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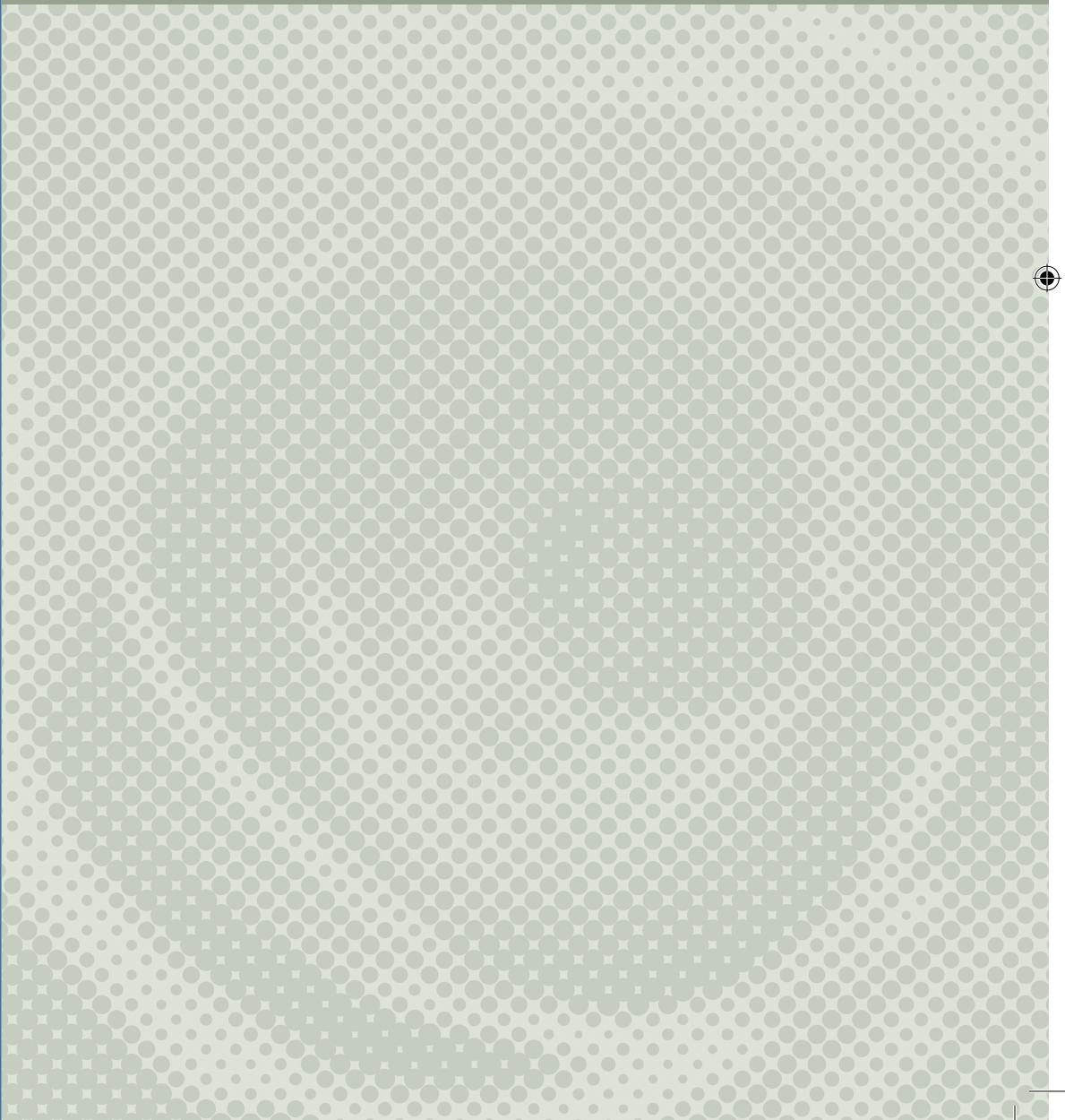
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# TREATMENT OF UNDESIRABLE OUTPUTS IN PRODUCTION ANALYSIS: DESIRABLE MODELING STRATEGIES AND APPLICATIONS

BEHANDLING AV UØNSKEDE BIPRODUKTER I PRODUKSJONSANALYSE: ØNSKEDE  
MODELLERINGSSTRATEGIER OG ANVENDELSER

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Behandling av uønskede biprodukter i produksjonsanalyse:  
Ønskede modelleringssstrategier og anvendelser

Philosophiae Doctor (PhD) Thesis

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## Introduction

This thesis assesses the potential for extending conventional economic production models to accommodate for joint production of desirable and undesirable outputs. Model frameworks that allow treatment of undesirable outputs have recently received much attention in the production analysis literature. Once developed, they should among other things allow for assessments of environmental performance and marginal abatement costs. Being able to undertake such analysis will make the models a more desired tool for public agencies and polluting firms alike.

Most currently popular model approaches that seek to capture joint production of goods and bads include undesirable pollutants either as inputs or outputs in the traditional neoclassical production model. This procedure may be suitable when an output has the property of being both a good and a bad. An example of this is farmed salmon which is a desirable output when it is slaughtered and brought to the marketplace, but becomes an undesirable output if it escapes from the fish farm. Most of the undesirable outputs studied in the literature do not have this property, but are byproducts generated jointly with intended outputs. In this case, the popular extended production models are likely to neglect essential dynamics of polluting technologies. In the articles that constitute the core of this thesis I argue that they conceal both the ways in which byproducts come to existence as well as different producer strategies for reducing them. The models are therefore less likely to capture the least cost way of emission reductions, i.e. marginal abatement costs. Improper treatment of undesirable outputs will also render these models less suitable for other desirable applications.

The critique of currently popular models and the alternative framework for modeling joint production of desirable and undesirable outputs presented in this thesis is highly influenced by a few recent publications, namely Coelli et al. (2007), Førsund (2009), and Murty and Russell (2010). A similarity between the papers by Coelli et al. and Førsund is their emphasis on the materials balance condition as the pollution generating mechanism. The importance of materials balance for the generation of byproducts was treated in a seminal paper by Ayres and Kneese (1969), and has recently received much attention in the environmental economics literature. Coelli et al. argue that the currently most applied frameworks for modeling joint production of desirable and undesirable outputs are inconsistent with the materials balance condition. I find their conclusion to be unsatisfactory since it neglects an important aspect of polluting technologies, namely the firms' ability to involve in end-of-pipe abatement activities. I reconsider their critique of one of the most popular pollution models, the model of Färe et al. (1989; 2005), in the first paper of this thesis. I show that Coelli et al.'s conclusion, that technical inefficiency in Färe et al.'s model is physically inconsistent, is only correct in the absence of end-of-pipe abatement. Contrary to Coelli et al., I show that this result is caused by contradiction of the axiom of weak disposability of desirable and undesirable outputs that is crucial to the analysis of Färe et al.. I further show that the implicit treatment of abatement in Färe et al.'s model may lead to overestimation of firms' ability to reduce undesirable outputs. This result indicates why modeling of polluting firms should explicitly take into account the choices they have for reducing their emissions. Such choices include technical change, input substitution, output reductions, and end-of-pipe abatement or waste recycling.

Førsund (2009) and Murty and Russell (2010) propose a production model consisting of multiple production relations for modeling joint production of desirable and undesirable outputs. The model

structure allows for the existence of a unique output bundle for each input bundle. This intends to capture the impossibility to substitute undesirable outputs with desirable outputs for a given input vector. In other words, undesirable outputs are treated as unavoidable byproducts. Articles two to four in this thesis merge the model structure proposed by Førsund (2009) and Murty and Russell (2010) with the representation of the materials balance condition from Coelli et al. (2007). The contribution of these articles is to simplify the analysis of Førsund (2009) and Murty and Russell (2010), and to extend it to empirical applications. End-of-pipe abatement is treated as an output in Førsund and Murty and Russell's papers, and their analysis requests explicit information on the abatement output and on inputs allocated to this activity. Such information is often unavailable to economic researchers. To overcome this problem, I treat abatement implicitly by recognizing that polluting firms will involve in abatement activities when the gain from involvement exceeds its costs. The result is a simple model framework that takes the materials balance condition into account. The framework is applied to illustrate the importance of accounting for environmental regulations in efficiency measurements (article two), for deriving marginal abatement costs (article three), and for estimating secondary benefits of environmental regulations (article four).

I believe that the theory and applications presented in this thesis offer new insight to economists interested in pollution modeling. The theory resembles that of expenditure constrained profit maximization which makes it easy accessible to most economists, as well as allowing for simple empirical treatment. The framework further offers applications to new areas of pollution modeling, including treatment on costs and benefits of end-of-pipe abatement and secondary benefits of emission reductions. That said, economic pollution modeling is in several aspects uncharted ground. More work needs to be done to establish the usefulness of the modeling proposed in this thesis. Only future efforts can determine how useful new modeling approaches turn out to be.

The rest of the introduction is structured as follows. Section 1 gives a brief overview of the conventional set theoretical production model. This section presents the fundamental axioms of production analysis on which the following analysis rests. Section 2 describes previous attempts to extend the conventional production model to account for undesirable outputs. The analysis discusses the practice of treating undesirable outputs as inputs and weakly disposable outputs. Section 3 presents the materials balance condition, its importance to environmental economics, and its recent recognition as a standard that all models treating undesirable outputs should satisfy. Section 4 follows up on section 3, by utilizing the materials balance principle for assessing the models presented in section 2. It is then suggested that polluting technologies are better captured by production models that consist of multiple production relations. These models are presented in section 5. Section 6 reviews popular applications for polluting technologies while section 7 suggests some new areas of pollution modeling. Section 8 concludes.

## 1. THE CONVENTIONAL PRODUCTION MODEL

This section follows Chambers (1988) and Färe and Primont (1995). Let  $x \in \mathfrak{X}_+^N$  denote a vector of inputs and  $y \in \mathfrak{X}_+^M$  denote a vector of desirable or intended outputs. The technology set can then be defined as equation 1:

$$T1 = \{(x, y) : x \text{ can produce } y\} \quad (1)$$

There exist two other useful and equivalent representations of the technology. The first is the output set,  $P(x)$ , which defines feasible output combinations for each input vector,  $x$ .

$$P(x) = \{y : (x, y) \in T1\} \quad (2)$$

Furthermore, the technology can be represented by the input set,  $L(y)$ , which defines required inputs for each output vector,  $y$ .

$$L(y) = \{x : (x, y) \in T1\} \quad (3)$$

The following axioms impose structure on the production model:

- (i)  $T1$  is nonempty
- (ii)  $T1$  is closed
- (iii) For every finite  $x$ ,  $T1$  is bounded from above ( $P(x)$  is bounded)
- (iv) Inactivity is feasible, i.e.  $(0, 0) \in T1$
- (v) There is no free lunch, i.e. if  $(x, y) \in T1$  and  $x=0$ , then  $y=0$
- (vi) Inputs and outputs are freely disposable,  
i.e. if  $(x, y) \in T1$  and  $(x', -y') \geq (x, -y)$ , then  $(x', y') \in T1$
- (vii)  $T1$  is a convex set

The first axiom secures the existence of at least one feasible input-output combination. The technology set includes a boundary that contains “no holes” by axiom (ii). Axiom (iii) implies that only finite amounts of outputs can be produced by each finite input vector. Together, axiom (ii) and (iii) secure the existence of a “maximal feasible” output vector for each finite input vector. In the first article of this thesis, I argue that the implicit treatment of end-of-pipe abatement in Färe et al.’s (1989; 2005) model makes it unable to determine “maximum feasible abatement efforts” for each input vector. The abatement efforts proposed by their model may therefore be biased upwards relative to the actual production potential.

Axiom (iv) states that doing nothing is feasible while axiom (v) states that doing something for nothing is infeasible. The axioms of free disposability secure that production takes place in the economic region of the technology, i.e. where there are no congestion of inputs. Simply put, the axiom of free disposability of inputs implies that if an input vector can produce a certain output vector, then a larger input vector is also capable of producing that output vector. This generalizes the concept of positive marginal productivities of inputs. The final axiom, convexity, secures that an average of two technically feasible input-output allocations is also feasible. The axiom generalizes the concept of diminishing marginal rate of technical substitution.

The use of set theory allows for more flexible representations of technology as compared to standard textbook production functions. First, it allows for treating multiple outputs rather than the

single output which is enforced by the production function. This is desirable as most production processes result in multiple desirable and undesirable outputs. Second, set theory does not require that all firms are optimally allocated or efficient. This is not the case for the production function  $f(x)$  which, for a given  $T1$  and the assumption that  $M=1$ , is defined as the efficient output for each input vector,  $x$ :

$$f(x) = \max_y \{y : (x, y) \in T1\} \quad (4)$$

Distance functions provide suitable function representations for multi-output technologies. The most common distance functions are the input and output distance functions (Shephard, 1953, 1970). These functions proportionally decrease inputs or expand outputs, respectively, to the technology frontier. Recently, the hyperbolic (Färe et al., 1985) and the directional distance function (Chambers et al., 1996; Chambers et al., 1998) have been introduced. They offer more flexibility in choosing the direction in which a datapoint is projected to the frontier. Consider a direction vector  $g=(g_x, g_y)$  in  $\mathbb{R}_+^N \times \mathbb{R}_+^M$ . The directional distance function,  $\bar{D}(x, y; -g_x, g_y)$ , the directional output distance function,  $\bar{D}_O(x, y; g_y)$ , and the directional input distance function,  $\bar{D}_I(x, y; -g_x)$ , are then defined formally by equation 5:

$$\begin{aligned} \bar{D}(x, y; -g_x, g_y) &= \sup \left\{ \beta \in \mathbb{R} : (x - \beta g_x, y + \beta g_y) \in T1 \right\} \\ \bar{D}_O(x, y; g_y) &= \sup \left\{ \beta \in \mathbb{R} : (y + \beta g_y) \in P(x) \right\} \\ \bar{D}_I(x, y; -g_x) &= \sup \left\{ \beta \in \mathbb{R} : (x - \beta g_x) \in L(y) \right\} \end{aligned} \quad (5)$$

Under the axiom of free disposability of inputs and outputs<sup>1</sup>, the distance functions completely characterize the underlying technology in the sense that:

$$\begin{aligned} \bar{D}(x, y; -g_x, g_y) &\geq 0 \text{ if and only if } (x, y) \in T1 \\ \bar{D}_O(x, y; g_y) &\geq 0 \text{ if and only if } y \in P(x) \\ \bar{D}_I(x, y; -g_x) &\geq 0 \text{ if and only if } x \in L(y) \end{aligned} \quad (6)$$

Distance function representations of the technology are desirable when treating undesirable outputs, since they only require information on quantities of inputs and outputs. Dual representations of the technology do, on the other hand, require price information. This is problematic for undesirable outputs and other non-market commodities for which prices are unavailable. Shadow prices for the undesirable outputs may, on the other hand, be obtained by exploiting the distance functions' duality to the cost, revenue, and profit function (Färe et al., 1993).

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<sup>1</sup> The weaker axiom of  $g$ -disposability is sufficient for the distance functions to completely characterize the underlying technology (Färe and Primont, 2006).

Article 3 in this thesis uses this insight to propose two new ways of deriving marginal abatement costs for undesirable outputs.

## 2. POLLUTION MODELING IN THE PRODUCTION ANALYSIS LITERATURE

This section provides a brief overview of the two main approaches to modeling undesirable outputs that is proposed in the production analysis literature. The first approach treats undesirable byproducts as freely disposable inputs. The second approach treats undesirable outputs as weakly disposable outputs.

### a. Byproducts as freely disposable inputs

Some studies account for byproducts by including them in the input vector. Let  $b \in \mathbb{R}_+^K$  denote a vector of undesirable byproducts and define the extended input set as:

$$V(y) = \{(x, b) : (x, b) \text{ can produce } y\} \quad (7)$$

The axiom of freely disposable inputs generalizes the concept of positive marginal productivities of inputs. Modeling undesirable byproducts as freely disposable inputs is thus equivalent to assuming increases in desirable outputs from increases in the level of byproducts. This idea is applied in several pioneering contributions to pollution modeling, e.g. Baumol and Oates (1975), Pittman (1981), and Barbera and McConnell (1990), but also appears in more recent publications, e.g. Lee (2005) and Bye et al. (2009). Its rationale is to capture increases in desirable outputs due to transfer of resources from abatement activities to the production of desirable outputs.

### b. Byproducts as weakly disposable outputs

Färe et al. (1989; 2005) treat undesirable byproducts as outputs rather than inputs:

$$Y(x) = \{(y, b) : x \text{ can produce } (y, b)\} \quad (8)$$

Axiom (vi), free disposability of outputs, implies that the producers can dispose all unwanted byproducts at no costs when byproducts are treated as outputs. In other words, complying with environmental regulations is free by assumption. To account for costly reduction of undesirable outputs, Färe et al. perceive desirable and undesirable outputs as weakly disposable (Shephard, 1970):

$$\text{if } (y, b) \in Y(x) \text{ and } 0 \leq \theta \leq 1, \text{ then } (\theta y, \theta b) \in Y(x) \quad (9)$$

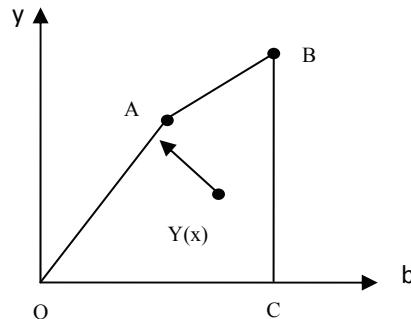
Weak disposability imposes that, for each input vector,  $x$ , reductions in undesirable outputs are always feasible when proportionally reducing the desirable outputs. The reduction in desirable outputs is explained by “*the imposition of a fine, or more likely in the present case through diversion of given inputs to cleanup of the bad output*” (Färe et al., 2005).

Secondly, Färe et al. impose unavoidability of undesirable outputs in the production process through the axiom of null-jointness (Shephard and Färe, 1974).

$$\text{if } (y, b) \in Y(x) \text{ and } b = 0, \text{ then } y = 0 \quad (10)$$

Baumgärtner et al. (2001) and Baumgärtner and Arons (2003) apply the first and second law of thermodynamics to show that null-jointness must hold in any conventional production process. The axiom of weak disposability is more questionable. It will therefore be thoroughly discussed in the first article of this thesis.

Figure 1 illustrates the extended output set  $Y(x)$  under the assumption of null-jointness and weak disposability of desirable and undesirable outputs. For simplicity, I here consider the case with one desirable output and one undesirable output. The piece-wise linear weakly disposable technology is bounded by OABCO. The axiom of weak disposability generates the positively sloped region OAB of the production frontier, implying that firms allocated in this region can only reduce the undesirable output when simultaneously reducing the desirable output. However, a frontier which is everywhere positively sloped<sup>2</sup> as in figure 1 is, in general, not always supported by empirics. Parts of the frontier may, in fact, be negatively sloped, implying that it is profitable to increase the undesirable byproduct for firms located in this region of the frontier. This is for example experienced by Färe et al. (1989) who find that 12 out of the 30 firms they study do not face any opportunity costs, in terms of forgone desirable outputs, when reducing emissions. The problem of negatively sloped regions is discussed by Picazo-Tadeo and Prior (2009) and Førsund (2009) and will not be treated further here.



**Figure 1:** Output set for the Färe et al. technology

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<sup>2</sup> The claim that the production frontier is everywhere positively sloped is incorrect if one account for the vertical line segment BC. This segment is caused by the axiom of free disposability of desirable outputs. However, this region is not important for the current discussion, since relevant distance function representations are assigned to project a datapoint in the direction of the segment OAB.

The introduction of the hyperbolic and the directional distance function has been of vital importance to the popularity of Färe et al.'s framework. These measures allow for evaluating firms' performance by their potential for simultaneously increasing desirable outputs and reducing undesirable outputs. The directional output distance function is illustrated by the arrow in figure 1 that projects a point in the interior of  $Y(x)$  to the frontier in a way which increases  $y$  and reduces  $b$ .

### 3. THE RETURN OF THE MATERIALS BALANCE CONDITION

The materials balance condition, or the first law of thermodynamics, states that matter can neither be created nor be destroyed, but that it may only change its form. In their seminal paper, Ayres and Kneese (1969) show that this rather simple conservation rule is of crucial importance to joint production of desirable and undesirable outputs: Mass conservation implies that material inputs which are not recuperated by intended outputs end up as (undesirable) byproducts. Ayres and Kneese use the phrase materials balance to point out the inevitability of byproducts when applying materials in the production process.

The contribution of Ayres and Kneese has recently been revisited by several economic researchers. They are occupied with the implications materials balance has for economic theory and modeling. Baumgärtner (2004) shows that the materials balance condition imposes marginal products of materials that are bounded from above. He ties this observation to the Inada conditions of economic growth theory and argues that they are violated when materials are applied. Krysiak and Krysiak (2003) show that materials balance limits substitution possibilities in production. They find conventional textbook production functions that assume full degree of substitutability between inputs to be inconsistent with the materials balance principle. In a recent article, Ebert and Welsch (2007) suggest that this problem can be overcome by explicitly accounting for materials balance. According to their analysis, undesirable byproducts may then equivalently be treated as inputs or outputs in the conventional production model. A similar conclusion is reached by Pethig (2003; 2006) who starts his analysis by showing that the model in equation 7, the model that incorporates byproducts as inputs, is inconsistent with the materials balance principle. He proposes to integrate the production function with "undesirable inputs" in a more comprehensive technology that explicitly accounts for material flows in production and abatement processes. Although theoretically appealing, his analysis would be highly data demanding and computationally intensive when extended to an empirical setting.

Coelli et al. (2007) and Lauwers (2009) propose a new way in which eco-efficiency can be calculated by applying the materials balance condition. In their approach, the concept of emission factors is used for representing materials' flows. This is especially convenient for non-point byproducts that cannot be directly measured. Let  $n$  be a  $(K \times N)$  matrix of input emission factors and  $m$  be a  $(K \times M)$  matrix of output recuperation factors. The materials balance conditions for the undesirable byproducts,  $b$ , is then defined:

$$b^{uc} = [nx - my] \quad (11)$$

The analyses of Coelli et al. (2007) and Lauwers (2009) are only concerned with uncontrolled byproducts ( $b^{uc}$ ). Regulated firms are, however, often involved in end-of-pipe abatement processes that allow them to diminish the level of targeted byproducts below uncontrolled emissions. Note that abatement does not reduce the amount of byproducts generated, but changes their characteristics. Since I am only concerned with their original characteristics, I perceive the abatement activity as reducing undesirable byproducts. Byproducts remaining after subtracting abatement efforts,  $A \in \Re_+^K$ , from the uncontrolled byproducts are called controlled byproducts:

$$b^c = [nx - my] - A \quad (12)$$

I omit the superscripts  $uc$  and  $c$  in the following.

#### 4. COMMENTS TO SINGLE EQUATION POLLUTION MODELING

Section 3 discussed the fundamental importance of materials balance for pollution generation. It is therefore reasonable to assess the two single production relation models from section 2 in its context. As mentioned above, Pethig (2003; 2006) points out that the production model which incorporates undesirable byproducts as inputs is inconsistent with the materials balance condition. This result is easily derived from equation 11: The materials balance equations can, for a fixed input vector,  $x$ , only hold if desirable outputs are reduced while increasing undesirable byproducts. In other words, positive marginal productivities of byproducts, imposed by the axiom of free disposability of conventional and undesirable inputs, are not consistent with materials balance.

Coelli et al. (2007) utilize equation 11 to show that Färe et al.'s (1989; 2005) model is inconsistent with the materials balance condition. Although Coelli et al. use the hyperbolic distance function for their proof, I here restate their arguments in terms of the directional distance function. Consider the directional output distance function that seeks simultaneous reductions of undesirable outputs and expansions of desirable outputs:  $\bar{D}_O(x, y, b; g_y, -g_b) = \sup\{\beta \in \Re : (y + \beta g_y, b - \beta g_b) \in Y(x)\}$ . Inserting the distance function into the equation 11,  $b - \beta g_b = nx - m(y + \beta g_y)$ , provides the result:

$$b + my - nx = \beta(g_b - mg_y) \quad (13)$$

To secure materials balance, the right hand side of equation 13 must be equal to zero. This is satisfied when all firms are technically efficient, i.e. when  $\beta=0$ . This is the result obtained by Coelli et al. for the hyperbolic distance function. Färe et al.'s model is, however, defined as a set theoretical production model to allow for technical inefficient observations. Coelli et al. therefore conclude that the Färe et al. model is not viable in the materials balance context. In our case, however, materials balance also holds if the direction vector satisfies  $g_b=mg_y$ . That is, when the direction vector is determined such that reductions in undesirable outputs equal the recuperation by desirable

outputs. Coelli et al.'s result may thus be avoided by appropriately choosing the direction vector. The costs are severe constraints on the choice of direction vector.

The results from the two previous paragraphs do only hold in absence of end-of-pipe abatement. It should be accounted for since the rationale behind modeling byproducts as inputs or weakly disposable outputs is to capture increases in desirable outputs due to transfer of resources from abatement purposes to production. Using equation 12, it is straightforward to show that simultaneous increases in both desirable and undesirable outputs are physically feasible, for a fixed input vector, when simultaneously reducing abatement efforts to maintain balance. The directional output distance function is further allowed to take other values than  $\beta=0$  for the Färe et al. model, independent of the choice of direction vector. Abatement efforts can now be adjusted to maintain materials balance.

Førsund (2009) points out that Färe et al.'s (1989; 2005) model only considers one way in which firms can reduce their emissions, namely by reducing intended outputs. This follows from the axiom of weak disposability of desirable and undesirable outputs which secures that desirable outputs must be forgone in order to reduce undesirable outputs. Applying equation 12 and assuming a fixed input vector, it is evident that a simultaneous reduction in desirable and undesirable outputs is only physically consistent with increases in abatement. In other words, the Färe et al. model implicitly assumes that emission reduction takes place by diverting resources from desired production to end-of-pipe abatement. The fact that the abatement processes are implicitly rather than explicitly modeled may lead to overestimation of firms' potential to reduce undesirable byproducts. This is further discussed in the first article of this thesis.

Although the models from section 2 may be justified when accounting for end-of-pipe abatement, it is desirable to treat uncontrolled emissions and abatement efforts explicitly. Emission reductions can be undertaken in many other ways than through end-of-pipe purifications. Førsund and Strøm (1988) consider input substitution, output reductions, technical change, and waste recycling as other important possibilities. The three first mentioned measures are related to reductions in uncontrolled emissions. Their influence on undesirable byproducts can easily be seen by inspecting the materials balance conditions from equation 11: Output reductions and technical change that lower the demand for polluting inputs will reduce emissions accordingly. Switching from inputs which emission factors are high to inputs which emission factors are low will further reduce the generation of undesirable outputs.

The severe restriction on firms' choice between different measures for emission reductions, implicitly assumed by the single production relation models from section 2, makes it impossible to identify the least cost way of emission reduction. These models are thus less likely to produce desirable estimates of marginal abatement costs. This observation is the point of departure of article three in this thesis that defines a new way of estimating marginal abatement costs in which the firms are allowed to select the least cost way of emission reduction.

Assuming that increased emissions always is beneficial for the polluting firms, both when treating byproducts as inputs in equation 7 or as weakly disposable outputs in equation 8, directly contradicts the non-controversial parts of the well-known Porter hypothesis (Porter and Van Der Linde, 1995). Porter and Van der Linde point out that scrap, harmful substances, or energy forms that are discharged into the environment is an indicator of resources being used incompletely, inefficiently, or ineffectively. Reducing emissions can accordingly be achieved by using inputs more productively, thereby improving the firms' competitiveness. Porter and van der Linde use the term

improved resource productivity for characterizing such achievements. They provide several examples where firms seem to have gained from reducing their emission. The Porter hypothesis has been heavily debated and criticized. However, a framework which allows for possible empirical explanation of the hypothesis beyond single anecdotes, rather than plain rejection that follows from applying standard models in settings where they may not be applicable, is highly desirable.

The materials balance principle clearly shows that inputs used in the production process are the source of byproducts. Furthermore, various inputs contribute differently to the total level of emissions, depending on their emission factors. Think for example of sulfur emissions from fossil fuel fired power plants. Whenever coal can be substituted for natural gas, the plants can reduce their emissions. Consider now treating sulfur emissions as an input in a standard production model for electricity. By the axioms from section 1, the model allows for substitution between the undesirable byproducts and conventional inputs. That is, sulfur can for example be substituted with coal. This is a rather counterintuitive result since coal is the main reason for sulfur emissions.

The recognition that byproducts are functions of both inputs and intended outputs suggests that a proper model of polluting firms should thoroughly describe the production of desirable outputs. Knowledge about substitutability of inputs and efficiency in conversion of inputs to desirable outputs is likely to provide more information about polluting technologies than the reduced form technologies in equation 7 and 8. This calls for a different way of modeling undesirable outputs that does not explicitly incorporate pollutants in the technology set. A framework that accommodates this requirement is presented in the following section.

## 5. ASSORTED PRODUCTION

Førsund (2009) and Murty and Russell (2002) suggest treating undesirable byproducts as outputs while taking into account the physical restrictions imposed by materials balance. To achieve physical consistency they propose the use of a Frisch (1965) multi-output model. This production model provides the opportunity of assigning multiple production relations, and thereby varying the degree of assortment for outputs for a given input vector. Let  $\mu$  be the number of production relations in the model. The degree of assortment is then defined by  $\alpha = (M + K) - \mu$ , i.e. the number of production relations subtracted from the number of outputs. In the case of one production relation, there is full degree of assortment. The Frisch model then collapses into the conventional production model from equation 1, allowing for transformation possibilities between all outputs for fixed input vectors. While this may be realistic for desirable outputs, the assumption is more questionable when the output vector includes undesirable byproducts. Consider equation 11,  $b = nx-my$ , and assume that  $m$  is the zero-matrix. That is, byproducts are not recuperated in intended outputs. This is for example the case for air pollutants from fossil fuel power generation, the case study of articles two and four of this thesis. It is here evident that undesirable outputs cannot be substituted with desirable outputs for a fixed input vector, since the undesirable outputs are directly determined by the input use. To overcome this problem, I model the polluting technology,  $T$ , as the intersection of  $T_1$ , the conventional technology set from equation 1, and the pollution generating mechanism  $T_2$ , namely controlled emissions.

$$\begin{aligned}
T(b+A) &= T1 \cap T2 \\
T1 &= \{(x, y) : x \text{ can produce } y\} \\
T2 &= \{(x, y) : [nx - my] = b + A\}
\end{aligned} \tag{14}$$

In a Frisch sense, the model in equation 14 has  $M$  degrees of assortment. That is, there exists transformation possibilities between all desirable outputs for given inputs. When the inputs,  $x$ , and intended outputs,  $y$ , are determined, the undesirable outputs follow directly. The undesirable outputs have become unavoidable byproducts that only can be influenced by selecting a different mix of inputs or desirable outputs, or by altering the abatement efforts,  $A$ .

The model in equation 14 assumes that the abatement production is separate from the production of desirable outputs. This follows Førsund (2009). In contrast to Førsund (2009) and Murty and Russell (2002), I do not explicitly model end-of-pipe abatement technologies. Although theoretically desirable, explicit modeling of end-of-pipe purifications requires a substantial amount of data that makes the model less attractive for applied work. This is related to the demand for information about abatement efforts and inputs that go into the abatement processes. Such information is hard to acquire and, when available, potentially biased by firms' strategic decisions. The implicit treatment of abatement is discussed further in articles two to four, where it is proposed that economic intuition justifies the implicit modeling.

The model proposed in equation 14, unlike the models from section 2, provides an intuitive way of capturing Business as Usual (BaU) scenarios. Assume that byproducts are not regulated. Equation 14 then adds nothing new to the conventional production model from equation 1. Firms are only concerned with their technological constraints in  $T1$ , and optimize their profits accordingly. Let  $w \in \mathbb{R}_{++}^N$  and  $r \in \mathbb{R}_{++}^M$  be vectors of prices for inputs and desirable outputs, respectively. The conventional profit function:

$$\pi(r, w) = \sup_{x, y} \{ry - wx : (x, y) \in T1\} \tag{15}$$

can then be considered as the BaU scenario, i.e. the allocation that will occur in absence of environmental regulations.

Consider now an environmental regulation that restricts the level of byproducts,  $b$ . From the materials balance conditions in equation 11,  $b = nx - my$ , it is evident that bounds on byproducts must similarly limit feasible allocations of inputs and desirable outputs. By treating  $b$  as exogenously determined and replacing the equality in  $T2$  with an inequality, the regulated technology becomes a restricted subset of technology  $T1$ . This changes the profit maximization problem in equation 15, leading to forgone profits due to the regulatory constraints.

The recognition that environmental regulations implicitly restrict feasible input-output allocations through the materials balance condition indicates that emission restricted firms can be treated similar to expenditure constrained firms, see e.g. Lee and Chambers (1986) and Färe et al. (1990). This is desirable, since the theory of expenditure constrained firms is already well established; it is an extension of Shephard's (1974) indirect production theory. The theory of an

emission constrained firm should thus be easy accessible to economists that already are familiar with these concepts.

## 6. DESIRABLE APPLICATIONS

The increasing interest in polluting technologies, indicated by the number of recent publications on this topic, is a result of the desirable information the pollution models can provide for regulators, consumers, and producers that are concerned about environmental impacts of production. The environmental productivity literature has mainly been occupied with identifying marginal abatement costs, eco-efficiency measurement, and evaluating the effects on technical efficiency and technical change of reducing byproducts. In article two of this thesis, I argue that the literature should also be occupied with the measurement of allocative efficiency. In the following, I present a review of production studies that covers these areas of application.

### a. Marginal abatement cost estimation

Marginal abatement costs is an important concept in environmental economics as it reflects the least cost way in which byproducts can be reduced. It is thus crucial for determining net benefits of environmental regulations. Marginal abatement costs are not directly observable and therefore provide the firms with incentives for strategic reporting. Consequentially, their stated costs do not necessarily coincide with the actual costs. This makes it preferable to be able to determine the actual abatement costs, which is what recent modeling of polluting technologies sets out to do.

The idea of applying the duality of the output distance function to the revenue function to obtain estimates of marginal abatement costs was introduced by Färe et al. (1993). They show that shadow prices for undesirable outputs can be determined from the distance function derivatives, by assuming that the shadow price of one output equals its observed sales price. The shadow prices for undesirable outputs can readily be interpreted as "*the value of desirable output that must be forgone (in order to reduce an undesirable output) once all inefficiency has been eliminated and the firm produces on the frontier of Y(x)*" from equation 8 (Färe et al., 2005). This approach to abatement cost modeling has become increasingly popular and can for example be found in Coggins and Swinton (1996), Reig-Martinez et al. (2001), and Färe et al. (2006).

When treating undesirable byproducts as inputs, marginal abatement costs can be obtained by exploiting the duality of the input oriented distance function to the cost function (Shephard, 1953). This idea is pursued by Lee (2005), who identifies the marginal rate of technical substitution between capital and sulfur emissions from the input oriented distance function and use the price of capital to identify marginal abatement costs for sulfur. The marginal abatement costs are here interpreted as the increase in capital expenses that is required to compensate a marginal reduction in sulfur emissions. The problem, however, is that the same procedure allows defining marginal abatement costs in terms of increases in coal that are necessary for a marginal reduction in sulfur. This cannot be physically consistent.

## b. Eco-efficiency measurement

Eco-efficiency is usually evaluated by a simple ratio measure of economic outcome of the production process to the environmental impact. This measure runs into problems when multiple inputs and outputs are concerned, since aggregation into a single numerator and denominator requires the selection of aggregation weights. In this case, it is hard to establish an appropriate criterion for the selection of weights. To resolve this issue, three classes of eco-efficiency measures based on production frontier techniques have been developed. Lauwers (2009) categorizes these as environmentally adjusted production efficiency models, frontier eco-efficiency models, and materials balance based eco-efficiency models. All three classes of models avoid arbitrary selection of aggregation weights.

The first class of models, the environmentally adjusted production efficiency models, is already well-known to the reader. This is simply technical efficiency evaluations for the single production relation technologies from section 2. The flexibility of the directional distance function allow for performance measurement in terms of reductions in undesirable outputs and increases in desirable outputs. Various versions of environmentally adjusted production efficiency is found in the literature, see e.g. Färe et al. (1996; 2004) and Tyteca (1997).

The second class of models, the frontier efficiency models, uses the original measure of eco-efficiency as point of departure. To overcome the issue of arbitrary selected aggregation weights, Kuosmanen and Kortelainen (2005) and Kortelainen (2008) propose to apply a Data Envelopment Analysis (DEA) procedure for the weight selection. The weights are then chosen in a way that maximizes the evaluated unit's performance in comparison to similar units. This procedure deviates from standard production modeling as it does not consider inputs and outputs directly. Outputs are replaced by economic value added and inputs are replaced by undesirable byproducts.

Coelli et al. (2007) introduce a new measure of eco-efficiency that explicitly considers the materials balance condition from equation 11, for the case where  $b \in \mathbb{R}_+^1$ . They recognize that the emission factors can be interpreted as prices and that the byproducts, following from the choice of input vector, can be interpreted as costs. They therefore define the environmental "cost" function:

$$c(n, y) = \min_x \{nx : x \in L(y)\} \quad (16)$$

The environmental cost function determines the minimum level of uncontrolled emissions that is consistent with an output level,  $y$ . Similar to the well-known measure of cost efficiency, eco-efficiency ( $EE$ ) can now be obtained as the ratio of the minimum polluting input bundle to the observed input bundle.

$$EE = \frac{c(n, y)}{nx} \quad (17)$$

Following Farrell (1957), the eco-efficiency measure from equation 17 can be decomposed into technical and allocative inefficiency. The eco-efficiency measure has been applied to assessments of environmental performance in agriculture (Coelli et al., 2007) and fossil fuel electricity generation

(Welch and Barnum, 2009). The framework has also been extended to account for the second law of thermodynamics, i.e. to accommodate for exergy analysis (Hoang and Rao, 2010).

### c. Technical and allocative efficiency

The directional distance function allows for performance evaluations in terms of firms' ability to increase desirable outputs and reduce undesirable outputs. Its extension to productivity indices, the Luenberger productivity indicator (Chambers et al., 1996) and the Malmquist-Luenberger index (Chung et al., 1997), allow for joint assessment of production and environmental performance when taking intertemporal patterns into account. The indices can further be decomposed into changes in technical efficiency and technical change.

Recent papers, e.g. Färe et al. (2001; 2007) and Ball et al. (2005), focus on the effects of neglecting joint production of desirable and undesirable outputs on technical efficiency and technical change. They argue that conventional performance measures for the technology from equation 1 are biased when undesirable byproducts are not taken into account. The rationale is that inputs which go into end-of-pipe abatement do not contribute to production of desirable outputs. In other words, environmentally regulated firms will appear technical inefficient compared to unregulated firms, since abatement inputs are perceived as unproductive. This is obviously a problem since data that separate "production inputs" from "abatement inputs" are hard to come by. On the other hand, the concern for non-producible "abatement inputs" largely overestimates the resort to end-of-pipe abatement for complying with environmental regulations. In section 4 of this introduction, I pointed out that there exist multiple measures at the firms' disposal: Input substitution, output reductions, technical change, waste recycling, and end-of-pipe abatement activities were proposed as potential ways of compliance. While the abatement activities require additional resources, the three first mentioned measures only concern technology  $T_1$  from equation 1 and should not affect technical efficiency. Input substitution and output reductions are, on the other hand, likely to influence the measurement of allocative efficiency. In section 5, I discussed how restrictions on emissions induce bounds on the use of polluting inputs. Input use and supply of desirable outputs are therefore likely to differ for a regulated and a non-regulated firm. Failure to take this into account may lead to underestimation of regulated firms' overall performance.

The Frisch model from equation 14 allows for evaluating optimal profits for the regulated and unregulated firm. The difference between the two optima can be considered forgone profits due to regulation, which in a wider setting is the appropriate measure of the difference in abatement costs. Assume a command and control policy that restricts the generation of undesirable byproducts. Let  $b^*$  be the vector of emission targets. Technology  $T$  now becomes the restricted subset of technology  $T_1$  that secures regulatory compliance. Assume, for simplicity, that the firm does not have access to end-of-pipe abatement. Optimal profits for the regulated firm,  $\pi^C$ :

$$\begin{aligned}\pi^C(r, w, b^*) &= \sup_{x,y} \left\{ ry - wx : (x, y) \in T(b^*) \right\} \\ &= \sup_{x,y} \left\{ ry - wx : (x, y) \in T_1, nx - my \leq b^* \right\}\end{aligned}\tag{18}$$

can then be compared to the unrestricted profit maximum,  $\pi$ , from equation 15. The difference between the two profit maxima constitutes forgone profits due to environmental regulations. If forgone profits are not accounted for, the regulated firm may appear allocative inefficient even if it is operating efficiently under the regulatory constraints.

The influence of environmental regulations on the measurement of allocative efficiency has received little attention in the production analysis literature. A few exceptions exists, e.g. Brännlund et al. (1995; 1998) who evaluate the impact of environmental regulations for the profitability of the Swedish pulp and paper industry, using the assumption of weak disposability of desirable and undesirable outputs. In articles two to four of this thesis I review the importance of rationalizing allocative inefficiency for firms that comply with environmental regulations. I show that it is of crucial importance for appropriate marginal abatement cost estimation as well as unbiased efficiency measurement.

The existence of multiple ways in which the producers can comply with environmental regulations reduces the potential for biased measurement of technical efficiency due to “unproductive inputs”. However, this problem should not be neglected. In cases where “abatement inputs” cannot be directly separated from the “production inputs” the researcher may consider excluding inputs which use is ambiguous. For example, empirical models for power generation can contain fuel inputs and generating capacity as inputs without concern for biased measurement of technical efficiency. These inputs do not play a role in the abatement technology. The use of labor, on the other hand, is more ambiguous. This input may be excluded from the analysis to avoid the issue of “unproductive inputs” caused by extensive abatement efforts. The use of labor is highly complementary to the production capacity, and exclusion of labor is likely to have little impact on the overall results.

## 7. FUTURE DEVELOPMENTS<sup>3</sup>

In the previous section I described the currently most popular applications for production models that take undesirable byproducts into account. However, the model framework applied in this thesis also allows for the introduction of new areas of pollution modeling. In the following, I present a list of new areas that, to my knowledge, has not been assessed by the production analysis literature. One of the areas, secondary benefits of environmental regulations, is developed further in article four of this thesis. The others are included to suggest some appropriate directions to future contributions of the environmental production analysis literature.

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<sup>3</sup> In the first draft to this introduction, I proposed the humoristic title “Resources for the future” for the current section. This headline was strongly rejected by my supervisor, although it may seem appropriate in the current context.

### a. Treatment of abatement

In environmental economics, and in the production models from section 2, the concept of abatement is generally treated as an aggregate for emission reductions. In contrast, the analysis based on equation 14 focuses on the multiple measures that firms have at their disposal for reducing emissions. Applying the materials balance principle, abatement can be decomposed into reductions in emissions due to reduced uncontrolled byproducts and due to end-of-pipe abatement. The potential to assess firms' selection between improvements of uncontrolled and controlled emission can prove to be important for evaluating environmental outcome of regulations. This is especially the case if one takes a Georgescu-Roegen (1971) view of the world. By evoking the second law of thermodynamics, Georgescu-Roegen recognize that there are limits to economical growth due to bounds on the amount of work material resources can supply. This proposal has spurred a substantial debate regarding the possibility for substituting material resources for non-material resources, see e.g. Daly (1997) and Stiglitz (1997).

While reductions in uncontrolled emissions stem from input substitution, reduction in outputs, or even improved efficiency of input conversion, the reductions in controlled emissions must come from extended resource use. End-of-pipe abatement processes are often highly resource-demanding. Thus, if one assesses the environmental impact of resource depletion in addition to reductions in regulated pollutants, it is evident that end-of-pipe purifications are inferior to improvements in uncontrolled emissions. This calls for an analysis that allows assessing both firms' incentives to choose abatement over other measures for emission reductions, as well as the gains that firms have from involving in abatement activities. The latter allows for comparing the gains from abatement to the cost of resource depletion. The framework established in equation 14 will allow for undertaking both these assessments.

Consider the emission constrained profit maximization problem in equation 18 and allow the producers to involve in end-of-pipe abatement,  $A$ . Increases in abatement will relax the emission constraint imposed by environmental regulations,  $nx - my \leq b^* + A$ . The technology set,  $T$ , is expanded accordingly and input-output allocations that were previously unavailable for the regulated firms can now be achieved. This will allow firms to approach the unconstrained profit maximum from equation 15. The prosperity of increased profits thereby constitutes firms' gains from involvement in end-of-pipe abatement. Assume now a dataset that identifies both controlled and uncontrolled emissions. It is then straightforward to develop, say, emission constrained DEA profit functions that define optimal profits when firms are constrained by their level of controlled emissions and uncontrolled emissions. Differences in profits when firms are evaluated with and without access to end-of-pipe purification can readily be interpreted as their benefits from involvement in abatement activities.

A similar reasoning can be applied for evaluating the choice between technical change and end-of-pipe abatement as measures for reducing undesirable byproducts. It is already established that abatement is potentially beneficial for the firms as it expands feasible allocations of inputs and desirable outputs. Technical change will also expand the feasible set of inputs and desirable outputs by expanding the conventional technology set,  $T_1$ . Consider technical change and abatement as perfect substitutes. The shadow prices on the technological constraint(s),  $T_1$ , and the emission constraint,  $T_2$ , determine their rate of substitution. The relative costs of investments in technology

improvements and abatement will then determine which of the two measures that is optimal for compliances.

b. Secondary benefits

Secondary benefits from environmental regulations take the form of unintended reductions in non-regulated pollutants that arise because of jointness in the generation of regulated byproducts and non-regulated byproducts. Reductions in environmental damages due to reductions in non-target pollutants may thus justify tight regulation standards for certain pollutants. One example is the case of secondary benefits in terms of reduced sulfur and nitrogen emissions that may arise from regulations targeted at reductions in carbon dioxides, see e.g. Ekin (1996). Article four of this thesis argues that benefits may be limited in this case, if suitable end-of-pipe abatement equipment is developed for carbon dioxide.

The use of materials balance conditions for multiple pollutants in the model from equation 14 makes it easy to identify secondary benefits. All byproducts depend on the selection of inputs and desirable outputs. Bounds on a subset of byproducts, that induce bounds on feasible allocations for inputs and desirable outputs, must thereby also restrict occurrences of non-regulated byproducts. Emission constrained profit DEA models can be developed and calculated with and without the emission constraints imposed on the subset of the byproducts in *b*. The changes in the unconstrained emissions for non-regulated byproducts across the two scenarios can readily be interpreted as secondary benefits from the environmental regulation.

c. Eco-efficiency

The eco-efficiency measure proposed by Coelli et al. (2007) is desirable due to its simplicity. There are, however, two areas where it can be improved. The first is connected to end-of-pipe abatement, as biased eco-efficiency measurement may arise due to Coelli et al. solely concern for “environmental friendly” input mix and neglect of end-of-pipe abatement. It is likely that firms which are involved in abatement apply a less environmental friendly input mix since they are able to reduce emissions below their uncontrolled emissions. They may, however, emit less of evaluated pollutants as compared to firms with “correct” input mix. By taking abatement into account, and preferably explicitly model this activity as proposed by Førsund (2009) and Murty and Russell (2010), the Coelli et al. measure will give a more proper evaluation of eco-efficiency.

The standard measure of eco-efficiency evaluates economic outcome to environmental impacts. To accommodate for economic evaluations, Coelli et al. (2007) propose to apply observed input prices in order to calculate conventional cost efficiency. Afterwards they compare the cost efficiency to the eco-efficiency measure in equation 17. However, Coelli et al.’s measure can easily be extended to take both economical and environmental impact directly into account, by maximizing the difference between economic outcome from equation 15 and the environmental impact. This leads to the profit maximization problem in equation 19, where prices are adjusted by emission factors. Prices for high-polluting inputs will then increase relatively to low-polluting inputs, while

prices for high-recuperating outputs increase relative to low-recuperating outputs. This analysis bears resemblance to the modeling procedure in article three of this thesis.

$$\begin{aligned}
 E(r, w, n, m) &= \sup_{x,y} \{(ry - wx) - (nx - my) : (x, y) \in T1\} \\
 &= \sup_{x,y} \{(r + m)y - (w + n)x : (x, y) \in T1\} \\
 &= \sup_{x,y} \{P_y y - P_x x : (x, y) \in T1\}
 \end{aligned} \tag{19}$$

where  $P_y \in \mathfrak{R}_{++}^M$  and  $P_x \in \mathfrak{R}_{++}^N$  are pollution adjusted unit prices. The function  $E(r, w, n, m)$  is thus the conventional profit function with emission-adjusted prices. Eco-efficiency can now be calculated by applying the Nerlovian profit efficiency measure (Chambers et al., 1998). Nerlovian profit efficiency is further treated in article two of this thesis.

## 8. SUMMARY AND CONCLUSION

This thesis proposes some new directions for the treatment of undesirable outputs in a production framework. A theory of emission constrained firms that resembles the theory of expenditure constrained firms is proposed, along with some new areas of application for production analysis based pollution modeling. The suggested model framework applies the materials balance condition as the pollution generating mechanism, and thereby overcomes the critique existing production frameworks have faced. The analysis further seeks to provide the polluting firms with a larger degree of freedom in choosing the way in which emission targets are met, compared to what is usually done in the environmental production analysis literature.

The framework proposed in this thesis can easily be adapted by classical trained economists who already are familiar with the theory of expenditure constrained firms. It may also be approved by environmental economists by allowing for the treatment of Business as Usual scenarios, and for evaluating costs of regulatory compliance in terms of induced deviation from the Business as Usual allocation. My emphasis on rationalizing allocative inefficiency implies a treatment of polluting firms that closely resembles conventional environmental economics, as compared to other production framework that accommodate for pollutants.

I believe that pollution modeling is a viable branch of the production analysis literature. The development of “easy to use” models that allow estimating net benefits of proposed and existing environmental regulations is important. This is reflected by the number of recent publications that are occupied with these issues. Whether the model framework proposed in this thesis will be a part of further developments in the literature will only be determined in the future. More efforts are needed to establish whether the proposed framework a viable tool for the environmental economist’s toolkit.

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# Paper I



# Weak Disposability: Bad Assumption for Bad Outputs?

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**Abstract:** This paper utilizes the materials balance condition for assessing the popular axiomatic production model that applies the axiom of weak disposability to characterize joint production of desirable and undesirable outputs. The analysis shows that end-of-pipe abatement is crucial for the model's physical consistency. Its implicit treatment of abatement is, however, likely to suggest expansions of abatement efforts beyond observed best practices. This may bias estimated efficiency scores as well as pollutant shadow prices.

**Keywords:** *Joint production; Weak disposability axiom; Materials balance*

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## 1. INTRODUCTION

In recent years there has been increasing concerns for treating undesirable outputs in the production analysis literature. A popular modeling approach is that of Färe et al. (1989; 2005) who suggest that pollutants be included in the technology set. Two non-conventional axioms, null-jointness and weak disposability of desirable and undesirable outputs, are proposed in order to modify the traditional neoclassical production model to account for special properties of pollutants. Distance function representations for the polluting technology are developed, which allow for environmental efficiency measurement and estimation of pollutant shadow prices.

Production activities are subordinate to the laws of physics. In the current article I assess Färe et al.'s model in light of the materials balance condition, a law of physics that is fundamental to the generation of undesirable outputs (Ayres and Kneese, 1969). First, by using the materials balance requirement, I show that involvement in end-of-pipe abatement activities is crucial for the validity of the weak disposability axiom. Second, an alternative model that explicitly treats abatement as an output is proposed. The abatement model conveys the trade-off between cleaning up emissions and producing desirable outputs proposed in the Färe et al. framework, but allows for imposing the conventional axiom of free disposability of outputs. As in the case of the former model, the abatement model can be represented by output sets containing desirable and undesirable outputs. I show that the output sets of the two models will in general differ. Estimates obtained for the two models must then differ accordingly, indicating that the implicit treatment of abatement in Färe et al.'s model potentially causes biased estimates of efficiency scores as well as pollutant shadow prices.

## 2. THE WEAK DISPOSABILITY MODEL

This section provides a short summary of the framework of Färe et al. (2005). Let  $x \in \mathfrak{R}_+^N$  denote a vector of inputs and let  $y \in \mathfrak{R}_+^M$  denote a vector of desirable outputs. In general, this model allows

for treating multiple pollutants. Without loss of generality I only consider one undesirable output,  $b \in \mathfrak{R}_+$ . The polluting technology is represented by the output set  $P(x)$ :

$$P(x) = \{(y, b) : x \text{ can produce } (y, b)\} \quad (1)$$

Conventional axioms of no free lunch, compact and convex output sets, and free disposability of inputs and desirable outputs apply. However, to capture that one of the outputs is undesirable, two non-conventional axioms are imposed:

- (i) if  $(y, b) \in P(x)$  and  $b = 0$ , then  $y = 0$
- (ii) if  $(y, b) \in P(x)$  and  $0 \leq \theta < 1$ , then  $(\theta y, \theta b) \in P(x)$ <sup>1</sup>

Axiom (i), null-jointness, implies that generation of the undesirable output is unavoidable in the production process. Evoking the first and second law of thermodynamics, Baumgärtner et al. (2001) and Baumgärtner and Arons (2003) show that this holds for any conventional production process. Axiom (ii) is known as weak disposability of desirable and undesirable outputs, and imposes feasibility of proportional reduction of the two. This implies costly emission reductions since desirable outputs must be forgone in order to reduce the undesirable byproduct.

### 3. MATERIALS BALANCE

The implications of materials balance was thoroughly treated in a seminal paper by Ayres and Kneese (1969). Recently several authors, including Pethig (2003; 2006), Coelli et al. (2007), Lauwers (2009), and Førsund (2009), have revisited materials balance and its importance to economic modeling. The materials balance condition states that materials can neither be created nor destroyed, but may only change their form. This conservation property implies that material inputs which are not recuperated in intended outputs end up as (undesirable) byproducts. One example can be found in the pig finishing industry. Feed and piglets represent nitrogen inflows to the production process. However, only approximately one third of the nitrogen input is recuperated in the desirable output, pork. The residual is manure byproducts that potentially cause negative environmental effects (Lauwers, 2009).

Many non-point byproducts cannot be directly measured. In such cases, it is convenient to apply emission factors for determining the generation of byproducts. They report the amounts of the undesirable byproduct contained in the inputs, as well as the amount recuperated in the desirable outputs. Denote emission factors by  $n \in \mathfrak{R}_+^N$  and output recuperation factors by  $m \in \mathfrak{R}_+^M$ . The materials balance condition for the undesirable byproduct,  $b$ , is then defined:

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<sup>1</sup> Färe et al (2005) define the weak disposability axiom such that the scalar  $\theta$  may take the value 1. I omit this special case to allow for a simple proof of proposition 1. This does not change the properties of the weak disposability axiom since  $\theta=1$  does not represent a proportional reduction of  $y$  and  $b$ , i.e.  $(1y, 1b)=(y,b)$ .

$$b^{uc} = [nx - my] \quad (2)$$

Equation 2 reports the uncontrolled byproduct. Firms often involve in end-of-pipe abatement activities in order to reduce emissions<sup>2</sup>. This can be represented by subtracting  $A \in \Re_+$  from equation 2. Emissions remaining after abatement,  $b^c$ , is called the controlled byproduct.

$$b^c = [nx - my] - A \quad (3)$$

The superscripts  $c$  and  $uc$  are dropped in the following for notational convenience.

#### 4. ABSENCE OF ABATEMENT ACTIVITIES

Equations 2 and 3 represent the underlying data generating mechanism for the undesirable output in equation 1. This relationship readily allows reviewing the model of Färe et al. (1989; 2005) in the context of materials balance. I start by considering the validity of their model in absence of end-of-pipe abatement.

*Proposition 1:* If  $b = [nx - my]$ ,  $(y, b) \in P(x)$ , and  $0 \leq \theta < 1$ , then  $(\theta y, \theta b) \notin P(x)$

Proposition 1 suggests that the axiom of weak disposability of desirable and undesirable outputs is not satisfied in absence of abatement activities.

*Proof:* Consider  $(y, b) \in P(x)$ . Rewriting equation 2 and scaling the outputs result in  $\theta[b + my] = nx$ . The only solution for the scalar  $\theta$  that allows maintaining materials balance is  $\theta = 1$  for the input vector  $x$ . Thus,  $(\theta y, \theta b) \notin P(x)$  when  $0 \leq \theta < 1$ .

*Q.E.D.*

$P(x)$  is defined for each input vector  $x \in \Re_+^N$ , implying that  $nx$  in equation 2 is defined accordingly. The only way in which a firm can reduce its byproduct, given the input vector, is to increase the level of desirable outputs provided that the  $m$  vector is not the zero-vector. This contradicts the weak disposability axiom, and the output sets suggested by Färe et al.'s model are thus physically inconsistent.

It should be added that Coelli et al. (2007) utilize the hyperbolic distance function for establishing that materials balance does not allow for technical inefficiency in Färe et al.'s model. Although not discussed by Coelli et al., this is a direct consequence proposition 1.

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<sup>2</sup> Notice that end-of-pipe abatement does not reduce the amount of byproduct generated, but changes its characteristics. Since I am concerned with the byproduct's initial (harmful) characteristics, I perceive abatement as reducing the undesirable output.

## 5. ABATEMENT ACTIVITIES

I define a production model which, contrary to Färe et al. (1989; 2005), explicitly models abatement efforts:

$$V(x) = \{(y, A) : x \text{ can produce } (y, A)\} \quad (4)$$

When treating abatement as an output conventional axioms, including free disposability of all outputs, apply. Equation 4 may equally be represented by the output set  $S(x)$  by applying equation 3:

$$S(x) = \{(y, b) : (y, A) \in V(x), b = [nx - my] - A\} \quad (5)$$

*Proposition 2:  $S(x) \subseteq P(x)$*

*Proof:*

1. For all  $(x, y, b) \in \mathfrak{R}_+^{N+M+1}$ , if  $(y, b) \in S(x)$ , then  $(y, b) \in P(x)$  by definition.
2. There exists at least one point  $(y^0, A^0)$  such that  $(y^0, A^0) \notin V(x)$  by the axiom of bounded output sets. For any  $(x, y, A) \in \mathfrak{R}_+^{N+M+1}$ , if  $(\theta y, \gamma A) \notin V(x)$ ,  $0 \leq \theta < 1, \gamma > 0$ , and  $\theta b = [nx - m(\theta y)] - \gamma A$ , then  $(\theta y, \theta b) \notin S(x)$ . However,  $(\theta y, \theta b) \in P(x)$  if  $(y, b) \in S(x)$ , by step 1 of this proof and the axiom of weak disposability.

*Q.E.D.*

The key to understanding proposition 2 is recognizing that any proportional reduction of  $y$  and  $b$ ,  $(\theta y, \theta b)$ , is only consistent with materials balance if abatement efforts are increased by  $\gamma$ . The output set  $S(x)$  reflects the levels of desirable and undesirable outputs that are producible given current output and abatement technology. Since  $P(x)$  is not explicitly treating these technological boundaries, it is likely to suggest expansions of abatement efforts beyond its observed best-practice counterparts.

The directional output distance function may be applied as a function representation of  $S(x)$  and  $P(x)$ . Define the direction vector  $(g_y, g_b)$  in  $\mathfrak{R}_+^M \times \mathfrak{R}_+^1$  and consider:

$$\begin{aligned} \bar{D}_O^P(x, y, b; g_y, -g_b) &= \sup \left\{ \beta \in \mathfrak{R} : (y + \beta g_y, b - \beta g_b) \in P(x) \right\} \\ \bar{D}_O^S(x, y, b; g_y, -g_b) &= \sup \left\{ \beta \in \mathfrak{R} : (y + \beta g_y, b - \beta g_b) \in S(x) \right\} \end{aligned} \quad (6)$$

Since  $\bar{D}_O^P(x, y, b; g_y, -g_b) \geq 0$  if and only if  $(y, b) \in P(x)$  and  $\bar{D}_O^S(x, y, b; g_y, -g_b) \geq 0$  if and only if  $(y, b) \in S(x)$  under  $g$ -disposability, it follows from equation 6 and proposition 2 that:

$$\vec{D}_O^S(x, y, b; g_y, -g_b) \leq \vec{D}_O^P(x, y, b; g_y, -g_b) \quad (7)$$

The directional distance function can be thought of as a combined environmental and technical efficiency measure (Färe et al., 2005). Equation 7 establishes that the distance function representation for the weak disposability technology is likely to provide downward biased estimates of efficiency, since technological constraints for abatement are not explicitly considered.

Following Färe et al. (2005) I specify the revenue function in terms of the directional output distance function. Let  $P_y \in \Re_+^M$  and  $P_b \in \Re_+$  be vectors of (shadow) prices and define:

$$R(x, P_y, P_b) = \max_{y, b} \left\{ P_y y - P_b b : \vec{D}_O(x, y, b; g_y, -g_b) \geq 0 \right\} \quad (8)$$

Following Chambers et al. (1998), the duality between the revenue function and the directional output distance function may be developed as:

$$\vec{D}_O(x, y, b; g_y, -g_b) = \inf_{P_y, P_b} \left\{ \frac{R(x, P_y, P_b) - (P_y y - P_b b)}{P_y g_y + P_b g_b} \right\} \quad (9)$$

Färe et al. (2005) suggest that the shadow price of the pollutant,  $P_b$ , can be interpreted as marginal abatement costs. From equation 7 follows that the shadow prices which solve the optimization problem in equation 9 must, in general, differ for  $P(x)$  and  $S(x)$ , suggesting that marginal abatement costs derived from Färe et al.'s model is potentially biased.

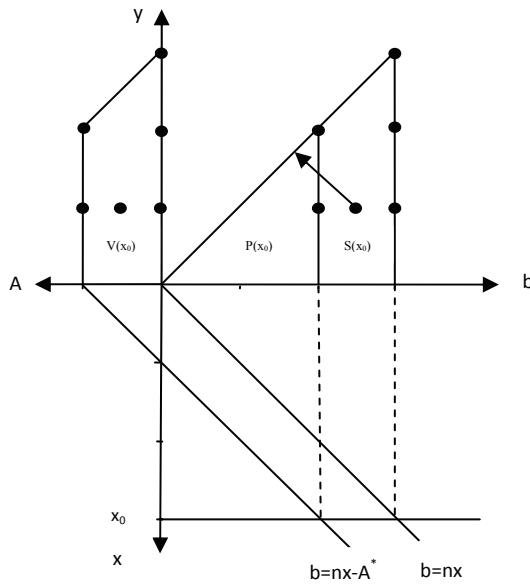
## 6. GRAPHICAL EXAMPLE

I consider six firms that all use the same input bundle,  $x_0$ . The input emission factor is arbitrarily set equal to 0.5 and the  $m$  vector is the zero-vector. The upper right panel of figure 1 depicts the output sets  $S(x_0)$  and  $P(x_0)$ , while the output set  $V(x_0)$  is represented in the upper left panel.

Proposition 1 can be assessed by considering the subset of firms that are not abating. The materials balance condition, illustrated in the lower panel of the graph, secures that these firms generate the same level of undesirable byproducts,  $b=nx_0$ . In situations where abatement techniques are absent it is not feasible for the firms to reduce their emissions beyond this level without adjusting the input level,  $x_0$ .

The set  $V(x_0)$  is mapped to the  $(y, b)$  dimension by applying the materials balance condition in the lower panel. The corresponding set  $S(x_0)$  is smaller than the set  $P(x_0)$ , since the weak disposability axiom allows contractions of the boundary points of  $S(x_0)$ . Consider the directional output distance function represented by the arrow in figure 1. The distance from the interior datapoint to the frontier of  $P(x_0)$  largely exceeds the distance to the frontier of  $S(x_0)$ . The consequence is downward biased efficiency estimates. The steeper surface of  $S(x_0)$  as compared to  $P(x_0)$  implies that the

shadow price of the pollutant, which is defined by the rate of transformation at the frontier, is underestimated for this firm.



**Figure 1: The polluting technologies**

## 7. SUMMARY AND CONCLUSION

Polluting technologies have recently received much attention in the production analysis literature. A popular framework for modeling joint production is that of Färe et al. (1989;2005). The current article assesses their framework in the context of the materials balance condition. I show that acceptance of the axiom of weak disposability, currently fundamental to the analysis of Färe et al., is contingent on end-of-pipe abatement. Their model's implicit treatment of abatement will potentially cause underestimation of both technical efficiency and shadow prices of pollutants. This suggests that the generation of undesirable byproducts, as well as measures for reducing them, should be explicitly rather than implicitly modeled.

## 8. ACKNOWLEDGEMENT

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# Paper II



# Emission Constrained Firms: A Materials Balance Approach to Pollution Modeling

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**Abstract:** This paper proposes a new framework for modeling environmentally regulated firms. It offers simple empirical treatment while capturing dynamics of polluting technologies often neglected by competing models. The impact of environmental regulations on the costs of employing polluting inputs that leads to forgone profits is emphasized. This is crucial for determining producers' costs of compliance, and for unbiased efficiency measurement. Empirical assessment of economic performance in the realm of changing prices for fossil fuels is performed for a sample of American power plants to illustrate this point. Furthermore, marginal abatement cost estimates that lie close to allowance prices are obtained.

**Keywords:** *Polluting technologies; Materials balance; Nerlovian efficiency; Marginal abatement costs*

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## 1. INTRODUCTION

There are many desirable features of production models that accurately capture the joint production of desirable and undesirable outputs. An appropriate model will, among other things, allow for eco-efficiency measurement and estimation of marginal abatement costs. Not surprisingly, treatment of undesirable byproducts has become increasingly important in the production analysis literature. The current literature, e.g. Färe et al. (1989; 2005), primarily suggests treating byproducts either as inputs or outputs in the technology set. This allows for easy empirical implementation, something which has lead to a tremendous increase in the number of published articles that apply these models. However, some recent publications raise questions about the validity of the current modeling of undesirable outputs. Coelli et al. (2007) show that the popular models are inconsistent with the materials balance condition. The fundamental importance of this law of physics for the generation of undesirable byproducts was thoroughly illustrated in a seminal paper by Ayres and Kneese (1969) and should not be disregarded. Førsund (2009) further points out that current models conceal the various ways in which a firm can reduce its byproducts. The models are thus less likely to be capable of determining the least cost way of regulatory compliance.

This paper proposes a set theoretical production framework that overcomes the critique of the existing pollution models, but which still allows for simple empirical treatment. The framework is applied to illustrate how environmental regulations influence the costs of employing polluting inputs and lead to forgone profits. Consequentially, measuring overall efficiency without taking environmental regulations into account erroneously makes firms that operate efficient under their regulatory constraints appear allocative inefficient. The existing literature on polluting technologies, on the contrary, primarily focuses on the effect on technical efficiency and technical change of including undesirable byproducts in the technology set. This information provides limited insight about the economic implications of environmental regulations, since regulatory induced measurement biases in technical efficiency scores only arise under end-of-pipe abatement. It is only

one of many tools that firms have at their disposal for complying with environmental regulations. Since the model framework applied in this article allows for multiple producer responses to environmental regulations, I emphasize the importance of accounting for both technical and allocative efficiency when evaluating polluting firms' performance.

I build on previous studies by Krysiak and Krysiak (2003), Førsund (2009), and Murty and Russell (2010), and I define the polluting technology as the intersection of the conventional technology set and a pollution generating mechanism, where the latter is explicitly modeled through the materials balance principle. Recognizing that firms which are constrained in terms of undesirable byproducts also are implicitly constrained in terms of input usage through the materials balance condition, I treat environmentally regulated producers similar to expenditure constrained producers (Lee and Chambers, 1986; Färe et al., 1990). In contrast to the exogenous treatment of expenditure constraints, environmentally regulated firms may weaken the regulatory constraints through end-of-pipe abatement or purchases of emission allowances. These choices come at a cost. A menu of tools for complying with environmental regulations, i.e. input substitution, reductions in desirable outputs, and abatement activities, is now identified. This offers more flexible producer responses as compared to current modeling approaches, and allows for evaluating the least cost way of compliance. Hence, marginal abatement costs may readily be identified. Comparing optimal allocations with and without regulatory constraints further allows identifying biases in the measurement of efficiency.

Fossil fuel based power generation is one of the main sources for nitrogen oxides ( $\text{NO}_x$ ) and sulfur dioxide ( $\text{SO}_2$ ) emissions. In the United States, power plants reduced their emissions of the two pollutants nationally by 67 and 54 percent in the period 1995-2009 due to the Acid Rain Program. In recent years the prices for fossil fuels, in particular for natural gas, have had a strong upward trend. This may have economical implications for the electricity producers since coal to gas switching is considered an important instrument for regulatory compliance. Programming of emission restricted profit Data Envelopment Analysis (DEA) models is performed for a balanced panel of 67 power plants with fuel switching options to disentangle the impact of the regulatory reforms under changing fuel prices. Comparisons of Nerlovian efficiency scores calculated with and without regulatory constraints show that emission restrictions have significant impact on the measurement of allocative efficiency. Marginal abatement cost estimates that lie close to allowance prices for  $\text{NO}_x$  and  $\text{SO}_2$  are further obtained from the dual problem of linear programming.

This paper is structured as follows. The following section reviews previous treatment on byproducts in the production analysis literature. In section 3, I discuss the materials balance condition and its implications for pollution modeling. In section 4, I integrate the materials balance condition in a production framework and evaluate profit maximization under environmental regulations. I show that the solution to the emission restricted profit problem leads to forgone profits due to additional costs of employing polluting inputs. Section 5 discusses Nerlovian profit efficiency while section 6 defines the emission restricted DEA technology. Section 7 provides an overview of the U.S power generation industry and compilation of the dataset. Section 8 discusses the empirical results while section 9 concludes.

## 2. BACKGROUND

Pioneering literature on pollution modeling, e.g. Baumol and Oates (1975), Pittman (1981), and Barbera and McConnell (1990), suggest treating undesirable byproducts as if they are inputs. The rationale for this procedure is to utilize the axiom of free disposability of inputs, implying positive marginal productivities of undesirable byproducts, to capture firms' gain from increasing their byproducts. Positive marginal products are explained by the transfer of resources from abatement activities to production of desirable outputs that occurs when regulations are weakened. A problem arises, however, due to input substitution. The treatment of byproducts as inputs must imply that there exists substitution possibilities between the inputs that cause the byproducts and the byproducts themselves. Despite this questionable assumption, some recent publications, e.g. Lee (2005) and Bye et al. (2009), still apply the procedure.

A new era of polluting modeling started with the ideas of Färe et al. (1989; 2005): a set theoretical production framework that incorporates byproducts as outputs. They propose two non-conventional axioms, namely null-jointness and weak disposability of desirable and undesirable outputs, to capture the joint production of desirable and undesirable outputs. The implication of the latter is that undesirable outputs can be reduced when simultaneously reducing desirable outputs for a fixed input vector. This mechanism captures that emission reductions are costly, but it also neglects that firms often find less expensive ways of reducing emissions than output reductions. Nevertheless, the Färe et al. framework is used in a substantial amount of publications. Several studies, e.g. Coggins and Swinton (1996), Reig-Martinez et al. (2001), and Färe et al. (1993; 2006), derive marginal abatement costs as the value of forgone desirable outputs due to reductions in undesirable outputs. Application for eco-efficiency estimation is also popular, see e.g. Färe et al. (1996; 2004) and Tyteca (1997). Recent papers, e.g. Chung et al. (1997), Färe et al. (2001; 2007), and Ball et al. (2005), focus on the effects of neglecting joint production of desirable and undesirable outputs on the measurement of technical efficiency and technical change. Brännlund et al. (1995) apply the weak disposability framework to evaluate the impact of environmental regulations on the profitability of the Swedish pulp and paper industry. This is one of few exceptions in the environmental productivity literature that focus on allocative efficiency rather than technical efficiency measurement.

Recently, some authors have started questioning the validity of Färe et al.'s model. Coelli et al. (2007) argue that it is inconsistent with materials balance, and therefore not appropriate for modeling undesirable outputs. Førsund (2009) criticizes the limitation to producer responses implied by the framework, while Murty and Russell (2010) question the trade-off between desirable and undesirable outputs implied by a single production relation framework. Coelli et al. (2007) suggest using emission factors to characterize the materials balance condition. These factors can be treated similar to prices, implying that eco-efficiency evaluations can be performed similar to cost efficiency evaluations. The papers by Førsund and Murty and Russell suggest using a model structure containing multiple production relations when treating undesirable outputs. The modeling approaches they propose require extensive knowledge about the polluting technology, in particular by demanding that the researcher should readily be able to separate inputs used for desired production from inputs used for abatement purposes. This type of information is often hard to obtain.

In the current paper I combine the ideas of Coelli et al. with those of Førsund and Murty and Russell to propose a new framework for modeling undesirable outputs that is consistent with materials balance. Like Färe et al.'s model, my approach offers simple empirical implementations. Simplicity is achieved by recognizing that economic intuition allows for treating abatement efforts implicitly.

### 3. THE MATERIALS BALANCE CONDITION

Ayres and Kneese (1969) demonstrated the fundamental importance of the materials balance condition for joint production of desirable and undesirable outputs. Recently several authors, e.g. Pethig (2003; 2006), Ebert and Welsch (2007), Coelli et al. (2007), Lauwers (2009), and Førsund (2009), have revisited Ayres and Kneese's analysis and its applicability to pollution modeling. The materials balance condition postulates that materials can neither be created nor destroyed, but may change their form. Material inputs which are not recuperated by desirable outputs will therefore remain as (undesirable) byproducts from the production process. An obvious example is air pollution emissions from fossil fuel based power plants. Different fuels contain various amounts of SO<sub>2</sub> and NO<sub>x</sub> that is converted to gas form during combustion. Since there is no recuperation of these air pollutants in the electricity output, the amount converted during combustion constitutes undesirable byproducts from the production process.

The materials balance condition can be represented by emission factors. This is particularly convenient for non-point byproducts that cannot be directly measured. Emission factors report the amount of undesirable byproducts released per unit of inputs used, as well as the amount recuperated per unit of desirable outputs produced. Let  $x \in \mathfrak{R}_+^N$  denote a vector of inputs,  $y \in \mathfrak{R}_+^M$  denote a vector of desirable outputs, and  $b \in \mathfrak{R}_+^K$  denote a vector of undesirable byproducts. Let  $n$  be a  $(K \times N)$  matrix of input emission factors and  $m$  be a  $(K \times M)$  matrix of output recuperation factors. The materials balance conditions for the undesirable byproducts,  $b$ , are then defined:

$$b^{uc} = [nx - my] \quad (1)$$

Equation 1 reports uncontrolled byproducts. Firms often involve in end-of-pipe abatement activities in order to reduce the level of generated emissions. This can be represented by subtracting  $A \in \mathfrak{R}_+^K$  from equation 1<sup>1</sup>. Emissions remaining after abatement,  $b^c$ , are called controlled byproducts:

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<sup>1</sup> While giving comments to an early draft of this paper, Finn R. Førsund pointed out that I do not explain how emissions magically disappear in the abatement process. I believe this is not called for in the current article, but I recommend Førsund (2009) to the interested reader. However, following Førsund (2009) I consider abatement as a separate production process where abatement inputs are not used at the expense of production inputs. This allows interpreting abatement as a service or an input that firms purchase in order to reduce undesirable byproducts.

$$b^c = [nx - my] - A \quad (2)$$

The analysis in the article's next section uses equation 2 as point of departure. For notational convenience, the superscript  $c$  is dropped in the following.

#### 4. A POLLUTING TECHNOLOGY

The theory presented in this section is influenced by recent studies of Krysiak and Krysiak (2003), Førsund (2009), and Murty and Russell (2010). They all propose to model polluting technologies consisting of multiple production relations rather than the conventional single production relation framework. The use of multiple production relations and its implications for the freedom of assigning inputs to outputs is due to Frisch (1965). In a Frisch setting, the single production relation framework defines a production with full degree of assortment. That is, there exist transformation possibilities between all outputs for each input vector. Consider now the previous example of air pollutants from fossil fuel based power plants. A model that incorporates the pollutants as outputs, and assumes full degree of assortment, implies the existence of transformation possibilities between the air pollutants and the electricity output for each input vector. This is not consistent with equation 1 that ties the generation of pollutants to the employment of inputs. In other words, the single production relation models allow for too large degree of freedom in determining the output mix. I therefore define the polluting technology as the intersection of the conventional technology set and the materials balance conditions:

$$\begin{aligned} T(b + A) &= T1 \cap T2 \\ T1 &= \{(x, y) : x \text{ can produce } y\} \\ T2 &= \{(x, y) : [nx - my] = b + A\} \end{aligned} \quad (3)$$

Heuristically, one may think of this model as recursive, where decisions about the employment of inputs and desirable outputs directly determine the occurrences of undesirable outputs through the materials balance conditions. Undesirable outputs have become byproducts, and there exists no transformation possibilities between desirable outputs and undesirable outputs for a given input vector<sup>2</sup>.

$T1$  is assumed to be a nonempty, closed, and convex set. No fixed costs, no free lunch, and free disposability of inputs and desirable outputs are assumed to prevail. See Chambers (1988) for a discussion of these properties.

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<sup>2</sup> This is particularly important for the cases where the output recuperation matrix is the zero matrix. In the case of positive recuperation factors, byproducts can be decreased when simultaneously increasing desirable outputs. An example of the latter is found in the salmon farming industry where feed conversion ratio has declined substantially since the industry's infancy, leading to reductions in effluent discharges from the farms (Tveterås, 2002).

Next, I introduce environmental regulations. For simplicity, I use the example of a command-and-control policy where the producers treat emission targets,  $b^*$ , as exogenously determined. Let  $T2$  be redefined as the set of inputs and desirable outputs which satisfy the emission targets through equation 2<sup>3</sup>.

$$T2 = \{(x, y) : [nx - my] \leq b^* + A\} \quad (4)$$

Bounds on emissions must similarly imply bounds on producers' input use through the materials balance equations. From equation 4, it follows that there exists a limited set of inputs and desirable outputs that are consistent with regulatory constraints,  $b^*$ , for a fixed  $A$ . Environmentally regulated firms can therefore be treated similar to expenditure constrained firms. The theory discussed in this section thus resembles the treatment on expenditure constraints in Lee and Chambers (1986) and Färe et al. (1990). However, my analysis deviates from these studies by allowing emission constraints to be endogenously rather than exogenously determined.

Firms have multiple tools at their disposal for complying with environmental regulations. Førsund and Strøm (1988) consider input substitution, output reductions, technical change, waste recycling, and end-of-pipe abatement activities as potential strategies. The three first mentioned strategies concern technology  $T1$ , while the latter strategies expand the set  $T2$  from equation 4: Since abatement is modeled by subtracting  $A \in \mathfrak{R}_+^K$  from the uncontrolled emissions, it means that increased abatement is equivalent to weakening the emission targets and expanding the set of feasible inputs and desirable outputs. This generates a trade-off for the producers: Involvement in abatement activities may allow them to reach more profitable input-output allocations. However, abatement is costly.

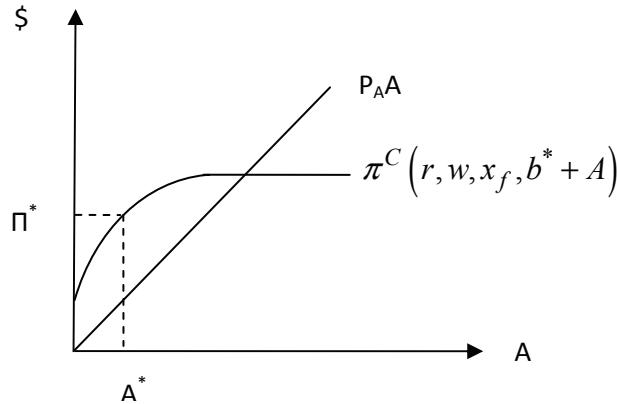
Considering short run profit maximization, I partition the input vector into  $V$  variable inputs and  $F$  (quasi)fixed inputs, i.e.  $x = (x_v, x_f)$ . Let  $w \in \mathfrak{R}_{++}^V$ ,  $r \in \mathfrak{R}_{++}^M$ , and  $P_A \in \mathfrak{R}_+^K$  be vectors of prices for variable inputs, desirable outputs, and abatement, respectively. The model structure, with separate abatement production and regulatory decisions, allows defining the short run profit maximization problem for a producer that complies with environmental regulations as:

$$\begin{aligned} \pi(r, w, P_A, x_f, b^*) &= \sup_{x_v, y, A} \left\{ ry - wx_v - P_A A : (x, y) \in T1, [nx - my] \leq b^* + A \right\} \\ &= \sup_A \left\{ \sup_{x_v, y} \left\{ ry - wx_v : (x, y) \in T1, [nx - my] \leq b^* + A \right\} - P_A A \right\} \quad (5) \\ &= \sup_A \left\{ \pi^C(r, w, x_f, b^* + A) - P_A A \right\} \end{aligned}$$

---

<sup>3</sup> Equation 4 can be interpreted as violating the axiom of free disposability of inputs for the polluting technology,  $T$ . This resembles Färe et al.'s weak disposability assumption, since polluting inputs cannot be freely disposed. However, I refrain from using this terminology since free disposability concerns technological characteristics, while equation 4 defines a constraint on the use of the technology.

$\pi^C$  represents the emission constrained short run profit function. The solution to the profit maximization problem from equation 5 is characterized by the level of abatement that maximizes the difference between restricted profits and abatement costs. At this level of abatement, the marginal benefit of expanding the restricted technology equals the marginal abatement costs. This concept is illustrated in figure 1:



**Figure 1: Emission restricted profit maximization**

Figure 1 shows that the solution to the emission restricted profit problem leads to forgone profits relative to the maximum of profits for a firm that is not complying with environmental regulations. The unrestricted profit maximum is defined by the conventional short run profit function:

$$\pi(r, w, x_f) = \sup_{x_v, y} \{ry - wx_v : (x, y) \in T1\} \quad (6)$$

This unrestricted profit maximum is represented by the maximum of  $\pi^C$  in figure 1. Define the inequality:

$$\pi(r, w, x_f) \geq \pi^C(r, w, x_f, b^* + A) \quad (7)$$

Following the argument by Färe and Logan (1983), the unrestricted profit function can be retrieved from the restricted profit function by applying the inequality in equation 7:

$$\pi(r, w, x_f) = \sup_A \{\pi^C(r, w, x_f, b^* + A)\} \quad (8)$$

When increasing abatement,  $A$ , the set of feasible combinations of inputs and desirable outputs is expanded. By sufficiently expanding the set, the unconstrained profit maximum can be achieved. Consider now the case where the abatement costs in equation 5 is zero. The profit maximization problem for the emission restricted producer is then reduced to:

$$\begin{aligned}\pi(r, w, 0, x_f, b^*) &= \sup_A \left\{ \pi^C(r, w, x_f, b^* + A) \right\} \\ &= \pi(r, w, x_f)\end{aligned}\tag{9}$$

where the last equality follows from equation 8. Equation 9 shows that the constrained and the unconstrained profit problem coincide when abatement costs are zero. In this case, the producer can employ polluting inputs without facing additional costs related to their cleanup.

Cap and trade regulations offer additional possibilities for expanding the emission restricted set  $T_2$  through purchases of emission allowances. Rather than expanding abatement,  $A$ , this will augment the restricted emission level,  $b^*$ . Purchases of allowances can be incorporated in equation 5 in the same way as abatement. I treat abatement efforts and allowance purchases as perfect substitutes in the following. That is, whenever a producer seeks to expand the set of feasible input-output combinations he will choose the least cost way of doing so.

The main insight of the previous discussion is that environmental regulations impose additional costs on the use of polluting inputs. Their employment induces costs related to cleanup of corresponding emissions, which results in forgone profits for regulated firms. A measure of overall efficiency that does not take this into account will assess regulated firms as inefficient, even if they are optimally allocated under their regulatory constraints. This obvious point is, in general, neglected by current literature on pollution modeling. Studies such as Färe et al. (2001; 2007) and Ball et al. (2005) have primarily focused on the effect of including pollutants in the technology set on technical efficiency and technical change. The rationale for doing this is to avoid that inputs used for preventing emissions appear as unproductive in the analysis. This idea neglects that there are multiple ways in which a firm can comply with environmental regulations. The assumption of weak disposability implies that firms must divert resources from the production of desirable outputs for reducing undesirable outputs. This can only be consistent with enhanced end-of-pipe abatement. Other tools for complying with environmental regulations, e.g. input substitution, output reductions, technical change, or purchase of emission permits, do not require extended input use and will therefore not make the inputs come off as unproductive in the analysis. Regulatory induced allocations are, however, likely to appear as allocative inefficient if regulatory constraints are not accounted for.

## 5. NERLOVIAN PROFIT EFFICIENCY AND THE DIRECTIONAL DISTANCE FUNCTION

The directional distance function and its duality to the profit function were introduced by Chambers et al. (1998). The directional distance function encompasses all known distance functions as special cases. This is due to the flexibility of selecting the direction in which inputs and outputs are projected to the technology frontier through the choice of the direction vector  $g = (g_x, g_y)$  in  $\mathbb{R}_+^N \times \mathbb{R}_+^M$ . In my case, the direction vector is set equal to  $g_f = 0$  for the (quasi)fixed inputs. The directional distance function is then defined as:

$$\vec{D}(x, y; -g_v, g_y) = \sup \left\{ \beta \in \mathfrak{R} : (x_v - \beta g_v, y + \beta g_y) \in T_1 \right\} \tag{10}$$

The directional distance function inherits the properties of the parent technology. It satisfies the translation property and is homogeneous of degree minus one in  $g$ , nondecreasing in  $x_v$ , nonincreasing in  $y$  and concave in  $(x_v, y)$ . Under free disposability, the directional distance function is a complete characterization of the underlying technology in the sense that:

$$\vec{D}(x, y; -g_v, g_y) \geq 0 \text{ if and only if } (x, y) \in T_1 \quad (11)$$

The short-run unrestricted profit function from equation 6 may be defined in terms of the directional distance function:

$$\pi(r, w, x_f) = \sup_{x_v, y} \left\{ ry - wx_v : \vec{D}(x, y; -g_v, g_y) \geq 0 \right\} \quad (12)$$

Chambers et al. (1998) show how the optimization problem in equation 12 can be written as an unconstrained problem. It follows that:

$$\pi(r, w, x_f) \geq ry - wx_v + (rg_y + wg_v) \vec{D}(x, y; -g_v, g_y) \quad (13)$$

Rewriting expression 13 and adding allocative inefficiency,  $AI$ , to secure equality, the Nerlovian profit efficiency and its decomposition into technical and allocative inefficiency is defined:

$$\frac{\pi(r, w, x_f) - [ry - wx_v]}{(rg_y + wg_v)} = \vec{D}(x, y; -g_v, g_y) + AI \quad (14)$$

The Nerlovian profit efficiency measure is invariant to proportional price changes due to the normalization  $(rg_y + wg_v)$ . The normalization allows maximum profits to be zero or negative, which gives the Nerlovian measure an advantage over other profit efficiency measures that use maximum profits as normalization. Furthermore, the measure allows for identifying profit inefficiency due to misallocations of inputs and outputs. This is desirable since the purpose of the current paper is to consider the effect of environmental regulations on the measurement of allocative efficiency.

## 6. EMPIRICAL IMPLEMENTATION

I apply linear programming techniques for computing profit efficiency. Assume that there are  $l=(1, \dots, L)$  firms in the dataset. Each firm uses inputs  $x^l = (x_1^l, \dots, x_N^l) \in \mathfrak{R}_+^N$  to produce desirable outputs  $y^l = (y_1^l, \dots, y_M^l) \in \mathfrak{R}_+^M$ . The inputs are partitioned into  $V$  variable inputs and  $F$  quasifixed

inputs, i.e.  $x^l = (x_v^l, x_f^l)$ . Let  $\lambda^l$ ,  $l=(1,..,L)$ , be the intensity variables. The emission constrained DEA profit function for firm  $l'$  can then be defined as equation 15.

$$\begin{aligned} \pi(r^{l'}, w^{l'}, x_f^{l'}, b^{l'} + A^{l'}) = \max_{y, x_v, \lambda} & \sum_{m=1}^M r_m^{l'} y_m - \sum_{n=1}^V w_n^{l'} x_n : \quad \sum_{l=1}^L \lambda^l y_m^l \geq y_m, \quad m=1, \dots, M \\ & \sum_{l=1}^L \lambda^l x_n^l \leq x_n, \quad n=1, \dots, V \\ & \sum_{l=1}^L \lambda^l x_n^l \leq x_n^{l'}, \quad n=V+1, \dots, N \\ & \lambda^l \geq 0, \quad l=1, \dots, L, \\ & \sum_{l=1}^L \lambda^l = 1, \\ & \sum_{n=1}^N n_k^{l'} x_n - \sum_{m=1}^M m_{km}^{l'} y_m \leq b_k^{l'} + A_k^{l'}, \\ & k=1, \dots, K \end{aligned} \tag{15}$$

Since some of the firms in the sample have negative short run profits, I assume that the intensity variables sum to one to allow for positive, negative, or zero maximal profits. The DEA model in equation 15 is computed twice for each firm. In the first computation, the materials balance constraints from equation 4 are included in the model. Equation 15 then determines the maximal short run profits for the emission restricted technology. In the second computation, the materials balance constraints are omitted. The optimization problem then defines maximal profits for the unrestricted technology  $T1$ , when the producer faces no cleanup costs related to employment of polluting inputs. A similar procedure is applied by Färe et al. (2004) who assess the effect of risk-based capital requirements in banking on Nerlovian profit scores.

The directional distance function is similarly calculated for each firm in the sample. For firm  $l'$ , the directional distance function is defined according to equation 16.

$$\begin{aligned} \bar{D}(x^{l'}, y^{l'}; -g_v, g_y) = \max_{\beta} \beta : & \sum_{l=1}^L \lambda^l y_m^l \geq y_m^{l'} + \beta y_m^{l'} \quad m=1, \dots, M \\ & \sum_{l=1}^L \lambda^l x_n^l \leq x_n^{l'} - \beta x_n^{l'} \quad n=1, \dots, V \\ & \sum_{l=1}^L \lambda^l x_n^l \leq x_n^{l'} \quad n=V+1, \dots, N \\ & \lambda^l \geq 0, \quad l=1, \dots, L, \quad \sum_{l=1}^L \lambda^l = 1 \end{aligned} \tag{16}$$

I apply observed levels of variable inputs and outputs as the direction vector. The profit difference in equation 14 is then normalized by the sum of observed revenue and variable costs, which may be perceived as a proxy for size of the firm (Färe et al., 2004). In contrast to the profit function, the distance function is only calculated without the materials balance condition. The materials balance constraints prevent expansions of inputs, while the directional distance function is characterized by contractions of inputs. The materials balance constraints cannot restrict this optimization problem and are therefore not relevant when estimating the directional distance function.

The dual problem of linear programming offers a way of identifying shadow prices and, thus, to determine the impact on profits from marginally relaxing the constraints in equation 15. Of particular interest to this paper are the shadow prices on the materials balance constraints. The constrained optimization problem in equation 5 suggests that the producers will allocate where the marginal benefits of expanding technology  $T$  equal the marginal costs of abatement. The marginal benefits of relaxing the emission constraints are identified by the shadow prices on the materials balance constraints, implying that marginal abatement costs can readily be identified from the dual problem under the assumption that firms are optimally allocated.

## 7. EMISSION REDUCTIONS IN U.S. ELECTRICITY GENERATION

Fossil fuel based power generation is one of the main sources of  $\text{NO}_x$  and  $\text{SO}_2$  emissions. The Acid Rain Program was introduced in 1995 in order to reduce American power plants' emissions of the two air pollutants. The program was implemented in two phases, and called for reductions of 10 million tons in annual  $\text{SO}_2$  emissions and 2 million tons in annual  $\text{NO}_x$  emissions from 1980 levels. In the period from 1995 to 2005, annual emissions of  $\text{SO}_2$  fell from 13,000 thousand metric tons to 10,000 thousand metric tons while  $\text{NO}_x$  emissions fell from 6,000 thousand metric tons to 4,000 thousand metric tons. An additional cap and trade program for  $\text{NO}_x$ , the Ozone Transport Commission program, was introduced in 1999. It was later replaced by the federal  $\text{NO}_x$  Budget Trading Program in 2003. In 2005, the Environmental Protection Agency issued their Clean Air Interstate Rule (CAIR) that uses a cap-and-trade system to achieve the goal of reducing  $\text{NO}_x$  and  $\text{SO}_2$  emissions by 60 and 70 percent, respectively, from 2003 levels. The rule was temporary suspended in 2008, but was reinstated the same year. Emissions to air declined further, reaching 6,000 thousand metric tons of  $\text{SO}_2$  emissions and 2,000 thousand metric tons of  $\text{NO}_x$  emissions in 2009.

Fuel switching has been an important measure for complying with the Acid Rain regulation. This includes switching between different types of coal or substituting coal with other fossil fuels. The latter often implies switching to natural gas that emits substantially less  $\text{NO}_x$  and  $\text{SO}_2$  per unit of fuel than coal. However, recent years have seen steep increases in the prices for fossil fuels. The average price for coal rose from 120 cent/mmbtu in year 2000 to 210 cent/mmbtu in 2009. The average price for natural gas rose from 430 cent/mmbtu in year 2000 to 821 cent/mmbtu in 2005, causing a major change in the relative price between coal and gas. The price remained high and volatile until it fell to 474 cent/mmbtu in 2009. This calls for an assessment of changing fuel prices' implication for producer profits, as increasing gas prices raise the costs of regulatory compliance. When coal to gas switching becomes expensive the producers are likely to choose other tools for compliance, e.g. end-of-pipe abatement or purchases of emission allowances. If these tools are unavailable or too costly, the result is likely to be a slowdown in the generation of electricity.

I apply the model framework outlined in section 4 to a sample of regulated American power plants with coal to gas switching capacity for this inquiry. The sample consists of 67 fossil fuel fired power plants observed in the years 2002-2009. To model a homogeneous production technology with fuel substitution possibilities I follow Welch and Barnum (2009) and only include firms that obtain at least 1 percent of their energy input from both coal and gas in year 2002 in the sample. The price for gas was low in 2002, implying that firms with coal to gas switching capacity are likely to have exploited this opportunity in that particular year. Firms that satisfy the selection criterion in 2002 and use both coal and gas as energy inputs the following years are kept in the sample. Producers that convert to single fuel production are excluded to avoid corner solutions.

I define the DEA technology in equation 15 and 16 to consist of one desirable output, electricity, and two variable inputs, coal and gas. Coal and gas capacity are treated as quasifixed inputs. The form EIA-906/920 provides monthly information on fuel consumption and net generation. This information is aggregated up to annual levels. Prices and sulfur content of the fuels are obtained from EIA-423/923. Existing generating capacity is obtained from EIA statistics on capacity. Generating capacity is further divided into coal capacity and gas capacity by applying information on each boiler's primary and secondary fuels. The sales price for electricity is calculated from retail and resale revenues reported in EIA-861. Following Färe et al. (2005), the sales price is taken to be the average of the retail and resale price.

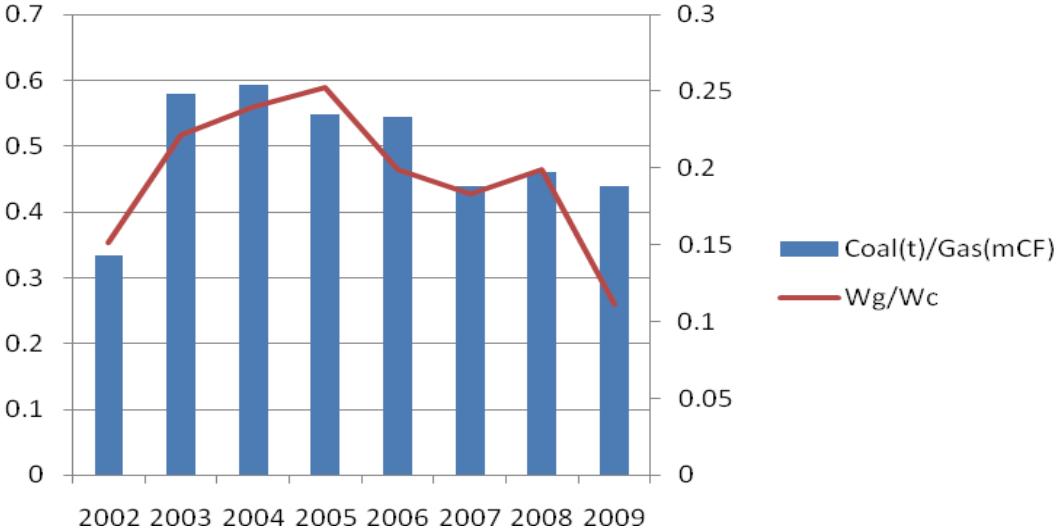
The emission factors in equation 1 and 2 are of crucial importance to this analysis. Appendix A of EIA's Electric Power Annual provides an overview of emission factors for various sub-groups of coal, gas, and petroleum. I adjust the emission factors with corresponding sulfur content of the fuels when specified by EIA. The factors are further adjusted to report tons of SO<sub>2</sub> emissions per unit of weight of fuels. Uncontrolled emissions of SO<sub>2</sub> and NO<sub>x</sub> are calculated by applying the emission factors and the firms' fuel inputs. I assume that firms are optimally allocated in terms of the amount of emissions they generate, and therefore use the calculated uncontrolled emissions as the emission constraints,  $b^* + A$ , in equation 15. The constraints thus reflect the firms' emission targets, allowance purchases, and abatement efforts.

I define aggregate variables for coal and gas rather than using the fuels' subgroups as inputs. The purpose is to avoid the problem of missing observations for disaggregated inputs. To capture that different subgroups generate different amounts of undesirable outputs, I define aggregate emission factors for coal and gas as the weighted sum of the emission factors for the fuels' subgroups, using the share of fuels purchased from the various subgroups as weights. Summary statistics for the complete dataset is provided in table 4 in appendix B<sup>4</sup>.

I plot the ratio of average coal to gas quantities and corresponding relative prices in figure 2, to highlight the sample's capacity to substitute between coal and gas. In 2002, the amount of gas used for electricity production is the highest observed due to its low price. In the following years, the use of gas declines substantially as the gas price doubles from year 2002 to 2005, making coal to gas switching a less attractive instrument for regulatory compliance. Increasing prices for coal in the period after 2004 distort the price ratio and again provides the firms with incentives to increase the gas share of fuels.

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<sup>4</sup> The dataset is available from the author upon request.



**Figure 2:** Ratio of coal to gas for firms in the sample

## 8. RESULTS

Table 1 presents mean Nerlovian efficiency scores for the models with (*NE cons*) and without (*NE unc*) the materials balance constraints. Standard deviations for the estimates are reported in brackets. The efficiency scores are further decomposed into technical efficiency (*TE*) and allocative efficiency (*AE*) for both models. The difference between the allocative efficiency scores for the constrained and the unconstrained model is a measure of forgone profits due to regulatory constraints (*AE reg*).

**Table 1: Nerlovian efficiency**

YEAR	NE unc	NE cons	TE	AE unc	AE cons	AE reg
2002	0.34 (0.50)	0.17 (0.21)	0.08 (0.11)	0.27 (0.44)	0.09 (0.13)	0.18 (0.41)
2003	0.44 (0.58)	0.22 (0.24)	0.14 (0.13)	0.30 (0.53)	0.08 (0.13)	0.22 (0.53)
2004	0.35 (0.45)	0.21 (0.21)	0.13 (0.13)	0.22 (0.41)	0.08 (0.11)	0.13 (0.38)
2005	0.34 (0.34)	0.24 (0.22)	0.15 (0.15)	0.20 (0.24)	0.09 (0.11)	0.10 (0.18)
2006	0.38 (0.64)	0.24 (0.30)	0.13 (0.13)	0.25 (0.57)	0.11 (0.21)	0.14 (0.46)
2007	0.40 (1.15)	0.20 (0.22)	0.11 (0.12)	0.29 (1.15)	0.09 (0.14)	0.10 (1.14)
2008	0.30 (0.43)	0.22 (0.31)	0.10 (0.12)	0.20 (0.37)	0.11 (0.24)	0.08 (0.17)
2009	0.96 (2.65)	0.43 (0.90)	0.12 (0.16)	0.84 (2.59)	0.31 (0.79)	0.53 (2.20)

I first consider the importance of accounting for environmental regulations for unbiased efficiency measurement. The ANOVA and Kolmogorov-Smirnov tests are applied for each year from 2002 to 2009, to test whether the constrained allocative efficiency scores (*AE cons*) are significantly smaller than the unconstrained allocative efficiency scores (*AE unc*). P-values for the two tests are reported in appendix A. Both tests indicate significant differences in the magnitude of the efficiency scores for the years from 2002 to 2006. For the years from 2007 to 2009 there is no statistical significant difference.

Following Färe et al. (1990), I calculate the ratios of variable inputs and outputs that solve the profit maximization problems with and without emission constraints:

**Table 2: Ratio of optimal input use and supply**

YEAR	Coal	Gas	Electricity
2002	0.70 (0.30)	1.03 (0.57)	0.82 (0.24)
2003	0.64 (0.33)	1.91 (1.95)	0.78 (0.26)
2004	0.74 (0.27)	2.22 (2.96)	0.86 (0.19)
2005	0.72 (0.29)	1.23 (0.66)	0.82 (0.22)
2006	0.76 (0.29)	1.87 (2.06)	0.85 (0.20)
2007	0.81 (0.26)	3.84 (7.44)	0.91 (0.20)
2008	0.77 (0.28)	1.47 (1.00)	0.88 (0.18)
2009	0.73 (0.33)	1.69 (2.66)	0.84 (0.24)

The ratios indicate that optimal coal use and electricity supply are lower under constrained profit maximization than under unconstrained profit maximization. Gas, on the other hand, is over-utilized relative to the unconstrained maximum due to its corresponding low emissions. The large standard deviations for the gas ratio indicate that the incentives to switch from coal to gas differ across firms in the sample.

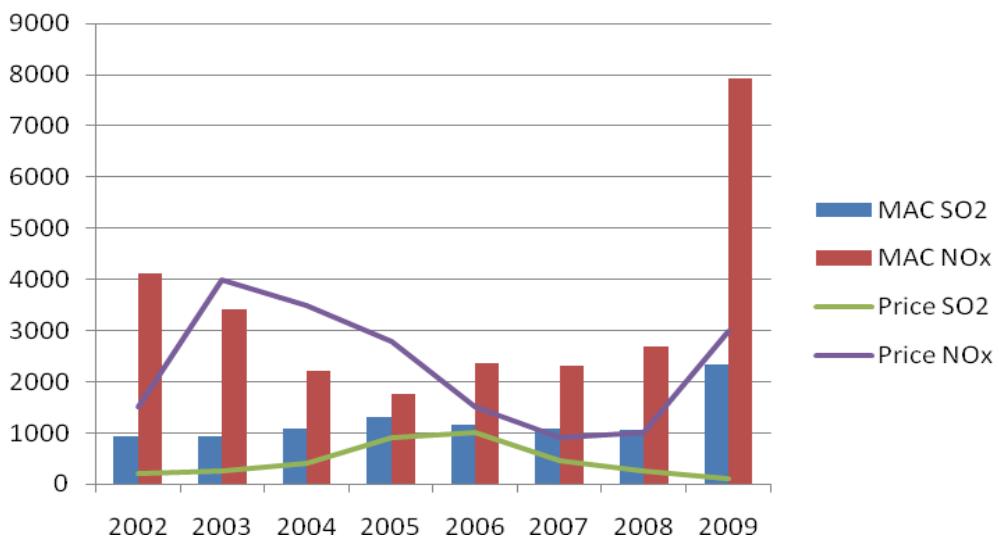
Both the ANOVA and Kolmogorov-Smirnov tests indicate that the coal use and electricity supply are closer to their unconstrained levels in the period 2007-2009 compared to the period 2002-2006. For gas, there is no significant difference. Table 4 from appendix B shows that mean uncontrolled emissions increase in the period after 2004, indicating that producers have increased their end-of-pipe abatement efforts or the purchase of emissions allowances to reach more profitable input-output allocations. This is potentially affected by the increasing price for natural gas which makes coal to gas switching an expensive strategy.

The significant difference in allocative efficiency scores calculated with and without regulatory constraints indicates the importance of explicitly accounting for environmental regulations when evaluating firms' overall performance. The measure of forgone profits due to regulation,  $AE_{reg}$ , provides the largest contribution to allocative inefficiency for the unregulated technology. Technical efficiency scores, on the other hand, appear as reasonably stable for all years between 2002 and 2009. If increases in uncontrolled emissions in the period following 2004 are related to abatement efforts, the current literature on pollution modeling postulates that one should observe a decline in technical efficiency as more resources are devoted to this activity. This is, however, not the case in this study. Since end-of-pipe abatement is only one of many potential responses to environmental regulations, my results indicate the importance of accounting for both technical and allocative efficiency when evaluating the economic impact of environmental regulations.

Nerlovian efficiency scores indicate that approximately 18 percent of the sample is profit efficient. The number of efficient firms is largely unaffected by inclusion of the materials balance constraints. Note, however, that mean emission constrained allocative efficiency ( $AE_{cons}$ ) from table 1 is close to zero in most years. The corresponding standard deviations are also low and reasonably stable across all years. This indicates that most firms are operating close to allocative efficient under the existing environmental regulations. Both mean allocative efficiency and corresponding standard deviations are substantially higher when the emission constraints are not accounted for, i.e. for  $AE_{unc}$ . The large and fluctuating standard deviations for forgone profits ( $AE_{reg}$ ) indicate that the costs of regulatory compliance vary across firms as well as over time. Local

differences in regulations for air pollutants, as well as changes in regulations over time, are likely to play an important role in determining these differences<sup>5</sup>.

The dual problem of linear programming is applied to obtain marginal abatement costs (MAC) for NO<sub>x</sub> and SO<sub>2</sub>. Mean abatement cost estimates and corresponding allowance prices are reported by figure 3.



**Figure 3: Marginal abatement costs**

High marginal abatement costs for NO<sub>x</sub> in 2002 and 2003 reflect the substantial levels of forgone profits due to environmental regulations (*AE reg*) from table 1. A high price for NO<sub>x</sub> allowances, making it costly to involve in coal intensive production in these years, may have caused the loss in profits. The NO<sub>x</sub> Budget Trading Program was launched in 2003 and was the main driver of the high price level. The allowance price fell in the following period, coinciding with declining marginal abatement costs. In 2009, both the allowance price and marginal abatement costs increase dramatically. This is due to the implementation of the new CAIR regulation, but is probably also influenced by external factors. In particular, the recent financial turmoil is likely to have had an impact on firms' economic performance.

The influence of allowance prices on marginal abatement costs is also visible for SO<sub>2</sub>. The allowance price for SO<sub>2</sub> was steadily increasing in the period from 2004 to 2006, peaking at \$1600/ton in early 2006. This price increase can be attributed to the proposition of CAIR<sup>6</sup>. A similar pattern is detected in the estimates of marginal abatement costs for SO<sub>2</sub>. The steep increase in

<sup>5</sup> Whenever these and other explanatory variables are available one may regress them on the AEreg efficiency scores to explain the variation in the costs of compliance. The Tobit regression is preferable, since the efficiency scores are censored at zero. I refrain from doing this in the current paper, since I am primarily interested in the differences in allocative efficiency when calculated with and without environmental constraints. See Blancard et al. (2006) for an example of a second stage regression procedure for Nerlovian efficiency.

<sup>6</sup> See <http://www.epa.gov/airmarkt/resource/docs/marketassessmnt.pdf>

abatement costs in 2009 may reflect that the new emission standards following the CAIR regulation are so strict that they require investments in abatement equipment for compliance, thereby leading to a deviation between allowance prices and abatement costs.

## 9. SUMMARY AND CONCLUSIONS

This paper presents a new framework for studying firms that comply with environmental regulations. A polluting technology defined by the intersection of the conventional technology set and a pollution generating mechanism, where the latter is modeled through the materials balance condition, is the point of departure. Recognizing that firms which are constrained in terms of emissions also are constrained in terms of input use, I propose a theory of emission constrained firms that resembles the well-known theory of expenditure constrained firms. An emission constrained profit problem that captures the dynamics of environmental regulations, in terms of increased costs of polluting inputs and forgone profits, is defined. It points to the crucial role of accounting for regulatory constraints in the measurement of overall efficiency, to avoid underestimation of firms' performance.

The Acid Rain program was implemented in 1995 in order to reduce NO<sub>x</sub> and SO<sub>2</sub> emissions from U.S power plants. The electricity producers' key responses to the regulation are fuel switching and end-of-pipe abatement. I assess the regulation's economic impact for a sample of 67 power plants with coal to gas switching capacity for the period from 2002 to 2009. The price for gas was high and volatile in this period, making coal to gas switching less attractive. It provides firms with incentives to choose other strategies for regulatory compliance that allow them to use more of the relatively cheaper coal inputs. The ability to increase coal firing reduces forgone profits due to regulatory compliance. Marginal abatement cost estimates that lie close to observed allowance prices for the two pollutants further support this conclusion.

## 10. ACKNOWLEDGEMENT

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## 12. APPENDIX A

**Table 3:** Tests for differences in allocative efficiency scores (P-values)

YEAR	Kolmogorov-Smirnov	ANOVA
2002	0.003	0.0020
2003	0.003	0.0012
2004	0.005	0.0108
2005	0.013	0.0015
2006	0.035	0.0618
2007	0.117	0.1623
2008	0.299	0.1155
2009	0.225	0.1145

### 13. APPENDIX B

**Table 4: Summary Statistics**

		2002	2003	2004	2005	2006	2007	2008	2009
<i>COAL</i>	(t)	1207965.0 (1219278.0)	1184212.0 (1225455.0)	1272225.0 (1257144.0)	1282352.0 (1287343.0)	1292302.0 (1294303.0)	1285967.0 (1226576.0)	1243757.0 (1239058.0)	1099866.0 (1215551.0)
<i>GAS</i>	(mCF)	3609969.0 (7113197.0)	2045929.0 (4443897.0)	2143212.0 (4817720.0)	2342252.0 (5286427.0)	2371563.0 (4996507.0)	2928845.0 (5701523.0)	2700399.0 (6226449.0)	3162203.0 (7405526.0)
<i>EL</i>	(MwH)	2840733.0 (2997614.0)	2643367.0 (2807723.0)	2813297.0 (2814453.0)	2836137.0 (2895691.0)	2817207.0 (2898768.0)	2880542.0 (2906832.0)	2735781.0 (2855661)	2503911.0 (2858071.0)
<i>COAL</i>	<i>CAP</i>	364.2 (377.1)	364.2 (365.4)	351.5 (365.7)	350.9 (365.7)	350.6 (365.4)	360.5 (393.0)	379.7 (408.2)	379.7 (408.2)
<i>GAS</i>	<i>CAP</i>	327.6 (331.8)	327.6 (331.8)	355.9 (347.7)	356.6 (347.9)	366.5 (361.6)	363.3 (338.1)	355.0 (336.6)	355.0 (336.6)
<i>b<sub>UC</sub></i>	(t)	15368.8 (19200.0)	14446.7 (18264.6)	15797.5 (18645.7)	16849.9 (21840.7)	17171.0 (22536.4)	16684.6 (22033.1)	16236.3 (22208.4)	13577.8 (18935.7)
<i>b<sub>NOX</sub><sup>UC</sup></i>	(t)	7054.9 (7089.2)	6669.1 (7039.0)	7181.8 (6989.6)	7377.1 (7350.7)	7445.7 (7394.0)	7530.2 (7317.3)	7204.0 (7086.3)	6332.9 (6665.8)
<i>n<sub>COUL</sub></i>	<i>n<sub>S02</sub></i>	1.51e <sup>-02</sup> (1.28e <sup>-02</sup> )	1.48e <sup>-02</sup> (1.25e <sup>-02</sup> )	1.51e <sup>-02</sup> (1.27e <sup>-02</sup> )	1.59e <sup>-02</sup> (1.44e <sup>-02</sup> )	1.56e <sup>-02</sup> (1.31e <sup>-02</sup> )	1.48e <sup>-02</sup> (1.22e <sup>-02</sup> )	1.56e <sup>-02</sup> (1.40e <sup>-02</sup> )	1.57e <sup>-02</sup> (1.36e <sup>-02</sup> )
<i>n<sub>COUL</sub><sup>GAS</sup></i>	<i>n<sub>NOX</sub></i>	5.70e <sup>-03</sup> (1.25e <sup>-03</sup> )	5.68e <sup>-03</sup> (1.38e <sup>-03</sup> )	5.73e <sup>-03</sup> (1.24e <sup>-03</sup> )	5.77e <sup>-03</sup> (1.22e <sup>-03</sup> )	5.76e <sup>-03</sup> (1.23e <sup>-03</sup> )	5.76e <sup>-03</sup> (1.23e <sup>-03</sup> )	5.75e <sup>-03</sup> (1.23e <sup>-03</sup> )	5.74e <sup>-03</sup> (1.23e <sup>-03</sup> )
<i>n<sub>S02</sub><sup>GAS</sup></i>		3.00e <sup>-07</sup> (0.00e <sup>+00</sup> )							
<i>r<sub>EL</sub></i>		45.7 (9.8)	47.3 (10.7)	48.2 (11.1)	57.4 (15.4)	60.7 (16.6)	61.6 (13.2)	66.8 (15.8)	64.2 (14.1)
<i>W<sub>COUL</sub></i>		28.1 (10.5)	27.8 (9.9)	30.2 (12.8)	36.6 (16.6)	40.1 (17.6)	42.9 (18.5)	50.3 (24.2)	56.2 (27.9)
<i>W<sub>GAS</sub></i>		4.3 (1.4)	6.1 (1.2)	7.3 (2.9)	9.2 (1.3)	8.0 (0.9)	7.9 (1.1)	10.0 (2.4)	6.2 (3.4)



# Paper III



# True Abatement Costs

Kenneth Løvold Rødseth

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**Abstract:** In recent years the production analysis literature has been increasingly concerned with estimating marginal abatement costs. Previous studies have, however, treated both pollution generation and emission reduction as a black box. This paper conceptualizes the polluting technology as the intersection of the conventional technology set and a pollution generating mechanism, where the latter is modeled through the materials balance principle. The framework allows the producers to reduce their emissions through input substitution or output reductions, as well as by abatement. By capturing flexible producer responses, the least cost way of regulatory compliance and, thus, marginal abatement costs can be identified. A constrained profit maximization problem is proposed in the case where the abatement technology is separate from the polluting production process. Using the directional distance function, marginal abatement costs can be identified when having information about emission factors, input or output prices, and quantities of inputs and desirable outputs. The directional output distance function and its duality to the revenue function allow for obtaining abatement cost estimates in the case where abatement is integrated in the production of intended outputs. This procedure is shown to be the reciprocal of Färe et al.'s (1993) marginal abatement cost estimation, except for its explicit rather than implicit modeling of the emission reducing mechanism.

**Keywords:** *Marginal abatement costs; Materials balance; Distance functions*

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## 1. INTRODUCTION

In recent years the production analysis literature has been increasingly concerned with environmental issues, in particular estimating marginal abatement costs. These estimates can play a significant role in determining net benefits of environmental regulations. Their applicability for guiding the design of regulatory regime hinges on the estimates' quality and validity. In turn, that also influences whether socially optimal outcomes or welfare increases are achieved. Models that are unable to capture the actual dynamics of pollution generation as well as producer choices for complying with environmental regulations are unlikely to reveal the firms' actual abatement costs. Hence, the corresponding estimates are less likely to be applicable for policy decisions.

In the current paper I conceptualize the polluting technology as the intersection of the conventional technology set and a pollution generating mechanism, where the latter is modeled through the law of mass balance. The materials balance condition allows identifying both uncontrolled (prior to abatement) and controlled (post abatement) emissions. Abatement is here processes where production byproducts are transformed into different, less harmful products. Whenever information on input quantities, output levels, and pollutants is provided, uncontrolled and controlled emissions as well as abatement efforts can be quantified.

Application of the materials balance principle gives my model a benefit over other comparable models used for identifying marginal abatement costs. In previous studies, e.g. Färe et al. (1993;

2005), Coggins and Swinton (1996), Reig-Martinez et al. (2001), and Cuesta et al. (2009), the generation of pollution as well as possible ways in which firms can reduce their emissions are, in general, treated as a black box. The model utilized in this paper does, on the contrary, allow for identifying these relations explicitly. It provides the producers with an opportunity of reducing their emissions through input substitution, output reductions, or by abatement. The current most common framework for estimating marginal abatement costs (Färe et al., 1993; Färe et al., 2005) considers reductions in desirable outputs as the producers' only choice for complying with environmental regulations. However, output reduction is only one of the tools that polluting firms have at their disposal for reducing emissions. Førsund (2009) points out that this option is likely to be undesirable for the firms as it is often one of their most unprofitable choices. Thus, estimating marginal abatement costs solely based upon the single alternative of output reductions is likely to overstate the true abatement costs experienced by the producers.

I consider two distinct cases for modeling polluting firms. In the first case, I treat abatement as a conventional output. Hence, usual technology properties apply. This deviates from the established literature, where non-conventional axioms are utilized when pollutants are accounted for. In the second case, I consider the production of abatement to be separate from the production of desirable outputs. This is an important case as inputs that go into abatement processes often deviate strongly from inputs that go into the production of desirable outputs.

Applying axiomatic production theory to model polluting firms is desirable due to the duality of revenue and profit maximization. I exploit the duality of the directional output distance function to the revenue function for estimating marginal abatement costs in the case where abatement is treated a non-separate production. Following Färe et al. (1993), I obtain the shadow price of the abatement output from the distance function derivatives. The shadow price can readily be interpreted as the value of desirable outputs that must be forgone in order to reduce emissions. This procedure for estimating abatement costs is shown to be the reciprocal of Färe et al.'s (1993) procedure, with the exception of explicit rather than implicit modeling of the emission reducing mechanism. The method thus provides a benchmark to which abatement cost estimates based on Färe et al.'s method may be compared.

Despite the desirable simplicity of the two methods previously discussed, they only consider one way in which producers can reduce their emissions. This imposes too much restriction on producers' responses to environmental regulations. In the second case considered in this paper I attempt to capture flexible producer responses. Polluting firms are here perceived as operating under emission constraints that may be weakened by abatement activities. The costs of abatement are then weighed against the benefits of employing polluting inputs. The duality of the directional distance function to the profit function is exploited for estimating marginal abatement costs. The solution to the emission constrained profit problem is further shown to rationalize allocative inefficiency for firms that comply with environmental regulations. This recognition is important for properly understanding the dynamics of environmentally regulated firms.

The paper is organized as follows. I discuss existing literature on marginal abatement costs estimation in the following section. The method reviewed in section 2 makes up the foundation of the proposed procedures for abatement cost estimation in this paper. Section 3 discusses the materials balance principle, while section 4 incorporates it in an economic model. The derivation of the shadow price for abatement is further provided. Section 5 discusses profit maximization for the environmentally regulated firm. Marginal abatement costs are derived from the first order

conditions for profit maximization. It is shown that environmental regulations induce optimal allocations that deviate from unrestricted allocations as the producers must account for the costs of cleaning up pollutants. Section 6 briefly discusses social optimal allocations and the importance of reliable marginal abatement cost estimates for its achievement. Section 7 discusses computational approaches while section 8 concludes.

## 2. MARGINAL ABATEMENT COST ESTIMATION IN THE LITERATURE

Existing literature on pollution modeling usually treats pollutants as inputs or outputs to be included in the technology set. In an early attempt to estimate marginal abatement costs, Pittman (1981) incorporates pollutants as inputs in the technology. This treatment is contingent on the assumption that positive marginal productivities of pollutants characterize transformation of resources from emission controls to intended productions. Pittman defines an environmentally restricted profit function and applies the Lagrangian multiplier on the regulation constraint to obtain estimates of marginal abatement costs for a sample of pulp-and paper mills. This modeling approach has not been followed up in the literature. However, his restricted profit problem bares resemblance to that found in section 5 of this paper.

Pittman's dataset was later applied by Färe et al. (1993), who introduced a new and innovative method for estimating marginal abatement costs. In their approach, pollutants are treated as outputs. Let  $x \in \mathfrak{R}_+^N$  denote a vector of inputs and  $y \in \mathfrak{R}_+^M$  denote a vector of desirable outputs. Consider, for simplicity, only one undesirable output,  $b \in \mathfrak{R}_+$ . An extended output set may then be defined:

$$P(x) = \{(y, b) : x \text{ can produce } (y, b)\} \quad (1)$$

Färe et al. assume that the polluting technology satisfies the conventional axioms of no free lunch, compact and convex output sets, and free disposability of inputs and desirable outputs. See Färe and Primont (1995) for a discussion of these properties. In addition, two non-conventional axioms are imposed to accommodate for the production of bads:

- (i) if  $(y, b) \in P(x)$  and  $b = 0$ , then  $y = 0$
- (ii) if  $(y, b) \in P(x)$  and  $0 \leq \theta \leq 1$ , then  $(\theta y, \theta b) \in P(x)$

Axiom (i), null-jointness (Shephard and Färe, 1974), imposes unavoidable pollution. Axiom (ii), weak disposability (Shephard, 1970), secures that reduction of the undesirable output can be achieved when simultaneously reducing some desirable outputs. According to the authors, this is consistent with regulations which require abatement or cleanup of pollutants as resources are diverted from producing desirable outputs to emission reductions.

The directional output distance function is a suitable function representation for the polluting technology from equation 1 (Färe et al., 2005). The directional distance function was introduced in Chambers et al. (1996); Chambers et al. (1998) and allows for defining maximum feasible translation

of inputs and outputs in any preassigned direction. Here, it seeks the simultaneous maximal reduction in the undesirable output and expansions of desirable outputs. Define the direction vector  $\mathbf{g} = (g_y, -g_b)$  where  $g_y \in \mathfrak{R}_+^M$  and  $g_b \in \mathfrak{R}_+$ , and consider:

$$\bar{D}_O(x, y, b; g_y, -g_b) = \sup \left\{ \beta \in \mathfrak{R} : (y + \beta g_y, b - \beta g_b) \in P(x) \right\} \quad (2)$$

The directional distance function inherits the properties of the parental technology. Under  $\mathbf{g}$ -disposability<sup>1</sup> the directional distance function completely characterizes the underlying polluting technology in the sense that:

$$(y, b) \in P(x) \text{ if and only if } \bar{D}_O(x, y, b; g_y, -g_b) \geq 0 \quad (3)$$

It satisfies the translation property, which is the translation counterpart to the homogeneity property of the output distance function (Shephard, 1970):

$$\bar{D}_O(x, y + \alpha g_y, b - \alpha g_b; g_y, -g_b) = \bar{D}_O(x, y, b; g_y, -g_b) - \alpha, \quad \alpha \in \mathfrak{R} \quad (4)$$

The distance function is further homogenous of degree minus one in  $(g_y, -g_b)$ , non-decreasing in  $b$ , non-increasing in  $y$ , and concave in  $(y, b)$ .

Equation 3 allows defining the revenue function in terms of the distance function. Let  $P_y \in \mathfrak{R}_+^M$  and  $P_b \in \mathfrak{R}_+$  be vectors of (shadow) prices and define the revenue function:

$$\begin{aligned} R(x, P_y, P_b) &= \max_{y, b} \left\{ P_y y - P_b b : \bar{D}_O(x, y, b; g_y, -g_b) \geq 0 \right\} \\ &= \max_{y, b} \left\{ P_y y - P_b b + (P_y g_y + P_b g_b) \bar{D}_O(x, y, b; g_y, -g_b) \right\} \end{aligned} \quad (5)$$

where the last equality is due to Chambers et al. (1998). The first order conditions for revenue maximization are:

$$(P_y g_y + P_b g_b) \nabla_y \bar{D}_O(x, y, b; g_y, -g_b) = -P_y \quad (6)$$

and

$$(P_y g_y + P_b g_b) \nabla_b \bar{D}_O(x, y, b; g_y, -g_b) = P_b \quad (7)$$

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<sup>1</sup> If  $(y, b) \in P(x)$  then  $(y - g_y, b + g_b) \in P(x)$

For the output  $m$  and the undesirable output  $b$ , it follows that their relative price equals the corresponding ratio of distance function derivatives. Hence:

$$P_b = -P_{y_m} \frac{\partial \vec{D}_O(x, y, b; g_y, -g_b) / \partial b}{\partial \vec{D}_O(x, y, b; g_y, -g_b) / \partial y_m} \quad (8)$$

The shadow price  $P_b$  can now be obtained from equation 8, by assuming that the observed sales price of the output  $y_m$  equals its shadow price (Färe et al., 1993). The shadow price is here interpreted as the value of desirable output that must be forgone in order to marginally reduce the undesirable output. In other words, it defines marginal abatement costs.

Färe et al.'s approach to abatement cost modeling benefits from the use of distance functions. They do not rely on price information, and are therefore suitable in cases with missing prices for pollutants. Consequentially, the procedure is very popular and has been employed in several studies of polluting industries, e.g. electricity generation (Coggins and Swinton, 1996; Färe et al., 2005), agriculture (Färe et al., 2006), ceramic pavement industry (Reig-Martínez et al., 2001), and aquaculture (Liu and Sumaila, 2010). The framework of Färe et al. (1993) is also applicable when pollutants are treated as inputs. One example is Lee (2005), who calculates the shadow price of sulfur dioxide as the cost of reducing sulfur emissions in terms of increased capital expenses.

Recently, several authors have started questioning Färe et al.'s approach to pollution modeling. Coelli et al. (2007) argue that their model is inconsistent with the materials balance condition, a law of physics to be treated in the subsequent section. The approach of Färe et al. further imposes severe constraints on producers' responses to environmental regulations since the axiom of weak disposability of desirable and undesirable outputs only allows for output reduction as a measure for regulatory compliance (Førsund, 2009). The purpose of the current paper is therefore to propose some new procedures for estimating marginal abatement costs that maintain the desirable features of Färe et al.'s method, but which overcome the critique of Coelli and Førsund. These procedures rely on the duality of the directional distance function to the revenue and profit function, and, thus, resemble the method for marginal abatement cost estimation reviewed in the current section. It should also be noted that my perspective is more in line with the interpretation of abatement costs in the environmental economics literature – the least cost approach to satisfying environmental regulations.

### 3. CONTROLLED AND UNCONTROLLED EMISSIONS

Most production processes involve transformations of materials which have low economic values into final products which have higher economic values. Usually, the energy required to perform the transformations is supplied by material fuels. Baumgärtner et al. (2001) and Baumgärtner and Arons (2003) show that byproducts are inevitable in such production processes. This is a consequence of physical limits to production, imposed by the first and second law of thermodynamics. The first law of thermodynamics, often called the materials balance condition, secures that materials can neither be created nor destroyed, but may only change their form. Its implication for pollution generation is evident: the share of material inputs that is not recuperated in intended products ends up as

(undesirable) byproducts. Whenever such byproducts affect the welfare of external agents, they are dubbed externalities or pollutants. See Ayres and Kneese (1969) for a brilliant discussion on the subject.

The importance of materials balance is clearly illustrated by sulfur emissions from fossil fuel fired power plants. The sulfur content of fuels is transformed to gas form during combustion and emitted to air. However, the amount of sulfur released is the equivalent of the sulfur content stored in the fuels prior to combustion. Sulfur is not recuperated in the electricity output, and its release during fossil fuel based electricity generation is unavoidable.

A specific form of the materials balance condition - that has gained popularity in the production analysis literature (Coelli et al., 2007; Lauwers, 2009) - is reproduced in equation 9. Let  $u \in \mathfrak{R}_+^N$  be a vector of emission factors and  $v \in \mathfrak{R}_+^M$  be a vector of recuperation factors for the desirable outputs, and define:

$$b = [ux - vy] \quad (9)$$

Equation 9 is a representation of the producer's uncontrolled byproduct. By uncontrolled I mean the byproduct that is generated if the producer is not involved in any abatement activities. In the case of electricity production and sulfur emissions, the emission factors report the specific sulfur content of fossil fuels. Non-polluting inputs such as labor and capital inputs receive emission factors of zero since they do not contribute to the generation of sulfur emissions. Similarly,  $v$  is the zero-vector since there is no sulfur content embedded in the final product, i.e. electricity. The benefit of using equation 9 to represent uncontrolled emissions is that information on emission- and recuperation factors is easily applicable and available from a wide range of sources. This is discussed further in section 7 of this paper.

Equation 9 allows for assessing feasible producer responses to regulations that limit the generation of the byproduct,  $b$ . It is clear that uncontrolled emissions can be reduced both by decreasing overall input use and by substituting high-polluting inputs, i.e. inputs which emission factors are relatively high, with low-polluting inputs. Decreases in input use can take place through efficiency improvements or technical change. Farrell (1957) input oriented measure of technical efficiency, which compares inputs used to produce a vector of desirable outputs is, thus, suitable for evaluating uncontrolled emission performance. Similarly, whenever at least one of the elements in the  $v$  vector is non-zero, uncontrolled emissions can be evaluated by the output oriented measure of technical efficiency.

So far I have only considered the uncontrolled byproduct. Firms do, however, often involve in abatement activities. Its purpose is to reduce the emissions of certain pollutants without reducing their uncontrolled emissions. Notice that abatement processes do not diminish the level of byproduct generated by the firms, but transform it into different byproducts. In fact, abatement activities actually increase the overall amount of byproducts generated by the firms. See Pethig (2003; 2006) for a detailed discussion. The explanation is, of course, that abatement processes also are subordinate to the laws of physics, and thus generate their own undesirable byproducts.

In the case of electricity production, the generated sulfur byproducts can be applied in the production of gypsum. This does not reduce the level of uncontrolled sulfur emissions, but parts of

the emissions are absorbed by end-of-pipe scrubbers installed in the power plants. The byproduct which is left after the abatement process is called the controlled byproduct, which is the level of sulfur that is emitted to air. Let  $A \in \mathbb{R}_+$  denote the amount of byproduct that is absorbed or transformed during abatement. The controlled byproduct is then defined by equation 10.

$$b = [ux - vy] - A \quad (10)$$

Abatement offers an alternative to input substitution or productivity improvements that allows reducing regulated pollutants. This leaves the producers with a menu of tools for complying with environmental regulations. Following conventional economic theory, producers are likely to choose the least cost tool or combination of tools that meet regulation standards. Thus, assessment of marginal abatement costs that does not take these options into account is likely to lead to overestimation of the costs of compliance. This is my point of departure when I now propose some new approaches to marginal abatement cost estimation.

#### 4. NON-SEPARATE ABATEMENT

My treatment of polluting firms diverges from that of Färe et al. (1993; 2005), as I conceptualize the polluting technology as the intersection of the pollution generating mechanism in equation 10 and a conventional production model. This resembles the model structure proposed by Krysiak and Krysiak (2003) and Murty and Russell (2010) for modeling polluting firms. I am, in principle, merging the physical model in equation 10 with an economic model. This allows assessing the economic aspects of pollution generation through the application of duality theory.

In the first model specification considered I define abatement as an output in the technology set,  $T_1$  (Coelli et al., 2007; Murty and Russell, 2010). In contrast to Färe et al.'s model, there is no need for imposing non-conventional axioms since abatement is a desirable output. The technology  $T_1$  is thus assumed to satisfy the usual regularity conditions of no fixed costs and no free lunch, free disposability of inputs and outputs, and that  $T_1$  is a non-empty, closed, and convex set. See Chambers (1988) for a discussion of these properties. The polluting technology,  $T$ , is then defined:

$$\begin{aligned} T(b) &= T_1 \cap T_2 \\ T_1 &= \{(x, y, A) : x \text{ can produce } (y, A)\} \\ T_2 &= \{(x, y, A) : [ux - vy] - A = b\} \end{aligned} \quad (11)$$

The multi-output theory of Frisch (1965) emphasizes the concept of assorted production, which concerns the freedom of directing inputs into outputs. The model in equation 11 limits the freedom of choosing the output mix for a fixed input vector. Here, the undesirable output,  $b$ , is directly determined by the inputs and outputs in  $T_1$ . The model thereby captures the fact that  $b$  is an unavoidable byproduct from the production process.

There are several benefits from choosing the model structure in equation 11 for studying polluting firms. First of all, there is no need for introducing non-conventional axioms for the

undesirable output. Secondly, the polluting technology explicitly accounts for the underlying factors of pollution generation, as well as separates uncontrolled emissions from controlled emissions. This avoids the usual black-box treatment of pollutants found in the literature from section 2. Explicit modeling allows evaluating the various ways in which a producer can reduce undesirable byproducts and, thus, to identify the least cost way of achieving these reductions.

Färe et al. (1993; 2005) apply the axiom of weak disposability to impose that some level of desirable outputs must be forgone in order to reduce emissions. Utilizing the materials balance condition from equation 10, it is straightforward to show that such reductions must be related to transfers of resources from intended production to abatement activities. Consider  $(y, b) \in P(x)$  from equation 1, and define the scalar  $\theta$  such that  $0 \leq \theta \leq 1$ . Inserting the scalar into equation 10 gives  $\theta[b + vy] = ux - A$ . It is evident that the weak disposability axiom,  $(\theta y, \theta b) \in P(x)$ , is only physically consistent when abatement,  $A$ , is increased to allow for a proportional decrease of desirable and undesirable output. This has important implications for empirical work: Marginal abatement costs estimated by equation 8 cannot represent firms' actual abatement costs when they are not involved in abatement activities. It therefore seems reasonable to explicitly account for firms' abatement efforts, in order to avoid this issue.

In the model from equation 11, the undesirable output,  $b$ , is solely determined by the producer's choice of inputs and outputs in technology  $T1$ . When resources are devoted to abatement,  $A$ , the undesirable output decreases accordingly. In the first procedure I consider for estimating marginal abatement costs, I thus establish the trade-off between desirable outputs and abatement. Define the direction vector  $(g_y, g_A)$  in  $\mathfrak{R}_+^M \times \mathfrak{R}_+^1$ , and consider the directional output distance function that seeks the maximal expansions of outputs:

$$\bar{D}_O(x, y, A; g_y, g_A) = \sup \left\{ \beta \in \mathfrak{R} : (y + \beta g_y, A + \beta g_A) \in T1 \right\} \quad (12)$$

Since the distance function completely characterizes the underlying technology  $T1$ , the revenue function can be defined similar to equation 5. Let  $P_A \in \mathfrak{R}_+$  be the (shadow) price for the abatement output and consider:

$$R(x, P_y, P_A) = \max_{y, A} \left\{ P_y y + P_A A : \bar{D}_O(x, y, A; g_y, g_A) \geq 0 \right\} \quad (13)$$

For the abatement output  $A$  and the desirable output  $m$ , it follows that the relative price equals the marginal rate of transformation in optimum:

$$\frac{P_A}{P_{y_m}} = \frac{\partial \bar{D}_O(x, y, A; g_y, g_A) / \partial A}{\partial \bar{D}_O(x, y, A; g_y, g_A) / \partial y_m} \quad (14)$$

The abatement cost,  $P_A$ , can now be identified from the derivatives of the distance function, by assuming that the observed sales price for output  $y_m$  equals its shadow price.

Equation 14 establishes a simple method for estimating abatement costs that resembles Färe et al.'s procedure, but which explicitly treats abatement efforts. It may therefore prove useful for testing the validity of marginal abatement cost estimates obtained from the established approach. The interpretation that the producers have to give up desirable outputs in order to reduce the undesirable output is maintained, but the loss of desirable outputs is explicitly rather than implicitly related to transfer of resources to the abatement activity.

The output recuperation factors,  $v$ , from equation 9 and 10 are important for the interpretation of the abatement costs from equation 14. Whenever they are non-zero, the producers can also reduce their emissions by increasing the production of desirable outputs. Hence, the trade-off between desirable outputs and abatement is of less relevance since increases in both lead to decreases in undesirable outputs. This is, of course, also a problem for Färe et al.'s model, since it only allows for proportional contractions of desirable and undesirable outputs. Output recuperation may be the reason why some empirical studies find negatively sloped segments of the frontier of  $P(x)$  from equation 1, implying that emissions reductions are beneficial for firms located in these regions. See Picazo-Tadeo and Prior (2009) for a more detailed discussion.

## 5. SEPARATE ABATEMENT

Despite their attractive simplicity, the estimation procedures from section 2 and 4 may not be able to provide the least cost way of emission reductions. These measures implicitly or explicitly consider abatement as the preferred producer response to environmental regulations. This is likely to impose too much restrictions on producers' choices, thereby leading to overestimation of marginal abatement costs.

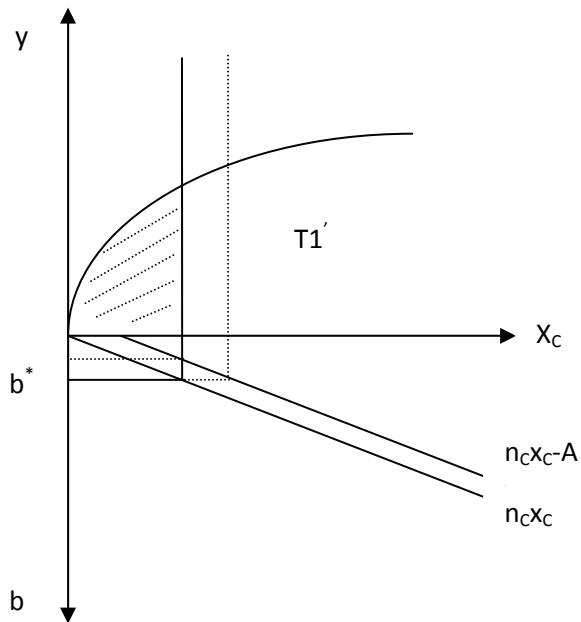
A second issue is the specification of the polluting technology. Equation 11 treats abatement as an ordinary output in  $T_1$ . This allows for full flexibility in selecting the output mix of abatement and desirable outputs for a fixed input vector. However, inputs which go into the abatement process are likely to differ from those employed in the production of desirable outputs. Consider for example Färe et al. (2005) who estimate marginal abatement costs for a sample of electricity generating units by applying equation 8. In their study, the input vector is made up of fuel, labor, and generating capacity. These inputs, with a potential exception for labor, are not likely to be employed for abatement purposes, but are related to electricity generation itself. The interpretation of marginal abatement costs as desirable outputs that must be forgone when inputs are allocated to the cleanup of pollutants is questionable, especially since fuels are the main source of the pollutant in question (sulfur dioxide). To follow up this discussion, I now consider the abatement production to be separate from the production of desirable outputs. The employment of inputs that go into the abatement process is thus a separate decision from the production inputs. Abatement can now be treated as a commodity or a service which the firm may purchase in order to reduce its emissions. The polluting technology is here defined:

$$T(b+A) = T1' \cap T2'$$

$$T1' = \{(x, y) : x \text{ can produce } y\} \quad (15)$$

$$T2' = \{(x, y) : [ux - vy] \leq b + A\}$$

A graphical representation of the polluting technology is provided by figure 1, which illustrates the case of electricity generation and sulfur emissions. For simplicity, I only consider coal inputs,  $x_c$ , used for producing electricity,  $y$ . The materials balance condition is represented pictorially by the lines  $n_c x_c$  and  $n_c x_c - A$  in the figure's lower panel. The line  $n_c x_c$  plots combinations of the quantity of coal and corresponding uncontrolled sulfur emissions. Abatement efforts allow reducing sulfur emissions below the level of uncontrolled emissions. Consider the line  $n_c x_c - A$ , which plots combinations of coal inputs and controlled emission for an arbitrary level of abatement. The controlled sulfur emissions are lower than the uncontrolled emissions for all levels of coal inputs.



**Figure 1: The polluting technology**

A firm which is restricted in terms of sulfur emissions is implicitly restricted in terms of input use. More specifically, a firm complying with the emission target  $b^*$  in figure 1 can only choose input bundles which lie in the shaded area of figure 1, as long as the firm is not involved in abatement activities. Whenever the use of coal exceeds this threshold, uncontrolled emissions will similarly exceed the emission target. Abatement allows the producer to extend the use of coal to levels outside of the restricted set without violating the sulfur regulation. Consider the line  $n_c x_c - A$  in figure 1. By choosing the specific abatement level that the line represents, the constraint for coal use is shifted to the dotted vertical line in the figure. As long as the coal quantity stays below this boundary, the controlled emissions will meet the emission target  $b^*$  even if the uncontrolled emissions exceed it. This gives the producer an obvious trade-off: By involving in abatement activities he can expand the set of feasible input-output combinations, an option which may contribute to increases in profits. On the other hand, involvement in abatement activities is costly.

The optimal producer response to the environmental regulation is then defined by an extended profit maximization problem that takes the cost of abatement into account. The firm now weighs the benefits of maintaining a Business as Usual<sup>2</sup> (BaU) production against the costs of abatement. The producer will only benefit from increased abatement as long as the potential increase in profits exceeds the marginal abatement costs. Hence, it is profitable to employ less polluting inputs compared to the BaU scenario when the costs of input substitution are less than abatement costs.

I exploit the duality of the directional distance function to the profit function (Chambers et al., 1998) to derive marginal abatement costs. The directional distance function determines the maximal simultaneous expansion of outputs and contraction inputs that is technologically feasible. Define the direction vector  $g=(-g_x, g_y)$  where  $g_x \in \mathfrak{R}_+^N$  and  $g_y \in \mathfrak{R}_+^M$ , and:

$$\bar{D}(x, y; -g_x, g_y) = \sup \left\{ \beta \in \mathfrak{R} : (x - \beta g_x, y + \beta g_y) \in T1' \right\} \quad (16)$$

Similar to the directional output distance functions, the directional distance function in equation 16 completely characterizes the underlying technology  $T1'$ . Let  $w \in \mathfrak{R}_+^N$  be a vector of input prices. The profit maximization problem for an environmentally regulated firm can now be defined as:

$$\pi(P_y, w, P_A, b^*) = \sup_{x, y, A} \left\{ P_y y - wx - P_A A : \bar{D}(x, y; -g_x, g_y) \geq 0, [ux - vy] \leq b^* + A \right\} \quad (17)$$

The profit problem in equation 17 resembles the well-known expenditure constrained profit maximization problem, see Chambers and Lee (1986) and Färe et al. (1990). This is reasonable, since the emission constraint is similar to a cost constraint where prices are replaced by emission factors. The current analysis deviates from the expenditure constraint literature by allowing for endogenous constraints through the choice of abatement. By solving for abatement, equation 17 may be rewritten as:

$$\begin{aligned} \pi(P_y, w, P_A, b^*) &= \sup_{x, y} \left\{ P_y y - wx - P_A (ux - vy - b^*) : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\} \\ &= \sup_{x, y} \left\{ (P_y + vP_A) y - (w + uP_A) x + P_A b^* : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\} \\ &\quad \sup_{x, y} \left\{ ry - sx + P_A b^* : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\} \end{aligned} \quad (18)$$

The maximization problem in equation 18 is a conventional profit maximization problem, extended by a fixed income,  $P_A b^*$ . The fixed income reflects that abatement is only required when uncontrolled emissions exceed the emission target,  $b^*$ . Heuristically, it can be thought of as a discount on compliance costs, amounting to the cleanup costs of the firm's legal emissions.

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<sup>2</sup> By BaU I mean optimal allocations of inputs and desirable outputs under unconstrained or conventional profit maximization.

Notice that input prices,  $s$ , and output prices,  $r$ , deviate from their observed market prices by  $mP_A$  and  $nP_A$ , respectively. The vector  $mP_A$  determines the reduction in abatement costs due to the desirable outputs' recuperation of the undesirable output. This is related to the reduced need for abatement when pollutants are recuperated. The vector  $nP_A$  determines the costs of cleaning up a unit of undesirable output caused by the corresponding inputs. Highly polluting inputs now becomes more expensive to employ since they require additional cleanup of corresponding emissions.

Following the treatment in equation 8 and equation 14, I now consider the first order conditions for profit maximization. For two desirable outputs,  $m$  and  $m'$ , it follows that their relative price equals the ratio of distance function derivatives:

$$\frac{P_{y_{m'}} + v_{m'} P_A}{P_{y_m} + v_m P_A} = \frac{\partial \vec{D}(x, y; -g_x, g_y) / \partial y_{m'}}{\partial \vec{D}_O(x, y; -g_x, g_y) / \partial y_m} \quad (19)$$

For inputs  $n$  and  $n'$ , the first-order conditions are defined:

$$\frac{w_{n'} + u_{n'} P_A}{w_n + u_n P_A} = \frac{\partial \vec{D}(x, y; -g_x, g_y) / \partial x_{n'}}{\partial \vec{D}_O(x, y; -g_x, g_y) / \partial x_n} \quad (20)$$

Equation 19 and 20 establish an alternative route to marginal abatement cost estimation. The directional distance function from equation 16 is estimated by applying quantities of inputs and desirable outputs. The ratio of distance function derivatives are further calculated as in Färe et al.'s procedure. The marginal abatement cost,  $P_A$ , can thus be obtained from equation 19 or 20, when prices and emission factors are known.

In the efficiency measurement literature, equation 19 and 20 are usually considered as expressions of allocative inefficiency. Here, the ratio of shadow prices deviates from ratio of observed sales- and factor prices. Intuitively, equation 19 and 20 rationalize allocative inefficiency for producers that comply with an environmental regulation. Environmental regulations raise the costs of employing polluting inputs as the producers must take into account the costs of cleaning up emissions related to their use. As a consequence, the input mix that solves the emission constrained profit problem will employ a less amount of polluting inputs relative to the unconstrained profit maximum. For the outputs, on the other hand, the sales prices increase in the recuperation factors, as the need for abatement becomes less when emissions are recuperated by intended outputs. The optimal output mix is thus composed of outputs with high recuperation factors, compared to the optimal output mix under unconstrained profit maximization.

Next, I evaluate the marginal abatement cost. Solving equation 20 with respect to the abatement cost one obtains:

$$P_A = \frac{(MS * w_n) - w_{n'}}{u_{n'} - (MS * u_n)}, \text{ where } MS = \frac{\partial \vec{D}(x, y; -g_x, g_y) / \partial x_{n'}}{\partial \vec{D}_O(x, y; -g_x, g_y) / \partial x_n} \quad (21)$$

The numerator in equation 21 defines the marginal costs of substituting factor  $n'$  with  $n$ : The marginal rate of technical substitution,  $MS$ , determines the increase in factor  $n$  that is sufficient to compensate a marginal reduction in factor  $n'$ . The product  $MS \cdot w_n$  thus determines the increase in costs related to the increase in the use of input  $n$ . Subtracting the reduced unit costs for  $n'$  provides the costs of substitution.

The nominator in equation 21 defines the marginal change in emissions caused by substituting factor  $n'$  with factor  $n$ . This follows the same arguments as for the substitution costs. Marginal abatement costs are, in other words, determined by weighing the costs of input substitution against the environmental damage, i.e. against the need for abatement. A similar interpretation can be provided for equation 19 in terms of output transformation. By accommodating for flexible producer responses, abatement costs are now defined in terms of the least cost way of compliance. This is likely to produce more reliable estimates of abatement costs as compared to the two models from equation 8 and 14, which only considers one producer response to environmental regulations.

So far the analysis has only concerned one undesirable output. However, inputs often generate multiple pollutants and producers face several regulations. In this case, the first order conditions in equations 19 and 20 are extended to accommodate for the sum of abatement costs. Assume that there exists  $K$  regulated pollutants. The first order conditions for the regulated profit problem, corresponding to equation 20, are then extended to<sup>3</sup>:

$$\frac{w_{n'} + \sum_{k=1}^K u_{n'k} P_{A_k}}{w_n + \sum_{k=1}^K u_{nk} P_{A_k}} = \frac{\partial \bar{D}(x, y; -g_x, g_y) / \partial x_{n'}}{\partial \bar{D}_O(x, y; -g_x, g_y) / \partial x_n} \quad (22)$$

In the case with two regulated pollutants, and where allowance prices exist for one of the two pollutants, the shadow price of the residual pollutant can be determined by assuming that the known allowance price equals the marginal abatement costs. In the case with more than two pollutants, each marginal abatement costs can no longer be directly determined from the derivatives of the distance function. Alternative procedures, such as estimating both the distance function and shadow prices directly, have to be employed.

## 6. SOCIAL OPTIMALITY

The classical policy instrument for achieving a socially optimal solution is to charge the polluting firms for the environmental damage they cause. I follow Førsund (2009) and specify a fixed unit tax,  $t$ , levied on the undesirable output,  $b$ . The producer profit problem then converges into:

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<sup>3</sup> Equation 22 bears resemblance to equation 38 in Førsund (2009). He shows that perceived input prices are adjusted according to their environmental impact when environmental policy instruments are in place. I am grateful to Finn R. Førsund for making me aware of the similarity between the two equations.

$$\begin{aligned}
\pi(P_y, w, t) &= \sup_{x,y} \left\{ P_y y - wx - tb : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\} \\
&= \sup_{x,y} \left\{ P_y y - wx - t(ux - vy - A) : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\} \\
&= \sup_{x,y} \left\{ (P_y + vt)y - (w + ut)x + tA : \bar{D}(x, y; -g_x, g_y) \geq 0 \right\}
\end{aligned} \tag{23}$$

The second equality follows by applying the materials balance condition from equation 10. The corresponding first order condition for profit maximum for inputs  $n$  and  $n'$  is:

$$\frac{w_{n'} + u_{n'}t}{w_n + u_nt} = \frac{\partial \bar{D}(x, y; -g_x, g_y) / \partial x_{n'}}{\partial \bar{D}_O(x, y; -g_x, g_y) / \partial x_n} \tag{24}$$

Comparing equation 24 with equation 20, it is straightforward to show that they coincide when abatement costs,  $Pa$ , equal the unit tax. However, in their treatment on polluting firms, Pethig (2003; 2006) and Førsund (2009) point out that abatement activities further generate undesirable outputs that should be accounted for when designing the optimal environmental policy. In this case, the marginal abatement cost from equation 20 provides a social optimal solution if it also accounts for secondary pollutants from the abatement processes.

The estimation of marginal abatement costs provides an important tool for evaluating and designing optimal regulatory schemes. Establishing the economical implications that environmental regulations have for the producers is important for quantifying the costs of achieving a cleaner environment. This point to the crucial role of accounting for multiple producer responses to environmental regulations in marginal abatement cost estimation. Only when the producers' options and the least cost response are fully considered can the corresponding estimates be considered a desirable tool for guiding policy decisions.

## 7. COMPUTATIONAL APPROACHES

To this point, my discussion on marginal abatement costs has been very general. I therefore briefly discuss computational approaches in this section. The estimation procedures from previous sections require a differentiable distance function. In addition, emission factors are essential for identifying the abatement costs in equations 19 and 20, and for computing the abatement output from equation 11. Both emission factors and the computation of distance functions are therefore treated in the following.

Emission factors are easy applicable and available from a wide range of sources. Some examples include the intergovernmental panel on climate change's (IPCC) emission factor database, the U.K. National Atmospheric Emissions Inventory, and AP 42 Compilation of Air Pollutant Emission Factors. The emission factors relate the quantity of an undesirable byproduct released to the environment with a certain activity. Common examples are air pollutant and greenhouse gas emissions from the combustion of fossil fuels or number of animals in animal husbandry. The factors are usually defined

as the weight of pollutants divided by a unit weight, e.g. tons of sulfur dioxide emitted per tons of coal burned.

Uncontrolled emissions are calculated according to equation 9, by multiplying emission- and recuperation factors with the input and output vectors. Abatement efforts can further be quantified if data on controlled emissions are available<sup>4</sup>. They are calculated according to equation 10, by subtracting the controlled emissions from the estimated uncontrolled emissions. Data on abatement effort is required for estimating the directional output distance function from equation 12.

I now turn to the computation of the distance function derivatives. The estimation of a parametric distance function requires the selection of a functional form. Chambers (1998) suggests the quadratic functional form for the directional distance function, which has been followed up in the literature. See for example Färe et al. (2005; 2006). On the contrary, studies that estimate the radial input- and output distance functions (Shephard, 1953, 1970) usually prefer the translog functional form (Christensen et al., 1971). Economic theory provides guidance to the selection of functional form as the quadratic form can be restricted to satisfy the translation property while the translog form can be restricted to satisfy the homogeneity property. The converse does not apply.

Parametric distance functions can be estimated by econometric techniques or by linear programming. I consider two econometric techniques and one linear programming technique in the following.

Suppose that there are  $l = (1, \dots, L)$  observations in the dataset. A technique developed by Aigner and Chu (1968), that minimizes the sum of deviations of the estimated distance functions from the frontier, can now be employed to estimate the directional distance function. Formally,

$$\min \sum_{l=1}^L (\bar{D}(x^l, y^l; -g_x, g_y) - 0) \quad (25)$$

Equation 25 forms a linear programming problem to be solved under a set of regularity constraints. They require the value of the distance function to be nonnegative, and secure that monotonicity, translation, and symmetry properties are satisfied. The program then chooses the parameters of the distance function such that the firms appear as efficient as possible.

Stochastic frontier analysis is an alternative to the deterministic programming method of Aigner and Chu. A suitable stochastic specification for the directional distance function takes the form:

$$\bar{D}(x^l, y^l; -g_x, g_y) + \varepsilon^l = 0 \quad (26)$$

Where  $\varepsilon^l$  is the composite error term  $\lambda^l - \mu^l$ ,  $\lambda^l \sim N(0, \sigma_\lambda^2)$  and  $\mu^l \sim N^+(\mu, \sigma_\mu^2)$ . Applying the translation property from equation 4, equation 26 can be rewritten as:

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<sup>4</sup> One example where the required information on controlled and uncontrolled emissions is readily available is the case of U.S. electricity generation, where Continuous Emission Monitoring (CEMS) provides detailed data on regulated plants' emissions of air pollutants.

$$-\alpha^l = \vec{D}(x^l - \alpha^l, y^l + \alpha^l; -g_x, g_y) + \epsilon^l \quad (27)$$

An arbitrary output or input is chosen as the scalar,  $\alpha^l$ . The selected output or input is applied as the dependent variable in the regression, after multiplying it with minus 1. All independent variables are transformed according to equation 27 prior to estimation. The directional output distance function is estimated accordingly, but inputs are not transformed as in equation 27. Ordinary least squares or maximum likelihood techniques can be applied to estimate the function.

A third option for estimating marginal abatement costs is the econometric shadow price approach. See Kumbhakar and Lovell (2000) for an overview. This technique allows for estimating the profit function from section 5, rather than the distance function. Since the profit function here only includes desirable inputs and outputs, there is no problem of missing prices for undesirable outputs. The shadow price approach allows the producers to fail to optimize profits with respect to market prices of inputs or outputs, but allow them to optimize profits with respect to shadow prices. Both technical and allocative inefficiency are modeled parametrically by introducing additional parameters in the profit function. Marginal abatement costs may be derived directly from the estimates of allocative inefficiency.

## 8. SUMMARY AND CONCLUSION

This paper proposes two new procedures for estimating marginal abatement costs. A model framework that utilizes the materials balance condition as the pollution generating mechanism is applied to address how undesirable byproducts come into existence, as well as potential ways of reducing them. Such dynamics have generally been neglected in previous comparable abatement cost studies. Hence, these studies are potentially overstating the restrictions imposed by environmental regulations, and therefore less likely to provide abatement cost estimates that can be useful for guiding policy decisions. By recognizing the producers' multiple choices for complying with environmental regulations I am able to define marginal abatement costs as the least cost way of satisfying regulatory constraints.

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# Paper IV



# Environmental Regulations, Producer Responses, and Secondary Benefits: Carbon Dioxide Reductions under the Acid Rain Program

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**Abstract:** This paper derives a framework for modeling the secondary benefits, i.e. changes in yet unregulated pollutants, from various abatement strategies. We use the materials balance condition coupled with the tools of production analysis to derive the conditions for such effects to occur. An application on the US electricity generation sector illustrates our main point: In our case, end-of-pipe solutions appear to be among the least costly firm responses to sulfur and nitrogen regulations, but at the expense of increased carbon dioxide emissions.

With new environmental issues emerging over time, our findings suggest that regulators should signal the possibilities of new regulations on connected pollutants to firms. Such information may be relevant for firms when choosing current abatement strategies – with minor cost increases to deal with today's issues, overall compliance costs for near-future environmental problems may be lower.

**Keywords:** *Multiple pollutants, Secondary benefits, Partial abatement costs, Production analysis, Environmental regulation*

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## 1. INTRODUCTION

Several pollutants are connected in the sense that efforts aimed at reducing one or few pollutants potentially lead to unintended reductions or increases in others. Additional environmental gains caused by linkages between pollutants may thus justify strict environmental regulations for certain pollutants, while the opposite may be the case when reductions in one pollutant lead to increased environmental damages from other pollutants. In this paper we primarily focus on settings where regulations for some pollutants may lead to reductions in other pollutants.

However, the magnitudes of secondary or ancillary benefits are likely to depend upon how regulations are formulated, and hence also how polluters respond to regulations. Higher secondary benefits can be expected when compliance is achieved by substitution of polluting inputs, output reductions, or technical change, compared to cases where producers prefer end-of-pipe abatement. In the latter case one would expect that emissions of the regulated pollutant(s) are reduced while emissions of other pollutants remain relatively unchanged.

These issues of secondary benefits are further accentuated by the fact that harmful effects from economic activities do not emerge simultaneously over time, but tend to appear gradually as the economy changes. Hence, a fix on one issue may alter emissions and hence damages from other pollutants. Often, the sum of piecemeal fixes costs more than integrated fixes. Regulatory agencies should therefore be forward looking, signaling to producers potential regulations on yet unregulated pollutants so that producers can make better informed abatement strategy choices. Our paper provides a simple model framework that allows for undertaking such assessments. We use data from the U.S. electricity sector to illustrate our points.

The combustion of fossil fuels emits a wide range of damaging pollutants including sulfur dioxide ( $\text{SO}_2$ ), nitrogen oxides ( $\text{NO}_x$ ), and carbon dioxide ( $\text{CO}_2$ ). Sulfur and nitrogen compounds from human sources such as electricity generation, industry, and motor vehicles are the main contributors to acid rain. Carbon dioxide is, on the other hand, a greenhouse gas that is the principle cause of global warming. Contrary to the benefits of reducing carbon emissions, which relates to future benefits of preventing global warming, the reductions of sulfur and nitrogen yield instantaneous benefits. The linkages between these three pollutants suggest that benefits from reduced climate gas emissions on sulfur and nitrogen emissions may justify stricter environmental standards on carbon emissions (see for example Burtraw et al.(2003)).

Electricity generation is one of the main sources for air pollution. In U.S., power plants are responsible for approximately 65 percent of national sulfur emissions, 20 percent of national nitrogen emissions, and 30 percent of national carbon emissions. The power producers face strict environmental standards on sulfur and nitrogen, while U.S. carbon emissions are currently left unregulated. A consequence of the stringent standards for sulfur and nitrogen is a widespread adoption of end-of-pipe technologies that has lead to substantial cost reductions for end-of-pipe abatement (Riahi et al., 2004). Similar abatement technologies that apply chemical sorbents to remove carbon from gas mixtures are currently being developed. Although they are not economically viable today, one cannot disregard that they will play an important role in future producer compliance strategies. With prospects of maturing end-of-pipe technologies for carbon it seems justifiable to assess secondary benefits in terms of reduced carbon emissions related to current regulations of sulfur and nitrogen, to evaluate the hypothesis of the converse.

Previous studies of secondary benefits have primarily applied simulations techniques (see for example Farrell et al. (1999)). The results from these studies are uncertain as they hinge on the choice of assumptions and parameterization of the models used. We therefore choose to assess secondary benefits from environmental regulations by applied production analysis rather than by simulations. In recent years, the production analysis literature has had several advances in modeling joint production of desirable and undesirable outputs. New frameworks developed for treating polluting technologies have mainly been employed to three areas of empirical analysis; estimation of marginal abatement costs, evaluation of abatement activities' influence on the measurement of productivity growth, and eco-efficiency assessments. A framework that treats joint production should also allow for evaluating secondary benefits. The current paper makes an attempt at doing just that. This provides a new area of application that should be of great interest to environmentally concerned policy makers.

Our analysis applies a new framework for polluting technologies that is influenced by recent publications by Førsund (2009) and Murty and Russell (2010), and by recent advances in the estimation of eco-efficiency (Coelli et al., 2007). The framework applies the materials balance condition as the pollution generating mechanism and separates emissions into uncontrolled emissions (pre-abatement emissions) and controlled emissions (post-abatement emissions). This partition is desirable since it allows evaluating how different producer responses – input substitution, output reductions, or end-of-pipe abatement – influence secondary benefits in the sense of reduced emissions.

Our case study deals with secondary benefits from sulfur and nitrogen regulations on carbon emissions, where coal is an input factor that scores poorly on sulfur, nitrogen, and carbon emissions compared to natural gas. That is, we have a situation where input substitution is expected to have

more desirable impacts on the secondary pollutant than technical change or end-of-pipe solutions. We use a sample of 62 regulated power plants with coal to gas switching capacity to evaluate secondary carbon dioxide benefits from the Acid Rain Program. We use data from 2002 to 2005, a period with large fluctuations in relative prices for fossil fuels. Changes in the relative price for coal and gas are important for the producer responses since coal to gas switching is a central measure for regulatory compliance on sulfur and nitrogen. Emission constrained Data Envelopment Analysis (DEA) models are calculated in order to determine regulatory induced reductions in carbon emissions. We find that end-of-pipe abatement is an economically viable response to current environmental regulations, but that is significantly reduces secondary benefits.

Our paper is structured as follows. The following section includes a brief discussion of secondary benefits and how they are affected by producers' choice of regulatory compliance. Section 3 discusses the materials balance condition and its importance for the generation of byproducts. In section 4, we describe how a traditional axiomatic production model can be extended to take the materials balance into account, where we also show how this extension allows for calculating secondary benefits of emissions reductions. Section 5 specifies the empirical implementation of the emission constrained DEA model, while section 6 evaluates secondary benefit in terms of carbon reductions under the Acid Rain Program. Section 7 summarizes the discussion in the paper.

## 2. POWER GENERATION AND SECONDARY BENEFITS

The threat of global warming has lead to enhanced focus on carbon emissions from human activities. Costs and benefits of preventing carbon emissions have been much debated accordingly. Two of the best known cost-benefit analysis, that of Nordhaus (1991) and Cline (1992), have been criticized for not taking into account that the generation of carbon is joint with sulfur and nitrogen, and thereby not accounting for secondary benefits from reducing carbon emissions (Ayres and Walter, 1991; Ekin, 1996). The idea is simple: Slowing carbon emissions will reduce conventional air pollutants, leading to health benefits and potentially reduced costs of meeting environmental standards for other air pollutants. A failure to adequately account for these secondary benefits will lead to incorrect assessments of the net cost of mitigation policies (Burtraw et al., 2003).

Secondary benefits that arise as a result of environmental regulations are contingent on the way in which producers respond to the environmental restrictions. Textbooks in environmental economics, for example Førsund and Strøm (1988), list input substitution, output reduction, efficiency improvements, waste recycling, and end-of-pipe abatement as measures that firms have at their disposal for complying with environmental regulations. The three first measures are likely to produce secondary benefits: Switching from coal to gas has for example been a central producer response in the case of U.S electricity generation, since gas firing generates less sulfur and nitrogen emissions as compared to coal. A switch towards a gas intensive production will also lead to reduced carbon emissions. Output reductions and fuel efficiency improvements that lead to reduced demand for fossil fuels will further lead to secondary benefits. On the other hand, the use of abatement technologies is less related to secondary benefits since it allows for reducing emissions to air without altering the underlying polluting production process.

Current sulfur and nitrogen regulations for U.S power plants have lead to a widespread adoption of end-of-pipe abatement technologies. Abatement techniques for sulfur, flue gas desulfurization

technologies, were introduced in Japan in the 1960s. In the 1970s, these techniques were adopted in the U.S and later in Germany and Europe. Investment costs declined accordingly by 13 percent for each doubling of abatement capacity (Riahi et al., 2004). Currently, similar abatement technologies are being developed for abatement of carbon from point sources. If learning in the application of these carbon capture technologies occurs at a pace similar to that of sulfur abatement, their future potential is vast (Riahi et al., 2004).

If secondary benefits from carbon reductions are evaluated without taking into account the potential widespread of carbon abatement technologies, the suggested benefits are likely to exceed experienced benefits. One way of shedding light on secondary benefits for joint pollutants while taking this into account, is to study a case where economically viable abatement techniques already are in place. In the current article, we assess secondary benefits in terms of carbon reductions related to regulations for sulfur and carbon emissions. This is, of course, an interesting study on its own for evaluating the current regulations, but also as a what-if scenario in the case of targeting carbon emissions.

### 3. MATERIALS BALANCE

Materials balance is a fundamental law of physics which states that materials can neither be created nor destroyed, but only change their form. The implication of this simple conservation property, brilliantly shown in the seminal paper by Ayres and Kneese (1969), is that material inputs which are not recuperated in desirable outputs end up as (undesirable) byproducts from the production process. Recently several authors, e.g. Pethig (2003; 2006), Ebert and Welsch (2007), Coelli et al. (2007), Lauwers (2009), and Førsund (2009), have revisited Ayres and Kneese's analysis and its applicability to pollution modeling.

The materials balance condition can be represented by emission factors. This is in particular convenient for non-point byproducts that cannot be directly measured. Emission factors report the amounts of undesirable byproducts generated per unit of inputs used as well as the amount recuperated per unit of outputs produced. Let  $x \in \mathfrak{R}_+^N$  denote a vector of inputs,  $y \in \mathfrak{R}_+^M$  denote a vector of desirable outputs, and  $z \in \mathfrak{R}_+^K$  denote a vector of undesirable byproducts. Let  $n$  be a  $(K \times N)$  matrix of input emission factors and  $m$  be a  $(K \times M)$  matrix of output recuperation factors. The materials balance conditions for the undesirable byproducts are then defined:

$$z^{uc} = [nx - my] \quad (1)$$

Equation 1 reports uncontrolled byproducts. Firms are often involved in end-of-pipe abatement activities in order to reduce their emissions below their uncontrolled byproducts<sup>1</sup>. This is represented by subtracting  $A \in \mathfrak{R}_+^K$  from equation 1. Emissions remaining after abatement,  $z^c$ , are called controlled byproducts.

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<sup>1</sup> Abatement does, of course, not reduce the amount of byproducts generated, but transforms them into different, less harmful byproducts. Our analysis does not concern the post-abatement byproducts.

$$z^c = [nx - my] - A \quad (2)$$

The analysis in the next section uses equation 1 and 2 as point of departure. For notational ease the superscripts  $uc$  and  $c$  are omitted in the following.

#### 4. A POLLUTING TECHNOLOGY

The multi-output theory of Frisch (1965) emphasizes the concept of assorted production, which concerns the freedom of directing inputs into outputs. A maximal degree of assortment means that there exist transformation possibilities between all outputs for a given input vector. The other extreme is no assortment, where there is no choice of output mix for a given input vector. The usual way of treating undesirable outputs, by incorporating them into a single production-relation model framework, imposes too large freedom of assigning the output mix. It enforces maximal degree of assortment, and thus transformation possibilities between desirable and undesirable outputs for each input vector. A maximal flexibility in choosing the output mix is not consistent with equation 1 where byproducts are directly determined by the producers' decisions on inputs and desirable outputs<sup>2</sup>.

Førsund (2009) and Murty and Russell (2010) suggest applying a multi production-relation structure when modeling joint production of desirable and undesirable outputs. This structure limits the degree of assortment, securing that uncontrolled pollutants are directly determined by the producers' decisions on inputs and desirable outputs. Pollutants are byproducts, and there exists limited or no transformation possibilities between the desirable outputs and undesirable byproducts for a given input vector.

In this paper, we define a model which resembles that of Førsund (2009) and Murty and Russell (2010). However, we deviate from these studies by defining restrictions on a subset of undesirable byproducts,  $b \in \mathbb{R}_+^L$ . These restrictions reflect the environmental regulations which the firms face. The additional (K-L) unregulated pollutants are assumed not to influence the economic decisions of firms, and are therefore not explicitly accounted for in the model. Emission factors for the  $L$  restricted pollutants are defined by the  $(L \times N)$  matrix  $n_l$  while the corresponding recuperation factors are defined by the  $(L \times M)$  matrix  $m_l$ . The restricted technology thus takes the form:

$$\begin{aligned} T(b^*) &= T1 \cap T2 \\ T1 &= \{(x, y) : x \text{ can produce } y\} \\ T2 &= \{(x, y) : [n_l x - m_l y] \leq b^*\} \end{aligned} \quad (3)$$

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<sup>2</sup> Recently, Färe et al. (1989; 2005) have suggested replacing the axiom of free disposability of outputs with weak disposability to impose that emissions can be reduced when simultaneously reducing desirable outputs. This does, however, not solve the problem of exact emissions for each input vector.

Technology  $T1$  is the well-known set theoretical production model. It is assumed to satisfy the conventional axioms of no free lunch and no fixed costs, free disposability of inputs and desirable outputs, and that  $T1$  is a nonempty, closed, and convex set. See Chambers (1988) for a discussion of these properties.

Technology  $T2$  defines the set of inputs and desirable outputs that are feasible given the regulatory constraints. These constraints have two important implications: First, restrictions on the use of polluting inputs may lead to forgone profits for the producers. Second, restrictions on input use may also lead to reductions in non-regulated byproducts that are jointly generated with the regulated byproducts. This is the source of secondary benefits.

A producer's response to environmental regulations is based on sound economic reasoning. Consider a short-run profit maximization problem for the restricted technology from equation 3. Let  $w \in \mathfrak{R}_{++}^V$  and  $r \in \mathfrak{R}_{++}^M$  be vectors of prices for variable inputs,  $x_v \in \mathfrak{R}_+^V$ , and desirable outputs, respectively. The additional  $F$  inputs,  $x_f \in \mathfrak{R}_+^F$ , are assumed to be (quasi)fixed. The restricted short run profit maximization problem is then defined:

$$\begin{aligned}\pi^{C_{NA}}(r, w, x_f, b^*) &= \sup_{x_v, y} \left\{ ry - wx_v : (x, y) \in T(b^*) \right\} \\ &= \sup_{x_v, y} \left\{ ry - wx_v : (x, y) \in T1, [n_l x - m_l y] \leq b^* \right\}\end{aligned}\tag{4}$$

From equation 4 it is evident that the theory of emission constrained firms largely resembles that of expenditure constrained firms (see Lee and Chambers (1986) and Färe et al. (1990)). This literature, which can be considered as an extension of Shephard's (1974) indirect production theory, focuses on the impact of exogenous credit constraints on producers' input and output decisions and corresponding forgone profits. In our case, the credit constraints are replaced by emission constraints. However, access to end-of-pipe abatement technologies allows for endogenously determined regulatory constraints. By extending the profit problem from equation 4 with abatement choices, we subsequently deviate from the expenditure constraint literature. Emission constraints are now defined according to equation 2. Notice that we treat abatement as a separate production process, in the sense that inputs employed for abatement is a separate decision from inputs employed in the polluting production process. This allows us to treat abatement as a service or an input that the firm can purchase to reduce its emissions. Hence, problems related to data requirement for abatement production are avoided<sup>3</sup>.

Involvement in abatement activities is attractive if it allows the producer to reach more profitable input-output allocations compared to the restricted allocation in equation 4<sup>4</sup>. Let  $P_A \in \mathfrak{R}_+^L$  be the vector of unit abatement costs and define the short run profit maximization problem for a producer that has access to abatement technologies as:

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<sup>3</sup> See Førsund (2009) for a discussion on explicit modeling of abatement processes.

<sup>4</sup> The same argument goes for purchases of allowances under emission trading schemes. Abatement efforts and purchases of allowances are thus treated as perfect substitutes in the current model. Purchases of allowances allow for increasing the emission target,  $b^*$ , rather than  $A$ .

$$\begin{aligned}
\pi^{C_A}(r, w, P_A, x_f, b^*) &= \sup_{x_v, y, A_l} \left\{ ry - wx_v - P_A A_l : (x, y) \in T1, [n_l x - m_l y] - A_l \leq b^* \right\} \\
&= \sup_{A_l} \left\{ \sup_{x_v, y} \left\{ ry - wx_v : (x, y) \in T1, [n_l x - m_l y] - A_l \leq b^* \right\} - P_A A_l \right\} \\
&= \sup_{A_l} \left\{ \pi^{C_A}(r, w, x_f, b^* + A_l) - P_A A_l \right\}
\end{aligned} \quad (5)$$

The solution to the profit maximization problem from equation 5 is characterized by the level of abatement that maximizes the difference between emission restricted profits,  $\pi^{C_A}$ , and abatement costs. The restricted profits from equation 5 exceed that of equation 4 as long as the benefits from involvement in abatement activities exceed its costs.

The unrestricted profit maximum can be obtained from the conventional short-run profit function. This function defines maximal profits when the producer neglects his environmental impact and is solely concerned with current technological boundaries imposed by T1.

$$\pi(r, w, x_f) = \sup_{x_v, y} \{ry - wx_v : (x, y) \in T1\} \quad (6)$$

The optimization problems in equation 5 and 6 coincide when abatement costs are zero, i.e. when the firm faces no additional cleanup costs related to the employment of polluting inputs. If abatement costs are positive, the costs of regulatory compliance induce forgone profits relative to the unrestricted profits in equation 6.

The difference between the three profit maxima allows for evaluating secondary benefits in terms of regulatory induced reductions in non-regulated pollutants. Let  $y^{C_{NA}}(r, w, x_f)$ ,  $y^{C_A}(r, w, x_f)$ , and  $y(r, w, x_f)$  be the output vectors that solve the optimization problems in equation 4, 5, and 6, respectively, and let  $x^{C_{NA}}(r, w, x_f)$ ,  $x^{C_A}(r, w, x_f)$ , and  $x(r, w, x_f)$  be the corresponding factor demands. Secondary benefits can now be evaluated for any non-regulated pollutant,  $z_k \in z \setminus b$ , by applying the materials balance condition. Let  $j \in \mathfrak{R}_+^N$  be the vector of input emission factors, and let  $h \in \mathfrak{R}_+^M$  be the vector of output recuperation factors for the non-regulated pollutant. The condition for existence of secondary benefits for this pollutant is then defined:

$$z_k > z_k^A \Leftrightarrow [jx(r, w, x_f) - hy(r, w, x_f)] > [jx^{C_A}(r, w, x_f) - hy^{C_A}(r, w, x_f)] \quad (7)$$

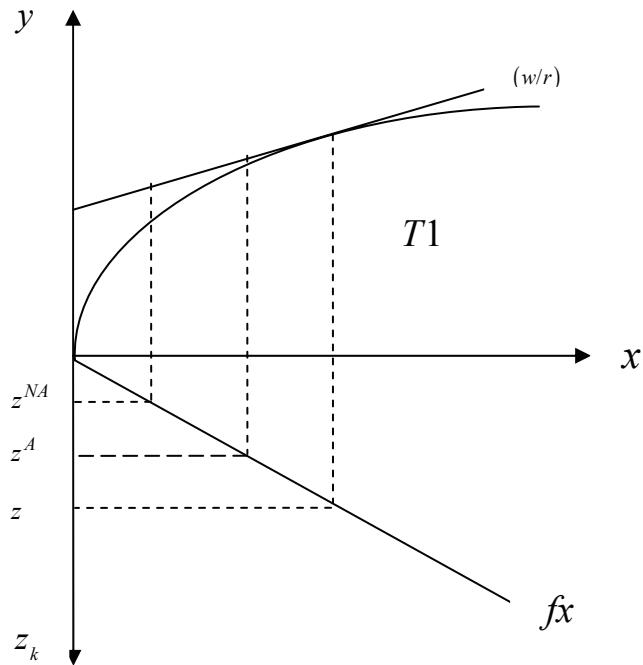
Loss of secondary benefits due to end-of-pipe abatement occurs when:

$$\begin{aligned}
z_k^A > z_k^{NA} \Leftrightarrow \\
[jx^{C_A}(r, w, x_f) - hy^{C_A}(r, w, x_f)] > [jx^{C_{NA}}(r, w, x_f) - hy^{C_{NA}}(r, w, x_f)]
\end{aligned} \quad (8)$$

Figure 1 illustrates the concept of secondary benefits in the case with one (variable) input,  $x$ , and one desirable output,  $y$ . For simplicity, the output recuperation factor concerning the non-regulated byproduct is here set equal to zero. Uncontrolled emissions of the secondary pollutant are then solely determined by the producer's input choice and the input emission factor, as depicted in the figure's lower panel.

The solutions to the optimization problems in equation 4, 5, and 6 lead to different levels of the secondary pollutant,  $z_k$ . The lack of abatement technology induces a low level of desired production and therefore also a low level of secondary emission,  $z^{NA}$ . In the case where the firm has access to abatement technologies, it has an opportunity to use more of the polluting input while cleaning up regulated emissions. This does, however, lead to higher secondary emissions,  $z^A$ . In the third case of absence of environmental regulations, the producer allocates optimally where the iso-profit line is tangent to the technology surface. Secondary emissions take the level  $z$  in this case.

Secondary benefits can now be assessed by applying equation 7 and 8: The difference between emission levels  $z$  and  $z^A$  determines the existence of secondary benefits under existing environmental regulations. The difference between  $z^A$  and  $z^{NA}$  indicates that secondary benefits are being crowded out by abatement. This difference represents potential increases in secondary benefits if the producer chooses to comply with the environmental regulation by improving uncontrolled emissions rather than controlled emissions.



**Figure 1:** Decomposing secondary benefits

## 5. DATASET AND EMPIRICAL IMPLEMENTATION

The Acid Rain program was implemented in 1995, aiming at reductions in annual sulfur and nitrogen emissions from American power plants of 10 million tons and 2 million tons, respectively, from 1980 levels. A cap and trade program for nitrogen was further introduced in 1999. Consequentially, sulfur emissions declined by 3,000 thousand metric tons and nitrogen emissions by 2,000 thousand metric tons in the period between 1995 and 2005.

The power producers have responded to the regulations in various ways: Lower costs for end-of pipe solutions, especially for flue gas desulfurization (FGD) technologies, have made this option attractive for compliance. The stringent environmental regulations have also increased the demand for low-sulfur coals and cleaner fossil fuels, in particular natural gas.

The average price for natural gas doubled in the period between 2002 and 2005. Changes in relative prices for fuels influence the attractiveness of fuel switching and potentially distort the producers' way of complying with regulatory constraints. A slowdown in switching to natural gas will have large implications for secondary carbon reductions since natural gas' carbon content is approximately half of that in coal and oil. We therefore utilize data for the period between 2002 and 2005 in this study: A balanced panel of 62 regulated power plants with coal to gas switching capacity is applied to assess secondary benefits of carbon reductions under the existing sulfur- and nitrogen regulations. Our dataset includes information on prices and quantities of coal, gas, and electricity, in addition to information on emission factors and controlled and uncontrolled emissions. Summary statistics of the data as well as information about the collection of variables can be found in appendix A<sup>5</sup>.

Next, we estimate the short run profit functions from equations 4, 5, and 6. We model the polluting technology as an activity analysis model or DEA model. Assume there are  $s=(1,..,S)$  firms in the dataset. Each firm uses inputs  $x^s = (x_1^s, \dots, x_N^s) \in \mathfrak{R}_+^N$  to produce desirable outputs,  $y^s = (y_1^s, \dots, y_M^s) \in \mathfrak{R}_+^M$ . The inputs are partitioned into  $V$  variable inputs and  $F$  quasifixed inputs, i.e.  $x^s = (x_v^s, x_f^s)$ . In this study, we consider coal and gas as variable inputs while generating capacity for coal and gas are treated as quasifixed factors. The inputs are employed to produce a desirable output, electricity. The firms operate under the emission targets  $b^s = (b_1^s, \dots, b_L^s) \in \mathfrak{R}_+^L$ . Let  $\lambda^s$ ,  $s=(1,..,S)$ , define intensity variables. We assume that the intensity variables sum to one to allow for increasing, constant, and decreasing returns to scale. The emission constrained short run profit DEA model for firm  $s$  is then defined by equation 9:

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<sup>5</sup> The dataset is available from the authors upon request.

$$\begin{aligned}
\pi(r^{s'}, w^{s'}, x_f^{s'}, b^{s'}) = \max_{y, x_v, \lambda} & \sum_{m=1}^M r_m^{s'} y_m - \sum_{n=1}^V w_n^{s'} x_n : \\
& \sum_{s=1}^S \lambda^s y_m^s \geq y_m, \quad m = 1, \dots, M \\
& \sum_{s=1}^S \lambda^s x_n^s \leq x_n, \quad n = 1, \dots, V \\
& \sum_{s=1}^S \lambda^s x_n^s \leq x_n^{s'}, \quad n = V+1, \dots, N \\
& \lambda^s \geq 0, \quad s = 1, \dots, S, \\
& \sum_{s=1}^S \lambda^s = 1, \\
& \sum_{n=1}^N n_{ln}^{s'} x_n - \sum_{m=1}^M m_{lm}^{s'} y_m \leq b_l^{s'}, \\
& l = 1, \dots, L
\end{aligned} \tag{9}$$

The materials balance constraints in equation 9 represent the regulatory constraints for sulfur and nitrogen emissions. Notice that they are specified without abatement efforts, A. Equation 9 is thus the empirical counterpart of equation 4. Here, the firm's controlled emissions are applied as emission constraints. The materials balance constraints will then be more stringent than those the firm currently is operating under, if the firm in question is involved in abatement activities. Notice further that emission factors are firm specific. This reflects differences in fuel quality, and thus unit emissions, across firms.

Next, we consider abatement efforts to calculate maximum profits under the firm's actual regulatory constraints. The materials balance constraints from equation 9 are now replaced by equation 10. The regulatory constraints are determined by the firm's uncontrolled emissions, which reflect its emission targets, allowance purchases, and abatement efforts.

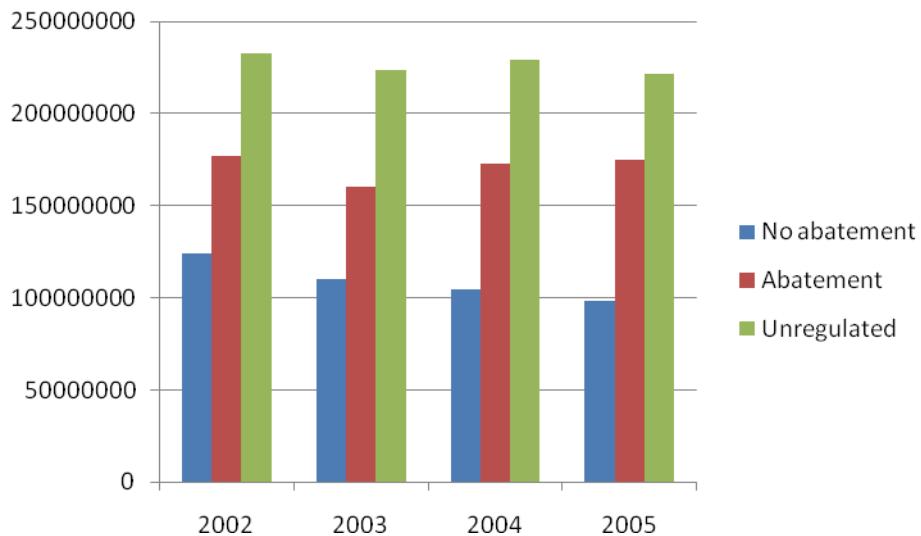
$$\sum_{n=1}^N n_{ln}^{s'} x_n - \sum_{m=1}^M m_{lm}^{s'} y_m \leq b_l^{s'} + A_l^{s'}, \quad l = 1, \dots, L \tag{10}$$

In the final modeled considered, the regulatory constraints are omitted from equation 9. The model now solves the unconstrained short run profit maximum corresponding to equation 6.

The two constrained profit maxima and the unconstrained profit maximum are solved for all firms in all four years. The results on variable inputs usage and desirable outputs that solve the profit maximization problems are further used to calculate secondary carbon emission, by applying carbon dioxide uncontrolled emission factors.

## 6. RESULTS

Figure 2 reports annual aggregated carbon emissions in tons for the 62 firms under the three scenarios. Years are specified along the horizontal axis. The green bar reports carbon emissions in absence of regulations for sulfur and nitrogen. The red and blue bars report carbon emissions generated under current regulatory constraints, with (red) and without (blue) access to abatement technologies.



**Figure 2: Secondary carbon benefits<sup>6</sup>**

Figure 2 can easily be related to figure 1. The difference between the green bar and the red bar indicates secondary benefits, in terms of reduced carbon dioxide, that is induced by current regulations for sulfur and nitrogen. The difference between the red bar and the blue bar indicates potential carbon reductions if compliance were to be achieved without end-of-pipe efforts.

The current regulation is estimated to produce secondary benefits, amounting to 25 percent lower carbon emissions than in the unregulated scenario. These secondary benefits should be accounted for when designing environmental regulations. In the current article, we have established a simple production framework that allows undertaking such evaluations. We believe this is an easy accessible tool for policy makers that allow for better tailoring of regulations.

Secondary benefits may be severely hampered by the existence of economically viable end-of-pipe abatement solutions. Carbon emissions calculated without end-of-pipe abatement are, on average, 36 percent lower than the emissions under existing regulatory constraints. This implies that carbon emissions, in absence of abatement, are 52 percent lower than unregulated carbon emissions. The absence of abatement technologies limits the producers' potential responses to environmental regulation substantially. They are now, to a larger extent, forced to involve in fuel substitution and/or reduce their activity levels, both of which contribute to secondary carbon benefits. To gain a better overview over the producers' responses, we calculate the ratio of variable

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<sup>6</sup> One firm is excluded from the calculations in 2003 due to missing observations of controlled emissions.

inputs and electricity that solve the profit maximization problem with ( $\pi^{CNA}$  and  $\pi^{CA}$ ) and without environmental regulations ( $\pi$ ). Table 1 reports mean values and standard deviations for the ratios.

**Table 1: Ratios of optimal supply and input use**

YEAR	$\Pi^{CNA} / \Pi$			$\Pi^{CA} / \Pi$		
	Coal	Gas	Electricity	Coal	Gas	Electricity
2002	0.49 (0.31)	0.97 (0.75)	0.65 (0.27)	0.72 (0.28)	1.02 (0.66)	0.83 (0.19)
	0.47 (0.29)	1.82 (2.40)	0.68 (0.24)	0.67 (0.31)	2.04 (1.99)	0.82 (0.19)
2003	0.42 (0.32)	2.59 (3.91)	0.60 (0.32)	0.73 (0.26)	2.34 (3.06)	0.86 (0.14)
	0.43 (0.29)	1.61 (1.76)	0.63 (0.29)	0.75 (0.27)	1.28 (0.63)	0.86 (0.16)

The results are in line with our expectations. The optimal coal use and electricity supply are lower under emission constrained profit maximization than under unconstrained profit maximization. Gas, on the other hand, is over-utilized relative to the unconstrained profit maximum since it causes less sulfur and nitrogen emissions compared to coal. The gas ratios increase over the period from 2002 to 2004 as a consequence of increasing unit costs for gas. Notice the large standard deviations for the gas ratios, implying that the firms' incentives to switch to gas differ in the sample. This may be due to regional differences in fuel costs and quality<sup>7</sup>, as well as differences in abatement efforts.

The mean gas ratios reported in table 1 are approximately equivalent when calculated with and without abatement. Coal use and electricity generation are, however, lower without abatement than with abatement. This implies that the missing abatement activities induce gas-intensive productions for some of the firms in the sample.

The reduction in flexibility induced by missing abatement technologies will, of course, come at a cost for the producers. We therefore calculate the ratio of profits in the three scenarios. This follows Brännlund et al. (1995) who evaluate environmental regulations' impact on the profitability of the Swedish pulp and paper industry. Our measure of forgone profits is useful as it allows weighing the costs of compliance against the primary and secondary benefits. The mean profit ratios are reported in table 2. Standard deviations are reported in brackets.

**Table 2: Average profit ratios**

YEAR	$\Pi^{CNA} / \Pi^{CA}$	$\Pi^{CA} / \Pi$	$\Pi^{CNA} / \Pi$
2002	0.76 (0.27)	0.88 (0.17)	0.70 (0.26)
2003	0.82 (0.25)	0.86 (0.17)	0.73 (0.24)
2004	0.71 (0.36)	0.91 (0.10)	0.65 (0.34)
2005	0.76 (0.33)	0.92 (0.10)	0.71 (0.31)

The ratio  $\Pi^{CA} / \Pi$  is a measure of forgone profits under existing regulations, when the producers have access to abatement technologies. It indicates that compliance is costly, as the restricted profits are, on average, 10 percent lower than the unconstrained profits. The relatively low standard deviations

<sup>7</sup> Prices are gross, including transport costs to the generation facility. This may also explain differences in coal qualities, as coal mining takes place in some regions of the U.S., making transport costs low for some qualities.

indicate that costs of compliance are distributed evenly across the producers. This is expected since the Acid Rain program is an allowance trading program that seeks to equalize the costs of compliance across firms.

$\Pi^{\text{CNA}} / \Pi^{\text{CA}}$  is a measure of forgone profits due to lack of access to abatement techniques. On average, constrained profits are 25 percent lower when calculated without abatement opportunities. Access to abatement, that allows reaching more profitable input-output allocations, is clearly desirable for the producers. This indicates that economically viable abatement technologies are also likely to be attractive in the case of carbon emissions: Stringent carbon regulations may enhance incentives to adopt new abatement technologies, potentially accompanied by decreasing costs due to accumulation of experience in constructing these technologies (Riahi et al., 2004). This raises questions about the prospects of secondary benefits for sulfur and nitrogen from regulations for point source carbon emissions.

## 7. SUMMARY AND CONCLUDING REMARKS

This paper demonstrates that our approach is well suited for analyzing the impacts of end-of-pipe solutions on secondary benefits. Our framework is consistent with the environmental economics literature's "least cost" understanding on how firms choose to reduce regulated pollutants. This is reflected in our empirical results from the U.S. electricity sector, where end-of-pipe abatement is part of current firm responses and profits decrease when we estimate producer responses without end-of-pipe solutions. As expected, end-of-pipe abatement of sulfur and nitrogen emissions leads to increased carbon dioxide emissions compared to other abatement strategies. That is, secondary benefits from end-of-pipe solutions decrease.

Our results also have profound implications for environmental policy. Once firms have chosen abatement strategies for currently regulated pollutants, the costs of added regulations for other pollutants may be higher. If the likelihood of new regulations is perceived high enough, the initial abatement strategies may have emphasized synergies with other pollutants to a larger extent than what is the case if regulations for other pollutants are not expected. Moreover, abatement costs are likely to be higher if the new regulations are inflexible, limiting firms' regulatory responses.

We see indications of this in our analyses. Carbon dioxide emissions are currently not regulated in the U.S., while they may be in the future. However, global climate change has been on the agenda for more than a decade, and signals from regulators that green house gases could face regulations could have influenced firms' choices of abatement strategies for sulfur and nitrogen.

This brings us back to the issue of forward looking environmental policies raised in the introduction, and the importance of regulatory agencies acting in predictable ways. The seminal paper "Rules Rather than Discretion: The inconsistency of optimal planning" by Kydland and Prescott (1977) primarily deals with creating predictable (macro) economic policies. Kydland's and Prescott's concerns about dynamic inconsistency arise from agents having expectations about ad-hoc regulations, thereby adjusting their regulatory responses. This consistency issue is further accentuated by the fact that polluter responses to a current environmental issue may differ if one knew which other issues were likely to surface in the future. Our case with U.S. power utilities, with the Acid Rain and nitrogen regulations put in place in the middle of the 1990s, and green house gas emissions, is an excellent example: If there had been some notion about a connected environmental

issue, polluter responses may have differed with less emphasis on end-of-pipe solutions for sulfur and nitrogen and more on fuel switching or efficient burning of fossil fuels (technological process change).

The quasi-option value (Arrow and Fisher, 1974; Henry, 1974) is one approach of dealing with this uncertainty. As noted by Conrad (1980), the size of quasi-option value depends on the probabilities used by agents when calculating expected payoffs of various alternatives. One way of reconciling quasi-option values and investment flexibility with the predictability issue raised by Kydland and Prescott (1977) is for the regulator to signal that other yet unknown issues may emerge, and will be followed by incentive based environmental regulations. This would induce polluters to consider the flexibility and robustness of their abatement responses to current pollution regulations.

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## 9. APPENDIX A

**Table 3:** *Summary statistics*

	YEAR			
	2002	2003	2004	2005
$COAL$ (t)	1301041.0 (1220806.0)	1275955.0 (1228863.0)	1370610.0 (1256180.0)	1381206.0 (1288359.0)
$GAS$ (mCF)	3894523.0 (7324023.0)	2206878.0 (4584085.0)	2314146.0 (4971411.0)	2528379.0 (5455743.0)
$EL$ (MwH)	3064065.0 (3007108.0)	2851715.0 (2817467.0)	3034895.0 (2810951.0)	3058841.0 (2897703.0)
$COAL$ CAP (MW)	391.9 (378.7)	391.9 (378.7)	378.2 (367.1)	377.6 (367.5)
$GAS$ CAP (MW)	350.3 (334.7)	327.6 (334.7)	380.9 (349.6)	381.6 (349.7)
$b_{SO_2}$ (t)	13894.7 (15938.1)	14115.1 (16548.3)	13397.6 (14935.6)	13931.4 (16618.9)
$b_{NO_X}$ (t)	5639.5 (4866.6)	5719.7 (4993.0)	5027.8 (4159.3)	5044.2 (4278.1)
$A_{SO_2}$ (t)	3925.0 (13020.3)	4223.5 (14131.1)	4916.7 (15471.2)	5177.9 (17185.3)
$A_{NO_X}$ (t)	2258.8 (2985.3)	2177.8 (3024.7)	3051.3 (3722.2)	3162.6 (3977.0)
$n_{COAL}^{SO_2}$	$1.51e^{-02}$ ( $1.31e^{-02}$ )	$1.47e^{-02}$ ( $1.28e^{-02}$ )	$1.50e^{-02}$ ( $1.30e^{-02}$ )	$1.58e^{-02}$ ( $1.48e^{-02}$ )
$n_{COAL}^{NO_X}$	$5.68e^{-03}$ ( $1.25e^{-03}$ )	$5.66e^{-03}$ ( $1.38e^{-03}$ )	$5.71e^{-03}$ ( $1.24e^{-03}$ )	$5.76e^{-03}$ ( $1.22e^{-03}$ )
$n_{COAL}^{CO_2}$	$2.20e^{00}$ ( $3.08e^{-01}$ )	$2.15e^{00}$ ( $3.87e^{-01}$ )	$2.15e^{00}$ ( $3.16e^{-01}$ )	$2.17e^{00}$ ( $2.94e^{-01}$ )
$n_{GAS}^{SO_2}$	$3.00e^{-07}$ ( $0.00e^{+00}$ )	$3.00e^{-07}$ ( $0.00e^{+00}$ )	$3.00e^{-07}$ ( $0.00e^{+00}$ )	$3.00e^{-07}$ ( $0.00e^{+00}$ )
$n_{GAS}^{NO_X}$	$1.40e^{-04}$ ( $0.00e^{+00}$ )	$1.40e^{-04}$ ( $0.00e^{+00}$ )	$1.40e^{-04}$ ( $0.00e^{+00}$ )	$1.40e^{-04}$ ( $0.00e^{+00}$ )
$n_{GAS}^{CO_2}$	$5.91e^{-02}$ ( $1.72e^{-03}$ )	$5.94e^{-02}$ ( $1.01e^{-03}$ )	$5.91e^{-02}$ ( $1.71e^{-03}$ )	$5.95e^{-02}$ ( $1.51e^{-03}$ )
$r_{EL}$	46.0 (9.5)	48.1 (10.4)	48.8 (11.0)	58.9 (14.8)
$w_{COAL}$	28.0 (10.9)	27.7 (10.3)	30.1 (13.3)	36.6 (17.3)
$w_{GAS}$	4.3 (1.4)	6.2 (1.3)	7.2 (3.0)	9.2 (1.4)

The sample applied in this study consists of 62 fossil fuel fired power plants operating in the period 2002-2005. To model a homogeneous production technology with fuel substitution possibilities we follow Welch and Barnum (2009) and only include firms that obtain at least 1 percent of their energy input from both coal and gas in year 2002 in the sample. The price for gas was low in 2002, implying that firms with coal to gas switching capacity are likely to have exploited this opportunity in that particular year. Firms that satisfy the selection criterion in 2002, and use both coal and gas as energy inputs the following years, are kept in the sample. Producers that convert to single fuel production are excluded to avoid corner solutions.

The form EIA-906/920 provides monthly information on fuel consumption and net generation. This information is aggregated up to annual levels. Prices and sulfur content of fuels is obtained from

EIA-423/923. Existing generating capacity is obtained from EIA statistics on capacity. Generating capacity is further divided into coal capacity and gas capacity by applying information on each boiler's primary and secondary fuels. The sales price for electricity is calculated from retail and resale revenues reported in EIA-861. Following Färe et al. (2005), the sales price is taken to be the average of the retail and resale price.

Appendix A of EIA's Electric Power Annual provides an overview of emission factors for various sub-groups of coal and gas. We adjust the emission factors with the corresponding sulfur content of the fuels when specified by EIA. To capture that different subgroups have different emissions factors, we define aggregate emission factors for coal and gas as the weighted sum of the emission factors for the fuels' sub-groups, using the share of fuel purchased from the various subgroups as weights. The factors are further adjusted to report tons of SO<sub>2</sub>, CO<sub>2</sub>, and NO<sub>x</sub> emissions per unit of weight of fuels. Controlled emissions of SO<sub>2</sub> and NO<sub>x</sub> are obtained from Environmental Protection Agency's (EPA) Acid Rain Program database. Summary statistics for the dataset is provided in table 3.



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Kenneth Løvold Rødseth was born in Ålesund, Norway, in 1982. He holds both a Bachelor's and Master's Degree in Economics from the University of Bergen, Norway (2005;2007).

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The thesis consists of an introduction and four related papers. Their common topic is production analysis when undesirable outputs are accounted for. The introduction establishes the research platform on which the following papers are built by reviewing existing literature, proposing a new way of modeling environmentally regulated firms, and introducing some new areas of model application.

The first article evaluates one of the most commonly applied axioms when some outputs are undesirable, namely weak disposability. The axiom is discussed in relation to the materials balance condition. It is shown that the two cannot hold simultaneously except when introducing abatement activities. Second, since abatement is not explicitly accounted for when applying the weak disposability axiom, the potential for reducing emissions may be overstated. This has important implications for efficiency measurement and estimation of marginal abatement costs.

The second article builds on the conclusions from the first article by explicitly accounting for both the ways undesirable outputs are generated and how they may be reduced. Profit maximization under environmental regulation is evaluated and it is shown how regulations increase the costs of polluting inputs and lead to forgone profits. Consequently, allocative efficiency may be underestimated if regulatory constraints are not accounted for. Using U.S. electricity data, I find empirical support for this proposal. This is contrary to the established literature that has mainly focused on undesirable outputs' influence on technical efficiency.

The third article's purpose is to extend an established method for abatement cost estimation to account for more flexible producer responses to environmental regulation than is currently being done. By applying the directional distance function, marginal abatement costs are derived that reflect the least cost way of reducing undesirable outputs.

The fourth article develops one of the new areas of model application proposed in the introduction, namely evaluating how efforts directed at reducing some undesirable output may influence other undesirable outputs due to jointness. The emphasis is on how different strategies for reducing undesirable outputs influence the occurrence of reductions in secondary undesirable outputs.

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