand and California (see e.g. Hood, 2010). A main justification for this switch is the fear of "carbon leakage" through the markets for emission-intensive, trade-exposed goods. The underlying problem is that lower output of such goods in one region, due to unilateral climate policy, leads to greater output and emissions in regions with more lenient climate policies.[17] With output-based allocations, an "emission intensity benchmark" is defined for each product, based on e.g. an average standard of all or the best firms in the industry. This would, at least in principle, make this "benchmark" independent of the emissions of any one given firm.[18]

To account for such effects we will in this section focus on output-based allocations, so that $\gamma > 0$ and $\alpha = 0$. From (3b), absent an offset market, emissions within the policy bloc are then "optimal" in the sense that marginal value of emissions equals the quota price, which is set equal to marginal damage cost. On the other hand, from (4b) and with no carbon leakage, output is excessive: Optimality would here entail (net) marginal value equal to zero. Emissions from fossil energy are then likely also excessive, given that (as is reasonable) output and energy use are complementary.

We now introduce offsets into this alternative model. Instead of (9a) we then have the following alternative objective function for the policy bloc:

[17] There is a large literature on emissions leakage (e.g., Hoel, 1996; Rosendahl and Strand, 2011; Böhringer et al., 2011). Leakage may occur through the international markets for fossil fuels, and through the markets for emission-intensive, trade-exposed goods. Here we focus on the latter channel.

[18] In the EU ETS, benchmarks for the period 2013-2020 are mainly determined based on the ten per cent least emission-intensive installations.

(14) $B_1(q,k) = R_1(E_1(q), X_1(q)) - cE_1(q) - (c-q)k\hat{E}_2(q, X_1(q)) - [c(1-k) + qk]E_{20}(X_1(q)).$

Following the discussion above, we assume that in the absence of free allocation (γ=0), $\partial X_1 / \partial q < 0$ and $\partial \hat{E}_2 / \partial X_1 < 0$. The first of these derivatives simply expresses that a higher quota price (or emission cost) reduces energy-intensive output in the policy bloc if no quotas are allocated for free. Note, however, that with output-based allocation (γ>0), the positive effect on output through higher implicit subsidy may dominate the negative effect through reduced emissions – this depends on the size of γ and the complementarity of E and X (we return to this below).

The second of the derivatives states that when output is reduced in energy-intensive sectors in the policy bloc, fringe emissions will shift up, presumably as related industrial activity is shifted to this region.

Both these conditions are intuitive for emission-intensive, trade-exposed industries, and are preconditions for having a leakage problem in our context. The size of $\partial X_1 / \partial q$ determines how sensitive domestic output is to the quota price, whereas $\partial \hat{E}_2 / \partial X_1$ determines "leakage exposure" for domestic firms. To simplify, we assume $\partial \hat{E}_2 / \partial X_1 = \partial E_{20} / \partial X_1$, so that leakage is independent of offset projects.

Maximizing (14) with respect to q and then reorganizing yields in this case:

(15) $$\frac{\partial B_1(q,k)}{\partial q} = (q-c)\frac{\partial E_1}{\partial q} - k(c-q)\frac{\partial \hat{E}_2}{\partial q} - k(E_{20} - \hat{E}_2) - \left(bq + c\frac{\partial \hat{E}_2}{\partial X_1} \right)\frac{\partial X_1}{\partial q} = 0$$

(12) still holds, so we must either have $q \geq c$ and $k = 0$, or $q < c$ and $k = 1$.

Reasonably, the value of free allocations is likely related to the costs of leakage exposure; this may be because the authorities are inclined to compensate firms, in terms of reduced quota costs, for their loss of competitive position when subject to climate policy. Such effects are captured by the two terms inside the parenthesis of the last term on the RHS. The first is the expected value of future allocations per unit output today (where $b=\gamma\beta\delta$), i.e., the implicit output subsidy. The second is the environmental cost of leakage due to a marginal reduction in domestic output. Assume first that this parenthesis is zero. Then we are back to (11) (remember that $R_{1E}' = q$ when $a = 0$). From Section 3 with a =0, we know that the optimal solution is characterized by $q < c$ and $k = 1$. This further implies that for the parenthesis to be zero (in the optimal solution), we must have $b > -\partial \hat{E}_2 / \partial X_1$. Let q^* denote the optimal quota price in this case.

If the last term in (15) is negative, e.g., because ($-\partial\hat{E}_2 / \partial X_1$) is large compared to b (and $\partial X_1 / \partial q < 0$), it is optimal to reduce the quota price below q^*. The reason is that leakage reduces the environmental effectiveness of climate policy in the policy bloc, and hence the optimal quota price falls.

If the last term is positive, the optimal quota price is higher than q^*. This occurs if firms are given free quotas even when leakage exposure is negligible ($b >> -\partial\hat{E}_2 / \partial X_1$), and output is declining in the quota price (despite γ being large). Intuitively, the free quota allocation stimulates output too much, and so the optimal (second-best) response is to increase the price of emissions to moderate output.[19] If this effect, represented by the last term of (15), is big compared to the offset potential, represented by the third term of (15), it is optimal to increase q at least up to $q = c$ (the two first terms in (15) are both positive for $q < c$). But then we know from above that $k = 0$, i.e., banning offsets is optimal.

As explained above, however, if γ is large the sign of $\partial X_1 / \partial q$ may turn positive, in which case the last term becomes negative when $b >> -\partial\hat{E}_2 / \partial X_1$. Hence, excessive allocation may not necessarily imply that the quota price should exceed q^* – hence banning offsets may not be optimal even if the offset potential is limited. In order for q^* to exceed c in (15), we must have a combination of excessive allocation, output decreasing in the quota price, and limited offset potential.[20]

Let us now discuss the sign of the last term in (15), and in particular the size of b and $-\partial\hat{E}_2 / \partial X_1$, based on the allocation rules of the EU-ETS. For the most highly exposed sectors in the EU ETS, which account for most of industry emissions in the EU, $\gamma = b/(\beta\delta)$ is set close to E_1/X_1 (almost 100% compensation at the sector level). This means that, if reductions in domestic output are replaced one-to-one by foreign output, and emissions intensities are similar inside and outside the policy bloc, $b/(\beta\delta) \approx -\partial\hat{E}_2 / \partial X_1$. More likely, however, the output replacement is less than 100%. But since emissions intensities are often higher in the fringe, it is still difficult to judge whether b could be higher or lower than $-\partial\hat{E}_2 / \partial X_1$, at least for the highly exposed sectors.

[19] Obviously, the first-best response would be to lower b, but this might be difficult for political reasons.

[20] The last term in (15) can also be positive if $0 < b < c(-\partial\hat{E}_2 / \partial X_1) / q$, and $\partial X_1 / \partial q > 0$, in which case a higher quota price reduces leakage. If offset potential is limited, banning offsets may be optimal. We find this alternative less realistic. As under case b) and c) in Section 3, we may also have cases where the internal solution to (15) entails $q < c$, but still $k = 0$ is better than $k = 1$. See the discussion in Section 3 for details.

The EU has been criticized for allocating too many free quotas also to sectors that are only slightly exposed to leakage (see e.g. Martin et al., 2012). This relates both to sectors given 100% compensation (e.g., fossil fuel extraction), and to the remaining sectors which initially receive 70% compensation. Hence, sectors can probably be found where *b* exceeds $-\partial \hat{E}_2 / \partial X_1$, and possibly also $b >> -\partial \hat{E}_2 / \partial X_1$. This could be explained by strong industry lobbying groups. Still, since $\partial X_1 / \partial q$ may turn positive when *b* becomes large, the sign of the last term in (15) is ambiguous.

To sum up, free output-based allocations to leakage-exposed sectors have ambiguous effects on the optimal quota price when an offset market is available. This price will most likely be below marginal environmental cost. However, if sectors with limited leakage exposure are granted substantial free quotas and the offset potential is limited, the policy bloc may choose to ban offsets completely.

We sum up our main findings from the discussion above in the following proposition:

Proposition 2. *Consider a policy bloc with an emissions trading system, free quota allocations based on firms' past-period output, and an offset market with free trading among market participants. Then:*

i) If allocation is not too generous relative to the leakage exposure ($b \leq -\partial \hat{E}_2 / \partial X_1$), and output is declining in the quota price ($\partial X_1 / \partial q < 0$), it is not optimal to put any restrictions on the use of offsets.

ii) If allocation is generous relative to the leakage exposure ($b >> -\partial \hat{E}_2 / \partial X_1$), output is declining in the quota price ($\partial X_1 / \partial q < 0$), and the offset potential $E_{20} - \hat{E}_2$ is sufficiently limited, it is optimal for the policy bloc to ban the use of offsets.

iii) If offsets are used, marginal damage costs strictly exceed the quota price and marginal abatement costs in the fringe and in the policy bloc (which are all equal).

We see by comparing Propositions 1 and 2 that banning offsets is less likely to be optimal under output-based allocation than under emissions-based allocation, even if leakage exposure is limited. To shed more light into this question, we have performed simulations on a simple numerical model with the following revenue function for region 1:

(14b) $$R(E_1, X_1) = \mu_0 + (1-\theta)\left[\varphi(E_1 - E_1^2/2) + X_1 - X_1^2/2\right] + \theta E_1 X_1$$

$\varphi > 0$ determines the relative importance of emissions in the revenue function, while $\theta \geq 0$ determines to what degree *E* and *X* are complements. The lower is θ, the easier emissions can be reduced without affecting output. Region 2 is assumed to have the same revenue function, except for the size of the region given by σ. Let $\zeta = \partial \hat{E}_2 / \partial X_1$ denote an exogenous leakage parameter. For more details, see the appendix.

We can now investigate the importance of respectively the leakage exposure (represented by ζ), the extent of free allocation (represented by b), the complementarity between X and E (represented by θ), and the offset potential (represented by σ).

As a benchmark, consider first $\sigma = 1$, $\varphi = 1$ and $\theta = 0.25$, and $\zeta = 0$, 0.5 and 1. The BaU emission intensity is then unity, so $\zeta = 1$ means substantial leakage exposure.

Figure 3 displays the optimal quota price, relative to the carbon externality for the policy bloc, c. This is shown under the three alternative levels of leakage exposure, for different degrees of free allocations, and both with offsets ($k = 1$) and without ($k = 0$). Note that even without offsets, the quota price is more often below than above c, as opposed to the case with emission-based allocation. Moreover, the optimal quota price seems to fall in b for high b, both with and without offsets. This is due to the indirect subsidy effect explained above.

Figure 4 shows the corresponding global emissions, relative to BaU emissions. We first note that global emissions are always biggest when offsets are banned. Moreover, global emissions increase in the free allocation rate when $\zeta = 0$; fall when $\zeta = 1$; and are U-shaped when $\zeta = 0.5$. This illustrates that the allocation factor should reflect the leakage exposure, if the aim is to reduce global emissions.

Figure 3. Quota price as a function of *b*, relative to the marginal damage cost of emissions *c*

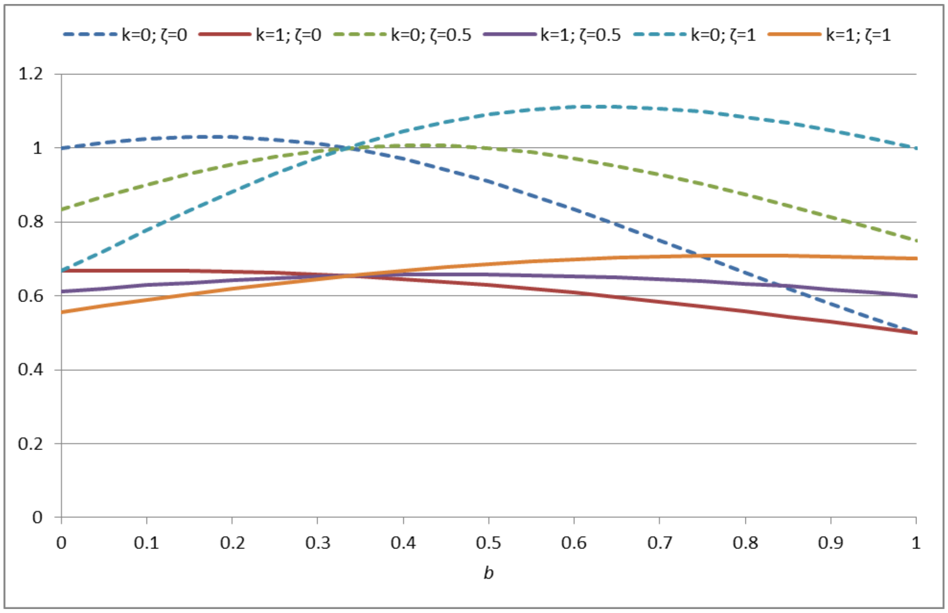


Figure 4. Global emissions as a function of *b*, relative to BaU emissions

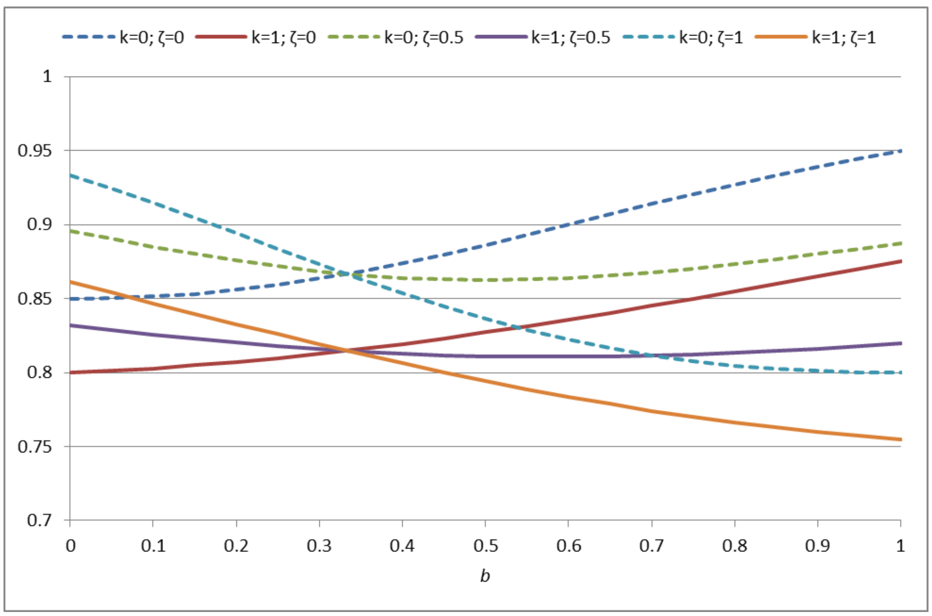
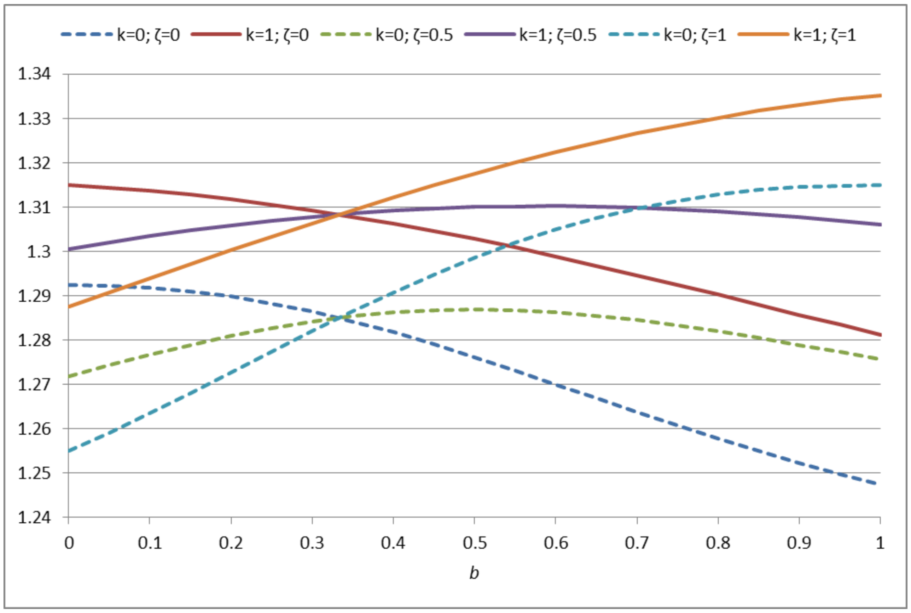


Figure 5 shows welfare in region 1. Welfare is always highest when offsets are allowed. This also holds when free allocations are very generous and there is no leakage ($b = 1$; $\zeta = 0$). Hence, some of the other conditions for banning offsets in Proposition 2ii) are not fulfilled. The explanation is that the optimal quota price is always below *c* when offsets are used (see above), and global emissions are always lower with offsets than without.[21]

Figure 5. Welfare in region 1 as a function of *b*



In appendix 2 we present results under alternative parameter assumptions. In particular, we consider the effects of larger complementarity between X and E (i.e., higher θ), and lower offset potential (i.e., lower σ). We also reduce the size of φ, which is likely less than one.

As long as regions 1 and 2 are equally large, banning offsets is not optimal as long as φ is not too small and θ close to its upper limit. However, if we scale down region 2 to e.g. one third of region 1, and set $\varphi = 0.5$ and $\theta = 0.35$ (with $\varphi = 0.5$, upper limit for θ is 0.41), banning offsets is optimal if b is between one third and one half of the BaU emission intensity *given no leakage*. If b is higher, the optimal quota price without offsets is lower (because it stimulates output), and offsets will again be used. If there is leakage, banning offsets is not optimal unless θ is even closer to its upper limit, meaning that reducing E without reducing X is very difficult. A higher quota price will then to lead to greater reduction in output even if b is large. If instead the size of region 2 is only one tenth of region 1, banning offsets is optimal with less restrictive assumptions about θ and φ, and moderate leakage.

To sum up, our numerical simulations suggest that unless the offset potential is very small, it is optimal for the policy bloc to allow offsets even when the amount of free quota allocations is very generous compared to the leakage exposure. Only if emissions and output are very closely

[21] Note that the optimal b seems to be very close to the size of $(-\zeta)$, i.e., the allocation factor should reflect the size of leakage exposure, almost one-to-one.

linked, offset potential is relatively small, and leakage is small, banning offsets may be optimal for certain levels of output-based allocation.

5. Optimal Offset Policy with Quota Price Discrimination

We now revert to the case of emissions-based free allocations in Section 3. We assume that the quota market is not necessarily unified, and that the quota price can be set at different levels in the policy bloc and the fringe. One such case is where all trading of offsets is done by a government agency representing all policy countries, and the offset price can be set lower in the fringe. Strand (2013) then shows, in a similar model except only with no free quota allocations, that such an agency would operate as a monopsonist in the offset market, and set the offset price below the quota price in the policy bloc.

We here model a similar case, assuming updated quota allocations. Assume that the government representing the policy bloc needs to set a single price for purchasing offsets from the fringe. Note that this is still not fully optimal for this bloc: price discrimination in the offset market, whereby quotas are purchased "cheaper" from "fringe" firms with lower abatement costs, is thereby precluded. Such price discrimination probably takes place at least to some degree in the CDM market today. Our assumption could reflect serious asymmetric information about abatement costs, where low-cost firms will in general have incentives to mimic as high-cost. If such mimicking is successful, no type revelation will take place in equilibrium.[22]

The problem is now formally similar to that in Section 3 except that we have two quota prices, q_1 for the policy bloc, and q_2 for the offset market (in the fringe), instead of just a single price, q. Define now the policy bloc's objective function in similar fashion to (9a), by

$$B(q_1,q_2,k)=R_1(E_1(q_1))-cE_1(q_1)-(c-q_2)k\hat{E}_2(q_2)-[c(1-k)+q_2k]E_{20}. \tag{16}$$

(16) is maximized with respect to q_1, q_2 and k, yielding the first-order conditions:

$$\frac{\partial B(q_1,q_2,k)}{\partial q_1}=(R_1'-c)\frac{\partial E_1}{\partial q_1}=0 \tag{17}$$

$$\frac{\partial B}{\partial q_2}=-k(c-q_2)\frac{\partial \hat{E}_2}{\partial q_2}-k(E_{20}-\hat{E}_2(q_2))=0 \tag{18}$$

$$\frac{\partial B(q_1,q_2,k)}{\partial k}=(c-q_2)(E_{20}-\hat{E}_2(q_2))\geq 0. \tag{19}$$

[22] For an introduction to the game theoretic basis for such an equilibrium, see e.g. Gibbons (1992), chapter 3.

From (17), $R_1' = c$: mitigation is optimal in the policy bloc. This implies that q_1, given from (13), exceeds marginal damage cost from emissions as long as $a > 0$.

Consider next q_2. From (18) we find

(20) $$q_2 = c - \frac{(E_{20} - \hat{E}_2)}{-\partial \hat{E}_2 / \partial q_2} < c \, .$$

The optimal quota price, at which quotas are purchased from the offset market, is now below marginal damage cost of emissions. This is a standard monopsony solution where a unified policy bloc government (acting as a monopsonist in the offset market) trades off an environmental efficiency aspect ("too little" mitigation) against a fiscal cost aspect (expenditure on the purchase of offset quotas). It results in too little mitigation through offsets, but the gain for the policy bloc is that quota expenditures are reduced.

The main difference between mitigation policy towards domestic firms versus firms in fringe countries is that government payments to domestic firms, but not to foreign firms, are part of net social welfare for the policy bloc. The government thereby wishes to limit the latter payments, doing so in (constrained) optimal, monopsonistic, fashion.

The quota price facing policy bloc countries, q_1, is here *greater than* marginal damage cost; while the quota price facing firms in fringe countries, q_2, is *lower than* this marginal damage. The difference between the internal and external quota prices, q_1 and q_2, can be substantial. In our model, quota prices can differ only because private actors are not allowed to trade in the offset markets. Admittedly, it is unrealistic to assume that governments-sponsored purchases of offsets will lead to such strong discrimination in disfavor of offset sellers; and that offsets are always purchased at a given price. Our analysis must be viewed as a first cut at this issue, so that further research is warranted.[23]

Using $c - q_2 > 0$, (19) must hold with inequality, so that $k = 1$ at the optimal solution. The reason is that the policy bloc, in implementing its desired volume of offsets at minimum cost, finds it preferable to reduce the offset price, rather than restrict offset purchases by setting $k < 1$.

We sum up these results as follows.

[23] Price differences exist today between the CDM market and the EU ETS, and within the CDM market, for a variety of reasons beyond our simple model. Among these are 1) transaction costs and other imperfections in the offset market; 2) uncertain delivery of effective offsets from the point of view of offset purchasers (not all offsets are actually achieved and credited); and 3) bilateral bargaining or monopsonistic power by buyers in the offset markets.

Proposition 3. *Consider a policy bloc with a domestic emissions trading system, free quota allocations based on firms' past-period emissions, and an offset market fully controlled by policy-bloc governments. Then:*

i) Within the policy bloc, the equilibrium quota price exceeds firms' marginal abatement costs which in turn equal marginal damage cost, so that abatement is efficient within the policy bloc.

ii) Offsets are always used.

iii) The equilibrium offset price is lower than marginal damage cost; thus abatement in the fringe is inefficiently low.

Proof: The results follow from the discussion above.

For the policy bloc countries, the solution in this section is preferable to a unified quota market. Efficiency loss still occurs due to the wedge between marginal damage cost and marginal mitigation cost in the fringe, leading to too little mitigation. On the other hand, the policy bloc now always finds it optimal to utilize the offset market, which is not necessarily the case under a unified market. Moreover, if offsets are used under a unified market, the amount of domestic mitigation is too low. Both types of solutions thus entail inefficiencies, which are quite different. Although price discrimination is preferred by the policy bloc, we cannot easily say which of the two solutions is preferable from a global efficiency perspective.

It is also ambiguous which solution is more favorable to fringe countries. The solution with different quota prices in the two regions is clearly more favorable to fringe countries than the solution with $k = 0$ in Section 3 above, i.e., when the offset market is not utilized at all by the policy bloc. Price discrimination can also be more favorable to fringe countries when the offset market is used in the case without price discrimination.[24]

The solution sketched in Section 3, with a unified quota price, is also easier to understand when viewed in the context of the current case. Given that the offset market is overwhelmingly important compared to the domestic mitigation market, as it may be under case b) in Section 3, the unified solution for q will be close to that from (23). The quota price will then be below the marginal damage cost of emissions, also for the domestic mitigation market. By contrast, when the domestic mitigation market is overwhelmingly important compared to the offset market, as under case a) in Section 3, the unified quota price is set by (13), and there will be no offset market.

[24] This depends among other things on whether the amount of offsets purchased is smaller or greater. This however needs further study, and we refer to the discussion in Strand (2013).

Can the current solution be implemented in a decentralized market where emitters in the policy bloc also trade with the offset market? Any such trading must be subject to a price difference $q_1 - q_2$ facing policy bloc actors versus fringe actors. Focusing on policy bloc actors, the price of all quotas facing these must be the same.

There are at least two ways of implementing such a solution. In the first, the government imposes an (excise) tax per quota purchased in the offset market, equal to $t_q = q_1 - q_2$.[25] Given such a tax, the government needs not impose any other restrictions on quota sales (all possible offsets ought to be realized given this tax).

The other solution, from Malik et al (1993), Mignone et al (2009), Castro and Michaelowa (2010), and discussed analytically by, among others, Bushnell (2011), Klemick (2012) and Strand (2013a), is to "discount" offsets by giving them less value to purchasers per offset ton CO_2. Such discounts can be justified by inherent problems in the offset markets (such as lack of additionality) not focused on here; see discussions in Montero (1999, 2000), Wear and Murray (2004), Gan and McCarl (2007), Wara (2008). Note also that a similar solution is implemented when the rate of "quota discounting" (reduction in value relative to domestic abatement), call it λ, equals $(q_1 - q_2)/q_1$. When buying one unit of offset from the fringe, a policy bloc firm is then credited with only q_2/q_1 units of offset.

One difference between the two solutions is that, in the first, the government would raise revenue $(q_1 - q_2)Q_2$, where Q_2 is the amount of offsets purchased from the fringe. In the second case, the government would raise no revenue. However, if the government auctions off λQ_2 quotas, we get identical outcomes with respect to emissions, costs and government revenues.[26] Both solutions are similar to the optimal price discrimination solution under a carbon tax, see Strand (2013a).

Note that giving the policy bloc the ability to discriminate between quota prices resolves some problems of inefficiency, but not all. There is still inefficiency due to the policy bloc not incorporating the fringe's utility in the objective function that is maximized; this will be discussed in section 6.

6. Global Welfare Considered

[25] We thank Ian Parry for suggesting this solution.

[26] Note that in the "discount" case, policy bloc emissions will be lower than in the "tax" case if the overall cap of the emissions trading system in the policy bloc is the same in the two cases. By increasing the cap by λQ_2 units, emissions will be the same both in the fringe and in the policy bloc. It is straightforward to see that the revenue from selling λQ_2 quotas is $(q_1 - q_2)Q_2$.

So far we have assumed that the objective function to be maximized is determined solely by the policy bloc, so that welfare in the fringe is ignored. We will now instead assume that welfare of the fringe is fully incorporated in the objective function being maximized. We return to the case of section 3, assuming a common quota price and free allocations according to past emissions. The relevant objective may now be formulated as

$$B(q,k) = R_1(E_1(q)) + (1-k)R_2(E_{20}) + kR_2(\hat{E}_2(q)) - c_T\left(E_1(q) + E_{20}(1-k) + k\hat{E}_2(q)\right) \tag{21}$$

where now c_T denotes the global climate externality (experienced by both the policy bloc and fringe), so that $c_T > c$. Note that, on the right-hand side of (21), two terms representing the fringe enter with weights 1-*k* (the share of firms not allowed to sell offsets) and *k* (the share of firms that sell offsets). Also, as different from (9), the fringe's net revenue function is included in the objective. Maximizing (21) with respect to *q* and *k* now yields

$$\frac{\partial B(q,k)}{\partial q} = (R_1' - c_T)\frac{\partial E_1}{\partial q} + k[R_2'(\hat{E}_2) - c_T]\frac{\partial \hat{E}_2}{\partial q} = 0 \tag{22}$$

$$\frac{\partial B(q,k)}{\partial k} = R_2(\hat{E}_2) - R_2(E_{20}) + c_T(E_{20} - \hat{E}_2)\ . \tag{23}$$

We still have $R_1' = (1-a)q$, and $R_2'(E_2) = q$. Since partial derivatives of E_1 and E_2 with respect to *q* are both negative, given $k > 0$, (22) must for any *a* where $0 < a < 1$, imply that $q > c_T$.

Consider next determination of *k*. Note that we here need to be careful in interpreting $R_2(E_2)$, since the offset payment $q(E_{20} - E_2)$ should not be included in this expression (since it is not part of the social surplus). Thus generally $R_2(E_{20}) - R_2(\hat{E}_2) > 0$. Moreover, as long as $R_2'' \leq 0$ (increasing marginal mitigation costs), $R_2(E_{20}) - R_2(\hat{E}_2) \leq q(E_{20} - \hat{E}_2)$. Given quadratic mitigation costs (cf sections 3-4), (23) becomes $\partial B(q,k)/\partial k = (c_T - 0.5q)(E_{20} - \hat{E}_2)$. Hence, if the quota price is close to marginal damage costs, the expression is positive, and $k = 1$ is optimal. This holds if *a* is not too high ($a \leq 0.5$ is a sufficient condition), or the offset potential is not too small. On the other hand, if *a* is large and the offset potential small, it may be optimal with a quota price more than the double of c_T. If so, offsets should be banned, as it is more important to set the incentives correctly in the policy bloc.[27] Simulations indicate that it is never optimal with an interior solution for *k* in this case either. It can here be shown that $\partial B(q,k)/\partial k = 0$ for $0 < k < 1$ corresponds to a *local minimum* point, meaning that both $k = 0$ and $k = 1$ are better solutions.

---

[27] Given the numerical specification in section 3, it is never optimal to ban offsets if the two regions are identical. On the other hand, if for instance the size of region 2 is one fifth of region 1 ($\mu_{2C} = 5$), it is optimal to ban offsets if $a > 0.6$.

We can formulate the following Proposition:

Proposition 4: *Consider a policy bloc maximizing global welfare, but is otherwise subject to the same conditions as under Proposition 1. Then*

i) The optimal quota price always exceeds global marginal damage cost from emissions

ii) It may be optimal to ban offsets if allocation is very generous, and the offset potential is limited.

Here, the optimal quota price implies an optimal tradeoff between an inefficiently low (suboptimal) mitigation activity in the policy bloc (corresponding to a suboptimal emissions price), and an excessive mitigation activity in the fringe (corresponding to an over-optimal emissions price). The over-optimal emissions price in the fringe explains why offsets may be banned even when maximizing global welfare.

Overall, the quota price q is substantially higher here than in the case of $k = 1$ in subsection 3.1, for two reasons. First, in determining optimal mitigation one now considers the higher, global, carbon emissions cost c_T, instead of only c in the previous case. Secondly, the policy bloc is no longer constrained in its (now considerably higher) offset payments by any fiscal concern, as payments to the fringe are a pure transfer and not an expense.

Note that incorporating fringe welfare resolves some inefficiency problems, but not all. The problem that policy bloc and fringe firms choose different marginal abatement cost levels still persists, and follows from the requirement that the quota price be set at the same level in both regions. A fully efficient solution can be implemented only when quota price discrimination is allowed as in Section 5. It is easily verified that this results in an overall optimal (first-best) solution, where $q_2 = c_T$ for the fringe, and $q_1 = c_T/(1-a)$ for the policy bloc.

7. Conclusions

This paper studies (constrained) optimal climate policies of a bloc of countries concerned with enforcing a unified climate policy, using an emissions trading system with free quota allocations to its domestic firms, and an offset market where emissions reductions are purchased from a "fringe" of (non-policy) countries, as under the CDM. We consider two separate ways of organizing this market. In sections 2-5, only the policy bloc's utility enters the objective function of the policy maker, which is here a unified government of all countries in the policy bloc. Section 6 considers the corresponding global welfare-maximizing solution.

We also consider two alternative models for pricing, and trading, of emissions quotas. We first assume a unified market for emissions reductions in the policy bloc and fringe, allowing market participants to trade emissions quotas in both blocs. Secondly, all offsets are purchased directly from the fringe by a central unit in the policy-bloc countries, at a price below that charged to policy bloc emitters. A key feature of our analysis is that a large share of emission quotas are given away for free to participating firms, based on either their emissions, or output, in the preceding period.

We show that when the carbon market is organized in this way, it leads to inefficiencies. One reason for inefficiency is that free emissions rights tend to raise the preferred quota price above marginal mitigation cost of firms in the policy bloc, but not in the "fringe". When then imposing the requirement of a single quota price, the marginal abatement cost is higher in the fringe than in the policy bloc. Moreover, purchasing offsets from the fringe is a net fiscal outlay and thus expensive for the policy bloc. When the fringe dominates the overall quota market, and/or the effect of free allocations on the quota price is not too great, the policy bloc prefers to set a low quota price. Offsets are then inexpensive, and there is too little abatement within the policy bloc. When the fringe becomes less significant, and/or there is a large effect of free allocations on the domestic quota price, policy-bloc countries instead prefer to ban the offset market altogether if free quotas are given based on past emissions. Offsets are then too expensive to be worthwhile. If quotas are instead allocated in proportion to past output, and not too generously relative to leakage exposure, it is never optimal to completely ban the use of offsets.

In the second model, with full government control of offset purchases, the policy bloc acts as a monopsonist in limiting the number of offsets, and sets a lower quota price for offsets than that resulting in the domestic market. The inefficiency then takes the form of too little abatement in the fringe. The quota price in the policy bloc is then always higher than, and the offset price lower than, the marginal abatement cost in the policy bloc.

Section 6 considers maximization of global welfare, given a common quota price and free allocations tied to past emissions. It may then still be optimal to ban offsets if the offset potential is limited and allocation is generous. However, the quota price then always exceeds (global) marginal damage cost. Emissions are lower and the quota price higher than when the policy bloc's utility only is maximized, perhaps substantially so. There are two factors behind these differences. First, when global welfare is maximized, offset payments are not a net cost but instead a transfer from the policy bloc to the fringe; this makes offsets more attractive. Secondly, the global and not regional (for the policy bloc) carbon externality is considered in setting climate policy, making it stricter. Still, however, a first-best solution cannot result as long as there is a unified quota price for the policy bloc and fringe. This would require the quota price to be differentiated, as in section 5.

The analysis shows that providing free quota allocations to participating firms based on updating schemes is, quite generally, problematic for the functioning of offset markets, and can lead to various types of inefficiencies. When offsets are traded freely at a price identical to the emissions quota price in the policy bloc, the solution chosen by policy-bloc countries can then be to ban trading in the offset market entirely. Possible measures to reduce these problems are to eliminate or reduce the value of free allocations; make the updating rules less distortive; separate the domestic quota market and the offset market, thus not allowing free trading across these markets; or tax offsets making the policy bloc's optimal price discrimination solution implementable for the offset market.

Several extensions of this work can be visualized. One is to more explicitly consider non-competitive behavior among firms, and the processes by which entry and exit of firms are affected by features (including generosity) of the quota allocation market; see the related analyses by Rosendahl and Storrøsten (2011), and Anouliès (2013). Another is to more explicitly consider features of the offset markets that cause trouble here, including asymmetric information (adverse selection and moral hazard), e g in Bushnell (2011), and the related applications to REDD markets by Benthem and Kerr (2013) and Mason and Plantinga (2013); and basic reluctance of lower-income countries to engage in mitigation action, see Strand (2014). Additionality of GHG mitigation, which concerns both offsets and the basic policy bloc quota market (see, e. g., Fischer (2005), and Strand and Rosendahl (2012)), is then a key issue. Furthermore, a global welfare analysis is called for: this requires that the welfare effects of mitigation are evaluated based on a "global carbon cost" and not the cost only to the policy bloc region, as here. Regardless of the immediate apparent relevance of offset markets (which currently may appear moderate), such issues will be important for the foreseeable future.

References

Anouliès, L. (2013), Heterogeneous Firms in a Cap-and-Trade Program. Working Paper, Université de Paris Sud.

Benthem, A. V. and Kerr, S. 2013. Scale and Transfers in International Emissions Offset Programs. *Journal of Public Economics*, 107, 31-46.

Böhringer, C., Fischer, C. and Rosendahl, K.E. (2010): The Global Effects of Subglobal Climate Policies, *The B.E. Journal of Economic Analysis & Policy* 10 (2) (Symposium), Article 13.

Böhringer, C. and Lange, A. (2005), On the Design of Optimal Grandfathering Schemes for Emissions Allowances. *European Economic Review*, 49, 2041-2055.

Brechet, T., Meniere, Y. and Picard, P. M. (2011), The Clean Development Mechanism in a Globalized Carbon Market. Center for Research in Economic Analysis Working Paper, no 2011-12, University of Luxemburg.

Bushnell, J. (2011), Adverse Selection and Emissions Offsets. Working Paper, Department of Economics, Iowa State University.

Castro, P. and Michaelowa, A. (2010), The Impact of Discounting Emission Credits on the Competitiveness of Different CDM Host Countries. *Ecological Economics*, 70, 34-42.

Convery, F., De Perthuis, C. and Ellerman, D. (2008), The European Carbon Market in Action: Lessons from the First Trading Period. Interim Report, CEEPR Working Papers WP-2008-002, March 2008, MIT.

Fischer, C. (2005), Project-based Mechanisms for Emissions Reductions: Balancing Trade-offs with Baselines. *Energy Policy*, 33, 1807-1823.

Fowlie, M. (2010), Updating the Allocation of Greenhouse Gas Emissions Permits in a Federal Cap-and-Trade Program. NBER working paper no. 16307.

Gan, J. and McCarl, B. (2007), Measuring Transnational Leakage of Forest Conservation. *Ecological Economics*, 64, 423-432.

Germain, M., Magnus, A. and Steenberghe, V. van (2007), How to Design and Use the Clean Development Mechanism under the Kyoto Protocol? A Developing Country Perspective. *Environmental and Resource Economics*, 38, 13-30

Gibbons, Robert (1992): *A Primer on Game Theory*. New York: Prentice-Hall.

Goulder, L. H., Parry, I. W. H.,Williams, R. C. III and Burtraw, D. (1999): The cost-effectiveness of alternative instruments for environmental protection in a second-best setting, *Journal of Public Economics*, 72, 329-360.

Harstad, B. and Eskeland, G. (2010), Trading for the Future: Signaling in Permit Markets. *Journal of Public Economics*, 94, 749-760.

Hoel, M. O. (1996), Should a Carbon Tax be Differentiated Across Sectors?" *Journal of Public Economics*, 59, 17-32.

Hood, C. (2010): Reviewing Existing and Proposed Emissions Trading Systems, OECD/IEA, 2010. https://www.iea.org/publications/freepublications/publication/ets_paper2010.pdf

Klemick, H. (2012): What is the Optimal Offsets Discount under a Second-Best Can and Trade Policy? National Center for Environmental Economics Working Paper # 12-04. Washington, D. C.: U.S. Environmental Protection Agency.

Lecourt, S. (2012): The EU ETS Phase 3 Preliminary Amounts of Free Allowances. Information and Debate Series no 16, Climate Economics Chair, Paris-Dauphine University, June 2012.

Malik, A., Letson, D. and Cructhfield, S. (1993), Point/Nonpoint Source Trading of Pollution Abatement: Choosing the Right Trading Ratio. *American Journal of Agricultural Economics*, 75, 959-967.

Martin, R., M. M. Muûls, L.B. de Preux and U.J. Wagner (2012): Industry Compensation Under The Risk of Relocation: A Firm-level Analysis Of The EU Emissions Trading Scheme (April 3, 2012). Available at SSRN: http://ssrn.com/abstract=2033683.

Mason, C. F. and Plantinga, A. J. 2013. The Additionality Problem with Offsets: Optimal Contracts for Carbon Sequestration in Forests. *Journal of Environmental Economics and Management*, 66, 1-14.

Mignone, B., Hurteau, M., Chen Y. and Sohngen, B. (2009), Carbon Offsets, Reversal Risk and U.S. Climate Policy. *Carbon Balance and Management*, 4 no 3.

Montero, J.-P. (1999), Voluntary Compliance with Market-Based Environmental Policy: Evidcned from the U.S. Acid Rain Program. *Journal of Political Economy*, 107, 998-1033.

Montero, J.-P. (2000), Optimal Design of a Phase-In Emissions Trading Program. *Journal of Public Economics*, 75, 273-291.

Rosendahl, K. E. (2008), Incentives and prices in an emissions trading scheme with updating, *Journal of Environmental Economics and Management*, 56, 69-82.

Rosendahl, K.E. and Storrøsten, H. (2011): Emissions trading with updated allocation: Effects on entry/exit and distribution, *Environmental and Resource Economics* 49, 243-261.

Rosendahl, K. E. and Strand, J. (2011), Leakage from the Clean Development Mechanism. *The Energy Journal*, 32 no 4, 2011, 27-50.

Strand, J. (2013), Strategic Climate Policy with Offsets and Incomplete Abatement: Carbon Taxes Versus Cap-and-Trade. *Journal of Environmental Economics and Management*, 66, 202-218.

Strand, J. (2014), Mitigation Incentives with Future Climate Finance and Treaty Options. Unpublished, Development Research Group, World Bank.

Strand, J. and Rosendahl, K. E. (2012), Global Emissions Effects of CDM Projects with Relative Baselines. *Resource and Energy Economics*, 34, 533-548.

Wara, N, (2008), Measuring the Clean Development Mechanism's Performance and Potential. *UCLA Law Review*, 55, 1759-1803.

Wear, D. and Murray, B. (2004), Federal Timber Restrictions, Interregional Spillovers, and the Impact on U.S. Softwoord Markets. *Journal of Environmental Economics and Management*, 47, 307-330.

Wirl, F., Huber, C. and Walker, I. O. (1998), Joint Implementation: Strategic Reactions and Possible Remedies. *Environmental and Resource Economics*, 12, 203-224.

Appendix 1: Quadratic Mitigation Costs: Some More Specific Results Under Emissions-Based Allocations

In this appendix we assume quadratic mitigation costs, and derive some more specific results for the model in Sections 2-3. Assume, in particular, $R_j(E_j) = \mu_{jA} + \mu_{jB}E_j - (\mu_{jC}/2)(E_j)^2$, with μ_{jA}, μ_{jB}, $\mu_{jC} > 0$ (j = 1,2). To simplify notation, we assume that the two regions (the policy bloc, and the fringe) are identical except for their sizes, so that $\mu_{1A} = \mu_{2A} = \mu_A$, $\mu_{1B} = \mu_{2B} = \mu_B$, $\mu_{1C} = \mu_C/h$ and $\mu_{2C} = \mu_C/(1-h)$, with $0 < h < 1$. All results shown here also apply in the general case.

First, we have

(A1)-(A2) $$R_1'(E_1) = \mu_B - \mu_C E_1/h,\ R_2'(E_2) = \mu_B - \mu_C E_2/(1-h),\ E_{10} = h\mu_B/\mu_C$$

and

(A3) $$E_{20} = (1-h)\mu_B/\mu_C.$$

Equation (3b) then gives:

(A4) $$E_1(a,q) = \frac{\mu_B - (1-a)q}{\mu_C} h$$

and hence:

(A5) $$\frac{\partial E_1(a,q)}{\partial q} = -\frac{1-a}{\mu_C} h$$

whereas

(A6) $$E_{20} - \hat{E}_2(q) = \frac{q}{\mu_C}(1-h)$$

and hence:

(A7) $$\frac{d\hat{E}_2}{dq} = \frac{\partial \hat{E}_2(q)}{\partial q} = -\frac{1}{\mu_C}(1-h)$$

Equation (11b) now gives:

(A8) $$-aq\frac{-(1-a)}{\mu_C}h = \frac{q}{\mu_C}h + (c-q)\left(\frac{-(1-a)}{\mu_C}h - \frac{1}{\mu_C}(1-h)\right) = \frac{q}{\mu_C}h + (c-q)\frac{1}{\mu_C}(ah-1),$$

leading to:

(A9) $$q = \frac{1 - ah}{a^2 h - 2ah - h + 2} c$$

(in the special case with h = ½, this simplifies to $q = \frac{2 - a}{a^2 - 2a + 3} c$)

We can now examine the effects on q of increasing a:

(A10) $$\frac{dq}{da} = \frac{h\left(h + ha^2 - 2a\right)}{\left(a^2 h - 2ah - h + 2\right)^2} c$$

We see that this expression is positive for a sufficiently close to zero, but negative for a sufficiently close to one. Thus, the optimal quota price will increase when updating is (weakly) introduced, but may start to decline when a becomes higher (unless it has become optimal to switch to k = 0).

What about the effects on E_j of increasing a? $\hat{E}_2$ obviously follows the opposite direction of q. For E_1 the outcome is less clear as long as q increases in a, as higher a leads to higher emissions (for a given q).

We total differentiate the expression for $E_1(a,q)$ above, using the expression for q:

(A11)
$$\begin{aligned}
\frac{dE_1}{da} &= \frac{\partial E_1(a,q)}{\partial a} + \frac{\partial E_1(a,q)}{\partial q}\frac{dq}{da} = \frac{q}{\mu_C}h - \frac{1-a}{\mu_C}\frac{h + ha^2 - 2a}{\left(ha^2 - 2ha - h + 2\right)^2} hc \\
&= \frac{h}{\mu_C}\left[\frac{1 - ah}{ha^2 - 2ha - h + 2}c - \left(1 - a\right)\frac{h + ha^2 - 2a}{\left(ha^2 - 2ha - h + 2\right)^2}c\right] \\
&= \frac{hc\left[ha^2 - 2ha - h + 2 - h^2a^3 + 2h^2a^2 + h^2a - 2ha - h^2 + h^2a - h^2a^2 + h^2a^3 + 2ha - 2ha^2\right]}{\mu_C\left(ha^2 - 2ha - h + 2\right)^2} \\
&= \frac{\left(1 - h\right)\left(2 + h - ha^2 - 2ha\right)}{\mu_C\left(2 - h + ha^2 - 2ha\right)^2} hc > 0.
\end{aligned}$$

Thus, domestic emissions will always increase when a is increased (as long as k = 1). To find the effect on global emissions, we differentiate with respect to A, and find:

$$\frac{dE}{da} = \frac{dE_1}{da} + \frac{d\hat{E}_2}{dq}\frac{dq}{da} = \frac{(1-h)\left(2+h-ha^2-2ha\right)}{\mu_C\left(2-h+ha^2-2ha\right)^2}hc - \frac{1-h}{\mu_C}\frac{h\left(h+ha^2-2a\right)}{\left(2-h+ha^2-2ha\right)^2}c$$

$$\text{(A12)} \quad = \frac{(1-h)hc}{\mu_C\left(2-h+ha^2-2ha\right)^2}\left[2+h-ha^2-2ha-h-ha^2+2a\right]$$

$$= \frac{2(1-h)hc}{\mu_C\left(2-h+ha^2-2ha\right)^2}\left[1+a-ha^2-ha\right] > 0.$$

Hence, global emissions will unambiguously increase when *a* is increased as long as offsets are used. We can also show, given $a = 0$, that global emissions are lower if offsets are used than if they are not. Updating will then increase global emissions, regardless of the optimality of offsets.

Furthermore, we can show that welfare is decreasing in the level of *a* as long as offsets are used ($k = 1$), cf. equation (9):

$$\frac{dB}{da} = \left[\mu_B - \frac{\mu_C}{h}E_1 - c\right]\frac{dE_1}{da} - \left[c-q\right]\frac{d\hat{E}_2}{da} - \left[E_{20} - \hat{E}_2\right]\frac{dq}{da}$$

$$\text{(A13)} \quad = \left[\mu_B - \frac{\mu_C}{h}E_1 - c\right]\frac{dE_1}{da} - \left[(c-q)\frac{d\hat{E}_2}{dq} + \frac{q(1-h)}{\mu_C}\right]\frac{dq}{da}$$

$$= \left[(1-a)q - c\right]\frac{dE_1}{da} + \left[\frac{(1-h)(c-2q)}{\mu_C}\right]\frac{dq}{da}$$

The first term in the last line of (A13) is negative due to the 1.o.c. in (3b). The bracket in the second term is also negative, as $q > 2c$ (cf. the expression of *q* above). Thus, since *q* increases in *a* for sufficiently low levels of *a*, we know that *B* must decrease when *a* is increased starting at zero, and as long as dq/da is positive. Furthermore, consider a level of *a* where dq/da is negative. Assume that *a* is increased from a_1 to a_2, so that $q(a_1) > q(a_2)$. Consider the suboptimal pair a_1 and $q(a_2)$. We know that $B(a_1,q(a_1))$ is higher than $B(a_1,q(a_2))$, since $q(a_1)$ is the optimal quota price given a_1. What about the comparison between $B(a_1,q(a_2))$ and $B(a_2,q(a_2))$? Obviously, emissions in region 2 are the same in these two cases, and thus the two last terms in (10a) are identical. Moreover, emissions in region 1 are lowest when *a* is lowest (given the same *q*). Since $R_1'(E_1) < c$ whenever offsets are used, *B* must be highest when E_1 is lowest. Thus, $B(a_1,q(a_2)) > B(a_2,q(a_2))$. But then we have shown that *B* is decreasing in *a* also when *q* is decreasing in *a*. A higher level of *a* will therefore unambiguously reduce welfare whenever offsets are used.

Finally, we can show that it is optimal to switch to no offsets exactly when global emissions are the same with and without offsets. To see this, we first calculate the level of *a* where global emissions are identical with and without offsets. Then we must have:

(A14)

$$\begin{aligned}
&E_1^{k=1} - E_1^{k=0} = E_{20} - \hat{E}_2 = \frac{q}{\mu_C}(1-h) \\
&\frac{\mu_B - (1-a)q}{\mu_C} h - \frac{\mu_B - c}{\mu_C} h = \frac{q}{\mu_C}(1-h) \\
&q\left[1 - h + (1-a)h\right] = ch \\
&\frac{1-ah}{a^2h - 2ah - h + 2} c\left[1 - h + (1-a)h\right] = ch \\
&1 - 2ha + h^2a^2 = h^2a^2 - 2h^2a - h^2 + 2h \\
&a = \frac{1-h}{2h}
\end{aligned}$$

We denote this level of a as $\hat{a}$. It follows straightforwardly that:

(A15)
$$q(a) = \frac{2h}{h+1} c$$

We next calculate the difference in welfare with and without offsets when global emissions are the*e* same:

$$\begin{aligned}
&B^{k=1} - B^{k=0} = R_1(E_1^{k=1}) - R_1(E_1^{k=0}) - q\left(E_{20} - \hat{E}_2\right) \\
&= \mu_B\left(E_1^{k=1} - E_1^{k=0}\right) - \frac{\mu_C}{2h}\left(\left(E_1^{k=1}\right)^2 - \left(E_1^{k=0}\right)^2\right) - q\frac{q}{\mu_C}(1-h) \\
&= \left(\mu_B - q\right)\frac{q}{\mu_C}(1-h) - \\
&\frac{\mu_C}{2h}\left[\frac{\mu_B{}^2\mu_C{}^2h^2 + c^2\mu_C{}^2h^2 + q^2\mu_C{}^2(1-h)^2 - 2c\mu_B\mu_C{}^2h^2 + 2q\mu_B\mu_C{}^2h(1-h) - 2cq\mu_C{}^2h(1-h)}{\mu_C{}^4}\right. \\
&\left. - \frac{\mu_B{}^2h^2 - 2c\mu_Bh^2 + c^2h^2}{\mu_C{}^2}\right] \\
&= \frac{1}{2h\mu_C}\left[2\mu_Bhq - 2\mu_Bh^2q - 2hq^2 + 2h^2q^2 - \mu_B{}^2h^2 - c^2h^2 - q^2(1-h)^2\right. \\
&\left. + 2c\mu_Bh^2 - 2q\mu_Bh(1-h) + 2cqh(1-h) + \mu_B{}^2h^2 - 2c\mu_Bh^2 + c^2h^2\right]
\end{aligned}$$

(A16)
$$= \frac{q\left(-q + h^2q + 2ch(1-h)\right)}{2h\mu_C} = \frac{q\left((-1+h^2)\frac{2h}{h+1}c + 2ch(1-h)\right)}{2h\mu_C} = 0$$

Thus, we have shown that, when global emissions are the same with and without offsets, welfare is also the same with and without offsets. As welfare is monotonically decreasing in the level of *a*, and global emissions are monotonically increasing in the level of *a*, it must be optimal to switch to no offsets exactly when global emissions are the same in the two cases.

Appendix 2: Output-Based Allocation and Offsets – Numerical Illustration

We consider the following revenue function for region 1, building on the revenue function $R(E)$ in the Appendix, but now a function of both E and X:

(A17) $$R(E_1, X_1) = 1 + (1-\theta)\left[\varphi(E_1 - E_1^{\,2}/2) + X_1 - X_1^{\,2}/2\right] + \theta E_1 X_1$$

$\varphi > 0$ determines the relative importance of emissions in the revenue function, while $\theta \geq 0$ determines to what degree E and X are complements. The lower is θ, the easier emissions can be reduced without affecting output. Note that we must have $\varphi(1-\theta)^2 - \theta^2 > 0$ to get a finite solution (cf. the expressions for X and E below).

We assume that region 2 has the same revenue function as region 1, except for the size of region 2 relative to region 1, represented by the parameter σ.

Under BaU, it is straightforward to show that:

(A18) $$E_1 = 1 + \frac{\theta}{\varphi(1-\theta)^2 - \theta^2}$$

(A19) $$X_1 = \frac{\varphi(1-\theta)}{\varphi(1-\theta)^2 - \theta^2}$$

It follows that the emission intensity under BaU is $(E_1 / X_1)^{BaU} = (\varphi(1-\theta) + \theta)) / \varphi$, and we assume that the OBA parameter $b \leq (E_1 / X_1)^{BaU}$

The 1.o.c. for a given q and b are (for region 1):

(A20) $$\theta X_1 + (1-\theta)\varphi(1-E_1) = q$$

(A21) $$\theta E_1 + (1-\theta)(1-X_1) = -bq$$

This gives:

(A22) $$E_1 = E_1(q) = 1 + \frac{\theta + \theta bq - (1-\theta)q}{\varphi(1-\theta)^2 - \theta^2}$$

(A23) $$X_1 = X_1(q) = \frac{\varphi(1-\theta) - \theta q + (1-\theta)\varphi b q}{\varphi(1-\theta)^2 - \theta^2}$$

We then get:

(A24) $$\partial E_1 / \partial q = \frac{\theta b - (1-\theta)}{\varphi(1-\theta)^2 - \theta^2}$$

(A25) $$\partial X_1 / \partial q = \frac{-\theta + (1-\theta)\varphi b}{\varphi(1-\theta)^2 - \theta^2}$$

Emissions decline in the quota price as long as $b < (\varphi(1-\theta)+\theta))/\varphi$, i.e., the BaU emission intensity. Note that output declines in the quota price as long as $b < \theta/(\varphi(1-\theta))$. However, if θ is very small (and φ is not small), output increases in the quota price if *b* is positive. The intuition is that when θ is very small, output is only marginally affected by abatement measures, and hence the positive effect through higher implicit subsidy (when $b > 0$) dominates the negative effect through reduced emissions (caused by a higher *q*).

Let ζ denote the change in emissions in region 2 due to an increase in production in region 1 (leakage), i.e., $\zeta = \partial \hat{E}_2 / \partial X_1 = \partial E_{20} / \partial X_1$. To simplify, we assume that ζ is exogenous and in particular independent of *q*.

For region 2 and $k = 1$ we get (by setting $b = 0$ into the corresponding expressions for region 1, and adding σ):

(A26) $$\partial \hat{E}_2 / \partial q = \sigma \frac{-(1-\theta)}{\varphi(1-\theta)^2 - \theta^2} < 0$$

and:

(A27) $$E_{20} - \hat{E}_2 = \sigma \frac{(1-\theta)q}{\varphi(1-\theta)^2 - \theta^2}$$

From equation (15) we can then derive the optimal quota price as a function of φ, θ, ζ, *b* and *c* (for *k*=1)

(A28) $$q = \frac{\theta b - (1+\sigma)(1-\theta) + \left((1-\theta)\varphi b - \theta\right)\zeta}{2\theta b - (1+2\sigma)(1-\theta) - (1-\theta)\varphi b^2} c$$

Without offsets (*k*=0), the optimal quota price becomes:

(A29)
$$q = \frac{\theta b - (1-\theta) + \left((1-\theta)\varphi b - \theta\right)\zeta}{2\theta b - (1-\theta) - (1-\theta)\varphi b^2} c$$

We can next calculate emissions and output in region 1 from the expressions $E_1(q)$ and $X_1(q)$ above. We can also calculate emissions in region 2 – here we must add the leakage effects given by ζ and the change in X_1 (vis-à-vis BaU). Then we can calculate revenue $R_1(E,X)$, and finally benefits for region 1 from equation (9).

We first consider the "benchmark" case where $\sigma = 1$ (equally large regions), $\varphi = 1$ (which implies $\theta < 0.5$ and $b \leq 1$), $\theta = 0.25$, and $\zeta = 0$, 0.5 or 1 (i.e., 0%, 50% or 100% leakage). We always calibrate c to equal the emissions price that leads to an emissions reduction of 30% (vis-à-vis BaU).[28] In the main text we showed the effects on the quota price, global emissions and welfare in region 1 for different levels of b.

It is more reasonable to assume that $\varphi < 1$, say $\varphi = 0.5$. This implies that $\theta < 0.41$. Assume e.g. that $\theta = 0.35$, which means that the complementarity between E and X is quite strong (but not Leontief structure). Emissions intensity under BaU is now 1.35. Moreover, assume that the size of region 2 is one third of region 1, i.e., $\sigma = 1/3$. We consider no leakage exposure, i.e., $\zeta = 0$, as non-negligible leakage makes offsets optimal for any b with these parameters.

Figure A1 shows the effects on global emissions, the quota price and welfare in region 1. We see that the quota price under $k = 0$ is now above one for all b. That is, the second-best quota price now exceeds the marginal damage costs c as output to a lesser degree is stimulated through b when q increases. Further, we see that global emissions are almost the same with and without offsets, and are slightly lower without offsets for medium levels of b. Moreover, welfare in region 1 is higher with offsets for small and high levels of b, but slightly lower when b is between 0.45 and 0.7, or between one third and one half of the BaU emissions intensity.

[28] This is simply to make sure that the interpretation of c is not changed when we change the parameters in the revenue function.

Figure A1. Effects on global emissions (*E*), the quota price (*q*) and welfare in region 1 (*B*1) as a function of *b*