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# Dynamic Factor Portfolios in the Norwegian Stock Market

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Master in Business Administration

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Ås - May 10, 2016

# Dynamic Factor Portfolios in the Norwegian Stock Market

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May 10, 2016

## Abstract

We test how dynamic factor portfolios utilizing acknowledged market anomalies perform on Oslo Stock Exchange in the period 1998 to 2015. The individual factor portfolios have varying performance over the market through time, but carry a significantly lower level of risk and higher risk-adjusted return. Together with low correlation and cross-exposure in factors, they clearly give a diversification effect. Our equally weighed factor portfolio produces a higher risk-adjusted return over the market (M2). Even though the use of leverage is controversial, an investor could achieve the same performance as the market with half the risk of an index fund by adding leverage. This study is especially interesting for long term institutional investors.

**Keywords:** *Factor Analysis, Multi-factor Portfolios, Quant Equity*

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# 1 Introduction

Quantitative equity investing has been around for a long time, but the recent years increased interest<sup>1</sup> is much because traditional diversification met a brick wall under the financial crisis. The diversification failed when investors needed it the most, and since then there has been an increased demand for alternative diversification methods. Factor investing is initially quantitative management that seeks to exploit known market anomalies, recently relabeled as smart beta. The goal for a smart beta investor is to capture a better risk reward than the market cap weighted index by tilting the portfolio towards other weightings, based on ranking of measures like company size or volatility.

Our fascination for factor investing has been influenced by the ongoing debate in the financial world about smart beta, and that the theory of harvesting risk premium goes against well-known financial theories. The first sight of smart beta came in 2005, when Research Affiliates created a fundamentally weighed index based on book value, sales, cash flow and dividends (Arnott, Hsu, and Moore (2005)). Smart means that you are investing simply in an alternative or a non-traditional index. Beta means that you are still investing in a passive index, and not in the discretionary judgment of a single manager. A smart beta fund can be seen as a combination of an active and passive strategy, where given rules decides trades of action while still investing in a broad selection of stocks, like an index. Active funds are already exposed to risk factors trough active management with bottom-up firm picking or top-down country and market timing. However, this is achieved at much higher cost compared to smart beta funds and ETFs<sup>2</sup>. David Blitz, Head of Quantitative Equities Research at Robeco, argues that much of the reason for the underperformance by many actively managed funds is their lack of ability to exploit these factor premiums efficiently<sup>3</sup>.

A way to try outperform the market is by utilizing a dynamic factor portfolio that aim to use stock characteristic to exploit anomalies and set them together in a multi-factor portfolio. The goal is to obtain a better performance than the market by tilting the portfolio onto known risk factors, proven historically to yield a higher performance. The reason for the boom is related to the debate and theory of efficient markets and whether it is a lack of this in practice. For a fully efficient market, no one should obtain a higher return than the market portfolio given the risk level. All information is given and prices follow a random walk. In the real-world however, the opinions are divided between none-efficient and efficient market situations. Lasse Pedersen explains it as something in between, an efficiently inefficient market.

*“Competition among professional investors makes market almost efficient, but the market remains so inefficient that they are compensated for their costs and risks”*

*Efficiently Inefficient, Pedersen (2015)*

A quick search on the Financial Times reveals that factor investing and smart beta is one of the most heavily debated themes in the financial world at the moment. Moreover, the practitioners and academics argue about its diversification effect and tilting towards specific risks. At the same time, more money is

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<sup>1</sup>Smart Beta was the single most searched topic on Investopedia in 2015. <http://www.investopedia.com/articles/investing/120715/investopedias-top-10-terms-2015.asp>. Written 07.12.15, extracted 25.04.2016.

<sup>2</sup>Exchange Traded Fund

<sup>3</sup><http://www.robeco.com/en/professionals/insights/quantitative-investing/factor-investing/Strategic-allocations-to-factor-premiums-the-next-big-thing.jsp>. Written 10.07.2012 extracted 02.05.2016.

pouring into factor-based investment firms, and more smart beta funds and ETFs are being offered to the public. In the ETF market, 24 percent of all institutional decision makers use smart beta products (Clarke, De Silva, and Thorley (2016)). According to Financial times, smart beta ETFs accounts for almost one fourth of the \$ 1.7 trillion ETF industry, and the industry is gearing up for a “smart beta war”<sup>4</sup>. The financial world is divided in their belief of smart beta, and its ability to outperform the market. Malkiel (2014) argues that a majority of smart beta funds and ETF have failed to produce reliable excess returns. Bruce and Levy (2014) argues that the factors have broad support in the literature, but that the strategy is vulnerable to overcrowding and specific risks from tilting the portfolio onto the risk anomalies.

The Norwegian Government Pension Fund Global ordered a report based on the big losses experienced during the financial crisis. The findings, afterwards called “the Norway Model”, revealed that around 70 percent of the active returns since inception could be explained by exposure to factors (Ang (2009)). In the report, they stated that the fund, as a long term investor, should approach a more systematic factor harvesting, through a higher deviation from the benchmark. The more recent report<sup>5</sup> from the fund, covering performance and risk for 2015, shows that 47 percent of the active returns adjusted for management costs, could be explained by exposures to risk factors. The report was based on Fama and French (2015) five-factor model. In other words, a great deal of the active returns can be associated with exposure in risk factors.

Existing literature on the Norwegian market is based on explaining the cross-sectional variation in returns, and testing for existence of single anomalies. In our study, we will take advantage of the findings by utilizing the factors in practice by creating factor portfolios and see if it is possible to harvest the risk premium in practice. We will test how well such a strategy would have worked in Norway from 1998 to 2015, and further test how the factors combined in a portfolio performs over time. The factors we have chosen to further incorporate are generally recognized, with decades of confirming evidence during different time periods and market types. The four factors presented further are size, value, momentum and low volatility. This study is especially interesting for large institutional investors; such as pension or wealth funds with a long-term investing perspective.

In the next chapter, We will start by going through the related literature on the different factors and factor investing, focusing on their main findings, and methodology used, especially for the Norwegian market. Thereafter, we will present the data and the main characteristics of Oslo Stock Exchange. Furthermore, we will go through the methodology used in constructing the factor portfolios and portfolio optimization. We then present our analysis of the factors and multi-factor portfolios. Finally, we draw conclusions and suggest further research on the topic.

## 2 Related Literature

Factor investing strategies is based on the assumption that investing in well-documented factors will yield a risk premium. In this chapter, we will take the reader trough the central theory of factor investing. We start with an overview of central pricing theories and how factor investing goes against them, before

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<sup>4</sup><http://on.ft.com/1SDFAub>. Written 08.02.2016, extracted 15.04.2016.

<sup>5</sup>Performance and risk, Government Pension Fund Global, Report 2015, <http://www.nbim.no/en/transparency/reports/2015/performance-and-risk-2015/>. Written: 16/03/2016, extracted: 07/04/2016.

explaining the theories behind the existence of each systematic factor.

The efficient market hypothesis (EMH) states that all share prices reflect all relevant information. It is in that respect not possible to beat the market given the risk level (Malkiel and Fama (1970)). The factor investing strategy seeks to exploit the anomalies around EMH by utilizing available information to find mispriced securities and thereby earn an excess return. For factor investing to yield an abnormal return, the market must be in some way inefficient in the way information is used for pricing securities and thereby a chance to buy (sell) cheap (expensive) securities. The main tool for pricing securities in the financial world is the capital asset pricing model (CAPM) introduced independently by Sharpe (1964), Lintner (1965) and Mossin (1966). The CAPM explains the relationship between risk and expected return for an asset. According to Fama and French (2004), it is still the most used framework for pricing assets, and the main reason for this is its simplicity and practical usefulness. The theory builds on the assumption that an investor must accept higher risk in order to obtain a higher expected return. The general idea behind the capital asset pricing model is that an investor has to be compensated in two ways. The time value of money denoted as the risk-free rate, and the risk associated with the investment. A number of studies have challenged the CAPM. Jensen, Black, and Scholes (1972) was some of the first to put forward critique on the CAPM. They argue that realized returns is significantly impacted by the fact that most investors are not able to borrow at the risk free rate. This implies that the CAPM overestimates the expected return, meaning that the relationship between beta and the expected return in reality is flatter.

The CAPMs limited ability to perform empirically has resulted in the evolution of alternative asset pricing models to better explain returns. Fama and French (1992) found that size and book value to equity better explained cross-sectional variations in average stock returns associated with market beta. In 1993, Fama and French introduced the famous Fama-French three-factor model. The model seeks to better explain returns by including a SMB (small minus big) and HML (high minus low) risk factor. The model states that small companies on average have a higher risk-adjusted return than big companies. It further states that value companies (high BE/ME ratio) have a higher risk-adjusted return than growth companies (low BE/ME ratio). Carhart (1997) further expanded the model taking into account the momentum effect based on Jegadeesh and Titman (1993) findings.

As a result of this, research on factors and the ability to implement this as a part of an investment strategy has increased rapidly. When testing the explanatory power of three multi-factor models; macroeconomic, statistical and fundamental, Connor (1995) finds that the fundamental factor model gives the best explanatory power of returns. Explaining that a factor is constructed from a set of mimicking portfolios to capture the marginal returns associated with the exposure in the factor. Cochrane (1999) finds that investors can earn a substantial premium from holding known risk factors, and implementing them in their investment strategy. After the devastating effects the financial crisis had on the equity market, factor investing got additional interest. Many refer to especially Ang (2009) report for the Norwegian Government Pension Fund Global, as the start of the increased international interest for the subject. As mentioned, they conclude that much of the active return generated by the fund could be explained by tilting towards known risk factors, and in that sense explain the terrible performance the fund experienced during the financial crisis. They further argued that the fund, as a long-term investor should continue the approach, and that this over the long-term yield a risk premium. Bender, Briand, Nielsen,

and Stefek (2010) argues that factor risk premiums already exists in traditional diversified portfolios, but are dominated by broad equity returns and that the solution is to separate them. Ilmanen and Kizer (2012) states that investors in light of the financial crisis started to look at alternative ways of diversifying their portfolio, and that the solution might be to shift focus from asset class diversification to factor diversification. Further arguing that long/short factor diversification has been more effective than asset class diversification during times of financial distress, and that a long-only factor tilt approach is a good way of enhancing returns. When studying the practical aspects of factor investing, Koedijk, Slager, and Stork (2014) presents three different ways for institutional investors to implement factor investing in their investment strategy. 1. Risk due diligence, that refers to using risk factors to check for unwanted concentration in the portfolio and adjust accordingly. 2. Use of long-only factor tilts to underweighted factors in the asset allocation, through investment styles like momentum, value or alternative indexing. 3. Factor optimization based on the risk factors, with a long/short exposure in a factor to harvest the pure factor premium. In this paper, we will focus on the second and third approach. The EDHEC Business school argue that factors add value in both single and multiple asset class portfolios when they are used as an alternative to broad asset class indices, focusing on the size, value, momentum and low volatility factors for equities (Martellini and Milhau (2015)).

In the following sub-chapters, we will present the relevant literature for the chosen factors, and at the end take a look on research that has been done on the Norwegian market and their main findings. The research done on the factors internationally is vast. We have therefore focused on presenting the initial findings and the evolution up to the most recent literature. We will go through the factors in the fashion they were systematically documented, from value and size to momentum and finally the low volatility factor.

## 2.1 Value Stocks Outperform Growth Stocks

Value investing has been an established strategy for a long time, utilized by analyzing a firm's fundamentals to decide whether it is underpriced. Benjamin Graham first mentioned the strategy in 1934 together with David Dodd (Graham, Dodd, Cottle, et al. (1934)), and the strategy has been further developed, and used by many famous investors like Warren Buffett, William J. Ruane, Irving Kahn and Charles Brandes.

Basu (1977) was the first to emphasis this theory further. By taking systematic investment decisions based on the price/earnings relation relative to the market, he observed that low P/E-ratio portfolios had a better risk-adjusted return than high P/E-ratio portfolios during the period 1957 - 1971. Later on Stattman (1980) found that returns on stocks are positively correlated with the book value and the market value of equity. Both Basu (1983) and Rosenberg, Reid, and Lanstein (1985) further elaborated this factor, affirming that buying low priced stocks (high BV/P) and selling high priced stocks (low BV/P) relative to the market gained significant abnormal return on the US stock market.

Fama and French (1993) combined the size and value factor into a three-factor model, and the results concluded with a significant capture of cross-sectional variation in the average return. Their findings has been confirmed by studies on the Japanese market by Chan, Hamao, and Lakonishok (1991), and by Fama and French (1998) on the global equity market. They found that growth stocks were outperformed on an average of 7.68 percent compared to value stocks in 12 of 13 markets in the period of 1975 to 1995.

New research on the field by Alighanbari, Subramanian, and Kulkarni (2014) showed an outperformance for portfolios tilted towards value stocks relative to their market weighed indices over the last 40 years when examining US, European, emerging and international markets. The efforts to explain the value effect have divided the theorists between the rational and the behavioral. Fama and French (1993) argue that the value effect captures the increased exposure to distress risk - including liquidity, cash flow and business cycle risk. Bondt and Thaler (1985) argue that value stocks often turn out to be long-term poor performers, and that the value effect is due to overestimation (growth) / underestimation (value) of fundamentals. The positive findings in value have been consistent throughout the period from the first theory of Graham up to today's research.

## 2.2 Small Cap Stocks on Average Outperform Large Caps

Banz (1981) was the first to highlight the size factor, with a significant excess return associated with small cap firms in the US stock market from 1926 to 1975. Banz stated that small firms have a higher risk-adjusted return than large firms, and that this effect is not linear with respect to market value. In other words, the effect is non-present between average-sized and large-sized firms. Ever since discovery, the size effect has been heavily debated. Basu (1983) found a significant size effect. However, by controlling for value, the size effect gave no additional explanation in returns. Chan and Chen (1988) further investigated the size effect by looking at the US stock market from 1949 to 1983. They found a short-term size effect (5 years) that diminished over time. After the release of the three-factor model by Fama and French (1993), the research increased dramatically, and the conclusions are many and diverse. When studying the size effect between 1963 and 1997, Horowitz, Loughran, and Savin (2000) found a significant size effect of 13 percent a year, in line with Banz. Contrary, they found no size effect between 1981 - 1997. Furthermore, when removing the smallest firms from the sample (less than 5 mill \$ market value) the size effect was nonexistent in the data sample 1963 - 1997, which led to the conclusion that the size effect had disappeared. Van Dijk (2011) points out that the size effect has been positive and large in recent years. He undermines Banz initial statement *"it is not known whether size per se is responsible for the effect or whether size is just a proxy for one or more unknown factors correlated with size"*. He argues that it is too early to conclude that the size effect has disappeared, and pointed out that more research was necessary on the subject. To confirm van Dijk's statement, Asness, Frazzini, Israel, Moskowitz, and Pedersen (2015) found that even though the size effect has been challenged for its weak historical record and variance in significance over time, the effect is significant and persistent if we control for junk. Raina, Anil, Lokesh, and Raman Aylur (2016) examine the three main arguments against the size effect; it has disappeared and no longer exists; it exists only in the US; the effect disappears when filtering out smaller stocks for investability. Their study dismisses all the arguments and claims that the size effect exists globally.

There is also an ongoing debate regarding the reason for the existence of the size factor, and three main explanations are in focus. 1. A result of data mining, and measurement errors (Black (1993)). 2. A proxy for non-diversifiable risk, such as firm distress, macroeconomic factors and liquidity (Fama and French (1992, 1993, 1995)). 3. A result of irrational investors or institutional constraints (Bondt and Thaler (1985)). In addition, many papers highlight that the size effect is sensitive to time periods, meaning it is not time-consistent. After concluding that the size effect had disappeared in the 1980s, several recent papers have stated that the size effect has been large and positive in recent years (Van Dijk

(2011)), followed by a similar statement by Asness, Frazzini, Israel, Moskowitz, and Pedersen (2015), and Raina, Anil, Lokesh, and Raman Aylur (2016).

### 2.3 Past Winner Stocks Outperform Past Losers

The momentum strategy in the literature is defined as a self-financing strategy consisting of going long positive momentum and short negative momentum. The momentum factor goes against the initial statement of Malkiel and Fama (1970) and the efficient market hypothesis. When testing for the short-run persistence of mutual fund performance from 1974 - 1988, Hendricks, Patel, and Zeckhauser (1993) found that funds with both poor and good past performance were likely to do similar in the future. Confirming this, Jegadeesh and Titman (1993) found that past winners are more likely to be future winners, arguing that a strategy consisting of buying past winners and selling past losers yield significant returns with holding periods of 3 - 12 months, called the one-year momentum effect. They assigned the momentum effect to investors over/under reaction to news. Asness (1997) found that the momentum effect was most noticeable over a short time of 3 - 12 months. Carhart (1997) argues that buying the top performing, and selling the bottom performing funds will yield a significant return of 8 percent per year. He also argues that most of the difference in the spread can be assigned to the value and momentum factor with 4.6 percent. By combining Jegadeesh and Titman's methodology with Fama and French three-factor model, Carhart managed to create a parsimonious model explaining returns including momentum. Moskowitz (1999) further states that there is a significant momentum effect observed in the intermediate investment horizon (6 to 12 months). He also states that most of the momentum effect can be assigned to common industry or sector movement. When examining the time-series momentum effect in a vast number of liquid instruments, Moskowitz, Ooi, and Pedersen (2012) finds a significant time series momentum effect and abnormal returns in all asset classes, covering equities, currencies, commodities and bond futures, across all markets. They confirmed that this effect is consistent with theories of over- or under-reaction to market news. Furthermore, they argue that this trading strategy is most effective on volatile markets.

Many explanations for the momentum effect has been put forward. Hong and Stein (1999) argue that the momentum effect is due to investors under-reaction to news in the short run, due to scattered information across the population. Some even argue that the momentum effect can be assigned to reversals between investment funds, meaning funds enter past good performers, and leave bad performers pushing the prices away from their fundamental value (Vayanos and Woolley (2013)). Daniel, Hirshleifer, and Subrahmanyam (1998) assign the momentum effect to over/under confidence to news, much in line with Jegadeesh and Titman (1993). The empirical consensus is that there is a momentum effect in international and regional markets. However, as with the other factors it is sensitive to time periods and varies over time.

### 2.4 The Low Volatility Anomaly

After the unveiling of the CAPM-model there has been research challenging many of the underlying assumptions. The first tilt towards the low volatility anomaly came in 1972 from Haugen and Heins, challenging the CAPM assumptions that higher risk gives higher return. They examined the New York Stock Exchange from 1926 to 1971 and concluded, “*Over the long run stock portfolios with lesser variance in monthly returns have experienced greater average returns than their riskier counterparts*”. Later on

Haugen together with Baker tested the same hypothesis in the period 1972 - 1989, for US and international markets, yet again confirming the anomaly (Haugen and Baker (1991)). Furthermore, they verified the result by using the same methodology between 1990 - 2011 (Haugen and Baker (2012))). More recent studies on different markets have boosted up the popularity of low volatility stocks. Clarke, De Silva, and Thorley (2006) performed an empirical analysis of minimum variance portfolios on the 1 000 largest stocks in the US from 1968 to 2005. They discovered that the volatility and beta declined by 25 and 33 percent respectively, compared to the capitalization-weighted market benchmark. Ang, Hodrick, Xing, and Zhang (2006) also tested the hypothesis on low volatility portfolios, and the results were consistent with the findings of Clarke, De Silva, and Thorley (2006). Frazzini and Pedersen (2014) took another approach by assuming low volatility stocks were consistent with low beta stocks. They created a betting against beta factor, going long low beta stocks and short high beta stocks, and levering and de-levering them accordingly to a beta of one. The portfolio showed significant positive risk-adjusted return over the time period 1926 - 2012.

Contributing to a more optimal portfolio strategy, with rebalancing and transaction cost in mind, Blitz and Van Vliet (2007) tested the hypothesis with a long-term volatility sample (past 3 years), in contrast to Ang, Hodrick, Xing, and Zhang (2006) where the sample period was on a very short term (1 month). The results were even better with a clear outperformance of the market portfolio with a volatility of approximately 1/3 lower than the market portfolio and a Sharpe ratio of 0.72 compared to the market portfolio of 0.4. Blitz and Van Vliet (2007) and Clarke, De Silva, and Thorley (2006) also tested if there were any effects bundled in other factors in the low volatility factor portfolio. After controlling for the value and size factor by Fama and French, they still captured a significant effect in explaining the variation in returns. The critics of the low volatility effect mainly comes from the standpoint that the portfolio often consist of illiquid and small-sized firms. Ibbotson, Chen, Kim, and Hu (2013) finds that illiquid stocks have gained excess return and risk reduction compared to liquid stocks, and states that the excess return comes from a liquidity premium. Haugen and Baker (1996, 2008) concluded that the liquidity and size of the stocks represented in the low volatility portfolios are not unambiguous, but a mix of both large and small companies with different levels of liquidity. Furthermore, low volatility portfolios are mainly bundled with large and stable companies.

The consensus around the low volatility factor is quiet broad on all international markets and both Ang, Hodrick, Xing, and Zhang (2009) and the earlier mentioned Haugen and Baker (2012) state that this also existing in Norway. The relationship between return and risk is negative and the low volatility factor has a distinct effect. The explanations by Blitz and Van Vliet (2007) for the existence of the low volatility can be assigned to three reasons. Firstly, leverage restrictions of investors meaning they overpay for risky assets in an effort to drive up expected returns. Secondly, a result of an inefficient investment process where a manager tilts towards high volatility stocks because he is more interested in outperforming the market in up periods than down periods (Binsbergen, Jules, Brandt, and Koijen (2008)). Thirdly, it can be assigned to behavioral bias of private investors, where they buy a few volatile stocks they believe will shoot up in value rather than a broad well-diversified portfolio (Shefrin and Statman (2000)). The reasons mentioned above all result in underpricing of low volatility stocks.

## 2.5 Empirical findings at Oslo Stock Exchange

In this section we will present findings on the Norwegian market. Through several papers, Bernt Arne Ødegaard has examined the existence of factors, and their ability to explain returns on OSE. Ødegaard (2015) finds a significant size effect in 1990 - 1999, and a significant momentum and size effect in 2000 - 2012. He further finds size, value and momentum effects for the whole sample period (1980 - 2012). When examining the value premium, Aadland and Hansen (2012) finds a value effect in the Norwegian stock market for the period between 1983 - 2010, though not significant for P/B.

Kloster-Jensen (2006) examined OSE for a possible momentum effect from 1996 to 2005. He found a positive and significant momentum effect, where the effect was strongest on the short side of the strategy. By adjusting for systematic risk, they found that it explained almost all of the momentum effect. Myklebust (2007) found a significant momentum effect for the whole sample period from 1984 and 2006, also after controlling for size and firm beta. Vas and Absalonsen (2014) examined the Norwegian stock market from 2005 to 2013. They found a significant effect in their zero cost portfolios, with the most successful being a portfolio based on the past 12-month winners versus losers with a 3 month holding period. When studying the OSE from 2004 - 2012 with the methodology of Jegadeesh and Titman (1993), Reiersrud (2013) found that the momentum effect was present after taking transaction costs and systematic risk into account. She further found that the momentum effect had a decreasing effect after the financial crisis of 2009. Examining the low volatility effect on OSE, Dingsør and Sørgaard (2014) finds a persistent low volatility effect during their sample period 1985 to 2013. Low volatility stocks outperforms high volatility stocks, after adjusting for risk. Further, as mentioned both Ang, Hodrick, Xing, and Zhang (2009) and Haugen and Baker (2012) found a significant low volatility effect in Norway.

The literature and the consensus around the existence of the factors on international markets is broad. The picture is not quite as clear when we look at the Norwegian market, which is interesting and further motivates this paper. We especially note that the value factor has to some degree not the same backing as the other factors with both Ødegaard (2009) arguing it is not relevant for the Norwegian market and Aadland and Hansen (2012) finding a non-significant effect for P/B. We are aware of this but want to incorporate it based on the broad acceptance it has in international markets. Furthermore, we see that the findings in the Norwegian market are quite affected by the observed time periods, especially in the studies regarding momentum.

## 2.6 Contribution to Existing Literature

As mentioned, quite a few papers have been looking at single factors in the Norwegian stock market, and their ability to explain cross-sectional returns. On the other hand, few papers look at the Norwegian stock market as a whole, and apply a multi-factor strategy frequently used in the institutional international investment world. This paper will apply the utilization of factor theory in practice, to generate dynamic factor portfolios. We will analyze their individual performance and risk characteristic to get a view of their behavior in contrast to the market. By combining the factors into multi-factor portfolios, we will utilize a full four-factor exposure in the Norwegian stock market and see how it performs relative to the market. This will give us an indication of whether the Norwegian stock market is suitable for a 100 percent exposed dynamic factor investment strategy. The paper is rare in the fact that it is not subject to

survivorship bias (Elton, Gruber, and Blake (1996)), meaning the data sample exists of all stocks traded during the period. This includes both new listings and delisted companies due to merger, acquisition or bankruptcy.

### 3 Data

The data used are primarily financial data and book values on all stocks listed at Oslo Stock Exchange. For comparison, we have a benchmark index, replicating the broad market and a risk-free rate, for risk adjustments.

#### Stock Data

We have two sources of data related to stocks. One for daily financial data for all the stocks on Oslo Stock Exchange and one for the corresponding book values. The financial data is delivered from the database “Børsprosjektet” administrated by the Norwegian School of Economics. We extract daily adjusted price, nominal price and shares issued for every stock listed in the period March 1995 to June 2015. Adjusted prices take into account the investors actual change in value based on adjustments for dividends, splits and mergers. Nominal price is the actual trading price on a given day. Multiplied with shares issued, we get the market equity (ME) to a firm. For book equity (BE), we use Thomson Reuters DataStream.

#### Selection of Stocks

The dataset contains 568 stocks on Oslo Stock Exchange in the period from 1995 to 2015, including new and delisted stocks during the period. This makes the results, as mentioned free of the survivorship bias (Elton, Gruber, and Blake (1996)), and includes information in delisted stocks (Taleb (2007)). We exclude all stocks with less than one year of data from the sample, since all stocks included must have data for the minimum of the formation period and the following holding period.

We filter out the class B stocks of double listed companies and all kinds of equity certificates, mainly savings bank under Sparebank 1 Gruppen ASA. All stocks have gone through a screening for illiquidity and been accordingly taken out. When screening for illiquidity we have looked at weekly returns. Stocks without regularly weekly trading is taken out of the data sample. Due to missing volume data, we have not had the opportunity to check for liquidity besides if it is traded or not. There has been a trade-off in mind when removing stocks and the fact that we need a reasonable set of stocks to construct the factor portfolios.

In conjunction with the creation of the different factors, they require different data and timespans to be included. We end up with a total of 366 stocks that forms the stock pool we use to construct the factor portfolios. A full overview is presented in Table 1.

Each factor has different constraints and selection criteria's. At the time of formation, all traded stocks with known number of stocks issued, are included in the size factor. For the value factor all stocks with known BE and ME as of the end of last year prior to the time of rebalancing is included. No restrictions

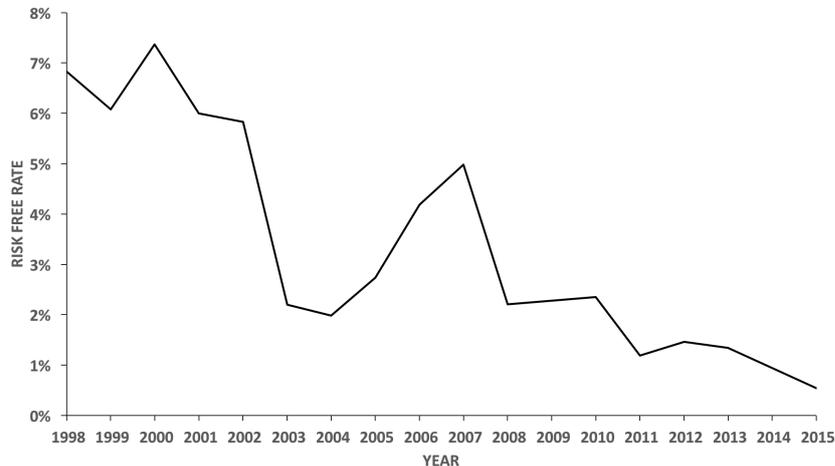
**Table 1:** Pool of stocks in each factor

Number of Stocks	Size	Value	Momentum	Low Volatility
Total (1995-2015)	360	313	363	305
Yearly average (1998-2015)	154	123	153	126

have been set on the minimum or maximum market cap required to get included in the sample - in contrast to Ødegaard (2009) who excludes stocks with less than one million NOK in ME, and companies with a stock price below 10 NOK (penny stocks). He argues that removing penny stocks will reduce extreme price and return movements that will affect the results. We have chosen not to do this because we want the presorted dataset to include all liquid and investable stocks on the exchange. When selecting stocks for the momentum factor, a minimum of 1 year of historical data is required at the time of rebalancing. In constructing the low volatility factor all stocks included in the sample at the time of rebalancing must have at least three years of weekly observations to be included.

## The Market and Risk-free Rate

To replicate the market, we have used Oslo Stock Exchange All-Share Index (OSEAX). This is a value-weighted index comprised of all stocks listed on Oslo Stock Exchange adjusted for dividends (Figure 2). The reason for choosing this index is that we wanted an index that replicates the whole market. This is the best for comparative purposes when examining the factor portfolios. As risk-free rate we use Bernt Arne Ødegaards estimated monthly risk free rate available on his website<sup>6</sup>. The rate is estimated from government securities and NIBOR<sup>7</sup>, and are available as annually, monthly and daily rates. As seen by Figure 1 the risk free rate has fallen considerably over the period examined. As a result of this, risky securities gets more attractive. For full details regarding risk-free rate, see Appendix A.8.

**Figure 1:** Yearly risk-free rate calculated from NIBOR and Government securities

<sup>6</sup><http://finance.bi.no/~bernt/>

<sup>7</sup>Norwegian Interbank Offered Rate (money market interest rate)

## Characteristics of Oslo Stock Exchange

In this sub-chapter, we will explain some of the key features of the Norwegian stock market that is relevant for this paper. OSE has during the period 1998 - 2015 been through a big transformation. We want to look closer at some of the market changes and what relevance this have for our analysis. Ødegaard (2016) provides a good overview of the Norwegian stock market for the period 1980 - 2014. OSE is a small exchange in international standards, but has seen a remarkable growth in the period we examine (Figure 2). The exchange has gone from around 150 active listed shares in 1995 to 220 in June 2015 (Ødegaard (2016)). OSE has always been dominated by a few large firms. We can see by Table 2 that the 10 most traded stocks in June 2015, accounts for over 60 percent of both the total market capitalization and the trading volume of the exchange. This implies that the exchange is considerably affected by firm specific risks, in the biggest and most liquid stocks on the exchange.

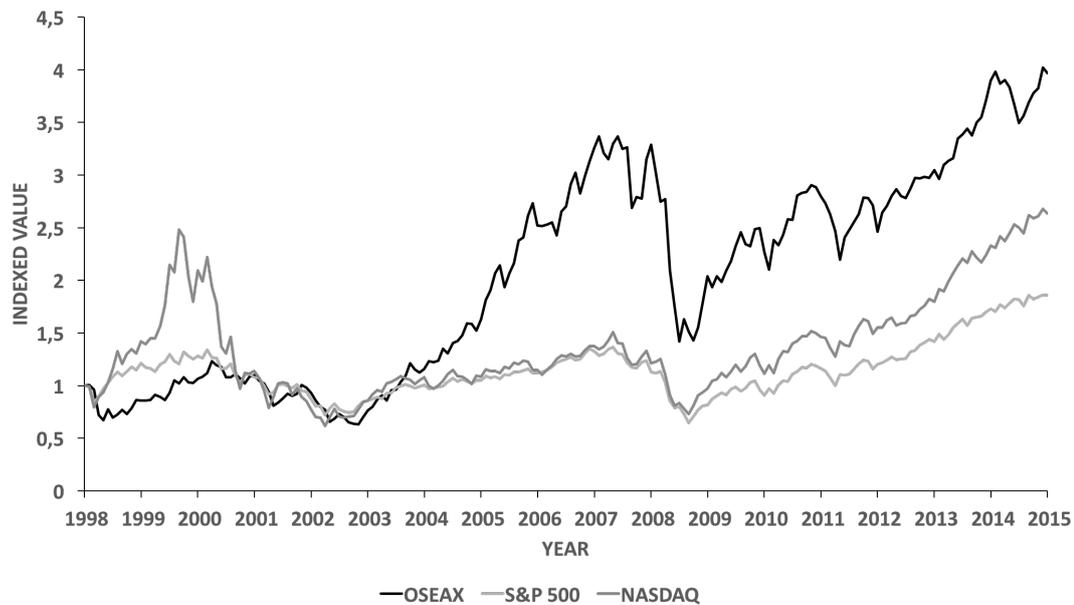
**Table 2:** Turnover and market equity of the ten most traded stocks June 2015<sup>8</sup>

Most traded June 2015	Turnover(Mill NOK)	%	ME (Mill NOK)	%
Statoil	11 664	12	446 729	23
Telenor	7 465	8	257 950	13
Norsk Hydro	6 561	7	68 380	4
DNB	6 354	7	213 047	11
Yara International	6 267	7	112 811	6
Seadrill	5 381	6	16 877	1
Europris	4 669	5	6 528	0
Marine Harvest	4 196	4	40 463	2
Orkla	2 661	3	62 868	3
Subsea 7	2 641	3	25 494	1
Sum top 10	57 860	61	1 251 149	65
Total	95 017	100	1 923 917	100

The liquidity on OSE has improved significantly over the period, and the average number of trading days per stock has risen from 151 (250) in 2003 to 216 (251) in 2015<sup>8</sup>. Further the average bid-ask spread has been significantly reduced over the period, but varies significantly between sectors and stocks as reported by Ødegaard (2013). However, the methodology examined are vulnerable to investment in stocks that are less liquid then the average, and thus increase the transaction costs. Especially the size factor and small caps will be affected. As stated by Pedersen (2015), today's electronic markets are quite liquid and the bid-ask spreads are low, but the amount you can buy at these levels are limited, especially in a small market like OSE.

As we see in Figure 2 the period examined has been quite volatile, significantly more than the American markets, represented by NASDAQ and S&P 500. When examining further, we can divide the period examined into different market conditions. The internet bubble and subsequent crash in 2001 is evident in the NASDAQ index, which is heavily weighted in tech firms. Followed by a sharp rise up to the financial crisis of 2008. After the financial crisis the OSE has recovered and are now, well above the pre-crisis values. We see that the recent tumble caused by the fall in the oil price, is also partly evident in the

<sup>8</sup><http://www.oslobors.no/Oslo-Boers/Statistikk>. Data extracted: 24.04.16 Actual trading days in parenthesis.



**Figure 2:** Development of OSEAX compared to S&P 500 and NASDAQ. Indexed from 1998 to 2015.

examined period. Based on the characteristics, we have chosen to divide the analysis of the factor into four sub-periods, to test the robustness during different market conditions. 1998 - 2003 including the oil price drop of 1998 and the internet bubble of 2001, the growth period of 2003 - 2008, followed by the financial crisis and recovery in 2008 - 2011. Finally, the period of 2011 - 2015 including the effects of the European debt crisis and the oil price tumble. The periods chosen gives a good insight into how factor investing will work under different market conditions, and shows the performance of the factor portfolios over a long period (Ang (2009)).

Shorting stocks is a big part of a factor investing strategy, with this in mind we have looked at the possibility for shorting, and the different providers in Norway. In the Norwegian stock market, a market participant can only make a covered short. This means that the participant must borrow the stock before selling it. This implies additional costs associated with the borrowing, at the same time there is a risk that a lender will call back the stock causing unintended transaction costs and instability. Nordnet reports an overnight cost of a holding a short position starting with a rate plus a fixed cost<sup>9</sup>. An institutional investor will achieve much lower costs, and this is just an example of potential costs. The other provider we have looked into is Pareto Securities. They do not present the costs of a short position, as this varies significantly between the stocks due to liquidity. For a full overview of the stocks available for short selling by Pareto securities and Nordnet, see Appendix A.9. We note that the stocks available for short selling varies considerably between the providers, and are reasonably limited compared to other international markets. In this analysis we assume that cost associated with shorting are covered by lending out the inventory from the long portfolio to other market participants, affirmed by portfolio managers at Nordea. Further, we assume that we can finance buying the stock from the short selling, these are simple assumptions but a necessity. An alternative method of achieving the long/short factor exposure is to short an index tilted against the factors. This has showed to give sufficient results in the US market.

<sup>9</sup><https://www.nordnet.no/tjenester/prisliste.html>. Extracted: 10.04.16.

The grade of availability of such factor indexes in Norway and the fact that this is out of the scope of this paper, we do not examine this further.

The challenges mentioned of implementing a factor investing strategy in the Norwegian market comes from the fact that the market is small in international standards - both in terms of market size, depth, number of participants and available stocks for trading. We are aware of this, but as this is a theoretical approach and the fact that this is out of the scope of this paper we have chosen not to look further into the market restrictions and transaction costs associated with the strategy.

## 4 Method

In this section, we will go through the different methods used to establish portfolios exposed to the described factors. Thereafter, an explanation of the portfolio optimization techniques - followed by an explanation of risk and performance measures beyond standard.

### Creation of Factor Portfolios

Our methodology for constructing the factor portfolios are based on Fama and French (1992) for the size and value factor, Jegadeesh and Titman (1993) for momentum and Blitz and Van Vliet (2007) for low volatility. In the formation of the portfolios, we use their ranking procedure to pick which stocks to include in the portfolios, as described below. Any deviation from this, will be mentioned when occurred. Regarding the portfolio size, most studies use a decile break point. In our approach, we use a fixed number of stocks we choose to incorporate into the different portfolios. The reason for this is to give the factor portfolios a more stable size, together with the varying size of OSE in terms of numbers of stock. At the same time, it makes the factors easier to compare. If we look back to Table 1 presented under data, choosing 15 stocks in each portfolio, is not far from the decile breakpoints. The portfolio formation period for all factors begins in June 1998 and continues to May 2015<sup>10</sup>.

At the time of rebalancing, we begin the process by calculating a ranking factor for each stock in the pre-selected pool of stocks for the given period. The historical data needed for each factor, vary from a year for momentum, to three years for low volatility, while size and value uses a one-time measuring point to determine its ranking value for the forthcoming period. The top and bottom 15 ranking stocks are sorted into one top and one bottom portfolio. The portfolios are made up of equally weighted investments in each stock, to avoid large exposure and impact from large-cap firms in a market capitalized weighing regime. A more detailed description for each factors portfolios follows below.

The size factor is calculated every first of June, based on outstanding shares and nominal price to determine its market equity (ME) (Fama and French (1992)). Here we seek to capture an excess return based on the anomaly that small-cap firms (long) on average has a higher return then large-cap firms (short).

Value and growth stocks are found by creating a BE/ME ratio in June each year using a stock year-

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<sup>10</sup>Size and value is rebalanced in June 2014, momentum in December 2014, while low volatility has its last rebalancing period in May 2015.

end book equity divided by the nominal price times outstanding shares ( $t - 6$  months) (Fama and French (1992)).  $BE/ME$  exceeding 10 and stocks with negative  $BE$  is excluded from the selection, in line with the methodology used by Fama and French (1993). The anomaly seeks to buy underpriced stocks (*high  $BE/ME$* ) and sell overpriced (*low  $BE/ME$* ) in hope for correction in the price.

Low volatility is measured as 3-year weekly standard deviation and are sorted from low to high volatility. The factor is the difference in the low and high volatility portfolio. The long sample of 3 years, has shown in previous research (Blitz and Van Vliet (2007)) to give a more stable holding of stocks that minimize the rebalancing and transaction costs.

In contrast to the other factors, momentum has a two-step ranking procedure before final decision. First, we rank the stocks based on a 12 months minus the last month ( $12 - 1$ ) total return. This is in line with Jegadeesh and Titman (1993), in addition, we have a 6 months proxy as a dummy to capture recent reversion effects. If a stock experience an opposite drawdown, it will be excluded from the ranking for that period (Gupta, Balint, Jain, and Melas (2015)).

When portfolios are established, we go long (short) the bottom (top) portfolio to get an exposure in the factor. All factors besides the low volatility is a zero-sum investment portfolio, where we have a long-only exposure in low volatility stocks. The reason for not shorting the high volatility portfolio is due to the fact that the performance is highly volatile together with low levels of return over the long run (Blitz and Van Vliet (2007)). We are at any time exposed in a maximum of 105 stocks (30 in size, 30 in value, 30 in momentum and 15 in low volatility), but the actual number is considerably lower due to overlapping exposures in the different factors. The holding periods vary across factors and goes from monthly holding period for low volatility, 6 months for momentum to one year for size and value. All factors have non-overlapping holding periods, as this reduces transaction costs (Gupta, Balint, Jain, and Melas (2015)).

### **BOX 1: Correlation in Exposure between Factors**

The total number of stocks used in the portfolios count up to 297 of the total sample pool of 366 stocks. By investing in one factor, we tend to get an exposure in other factors, both on the long side and the short side. The zero-sum investment between factors, where the same stock is long and short in two different factors, has a 12 percent average monthly cover, indicating that almost 4 stocks in each portfolio does not need to be traded to fulfil its function. The double long and double short exposure is also present with an average of 7 percent and 11 percent respectively. For full overview of yearly exposure between factors, see Appendix A.3.

The return each portfolio generates in their respective holding periods is the sum of simple returns for each stock times their weighing. As stated by Meucci (2010), simple returns are best to use considering analysis and construction of portfolios. Furthermore, when combined portfolios of the factors are made we keep using simple returns as they best state the actual change. As stated, transaction costs are ignored.

**Table 3:** Summary of factors

Factor	Input	Data period	Holding/Rebalancing
Size	Market Equity	t	1 year / Annually
Value	Book Equity/Market Equity	t - 6m	1 year / Annually
Momentum	Absolute total return	Last 12m - 1m	1/2 year / Semi-Annually
Low Volatility	Weekly volatility	3 years	1 month / Monthly

After the holding period, all ranking calculations are moved forward in accordance with the length of the holding period and thereafter recalculated. This is the new basis for the ranking process, and all portfolios are rebalanced in accordance with the results from the ranking process mentioned above. This means we are running a rolling sample period of the data for determination of the ranking factors, while testing the results in the forthcoming period out-of-sample. A summary of the input for rankings, data period and holding periods are outlined in Table 3.

## BOX 2: One Year in the Value Factor

As an example, we will give you an insight into the practical investment procedure for the long side of the value factor. The long side of the value factor portfolio invests in underpriced stocks. In theory, we would go long 15 stocks to establish an exposure in the factor against the bottom portfolio of 15 stocks. A list of the stocks we are invested in follows in Table 4. Due to stocks included in other factor portfolios, the value factor may be indirectly exposed to both long and short positions in other factors. If a stock is short in another factor we get a zero-sum investment, meaning that we indirectly get an exposure in both factors but either buys or sells the stock, saving transactions cost. If a stock is long in another factor, we get a double exposure by just holding one stock. You can also get a situation where you are both short and long in another factor, making it a three-factor exposure while just holding one stock. For instance, ATEA, which is long value, is also long the momentum factor and short the size factor at the same time.

**Table 4:** Stock positions value factor long side 1998 - 1999

Time period	1	2	3	4	5	6	7	8	9	19	11	12	13	14	15
1998-1999	NSG	BOR <sup>I</sup>	SCI	KVI <sup>0</sup>	PGS <sup>II</sup>	KEN <sup>0</sup>	TTS <sup>0</sup>	TCA <sup>0</sup>	ATEA <sup>II</sup>	IGNIS <sup>I</sup>	SEN <sup>0</sup>	HEX <sup>II</sup>	ASC <sup>0</sup>	FRO	HJE

*0 indicates a short position in another factor, causes a zero-sum investment*

*I indicates a long position in another factor*

*II indicates both a long and short position in another factor*

After a one-year holding period, we do the same procedure and rank the stocks based on new values and get the portfolio of stocks as seen in Table 5. Some of the stocks are persistent from the previous year, with a rebalancing rate of 60 percent.

**Table 5:** Stock positions value factor long side 1999 - 2000

Time period	1	2	3	4	5	6	7	8	9	19	11	12	13	14	15
1999-2000	NSG <sup>∞</sup>	BOR <sup>∞</sup>	SCI <sup>∞</sup>	KVI <sup>∞</sup>	PGS <sup>∞</sup>	KEN <sup>∞</sup>	DNO	BEL	DNB	IMSK	HSU	NIS	DOF	FOE	ODF

*∞ indicates a none-rebalancing position*

A full overview of long and short positions in all the factors for the period 1998 to 1999 are presented in Appendix A.5. In addition, a list of all stocks used in the factor portfolios from 1998 to 2015 with corresponding exposure time is included in Appendix A.7.

Average annual turnover is presented in Table 6. As expected the momentum factor has a large turnover rate. The low volatility with monthly rebalancing has a larger turnover rate than the value and size factor, but considering their yearly rebalancing the low volatility has a consistent turnover rate. This states that the stocks included in the portfolio are mostly large, liquid and stable companies in consensus with Haugen and Baker (1996, 2008), making the turnover relatively low hence the opportunity to rebalance the portfolio every month. When combining the factors into multi-factor portfolios, we get an even lower turnover rate due to overlapping investments in several stocks as seen in Box 1.

**Table 6:** Average annual turnover rate for factors 1998 - 2015

	Size	Value	Momentum	Low Volatility
Turnover rate (%)	30.4	46.9	129.2	73.3

*Annual turnover for size and value. Momentum and low volatility are annualized turnover.*

### Portfolio Optimization and Measures

In our optimization of the factors, we will create portfolios based on well-known techniques and analyze the results. Our three portfolios will be based on two full exposure portfolios, respectively naïve diversification and risk parity optimization. In addition, we create a factor tilt portfolio with a long-only naïve approach. There are several reasons for deselection of other portfolio optimizations techniques. Firstly, there is a large parameter risk in the prediction of returns, leading to undesired weightings in a mean-variance portfolio (Best and Grauer (1991)). Secondly, the mean-variance approach will give different concentrations in the factors along the efficient frontier. For example, low volatility will dominate the minimum variance portfolio and the factor with the highest Sharpe ratio will dominate the maximum Sharpe portfolio, implying a low diversification effect. Risk parity on the other hand uses covariance, which is a more stable measurement over time, making it easier to do accurate predictions. And as stated by Chopra and Ziemba (1993), enhanced measures have little influence on the allocation between factors relative to enhancements in returns. In recent years, many have argued that the naïve approach shows superior performance over other more complex optimizations. Pflug, Pichler, and Wozabal (2012) shows that using naïve optimization is best when faced with model uncertainty. Since this is an out-of-sample test with a high degree of uncertainty and our focus is on diversification, we choose to implement both the naïve and risk parity approach. A value-weighted approach would give large exposure in big and expensive companies, resulting in a low factor exposure for a fund which aims for high diversification with a broad factor exposure.

The naïve portfolio is a fixed weight distribution between factors with no presence of optimization rules based on model inputs. In our approach, 1/4 is equally invested in each factor. The risk parity optimization is an allocation strategy where each asset class have an equally marginal contribution to the total risk of the portfolio (Equation 1). The marginal contribution is based on the assets standard deviation and the covariance between the assets. The rebalancing of the risk parity portfolio is performed yearly in June together with the rest of the main rebalancing events, with the former year's performance as input. The factor tilt portfolio uses the same approach as the naïve portfolio, with all factors being long-only.

$$MC_i = (\text{Weight of Asset Class } i) * \frac{\Delta \text{Total Risk of Portfolio}}{\Delta \text{Weight of Asset Class } i} \quad (1)$$

Beyond standard measures of performance, risk and significance, we use a Z-test to determine if a Sharpe ratio is significant, relative to the Sharpe ratio of the market. We also do a calculation to directly compare the portfolios return, by adjusting them to the market risk (M2). Other performance measures elaborated further are the MAR ratio and the Sortino ratio.

To test whether the Sharpe ratios differ, we use a Z-test based on Jobson and Korkie (1981) with the Memmel (2003) correction. The difference between the two Sharpe ratios is divided by the asymptotic variance (V) of the difference in the Sharpe ratios (Equation 2 and 3). The Z-test is a two-sided test and has a critical value of 1.96 on 5 percent significance.

$$Z = \frac{SR_1 - SR_2}{V} \quad (2)$$

$$V = \sqrt{\frac{1}{T} [2(1 - \rho_{1,2}) + \frac{1}{2}(SR_1^2 + SR_2^2 - 2SR_1SR_2(1 + \rho_{1,2}))]} \quad (3)$$

The M2 is a risk-adjusted performance measure, which allows for direct comparison between multiple investments derived on the same risk level as the market (Modigliani and Modigliani (1997)). It uses the risk-free rate as a basis for the return and then scales the portfolio's excess return over or under the risk-free rate relative to its risk levels compared to the market (Equation 4). This measure has an advantage in the fact that it lets us compare the risk-adjusted returns between investments directly.

$$M2 = R_f + \frac{\sigma_M}{\sigma_A} * (R_A - R_f) \quad (4)$$

Managed Account Reports (MAR) ratio compares the performance of hedge funds and trading strategies. The higher the ratio the better the risk-adjusted return. It is calculated by dividing the compounded annual growth rate (CAGR) (Equation 5) over the sample period by the maximum drawdown (Equation 6). Maximum drawdown refers to the maximum loss obtained from a peak during the sample. It can be used as a stand-alone measure of downside risk.

$$CAGR = \frac{Price_t}{Price_{t-1}}^{\frac{1}{years} - 1} \quad (5)$$

$$MAR = \frac{CAGR}{MAXDD} \quad (6)$$

As a modification of the Sharpe ratio, the Sortino ratio takes only into account the volatility of the negative asset returns (Equation 7). A large Sortino ratio indicates a small chance of a big loss. This helps us compare the downside risks of the different factors and the market.

$$SortinoRatio = \frac{R_p - R_f}{\sigma_{Downside}} \quad (7)$$

## 5 Factor Results

In this chapter, we present the results of the individual factor portfolios. Firstly, we look at the returns for the top and bottom portfolios of each factor exposure to see if the return patterns are consistent with the anomalies. Thereafter, we present the return and risk characteristics, and performance measures against the market portfolio. As risk measures we use the realized standard deviation, value at risk (VaR), conditional value at risk (CVaR) and maximum drawdown. VaR is a commonly used measure that captures the worst possible loss over a period at a given probability. CVaR is the average expected loss, if exceeding the VaR limit. Maximum drawdown shows the largest loss from the highest peak over the period. As performance measures we use the Sharpe ratio, Sortino ratio, MAR ratio and the M2.

### Top versus Bottom Portfolio

In this section we look at the difference in return between the top and bottom portfolio. Our factor portfolios in term of excess return between the top and bottom portfolio delivered an overall significant outperformance for the size, value and momentum effect, shown in Table 7. The fact that the low volatility factor portfolio underperforms is mainly because of the top portfolio position of high volatile stocks, drawing the excess return down in upward going periods. As mentioned, and in line with Blitz and Van Vliet (2007) we will only denote the low volatility as the long side. The two positive periods of 1998 - 2003 and 2008 - 2011, indicates that the low volatility stocks outperform high volatile stocks in turmoil markets. The significant size and value effect for the sample period is in line with Ødegaard (2015) findings on OSE. We see that small cap firms outperform large cap firms in all but one of the sub-periods. The two periods with none significance is hence because of the large firms stable behavior in troubled times. The top (*value stocks*) portfolio of the value factor outperforms the bottom (*growth stocks*) during the first two sub-periods, yet the effect turns and the bottom portfolio outperforms during the last two sub-periods, turning into an overall positive effect for the whole sample period. We also see a

**Table 7:** Performance between top and bottom portfolio 1998 - 2015

	Size			Value		
	Top	Bottom	Diff	Top	Bottom	Diff
<i>1998-2003</i>	-8,36	-11,57	3,21	-12,00	-78,30	66,30*
<i>2003-2008</i>	55,57	32,77	22,81*	48,83	15,78	33,04*
<i>2008-2011</i>	-12,61	-0,30	-12,31	-17,00	-0,56	-16,45
<i>2011-2015</i>	21,58	2,30	19,28*	-0,97	5,33	-6,31
<i>1998-2015</i>	17,47	7,06	10,41*	7,87	-15,18	23,05*
	Momentum			Low Volatility		
	Top	Bottom	Diff	Top	Bottom	Diff
<i>1998-2003</i>	17,51	-4,53	22,03*	-3,20	-28,03	24,84*
<i>2003-2008</i>	46,98	52,60	-5,63	24,28	55,70	-31,41
<i>2008-2011</i>	-19,26	-17,15	-2,11	-4,45	-14,12	9,67
<i>2011-2015</i>	8,87	-25,48	34,35*	2,29	16,74	-14,45
<i>1998-2015</i>	17,44	4,60	12,84*	6,09	10,68	-4,59

*Annualized excess return in bottom versus top portfolio. \*indicates a significant present factor effect in our portfolios. Note that returns are not accumulated.*

significant underperformance under the financial crisis. The outperformance in the first two periods can be explained by the large fall in growth stocks under the internet bubble, where the portfolio is short, and the continued confidence in value stocks until the financial crisis. The underperformance of value stocks during the financial crisis, reflects that underpriced stocks may be in distress, and suffer higher loss during turmoil markets. The top (*past winners*) momentum portfolio significantly outperforms the bottom (*past losers*) over the whole sample period, first and last sub-period, also in line with Ødegaard (2015). The significant first period is in line with the finding of Kloster-Jensen (2006) and Myklebust (2007).). Further, we see that momentum benefits from volatile periods in line with Moskowitz, Ooi, and Pedersen (2012). It is clear that the factors are changing over time, and is sensitive to sample periods. We clearly see the cyclicalities in factors as mentioned in earlier literature. Bender, Briand, Melas, and Subramanian (2013), argued that the reason for the style factors not being arbitrated away might be their cyclicalities.

### Factor Portfolio Performance

An overview of the main results for our factor portfolios are outlined in Table 8. The table contains return characteristics, risk and performance measures for the whole period and sub-periods compared to the market. None of the pure factor portfolios beat the market in terms of annualized returns over the sample period, in fact value and low volatility significantly underperforms the market. In the sub-periods, we see various movements in both directions compared to the market, some of them significant. In the period 1998 - 2003, the market went through a period of both extreme gains and falls. All factors beat the market, and both the momentum (*t-test: 4.90*) and size factor (*t-test: 2.12*) by statistical significance. In the bullish period of the mid 2000, no factors could cope with the return of the market portfolio. However, size and value performs relatively good in contrast to the negative return for momentum. During the financial crisis and recovery period (2008 - 2011) value and size performs poorly, with low volatility performing similar to the market. In the volatile post-financial crisis's period, we see a variety of different outcomes. Low volatility and value once again significantly underperforms, while the size and momentum both outperformed the market, momentum significantly (*t-test: 2.77*). Low volatility was clearly outperformed by the market in both the upwards trending periods of 2003 - 2008 (*t-test: 2.92*) and 2011 - 2015 (*t-test: 2.86*). The low volatility factor operate in opposite direction of the market throughout most of the time, with two significant sub-periods of poorer performance and one outperformance in the first sub-period, though not significant.

We see by Table 9 that all factors except low volatility are negatively correlated over the sample period, which is in line with the findings of Asness, Moskowitz, and Pedersen (2013), when studying markets outside the US. The reason for the positive correlation between the market and the low volatility factor, is mainly due to the holding of long-only stocks. We see that both size and momentum have a correlation approximately equal to zero. This implies a low to zero correlation between the factors and the market, and in that contexts a portfolio combining the factors might add a diversifying effect. Koedijk, Slager, and Stork (2014) argues that having two imperfectly correlated assets in a portfolio, yields the possibility to earn the same return at a lower risk, or a higher return at a lower risk. When examining 24 months rolling correlation (Appendix A.1), we clearly see a considerable time varying correlation. The correlation between the factors and the market is more stable than the correlation between the factors. This implies a more stable diversification effect between the factors and the market over time. The correlation falls considerable in the tumbles of the financial crisis, before rising again. We see by the correlation that it

**Table 8:** Overview of factors performance for 1998 - 2015 including sub-periods

	Mkt	Size	Value	Mom	LowVol	Mkt	Size	Value	Mom	LowVol
<i>Return (%)</i>						<i>Return / Risk</i>				
1998 - 2003	-4.65	3.99*	-0.68	15.70*	-3.20	-0.19	0.23	-0.04	0.85	-0.24
2003 - 2008	32.91	16.64*	20.37*	-0.48*	24.28*	1.79	0.94	1.42	-0.03	1.78
2008 - 2011	0.95	-4.55	-3.14	1.44	-4.45	0.03	-0.32	-0.34	0.12	-0.26
2011 - 2015	8.75	12.19	-2.54*	16.50*	2.29*	0.66	0.70	-0.25	1.07	0.23
1998 - 2015	10.87	8.33	4.59*	8.63	6.09*	0.47	0.49	0.32	0.53	0.44
<i>Standard Deviation (%)</i>						<i>Maximum Drawdown (%)</i>				
1998 - 2003	24.71	17.36*	18.90*	18.38*	13.36*	46.54	25.76	50.51	41.31	56.25
2003 - 2008	18.36	17.69	14.36*	16.26	13.61*	11.10	19.32	10.06	29.76	6.66
2008 - 2011	33.79	14.07*	9.31*	12.48*	17.23*	56.87	19.49	13.27	22.71	36.79
2011 - 2015	13.18	17.36*	10.38*	15.49	9.79*	24.39	17.82	21.65	12.62	16.36
1998 - 2015	23.01	17.07*	14.45*	16.24*	13.85*	57.94	36.14	47.78	29.76	44.37
<i>Skewness</i>						<i>MAR Ratio</i>				
1998 - 2003	-0.60	1.83	-0.93	-0.44	-0.94	-0.17	0.10	-0.05	0.37	-0.07
2003 - 2008	-0.32	1.76	0.80	-0.08	-0.40	3.42	0.72	2.06	-0.08	3.98
2008 - 2011	-0.55	0.40	0.93	-0.71	-0.56	0.02	-0.36	-0.24	0.00	-0.08
2011 - 2015	-0.65	1.34	-0.31	0.66	-0.57	0.42	0.73	-0.29	1.87	0.10
1998 - 2015	-0.75	1.51	-0.27	-0.05	-0.59	0.15	0.20	0.08	0.25	0.12
<i>Excess Kurtosis</i>						<i>Sortino Ratio</i>				
1998 - 2003	1.70	5.90	7.76	1.21	2.06	-0.61	-0.27	-0.40	0.71	-0.85
2003 - 2008	-0.73	4.31	1.30	-0.37	-0.09	3.33	2.15	2.71	-0.40	2.73
2008 - 2011	-0.01	-0.54	1.77	1.62	0.27	-0.06	-1.02	-1.09	-0.09	-0.53
2011 - 2015	1.21	2.60	3.01	1.53	0.51	0.78	1.79	-0.47	2.30	0.15
1998 - 2015	1.81	3.80	7.49	0.91	1.03	0.42	0.69	0.09	0.49	0.25
<i>VaR (95 %)</i>						<i>Sharpe Ratio</i>				
1998 - 2003	14.18	6.52	4.85	7.44	8.46	-0.43	-0.12	-0.36	0.52*	-0.69*
2003 - 2008	6.46	5.68	4.67	9.13	5.84	1.62	0.76*	1.19	-0.23*	1.55
2008 - 2011	18.94	6.92	4.45	8.06	10.57	-0.04	-0.49	-0.58*	-0.07	-0.39*
2011 - 2015	6.15	5.40	5.35	5.35	5.68	0.57	0.63	-0.36*	0.99	0.11*
1998 - 2015	9.68	5.72	4.83	7.14	6.95	0.32	0.28	0.07*	0.31	0.18*
<i>CVaR (95 %)</i>						<i>M2 (%)</i>				
1998 - 2003	18.01	7.25	14.95	11.04	10.75	-10.73	3.09	-2.77	19.01	-11.08
2003 - 2008	7.91	6.10	5.70	9.59	6.26	29.69	17.15	25.15	-0.96	31.63
2008 - 2011	21.72	7.04	4.78	9.48	11.85	-1.33	-14.12	-17.39	0.00	-10.93
2011 - 2015	8.77	6.15	7.55	6.05	6.65	7.52	9.55	-3.56	14.22	2.66
1998 - 2015	16.22	6.72	8.99	9.66	9.44	7.29	9.99	5.19	10.73	7.75

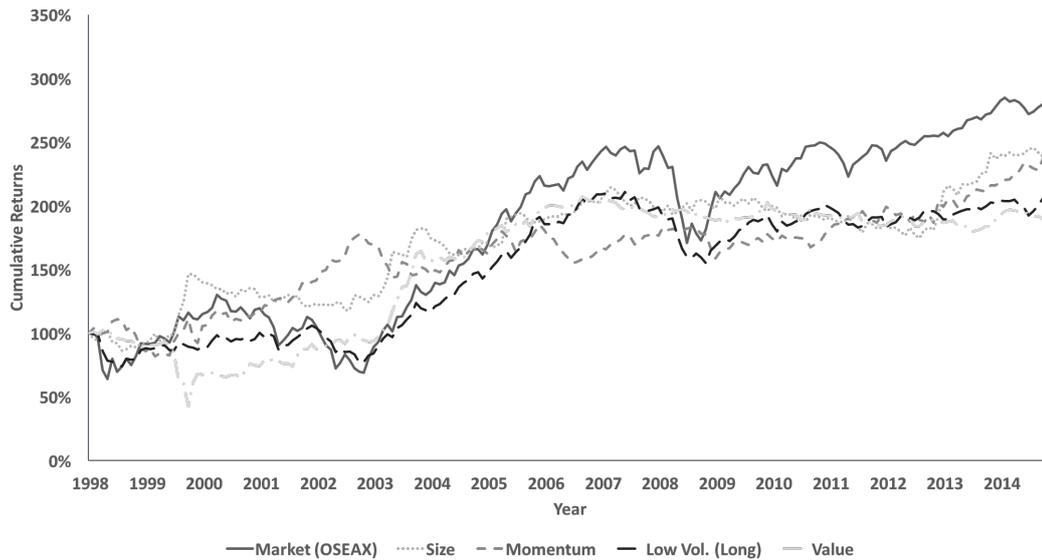
Results are based on monthly data. The sample period runs from June 1998 to June 2015. Return and standard deviation are annualized. \* indicates 5 % statistical significance. Skewness, Excess Kurtosis, VaR and CVaR are on monthly basis. The risk-free rate used for Sharpe ratio, Sortino ratio and M2 are for the periods 6.09% (1998-2003), 3.22% (2003-2008), 2.28% (2008-2011), 1.23% (20011-2015) and 3.58% for the total period (1998-2015). For comparative purposes is the risk-free rate subtracted the markets return under M2.

**Table 9:** Correlation in return between factors 1998 - 2015

	Market	Size	Value	Momentum
Size	-0.05			
Value	-0.22*	-0.03		
Momentum	-0.02	-0.18*	-0.12	
Low volatility	0.77*	0.13	-0.12	-0.01

follows the cycles of the market, with lower correlation in times of distress, and higher in normal market conditions. This is interesting and indicates a significantly reduced downside risk in times of distress in contrast to the market. The low degree of correlations in the factors also indicates a low degree of overlapping factor exposures.

When looking back at Table 8 we see that the market has considerable risk compared to the factors. It has the highest standard deviation, VaR, CVaR and maximum drawdown of the sample. In contrast to the factors, we see that the market consistently has a negative skewness and the highest over the sample period, meaning that the market has a persistent overweight of negative returns compared to the factors. All factors and the market have a positive kurtosis over the sample period, which implies fat tailed distributions. Extreme returns happen more often than in a normal distribution, and the returns of a fat tailed distribution tends to overestimate the mean. As assumed the Jarque-Bera test (Appendix A.6) were rejected for normality on the market and the factors for the whole sample period, thus some sub-periods were normally distributed. This means that the risk measures must be analyzed with caution and emphasize the use of alternative risk measures, such as VaR, CVaR and maximum drawdown. All factors have significantly lower variance in returns in contrast to the market portfolio, which also has the largest maximum drawdown. This implies that the market has significant downside risk compared to the factors. We note especially the large and consistent differences in maximum drawdown during the financial crisis. This is partly explained by the offsetting short positions that helps reduce drawdown, which is further undermined by looking at the long-only low volatility factor. This is in line with the dip in correlation between the factors and the market during the period. On the other hand, as we see in the returns, the short positions cap the returns of the factors during bull periods. Unsurprisingly, low volatility has the lowest standard deviation over the sample period, with a consistently lower drawdown and value at risk compared to the market. The size portfolio delivers the most stable risk characteristics over the period. We see that both the size and momentum factor have a lower maximum drawdown than the market and the other factors. Size and momentum reports a higher MAR ratio over the sample period, but the ratio varies significantly when looking at the different sub-periods. Size has the best ratio during bull periods, and momentum as mentioned benefits from volatile periods. Further we see that the momentum and size factor have a better Sortino rate than the market, momentum only marginally though. In terms of Sharpe ratio, the only significant outperformance is by the momentum factor between 1998 - 2003. Also note that the negative Sharpe ratio for the period 1998 - 2003, is mainly due to the high risk-free rate (6.09 %) relative to the performance of the portfolios. Size, momentum and low volatility outperforms the market over the sample in terms of M2. The negative M2 for the factors and the market during the financial crisis is due to the loss associated with not holding the risk-free rate in relation to the risk exposed to by holding the factor with the same risk as the market. In other words, the loss you would have endured if leveraging your portfolio to the market risk.



**Figure 3:** Cumulative returns of factors against index, 1998 - 2015

### Summary

The factors have clearly been outperformed by the market during the period examined in terms of pure returns. As mentioned, the factors have done well in some of the sub-periods with higher returns than the market. This combined with the fact that the factors have considerably lower risk than the market both in terms of variance and in terms of downside risk gives an interesting result. A big part of the reduction in risk can be assigned to the short side of the factors, as it reduces the shortfall, but also caps the upside in bull markets. Leading to a more stable accumulation of returns over time, clearly shown in the period between 2003 - 2008. This is evident in Figure 4 during the financial crisis where the market plummets, but the factors remain fairly stable. Adjusting for market risk, we get better performance for size, momentum and slightly in low volatility. Together with lower risk in all factors contra the market and a diversification effect in the correlation between factors, a portfolio combining the strategies could yield an improved risk-adjusted performance. Investing in a single factor entails an exposure in timing risk as a result of the cyclicity of single factors. By investing in a set of factors much of this effect gets diversified away due to the low correlation. This is in line with Bambaci, Bender, Briand, Gupta, Hammond, and Subramanian (2013) arguing that investing in a set of factors is a good way to reduce timing risk.

When looking at the factors overall performance, much of the cyclicity can be explained by the varying market conditions. Size performs well in all periods except the financial crisis, clearly the small sized companies are more sensitive to market turmoil's. The value factor clearly struggles after the financial crisis, which can be explained by the crash of the growth stocks in the IT-bubble and that undervalued stocks increased more than overvalued growth stocks during the following bull period. The momentum factor performs well in the volatile first, third and last sub-period. Low volatility clearly goes against the market as earlier stated, with an underperformance in bull periods and a more stable behavior in bear periods.

## Factor Tilts Portfolios

In this section, we will go through the return characteristics and performance for the factor tilt portfolios. The long-only factor tilts are constructed by imposing a short constraint, meaning that the strategy only are exposed in the long side of the factor. In Table 10 we see that with a long-only factor exposure the correlations all gets positive and significant. As expected, low volatility has the highest correlation to the market, and size the lowest. Furthermore, we see that the correlations are much higher than previously shown in Table 9. This shows the added diversification effect of the short side in the long/short factor portfolios. A portfolio consisting of long-only factor tilts will as indicated give some diversification effects, though not as significant as earlier shown. This makes the strategy more vulnerable to cyclical performance of individual factors, clearly evident by the enhanced risk levels. The 24 month rolling correlation between the factors (Appendix A.1), gets a spike in the post financial crisis, contradictory to the effect seen in the long/short factor portfolio. Nevertheless, more in line with the effects of traditional diversification failing during the financial crisis.

**Table 10:** Correlation in returns for long-only factor tilts, 1998 - 2015

	Market	Size	Value	Momentum
Size	0.44*			
Value	0.73*	0.61*		
Momentum	0.77*	0.59*	0.73*	
Low volatility	0.77*	0.45*	0.77*	0.69*

*\*indicates significance at 5 percent level*

**Table 11:** Performance for factor tilt portfolios, 1998 - 2015

	Market	Size	Value	Mom	LowVol
Average return (%)	10,87	17,47*	7,87	17,44*	6,09*
Standard Deviation (%)	23,01	34,76*	28,80*	30,25*	13,85*
Return/risk	0,47	0,50	0,27	0,58	0,44
Skewness	-0,75	0,86	-0,35	0,32	-0,59
Exess Kurtosis	1,81	1,66	0,49	2,58	1,03
JB-Test	46,85	6,12	48,36	60,22	20,60
VaR (95%)	9,68	13,57	14,36	14,63	6,95
CVaR (95%)	16,22	16,75	18,20	18,23	9,44
Maximum Drawdown (%)	57,94	73,55	76,30	70,97	44,37
MAR Ratio	0,15	0,17	0,05	0,19	0,12
Sortino Ratio	0,42	0,83	0,21	0,72	0,25
Sharpe Ratio	0,32	0,40	0,15	0,46	0,18
M2 (%)	7,29	12,77	7,01	14,12	7,75

*Results are based on monthly data. The sample period runs from June 1998 to June 2015. Return and standard deviation are annualized. \* indicates 5 % statistical significance. Skewness, Excess Kurtosis, VaR and CVaR are on monthly basis. The risk-free rate used for Sharpe ratio, Sortino ratio and M2 are for the periods 6.09% (1998-2003), 3.22% (2003-2008), 2.28% (2008-2011), 1.23% (20011-2015) and 3.58% for the total period (1998-2015). For comparative purposes is the risk-free rate subtracted the markets return under M2.*

As presented in Table 11, the long-only factor tilt portfolios yield significant returns over the markets for both size and momentum. We see that the overall downside risk has risen considerably compared to the earlier long/short factor portfolios. This is especially evident in the maximum drawdowns. Furthermore, all factor tilts have significantly higher variance in returns, except for low volatility. The Sortino ratio is considerably larger for size and momentum. We see that the value factor yet again underperforms. The cumulative performance over the period are presented for all individual factors in Appendix A.2. For a full overview of the factor tilts performance during the sub-periods, see Appendix A.4. Based on the findings so far, and the fact that long-only factor tilts are widely used to obtain factor premiums, we take a portfolio consisting of long exposures to the factor into consideration when analyzing the multi-factor portfolios performance.

## 6 Portfolio Performance

To test whether a multi-factor portfolio is suitable as an investment strategy we form three portfolios in light of the results above. A naïve portfolio with equally weighed positions in all factors, a risk parity portfolio where each factor contributes to the same marginal amount of risk to the total portfolio risk, and a naïve portfolio with a long-only factor tilt. We will base our analysis on the same risk and performance measures as noted under the factor results.

The performance and risk characteristic for the whole period presented in Table 13 states that both the multi-factor portfolios, naïve and risk parity, have a significantly lower average return than the market for the sample period. Further, we see that all portfolios are significantly and positively correlated with the market over the sample (Table 12). The factor tilt portfolio, as expected, has the highest correlation with the market. The low correlation between the multi-factor portfolios and the market is mostly due to the short side of the portfolios as described in chapter 6. The risk of the portfolios is remarkably lower than the market, mainly assigned the added diversification effect caused by the low correlation between the individual factors and the offsetting short positions. The factor tilt portfolio is the only one that outperforms the market in terms of average returns with a risk level in line with the market, making the return/risk ratio marginally better. The overall risk of the multi-factor portfolios are over twice as good as the market when it comes to return-risk. The risk parity performs almost identically as the naïve portfolio, in form of all the risk measures and risk-adjusted ratios. In addition, the correlation between the naïve and risk parity portfolio is virtually equal 1 which indicates that the naïve portfolio already is a well-diversified portfolio with risk contribution in mind. To simplify, we only denote the equally weighed factor portfolio and the factor tilt portfolio.

**Table 12:** Correlation in returns for portfolios, 1998 - 2015

	Market	EW	RP
EW	0.22*		
RP	0.21*	0.96*	
Factor tilt	0.76*	0.63*	0.60*

*\*indicates significance at 5 percent level*

**Table 13:** Overview of Portfolio Performance 1998 - 2015 including sub-periods

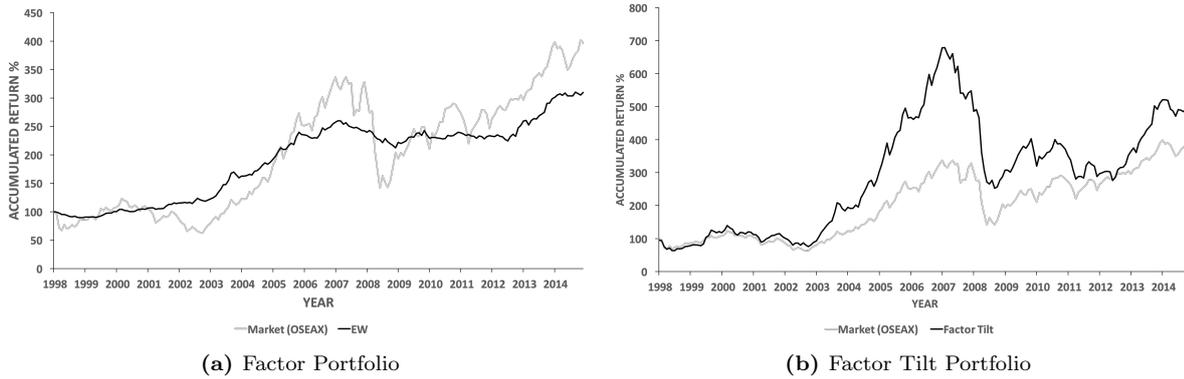
	Market	EW	RP	Factor Tilt	Market	EW	RP	Factor Tilt
<i>Return (%)</i>					<i>Return / Risk</i>			
1998 - 2003	-4.65	3.95*	4.13*	-1.51	-0.19	0.71	0.72	-0.06
2003 - 2008	32.91	15.20*	15.91*	43.91*	1.79	1.84	1.85	2.04
2008 - 2011	0.95	-2.68	-1.91	-13.33*	0.03	-0.42	-0.30	-0.53
2011 - 2015	8.75	7.11	5.77	7.94	0.66	1.16	0.97	0.49
1998 - 2015	10.87	6.91*	6.95*	12.22	0.47	0.99	0.98	0.53
<i>Standard Deviation(%)</i>					<i>Maximum Drawdown(%)</i>			
1998 - 2003	24.71	5.58*	5.70*	25.50	46.54	10.81	10.66	42.66
2003 - 2008	18.36	8.27*	8.62*	21.56	11.10	6.03	5.86	13.27
2008 - 2011	33.79	6.38*	6.38*	25.22*	56.87	14.74	13.64	59.49
2011 - 2015	13.18	6.14*	5.93*	16.26	24.39	6.27	5.04	31.06
1998 - 2015	23.01	6.98*	7.09*	23.06	57.94	18.37	17.85	62.85
<i>Skewness</i>					<i>MAR Ratio</i>			
1998 - 2003	-0.60	0.35	0.31	0.26	-0.17	0.36	0.39	-0.11
2003 - 2008	-0.32	0.44	0.56	-0.18	3.26	2.64	2.85	3.81
2008 - 2011	-0.55	0.78	0.57	-1.11	-0.09	-0.19	-0.15	-0.26
2011 - 2015	-0.65	0.92	0.49	0.19	0.43	1.45	1.45	0.28
1998 - 2015	-0.75	0.71	0.72	-0.26	0.15	0.37	0.39	0.16
<i>Excess Kurtosis</i>					<i>Sortino Ratio</i>			
1998 - 2003	1.70	-0.03	0.09	1.39	-0.61	-0.72	-0.65	-0.48
2003 - 2008	-0.73	-0.00	0.16	-0.36	3.33	4.17	4.13	4.28
2008 - 2011	-0.01	0.35	0.20	0.72	-0.06	-1.57	-1.16	-0.72
2011 - 2015	1.21	1.73	0.70	0.94	0.78	2.45	1.85	0.65
1998 - 2015	1.81	0.68	0.92	1.05	0.42	1.14	1.10	0.55
<i>VaR (95 %)</i>					<i>Sharpe Ratio</i>			
1998 - 2003	14.18	2.44	2.44	12.13	-0.43	-0.38	-0.34	-0.30
2003 - 2008	6.46	2.21	2.51	8.33	1.62	1.45	1.47	1.89
2008 - 2011	18.94	3.02	2.79	14.66	-0.04	-0.78*	-0.66*	-0.62*
2011 - 2015	6.15	1.66	1.95	8.52	0.57	0.96	0.77	0.41
1998 - 2015	9.68	2.44	2.51	11.31	0.32	0.48	0.48	0.37
<i>CVaR (95 %)</i>					<i>M2 (%)</i>			
1998 - 2003	18.01	2.52	2.67	15.38	-10.73	-3.37	-2.41	-1.28
2003 - 2008	7.91	2.78	2.82	8.81	29.69	29.83	30.26	37.88
2008 - 2011	21.72	3.06	3.09	18.67	-1.33	-23.96	-19.90	-18.64
2011 - 2015	8.77	2.24	2.46	9.19	7.52	13.84	11.33	6.67
1998 - 2015	16.22	2.80	2.90	14.27	7.29	14.56	14.54	12.20

Results are based on monthly data. The sample period runs from June 1998 to June 2015. Return and standard deviation are annualized. \* indicates 5 % statistical significance. Skewness, Excess Kurtosis, VaR and CVaR are on monthly basis. The risk-free rate used for Sharpe ratio, Sortino ratio and M2 are for the periods 6.09% (1998-2003), 3.22% (2003-2008), 2.28% (2008-2011), 1.23% (20011-2015) and 3.58% for the total period (1998-2015). For comparative purposes is the risk-free rate subtracted the markets return under M2.

As elaborated under the factor results, the market has a considerably riskier character, which becomes increasingly evident when combining the factors. The multi-factor portfolio has a larger amount of positive returns and a significantly lower variation in returns. Similar to the market, the factor tilt portfolio has a negative skewness, but at a lower level, making the distribution more normalized. All portfolios have lower positive excess kurtosis than the market, meaning they have a more stable return distribution. None of the portfolios are normally distributed according to the Jarque-Bera tests (Appendix A.6). Looking further at realized losses during the period, we see that the multi-factor portfolio maximum drawdown is dramatically reduced compared to the market, also confirmed by the small amount of value at risk and tail loss. As an effect of this we clearly see an anti-bubble behavior in Figure 4 (a) during the financial crisis (Ilmanen and Kizer (2012)). The factor tilt portfolio performed quite similar to the market, even being worse when it comes to drawdown and value at risk. The multi-factor portfolio underperforms the market in terms of accumulated returns, but the drawdowns during market collapse and financial crisis is considerably lower, due to short positions covering much of the falls in bear periods (Figure 4 (b)). The long-only has almost the same variation in returns as the market, but with a higher rate of return it gain momentum in bull periods. This is in line with the previous findings mentioned above.

### Sub-periods Analysis

To test the robustness of the portfolio performance, we take a closer look at the different sub-periods. The multi-factor portfolio significantly outperforms the market during the first period and significantly underperforms during the bull period of 2003 - 2008. As mentioned above, the factor tilt portfolio follows the market closely over the sample, but both significantly over- and underperforms over the sub-periods. The returns of the factor portfolio show a consistent positive skewness and an excess kurtosis close to zero over the mentioned periods. As indicated by the Jarque-Bera tests (Appendix A.6) we cannot reject the hypothesis of normally distributed returns for some of the sub-periods. When examining the risk characteristics of the portfolios, we note that the risk is consistently lower than the market, just like the individual factors. We see the effects of combining multiple factors reduces the realized risk considerably also in the sub-periods, with all periods yielding a significantly lower variance in returns. The factor tilt portfolio has in contrast consistently higher risk throughout the periods. The same trend is seen in VaR and CVaR. It is interesting to note that the VaR and CVaR measures is very consistent over all the periods, further stating the low degree of downside risk of the strategy. Moving over to performance measures, we see that all portfolios have a significantly lower Sharpe ratio as opposed to the market in the financial crisis, which is partly explained by the negative excess returns over the risk-free rate. By focusing on the downside risk, we see that the MAR ratio follow the same pattern, but with a much lower impact. This is explained with a lower annual growth rate (CAGR) over the period, and does not fully enlighten the performance of the factor portfolio as the market quickly re-bounces from the financial crisis, and has a much larger drawdown than the portfolio, as illustrated in Figure 4 (a). The effect is also evident in the second sub-period, where the market has a higher growth rate and in that respect gets a higher MAR ratio. We see that the factor gets a noticeably higher Sortino ratio in the periods from 2003 - 2008 and 2011 - 2015, implying that the factor portfolio yields a much better risk-adjusted return when only considering the downside risk. Examining the M2 measure, we see that the factor portfolio outperforms the market in all except one sub-period, though only marginally between 2003 and 2008. The effect is also evident in the factor tilt portfolio.

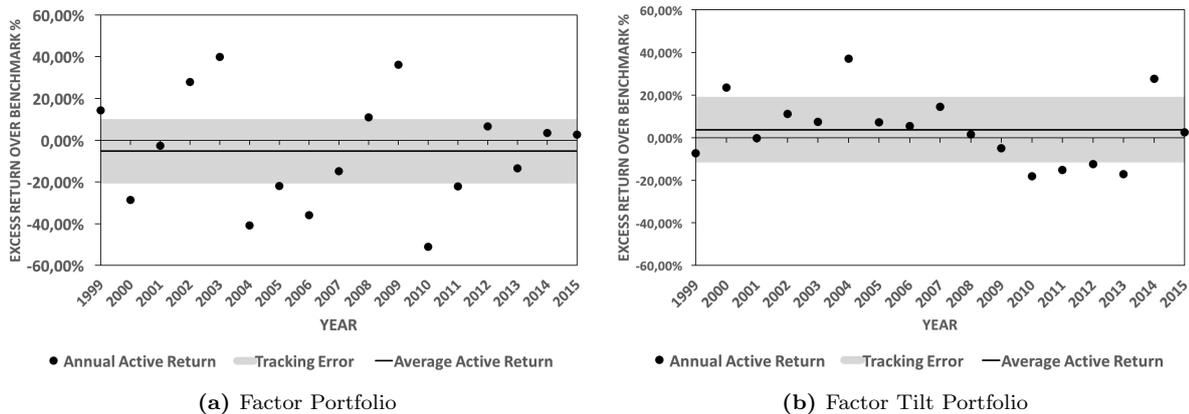


**Figure 4:** Accumulated return for the portfolios versus the market 1998 - 2015

**Table 14:** Leverage adjusted annualized return and standard deviation to the market 1998 - 2015

	Market	EW	Factor Tilt
Gross return (%)	10.87	6.91	12.22
Standard deviation (%)	23.01	6.98	23.06
Adjusted return to the standard deviation of the market (%)	-	22.77	12.19
Adjusted standard deviation to the return of the market (%)	-	10.98	20.51

*Adjusted return and standard deviation by leveraging the portfolio to the same return or risk level as the market.*



**Figure 5:** Tracking error for portfolios return versus market 1998 - 2015

## Performance Measures

Taking risk into account when analyzing the returns, all portfolios beat the market in form of the M2. With the naïve factor portfolio producing the best risk-adjusted return and the highest Sharpe ratio of all portfolios. The multi-factor portfolio has a considerably higher Sortino ratio with 1.14. The factor tilt portfolio has a Sortino ratio only marginally better than the market. The trend is similar in the MAR ratio with all portfolios yielding a ratio in excess of the market, factor tilt only marginally though. By leveraging up a low volatility well-diversified portfolio, you can achieve a higher rate of return with the same risk levels as the market or you could lever up to the same rate of return with lower risk. By leveraging up our naïve portfolio by 229 percent, we get a return of 22.77 percent with a standard deviation of the market. Elsewhere we could obtain the same return as the market with a standard deviation of 10.98 percent with a leverage of 57.3 percent (Table 14). In the process of leveraging the portfolio, we have not taken into account the costs related to borrowing. The risk regarding the borrowed money is accounted for in the borrowing costs from the lending institution and are assumed to increase with the level of leverage. An investor will basically only lose the initial investment.

## Tracking Error

Tracking error (TE) is a commonly used measure of a fund's performance against a benchmark, preferably an index, and indicates how much it diverges from the index. Our equally weighed factor portfolio has an annual tracking error of 25.85 percent and an annually average excess return of -5.29 percent. The high TE can be explained by the equally weighted structure of the portfolios in contrast to the value-weighted structure of the OSEAX index, and the short side of the strategy. Figure 5 illustrates the variance in performance to the market in context to performance for the portfolios against the market in Figure 4 (a). The high tracking error indicates that the portfolio does not follow the market, regardless the share size of the exchange. We see that the long/short portfolio has significantly positive deviations from the average TE in earlier examined bear periods (1998 - 2003 and 2008 - 2011), and significantly negative deviations in bull periods (2003 - 2008). Note that the mentioned active returns correspond with the periods where the portfolio over/underperforms the market in Figure 4 (a). TE of the factor tilt portfolio (Figure 5 (b)) has a considerable lower variation in active returns, and a lower tracking error over the sample period. This is in line with the previously mentioned risk and return characteristics. The portfolio follows the market more closely and has a similar risk and return profile in contrast to the long/short factor portfolio. We also see that the portfolio has positive active returns in excess of the average tracking error in bull periods, and significantly lower in the financial crisis and the period that follows. This is in some respect contradictory to the long/short portfolio.

## Summary

Summarized, we have surveyed the performance of the constructed multi-factor portfolios in contrast to the market portfolio. The portfolio offers a well-diversified, low risk alternative to the market. As illustrated by Table 14, the factor portfolio gets a drastic boost in performance when adjusting the risk of the portfolio up to the same level as the market. Naturally, the effect is almost nonexistent for the factor tilt portfolio. The factor portfolio has a higher Sortino ratio, and a higher Sharpe than the market. Much due to the lower risk, both in terms of total risk and downside risk, consistently illustrated in various risk measures, and as illustrated in Figure 4 (a). The factor tilt portfolio shows a higher return than the market, with a higher downside risk, illustrated in Figure 4 (b), yielding only a marginally better risk adjusted return. An alternative to a full factor exposure is to combine a position in the market index

with a position in the multi-factor portfolio. This will give a more stable return characteristic than the market, and a higher return than the multi-factor portfolio, with corresponding risk levels. In this sense, the investor is able to control the exposure, ranging from 1 to 99 percent exposure in the factor with the remanding in the market index.

## 7 Robustness and Drawbacks

This paper has mainly been a practical implementation of factor investing at OSE, and not a study on the existence of known international anomalies. This makes the study to our awareness special and puts focus on using the most reliable methodologies when capturing and harvesting the returns associated with the market anomalies.

When performing this study, we have carefully chosen the factors. Choosing anomalies that are well documented in literature, have shown persistent performance over time, and are followed by an economic rationale, in line with Hsu and Kalesnik (2014) key criteria's for choosing robust factors. In this sense, we feel that the study will give valuable insight into how stakeholders may harvest risk premium in the Norwegian market, much like institutional investors do in the international markets. In constructing the factors, we have carefully chosen the methodology such that the results are comparable with other studies on factor investing abroad. We have focused on the most consistent ways of capturing the anomalies, rather than those that indicate to give the best results in a short time span. It is important to stress that a factor investing strategy is a long term strategy that are dependent on being well advocated and supported by the practitioners due to the underperformance in certain periods.

To test the robustness, we have divided the period into sub-periods to test their performance and controlled that the findings are not a result of data mining or errors in estimation. We have also chosen to look at the factor premiums over a relatively long period. This is consistent with the long time horizon for a factor investing strategy, and will give better insight into the future performance as stated by Professor Elroy Dimson<sup>11</sup>. We see that the factors perform well during different market conditions, as the sample includes both financial distress (IT-bobble, financial crisis) and prolonged periods of upwards trending markets (2003 - 2008). In the analysis of the performance, we have used both measures that assumes normally distributed returns, and measures that gives more correct estimation of performance in non-normally distributed returns. We feel that this also strengthens the robustness of the analysis.

When examining the Norwegian stock market there is a few implications that makes this form of investing difficult. We especially note the small market size, relatively low market depth and unbiased shareholder structure in contrast to similar studies done abroad. OSE is a rather small market and most of the trading is concentrated around the largest most liquid stocks. Even though we have seen a considerable improvement on OSE over the period examined, most studies in the European, American and international markets use a stock pool of around the 1 000 largest companies. Despite this, we observe similar effects in the Norwegian market. On the other hand, because of a small number of stocks compared with international studies, we experience a moderate cross-exposure in the different factor portfolios, which implies that the market might be too small for such a strategy. A drawback with the

<sup>11</sup><http://www.robeco.com/en/professionals/insights/quantitative-investing/factor-investing/seminar/7-lessons-for-factor-investing.jsp>. Written: 29.09.2014. Extracted: 20.04.2016

study is the limited testing of different sizes on the portfolios. The use of one size of the mimicking factor portfolios (15), is due to the extensive data mining and results to go through. As mentioned in the method chapter, 15 stocks are on average quite close to the decile breakpoints for the portfolios. When it comes to returns we have not adjusted for extreme values, like many other studies, as we wanted the return to reflect the actual change in prices. In conjunction with extracting data used, we have taken precautions and consequently used the same method, but the result can be affected by adverse data mining. Due to the size of the market, the stocks available for short sale are limited, and the costs varies considerably, which makes it difficult to calculate costs accurately. Another important drawback is the exclusion of transaction costs and cost of leverage. Implementing transaction costs would not as we see it increase the validity, as the costs are individual between different types of investors. When it comes to leverage much of the same considerations apply. Finally, the choosing of methodology, and factors are based on robust international findings, which may not be the best choice for the Norwegian Market.

## 8 Conclusion

In this paper, we have constructed factor portfolios on Oslo Stock Exchange to utilize market anomalies and test how these factors perform over time in a practical portfolio by using rolling out-of-sample data periods. We also combine these factors into multi-factor portfolios to form an alternative index fund, to see whether this is a suitable investment case.

Our results for the four factors is mostly consistent with previous findings in the Norwegian stock market. All the long/short factors give excess return in their exposure between the top and bottom portfolio, affirming the effect of the anomalies and the possibility to harvest risk premium. The return over the sample period is varying for all the factors, with both over- and under-performance. When it comes to overall average gross return, none of the portfolios beat the market. The real effect of the factor portfolios is seen in the significantly reduced risk for all the sample periods and the low correlation between the factors, resulting in a considerable diversification effect. Especially drawdown and value at risk during financial distresses, makes the portfolios withstand large losses and thereby deliver stabilized long-term return. Looking at risk-adjusted return over the market (M2) for the factors, we clearly see that all factors except value performs better than the market.

By combining the factors into multi-factor portfolios, we can further reduce cyclical risk and achieve an even better diversification effect. Our naïve and risk parity portfolio technique deliver almost identical results, implying that the naïve portfolio already is well-diversified. We also see, as stated by Ilmanen and Kizer (2012), a clear anti-bubble behavior reducing potential downside risk in distress, making the strategy a good alternative for long-term investor like pension and wealth funds. The different portfolio optimizations all give risk-adjusted return beyond the market portfolio, seen through Sharpe, Sortino and MAR. Tracking error shows that the portfolio places itself away from index funds, delivering a stable and upward trending return, with few drops in contrast to the index. By implementing this strategy together with leverage, one could achieve returns higher than the market with lower risk and fewer shortfalls. This is far more important for many long-term investors and institutions, for example when it comes to reducing liability risk. In the analysis of the factor tilt portfolio, we see an ability to enhance returns, but the excess returns come at a price of higher risk. On the other hand, a factor tilt is a good choice

for institutional investors that have short or leverage constraints. In contrast, we cannot conclude that a long only factor tilt, significantly and persistently outperforms the market. We conclude that harvesting risk premium is possible, but due to the characteristics of the market may be difficult to implement in practice. We strongly urge the decision to implement factor investing as a part of an investment strategy to be well thought through, and knowing what risks you are taking.

As further research, we encourage others to look into alternative style factors that might be more suited for the Norwegian market, and alternative methods of ranking and sorting the factors, mentioned in the literature, and used by practitioners. Factors that might be of interest are quality, liquidity and dividend yield. Another direction to look further into, is the optimization of portfolio size for a practical implementation of factor portfolios adapted to the Norwegian market. We believe that factor investing will increase in the future and hope that others will examine systematic factor tilt exposure further. Both as a stand-alone investment strategy, and as part of an already existing strategy.

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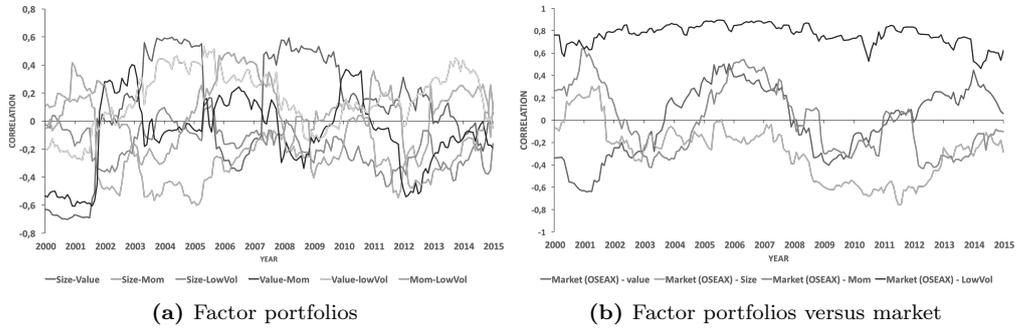
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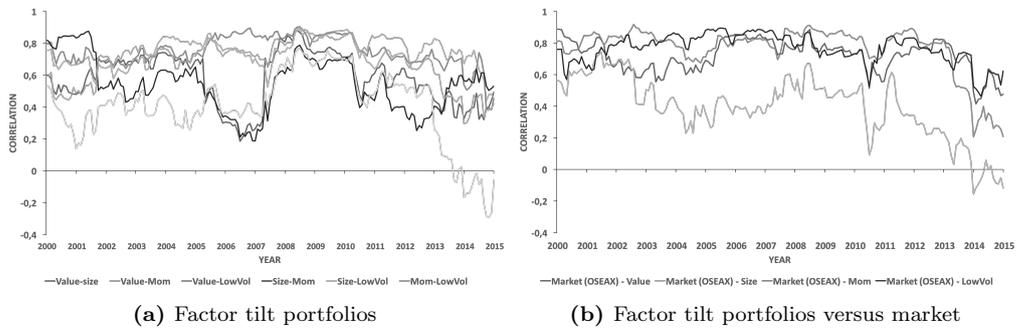
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# A Appendix

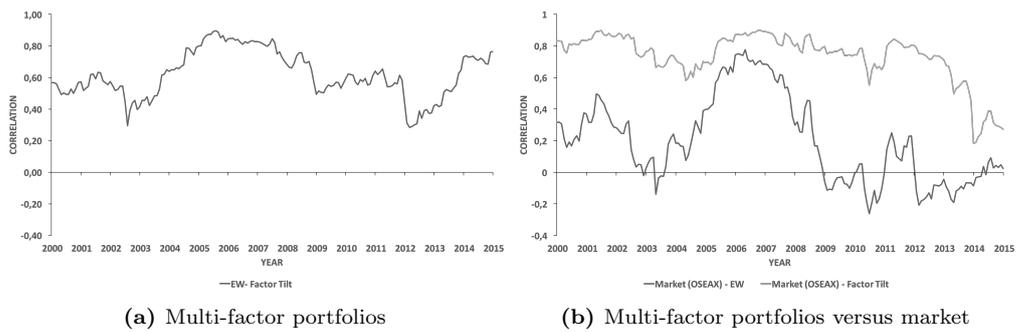
## A.1 24-month Rolling Correlation



**Figure 6:** 24-month rolling correlation for factor portfolios, 1998 - 2015

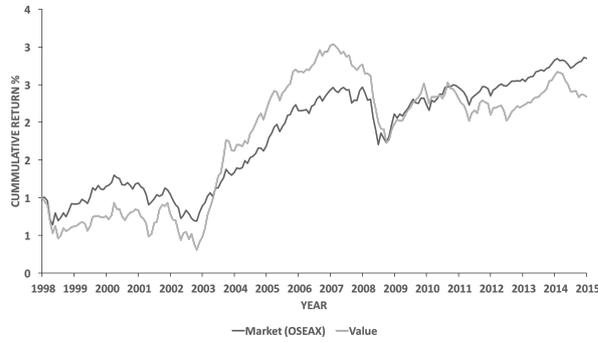


**Figure 7:** 24-month rolling correlation for factor tilt portfolios, 1998 - 2015



**Figure 8:** 24-month rolling correlation for multi-factor portfolios, 1998 - 2015

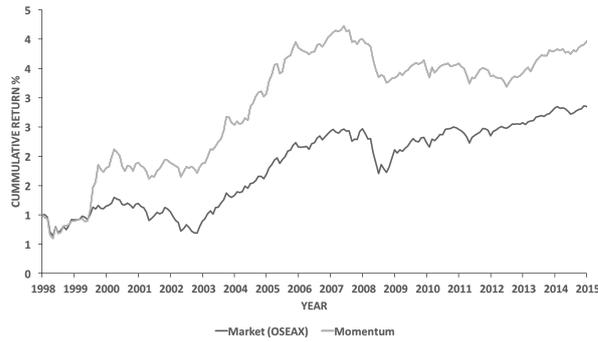
## A.2 Cumulative return for factor tilt portfolios



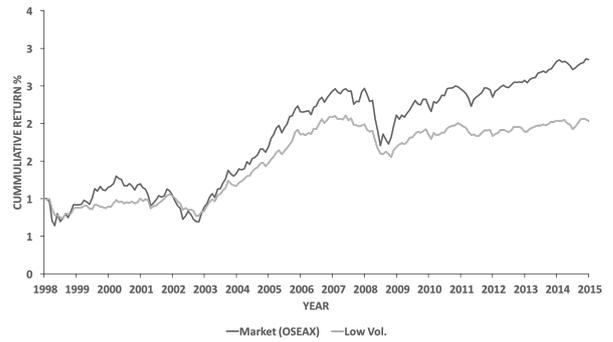
(a) Value versus market



(b) Size versus market



(c) Momentum versus market



(d) Low volatility versus market

**Figure 9:** Cumulative return for factor tilt portfolios, 1998 - 2015

### A.3 Factor exposure matrix

Table 15: Factor exposure matrix

	LONG / LONG						LONG / SHORT						SHORT / SHORT											
	LV		V		M		LV		V		M		LV		V		M							
	V	S	M	S	M	S	V	S	M	S	M	S	V	S	M	S	M	S						
1998	-	7	13	27	-	7	20	27	20	20	7	27	27	7	13	-	7	7	-					
1999	-	7	-	7	7	7	20	7	20	13	27	7	27	13	7	20	8	13	13					
2000	-	7	7	20	7	-	13	27	13	27	13	20	7	20	-	27	20	8	7					
2001	7	7	7	13	20	-	13	13	13	7	7	-	20	27	-	27	8	13	7					
2002	-	13	-	13	7	7	7	7	20	7	20	13	7	7	27	17	7	20	7					
2003	-	13	-	27	7	7	33	13	20	7	7	13	-	33	-	27	40	7	20					
2004	-	7	7	27	13	7	13	40	20	20	-	7	7	20	-	20	-	13	13					
2005	-	7	7	27	-	13	-	27	7	7	-	20	7	-	13	7	-	13	13					
2006	-	7	13	27	-	7	-	33	20	-	7	20	7	-	20	8	7	7	13					
2007	-	-	20	13	7	-	-	27	7	27	-	27	7	-	7	-	13	13	-					
2008	7	7	-	13	13	-	7	13	20	-	13	7	13	20	7	27	-	7	-					
2009	-	7	-	7	7	-	7	7	13	7	7	7	13	20	13	27	8	7	13					
2010	-	-	7	20	-	7	-	7	27	-	20	13	7	33	7	20	-	7	13					
2011	-	-	-	13	13	-	-	7	7	20	-	20	7	33	7	33	8	7	13					
2012	7	-	-	7	27	-	-	13	7	20	-	7	27	7	27	8	-	20	-					
2013	7	-	-	27	-	13	-	13	7	27	-	7	-	33	-	13	8	7	20					
2014	20	-	7	7	33	7	13	7	7	7	-	33	27	-	20	8	7	27	7					
<b>Average</b>	<b>1.2</b>	<b>4.7</b>	<b>2.4</b>	<b>10.4</b>	<b>20.4</b>	<b>5.7</b>	<b>9.2</b>	<b>1.4</b>	<b>14.5</b>	<b>15.3</b>	<b>15.9</b>	<b>5.1</b>	<b>12.2</b>	<b>10.0</b>	<b>11.4</b>	<b>25.2</b>	<b>3.5</b>	<b>23.3</b>	<b>7.9</b>	<b>24.7</b>	<b>5.4</b>	<b>11.0</b>	<b>14.9</b>	<b>2.7</b>

Percentage of individual portfolios total exposure in other factors at yearly basis and average for the whole sample. The first line indicates the form of relation. The second line indicates the factor portfolio for the actual stock investment. The third line indicates which corresponding portfolio that also is exposed in the same stock investment. S = Size, V = Value, M = Momentum, LV = Low volatility. 7 % exposure is equivalent to 1/15 of the stock investment.

## A.4 Factor tilts performance for 1998 - 2015 including sub-periods

**Table 16:** Overview of factor tilts Performance for 1998 - 2015 including sub-periods

	Mkt	Size	Value	Mom	LowVol	Mkt	Size	Value	Mom	LowVol
<i>Return (%)</i>						<i>Return / Risk</i>				
1998-2003	-4.65	-8.36	-12.00	17.51	-3.20	-0.19	-0.22	-0.41	0.46	-0.24
2003-2008	32.91	55.57	48.83	46.98	24.28	1.79	1.63	1.79	1.66	1.78
2008-2011	0.95	-12.61	-17.00	-19.26	-4.45	0.03	-0.41	-0.51	-0.65	-0.26
2011-2015	8.75	21.58	-0.97	8.87	2.29	0.66	0.69	-0.04	0.46	0.23
1998-2015	10.87	17.47	7.87	17.44	6.09	0.47	0.50	0.27	0.58	0.44
<i>Standard Deviation (%)</i>						<i>Maximum Drawdown (%)</i>				
1998-2003	24.71	37.25	29.22	37.71	13.36	46.54	71.50	53.27	42.62	24.18
2003-2008	18.36	34.19	27.34	28.36	13.61	11.10	25.72	20.08	19.79	6.66
2008-2011	33.79	30.92	33.02	29.86	17.23	56.87	59.86	68.45	55.59	36.79
2011-2015	13.18	31.49	22.05	19.12	9.79	24.39	47.29	43.52	35.71	16.36
1998-2015	23.01	34.76	28.80	30.25	13.85	57.94	73.55	76.30	70.97	44.37
<i>Skewness</i>						<i>MAR Ratio</i>				
1998-2003	-0.60	1.26	-0.25	0.75	-0.94	-0.17	-0.20	-0.29	0.27	-0.17
2003-2008	-0.32	1.05	-0.00	0.30	-0.40	3.42	2.19	2.68	2.63	3.98
2008-2011	-0.55	-0.63	-0.90	-0.70	-0.56	0.02	-0.22	-0.25	-0.28	-0.08
2011-2015	-0.65	1.28	-0.07	-0.54	-0.57	0.42	0.44	-0.18	0.25	0.10
1998-2015	-0.75	0.86	-0.35	0.32	-0.59	0.15	0.17	0.05	0.19	0.12
<i>Excess kurtosis</i>						<i>Sortino Ratio</i>				
1998-2003	1.70	2.60	0.04	2.88	2.06	-0.61	-0.88	-0.87	0.55	-0.85
2003-2008	-0.73	1.05	-0.09	0.39	-0.09	3.33	5.15	3.37	3.18	2.73
2008-2011	-0.01	0.89	0.36	0.28	0.27	-0.02	-0.18	-0.21	-0.27	-0.15
2011-2015	1.21	1.96	0.02	0.30	0.51	0.78	1.72	-0.16	0.58	0.15
1998-2015	1.81	1.66	0.49	2.58	1.03	0.42	0.83	0.21	0.72	0.25
<i>VaR (95%)</i>						<i>Sharpe Ratio</i>				
1998-2003	14.18	16.09	17.06	15.74	8.46	-0.43	-0.39	-0.62	0.30	-0.69
2003-2008	6.46	7.11	10.95	9.46	5.84	1.62	1.53	1.67	1.54	1.55
2008-2011	18.94	18.22	17.91	20.02	10.57	-0.04	-0.48	-0.58	-0.72	-0.39
2011-2015	6.15	9.61	12.26	9.46	5.68	0.57	0.65	-0.10	0.40	0.11
1998-2015	9.68	13.57	14.36	14.63	6.95	0.32	0.40	0.15	0.46	0.18
<i>CvaR (95%)</i>						<i>M2 (%)</i>				
1998-2003	18.01	16.44	18.81	20.65	10.75	-10.73	-3.50	-9.21	13.57	-11.08
2003-2008	7.91	9.90	11.94	11.66	6.26	29.69	31.34	33.86	31.55	31.63
2008-2011	21.72	22.44	23.16	21.29	11.85	-1.33	-14.00	-17.45	-22.10	-10.93
2011-2015	8.77	10.91	13.28	11.88	6.65	7.52	9.75	-0.09	6.50	2.66
1998-2015	16.22	16.75	18.20	18.23	9.44	7.29	12.77	7.01	14.12	7.75

Results are based on monthly data. The sample period runs from June 1998 to June 2015. Return and standard deviation are annualized. Skewness, Excess Kurtosis, VaR and CvaR are on monthly basis. The risk-free rate used for Sharpe ratio, Sortino ratio and M2 are for the periods 6.09% (1998-2003), 3.22% (2003-2008), 2.28% (2008-2011), 1.23% (2011-2015) and 3.58% for the total period (1998-2015). For comparative purposes is the risk-free rate subtracted the markets return under M2.

## A.5 Overview of stock positions in the factors from 1998 to 1999

Table 17: Stock positions in each factor from 1998 to 1999

		LONG															SHORT															
		VALUE															SIZE															
		MOMENTUM															MOMENTUM															
		LOW VOLATILITY															LOW VOLATILITY															
Jun 1998	TCA	ATEA	SCI	KVI	PGS	ASC	TTS	NSG	BOR	IGNIS	SEN	HEX	KEN	PRO	HJE	WBS	AFK	TOM	AVE	TAD	AXI	NAV	OPC	PFI	AGR	CAG	EKO	AMS	COV	VME		
Jun 1999	PGS	SCI	NSG	BEL	DNB	BOR	DNO	KVI	KEN	IMSK	HSU	NIS	DOF	FOE	ODF	WBS	HAG	TOM	NTC	NOD	AGR	OPC	PFI	AAV	EKO	COV	VME	OLT	NOW	OTR		
Jun 1998	NMG	PRO	BMA	ALX	ALV	IGNIS	HEX	ASC	VVL	BOR	NOCC	RGT	SUO	CRP	BEL	NHY	RCL	AHM	ORK	DNB	STB	CKR	PGS	SAG	KVI	ATEA	TOM	AMA	NTC	SCHA		
Jun 1999	NMG	ALV	EMS	RCT	BMA	ALX	DAT	IGNIS	TCA	TTS	HEX	REACH	BEL	PRO	NOW	NHY	RCL	AHM	ORK	DNB	CKR	STB	TOM	NTC	PGS	ATEA	NSG	SNOG	KVI	BEA		
Jun 1998	AXI	GOD	HIT	TAA	STN	NAV	TCA	TOM	TOD	TAD	HAG	ATEA	SME	SFJ	ARK	MOE	KLI	BEL	JIN	GYL	HUN	SOLV	DYN	BEA	ACL	ASC	AVA	AVE	EKJ	HES		
Dec 1998	DNO	SFJ	TAD	TAA	EKO	TOM	MSL	HAG	PGS	SME	ARK	STN	BON	AKE	GRO	AVE	HIT	GYL	SST	KOAL1	UTO	JIN	WBS	ASC	HNA	BNB	EKJ	GOD	HES	HUN		
Jun 1999	SEL	DNO	TAA	SFJ	TAD	EKO	KLI	ACL	NOV	TOM	ARK	NOD	FOK	AAV	ORK	VIS	STN	GOD	SEN	HNA	JIN	TCA	BEL	BSH	NAV	ASC	AVE	FOT	GRE	GYL		
Jun 1998	FIN	SNOG	SKI	VVL	VEI	OLT	HNA	NBK	WVI	AFK	RIE	NHY	STB	NOV	DNB																	
Jul 1998	SKI	FIN	SNOG	NBK	VVL	VEI	OLT	HNA	WVI	AFK	NHY	NOV	RIE	DNB	HAG																	
Aug 1998	SKI	FIN	SNOG	NBK	VVL	VEI	OLT	HNA	WVI	AFK	NHY	NOV	HAG	RIE	DNB																	
Sep 1998	FIN	FIN	NBK	VVL	HAG	SKI	NOV	AFK	VEI	HNA	OLT	RIE	WVI	NHY	HES																	
Oct 1998	FIN	NBK	VVL	AFK	SKI	SNOG	HNA	NOV	OLT	VEI	RIE	HAG	HES	LSW	ODF																	
Nov 1998	VVL	SKI	HNA	NBK	SNOG	OLT	HAG	RIE	LSW	ODF	AFK	ODF	HES	NHY	GYL																	
Dec 1998	VVL	SKI	HNA	NBK	SNOG	OLT	VEI	RIE	LSW	ODF	HAG	AFK	GYL	HES	WVI																	
Jan 1999	VVL	SKI	HNA	OLT	SNOG	NBK	LSW	VEI	RIE	HAG	AFK	AFK	WVI	ODF	NHY																	
Feb 1999	VVL	SKI	HNA	OLT	SNOG	HAG	NBK	LSW	AFK	RIE	VEI	WVI	GYL	DYN	BEA																	
Mar 1999	SKI	VVL	HNA	OLT	SNOG	HAG	NBK	LSW	AFK	RIE	VEI	AAV	BEA	WVI	DYN																	
Apr 1999	SKI	HNA	VVL	OLT	SNOG	HAG	NBK	LSW	AAV	AFK	GYL	VEI	BEA	AAV	RIE																	
May 1999	HNA	OLT	VVL	SNOG	HAG	SKI	NBK	AAV	AFK	LSW	WVI	BEA	GYL	VEI	RIE																	
Jun 1999	HNA	ENB	OLT	RNA	VVL	SNOG	SKI	AAV	NBK	LSW	HAG	AFK	WVI	AAV	RIE																	

## A.6 Jarque-Bera test

**Table 18:** Jarque-Bera test for market and factor portfolios

<b>Jarque-Bera test for factors</b>					
	<i>Market</i>	<i>Value</i>	<i>Size</i>	<i>Momentum</i>	<i>LowVol</i>
<i>1998-2003</i>	9.93	145.83	110.43	5.12	17.76
<i>2003-2008</i>	2.37	10.62	77.35	0.42	1.60
<i>2008-2011</i>	1.85	9.89	1.39	6.95	1.97
<i>2011-2015</i>	6.99	20.80	30.65	9.07	3.45
<i>1998-2015</i>	46.85	478.92	200.65	7.11	20.60

*Critical value for a Jarqu-Bera test at 5 percent significance is 5.99. A result over this value indicates that the distribution is not normally distributed.*

**Table 19:** Jarque-Bera test for portfolios

<b>Jarque-Bera test for portfolios</b>				
	<i>Market</i>	<i>EW</i>	<i>RP</i>	<i>Factor Tilt</i>
<i>1998-2003</i>	9.93	1.11	0.92	5.05
<i>2003-2008</i>	2.37	1.92	3.23	0.64
<i>2008-2011</i>	1.85	3.82	2.01	8.12
<i>2011-2015</i>	6.99	14.03	3.21	2.27
<i>1998-2015</i>	46.85	21.11	24.79	11.52

*Critical value for a Jarqu-Bera test at 5 percent significance is 5.99. A result over this value indicates that the distribution is not normally distributed.*

## A.7 Stocklist

*Table 20 and table 21 on the following two pages shows stock investments included in the factor portfolios for the whole sample period. Months refer to the number of theoretical months a single stock is invested in. The number includes both long and short positions, as well as zero-sum investments and represents the total number of months in all factor. Maximum number of months possible is 816. A position in one factor for the whole sample period equals 204 months.*

**Table 20:** Overview of stocks (1 of 2)

<b>Ticker</b>	<b>Name</b>	<b>Months</b>	<b>Ticker</b>	<b>Name</b>	<b>Months</b>
NHY	Norsk Hydro	319	HAG	HØG	99
DNB	DNB	312	SUO	SuperOffice	97
SUBC	Subsea 7	276	BON	Bonheur	97
COV	ContextVision	265	EIOF	Eidesvik Offshore	97
RCL	Royal Caribbean Cruises	265	IMSK	I.M Skaugen	97
STL	Statoil	255	DNO	DNO	96
NSG	Norske Skogindustrier	253	NGT	NextgenTel Holding	94
TEL	Telenor	247	SEVAN	Sevan Marine	90
SCI	Scana Industrier	246	NORD	Norda	90
VVL	Voss Veksel- og Landmandsbank	246	PFI	P4 Radio Hele Norge	90
PGS	Petroleum Geo-Services	234	AGA	Agasti Holding	90
SDRL	Seadrill	231	HND	Hands	90
BOR	Borgestad	229	TAD	Tandberg Data	84
PDR	Petrolia	228	EVRY	Evry	84
OLT	Olav Thon Eiendomsselskap	225	OFL	Office Line	84
BEL	Belships	218	TECO	Teco Maritime	84
KOG	Kongsberg Gruppen	218	NORMAN	Norman ASA	82
NOD	Nordic Semiconductor	216	SNOG	Gjensidige NOR Sparebank	81
STB	Storebrand	214	ITE	Itera	80
RGT	Rocksource	210	ELK	Elkem	80
ORK	Orkla	210	BIRD	Birdstep Technology	78
NOCC	Norwegian Car Carriers	210	OPERA	Opera Software	78
NMG	Nickel Mountain Group	204	EXE	Exense	78
SNI	Stolt-Nielsen	204	DOM	Domstein	78
IGNIS	Ignis	198	HAVI	Havila Shipping	78
TOM	Tomra Systems	198	GRO	Ganger Rolf	76
GOD	Goodtech	192	AHM	Amerstam	73
HJE	Hellegjerde	192	KVE	Kverneland	73
MHG	Marine Harvest	186	ECHEM	Eitzen Chemical	72
FRO	Frontline LTD	186	AMSC	American Shipping Company	72
ATEA	Atea	186	BIOTEC	Biotec Pharmacon	72
AFK	Arendals Fossekompani	185	ALV	Alvern	72
SCHA	Schibsted ser. A	182	KEN	Kenor	72
JIN	Jinhui Shipping and Transportation	180	APP	Apptix	72
SKI	Skien Aktiemølle	176	BERGEN	Bergen Group	72
PRO	Pronova	175	BOUVET	Bouvet	72
YAR	Yara International	170	GOL	Golar LNG	72
INM	Inmeta Crayon	168	ASC	ABG Sundal Collier Holding	71
EKO	Ekornes	166	MEDI	Medistim	70
EMS	EMS Seven Seas	162	BNB	Bolig- og Næringsbanken	69
TTS	TTS Group	162	PSI	PSI Group	66
OPC	Opticom	150	OCR	Ocean Rig	66
GOGL	Golden Ocean Group	150	AKER	Aker	66
TAA	Tandberg	150	NOW	Nordic Water supply	66
GIG	Gaming Innovation Group	144	GJF	Gjensidige Forsikring	65
NEL	NEL	144	WWI	Wilh. Wilhelmsen Holding ser. A	65
CSG	Component Software Group	144	SST	Steen & Strøm	61
NAVA	Navamedic	144	STP	StepStone	60
VEI	Veidekke	139	ITX	Intex Resources	60
ELT	Eltek	138	BMA	BYGGMA	60
HEX	Hexagon Composites	138	DAT	Data Respons	60
AKA	Akastor	138	NBK	Nordlandsbanken	60
REACH	Reach Subsea	132	GYL	Gyldendal	57
NEC	Norse Energy Corp.	132	DOF	District Offshore	55
DOLP	Dolphin Group	132	AKBM	Aker BioMarine	54
HNA	Hafslund Ser. A.	127	PAR	PA Resources	54
OTS	Oceansteam Shipping	126	FAKTOR	Faktor Eiendom	54
REPANT	Repant	126	HSU	Havila Supply	54
AFG	AF Gruppen	126	NTC	NetCom	54
KVI	Kværner	120	PRS	Prosafe	54
SINO	SinOceanic Shipping	120	SFM	Synnøve Finden	54
PHO	Photocure	118	ASD	Axsis-Shield	52
FAR	Farstad Shipping	114	ORO	Origio	48
SAS NOK	SAS AB	114	JSHIP	Janson Shipping	48
ODF	Odfjell ser. A	113	TST	Tandberg Storage	48
SIOFF	Siem Offshore	113	FAIR	Fairstar Heavy Transport	48
FARA	Fara	108	TGS	TGS-NOPEC Geophysical Company	48
FOE	Fred. Olsen Energy	108	AIK	Aktiv Kapital	43
TIDE	Tide	106	FSL	Fesil	42
ALX	Altinex	102	TCA	Telecast	42
FUNCOM	Funcom	102	AGR	Agresso Group	42
WEIFA	Weifa	102	EVE	Evercom Network	42
EMGS	Electromagnetic Geoservices	102	FAST	Fast Search & Transfer	42
COD	Codfarmers	102	GSF	Grieg Seafood	42
VME	Vmetro	102	OTR	Otrum	42

**Table 21:** Overview of stocks (2 of 2)

<b>Ticker</b>	<b>Name</b>	<b>Months</b>	<b>Ticker</b>	<b>Name</b>	<b>Months</b>
PLCS	Polarcus	42	PRON	Pronova BioPharma	18
QFR	Q-Free	42	SALM	SalMar	18
VIZ	Vizert	42	SIT	Simrad Optronics	18
BEA	Bergesen d.y ser. A	41	TREF	Trefoil	18
IMAREX	Imarex	39	POWEL	Powel	16
SOLV	Solvang	37	UNS	Ugland Nordic Shipping	14
BIONOR	Bionor Pharma	36	HIT	Hitec	12
CNR	CanAgro Energy Co.	36	NOF	Northern Offshore Gammel	12
ATG	Andvord Tybring-Gjedde	36	STN	Storm Real Estate	12
MBN	MediaBin	36	AXI	Axis Biochemicals	12
WBS	Western Bulk Shipping	36	JACK	Petrojack	12
SBX	SeaBird Exploration	36	SCORE	Scorpion Offshore	12
NER	Nera	36	AMA	Aker Maritime	12
AAV	Adresseavis	36	ARK	Ark	12
BWO	BW Offshore Limited	36	AWS	Awilco ser. A	12
EMAS	EMAS offshore	36	BAKKA	Bakkafrost	12
GRE	Gresvig	36	BSH	Bona Shipholding	12
NAS	Norwegian Air Shuttle	36	BWLPG	BW LPG	12
NEAS	Neas	36	FBU	Fornebu utvikling	12
STXEUR	STX Europe	36	IGR	IGroup	12
TAT	Tanderg Television	36	KVAER	Kværner	12
TCO	TeleComputing	36	LSW	Linstow	12
WILS	Wilson	32	MEF	Mefjorden	12
RXT	Reservoir Exploration Technology	30	MSL	Mosvold Shipping Ltd.	12
QEC	Questerre Energy Corporation	30	NAV	Navia	12
SEN	SensoNor	30	NIS	NAVIS	12
CRU	Crew Gold Corporation	30	NSTAT	Norstat	12
HRG	Hurtigruten	30	PEN	Panoro Energy	12
AVM	Avocet Mining	30	RIE	Rieber & Søn	12
NOR	Norman	30	SAG	Saga Petroleum	12
CHS	Choice Hotels Scandinavia	30	SIN	Sininvest	12
FJO	Fjord Seafood	30	SNS	Sense Communications International	12
KLI	Klippen Invest	30	TSH	Team Shipping	12
KOM	Komplett	30	FIN	Finansbanken	11
NOF	Northern Offshore	30	SRBANK	SpareBank 1 SR-Bank	11
SOI	Software Innovation	30	INFRA	Infratek	10
SOFF	Solstad Offshore	26	KOA2	Kongsberg Automotive	9
IOX	InterOil Exploration and Production	24	NOV	Norsk Vekst	9
AVE	Avenir	24	HLNG	Høegh LNG Holdings	7
CAG	Computer Advances	24	MOE	Moelven Industrier	7
AKFP	Aker Floating Production	24	VIS	Visma	6
ALGETA	Algeta	24	NCL	NCL Holdning	6
ARCHER	Archer	24	MDX	Mindex	6
CKR	Chr. Bank og Kreditkasse	24	AKVA	AKVA Group	6
COMROD	Comrod Communications	24	ASETEK	ASETEK	6
DESSC	Deep Sea Supply	24	AWO	COSL Drilling Europe AS	6
DETNOR	Det Norske Oljeselskap	24	BBA	Bergensbanken	6
DOCK	Dockwise	24	BRA	Braathens	6
GIPS	Glibal IP Solutions	24	BRG	Borregaard	6
NRS	Norway Royal Salmon	24	CEQ	Cermaq	6
ODIM	Odim	24	CNS	Conseptor	6
PROTCT	Protector Forsikring	24	FDR	Frontier Drilling	6
SIMTRO	Simtronic	24	FOP	Fred. Olsen Production	6
SONG	Songa Offshore	24	GGG	Grenland Group	6
TEC	Technor	24	IFC	InFocus Corporation	6
UNISON	Unison Forsikring	24	MIS	Maritime Industrial Services	6
NPRO	Norwegian Property	23	MORPOL	Morpol	6
RNA	Reitan Narvesen	23	NWS	Norway Seafoods	6
STORM	Storm Real Estate	22	ODL	Odjell Drilling	6
RISH	GC Rieber Shipping	21	OHI	OHI	6
AVA	Avantor	20	OILRIG	Odjell Invest	6
WAT	Waterfront Shipping	18	PBG	Petrobank Energy and Resources	6
VOI	Voice	18	RIG	Transocean Offshore	6
SFJ	DSND Subsea	18	SEVDR	Sevan Drilling	6
AUSS	Austevoll Seafood	18	SPC	SPCS-Gruppen	6
BWG	BWG Homes	18	TFDS	Troms Fylkes Dampskibsselskap	6
COP	Copeinca	18	UTO	Unitor	6
EKJ	Elkjøp	18	WBULK	Western Bulk	6
HFISK	Havfisk	18	WWASA	Wilh. Wilhelmsen	6
HYD	Hydralift	18	DYN	Dyno	5
ITC	Intelecom Group	18	STA	Stavanger Aftenblad	5
LHO	Leif Høegh & Co	18	APR	A-Pressen	4
LSG	Lerøy Seafood Group	18	HES	Helicopter Services Gr.	4
NEXUS	Nexus Floating Production	18			

## A.8 Risk-free rate

**Table 22:** Yearly risk-free rate

Year	Risk-free rate
1995	5.18
1996	4.13
1997	4.42
1998	6.83
1999	6.08
2000	7.37
2001	6.00
2002	5.83
2003	2.20
2004	1.98
2005	2.74
2006	4.19
2007	4.98
2008	2.21
2009	2.28
2010	2.35
2011	1.19
2012	1.46
2013	1.34
2014	0.94
2015	0.54

**Table 23:** Periodic yearly risk-free rate

Period	Risk-free rate
1998 - 2015	3.58
1998 - 2003	6.09
2003 - 2008	3.22
2008 - 2011	2.28
2011 - 2015	1.23

## A.9 Stocks available for short sale

Stocks available for short sale as of 13.04.16 by Pareto Securities<sup>12</sup> and Nordnet<sup>13</sup>.

**Table 24:** Stocks available for short sale by Pareto Securities

Ticker	Name	Ticker	Name
AKER	Aker	LSG	Lerøy Seafood Group
AKSO	Aker Solutions	MHG	Marine Harvest
AMSC	American Shipping Company	NAS	Norwegian Air Shuttle
ARCHER	Archer Limited	NHY	Norsk Hydro
ATEA	ATEA	NOD	Nordic Semiconductor
AUSS	Austevoll Seafood	OPERA	Opera Software
AVANCE	Avance Gas Holding	ORK	Orkla
BAKKA	Bakkafrost	PGS	Petroleum Geo-Services
BWLPG	BW LPG	QEC	Questa Energy Corporation
BWO	BW Offshore Limited	RCL	Royal Caribbean Cruises
DETNOR	Det Norske Oljeselskap	REC	Renewable Energy Corporation
DNB	DnB	SCHA	Scibsted A
DNO	DNO International	SCHB	Shibsted B
ENTRA	Entra	SDRL	Seadrill
EPR	Europris	SEVDR	Sevan Drilling
FOE	Fred. Olsen Energy	SONG	Songa Offshore
FRO	Frontline	STB	Storebrand
GJF	Gjensidige Forsikring	STL	Statoil
GOGL	Golden Ocean Group	SUBC	Subsea 7
HEX	Hexagon Composites	TEL	Telenor
KID	Kid	XXL	XXL
KOA	Kongsberg Automotive	YAR	Yara
KVAER	Kværner		

**Table 25:** Stocks available for short sale by Nordnet

Ticker	Name
DNB	DNB ASA
GJF	GJENSIDIGE FORSIKRING ASA
MHG	MARINE HARVEST ASA
NHY	NORSK HYDRO
ORK	ORKLA
STL	STATOIL ASA
STB	STOREBRAND
SUBC	SUBSEA 7 S.A
TEL	TELENOR ASA
TOM	TOMRA SYSTEMS
YAR	YARA INTERNATIONAL

<sup>12</sup><http://www.paretosec.no/aksjehandel-paa-nett/verdipapirfinansiering/shorthandel>

<sup>13</sup><https://www.nordnet.no/mux/page/blankninginl.html>